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ABSTRACT

This paper evaluates the performance of leading micro-founded pricing-to-market frictions vis-a-vis a set of robust stylized facts about international prices. In order to make that evaluation meaningful, we embed each friction into a unified IRBC framework and parameterize the models in a uniform way. Our goal is to evaluate the broad-based applicability of these frictions for policy-oriented DSGE modeling by documenting their strengths and weaknesses. We make three points: (i) the mechanisms generating pricing to market are not always neutral to business cycle dynamics of quantities, (ii) some mechanisms require producer markups at least 50% to account for the full range of estimates of the empirical exchange rate pass-through to export prices of 35%-50%, (iii) some frictions crucially depend on a particular driver of uncertainty in the underlying model.

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1 Introduction

It is a well known fact that frictionless international macro models, while successful in accounting for business cycle dynamics of quantities, fail to account for international prices. In particular, they are inconsistent with a vast empirical literature documenting international deviations from law of one price and incompleteness of exchange rate pass-through. Not surprisingly, reconciling the predictions of business cycle models for international prices with to the data has been on the forefront of research in international economics.

Thus far, the literature has identified two promising directions to remedy the problems with prices: (i) sticky prices in local currency, and (ii) pricing-to-market (PTM hereafter). In the first family of models, a friction of adjusting nominal prices in local currency directly limits the adjustment of all prices, and thus naturally leads to deviations from the law of one price across the border (LOP hereafter). In the second family, prices are flexible, but instead real frictions result in deviations from LOP.

In many ways, the above two directions should be regarded as complementary. It is a well established fact that sticky price models generate far too little persistence of the deviations from the law of one price on the international level (without additional sources of real rigidities)[1]. In addition, when volatile exchange rates are reproduced by these models, they typically imply enormous gains from price adjustment across the border, raising the questions about the size of the costs that could endogenously hinder nominal price adjustment. Since micro-founded PTM models can, in principle, remedy both problems when incorporated into a sticky price model, they thus seem a promising avenue from this perspective, but only to the extent that these real frictions, without nominal rigidities, deliver desirable properties for prices.

The literature has proposed several micro-founded frictions generating PTM. In various contexts, these frictions have been demonstrated to be capable of improving upon their respective frictionless benchmarks. However, since the definition of the frictionless benchmark as well as the set of analyzed data moments typically vary across the contributions, at this point it is hard to compare the performance of these mechanisms against each other, and

[1]Strategic complementarities can generate more persistent price stickiness and thus more persistent deviations from the law of one price (still falling short of the persistence the data, as in Johri & Lahiri (2008)). In addition, they are not a universal remedy, as they at the same time raise the level of predicted exchange rate pass-through to export prices. Only a particular mix of local currency pricing and producer currency pricing can deliver the empirical levels of pass-through to export prices.

thus evaluate their potential for policy-oriented work.

In an attempt to fill this gap, this paper performs a consistent comparison of several leading frictions in the literature that can generate PTM. To this end, as a point of our departure, we consider the same particular frictionless business cycle model that is relevant from an applied perspective, and embed all the frictions into this common framework. We then focus on the following three key features which, in our view, are likely to determine the broad-based applicability of these frictions to policy-oriented business-cycle work: (i) dependence of the mechanism generating PTM on a particular source of economic fluctuations in the model, (ii) neutrality of the mechanism generating PTM for the dynamic behavior of quantities, and finally, (iii) the degree of deviations from the law of one price generated by the friction.

Clearly, given the uncertainty about the actual shocks that drive the enormous exchange rate volatility in the data, the first feature determines to what a extent a given friction can be employed in DSGE models featuring different types of shocks. The second feature is important because accounting for dynamics of prices should not be achieved by sacrificing the quantitative fit of the model in other dimensions. Finally, the third feature determines how well a given friction accomplishes its primary goal of bringing prices closer to the data.

Our formal analysis focuses on the following four state-of-the-art PTM mechanisms widely cited in the literature.

1. *Industry aggregation* friction first introduced by Dornbush (1987), and further developed into a quantitative trade/macro model by Atkeson and Burstein (2008).
2. *Consumer search* friction developed by Alessandria (2009).
3. *Deep habits*, developed by Ravn Schmitt-Grohe and Uribe (2006), and extended in other papers by the same authors.
4. *Costly distribution* friction, proposed by Corsetti and Dedola (2005)[2]

All selected frictions are potentially amenable to quantitative analysis within large-scale DSGE models, and all are micro-founded[3]. Nevertheless, while guided by these criteria, our selection remains fairly arbitrary and incomplete.

[2]Or by the related work by the same authors that circulated since 2002 as an ECB working paper.

[3]Arise from a meaningful micro-level friction and thus allow for an enhanced quantitative discipline using micro-level evidence

The nature of our exercise is as follows. We embed each friction into the standard business cycle model a la Backus, Kehoe & Kydland (1995), and parameterize each model using a common set of data targets that are fairly standard in the literature. Whenever this is not possible, our preferred approach is to use the methodology or parameters from the original papers that introduced these frictions. To focus on the ability of each friction to generate deviations from the law of one price, in our qualitative analysis we focus on the predicted theoretical pass-through of exchange rates to export prices and contrast it with the available evidence. In our quantitative analysis, we study a set of simple moments pertaining to export prices deflated by the price of comparable domestic goods (basket of goods). These tests are typically most challenging for the theories to account for. Moreover, in most models, this approach additionally allows us to largely abstract from the exact mechanism generating real exchange rate movements—which are often counterfactual. Finally, to evaluate each PTM mechanism comprehensively, we also look at the predictions of the models for quantities and assess the importance of the exact specification of the forcing process.

Even though we often refer to each friction by citing the paper that introduced it, it is important to stress that our exercise modifies the models, and also takes a very selective look at the implications of the theory. Thus, our results should not be interpreted as a criticism of these original contributions. The exercise we perform here is intended to inform researchers about the differences between these mechanisms when modeled within a particular unified environment. From this perspective this is an imperfect exercise, but to our knowledge we are the first to perform it. We certainly do not claim that the frictions considered here would not perform differently in a different environment. We think of this paper as a first step toward popularizing the use of these frictions and helping to make informed modeling choices that suit best the intended applications. Clearly, by the very nature of our exercise, we devote here relatively more attention to uncover the weaknesses rather strengths of these theories. This, however, should not obscure the fact that all frictions we cite have been developed to work well within different environments, and within these environments have been shown successful[4]. Moreover, we focus here solely on the quantitative comparison of aggregate implications of the models, and do not discuss the micro-level evidence behind each

[4]In particular, Ravn et al. (2006) model has been developed to primarily deal with government shocks generating habit and develop counter-cyclical markups; Atkeson and Burstein (2008) is a trade model that was intended to bridge the gap between micro model and trade models with IO features and firm level heterogeneity; the original model features firm level heterogeneity and fixed cost of entry that we ignore here.

proposed PTM mechanism (which is clearly an important issue).

We make three points:

(1) Not all frictions generating PTM are neutral to business cycle dynamics of quantities. Moreover, some of the counterfactual implications are directly implied by the very mechanism generating PTM. First, the *costly distribution* and *deep habits* frictions are fully neutral to the predictions for quantities. The remaining two: *industry aggregation* friction due to Atkeson & Burstein (2008) and *consumer search* friction due to Alessandria (2009) significantly affect the fit of the model for quantities.

First, the *industry aggregation* friction was designed to work in a low-frequency-oriented trade model, as the PTM mechanism there requires a high elasticity between home and imported goods (consistent with the trade literature). However, the required high elasticity, while being useful to think about lower frequency deviations from the law of one price and incompleteness of pass-through, on higher frequencies hurts the implications of the model for quantities. In particular, the high elasticity resurrects some of the problems of the early Backus, Kehoe & Kydland (1992) setup with one homogeneous good, like excess comovement of TFP over output or excess comovement of consumption over output. Furthermore, the *industry aggregation* frictions delivers PTM through movements of importer's market shares that affect elasticity of demand. Consequently, in order to deliver deviations from LOP that are in line with the data, the model requires that the aggregate import shares be volatile. Our back-of-the-envelope calculation suggests that, for a plausible range of parameter values, they are more volatile in the model than in the data[5].

Second, the *consumer search* friction alters predictions for quantities when modeled as in Alessandria (2009) (we denote this formulation as *consumer search**). This original specification, featuring a linear utility function in labor with an exogenous labor-wedge shock, results in excess volatility of employment and output. With CRRA utility and only productivity shocks, similar problems arise[6]. We fix these problems in our baseline specification of the friction by modifying the original model and making search a market activity. This restores a good fit for quantities, while still delivering statistics for prices, but also alters the economic interpretation of the friction.

[5] This is less of a problem on lower frequencies—as suggested by much higher estimates of long-run price elasticities of trade.

[6] Employment is negatively correlated internationally, and there too little comovement of output.

(2) The equilibrium level of markups significantly affects the performance of the models for pricing. All frictions except for the case of *industry aggregation* friction require high equilibrium markups in order to reproduce a sufficiently high pass-through coefficient of exchange rates to export prices. In our benchmark comparison, we set markups equal to 30%, which turns out to be insufficient. In particular, the *consumer search* friction, and especially *costly distribution*, friction[7] deliver a subset of moments for prices at our targeted level of 30% markups, but in order to be consistent with the full range of empirical estimates of 35-50% of exchange rate pass-through to export prices, they require markups of at least 50%. The *industry aggregation* friction does well for the benchmark setting of markups, but quantity statistics deteriorate rapidly when markups are lowered significantly below this level (while still matching statistics on prices). In particular, when markups are set to 15%, as suggested by Basu & Fernald (1997), the country's import shares need to be much more volatile in the model to deliver PTM.

What is the appropriate target for markups? There is a lot of conflicting evidence about the level of markups in manufacturing. Industry studies point to numbers around 20-30% (as used by Atkeson and Burstein (2008) and Alessandria (2009)) or sometimes even higher, while on the other end, there are aggregate estimates by Basu & Fernald (1997) pointing to markups as low as 10-15%.

(3) Not all models are independent from the specifics of the forcing process. We find that the *deep habits* model with productivity shocks or standard demand shocks delivers deviations from LOP in the opposite direction to the desired one (export markups fall relative to home markups when the real exchange rate depreciates, and theoretical exchange rate pass-through to export prices is negative). This makes the model perform worse vis-a-vis our frictionless benchmark. It should be noted, however, that when demand shocks are modeled as stochastic government purchases that are additionally subject to deep habit formation, they can give rise to PTM in the right direction—as demonstrated by Ravn, Schmitt-Grohe & Uribe (2007).[8] We do not consider shocks of this kind here.

[7] In *costly distribution* friction only part of the markup is in the foreign unit, whereas in *consumer search* model the whole markup is. Thus, it is easier to get more PTM in *consumer search* model than in *costly distribution* model for the same level of markups.

[8] For such shocks, the model implies that following a government spending shock, the real exchange rate appreciates, rather than depreciates as in the standard models. According to some VAR-evidence, such correlation of the real exchange rate may be consistent with the data for some countries, including US and Japan (see Corsetti, Dedola & Leduc (2006)).

Last but not least, each friction delivers some additional desirable properties that we do not discuss, but which should be pointed out to complete our description. For instance, *industry aggregation* friction in the Atkeson and Burstein (2008) formulation is a parsimonious low frequency-oriented model with IO features that nicely fits into the modern trade theory. Alessandria (2009) not only delivers pricing to market, but also links international deviations from the law of one price to price dispersion of physically identical goods more broadly. Corsetti & Dedola (2005) show in a tractable and intuitive framework how the existence of a simple distribution cost on the retail level in a vertically integrated industry can lead to a time-varying elasticity of demand on the producer level and consequently change the dynamics of producer-level prices (wholesale prices). Finally, the *deep habits* model by Ravn et al. (2006) gives a convincing explanation of counter-cyclical markups over the business cycle.

2 Theory

In the next five sections, we set up the the models for later quantitative comparison. We start off by setting up the bare-bone framework into which we will later incorporate all the frictions—the Backus, Kehoe & Kydland (1995) model. Then, we move on to the modeling details of each particular friction considered by us.

2.1 Basic Frictionless Model

The overall structure of the model follows Backus, Kehoe & Kydland (1995). Time is discrete, $t = 0, 1, 2, ..., \infty$, and there are two ex-ante symmetric countries labeled *domestic* and *foreign*. Each country is populated by identical and infinitely lived households which supply labor and physical capital, consume goods, trade assets, and accumulate physical capital. Tradable intermediate goods are country-specific: d is produced in the domestic country, and f in the foreign country. The only source of uncertainty in the economy are country-specific productivity shocks.

Goods are traded at two levels: *intermediate* and *final*. At the intermediate good level, producers of goods (d at home and f abroad) sell their respective good to *final good producers* from each country. International trade happens only at the intermediate level. On the final good level, local final good producers combine d and f into a final consumption/investment good and resell them to the local households in a perfectly competitive market.

In terms of notation, we distinguish foreign country-related variables from the domestic ones using an asterisk. The history of shocks up to and including period t is denoted by $s^t = (s_0, s_1, ..., s_t)$, where the initial realization $s_0,$ as well as the time invariant probability measure μ over the compact shock space S are assumed given. In the presentation of the model, whenever possible, we exploit symmetry of the two countries and present the model from the domestic country's perspective only.

2.1.1 Uncertainty and Production

Each country is assumed to have access to a constant returns to scale production function $zF(k,l)$ that uses country-specific capital k and labor l, and is subject to a country-specific stochastic technology $\hat{z} \equiv \log(z)$ following an exogenous AR(1) process

$$\hat{z}(s^t) = \psi \hat{z}(s^{t-1}) + \varepsilon_t, \quad \hat{z}^*(s^t) = \psi \hat{z}^*(s^{t-1}) + \varepsilon_t^*, \tag{1}$$

where $0 < \psi < 1$ is a common persistence parameter, and $s_t \equiv (\varepsilon_t, \varepsilon_t^*) \in S$ is an i.i.d. normally distributed random variable with zero mean.

Since the production function is assumed to be constant returns to scale, we summarize the production process by an economy-wide marginal cost v. Given domestic factor prices w, r and domestic shock z, the marginal cost, equal to per unit cost, is given by:

$$v\left(s^t\right) \equiv \min_{k,l} \left\{ w\left(s^t\right) l + r\left(s^t\right) k \text{ subject to } z\left(s^t\right) F(k,l) = 1 \right\}. \tag{2}$$

2.1.2 Households

Each country is populated by a unit measure of identical and infinitely lived households. Households supply production factors to domestic producers, accumulate physical capital, and consume goods. After each history s^t, the stand-in household chooses the allocation, which consists of the level of consumption c, investment in physical capital i, labor supply l, and purchases of a set of one-period s_{t+1}-contingent bonds $b\left(s_{t+1}|s^t\right)$ to maximize the expected discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \int_{S^t} u\left(c\left(s^t\right), l\left(s^t\right)\right) \mu\left(ds^t\right). \tag{3}$$

Asset markets are complete, and the budget constraint of the domestic household is given by

$$
\begin{aligned}
& P\left(s^t\right)\left(c\left(s^t\right)+i\left(s^t\right)\right)+\int_S Q(s_{t+1}|s^t)b(s_{t+1}|s^t)\mu(ds_{t+1}) \qquad (4)\\
= \; & b(s^t)+w\left(s^t\right)l\left(s^t\right)+r\left(s^t\right)k\left(s^{t-1}\right)+\Pi\left(s^t\right),\\
k\left(t+1\right) = \; & \left(1-\delta\right)k\left(s^t\right)+i\left(s^t\right) \text{ all } s^t. \qquad (5)
\end{aligned}
$$

In the above budget constraints, we assume that the composite consumption good in each country is the numeraire. We do so by normalizing the level of prices $P\left(s^t\right)$ in each country to one so that the resulting ideal CPI price indexes in each country are equal to unity.[9]

The expenditure side of the budget constraint (4) consist of purchases of the consumption and investment goods and purchases of one-period-forward s_{t+1}-state contingent bonds. The income side consists of income from maturing bonds purchased at history s^{t-1}, labor income, rental income from physical capital, and the dividends paid out by local firms. The foreign budget constraint, due to a different numeraire unit, additionally involves a price $x(s^t)$ that translates the foreign numeraire to the domestic numeraire in the bond purchases term. By definition of the numeraire in each country, this price is the real exchange rate[10], which integrates the domestic and foreign asset markets into one world asset market.[11]

Summarizing, given the initial values for $k(s^{-1})$ and $b(s^{-1})=0$, households choose their allocations to maximize (3) subject to the budget constraint (4), the law of motion for physical capital (5), the standard no—Ponzi scheme condition, and the numeraire normalization. The first order conditions coming out of the household's problem are

[9] The ideal-CPI is defined by the lowest cost of acquiring a unit of composite consumption (c in the domestic country, c^* in the foreign country)

[10] In the data real exchange rate is measured using fixed-weight CPI rather than ideal CPI indices. Quantitatively, this distinction turns out not to matter in this particular class of models.

[11] Since the foreign budget constraint is expressed in the foreign country numeraire, and so is b^*, in order to use Q as the intertemporal price, the term $x(s^{t+1})b^*(s_{t+1}|s^t)$ first translates the purchase value of the foreign bonds to the domestic country numeraire units, and then $Q(s_{t+1}|s^t)/x(s^t)$ expresses the price of this purchase again in terms of the foreign numeraire.

$$
\begin{aligned}
Q\left(s^{t+1}|s^{t}\right) &= \beta\frac{\mu\left(s^{t+1}\right)}{\mu\left(s^{t+1}\right)}\frac{u_c\left(s^{t+1}\right)}{u_c\left(s^{t}\right)}, \\
\frac{x\left(s^{t+1}\right)}{x\left(s^{t}\right)}Q\left(s_{t+1}|s^{t}\right) &= \beta\frac{\mu\left(s^{t+1}\right)}{\mu\left(s^{t+1}\right)}\frac{u_c^{*}\left(s^{t+1}\right)}{u_c^{*}\left(s^{t}\right)} \\
x\left(s^{t+1}\right) &= x\left(s^{0}\right)\frac{u_c^{*}\left(s^{t}\right)}{u_c\left(s^{t}\right)}, \\
\frac{u_l\left(s^{t}\right)}{u_c\left(s^{t}\right)} &= -w\left(s^{t}\right), \\
u_c\left(s^{t}\right) &= E_{s^t}\beta u_c\left(s^{t+1}\right)\left[(1-\delta)+r\left(s^{t+1}\right)\right].
\end{aligned}
\tag{6}
$$

where $u_l\left(s^t\right), u_c\left(s^t\right)$ denote derivatives of the instantaneous utility function with respect to the subscript arguments.

Iterating backward up to state s^0 on (6) for the domestic and foreign country, we can derive under ex-ante symmetry between countries the real exchange rate,

$$
x\left(s^{t}\right) = \frac{u_c^{*}\left(s^{t}\right)}{u_c\left(s^{t}\right)}. \tag{7}
$$

The condition says that households fully share risk internationally, and equalize MRS from consumption across the borders with the relative price of their consumption x.

2.1.3 Final Good Producers

In each country, there is a unit measure of final good producers, which buy goods d and f from intermediate good producers in each country, and then aggregate them into the final consumption/investment good. Intermediate goods are aggregated according to a CES function given by

$$
G(d,f) = \left(\omega d^{\frac{\gamma-1}{\gamma}} + (1-\omega)\, f^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}, \tag{8}
$$

where γ is the elasticity of substitution (Armington elasticity) and ω is parameterizing home bias.

Given the aggregation technology above, instantaneous profits of an aggregation firm are

$$
P\left(s^{t}\right)G\left(d,f\right) - P_d\left(s^{t}\right)d\left(s^{t}\right) - P_f\left(s^{t}\right)f\left(s^{t}\right).
$$

The market for producing the final good is perfectly competitive, and hence the optimality conditions for the final good producers' problem are given by

$$\begin{aligned} P_d\left(s^t\right) &= G_d\left(s^t\right) P\left(s^t\right), \\ P_f\left(s^t\right) &= G_f\left(s^t\right) P\left(s^t\right), \end{aligned}$$

and the aggregation constraint (8).

2.1.4 Intermediate Good Producers

Tradable intermediate goods, d, and f, are country-specific and are produced by a unit measure of atomless competitive producers residing in each country. Producers employ local capital and labor to produce these goods using the technology specific to their country of residence. Their unit production costs are given by (2).

The instantaneous profit function Π of the producer is determined by the profits from sales in each market and is given by

$$\Pi\left(s^t\right) = \left(p_d\left(s^t\right) - v\left(s^t\right)\right) d\left(s^t\right) + \left(x\left(s^t\right) p_d^*\left(s^t\right) - v\left(s^t\right)\right) d^*\left(s^t\right) \quad \forall s^t.$$

By the perfectly competitive nature of the market for intermediate goods, the equilibrium conditions for the intermediate goods producers imply a zero profit restriction on prices (no markup and law of one price):

$$p_d\left(s^t\right) = x\left(s^t\right) p_d^*\left(s^t\right) = v\left(s^t\right).$$

2.1.5 Feasibility and Market Clearing

Equilibrium must satisfy the market clearing condition for bonds and the final good,

$$\begin{aligned} b\left(s^t\right) + x^*\left(s^t\right) b^*\left(s^t\right) &= 0 \\ c\left(s^t\right) + i\left(s^t\right) &= G(d, f) \ \ \forall s^t, \end{aligned}$$

and the aggregate resource constraint given by

$$d\left(s^t\right) + d^*\left(s^t\right) = z\left(s^t\right) F\left(k\left(s^{t-1}\right), l\left(s^t\right)\right) \ \ \forall s^t.$$

The definition of equilibrium is straightforward and will be omitted.

3 The Frictions

We now introduce the key frictions into the above base model that will give rise to pricing to market. Below, we only discuss the differences between the particular setup at hand and the base model's setup just described. For each model, we also briefly analyze the driving forces behind pricing to market.

3.1 Consumer Search

Here, we introduce into the frictionless setup the *consumer search* friction along the lines of Alessandria (2009). However, compared to the original paper, we crucially modify the setup so that the search occurs on the final good producer's level rather than being done by households. Such search is most naturally interpreted as business-to-business search friction, and looses the direct connection to *consumer search* of the original setup. We do so to improve the performance of the model on the quantity side without sacrificing prices, and offer an alternative application of the friction. Later we also consider a version of this model that adheres to the original more closely.

The production function and the household's problem are identical to the one in the frictionless benchmark and hence will be omitted (instead, Sections 2.1.1 and 2.1.2 apply).

3.1.1 Final Good Producers

The final good producer aggregates good d and f to produce the final goods using a standard CES aggregator, with elasticity of substitution γ:

$$G(d, f) = \left(\omega d^{\frac{\gamma-1}{\gamma}} + (1-\omega) f^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}.$$

Goods d and f are assumed to be acquired by the the process of search by atomless representatives of the final good firm—employed at the economy-wide marginal cost v. Specifically, it is assumed that in every period, the firm sends out measures s_d and s_f of its representatives to purchase $1/\theta$ units of good d and f, respectively. The total cost of this activity is given by $(s_d + s_f)v(s^t)$.

Each atomless representative observes one price quote of the commodity with probability q and two simultaneous price quotes with probability $1 - q$[12]. After observing the prices,

[12] The firms making the price quotes do not know what other prices the representative observes

she can purchase $1/\theta$ of the particular commodity, d or f, from one of the quoting firms (in the case of two quotes). Representatives are instructed by the headquarters to purchase the goods at the lowest price possible—conditional that such price does not exceed the reservation prices $r_d(s^t)$ and $r_f(s^t)$ set a priori by the headquarters. Since in equilibrium no posted price will ever exceed these reservation prices, without loss of generality we can assume that the representatives always make a purchase (i.e. all posted prices are lower than the reservation prices). Given measures of searching representatives, the total amount of goods purchased by the firm is thus given by:

$$\begin{aligned} d(s^t) &= \frac{s_d(s^t)}{\theta}, \\ f(s^t) &= \frac{s_f(s^t)}{\theta}. \end{aligned}$$

The distribution of prices at which goods are purchased depends on the distribution of prices posted by firms $F(p; s^t)$, and is given by (the lowest of two draws from F):

$$H_d(p; s^t) = qF_d(p; s^t) + (1 - q)\left[1 - F_d(p; s^t)\right],$$

The average price actually paid by the representatives searching for commodity d can be obtained by integrating over H[13]

$$p_d(s^t) = \int_{P_l}^{P_h} p \frac{dH_d(p; s^t)}{dp} dp,$$

where the bounds of the integration will be defined later.

3.1.2 Intermediate Good Producers

Intermediate good producers produce goods d and f that are purchased by the representatives of the final good producers from the home country and the foreign country. The production cost is equal to the marginal cost v.

The intermediate good producer, when making the price offer to any representative, does not know what prices and how many this representative has observed. This feature, as first demonstrated by Burdett & Judd (1982), is sufficient to give rise to a unique equilibrium

[13]The average price for f is straightforward by analogy.

featuring an endogenous price dispersion of physically identical goods. Burdett & Judd (1982) show that the support of the prices posted by all firms selling in a given market is compact and connected, and thus can be represented by a closed interval $[P_h, P_l]$. Moreover, the optimal probability distribution $F(\cdot)$ is uniquely pinned down by the condition that, given other producers draw from $F(\cdot)$, the intermediate good producer is indifferent between all prices from the support of $F(\cdot)$. Formally, for any $p \in [P_l(s^t), P_h(s^t)]$, the condition requires that the probability $F(p; s^t)$ that a posted price is lower than p satisfies:

$$(p - v(s^t))(q + 2(1-q)(1 - F(p; s^t))) = (P_h(s^t) - v(s^t))q. \tag{9}$$

This condition says that the expected profits from posting price p, factoring in the endogenous probability of making a sale, $(q + 2(1-q)(1 - F(p; s^t)))$, implied by the strategy F played by others, must be the same as the profits from posting the highest price P_h. At this highest price, by definition, the intermediate producer makes a sale if the representative has only one price quote (which happens with probability q).

Clearly, for given bounds, $P_h, P_l,$, equation (9) defines the function F. These bounds can be found as follows. The upper bound of the distribution P_h is determined by the condition that the final good producer must be indifferent between buying the good for P_h and instructing representatives who have a draw $P_h(s^t)$ to abort the purchase and instead send more searchers to purchase the good at the average price p_d, i.e.

$$\theta v(s^t) = P_h(s^t) - p_d(s^t).$$

The lowest bound can be found by plugging in P_l for p in (9).

The full characterization of equilibrium prices is thus given by

$$\begin{aligned}
P_h(s^t) &= v(s^t) + \frac{\theta}{1-q} v(s^t), \\
P_l(s^t) &= \frac{P_h(s^t)q + 2(1-q)v(s^t)}{2-q} = v(s^t) + \frac{q\theta}{2 - 3q + q^2} v(s^t), \\
F(p) &= 1 - \frac{1}{2}\frac{q}{1-q}\frac{P_h(s^t) - p}{p - v(s^t)},
\end{aligned}$$

and the expected price paid by a representative looking for good d is given by

$$p_d(s^t) = v(s^t) + \frac{\theta q}{1-\theta} v(s^t)$$

3.1.3 Feasibility

Each economy produces a different type of good sold at home and abroad. The feasibility implies:

$$\begin{aligned} d\left(s^t\right) + d^*\left(s^t\right) + \theta(d\left(s^t\right) + f\left(s^t\right)) &= z\left(s^t\right) k\left(s^t\right)^{\alpha} n\left(s^t\right)^{1-\alpha} \\ f\left(s^t\right) + f^*\left(s^t\right) + \theta(f^*\left(s^t\right) + d^*\left(s^t\right)) &= z^*\left(s^t\right) k^*\left(s^t\right)^{\alpha} n^*\left(s^t\right)^{(1-\alpha)} \end{aligned}$$

Assets must be in zero net supply, and so

$$b(s^t) + x(s^t) b^*(s^t) = 0,$$

Finally, the final good market must clear:

$$c\left(s^t\right) + i\left(s^t\right) = \left(\omega d\left(s^t\right)^{\frac{\gamma-1}{\gamma}} + (1-\omega) f\left(s^t\right)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \quad \forall s^t.$$

3.2 Consumer Search*

This is an extension of the baseline model that is very close to the original one from the published paper. In this version, following Alessandria (2009), we introduce search into the household's problem, and hence there are no final good producers in this formulation. The intermediate good producers' problem is identical to the one above. Below, we discuss the differences between the household's problem in the frictionless model and this specification below.

3.2.1 Households

The search friction requires that households need to search in order to purchase goods d and f. Specifically, it is assumed that the household can send measures n_d and n_f of searchers who can purchase $\bar{z}$ units of good d and f, respectively. These measures are counted against the total time endowment of the household.

Each searcher sent by the household observes one price of the good with probability q and two prices with probability $1-q$. Searchers are instructed to purchase the goods at the lowest price, conditional on not exceeding the reservation prices $r_d(s^t)$ and $r_f(s^t)$ set by the household. Since in equilibrium no posted price will ever exceed these reservation prices, we can assume for simplicity that the searchers always make a purchase (i.e. all posted prices are lower than the reservation prices).

The total amount of goods purchased by the searchers is given by:

$$
\begin{aligned}
d(s^t) &= n_d(s^t)\bar{z}, \\
f(s^t) &= n_f(s^t)\bar{z}.
\end{aligned}
$$

where $\bar{z}$ is the number of units purchased by each searcher.

Since each searcher purchases at the lowest price possible, the distribution of transacted prices is different from the distribution of posted prices by firms $F(\cdot)$, and is given by (the lowest of two draws from F):

$$
H_d(p;s^t) = qF_d(p;s^t) + (1-q)\left[1 - F_d(p;s^t)\right],
$$

The average price actually paid by the searchers (thus the household) is given by

$$
p_d(s^t) = \int_{P_l}^{P_h} p \frac{dH_d(p;s^t)}{dp} dp,
$$

where the function G and the bounds of the integration are defined as in Section 3.1.2.

Because search counts against time endowment of the household, the total amount of labor entering the utility function is given by

$$
l(s^t) = n\left(s^t\right) + \theta(d\left(s^t\right)/\bar{z} - f\left(s^t\right)/\bar{z}),
$$

where n is the time devoted to work in production of goods, θ is disutility from shopping, and $\bar{z}$ is defined as before.

Following the original paper, we also use a linear utility in labor

$$
u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \kappa l,
$$

and introduce shocks to κ (labor-wedge shock). The shock is backed out from the data on real wages and consumption, and labor-leisure choice first order condition implied the household's problem:

$$\kappa = \frac{w_t}{P_t} c^{-\sigma}.$$

Business cycle volatility of this object is about 4.5%, and it allows the model to match closely the volatility of the real exchange rate (for the US). In this version of the setup, we also assume a convex cost of capital adjustment, parameterized by ϕ, which gives rise to a new capital accumulation equation given by

$$k\left(t+1\right) = \left(1-\delta\right) k\left(s^t\right) + i\left(s^t\right) - \phi k(s^t)\left(\frac{i(s^t)}{k(s^t)} - \delta\right)^2 \text{ all } s^t.$$

The calibration of the model crucially differs from the baseline specification, as here shopping time is counted as home rather than market production. This imposed an additional restriction on the parameter values that search time is about 25% of work time, as implied by the time use survey of households. Since a lot of search is on B2B level, in our view, it is not the case that any of the two specifications is clearly superior independent of the context.

3.3 Costly Distribution

In this section, we modify the frictionless model to introduce monopolistic competition on the intermediate good level and a distribution-cost friction as in Corsetti & Dedola (2005). The friction here is that each producer is a monopolist over a country-specific variety of good, and for each good, a local distribution cost has to be incurred in order to deliver it to the final consumer. The existence of the distribution cost in this setup makes the demand for each variety of good depend not only on the price of that good charged by intermediate good producers, but also on the local cost of distribution, in effect delivering what is perceived by intermediate good producers as time-varying price elasticity of demand due to variation in the size of the distribution cost.

We first setup the baseline version of the model, and then extend it to introduce an explicit non-tradable sector as in the original formulation. The production function for each variety of good and the household's problem are identical to the one in the frictionless benchmark and hence will be omitted (instead, Sections 2.1.1 and 2.1.2 apply). For simplicity of exposition, we introduce *three* levels of production. First, at the lowest level of aggregation, imperfectly competitive intermediate good producers in each country produce a continuum of varieties of their respective goods ($d(i)$ and $f(i)$) and then sell their goods to *sectoral good producers*, who pay a fixed distribution cost per unit, aggregate them, and resell the country specific composite goods to the final good producers. The final good producers aggregate the domestic and foreign composite goods into a final consumption/investment good. Their problem is identical to the frictionless model and is described in Section 2.1.3. The problems of sectoral good producer and intermediate good producer are described below.

3.3.1 Sectoral Good Producers

The sectoral good producers aggregate a variety of differentiated intermediate goods purchased from measure one of intermediate good producers from each country. In addition to paying the purchase price for the good to the intermediate good producer, each sectoral producer has to pay a distribution cost denominated in local output[14]. In particular, in order to sell a good in the domestic market, each sectoral producer has to pay a distribution cost ξ denominated in units of the domestic output, and therefore having price equal to marginal cost $v(s^t)$. They aggregate different varieties of goods into composite goods d and f according to a CES production function with elasticity of substitution between varieties equal to θ:

$$\begin{aligned} d\left(s^t\right) &= \left(\int_0^1 d(i,s^t)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \\ f\left(s^t\right) &= \left(\int_0^1 f(i,s^t)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}. \end{aligned}$$

The instantaneous profit function of the sectoral producers is given by

$$P_d\left(s^t\right) d\left(s^t\right) + P_f\left(s^t\right) f\left(s^t\right) - \int_0^1 \left[p_d\left(i,s^t\right) + \xi v\left(s^t\right)\right] d(i,s^t) di - \int_0^1 \left[p_f\left(i,s^t\right) + \xi v\left(s^t\right)\right] f(i,s^t) di.$$

[14] We can think of the sectoral good producers as just employing capital and labor to produce distribution services according to the economy-wide technology.

Markets for composite goods are competitive, and hence the prices of composite goods charged by sectoral good producers are given by

$$P_d(s^t) = \left[\int_0^1 \left(p_d(i, s^t) + \xi v\left(s^t\right) \right)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

for composite good d and

$$P_f(s^t) = \left[\int_0^1 \left(p_f(i, s^t) + \xi v\left(s^t\right) \right)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

for composite good f.

Demand functions for goods $d(i)$ and $f(i)$ are given by

$$\begin{aligned} d(i, s^t) &= \left(\frac{p_d(i, s^t) + \xi v\left(s^t\right)}{P_d(s^t)} \right)^{-\theta} d(s^t), \\ f(i, s^t) &= \left(\frac{p_f(i, s^t) + \xi v\left(s^t\right)}{P_f(s^t)} \right)^{-\theta} f(s^t). \end{aligned}$$

The above equations capture the key implication of the introduced friction. Unlike in the frictionless Dixit-Stiglitz model, here demand for each variety of good depends not only on the price charged by each intermediate good producer ($p_d\left(i, s^t\right)$ and $p_f\left(i, s^t\right)$), but also on the local marginal distribution cost. Hence, intermediate good producers will perceive a time-varying price elasticity of demand for their goods and thus price-to-market in which they sell.

3.3.2 Intermediate Good Producers

In each country, there is measure one of imperfectly competitive intermediate good producers, indexed by i. Their unit production costs are given by (2).

A producer of variety $i \in [0, 1]$ chooses home wholesale price $p_d(i, s^t)$ and dock export price $p_d^*(i, s^t)$ to maximize profit

$$\left[p_d(i, s^t) - v\left(s^t\right) \right] d(i, s^t) + \left[x(s^t) p_d^*(i, s^t) - v\left(s^t\right) \right] d^*(i, s^t)$$

subject to demand equations coming from the sectoral level

$$
\begin{aligned}
d(i,s^t) &= \left(\frac{p_d(i,s^t)+\xi v(s^t)}{P_d(s^t)}\right)^{-\theta} d(s^t), \\
d^*(i,s^t) &= \left(\frac{p_d^*(i,s^t)+\xi v^*(s^t)}{P_d^*(s^t)}\right)^{-\theta} d^*(s^t).
\end{aligned}
$$

The wholesale prices that are the solution to the above problem are

$$
\begin{aligned}
p_d(i,s^t) &= \frac{\theta}{\theta-1} v(s^t) + \frac{\xi}{\theta-1} v(s^t), \\
p_x(s^t) &\equiv x(s^t) p_d^*(i,s^t) = \frac{\theta}{\theta-1} v(s^t) + \frac{\xi}{\theta-1} x(s^t) v^*(s^t).
\end{aligned}
\tag{10}
$$

3.3.3 Feasibility

Equilibrium requires several market clearing conditions and feasibility constraints. Representativeness of intermediate goods producers implies

$$
\begin{aligned}
d(i,s^t) &= d(s^t) \\
d^*(i,s^t) &= d^*(s^t) \\
f(i,s^t) &= f(s^t) \\
f^*(i,s^t) &= f^*(s^t)
\end{aligned}
$$

Resource feasibility with distribution cost implies

$$
\begin{aligned}
d\left(s^t\right) + d^*\left(s^t\right) + \xi(d(s^t)+f(s^t)) &= zk\left(s^t\right)^{\alpha} n\left(s^t\right)^{1-\alpha} \\
f\left(s^t\right) + f^*\left(s^t\right) + \xi(d^*(s^t)+f^*(s^t)) &= z^* k^*\left(s^t\right)^{\alpha} n^*\left(s^t\right)^{(1-\alpha)}
\end{aligned}
$$

Zero net supply of assets implies

$$
b(s^t) + x(s^t) b^*(s^t) = 0.
$$

The other feasibility conditions are embedded into notation.

3.4 Costly Distribution*

Here, we extend the baseline friction outlined above to incorporate non-tradable goods. This specification includes an explicit distinction between tradable and non-tradable sector and is set up and parameterized very closely to the original paper. Just like in the original model, we assume a symmetric Cobb-Douglas aggregator of tradable goods d and f and non-tradable goods. Specifically, the aggregation constraints in the household problem become

$$\begin{aligned} c\left(s^t\right)+i\left(s^t\right) &= 2\left(c^T\right)^{\frac{1}{2}}\left(c^N\right)^{\frac{1}{2}}, \\ c^T\left(d,f\right) &= 2d(s^t)^{\frac{1}{2}}f(s^t)^{\frac{1}{2}}. \end{aligned}$$

Just like the baseline friction, this version of the model also features monopolistic competition in production of a variety of non-tradable goods. The distribution cost in this version of the model is incurred in units of the non-tradable good. Specifically, sectoral good producers need to purchase ξ units of a local non-tradable good from the monopolistically competitive producers. There is no distribution cost of non-tradable goods.

The pricing formulas are very similar, but now involve an explicit distinction between tradable and non-tradable goods:

$$\begin{aligned} p_d(i,s^t) &= \frac{\theta}{\theta-1}v^T(s^t)+\frac{\xi}{\theta-1}P^N(s^t), \\ p_x(i,s^t) &\equiv x(s^t)p_d^*(i,s^t)=\frac{\theta}{\theta-1}v^T(s^t)+\frac{\xi}{\theta-1}x(s^t)P^{N*}(s^t). \end{aligned}$$

The specification of the forcing process is sector specific, and so the baseline period of the later quantitative specification of this model is 1 year.

3.5 Deep Habits

In this section, we modify the base model with monopolistic competition on the intermediate good level and 'relative deep habits', as proposed by Ravn, Schmitt-Grohe & Uribe (2006). The friction here is that each intermediate good producer is a monopolist over a country-specific variety of good, and for each good, consumers develop exogenous habits, i.e. the evolution of habits is driven by average purchases of each variety and not purchases of any individual household. For simplicity of exposition, we will actually model habits as being formed at the level of the producers, which is homomorphic to the setup in which it is directly incorporated into the consumer problem.

The production function for each variety of good and the household's problem are identical to the one in the frictionless benchmark and hence will be omitted (instead, Sections 2.1.1 and 2.1.2 apply). For simplicity of exposition, as in the case of the *costly distribution*, here we introduce three levels of production. First, at the lowest level of aggregation, imperfectly competitive intermediate good producers in each country produce a continuum of varieties of their respective goods ($d\,(i)$ and $f\,(i)$) and then sell their goods to *sectoral good producers*, who aggregate $d\,(i)\,s$ and $f\,(i)\,s$ into composite domestic and imported goods d and f, respectively, and then resell them to the final good producers. For clarity of exposition, we assume that habits here are formed at the level of the sectoral good producers. This assumption is equivalent to habit formation on the household level as assumed by the original paper[15] (first order conditions are the same). Final good producers aggregate the composite goods into a final consumption/investment good. Their problem is identical to the frictionless model's and is described in Section 2.1.3. The sectoral and intermediate good producers' problems are described below.

3.5.1 Sectoral Good Producers

Sectoral good producers are perfectly competitive and produce the composite goods d and f by aggregating varieties of *habit-adjusted* quantities $d^h(j)$ and $f^h(j)$ according to a CES aggregator with elasticity φ :

$$
\begin{aligned}
d(s^t) &= \left[\int_0^1 d^h\left(j, s^t\right)^{\frac{\varphi-1}{\varphi}} dj\right]^{\frac{\varphi}{\varphi-1}}, \\
f(s^t) &= \left[\int_0^1 f^h\left(j, s^t\right)^{\frac{\varphi-1}{\varphi}} dj\right]^{\frac{\varphi}{\varphi-1}}.
\end{aligned}
$$

For each good $j \in [0,1]$, the habit-corrected quantity $d^h\,(j)$ is assumed to be determined by the level of habit from yesterday, $h\,(j, s^{t-1})$, and the purchases of the good today, $D\,(j, s^t)$, according to the formula

$$
d^h\left(j, s^t\right) = \frac{D\left(j, s^t\right)}{h_d\left(j, s^{t-1}\right)^{\theta}},
$$

[15]This greatly simplifies the exposition as the household problem stays the same.

where θ parameterizes the strength of the habit (θ assumed to be less than zero, following the convention introduced by Ravn, Schmitt-Grohe & Uribe (2006)). Habit is formed according to the following law of motion:

$$h_d\left(j, s^t\right) = \rho h_d\left(j, s^{t-1}\right) + (1-\rho)\,\bar{d}^c\left(j, s^t\right)$$

where $\bar{d}^c$ is the *average* level of purchases of good j in the economy—introduced here so that the households do not internalize the effect of their purchases on their own level of habit (catching up with the Joneses specification of habit). Analogous equations hold for good f.

The instantaneous profit function of the sectoral producers is given by the expression

$$P_d\left(s^t\right) d\left(s^t\right) + P_f\left(s^t\right) f\left(s^t\right) - \int_0^1 p_d\left(i, s^t\right) d(i, s^t)di - \int_0^1 p_f\left(i, s^t\right) f(i, s^t)di,$$

where $p_d\left(i, s^t\right)$ and $p_f\left(i, s^t\right)$ are prices charged by the monopolistically competitive intermediate good producers.

The sectoral good producer's problem can be simplified by solving a temporal decision of allocating purchases across the j goods. For any amount of the aggregate goods desired, d and f, given prices, $p_d\left(j\right)$ and $p_f\left(j\right)$, it is straightforward to show that there is a unique expenditure-minimizing allocation of $D\left(j\right)$ and $F\left(j\right)$, given by

$$D\left(i, s^t\right) = \left(\frac{p_d\left(i, s^t\right)}{P_d\left(s^t\right)}\right)^{-\varphi} h_d\left(i, s^{t-1}\right)^{\theta(1-\varphi)} d, \tag{11}$$

$$F\left(i, s^t\right) = \left(\frac{p_f\left(i, s^t\right)}{P_f\left(s^t\right)}\right)^{-\varphi} h_f\left(i, s^{t-1}\right)^{\theta(1-\varphi)} f, \tag{12}$$

where the aggregate price indices are defined as

$$\begin{aligned} P_f\left(s^t\right) &\equiv \left[\int h_f\left(j, s^{t-1}\right)^{\theta(1-\varphi)} p_f\left(j, s^t\right)^{1-\varphi} dj\right]^{\frac{1}{1-\varphi}}, \\ P_d\left(s^t\right) &\equiv \left[\int h_d\left(j, s^{t-1}\right)^{\theta(1-\varphi)} p_d\left(j, s^t\right)^{1-\varphi} dj\right]^{\frac{1}{1-\varphi}}. \end{aligned}$$

This allows us to express the final good producer's problem[16] in a standard way as in Section

[16]To save on notation, the setup here is a simplified version of the setup that we use in the quantitative

2.1.3.

3.5.2 Intermediate Good Producers

In each country, there is measure one of imperfectly competitive intermediate good producers, indexed by i. Their unit production costs are given by (2) and are the same across varieties within a country. Producers take the demand relations (11)-(12) as given, as well as the prices charged by other producers. They are monopolists for their own variety of good j, and so they internalize the effect of their sales on the habit formation for their own good. Specifically, they take into account the laws of motion for habit

$$\begin{aligned} h_d\left(j, s^t\right) &= h_d\left(j, s^{t-1}\right)\rho + (1-\rho) D\left(j, s^t\right), \\ h_d^*\left(j, s^t\right) &= h_d^*\left(j, s^{t-1}\right)\rho + (1-\rho) D^*\left(j, s^t\right) \end{aligned}$$

into their optimization problem. The instantaneous profit function of a domestic producer is

$$\left(p_d\left(j, s^t\right) - v\left(s^t\right)\right) d\left(j, s^t\right) + \left(x\left(s^t\right) p_d^*\left(j, s^t\right) - v\left(s^t\right)\right) d^*\left(j, s^t\right).$$

The optimization problem of each intermediate good producer is to maximize the the present discounted stream of instantaneous profits, where the discount factor is implied by the consumer's stochastic discount factor, subject to demand equations and laws of motion for habit. The problem is inherently dynamic as the habit h is a state variable from the producer's perspective. We omit the formal definition of this problem as it is straightforward.

---

analysis section. There, we exclude investment i from the habit formation process, as it seemed more natural for us to think of investment as being denominated in physical units rather than habit adjusted units. Habit is only imposed on consumption goods. We have verified that this distinction would not make any significant difference for any of the result reported throughout the paper.

3.5.3 Feasibility

The following feasibility conditions have to hold in equilibrium at every s^t and for all j:

$$
\begin{aligned}
D\left(j,s^t\right)+D^*\left(j,s^t\right) &= zk\left(j,s^t\right)^{\alpha} l\left(j,s^t\right)^{1-\alpha}, \\
F\left(j,s^t\right)+F^*\left(j,s^t\right) &= z^*k^*\left(j,s^t\right)^{\alpha} l^*\left(j,s^t\right)^{(1-\alpha)}, \\
\int k\left(i,s^t\right)di &= k(s^t), \\
\int l\left(i,s^t\right)di &= l(s^t), \\
\int k^*\left(i,s^t\right)di &= k^*(s^t), \\
\int l^*\left(i,s^t\right)di &= l^*(s^t),
\end{aligned}
$$

Representativeness implies that for all $i,j : d\left(j\right)=d\left(i\right),\ f\left(j\right)=f\left(i\right),\ k\left(j\right)=k\left(i\right),$ $l\left(j\right)=l\left(i\right),\ h_d\left(j\right)=h_d\left(i\right),\ h_d\left(j\right)=h_d\left(i\right),\ h_d\left(j\right)=h_d\left(i\right),h_d\left(j\right)=h_d\left(i\right),\ h_f\left(j\right)=h_f\left(i\right).$

As before, zero net supply of assets on the international level implies

$$
b(s^t)+x(s^t)b^*(s^t)=0.
$$

3.6 Industry Aggregation

This section studies the friction developed by Atkeson and Burstein (2008). The key feature introduced by this friction is a monopolistic competition featuring a nested two-layer CES demand system. The price elasticity of demand perceived by firms is going to be an average of the two elasticities at different layers of aggregation, with the weight depending on a firm's market share. Variation of market shares over the business cycle will translate into variation in markups, and in particular, different variation in the home and foreign markets—driving PTM in the model.

The production function for each variety of good and the household's problem are identical to the one in the frictionless benchmark and hence will be omitted (instead, Sections 2.1.1 and 2.1.2 apply). In contrast to the setup in the original paper, here we exogenously fix the number of firms in the economy, and make them all identical. There are also no non-tradable goods. Production in the economy is divided into sectors and individual goods and will feature three levels of production. First, at the lowest level of aggregation, *a finite number* of imperfectly competitive (Cournot competitors) intermediate good producers in sector j in each country produce a variety of their respective goods k ($d\,(k,j)$ and $f\,(k,j)$) and then sell their goods to *sectoral good producers*, who aggregate $d\,(k,j)\,s$ and $f\,(k,j)\,s$ into composite sectoral goods $y\,(j)$ and sell them to final good producers. Final good producers aggregate the composite sectoral goods into a final consumption/investment good. The critical assumption of the model is that the elasticity between sectoral goods $y\,(j)$ is low relative to the elasticity within sector at the intermediate good level. We describe formally all three levels of production/ aggregation below.

3.6.1 Final Good Producers

Final consumption $c(s^t)$ and investment goods $i(s^t)$ in home country are produced by a competitive firm using the output of a continuum of identical sectors j, $y(j,s^t)$ according to the aggregation constraint

$$c(s^t)+i(s^t)=\left[\int_0^1 \left(y(j,s^t)\right)^{1-\frac{1}{\gamma}}\,dj\right]^{\frac{\gamma}{\gamma-1}}. \tag{13}$$

Given prices of each sectoral good, $P(j,s^t)$, the inverse demand equation for $y(j,s^t)$ is

$$\frac{P(j,s^t)}{P(s^t)}=\left(\frac{y(j,s^t)}{c(s^t)+i(s^t)}\right)^{\frac{-1}{\gamma}},$$

where $P\,(s^t)$ is the price index (lowest cost of acquiring a unit of the final investment/consumption good), given by

$$P(s^t)=\left(\int_0^1 \left(P(j,s^t)\right)^{1-\gamma}\,dj\right)^{1/(1-\gamma)},$$

and is normalized to serve as the numeraire.

3.6.2 Sectoral Good Producers

There is a unit mass of perfectly competitive sectoral good producers in each sector in each country. They purchase goods from intermediate good producers within the sector and aggregate them into sectoral output. We assume that there are n home and n_X intermediate good producers operating within each sector j, with intermediate good producers indexed by k.

The aggregation of individual goods into sectoral output good is guided by a CES aggregator with elasticity ρ:

$$y(j,s^t) = \left[\sum_{k=1}^{n}\left(d(k,j,s^t)\right)^{\frac{\rho-1}{\rho}} + \sum_{k=n+1}^{n_X}\left(f(k,j,s^t)\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}.$$

The instantaneous profit function of the sectoral producers is given by the expression

$$P\left(j,s^t\right)y\left(j,s^t\right) - \sum_{k=1}^{n} P_d\left(k,j,s^t\right)d(k,j,s^t) - \int_0^1 P_f\left(k,j,s^t\right)f(k,j,s^t),$$

and the inverse demand for a domestic intermediate good k in sector j is

$$\frac{P_d(k,j,s^t)}{P(j,s^t)} = \left(\frac{d(k,j,s^t)}{y(j,s^t)}\right)^{\frac{-1}{\rho}},$$

where the sectoral price index $P\left(j,s^t\right)$ is taken by them as given and it is defined by

$$P(j,s^t) = \left[\sum_{k=1}^{n}\left(P_d(k,s^t)\right)^{1-\rho} + \sum_{k=n+1}^{n_X}\left(P_f(k,s^t)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}.$$

Since the firm's problem is effectively static, sectoral producers maximize the instantaneous profit function given prices, and subject to the aggregation constraint. Note here that since all firms of the same type and all sectors within each country are identical, the subscripts j and k are redundant here once we restrict attention to type-identical allocations.

3.6.3 Intermediate Good Producers

We assume that there are n home and n_X intermediate good producers operating within each sector j, indexed by k. They employ local labor and capital to produce their respective variety of good. Their unit production costs are given by (2) and are the same across varieties within a country. They are Cournot competitors.

The problem of the domestic country producer k selling in sector j of the home market is given by choosing price p and quantity d to maximize profit (at all s^t):

$$\pi_d(j,s^t) = \left(p - v(s^t)\right)d$$

subject to demand equation implied by sectoral and final good producer demands

$$p=\left(\frac{d}{y(j,s^t)}\right)^{-\frac{1}{\rho}}\left(\frac{y(j,s^t)}{c(s^t)+i(s^t)}\right)^{-\frac{1}{\gamma}}.$$

The implicit solution to the maximization problem is given by

$$p=\frac{\varepsilon(j,s^t)}{\varepsilon(j,s^t)-1}v(s^t),$$

where the elasticity $\varepsilon(j,s^t)$ is a function of the market share of intermediate good producer k and involves the choices of all other producers (Cournot competition):

$$\begin{aligned}\varepsilon(j,s^t) &= \left[\frac{1}{\rho}\left(1-S(j,s^t)\right)+\frac{1}{\gamma}S(j,s^t)\right]^{-1}\\ S(j,s^t) &= pd\left[\sum_{\kappa=1}^{n}P_d(\kappa,j,s^t)d(\kappa,j,s^t)+\sum_{\kappa=1}^{n_X}P_f(\kappa,j,s^t)f(\kappa,j,s^t)\right]^{-1},\end{aligned}$$

where $P_d(k,j,s^t)d(k,j,s^t)=pd$.

The maximization for other markets is defined in an analogous fashion, and can be considered separately due to constant returns to scale. The only modification is that to produce for exports, a producer has to incur an exporting cost of τ units of own production per unit exported.

3.6.4 Feasibility

Representativeness implies that subscripts k and j are redundant, and all variables indexed by these subscripts must take the same value. Given this assumption, production feasibility is given by

$$\begin{aligned}nd(s^t)+n_Xd^*(s^t)\left(1+\tau\right) &= z(s^t)k(s^t)^{\alpha}l(s^t)^{1-\alpha},\\ nf^*(s^t)+n_Xf^*(s^t)\left(1+\tau\right) &= z^*(s^t)k^*(s^t)^{\alpha}l^*(s^t)^{1-\alpha},\end{aligned}$$

As before, the bond market clearing is given by

$$b\left(s^t\right)+x\left(s^t\right)b^*\left(s^t\right)=0.$$

4 Sources of PTM and Qualitative Analysis

4.1 Consumer Search

Formulas for prices depend here on the local search cost. Specifically, the export price of the good in the model is given by

$$p_x(s^t) = v(s^t) + \frac{\theta q}{1-q} x(s^t) v^*(s^t),$$

while the home price of the some good is given by

$$p_d(s^t) = v(s^t) + \frac{\theta q}{1-q} v(s^t).$$

Dividing these two prices, we note that the deviations from the law of one price generated by the model can be directly linked to the cost-based real exchange rate $x(s^t)v^*(s^t)/v(s^t)$:

$$p_d^x(s^t) \equiv \frac{p_x(s^t)}{p_d(s^t)} = \frac{1 + \frac{\theta q}{1-q} \frac{x(s^t)v^*(s^t)}{v(s^t)}}{1 + \frac{\theta q}{1-q}}.$$

Moreover, the magnitude of pricing to market generated by the model crucially depends on the level of producer markups $\frac{\theta q}{1-q}$. Below we discuss the quantitative implication of this feature of the model for different levels of markups.

4.1.1 Quantitative Potential for PTM

As noted above, the model can generate pricing to market and incomplete pass-through of exchange rates due to costly search denominated in local units. Here we asses how large these effects will be by deriving the theoretical pass-through coefficient implied by the model. We do so by evaluating elasticity of export price with respect to real exchange rate, evaluated at the steady state values, to define the theoretical pass-through coefficient:

$$PT \equiv \frac{\partial \log(p_x)}{\partial \log(x)} = \frac{\frac{\theta q}{1-q} \frac{v^*(s^t)x(s^t)}{v(s^t)}}{1 + \frac{\theta q}{1-q} \frac{x(s^t)v^*(s^t)}{v(s^t)}}|_{ss} = \frac{\frac{\theta q}{1-q}}{1 + \frac{\theta q}{1-q}}.$$

By definition, this number tells us by how much in percentage terms the real export price moves in response to a 1% change of the real exchange rate x.

As we can see, the theoretical pass-through in the model crucially depends on the level of producer markups. In particular, for markups equal to 30%, the PT coefficient generated by the model is 24%, for the level of markups equal to 50%, it is 33%, and for the level of markups as high as 100%, it is 50%.

The evidence on the degree of pass-through varies widely in the literature, but most studies, while controlling for costs and other factors, estimate the empirical pass-through coefficient to be in the interval[17] 35%-50%. Our calculation above suggests that the model needs markups of at least 50%.

4.2 Costly Distribution

As we can see from (10), the formulas for prices for in the case of *costly distribution* friction are similar to the ones implied by *consumer search* friction. However, there is one crucial difference that we should point out. In the case of *costly distribution*, as opposed to *consumer search*, only part of the markup is denominated in the local numeraire units, limiting pricing-to-market for the same level of markups across the two models. This conclusion follows immediately from an analogous derivation of the theoretical exchange rate pass-through to export prices, which in this case is given by

$$PT \equiv \frac{\partial \log(p_x)}{\partial \log(x)} = \frac{\frac{\xi}{\theta-1}\frac{x(s^t)v^*(s^t)}{v(s^t)}}{1+\frac{\xi}{\theta-1}\frac{x(s^t)v^*(s^t)}{v(s^t)}}|_{ss} = \frac{\frac{\xi}{\theta-1}}{1+\frac{\xi}{\theta-1}},$$

and thus is determined by a term that is lower than the total producer markup given by[18]

$$\frac{\theta}{\theta-1}+\frac{\xi}{\theta-1}.$$

[17] Most estimates are centered around 60% for import prices, which would imply 40% to export prices (=100%-60%). For example, Goldberg & Campa (2005) find pass-through to import prices in OECD to be around 46%. Goldberg & Knetter (1997) report number closer to .6 for import prices and thus .4 for export prices.

[18] In the original paper by Corsetti and Dedola (2005) the non-tradable sector is assumed less productive than the tradable sector—requiring a lower setting of ζ. This assumption, however, does not resolve the issue. In the quantitative model, the share of distribution cost in the final retail price is tightly disciplined by the available estimates by Burstein, Neves & Rebelo (2003) and effectively it does not matter whether it comes from a parameter or a productivity inflated price of the non-tradable input.

4.3 Industry Aggregation

Formulas for prices depend here on the firm's market shares—as it varies the perceived demand elasticity by the monopolistic intermediate good producers. Specifically, the export price of the home good is given by

$$p_x(s^t) \equiv x(s^t)P_d^*(s^t) = \frac{\varepsilon_d^*(s^t)}{\varepsilon_d^*(s^t)-1}\,(1+\tau)\,v(s^t)$$

where

$$\begin{aligned}\varepsilon(s^t) &= \left[\frac{1}{\rho}(1-S_d^*(s^t))+\frac{1}{\gamma}S_d^*(s^t)\right]^{-1} \\ S_d^*(s^t) &= \frac{P_d^*(s^t)d^*(s^t)}{nP_f^*(s^t)f^*(s^t)+n_X P_d^*(s^t)d^*(s^t)}\end{aligned}$$

while the home price of the same good is given by

$$p_d(s^t) \equiv P_d(s^t) = \frac{\varepsilon_d(s^t)}{\varepsilon_d(s^t)-1}v(s^t)$$

where

$$\begin{aligned}\varepsilon(s^t) &= \left[\frac{1}{\rho}(1-S_d(s^t))+\frac{1}{\gamma}S_d(s^t)\right]^{-1} \\ S_d(s^t) &= \frac{P_d(s^t)d(s^t)}{nP_d(s^t)d(s^t)+n_X P_f(s^t)f(s^t)}\end{aligned}$$

(τ stands for the iceberg cost of exporting).

As we can see from the above formulas, markups for each market depend on the elasticity of demand, $\frac{\varepsilon^*}{\varepsilon^*-1}$ in the export market and $\frac{\varepsilon}{\varepsilon-1}$ in the domestic market. Any variation in markups in the model will hence work through the variation in these elasticities. Formally this will work through the term $\frac{1}{\rho}(1-S_d^*)+\frac{1}{\gamma}S_d^*$, in the above expressions. One can see immediately that the variation in markups will be amplified by a larger difference between elasticities γ and ρ (for a given change in s_{iD}), and also by a larger variation in the market share S_d^*. Following AB, we can derive the elasticity of exporter's markup with respect the market share

$$\frac{d\log(\frac{p_x}{v})}{d\log(S_d^*)} = \frac{S_d^*\left(\frac{1}{\gamma}-\frac{1}{\rho}\right)}{1-\frac{1}{\rho}(1-S_d^*)-\frac{1}{\gamma}S_d^*}.$$

From the above expression, we can see that the size of the firm matters—the elasticity of markup with respect to market share is strictly increasing in S_d^*. That is, having large firms in the model makes their markups very elastic to changes in market share, giving more pricing-to-market. Atkeson and Burstein (2008) quantitatively meet this requirement by matching the size distribution of firms in the data. In our quantitative analysis, since all firms are identical, we satisfy the large firm requirement by making an extreme assumption of one exporting firm.

The model then works as follows. After a positive productivity shock in the domestic country, the market shares of domestic firms at home and abroad go up, which decreases the elasticity of demand in both markets and raises markups in both markets ($\frac{\varepsilon}{\varepsilon-1}$ is a decreasing function). Because initially there was aggregate home bias in consumption, consumption of good d abroad goes up by more in percentage terms than consumption of good d at home, which causes the elasticity of exports to go down by more than the elasticity in the domestic market. As a result, export markups rise relative to domestic markups for good d. Quantitatively, this increase is accompanied by a real exchange rate depreciation, and the export markup increase is large enough to raise the export price, restoring consistency with the data in terms of the correlation of the export price and the exchange rate. [19].

The pricing to market mechanism of this model connects the degree of pass-through of exchange rates to export prices through movement of market shares. We next investigate the quantitative implications of this feature of the model.

4.3.1 Quantitative Potential for PTM

As noted above, for pricing to market to arise in the model, it is critical that market shares move when the real exchange rate changes. In this section we ask how large these market share movements need to be for the model to be consistent with the estimates of the empirical pass-through of exchange rates into export prices of at least 35%, and the overall business cycle frequency volatility of the real exchange rate of 3.97% (US quarterly data, 1984-2009).

To answer this question, we assume that only market shares covary with the real exchange rates—an assumption justified by the fact that other components of the prices are typically

[19] In our setup, as opposed to the original, does not have an explicit link between entry and size generated by firm level heterogeneity and fixed cost of entry. This could potentially generate additional effects. However, the quantitative experiments presented in AB08 suggest that this is not a critical feature for any of the results (see AB's sensitivity analysis, Table 5, case F=0).

controlled for in the empirical estimates of exchange rate pass-through. We next derive the theoretical pass-through coefficient implied by the model (at the steady state),

$$PT \equiv \frac{d\log p_x}{d\log x} = \left.\frac{S_d^*\left(\frac{1}{\gamma}-\frac{1}{\rho}\right)}{1-\frac{1}{\rho}\left(1-S_d^*\right)-\frac{1}{\gamma}S_d^*}\right|_{ss}\frac{d\log\hat{S}_d^*}{d\log x},$$

and by plugging in the required value of pass-through ($PT = 0.35$), we obtain the lowest bound for the elasticity of market shares with respect to exchange rates

$$\frac{d\log\hat{S}_d^*}{d\log x} \geq 0.35 \times \left.\frac{1-\frac{1}{\rho}\left(1-S_d^*\right)-\frac{1}{\gamma}S_d^*}{S_d^*\left(\frac{1}{\gamma}-\frac{1}{\rho}\right)}\right|_{ss}.$$

The right-hand side of the above formula tells us how much the foreign market shares must move (in percentage terms) in response to a 1% change of the real exchange rate change to deliver exchange rate pass-through of 35% to export prices.

The derived lowest bound depends only on elasticity parameters and steady state market shares. Hence, we can evaluate it using the values of calibrated parameters and the targeted aggregate import shares. Since higher S_d^* helps the model to generate more PTM, in order to give it the best chance, we follow our parameterization by assuming $n_X = 1$—implying that the market share of an average exporting firm is equal to an aggregate import share of a country. Furthermore, we choose sectoral import share S_d^* of 16.5% based on the numbers reported by Atkeson and Burstein (2008)[20]. Note that this choice is favorable for the model, as AB have 5 foreign firms in their model, and the average share of a firm is thus overstated by our assumption that there is only one. The elasticity parameters are also taken from AB[21].

Given the aforementioned values of all the parameters, our evaluation implies that the market shares in the model must be at least 1.8 more volatile than the real exchange rate. As mentioned above, we think that this is the most favorable calculation to the model, as the same estimate using our baseline parameter values is as high as 4.4 ($S_d^* = .12, \rho = 8.7, \gamma = 1.52$). Comparing to the data, 1.8 is still a bit too high. In the US data, the relative volatility

[20] Average of exports and imports of manufacturing goods in the US divided by gross manufacturing output

[21] In AB, these elasticities have been chosen to align the quantitative model with the producer markups of 30% and an equal expenditure share across industries. In our specification, we target the same level of markups and predicted theoretical pass-through of 40%.

of the (aggregate) import share relative to the real exchange rate is about 1.20[22].

Our final conclusion is that this friction can deliver high levels of exchange rate pass-through to export prices for a reasonable range of producer markups, but on the business cycle frequency, this implication is subject to the caveat of volatile market shares. (This caveat is unlikely to be a problem on lower frequencies, which we do not study here.)

4.4 Deep Habits

The formulas for prices in the *deep habits* model crucially depend on the target market specific invisible shadow value of habit. Specifically, the real export price is given by

$$p_x(s^t) = v(s^t) + \Delta_d^*(s^t) - (1-\rho)\,\psi_d^*(s^t),$$

whereas the home price of the same good is given by

$$p_d\left(s^t\right) = v(s^t) + \Delta_d(s^t) - (1-\rho)\,\psi_d(s^t),$$

where

$$\Delta_d(s^t) = \frac{p_d(s^t)}{\phi}, \tag{14}$$

$$\Delta_d^*(s^t) = \frac{x(s^t)p_d^*(s^t)}{\phi}, \tag{15}$$

and

$$\psi_d\left(s^t\right) = \sum_{s_{t+1}} Q\left(s_{t+1}, s^t\right)\mu\left(s^{t+1}|s^t\right)\left[\rho\psi_d\left(s^{t+1}\right) + \Delta_d\left(s^{t+1}\right)\frac{\theta\left(1-\varphi\right)d^c\left(j, s^{t+1}\right)}{h_d\left(j, s^t\right)}\right], \tag{16}$$

$$\psi_d^*\left(s^t\right) = \sum_{s_{t+1}} Q\left(s_{t+1}, s^t\right)\mu\left(s^{t+1}|s^t\right)\left[\rho\psi_d^*\left(s^{t+1}\right) + \Delta_d^*\left(s^{t+1}\right)\frac{\theta\left(1-\varphi\right)d^{c*}\left(j, s^{t+1}\right)}{h_d^*\left(i, s^t\right)}\right]. \tag{17}$$

In the above expressions, Δ represents the shadow cost of selling an additional unit of

[22] To calculate this number we have rescaled nominal series for GDP by a constant equal to the share of non-service sectors in GDP (for year 2000). Then, we have subtracted exports of goods to obtain the domestic absorption as measure of domestic sectoral output. For imports we have used series for imports of goods to the US. The time period is 1984:2009. Volatility of the real exchange rate for this period is 3.97% (IMF IFS), and it has been used to relate it.

output today, and it is implied by the loss of markups on existing sales due to an underlying fall in the price. ψ represents the shadow value of habit, and due to the persistence of habit, and is given by a recursive asset pricing equation involving an exogenous depreciation of habit determined by $1-\rho$, and a per unit 'dividend payment' *equal to* the reduced future cost of sales by an additional unit of habit[23].

The above formulas for prices reveal that pricing to market crucially depends on the dynamics of the shadow value of habit. In fact, after substituting Δ into the expressions above, we can see that in the absence of ψ, the export price is proportional to the marginal cost as in the Dixit-Stiglitz model:

$$x(s^t)p_d\left(s^t\right) = \frac{\phi}{\phi-1}\left[v(s^t) - (1-\rho)\psi_d^*\right].$$

What then determines the value of habit? Qualitatively, many factors, including discount factor and future evolution of prices, but, quantitatively, the following effect seems most important in driving the response of prices to a positive productivity shock. When marginal cost v falls persistently in the model due to a positive shock, the home producers know today that due to persistently lower cost and prices (even if markups were constant) they will be likely selling more today and in the future. As a result, due to the higher expected future sales by them, habit today becomes more valuable (term $\theta\left(1-\varphi\right)d^c\left(j,s^{t+1}\right)/h_d\left(j,s^t\right)$ goes up) as it optimizes on the expensed cost of raising sales Δ. Consequently, they slash markups below the Dixit-Stiglitz benchmark and price to market. (It is fairly straightforward to illustrate how the described above effects arise from the above formulas for prices by considering a one time permanent productivity shock.)

4.4.1 Quantitative Potential for PTM

With productivity shocks, the model generates a negative pass-through of exchange rates to export prices. More precisely, in the aftermath of a persistent positive productivity shock in the home country, home firms expect in this model to sell more abroad in the future, which makes habit abroad more valuable today, and thus results in lower markups on exported goods. Moveover, the markups abroad fall more than markups at home, as under home-bias, in this class of models the increase in demand abroad is also larger than the increase at home

[23] Note that the expression $\theta\left(1-\varphi\right)d^c\left(j,s^{t+1}\right)/h_d\left(j,t\right)$ that appears in the formula for ψ is equal to the derivative of the demand function faced by the monopolist w.r.t. h_d.

(in percentage terms). The same conclusion applies to standard demand shocks multiplying the utility function of a representative consumer—the exception is a particular specification of demand shocks proposed by Ravn et al. (2007)[24].

Based on this analysis, we thus conclude that the model requires a particular correlation between real exchange rate movements and the value of habit. This property will not in general hold for all kind of shocks/model environments[25].

5 Parameterization

All models are parameterized in a uniform way (whenever possible). The common targets we use to parameterize all models are:

- Imports/GDP ratio of 12% (US data)
- 30% producer markups
- 30% work hours relative to time endowment
- Short-run elasticity of trade flows of 0.7 (except for the *industry aggregation* friction; see Appendix for details)

The parameters, such as $\beta, \sigma, \delta, \alpha$, are calibrated in a way standard to the literature. Productivity shock process is common across all models, and symmetric across countries. The persistence parameter is .91, volatility of measured TFP residuals is 0.00608, and the correlation between measured TFP residuals is .28. In the case of Corsetti & Dedola (2005) the process has been taken directly from Corsetti, Dedola & Leduc (2008). (See the Appendix for more details.)

We now turn to the description of the specifics of the calibration of each friction. The values of all parameters are summarized in Table 1.

Consumer Search We parameterize this friction by requiring that the following additional targets from the original paper are met:

[24] In the setup proposed by Ravn et al. (2007) additive demand shocks generate incomplete pass-through because they are critically combined with an additional habit formation imposed on the government consumption generating demand shocks. As a result, government expenditures creates an externality on lower markups in the entire economy. This effect flips the correlation of the real exchange rate with the shock—which in this case depreciates following a positive demand shock at home rather than appreciates.

[25] We do not report here the results from the additive formulation of the *deep habits* model, but we have studied this variation as well, and the same conclusions apply.

Table 1: Parameter Values in the Models.

Parameter	Baseline	Variation (model w/ asterisk)
Common parameters		
σ	2.0	
β	0.99	
α	0.36	
δ	0.025	
Consumer Search, Consumer Search*		
η	0.325	n.a.
ω	0.643	0.7562
θ	1.73	11.193
$\bar{z}$	n.a.	5.46
γ	2.0	1.76
q	0.148	0.1051
ϕ	n.a.	8.5
κ	n.a.	0.033
Deep Habits		
θ	-0.1	
ρ	0.85	
γ	0.675	
$\bar{l}$	10	
φ	3.48	
Costly Distribution, Costly Distribution*		
ω	0.6680	n.a.
θ	8.7	8.0
ξ	1.33	0.8
γ	1.63	1
δ	as above	.1 (annualized)
β	as above	.96 (annualized)
$\bar{z}^N$ (rel. prod. of NT-sector)	n.a.	0.5
Industry Aggregation		
n_D	4	
n_X	1	
ρ	8.7	
γ	1.52	
τ	0.1525	

- shopping to time spent working of 25% as reported by Alessandria (2009)
- coefficient of variation of the posted prices of 25%

Because our baseline specification of the friction differs from the original, we no longer target the share of search in time endowment of households as Alessandria does. An analog of this share in our model is the fraction of GDP that is trapped into the distribution sector. In our setup, it is endogenously implied by the model, and accounts for 60% of the total value added produced in the economy. Relative to the size of retail and distribution sectors (38% of total value added in non-service sector in the US data for year 2000), this is still a bit too large. However, business-to-business search frictions may not be confined to these sectors only—as producers in the data also search to purchase intermediate goods.

Consumer Search* The parameterization of this setup follows closely the original paper. All targets of the original paper are met, but the exact values of the parameters are slightly different because the value of the common parameters is different.

Costly Distribution We parameterize the friction by requiring that distribution costs constitute 50% share of 'non-tradable' inputs in retail prices (as implied by Burstein, Neves & Rebelo (2003)).

Costly Distribution* The model has been parameterized similarly to the original one. The baseline period is one year, and all parameters have been adjusted to this frequency. The stochastic process has been taken from Corsetti, Dedola & Leduc (2008). All targets of the original paper are met and all main parameters take the same value. However, unlike the original model, this version of the model does not include monetary shocks and sticky wages.

Industry Aggregation The most important parameters for pricing-to-market are: the number of firms chosen in the model, and the difference between the elasticities, γ and ρ. In terms of the number of firms, as pointed out by Atkeson and Burstein, the presence of large firms is crucial to generating pricing to market. In the original paper, there is a large number firms (40), but firm size heterogeneity implies that only a few large ones really matter for aggregate prices. In our specification of their friction, due to assumed representativeness, all firms need to be of equal size. Therefore, we set the total numbers of firms to a low number of $n = 4$, and have $n_X = 1$ firm that exports to be consistent with the ratio $(\frac{n_X}{n})$ taken from AB.[26]

To calibrate the value of elasticity parameters γ and ρ, we note that the difference between these elasticities maps onto the degree of pass-through generated by the model, and their weighted average determines the average level of producer markups. Consequently, we choose these numbers to match a coefficient of pass-through of exchange rates to export prices of 40% and producer markups of 30% (as in the original paper).

Deep Habits As for the habit parameters, θ and ρ, we chose them in consistency with Ravn et al. (2006).

6 Quantitative Analysis

6.1 Data

To compare prices in the models to the data, we focus here exclusively on the moments characterizing the dynamics of export prices. Our moments are motivated by the following decomposition of the export price movements:

$$p_x \equiv \frac{P_X}{P} = \underbrace{\frac{P_X}{P_D}}_{p_d^x} \times \underbrace{\frac{P_D}{P}}_{p_d}, \tag{18}$$

where P_X denotes the home currency based price of exported good (basket), P_D denotes a home currency based producer price of a comparable tradable good (basket) sold at home, and P is some measure of the overall price of aggregate consumption in the home country (we chose CPI for convenience).

This decomposition allows us to break down the movements of the export prices into the pure deviations from LOP p_d^x, and the residual relative price movements observed in the home market p_d. In most models, P_X and P_D will correspond to the price of the same commodity, and so p_d^x will be a good test of the model's capability to generate deviations from LOP. As we will see below, in the data p_d^x is highly volatile and highly positively correlated with the real exchange rate. Standard models will imply that this term is constant, and all our friction will move it in some way.

[26]It would be possible to incorporate several different firm sizes, but we do not think it would matter much for the result. We have not validated this claim.

Table 2: Moments summarizing deviations from LOP.

Statistic	Description
$\sigma(p_d^x)/\sigma(x)$	Relative magnitude of deviations from LOP
$\sigma(p_d)/\sigma(x)$	Relative volatility of the price of tradable goods at home
$\rho(p_d^x, x)$	Correlation of dev. from LOP w/ the real exchange rate
$\rho(p_d, x)$	Correlation of home prices w/ the real exchange rate

σ denotes the standard deviation of logged and HP filtered data; ρ denotes the correlation coefficient.

The particular moments of the data we will focus are listed in the table below.

6.1.1 Aggregate Evidence

To document the moments described above for the aggregate data, we use US data on producer export price index (EPI) from BEA to measure export prices, and a comparable index for producer prices of finished products excluding food and energy (PPI) to measure home prices of comparable tradable basket.[27] We have excluded food and energy because these components are likely to be volatile for reasons unrelated to business cycle fluctuations. To measure the price of aggregate consumption, we use CPI that excludes food and energy for consistency (the results the not depend on this). All our data is quarterly for the time period (1983-2010), and statistics are based on first logged and then HP filtered time series (smoothing 1600).

The table 3 summarizes all the results, which are additionally illustrated graphically in Figure 1. As we can see, Panel A of table 3 shows that aggregate real export price in the US is relatively volatile and highly positively correlated with the real exchange rate. On the basis of our decomposition, and the statistics included in panel B of the same table, we can conclude that most of the movements of the real export price movements are attributable to the deviations from LOP. **In fact, the deviations from LOP captured by the relative price p_d^x are at least 4 times more volatile than the residual p_d, and are almost the sole driver of the observed positive correlation between the aggregate real**

[27]The PPI data confounds both export prices and domestic prices, and is only an approximate measure of P_D. However, for our particular application this is sufficient, as it will only imply that our conclusions establish the lower bound for the underlying deviations from LOP implied by the aggregate data.

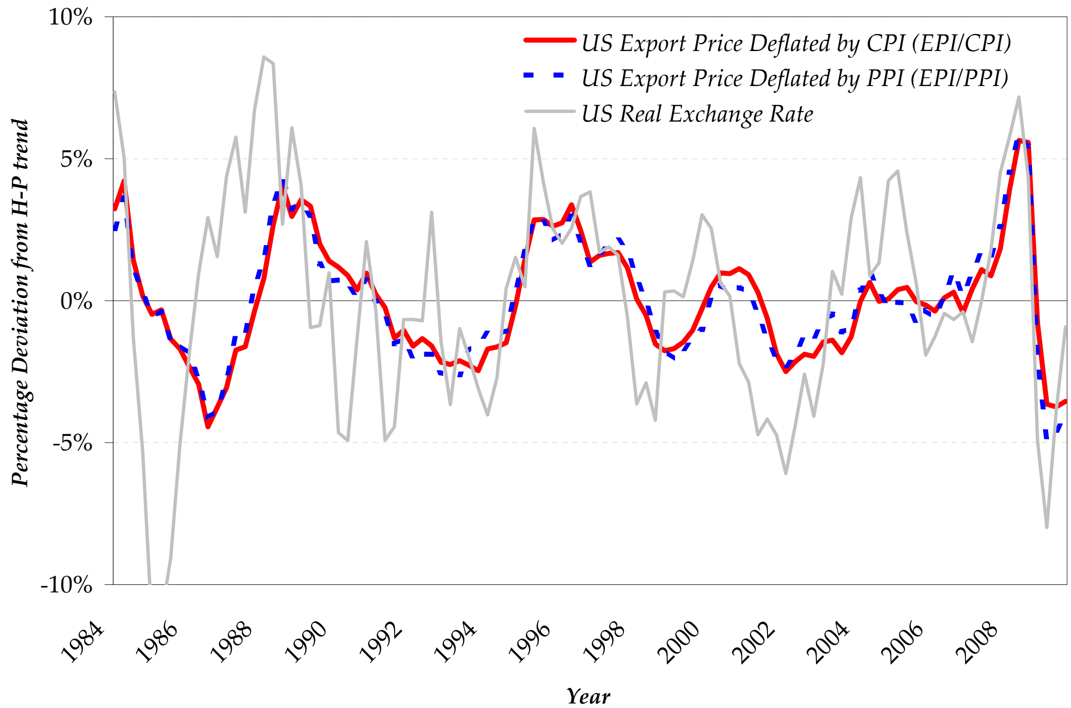

Figure 1: Dynamics of Aggregate Export Price in the US.

export price and the real exchange rate.

The above results are broadly consistent with the idea of a positive but incomplete pass-through of exchange rates to export prices.

6.1.2 Supporting Micro Evidence

To support our aggregate findings, here we offer an alternative look at the disaggregated data at a commodity level. The data-set we use comes from Bank of Japan and includes prices of 30 most heavily traded manufacturing commodities disaggregated at roughly 4 digit level[28]. The prices come from a producer level survey of the actual contractual agreements. The time period is 1995-2005.

The table 4 reports the results. **As we can see, the patterns we see on the individual commodity level are consistent with the aggregate evidence listed in the previous table.** Real export prices within the majority of commodity classifications turn out volatile and highly positively correlated with the Japanese real exchange rate. Moreover, just like in

[28] The examples of included commodity categories are: computers, ball bearing, agricultural tractors, silicon wafers etc...

Table 3: Deviations from LOP in Aggregate Data.

Statistic	Value
A. Properties of Aggregate Real Export Price	
$\sigma(p_x)/\sigma(x)$	.52
$\rho(p_x, x)$	.47
B. Deviations from LOP	
$\sigma(p_d^x)/\sigma(x)$	.53
$\rho(p_d^x, x)$	.51
C. Residual	
$\sigma(p_d)/\sigma(x)$	.13
$\rho(p_d, x)$	-.18

σ denotes the standard deviation of logged and HP filtered data, ρ denotes the correlation coefficient.

Table 4: Deviations from LOP in Disaggregated Data.

Statistics	Median Value	Quartile bracket $[Q_1, Q_3]$
A. Properties of disaggregated real export prices		
$\sigma(p_{x,i})/\sigma(x)$	.88	[.54, .99]
$\rho(p_{x,i}, x)$	.82	[.50, .89]
B. Deviations from LOP		
$\sigma(p_{d,i}^x)/\sigma(x)$	.90	[.55, .99]
$\rho(p_{d,i}^x, x)$	.84	[.67, .89]
C. Residual		
$\sigma(p_{d,i})/\sigma(x)$	.23	[.11, .33]
$\rho(p_{d,i}, x)$	-.14	[-.25, .07]

σ denotes the standard deviation of logged and HP filtered data, ρ denotes the correlation coefficient.

the case of aggregate data, the decomposition into a set of component statistics reveals that the movements of export prices are attributable to deviations from the law of one price. The relative price $p^r_{x,i}$ is highly volatile and positively correlated with the Japanese real (effective) exchange rate, whereas the home price of the same commodity classification is much less volatile and actually slightly negatively correlated with the real exchange rate.

We now turn to analyze the predictions of the theory.

6.2 Predictions of the Theory

Quantitative results are reported in Tables 5 and 6. As we can see from Table 5 (Row 4 of Panel B), all models generate some degree of pricing-to-market. Compared to US data, most statistics look well qualitatively, but the models typically fall short in generating enough incompleteness of pass-through (as pointed out in the sections above). Also, as pointed out in previous sections, the Ravn, Schmitt-Grohe & Uribe (2006) model generates pricing to market that gives rise to counterfactual correlations of the aggregate price indices (Panel A of Table 5).

Below, we briefly discuss each friction's quantitative predictions, and highlight the main predictions of each model.

Consumer Search The baseline model generates more pricing to market as compared to *costly distribution* friction. As discussed above, this is a robust feature of this friction and comes from the fact that in the case of *consumer search* the entire markup is denominated in the foreign unit, as opposed to only part of it in the case of *costly distribution*.

Interestingly, despite a low degree of theoretical pass-through predicted by the model, it implies large deviations from LOP as measured by p^x_d. At the same time, the real export price p_x is not volatile as measured relative to the real exchange rate. The reason for this behavior of prices is a relatively volatile home marginal cost v that is also strongly negatively correlated with the (counterfactually not too volatile) real exchange rate. Given the low overall volatility of x, this effect significantly boosts the volatility of cost-based real exchange rate affecting p^x_d relative to CPI-based real exchange rate affecting p_x. As we can see from column 4 of Table 5, this is not the case in for *consumer search**. This version of the model introduces an additional shock, and by matching volatility of the real exchange rate, aligns closely cost-based real exchange rate with the CPI-based real exchange rate.

Table 5: International Prices: Comovement and Relative Volatility[a]

		Predictions of the PTM Theories						
Statistic	Data[b]	Frictionless Model	Consumer Search	Consumer Search*	Costly Distribution	Costly Distribution*[d]	Industry Aggregation	Deep Habits
A. Correlations								
p_x, x	0.47	-1.00	1.00	0.82	1.00	0.52	0.98	-0.95
p_d^x, x	0.51	0.02	1.00	0.98	1.00	0.98	1.00	-0.83
p_d, x	-0.18	-1.00	-1.00	0.11	-1.00	0.27	-1.00	-0.97
B. Standard deviations								
x	3.97	0.45	0.55	3.67	0.54	1.86	0.31	0.51
relative to[c] x								
p_x	0.52	0.16	0.18	0.26	0.04	0.83	0.23	0.36
p_d^x	0.53	0.00	0.30	0.20	0.17	0.24	0.35	0.12
p_d	0.13	0.16	0.12	0.14	0.13	0.76	0.11	0.17
C. X-Rate Pass-through								
	35%-50%	0%	23%	18%	15%	20%	40%	n.a
D. Producer Markups								
	30%	0%	30%	22%	30%	40%	30%	30%

[a] All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter $\lambda = 1600$).
[b] Data for the US, 1984:1-2009:4.
[c] Ratio of corresponding standard deviation to the standard deviation of the real exchange rate x.
[d] The model has been calibrated to annual frequency and the statistics generated are not readily comparable to the ones listed in data column.

Table 6: Quantities - Comovement and Relative Volatility[a]

Statistic	Data[b]	Predictions of the PTM Theories						
		Frictionless Model	Consumer Search	Consumer Search*	Costly Distribution	Costly Distribution*[d]	Industry Aggregation	Deep Habits
A. Correlations								
domestic with foreign								
Measured TFP[c]	0.30	0.30	0.34	0.44	0.33	0.54	0.31	0.30
GDP	0.40	0.36	0.40	0.50	0.38	0.56	0.17	0.37
Consumption	0.25	0.33	0.30	0.61	0.32	0.54	0.73	0.40
Employment	0.21	0.49	0.52	0.43	0.50	0.55	0.05	0.55
Investment	0.23	0.19	0.24	0.56	0.23	0.31	0.32	0.26
GDP with								
Consumption	0.83	0.95	0.95	1.00	0.96	0.99	0.94	0.95
Employment	0.85	0.98	0.98	0.96	0.98	0.91	0.99	0.97
Investment	0.93	0.67	0.67	0.73	0.67	0.45	0.64	0.67
Net exports	-0.49	-0.57	-0.54	-0.49	-0.56	-0.87	0.58	-0.56
Terms of trade with								
Net exports	-0.17	-0.84	-0.86	-0.86	-0.86	-0.77	0.98	-0.93
B. Standard deviations								
GDP	1.33	1.13	1.18	2.45	1.15	2.05	1.55	1.08
relative to GDP[d]								
Measured TFP	0.60	0.70	0.67	0.32	0.69	0.89	0.51	0.74
Consumption	0.74	0.33	0.36	0.84	0.36	0.78	0.44	0.38
Investment	2.79	3.24	3.76	2.76	3.77	3.00	2.78	3.81
Employment	0.81	0.47	0.50	1.30	0.47	0.17	0.75	0.41
Net exports	0.30	0.14	0.13	0.04	0.13	0.11	0.21	0.13

[a]All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter $\lambda = 1600$).
[b]US data for the period 1980:1-2004:1.
[c]Calculated using the actual national accounting formulas; due to time varying markups measured TFP slightly differs from the TFP coefficient fed into the models.
[d]Ratio of corresponding standard deviation to the standard deviation of GDP.
[e]The model has been calibrated to annual frequency and the statistics generated are not readily comparable to the ones listed in data column.

The baseline model does exceptionally well in accounting for the quantity-related statistics. However, part this success is can be attributed to our particular specification of the friction that effectively makes a large share of GDP to be a non-tradable distribution/search sector. In our baseline specification more than half of the GDP is accounted for by the value added generated in the final good sector (search). Such large share would help all models to come closer to the data.

In contrast, the *consumer search** model, by treating shopping as home production and by adding an extra shock, is no longer neutral to quantities. The model results in excess output volatility and excess employment volatility. Moreover, we have also studied a specification of *consumer search* friction featuring *consumer search* with a CRRA utility function and only productivity shocks (no labor wedge shock), which we do not report here. This specification performed strictly worse than the frictionless model due to a negative international comovement of employment and counterfactually low international comovement of output. Our final conclusion is that the *consumer search* specification is not fully neutral to business cycle dynamics of quantities.

Costly Distribution, and Costly Distribution* The model results are similar to *consumer search* model, but due to a slightly different dependence of export prices on markups, the performance of the *costly distribution* model for prices is strictly worse.

Industry Aggregation While the model does well on prices by generating a high degree of exchange rate pass-through to export prices, quantity statistics look worse when compared to the standard model. This is not surprising due to a high setting of elasticity between home and foreign goods that is needed for pricing to market to arise. Specifically, the model implies a too low international comovement of GDP relative to TFP, and an excess international comovement of consumption. In addition, despite capital accumulation being present, it implies a positive correlation of net exports and output, versus a negative one in the data and in the standard model.

Finally, we also note that since pricing to market arises due to market share movements, once the model is required to match real exchange rate volatility from the data, it may be harder than in the case of other models to tame quantities by introducing additional features.

Deep Habits The prices look worse than in the standard model. Statistics for quantities look similar to the standard model, and in some dimensions even better.

7 Conclusions

In this paper, we provide the first unified study of several leading pricing to market frictions. We offer several conclusions as a guidance for future applied research on the topic. The key take away from our study is that the applicability of each particular friction crucially depends on the specifics of the particular application at hand.

Appendix

A1. National Accounts in the Model

Real GDP is real GDP in constant prices (≡steady state prices). Since the price of consumption/investment good is normalized to one, consumption and investment in period zero prices are c and i. Employment index is measured by $l_{i,t}$.

In deflation of the prices and measurement of real exchange rate, we have used the ideal CPI. This does not make any difference except for consumer search friction. In the case of this model, the transacted prices differ from the posted price (CPI measurement should be based on posted prices). However, since in the data such distinction does not make any difference for any of the patterns we focus on[29] we have decided to use the ideal CPI to abstract from any implications of the model coming from this channel.

A2. Estimation of the Productivity Shock Process

To construct the TFP residuals z from the data we follow a similar procedure to Heathcote & Perri (2004), and include physical capital. Physical capital has been constructed from the gross-fixed capital formation series using perpetual inventory method with exogenously assumed depreciation rate of $\delta = 0.025$. For the US we have used total hours worked, and for the rest of the world civil employment index instead. Given the quarterly data-set from 1980.1 to 2004.3 for the aggregate of main 15 European countries, Japan, Canada, Switzerland, and Australia, we have constructed the series of z from the following equation

$$\log(z) = \log(y) - 0.36 \log(k) - 0.64 \log(n),$$

where y denotes GDP in constant prices, and the coefficient 0.64 denotes the assumed share of labor income in GDP - consistent with the parameterization of the model and the values estimated for the developed countries. We linearly detrend the series for $\log(z)$, and estimate the parameters of the underlying productivity process. In the case of Corsetti & Dedola (2005) the process has been taken directly from Corsetti, Dedola & Leduc (2008)—which limits our analysis to annual frequency.

A1. Measurement of Short-run Price Elasticity of Trade

Short-run elasticity of trade flows measures how trade flows between countries respond to a relative price changes seen in the time-series. Here, we use the so called volatility ratio to assess the lower bound for this elasticity.

When the demand for domestic and foreign good is modeled by a CES aggregator of the form

$$G(d_t, f_t) = \left(\omega_t d_t^{\frac{\gamma-1}{\gamma}} + (1-\omega_t) f_t^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}},$$

the import ratio $\frac{f_t}{d_t}$ is intimately related to the relative price of domestic and imported goods $\frac{p_{d,t}}{p_{f,t}}$:

$$\log \frac{f_t}{d_t} = \gamma \log \frac{p_{d,t}}{p_{f,t}} + \log \frac{\omega_t}{1-\omega_t}. \tag{A1}$$

[29] I.e exactly analogous patterns can be documented by instead using PCE deflator or GDP deflator,

Thus, in the case of time-varying weights ω, the above approach gives the upper bound value for the value of this parameter:

$$\gamma = \sigma(\log \frac{f_t}{d_t})/\sigma(\log \frac{p_{d,t}}{p_{f,t}} + \frac{1}{\gamma}\log \frac{\omega_t}{1-\omega_t}) \leq \sigma(\log \frac{f_t}{d_t})/\sigma(\log \frac{p_{d,t}}{p_{f,t}}) = VR. \tag{A2}$$

Based on the median value for OECD countries, we this way obtain the upper bound on elasticity γ to be .7 on quarterly frequency. In models with frictions, VR does not map onto γ, and so we instead construct the analogous object in the model, and set γ so that the model implied VR is .7.

A1. Data Sources

Export prices and PPI data comes from BEA. Real exchange rate data comes from International Monetary Fund, International Financial Statistics Database, 2010. Prices for Japan come from Bank of Japan and have been compiled by the authors from flat files available online. To construct TFP residuals used in the estimation of the stochastic process, we have used nominal GDP data from World Development Indicators, World Bank, Gross Fixed Capital Formation, GDP in constant prices and Civil Employment from Source OECD.org, Quarterly National Accounts, series for physical capital have been constructed using the perpetual inventory method with a constant depreciation of 2.5%, and aggregate GDP for blocks of countries has been computed from growth rates of GDP in constant prices (recent years, varies by country) weighted by the nominal GDP of each country in 2000 (we applied the growth rates backwards). Statistics pertaining to quantities that appear in the paper have been calculated from the same data.

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