Standard errors of forecasts in dynamic simulation of nonlinear econometric models: some empirical results

Bianchi, Carlo and Calzolari, Giorgio

Centro Scientifico IBM, via S.Maria 67, 56100 Pisa, Italy

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STANDARD ERRORS OF FORECASTS IN DYNAMIC SIMULATION OF NONLINEAR ECONOMETRIC MODELS: SOME EMPIRICAL RESULTS

Carlo Bianchi and Giorgio Calzolari

Centro Scientifico IBM
Via S.Maria 67
56100 Pisa PI
Italy

In nonlinear econometric models, the evaluation of forecast errors is usually performed, completely or partially, by resorting to stochastic simulation. However, for evaluating the specific contribution of errors in estimated structural coefficients, several alternative methods have been proposed in the literature. Three of these methods will be compared empirically in this paper through experiments performed on a set of "real world" econometric models of small, medium and large size. This work extends to dynamic simulation of nonlinear econometric models, for which the authors have recently analysed the one-period (static) forecast errors empirically.

1. INTRODUCTION

Many economic forecasts are based on the solution or simulation of macroeconometric models, in the form of systems of simultaneous equations. Even if it is well known that these forecasts, as all others, involve uncertainty, estimates of the uncertainty are not usually presented.

Some papers have dealt with this problem in the last few years, proposing and experimenting with methods to evaluate forecast uncertainty in macroeconometric models and, in particular, evaluating the forecast error variance.

In a recent paper (Bianchi and Calzolari, 1982), the authors have performed some experiments on a set of small, medium and large size real world models, both linear and nonlinear, comparing the results and performances of three different methods:

- stochastic simulation and re-estimation, see Schink (1971);
- Monte Carlo on coefficients, see Fair (1980);

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- analytic simulation on coefficients, see Bianchi and Calzolari (1980).

The key statistic, to perform comparisons, was the variance of forecasts one-period ahead (usually, the first period beyond the sample estimation period); that is, for static forecasts. This paper aims at extending the empirical comparison of those methods to multiperiod dynamic simulations of some real world macroeconometric models. Its purpose is twofold. First, it will be shown that the risk of nonconvergence of the two Monte Carlo methods is not increased by dynamic simulation (it was shown by Bianchi and Calzolari, 1982, that there is a risk in practice). Then we show that the minor differences in the results produced sometimes, by the three methods in the static case, can be amplified by dynamic simulation; the empirical equivalence of the three methods being rarer for dynamic simulation.

The plan of the paper is the following. In section 2, the three methods for estimating the variance of forecasts in econometric models are briefly summarized. In section 3, some empirical evidence is provided on the convergence or nonconvergence of Monte Carlo methods for a small linear model; in particular, the differences from the static case are evidenced. Finally, in section 4, numerical results related to several nonlinear real world models are presented.

2. FORECAST ERRORS IN ECONOMETRIC MODELS

Let a structural econometric model, linear or nonlinear in the variables as well as in the coefficients, be represented as

$$(2.1) \qquad f(y_t, y_{t-1}, x_t, a) = u_t; \qquad t=1,2,\ldots,T$$

where:

$f' = (f_1, f_2, \ldots, f_m)$ is a vector of functional operators, continuously differentiable with respect to the elements of current and lagged y, x and a;

$y_t' = (y_{1t}, y_{2t}, \ldots, y_{mt})$, $x_t' = (x_{1t}, x_{2t}, \ldots, x_{nt})$ and y_{t-1} are the vectors of current endogenous, exogenous and lagged endogenous variables, respectively;

$a' = (a_1, a_2, \ldots, a_s)$ is the vector of the structural coefficients to be estimated (all the other known coefficients of the model are excluded from this vector and included in the functional operators);

$u_t' = (u_{1t}, u_{2t}, \ldots, u_{mt})$ is the vector of structural stochastic disturbances (or error terms) at time t, having zero mean and being independently and identically distributed over time, with finite contemporaneous covariance matrix, and independent of all the predetermined variables. In all the experiments described in this paper, the contemporaneous distribution of the error terms is assumed multivariate normal: $u_t \sim N(0,\Sigma)$.

It is usually assumed that a simultaneous equation system like (2.1) implicitly defines a single inverse relationship (reduced form) for relevant values of the coefficients, the predetermined variables, and any values of the disturbance terms:

$$(2.2) \qquad y_t = g(y_{t-1}, x_t, a, u_t),$$

g being, usually, an unknown vector of functional operators.

Let h be a time period not belonging to the sample estimation period 1,2,...,T, and let the model be used to forecast at times h+1, h+2, ..., h+r. Given the values of the endogenous variables at time h, y_h, and the values of the exogenous variables in the forecast periods, $x_{h+1}, x_{h+2}, \ldots, x_{h+r}$; then the values of the endogenous variables in the forecast periods can be obtained recursively as:

$$(2.3) \qquad \begin{aligned} y_{h+1} &= g(y_h, x_{h+1}, a, u_{h+1}); \\ y_{h+2} &= g(y_{h+1}, x_{h+2}, a, u_{h+2}) = g(g(y_h, x_{h+1}, a, u_{h+1}), x_{h+2}, a, u_{h+2}) \\ &= g_2(y_h, x_{h+1}, x_{h+2}, a, u_{h+1}, u_{h+2}); \\ &\vdots \\ y_{h+r} &= g(y_{h+r-1}, x_{h+r}, a, u_{h+r}) = g(g(\ldots), x_{h+r}, a, u_{h+r}) \\ &= g_r(y_h, x_{h+1}, \ldots, x_{h+r}, a, u_{h+1}, \ldots, u_{h+r}). \end{aligned}$$

Several sources of error can be identified at this point (see, for example, Klein and Young, 1980, p.81). First, if the model is incorrectly specified, equations (2.3) would not represent the time path for the endogenous variables. (Also, analysis of the forecast errors can be used to gauge misspecification, see Fair, 1980).

A major source of uncertainty are the unknown values of the exogenous variables in the forecast periods $x_{h+1}, x_{h+2}, \ldots, x_{h+r}$. If the model is used for ex-ante forecasting, that is h+1, h+2, ..., h+r are in the future, the exogenous variables must themselves be forecast, thus involving uncertainty. If ex-post forecasting is performed, some uncertainty still exists about the values of the exogenous variables (and of the initial values of the endogenous variables, y_h) especially if h is a sufficiently recent time period and preliminary data are used- but this problem is not considered here. The present analysis and comparison of methods are conditional on exact knowledge of the initial values of the endogenous variables and of the values of the exogenous variables in the forecast period.

At least two error sources still remain and, to help understand these, we summarise how model builders use their models to produce forecasts. The vector of structural coefficients, a, is unknown; modelers use data in the sample period, t=1,2,...,T, to get an estimated coefficients vector, $\hat{a}$, and an estimate of the contemporaneous covariance matrix for the structural error terms, $\hat{\Sigma}$. The model builder must then choose a starting point for the simulation experiment. Such a point (h) is usually the last time period for which "sure" information is available; in many cases it is h=T, the last point of the sample estimation period. For the purposes of this paper, it is simpler to start from a period h not belonging to 1,2,...,T (for example, h=T+1); when this will not be possible for lack of data, we shall choose h inside the sample period and a slight approximation will occur.

The model builder next introduces values for y_h and $x_{h+1}, x_{h+2}, \ldots, x_{h+r}$, assumed exact, sets the random error terms $u_{h+1}, u_{h+2}, \ldots, u_{h+r}$ to their expected value (zero) and solves simultaneously the dynamic system (2.1) at time h+1, h+2, ..., h+r. Using the reduced form notation (2.3), forecasts are obtained as

(2.4)
$$\hat{y}_{h+1} = g(y_h, x_{h+1}, \hat{a}, 0);$$
$$\hat{y}_{h+2} = g(\hat{y}_{h+1}, x_{h+2}, \hat{a}, 0) = g(g(y_h, x_{h+1}, \hat{a}, 0), x_{h+2}, \hat{a}, 0)$$
$$= g_2(y_h, x_{h+1}, x_{h+2}, \hat{a}, 0, 0);$$
$$\vdots$$
$$\hat{y}_{h+r} = g(\hat{y}_{h+r-1}, x_{h+r}, \hat{a}, 0) = g(g(\ldots), x_{h+r}, \hat{a}, 0)$$
$$= g_r(y_h, x_{h+1}, \ldots, x_{h+r}, \hat{a}, 0, \ldots, 0).$$

Forecasts $\hat{y}_{h+1}$, $\hat{y}_{h+2}$, ..., $\hat{y}_{h+r}$ differ from the values of the endogenous variables in the forecast period y_{h+1}, y_{h+2}, ..., y_{h+r}, because the estimated $\hat{a}$ is used instead of the unknown coefficients vector a, and due to the existence of the random error terms u_{h+1}, u_{h+2}, ..., u_{h+r}.

In order to assign forecasts an estimate of their degree of uncertainty, it would be useful to get, at least, an estimate of the first two moments of the forecast errors $\hat{y}_{h+1}-y_{h+1}$, $\hat{y}_{h+2}-y_{h+2}$, ..., $\hat{y}_{h+r}-y_{h+r}$. In most practical applications, however, the estimation of the first moment of the forecast errors does not appear particularly interesting, due to the mild nonlinearity of the econometric models used, which causes it to be very small, at least for static or short dynamic simulation periods (see, for example, Bianchi et al., 1976; Calzolari, 1979; Fair, 1980; Mariano and Brown, 1980).

Much more interesting, in practice, is the information which can be derived from an estimate of the second order moments; where standard errors allow one, in fact, to obtain confidence intervals for the single forecast endogenous variables, while an estimate of the covariance matrix allows the construction of joint confidence regions or the testing of hypotheses.

Three methods, to the authors' knowledge, have been proposed in the literature for the evaluation of the second order moments of forecast errors in nonlinear econometric models. Apart from the technical differences and the computational approximations which they involve, these three methods are conceptually equivalent in case of single equation models and for nonsimultaneous equation models. This equivalence does not extend to simultaneous equation systems (even linear). Experimental comparisons of results on real world models used for practical purposes aim at investigating to what extent the methods can still be considered as "practically" equivalent. The three methods can briefly be described as follows.

- Stochastic simulation and re-estimation.

With this method, a certain number of stochastic simulation runs over the sample period must be performed. In each run, T vectors (one for each period) of pseudo-random error terms, with zero mean and covariance matrix equal to the available $\hat{\Sigma}$, are inserted into the system (2.1) and the simultaneous solution of the system provides pseudo-random values of all the endogenous variables over the sample period.

Each set of pseudo-random values is used, like a new set of data, to re-estimate the model's vector of structural coefficients. Each vector is used for a new stochastic simulation run, this time not in the sample period, but in the forecast period, also introducing new pseudo-random error terms.

The values of the endogenous variables obtained as solutions of the model in the forecast period are affected either by errors in the estimated coefficients, which have been randomly generated by the stochastic simulation in the sample period and re-estimation, or by the structural error terms, which have been introduced by stochastic simulation in the forecast period. A convenient number of replications of this process allows one to compute a variance-covariance matrix of forecasts at times h+1, h+2, ..., h+r.

The method of stochastic simulation and re-estimation is often used to analyze the small sample behaviour of estimation methods, when the analitical investigation is difficult or impossible (see, for example, Hendry and Harrison, 1974; Mariano, 1980; Mikhail, 1972). Its use for investigating the forecast errors of nonlinear econometric models is proposed and described in detail in Schink (1971). This method can be applied with several variants, since re-estimation can be performed with different estimation procedures. Moreover, when the model is dynamic, at each re-estimation the lagged endogenous variables should be set at the same values of the corresponding stochastic simulation run. This means that, for each re-estimation procedure, the method can be applied with, at least, two different variants, since stochastic simulation over the sample period can be either static or dynamic. The experiments summarized in this paper have been performed with static stochastic simulation in the sample period. Some experiments have been repeated with dynamic simulation in the sample period and setting, in each re-estimation, the lagged endogenous variables at the values calculated in the simulation run. The results were, in most cases, quite close to those obtained by means of the static stochastic simulation and re-estimation, and are not presented here. The matter, however, would be worth a separate and detailed analysis; some experimental results can be found in Schink (1971).

- Monte Carlo on coefficients.

Instead of obtaining pseudo-random vectors of coefficients by means of a stochastic simulation and re-estimation process, this method gets them directly by sampling from the distribution of estimated coefficients. The small sample distribution of structural coefficients in a simultaneous equations system is usually very complicated, or even unknown. In several cases, like linear dynamic systems or nonlinear static systems, it can be proved that, under sufficiently wide conditions, the distribution of estimated coefficients is asymptotically multivariate normal. The experimenter, therefore, usually performs the generation of pseudo-random coefficients from a multinormal distribution (see, for example, Cooper and Fischer, 1974; Fair, 1980; Haitovsky and Wallace, 1972).

For this method, the covariance matrix of the structural coefficients is also required. An estimate of this matrix is standard output from system estimation methods, like three stage least squares or full information maximum likelihood; otherwise, if single equation methods are used, this matrix must be built block by block, as in Brundy and Jorgenson (1971,p.215) for limited information instrumental variables, or Theil (1971,p.500) for two stage least squares

estimates. Pseudo-random forecasts can then be produced by performing stochastic simulation of the forecast period. For each simulation run, a pseudo-random vector of coefficients must be generated from the multinormal distribution just mentioned, and r independent vectors of pseudo-random error terms (one for each period of forecast) must also be generated. Both error sources are, therefore, taken into account. As for the previous method, a convenient number of replications allows the computation of a variance-covariance matrix of forecasts at time h+1, h+2, ..., h+r.

- Analytic simulation on coefficients.

This method extends that proposed by Bianchi and Calzolari (1980), for static simulation of nonlinear models, to dynamic simulation. In turn, the earlier proposal extended the fully analytical methods developed, for linear models, in Goldberger et al. (1961), Dhrymes (1973) and Calzolari (1981). The case of dynamic simulation of linear models is treated in Schmidt (1974).

The random error terms have been assumed serially independent; therefore, assuming exact knowledge of all the predetermined variables (y_h, x_{h+1}, x_{h+2}, ..., x_{h+r}), the vector of coefficients, $\hat{a}$, which is obtained from an estimation procedure applied to the data of the sample period, is independent of the random error terms in the forecast period (which is outside the sample estimation period). We can now decompose the vector of forecast errors, in the generic forecast period h+k, as follows:

$$(2.5)\quad \hat{y}_{h+k} - y_{h+k} = g_k(y_h, x_{h+1}, \ldots, x_{h+k}, \hat{a}, 0, \ldots, 0) - g_k(y_h, x_{h+1}, \ldots, x_{h+k}, a, u_{h+1}, \ldots, u_{h+k})$$

$$= [g_k(y_h, x_{h+1}, \ldots, x_{h+k}, \hat{a}, 0, \ldots, 0) - g_k(y_h, x_{h+1}, \ldots, x_{h+k}, a, 0, \ldots, 0)] + [g_k(y_h, x_{h+1}, \ldots, x_{h+k}, a, 0, \ldots, 0) - g_k(y_h, x_{h+1}, \ldots, x_{h+k}, a, u_{h+1}, \ldots, u_{h+k})].$$

Having assumed exact knowledge of all the predetermined variables involved in our forecast, the two components of the forecast error vector are independent, since the former depends on $\hat{a}$, while the latter depends on u_{h+1}, ..., u_{h+k}. We can, therefore, calculate the variances or the covariance matrices of the two components separately, and sum them to get the final results.

The second component, which is due to the random error terms u_{h+1}, ..., u_{h+k}, can be computed as in the previous methods, by means of replicated stochastic simulations of the forecast period.

As far as the first component is concerned, its covariance matrix can be computed by means of a linear approximation which is, in many cases, asymptotically exact. If we assume that, as T (sample period length) increases, asymptotically

$$(2.6)\quad \sqrt{T}\,(\hat{a} - a) \sim N(0, \Psi),$$

and define G_{h+k} as the (m × s) matrix of first order partial derivatives of the vector of functions g_k with respect to the elements of a, computed at the point (y_h, x_{h+1}, ..., x_{h+k}, a, 0, ..., 0), then, asymptotically,

$$(2.7)\quad \sqrt{T}\,[g_k(y_h, x_{h+1}, \ldots, x_{h+k}, \hat{a}, 0, \ldots, 0) - g_k(y_h, x_{h+1}, \ldots, x_{h+k}, a, 0, \ldots, 0)] \sim N(0, G_{h+k}\Psi G'_{h+k})$$

An estimated covariance matrix of a multinormal distribution, which approximates the small sample distribution of the first component of the forecast errors, is obtained by calculating G_{h+k} at the point (y_h, x_{h+1}, ..., x_{h+k}, $\hat{a}$, 0, ..., 0), replacing Ψ with the available estimate $\hat{\Psi}$, and dividing $\hat{G}_{h+k}\hat{\Psi}\hat{G}'_{h+k}$ by the actual length of the sample period, T (see, for example, Schmidt, 1976, p.254). This approximation is asymptotically exact if the functions of the vector g_k are continuously differentiable (see Rao, 1973, p.388) and if the estimated structural coefficients are consistent and asymptotically normally distributed. The first condition is not usually restrictive for econometric models, while the second can be proved, with only weak restrictions, for linear dynamic models (see, for example, Schmidt, 1976) or for nonlinear static models. For the general case of nonlinear dynamic systems, formal proofs are not available, but the heuristic argumentations in Gallant (1977, pp.73-74) suggest that, even in the general case, it is reasonable to suppose that such a condition is satisfied; otherwise, the procedure should be considered approximate, not only for small samples, but even in the large sample case.

The computation of the first order derivatives has been performed, in all the experiments of this paper, by means of numerical differentiation, using finite differences between a control solution and disturbed solutions (with increments on the structural coefficients) in the forecast period.

3. ARE THE THREE METHODS EQUIVALENT?

First of all, it must be noticed that all three methods, just described, resort to stochastic simulation in the forecast period in order to evaluate the contribution of the error terms to the variance of forecasts. Alternatively, some mixed technique could be used; one of these techniques, based on the use of control variates (obtained from local linearizations of the model) is described in detail in Calzolari and Sterbenz (1982). These techniques, however, only aim at making convergence of the sample variances faster than for straightforward stochastic simulation, but they produce the same final results as stochastic simulation of the forecast period; furthermore, these techniques can be inserted into any of the three methods discussed in the previous section, without changing the final results of the computation. We can say, therefore, that the three methods do not differ from one-another in the treatement of the random error terms.

To investigate equivalence or nonequivalence of the three methods, therefore, we must analyze the different treatment given to the errors in the estimated coefficients.

For nonsimultaneous equation systems, the three methods are perfectly equivalent. The simple case of a linear regression model might help to clarify this point. Let the model be represented as

$$(3.1)\quad y_t = bx_t + u_t$$

and let $\hat{b}$ be the least squares estimate of b; furthermore, let $\hat{\sigma}^2$ be the estimated variance of u_t and $\hat{\sigma}^2/\Sigma x_t^2$ be the estimated variance of $\hat{b}$.

The analytic simulation method, applied to this model at time h (not belonging to the sample estimation period) produces an estimate of the forecast variance

$$(3.2) \qquad \hat{V}(\hat{y}_h) = x_h^2 \, \hat{\sigma}^2/\Sigma x_t^2 + \hat{\sigma}^2$$

which is the well known expression for regression models (see, for example, Klein, 1974, p.261).

If random coefficients are generated with mean equal to the available $\hat{b}$ (kept fixed throughout the experiment) and variance equal to the available $\hat{\sigma}^2/\Sigma x_t^2$, then the sample variance of a sufficiently large number of replicated solutions of the equation would be a value close to the first term on the right hand side of the equation. If, in addition, independent random error terms are introduced, the sample variance would converge to the sum of the two terms of equation (3.2).

We can now use equation (3.1), with estimated coefficient, to generate random values of y_t over the sample period, by introducing pseudo-random values of u_t (with zero mean and variance equal to the available $\hat{\sigma}^2$), and re-estimate each time a new (random) value of the coefficient. Since, in this case, the ordinary least squares estimation method is unbiased, after a convenient number of replications the sample mean of the re-estimated coefficients would converge to the original $\hat{b}$, and its sample variance would converge to $\hat{\sigma}^2/\Sigma x_t^2$. The procedure, therefore, becomes perfectly equivalent to the previous one.

However, as soon as simultaneity is introduced, the theoretical equivalence of the three methods ceases to hold. A simple two equations linear model can be used to explain why. The model consists of a consumption function (stochastic) and a national income identity; investment is an exogenous variable.

$$(3.3) \qquad \begin{cases} C_t = bY_t + u_t \\ Y_t = C_t + I_t \, . \end{cases}$$

The reduced form of this model is

$$(3.4) \qquad \begin{cases} C_t = b/(1-b)I_t + 1/(1-b)u_t \\ Y_t = 1/(1-b)I_t + 1/(1-b)u_t \, . \end{cases}$$

Let us now suppose b is estimated by means of some suitable estimation method, and let $\hat{\sigma}_b$ be the standard error of such an estimate. Let us now apply the Monte Carlo on coefficients procedure, generating random values of the coefficient from a normal distribution with mean and standard deviation equal to the available estimates. If we now look at the reduced form equations (3.4), it is clear that, since normally distributed values of b (and hence of 1-b) appear at the denominator, no term has finite moments (not even a first moment). This implies that, no matter how many replications, the sample means and variances of the forecasts should fluctuate abnormally as the number of replications increases, and would not be expected to converge.

It must be noticed, at this point, that the nonexistence of finite moments is due to the denominator (1-b), which can assume values close to zero with nonzero probability. If such a probability is sufficiently large, surely the phenomenon will be relevant in practice and we will observe that our Monte Carlo does not converge. However, this probability could be so small (for example if $1-\hat{b}$ is greater than 20 times $\hat{\sigma}_b$) that no value of (1-b) dangerously close to zero would be generated, even in millions of replications; in this case the risk would be purely theoretical, since any Monte Carlo experiment, with the currently available generators of random numbers, would always be successful. It is clear at this point that the risk of nonconvergence in any model, linear or nonlinear, can be actual or purely theoretical depending on the values of the coefficients and of their covariance matrix. It was shown in Bianchi and Calzolari (1982) that the risk was, in fact, actual in many real world models.

A quite different problem arises if b has a truncated distribution, such that (1-b) cannot assume zero value, but can assume values quite close to zero with high probability. In this case, the distribution of 1/(1-b) has finite moments, and the Monte Carlo converges after a sufficiently large number of replications. However, when computing the moments of 1/(1-b), the values of (1-b) close to zero dominate the final results. It might, therefore, happen that two distributions of (1-b) which differ substantially in the region close to zero, even though they may have the same mean and variance, lead to distributions of 1/(1-b) with quite different moments.

No risk of nonconvergence exists, for this particular model, if we apply a stochastic simulation and re-estimation procedure with a suitable estimation method. Let us suppose we know b and σ^2 with certainty and perform several stochastic simulation runs over the sample period, by introducing into system (3.3) values of u_t with normal distribution, zero mean and variance σ^2. Let $v_t = 1/(1-b)u_t$ and $d=b/(1-b)$. We can write the first reduced form equation in the form

$$(3.5) \qquad C_t = dI_t + v_t \, .$$

Since the equation is exactly identified, indirect least squares can be a suitable estimation method here (see Dhrymes, 1970, pp.279-289). For each stochastic simulation run, we use the calculated values of C_t to estimate, by ordinary least squares, a value of d; then a value of b can unequivocally be derived as $b=d/(1+d)$. The problem is clearly reversed with respect to the Monte Carlo on coefficients method. The random values of d have a normal distribution (therefore with finite variance), since equation (3.5) is estimated by ordinary least squares and v_t has, itself, a normal distribution. On the contrary, the random values of the structural form coefficient, b, have a distribution without finite moments, but this is not relevant for forecasting.

As we have estimated the reduced form directly by ordinary least squares, the sample variance of our random forecasts for C_h, after a large number of

replications, would converge to

$$(3.6) \qquad V(\hat{C}_n) = I_h^2 V(\hat{d}) + V(v_t) = I_h^2 \, \sigma^2/(1-b)^2 \, . \, 1/\Sigma I_t^2 + \sigma^2/(1-b)^2 .$$

Of course, b and σ^2 should be replaced by the available estimates $\hat{b}$ and $\hat{\sigma}^2$.

If we apply the analytic simulation method, since the estimated asymptotic variance of $\hat{b}$, estimated by indirect least squares, is $\hat{\sigma}^2(1-\hat{b})^2 / \Sigma I_t^2$, we would get for the forecast variance the same as equation (3.6). In this case, therefore, two of the three methods are equivalent, but one is not. It would be possible to make it equivalent to the others by performing the random sampling of coefficients from an appropriate distribution, different from the normal. In this way the problem could be solved for this particular example, but not in general, since the distribution should be "ad hoc" for any model and its derivation would usually involve overwhelming difficulties.

Let us now consider the general case of a static linear model,

$$(3.7) \qquad Ay_t + Bx_t = u_t ,$$

where A and B are matrices of structural coefficients (A is square and nonsingular), some of which are fixed a-priori (zero and one restrictions, for example), while others must be estimated. Defining $\Pi_1 = -A^{-1}B$ and $v_t = A^{-1}u_t$, the restricted reduced form can be represented as

$$(3.8) \qquad y_t = \Pi_1 x_t + v_t .$$

If $\hat{A}$, $\hat{B}$ and $\hat{\Pi}_1 = -\hat{A}^{-1}\hat{B}$ ($\hat{A}$ nonsingular) are the available estimates of the coefficients, the model can be used to forecast at time h+1, h+2, ..., h+r. For example, at time h+r, it will be

$$(3.9) \qquad \hat{y}_{h+r} = \hat{\Pi}_1 x_{h+r} .$$

The vector of forecast errors at time h+r is

$$(3.10) \qquad \hat{y}_{h+r} - y_{h+r} = (\hat{\Pi}_1 - \Pi_1) x_{h+r} - v_{h+r} .$$

Equation (3.10) shows the strict relation between forecast errors and errors in estimated coefficients of the resticted reduced form. When performing stochastic simulation and re-estimation, we reproduce, experimentally, the small sample distribution of either the structural form or the reduced form coefficients. When calculating the sample variances of the random forecasts, we could be sure that these variances converge only if the distribution of $\hat{\Pi}_1$ has finite moments, at least to the second order.

The existence of finite moments of reduced form coefficients is not a general property of simultaneous equations systems, but it strictly depends on the model and on the estimation method which is adopted. Detailed proofs can be found, for example, in Mariano (1980), McCarthy (1972), or Sargan (1976), but a simple intuitive explanation is given in McCarthy (1981). Some estimation methods, like full information maximum likelihood (or like indirect least squares, when it can be applied as in the two equations model) directly estimate $\hat{\Pi}_1$; they make use of any overidentifying restrictions of the structural form, but do not need inversion of the matrix $\hat{A}$. However, for other estimation methods, like two or three stage least squares, the reduced form coefficients are derived from the structural form coefficients through the inversion of the matrix $\hat{A}$, and "the model builder has no control on the determinant" of such a matrix. The determinant of $\hat{A}$ (appearing in the denominator of each element of the inverse) "can take a zero value with positive probability density associated with all points in the neighborhood of zero". This is quite similar to what happens when Monte Carlo on coefficients method is applied to the two equations model (and, in fact, the denominator $1-\hat{b}$ is the determinant of the matrix $\hat{A}$ in such a model). We see, in this way, that in the general case not only Monte Carlo on coefficients, but also stochastic simulation and re-estimation method is not equivalent to the analytic simulation method; the latter, in fact, calculates the covariance matrix of a distribution which asymptotically approximates the small sample distribution of our reduced form coefficients or forecasts; since this asymptotic distribution is normal, it always has finite moments.

Empirical evidence with a linear model based on real world data might help to clarify the above. The model is the well known model of the U.S. economy 1921-1941, described in Klein (1950), usually referred to as the "Klein-I model". It consists of 3 stochastic equations plus 3 identities; it includes 4 exogenous variables and 3 lagged endogenous variables.

The structural form of the model is:

$$(3.11) \qquad \begin{cases} C_t = a_1 + a_2 P_t + a_3 P_{t-1} + a_4 (W1+W2)_t + u_{1t} \\ I_t = a_5 + a_6 P_t + a_7 P_{t-1} + a_8 K_{t-1} + u_{2t} \\ W1_t = a_9 + a_{10}(Y+T-W2)_t + a_{11}(P+W1+T)_{t-1} + a_{12}t + u_{3t} \\ Y_t = C_t + I_t + G_t - T_t \\ P_t = Y_t - W1_t - W2_t \\ K_t = K_{t-1} + I_t \end{cases}$$

where the 6 endogenous variables are: C= consumption; I= net investment; W1= private wage bill; Y= national income; P= profits; K= end-of-year capital stock.

By substitution into the three stochastic equations, the three identities can be eliminated, obtaining:

$$(3.12) \qquad \begin{cases} C_t - a_1 - a_2 (C+I-W1-W2-T+G)_t - a_3 P_{t-1} - a_4 (W1+W2)_t = u_{1t} \\ I_t - a_5 - a_6 (C+I-W1-W2-T+G)_t - a_7 P_{t-1} - a_8 K_{t-1} = u_{2t} \\ W1_t - a_9 - a_{10}(C+I-W2+G)_t - a_{11}(P+W1+T)_{t-1} - a_{12}t = u_{3t} . \end{cases}$$

In the notation of equation (3.7), the matrix A is

$$(3.13) \qquad A = \begin{bmatrix} 1-a_2 & -a_2 & a_2-a_4 \\ -a_6 & 1-a_6 & a_6 \\ -a_{10} & -a_{10} & 1 \end{bmatrix}$$

The determinant of the matrix A is

$$(3.14) \qquad D = 1. - a_2 - a_6 + a_2 . a_{10} - a_4 . a_{10} + a_6 . a_{10}.$$

We can now assign, as initial values, to the coefficients, to their covariance matrix and to the covariance matrix of the error terms, the numerical values obtained by full information maximum likelihood estimates (FIML; most of these values can be found in Hausman, 1974). If we apply the Monte Carlo on coefficients method, with a sufficiently large number of replications, and calculate each time the determinant of the randomly generated matrix A, we get an experimental distribution of such a determinant as in the corresponding curve of Figure 1.

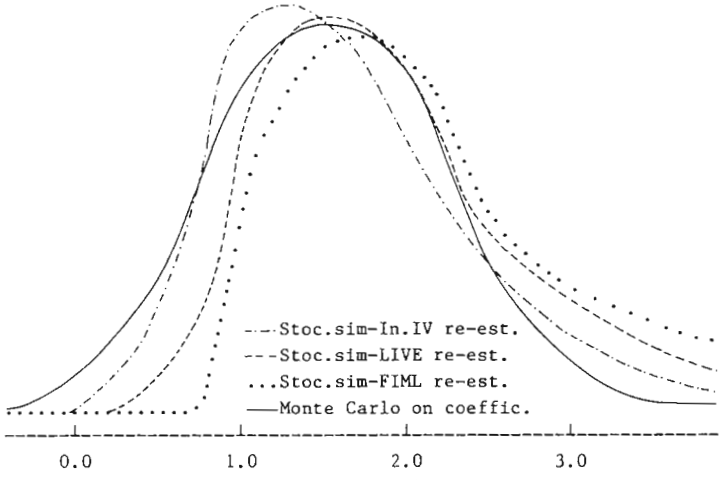

Fig. 1

Klein-I model. Initial estimates: FIML, 1921-1941.
Probability density of the determinant of matrix A

If we apply stochastic simulation over the sample period, re-estimating each time by full information maximum likelihood, we get, for the determinant of the matrix A, a probability density function as in the dotted line of Figure 1. We can now repeat the same experiment using different methods for re-estimating coefficients at each replication. We have done it twice, using two versions of the limited information instrumental variables method by Brundy and Jorgenson (1971, 1974): the efficient version, where an ordinary least squares estimation is followed by two iterations of the instrumental variables estimation (LIVE); and the inefficient (but still consistent) version, where the instrumental variables estimation is performed just once (In.IV).

The different behaviour of the methods is evidenced by the figure. All methods (including analytic simulation) lead to quite similar values for the first and second moments of the determinant of A (the mean is approximately 1.6, while the standard deviation is approximately 0.6). However, for stochastic simulation and re-estimation with FIML, there is no risk of generating random coefficients such that the matrix A is nearly singular. If re-estimation is performed with the efficient limited information instrumental variables method (LIVE), still there is no risk (some risk was evidenced in Bianchi and Calzolari, 1982, for this method applied to a shorter estimation period, 1921-1939). If re-estimation is performed with the inefficient instrumental variables method, the risk exists, even if small. For Monte Carlo on coefficients method, the risk is much higher.

Table 1

Klein-I model. Initial estimates: FIML 1921-1941
One-period forecasts and standard errors at 1948

	Obsrv.	Forec.	An.sim.	FIML	LIVE	In.IV	M.C.
C	82.8	76.5	2.7	2.6	3.0	5.7	9.5
I	6.4	6.3	2.2	2.2	2.3	2.6	7.2
W1	60.7	58.2	2.7	2.6	2.9	6.5	6.3
Y	97.4	91.0	4.7	4.7	4.9	7.6	17.
P	27.9	24.1	2.5	2.5	2.5	2.6	10.
K	204.	204.	2.2	2.2	2.3	2.6	7.2

Standard errors of forecasts produced by these four Monte Carlo experiments (1000 replications) and by analytic simulation, are displayed in Table 1. They are related to one-period forecasts at 1948; values of the predetermined variables have been taken from Goldberger et al. (1961). If we follow the way in which results of the Monte Carlo experiments change as the number of replications increases, we would observe large and frequent fluctuations of results in the case of Monte Carlo on coefficients, smaller and rarer (but still enough to inhibit convergence) fluctuations in case of inefficient instrumental variables re-estimation. On the contrary, results of stochastic simulation with LIVE or FIML re-estimation converge, and their results are quite similar to those obtained by analytic simulation.

We have concentrated our discussion mainly on the problem of a possible zero value of the determinant of matrix A, which causes the nonexistence of finite moments for our forecasts and, therefore, nonconvergence of the Monte Carlo experiments. The reason for stressing this point is that, in the large set of experiments on one-period forecasts performed in Bianchi and Calzolari (1982), an approximate equivalence of results was found in all cases in which Monte Carlo converged; in other words, no Monte Carlo experiment ever converged to values largely different from those produced by analytic simulation. The only cases in which large differences occurred were those cases in which Monte Carlo did not converge anywhere. It seemed, therefore, that the nonconvergence of Monte Carlo, due to some determinant close to zero (or for some other reason, presumably similar, but not so explicit for nonlinear models), could be the only source of large differences in the results produced by the three methods for models actually used for forecasting purposes. This, however, is not true for some models, when we

pass from one-period (static) forecasts to forecasts produced by multiperiod dynamic simulations. Then, even if the matrix which must be inverted is the same as in the static case, the dynamic simulation mechanism increases the risk of generating values of the denominator close to zero, as will soon be clear. Let

$$(3.15) \qquad Ay_t + Bx_t + Cy_{t-1} = u_t$$

be the structural form of the model. Defining $\Pi_0 = -A^{-1}C$, the reduced form of the model is

$$(3.16) \qquad y_t = \Pi_0 y_{t-1} + \Pi_1 x_t + v_t .$$

The solution of the model at time h+r, conditional on the value of y at time h, assumed known, is

$$(3.17) \qquad y_{h+r} = \Pi_0^r y_h + \sum_{k=1}^{r} \Pi_0^{r-k} \Pi_1 x_{h+k} + \sum_{k=1}^{r} \Pi_0^{r-k} v_{h+k} .$$

Using the model with estimated coefficients to forecast at times h+1, h+2, ..., h+r, the forecast error at time h+r is

$$(3.18) \qquad \hat{y}_{h+r} - y_{h+r} = (\hat{\Pi}_0^r - \Pi_0^r) y_h + \sum_{k=1}^{r} (\hat{\Pi}_0^{r-k} \hat{\Pi}_1 - \Pi_0^{r-k} \Pi_1) x_{h+k} - \sum_{k=1}^{r} \Pi_0^{r-k} v_{h+k} .$$

The matrix A which must be inverted to compute Π_0 and Π_1 is still the same as in the static case. Therefore, the risk of nonconvergence of Monte Carlo should be the same as in the static case. However, in a long dynamic simulation run the behaviour of the model crucially depends on the powers of Π_0 and, therefore, on the powers of A^{-1}. Values of the determinant of A not too close to zero, but smaller than one, may become sufficiently close to zero as soon as they are raised to a moderate power, as in dynamic simulation over a few periods; the probability density of the determinant raised to a power is higher, near zero, than for the determinant raised to one. In this case, Monte Carlo experiments would still converge after a sufficient number of replications, but the results would crucially depend on the form of the distribution of the determinant near zero, and not just on the first two moments of the determinant itself. The slight differences between the probability densities of the determinant, related to LIVE and FIML re-estimation, in the region between zero and one, are enough to cause sufficiently large differences in the dynamic simulation results.

In a set of experiments of dynamic simulation forecasts from 1931 to 1941 (unfortunately inside the sample period, due to lack of data) we always got convergence of stochastic simulation, with LIVE and FIML re-estimation, over the entire forecast period. (Of course, experiments with Monte Carlo on coefficients and with the other stochastic simulation and re-estimation method did not converge in any year of the forecast period, as they did not converge in the static case).

Results related to the first and last of the 11 years of forecasts (1931 and 1941) are displayed in Table 2, and empirically support the above discussion. In the first year of the forecast period, when simulation is still static, results

Table 2

Klein-I model. Initial estimates: FIML 1921-1941
Dynamic simulation forecasts 1931-1941.
Forecasts and standard errors at 1931.

	Obsrv.	Forec.	An.sim.	FIML	LIVE	In.IV	M.C.
C	50.9	54.5	2.4	2.3	2.5	6.5	9.6
I	-3.4	-.557	2.0	2.0	2.4	2.4	7.8
W1	34.5	37.2	2.3	2.3	2.5	8.5	6.8
Y	50.7	57.2	4.3	4.3	4.8	9.1	18.
P	11.4	15.2	2.3	2.2	2.6	2.7	10.
K	213.	216.	2.0	2.0	2.4	2.4	7.8

Forecasts and standard errors at 1941
(... indicate values greater than 1000).

	Obsrv.	Forec.	An.sim.	FIML	LIVE	In.IV	M.C.
C	69.7	63.5	3.8	3.8	6.0	...	...
I	4.9	-1.58	2.6	2.6	9.7	...	...
W1	53.3	45.4	3.6	3.6	5.8	...	...
Y	85.3	72.7	6.2	6.2	15.	...	...
P	23.5	18.7	3.0	2.9	9.9	...	...
K	209.	208.	10.	9.7	17.	...	...

obtained by stochastic simulation and re-estimation with FIML and LIVE methods are similar to each other; however, after 11 periods of dynamic simulation, results from the two methods are rather different, but we observe that the results related to FIML re-estimation are still sufficiently close to those produced by analytic simulation.

4. RESULTS ON NONLINEAR REAL WORLD MODELS

The set of experiments on the Klein-I model has been designed to evidence the possible situations which may arise when estimating the variance of forecasts in a linear model. When estimating the variance of forecasts in the dynamic simulation of a large nonlinear model, we cannot have a clear analytical insight of the mechanisms which cause similarity or differences in results obtained with the three methods. It seems, however, reasonable to believe that the mechanisms are similar to those which act in the linear case.

What we want to show, in this section, is that, even for medium or large scale nonlinear models, used in practice for forecasting purposes, we can encounter cases in which:

- all Monte Carlo methods converge and their results are close to each other and to analytic simulation results;

- some methods do not converge, while other methods converge to results similar to those produced by analytic simulation;

- some or all methods converge, but their results are close to those produced by analytic simulation in the first period of forecast, but diverge from each other and from analytic simulation after a few periods.

For computational simplicity, re-estimation has always been performed by means of the inefficient or efficient limited information instrumental variables methods, indicated as In.IV or LIVE, respectively. It must, however, be recalled that LIVE is efficient, in the class of limited information estimators, only when the model is linear. When the model is nonlinear, as discussed in Amemiya (1977), instrumental variables estimation methods can be inefficient, whatever number of iterations is performed; nevertheless, we have maintained the name LIVE even in case of nonlinear models, since the estimation procedure is exactly the same as in the linear case.

- ISPE model of the Italian economy

Table 3

ISPE model of Italian economy. Dynamic simulation forecasts 1960-1977. Initial estimates: 2SLS with principal components. Standard errors of forecasts

	at 1960 (percentage);				at 1977 (percentage).			
	An.sim.	LIVE	In.IV	M.C.	An.sim.	LIVE	In.IV	M.C.
CPNCF	1.7	1.7	1.7	1.8	5.8	6.7	6.7	8.2
DXML	3.5	3.2	3.2	3.5	6.7	7.6	7.6	9.9
IFIT	10.	10.	10.	10.	11.	12.	13.	17.
LI	2.3	2.3	2.3	2.3	3.4	3.9	4.0	4.7
MT	6.0	5.9	5.9	6.0	13.	15.	15.	18.
PCL	1.7	1.6	1.6	1.7	10.	11.	12.	15.
VAP	1.9	1.9	1.9	2.0	3.7	4.4	4.5	5.9
XT	3.5	3.2	3.2	3.5	8.8	10.	10.	13.

The results displayed in Table 3 are related to the annual model of the Italian economy developed by ISPE (Istituto di Studi per la Programmazione Economica, Roma). The model, described in Sartori (1978) and Bianchi et al. (1982), consists of 19 stochastic plus 15 definitional equations; there are 75 estimated coefficients. The initial estimates used for the experiments have been obtained by means of two stage least squares with principal components; the choice of the principal components and of the predetermined variables to be used in the first stage has been performed according to the method 4 described in Kloek and Mennes (1960). For lack of data, the forecast period (1960-1977) is not external to the sample estimation period (1955-1976).

The results in Table 3 are related to 8 of the main endogenous variables of the model. The variables are: CPNCF= private consumption; DXML= price deflator for exports; IFIT= private investment; LI= employees in industrial sector; MT= imports of goods and services; PCL= price deflator of private consumption; VAP= gross output of private sector; XT= exports of goods and services. The standard errors are displayed as percentages of the forecast value of each endogenous variable. It is clear from Table 3 that, for this model, most methods remain approximately equivalent both for static simulation (1960) and after a sufficiently long dynamic simulation period; only Monte Carlo on coefficients converges, in dynamic simulation, to values slightly larger than those produced by the other methods.

- IBM model of the United Kingdom

Table 4

IBM model of the United Kingdom. Dynamic simulation forecasts 1976/I-1978/IV. Initial estimates: iterative instrumental variables. Standard errors of forecasts

	at 1976/I (percentage);				at 1978/IV (percentage).			
	An.sim.	LIVE	In.IV	M.C.	An.sim.	LIVE	In.IV	M.C.
BI	3.2	3.2	3.2	3.2	8.3	14.	19.	28.
CPI	1.4	1.3	1.4	1.5	9.6	11.	11.	66.
EM	0.4	0.4	0.4	0.4	1.4	2.8	3.4	2.5
GNP	1.3	1.3	1.3	1.3	2.7	3.8	4.6	10.
GNPC	1.5	1.5	1.5	1.7	12.	15.	15.	133.
IM	2.7	2.7	2.7	2.7	4.6	6.0	6.8	15.
XIP	1.9	2.1	2.2	1.9	4.6	10.	18.	16.

The results displayed in Table 4 are related to the macroeconomic model of the United Kingdom developed by the IBM Economics Department. It is a quarterly model, with 120 equations, 21 of which are stochastic, and with 32 exogenous variables; there are 68 structural estimated coefficients. The start of the sample period varies from 1956/II to 1969/I but always ends at 1975/IV. The initial estimates used for the experiments have been obtained by means of iterative instrumental variables (obtained by iterating LIVE method till convergence is reached).

The results in Table 4 are related to 7 of the main endogenous variables of the model; standard errors are displayed as percentages of the value forecast for each variable. The forecast period is external to the sample estimation period: from 1976/I to 1978/IV. The variables are: BI= private fixed investment; CPI= consumer price index; EM= private sector employees; GNP= gross national product; GNPC= GNP in current prices; IM= imports; XIP= index of industrial production.

It is clear from Table 4 that, while for static simulation (1976/I) all methods are approximately equivalent, after some periods of dynamic simulation Monte Carlo on coefficients method and stochastic simulation and re-estimation with inefficient instrumental variables converge to results which are rather different

from each other and from the results produced by the other two methods. Smaller differences can be observed between results produced by analytic simulation and stochastic simulation with LIVE re-estimation.

- Bonn Forecasting System No.10 (real sector)

Table 5

Bonn Forecasting System No.10 (real sector).
Dynamic simulation forecasts 1970-1977.
Initial estimates: LIVE. Standard errors of forecasts

at 1970 (percentage); at 1977 (percentage).

	An.sim.	LIVE	In.IV	M.C.	An.sim.	LIVE	In.IV	M.C.
P'C	1.1	1.1	1.1	1.1	3.5	3.4	3.4	3.4
WR'P	2.6	2.6	2.6	2.6	8.7	9.1	9.1	9.3
C'PR	1.5	1.5	1.5	1.5	3.1	3.4	5.8	3.4
YDP'P	1.4	1.4	1.5	1.5	2.2	2.6	8.3	2.4
M'GSNO	3.2	3.2	3.2	3.2	5.3	5.9	19.	5.6
T	3.1	3.1	3.1	3.1	6.8	6.8	9.4	6.9
FW	11.	11.	11.	11.	25.	25.	24.	28.

The results displayed in Table 5 are related to the real sector sub-model of the Forecasting System No.10 of the German economy, developed by the University of Bonn. For a description of the model, reference should be made to Krelle (1976) and to Conrad and Kohnert (1980). The sub-model used for these experiments consists of 136 equations, 59 of which are stochastic; it includes 39 exogenous variables and 163 estimated coefficients (data are annual). For most of the equations, the estimation period is 1960-1977.

The initial estimates used for the experiments have been obtained by LIVE methods (two iterations after an initial ordinary least squares estimate). Since, for this model, the number of structural equations is considerably larger than the length of the sample period, the estimated covariance matrix of structural coefficients is not positive definite, so that the triangular decomposition of the matrix, required by the Monte Carlo on coefficients method, cannot be performed. Therefore, experiments with Monte Carlo on coefficients have been performed assuming the matrix to be block diagonal, as in Cooper and Fischer (1974), Fair (1980), Haitovsky and Wallace (1972); of course, this introduces an additional approximation.

Analytic simulation on coefficients has been performed both with the complete covariance matrix and with the block diagonal matrix; as the results did not change significantly, at least for most variables (see Bianchi et al., 1981 for other results on this topic), only results obtained with the complete matrix are displayed in Table 5.

The results are displayed for 7 of the main variables: P'C= price index of consumption; WR'P= wage rate private; C'PR= private consumption; YDP'P= gross domestic product private; M'GSNO= imports of goods and services; T= total tax payment; FW= foreign workers.

In the first forecast period (static simulation), the results produced by all methods are quite similar to each others. On the contrary, in the last forecast period (dynamic simulation from 1970 to 1977; also in this case, the forecast period is not external to the sample estimation period) three methods produce quite similar results, while results produced by stochastic simulation and inefficient instrumental variables re-estimation diverge from the others.

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