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14 January 2005

Seasonal and Periodic Long Memory Models in the Inflation Rates

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February 3, 2006

Abstract

This paper considers the application of long memory processes to describe inflation with seasonal behaviour. We use three different long memory models taking into account the seasonal pattern in the data. Namely, the ARFIMA model with deterministic seasonality, the ARFISMA model, and the periodic ARFIMA (PARFIMA) model. These models are used to describe the inflation rates of four different countries, USA, Canada, Tunisia, and South Africa. The analysis is carried out using the Sowell's (1992) maximum likelihood techniques for estimating ARFIMA model and using the approximate maximum likelihood method for the estimation of the PARFIMA process. We implement a new procedure to obtain the maximum likelihood estimates of the ARFISMA model, in which dummies variables on additive outliers are included. The advantage of this parametric estimation method is that all parameters are estimated simultaneously in the time domain. For all countries, we find that estimates of differencing parameters are significantly different from zero. This is evidence in favour of long memory and suggests that persistence is a common feature for inflation series. Note that neglecting the existence of additive outliers may possibly biased estimates of the seasonal and periodic long memory models.

Keywords *:* Long memory; Fractional integration; Seasonality; Periodic models; inflation.

JEL classification: C22, E31

1 Introduction

Inflation has been a major problem of many economics. In order to keep inflation in check the policy makers need to have good understanding of the dynamic properties of the inflation rates. In the literature, time series of inflation rates are highly persistent. Persistence refers to an important statistical property of inflation, namely the current value of the inflation rate is strongly influenced by its history. Despite extensive researches on the dynamic properties of inflation rates, there is still no agreement about the key question of persistence in inflation.

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These researches can be classified into tow major groups. The first group of papers test for the existence of unit root in the inflation rates, disagreement remains in these papers on the classification of inflation rates as stationary or nonstationary. Barsky (1987), MacDonald and Murphy (1989), and Ball and Cecchetti (1990) provided evidence in support of unit root in inflation rates. On the other hand, Rose (1980) found evidence of stationarity in inflation rates. Brunner and Hess (1993) claimed that the inflation rate was stationary before 1960, but it has become nonstationary since that time.

In response to this debate about the stationarity of inflation rates the second group papers provided an explanation by modelling inflation rates as fractionally integrated processes $I(d)$, Where the fractional order of differencing d is a real number. The fractionally integrated model implies that the autocorrelations of inflation exhibit very slow hyperbolic decay.

Baillie, Chung, and Tieslau (1996) used fractionally ARMA (ARFIMA) models with GARCH errors to test for long memory in the inflation rate of the G7 countries and they found significant evidence. Similar evidence of strong long memory in the inflation rate of the United States, United Kingdom, Germany, France, and Italy is also provided by Hassler and Wolters (1995). Baum, Barkoulas, and Caglayan (1999) found significant evidence of long memory in the inflation rates for the industrial as well as the developing countries. Baillie et al (2002) explore the long memory property in the first and second conditional moments of inflation rates simultaneously. Furthermore, Reisen, Cribari and Jensen (2003) suggest that the inflationary dynamics of Brazil are better modelled by a long memory process than by a unit root mechanism.

An additional characteristic of the monthly inflation rates, in major countries, is its marked seasonal pattern. In the literature on long memory model, it is practical to remove seasonal fluctuations by means of including seasonal dummies variables in the ARFIMA models or analyzing the seasonal fractionally differenced models as in Porter-Hudak (1990), where the differenced filter is $(1 - L^s)^d$, with s is the seasonal periodicity and d is a real number. More recently a new model was appeared, namely, the periodic ARFIMA model. This model yields a useful description for long memory time series characterized by a change in its dynamics across the seasons (see Franses and Ooms (1997)).

This study considers the nature of seasonality in the inflation series with long memory behaviour for four countries, USA, Canada, Tunisia, and South Africa. We use three approaches that take account of seasonality in monthly inflation rates, namely the ARFIMA models with seasonal dummies variables, the seasonal fractional integrated model, and the periodic ARFIMA (PARFIMA) models. To make comparison, we also estimate ARMA and periodic AR models for monthly inflation rates.

Many estimators of the fractional parameter d, based on parametric and semiparametric estimation, have been proposed in the literature. The parametric estimators of parameter d are usually obtained using maximum likelihood and approximate maximum likelihood methods. In this study, we use the sowell's (1992) maximum likelihood estimation method, for the ARFIMA model, and we use the approximate maximum likelihood estimation for the PARFIMA model. It is currently practice to estimate seasonal fractionally integrated model using semiparametric estimation technique due to Porter-Hudak (1990). One contribution of this study is to investigate whether we can estimate the ARFISMA (p, d, q) model by exact maximum likelihood. The advantage of this parametric estimation method is that all parameters of the ARFISMA (p, d, q) model, including the mean, the autoregressive and the moving average ones, can be simultaneously estimated. This is in contrast to the semiparametric estimation method, where the parameters are estimated in two steps. First, we estimate d. The autoregressive and the moving average parameters are estimated in a second step.

The plan of the rest of this paper is as follows. Section 2 briefly summarizes the standard long memory models. Section 3 discusses the data and present some descriptive analysis. An applications of additive outliers test reveal the exisitence of some outlying observations in the inflation. The ADF test reject the hypothesis of unit rout in the inflation series. In section 4, we test for long memory in the inflation rates. There is strong evidence of long-range dependence in the inflation for all countries. Section 5 reviews the non-periodic analysis of monthly inflation by estimating ARMA and ARFIMA models. In section 6, a maximum likelihood estimation procedure for the seasonal fractionally integrated model is proposed and monthly inflation rates are analyzed by this models. In section 7, we review the periodic analysis of monthly inflation by estimating PAR(1) and PARFIMA models. Section 8 summarize a comparison between various models employed in this study for the inflation rates. Finally, we conclude in section 9 with some remarks

2 Long memory time series

Over the last few years, a new model has been introduced for modelling data with long memory behaviour. This model is an extension of ARIMA models introduced by Box and Jenkins (1970). Recalling the ARMA (p,q) in which the process is stationary. However, if the process is nonstationary thus it is integrated. This process is known as ARIMA (p,d,q) models where d is an integer. As a generalization of this type of models to incorporate long-range dependence, Granger and Joyeux (1980) and Hosking (1981), independently, discuss fractionally integrated processes which is commonly referred to as "long memory" model and in which the difference parameter d is allowed to be a non-integer. The fractional integration in a time series y_t is defined as follows:

$$(1-L)^d(y_t-\mu)=\varepsilon_t \tag{1}$$

Where the parameter "d" is a real number and called the fractional degree of integration of the process , $\{\varepsilon_t\}$ is a sequence of uncorrelated random variables with zero means and constant variances σ^2, "L" is the backshift operator such that $Ly_t = y_{t-1}$, and μ is the expectation of y_t.and $(1-L)^d$ is the fractional difference operator and it can be expanded as:

$$(1-L)^d = 1 - dL - \frac{d(1-d)}{2!}L^2 - \frac{d(1-d)(2-d)}{3!}L^3 - ... - \frac{d(1-d)...(j-1-d)}{j!}L^j - ... \tag{2}$$

As a generalization to the fractionally integrated model in (1), The autoregressive fractionally integrated moving average process of order (p,d,q), denoted by ARFIMA(p,d,q), with mean μ , is defined as

$$\Phi_p(L)(1-L)^d(y_t-\mu)=\theta_p(L)\varepsilon_t \tag{3}$$

where $\Phi_p(L)=1-\Phi_1 L-......-\Phi_p L^p$ is the autoregressive operator, $\theta_q(L)=1+\theta_1 L+......+\theta_q L^q$ is the moving average operator.

The stochastic process y_t is both stationary and invertible if all roots of $\Phi_p(L)$ and $\theta_q(L)$ lie outside the unit circle and $|d|<0.5$.For $d\in \left]0,\frac{1}{2}\right[$,the process is said to be long memory stationary with non-summable autocorrelations, that is, $\sum_{k=0}^{\infty}|\rho_k|=\infty$,where the ρ_k is the aurocorrelation function of $\{y_t\}$ at lag k. For $d<0$; the model is an intermediate memory process, or long-range negative dependence with zero spectral density at frequency zero and summable autocorrelations, $\sum_{k=0}^{\infty}|\rho_k|<\infty$. For $0.5\leqslant d<1$,the process is said to be non stationary and is mean reverting and no long run impact of an innovation on future values of the process. For $d=1$, the time series corresponds to an autoregressive integrated moving average (ARIMA) model. And for $d=0$, the time series is short memory, corresponding to stationary and invertible ARMA process.

ARFIMA models are said to have long memory because their autocorrelations decay towards zero at a hyperbolic rate, that is, $\rho_k\sim|k|^{-\alpha}$, $\alpha>0$, for large k. On the other hand, ARMA models are called short memory processes since their autocorrelations converge to zero at an exponential rate, that is $\rho_k\sim e^{-a|k|}$, $a>0$,for large k.

If data are stationary, external shocks can have a short-term impact, but little long-term effects, as the data reverts to the mean of the series at an exponential rate. In contrast, integrated data do not decay; that is to say, do not return to the previous mean after an external shock. By allowing d to take fractional values, we allow data to be mean reverting and to still have long memory in the process.

A popular application of long memory time series models concerns inflation and returns on exchange rates and their volatility. See Cheung (1993), Hassler and Wolters (1995), among others.

3 Data and descriptive analysis:

3.1 The data:

Our data set consists of monthly Consumer Price Index (CPI) based inflation rates for four countries; USA, Canada, Tunisia, and South Africa. All the data were obtained from the IMF's International Financial Statistics except Tunisian data were obtained from the Tunisian National Statistics Institute (INS). All data series are seasonally unadjusted monthly observations beginning in 1978.02 and ending in 2002.12 for Tunisia and United States (for a total of 299 observations) and beginning in 1979.02 until 2002.12 for Canada and

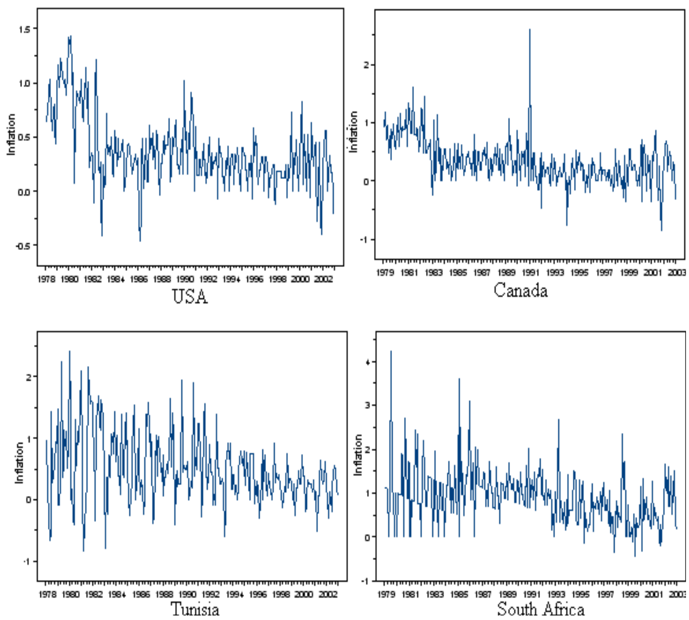

Figure 1: Time series plot of the inflations rates

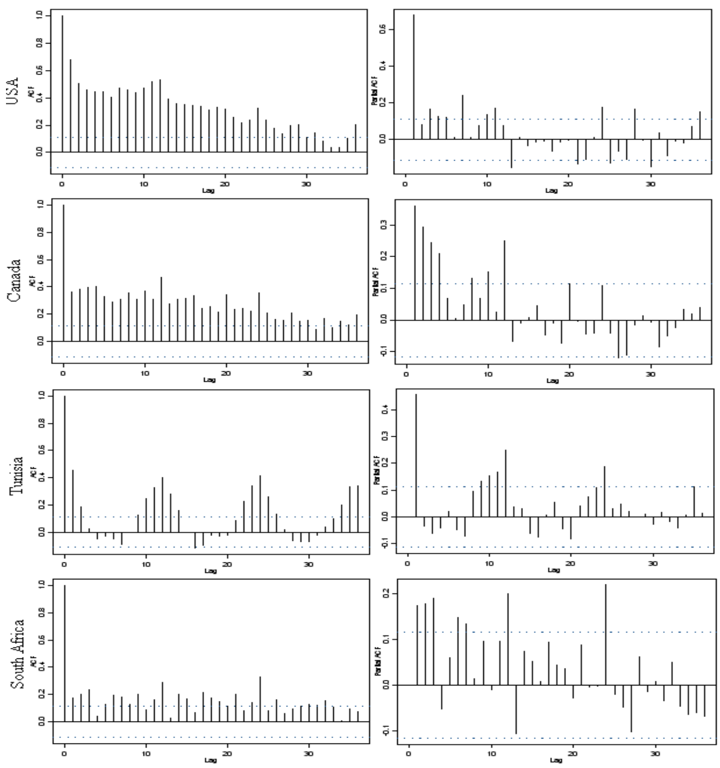

Figure 2: Autocorrelations (ACF) and Partial Autocorrelations (PACF) functions of monthly Inflation Rates

South Africa (for a total of 287 observations). Inflation rates are constructed by taking 100 times the first difference of the natural logs of CPI.

Figure 1 shows the plots of the monthly inflation rates in the different countries. Looking for these plots, there is indication of the existence of one or more outlying observations in each of the inflation series. Figure 2 shows the sample ACF and PACF of the inflation rates for each countries and it is clear that there is a marked seasonal pattern in all inflation series since the autocorrelation is highly significant at the seasonal lag. The ACF of Tunisian inflation rate exhibits a slow decay at the seasonal lags which is the behaviour of the seasonal fractionally differenced process. An additional characteristic in the ACF of the inflation is the persistence or the long memory properties, which is checked specially for USA and Canada. However, it is better to check the long memory property by plotting ACF of the seasonally adjusted data, adjustment of the data is derived from the application of monthly seasonal dummies.

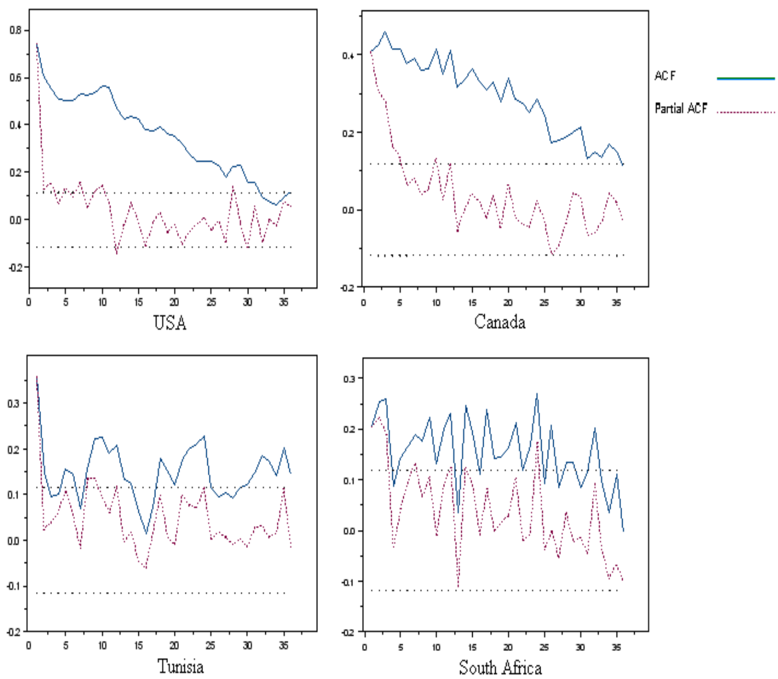

Figure 3: Autocorrelations (ACF) and Partial Autocorrelations (PACF) functions of monthly Inflation Rates (seasonally demeaned)

Figure 3 shows the ACF and PACF of the seasonally demeaned monthly inflation rates. A visual examination of the correlogram suggests that the data are nonstationory and possibly characterized by long memory behaviour since the sample autocorrelations do not die out quickly especially for United States and Canada. This may indicate long memory property of the inflation in these two counties. For Tunisia and South Africa, the persistence is less important but there are significant autocorrelations at a high order lag. The significance of the partial autocorrelation at a high order of lag may indicate the existence of some outlying observations. Furthermore, the ACF and the PACF of the differenced series, seasonally demeaned, are shown in figure 4. It is clear that the autocorrelation of the differ-

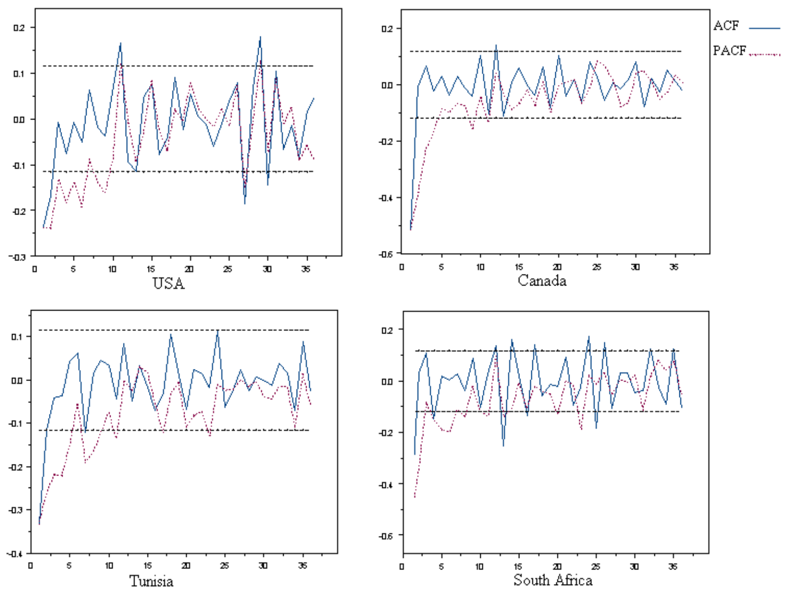

Figure 4: Autocorrelations (ACF) and Partial Autocorrelations (PACF) functions of first differences of monthly Inflation Rates (seasonally demeaned)

enced inflation series displays some negative values at low lags, which strongly suggestive of overdifferencing. Before testing for stationarity in the inflation rates, we must test for the existence of additive outliers in the data.

3.2 Testing for outlying observations:

To test for the presence of additive outliers in the data, we use the systematic testing procedure proposed by Vogelsang (1999), which is based on estimating

$$y_t = \alpha_0 + \alpha_1 D(T_{AO}) + \varepsilon_t \tag{4}$$

Where $D(T_{AO})$ is a dummy variable corresponding to an Additive Outlier in the data occurring at time T_{AO}. Then $D(T_{AO})$ takes 1 if $t = T_{AO}$ and zero otherwise. The statistic to test for an additive outlier is simply based on the t-ratio, which tests for the null hypothesis $\alpha_1 = 0$. The procedure is applied as follows: First we compute t for the entire series and we take $\tau = \max |t(T_{AO})|$ and if τ is significant, then the outlier and the corresponding row of the regressors are dropped from (4) and the equation is reestimated sequentially to test for a new outlier. We repeat these steps until no outlier is found.

The t-test statistic for α_1 is nonstandard since it is established under the assumption that y_t is nonstationary and contains unit root. The critical values for $t(\alpha_1)$ have been tabulated by Vogelsang (1999).

Table 1 shows the results from testing for additive outliers in the inflation. For all monthly inflation series, the obtained results show the existence of a number of additive outliers. Thus, the presence of outlying observations in the inflation rates for all countries must be taken into account in testing for unit root and in the various models that we will estimate in this paper.

Table 1: Test for outliers results

USA		Canada		Tunisia		South Africa	
$\hat{T}_{AO}$	t	$\hat{T}_{AO}$	t	$\hat{T}_{AO}$	t	$\hat{T}_{AO}$	t
1980.03	3,323	1991.01	6,133	1980.01	3,548	1979.07	5,488
1980.01	3,384	1981.06	3,526	1979.05	3,305	1985.02	4,649
1980.02	3.250	2001.11	3,350	1981.08	3,208	1986.01	3,858
		1982.05	3,219	1981.01	3,131	1980.09	3,289
		1994.02	3,183			1993.04	3,269
Significant levels :	5%	1%					
Critical values :	3.13	3.55					

3.3 Testing for unit root:

To test for the presence of unit root in the time series we use the Augmented Dickey Fuller (ADF) test. We take account of seasonality and of the existence of additive outliers in the data. Then the ADF statistic is based on the Ordinary Least Square (OLS) estimation of the auxiliary regression:

$$\Delta y_t = \mu_0 + \rho y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \sum_{s=1}^{S-1} \mu_s D_{t,s} + \sum_{j=1}^{m} \delta_j D(T_{AOj}) + \varepsilon_t \qquad (5)$$

Where p is the lag length. m is the number of outlying observations detected in the time series. S is the number of seasons within one period and is equal to 12 for our data. $D_{s,t}$ is a dummy variable for season s, being equal to 1 when y_t is an observation from that season and being 0 otherwise. $D(T_{AOj})$ is a dummy variable on additive outlier j.

To choose the lag length for (5) we use three criteria: the Akaike Information Criterion (AIC), the Schwarz information criterion (BIC), and the third criterion is based on the following procedure, we choose a sufficiently important value of $p = p_{\max}$, then we estimate the ADF regression with $p = p_{\max}$, If the last lagged difference is significant then we set $p = p_{\max}$ and we perform the unit root test. Otherwise, we reduce the lag length by one and we repeat the process. We initially check if at least two of three criteria agree at lag length, if there is no agreement, then we use the result of the criterion that provides us with the longest lag length since our objective is to remove any residuals autocorrelation.

In table 2 shows the ADF test results for the different inflation series and for both cases, with and without Additive Outliers dummies. These test results indicate that the null hypothesis of unit root in the inflation rates is strongly rejected for Tunisia, South Africa and Canada. For these countries, the estimation of the ADF regression gives highly significant parameters $\hat{\rho}$ (significant at 1% level) for both cases with and without Additive Outliers, except in Canada where $\hat{\rho}$ is significant at only 5% level when the Additive Outliers are not added to the ADF regression. In contrast, the ADF results of the United States inflation rates cannot reject the null hypothesis of unit root when ignoring Additive outliers, but when we take account for the outliers in the ADF regression, the null hypothesis of unit root is rejected at 5% level. In summary, the ADF results indicate a strong evidence for

Table 2: ADF test results

Countries	WithAO Dummies		WithoutAO Dummies			
	p	ADF	LB(24)	p	ADF	LB(24)
USA	6	-3.016**	26.989 (0.305)	6	-2.499	29.376 (0.206)
Canada	6	-4.217***	17.564 (0.824)	4	-3.146**	18.913 (0.757)
Tunisia	6	-5.426***	34.220 (0.081)	7	-3.863***	29.603 (0.198)
South Africa	6	-4.581***	31.575 (0.138)	6	-3.964***	24.770 (0.418)
Significant levels: 10% 5% 1%						** significant at 5% level
Critical values: -2,57- 2.87 -3,44						*** significant at 1% level

stationarity in the inflation rates.

4 Testing for Long memory in the inflation rates

To detect long-range dependence in time series, Hurst (1951) suggested the normalized rescaled range (R/S) test. Lo (1991) modified the R/S statistic to accommodate short-range dependence. In addition to the R/S test, Geweke and Porter-Hudak (1983) and Robinson (1995) suggested a frequency domain approach to test for long memory in the time series.

4.1 The R/S test

The R/S statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. Specifically, consider a time series y_t, for $t = 1, 2, ..., T$. Then the classical rescaled range (R/S) statistic proposed by Hurst (1951) is defined as:

$$Q_T = \frac{1}{s_T}\left[\max_{1\leq k\leq T}\sum_{j=1}^{k}(y_j - \bar{y}) - \min_{1\leq k\leq T}\sum_{j=1}^{k}(y_j - \bar{y})\right] \tag{6}$$

Where $\bar{y} = T^{-1}\sum_{i=1}^{T} y_i$ is the mean of the time series y_t and $s_T = \left[T^{-1}\sum_{i=1}^{T}(y_i - \bar{y})^2\right]^{-1/2}$

is the sample standard

deviation. If y_t are i.i.d normal random variables, then $\frac{1}{\sqrt{T}}Q_T$ converge to V, where V is the range of Brownian bridge on the unit interval.

4.2 The modified R/S test

The weakness of the standard R/S analysis is that is not robust to the short-range dependence. Thus, Lo (1991) modified the R/S statistic (6) by incorporating short-range dependence into the statistic. Then, the modified R/S statistic is written as

$$\tilde{Q}_T = \frac{1}{\hat{\sigma}_T(q)}\left[\max_{1\leq k\leq T}\sum_{j=1}^{k}(y_j - \bar{y}) - \min_{1\leq k\leq T}\sum_{j=1}^{k}(y_j - \bar{y})\right] \tag{7}$$

Where s_T in (6) is replaced by the square root of the Newey-West estimate of the long run variance with bandwidth q. Lo (1991) showed that in the presence of short memory but no long memory in y_t, $\tilde{Q}_T$ also converge to V, the range of Brownian bridge. When $q = 0$, $\tilde{Q}_T = Q_T$, the classical R/S statistic.

The GPH test

Geweke and Porter-Hudak (1983) suggest a semiparametric procedure in the frequency domain to testing for long memory. The spectral density of the fractionally integrated process y_t is defined as

$f(\xi) = [4\sin^2(\frac{\xi}{2})]f_\varepsilon(\xi) + \eta_\lambda$,

Where ξ is the Fourier frequency, and $f_\varepsilon(\xi)$ is the spectral density corresponding to ε_t in (1). The estimate of the fractional differencing parameter d is based on the slope of the spectral density function around the angular frequency $\xi = 0$. The spectral regression is defined by

$$\ln\{I(\xi_\lambda)\} = \beta - d\ln\{4\sin^2(\frac{\xi_\lambda}{2})\} + \eta_\lambda, \quad \lambda = 1, ..., \nu, \tag{8}$$

where $I(\xi) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} e^{it\xi}(y_t - \bar{y})\right|^2$ is the periodogram of the time series at the Fourier frequencies of the sample $\xi_\lambda = (2\pi\lambda/T)$, $(\lambda = 1, ..., (T-1)/2)$, T is the number of observations, and $\nu = g(T) << T$ is the number of Fourier frequencies included in the spectral regression.

Assuming that $\lim_{T\to\infty} g(T) = \infty$, $\lim_{T\to\infty}\{g(T)/T\} = 0$, and $\lim_{T\to\infty}\{\ln(T)^2/g(T)\} = 0$, the ordinary least squares (OLS) estimate of the slope coefficient in Equation (8) provides an estimate of d.

4.3 Testing for long memory results

Table 3 displays the results of R/S, Modified R/S, and GPH tests. The R/S and the modified R/S tests suggest that the inflation rates in all countries have long memory at 1% level. Similarly, for different choice of ν, the GPH test also shows that d is significantly different from zero. Hence, there is strong evidence of long memory and strong persistence in the inflation.

5 Non periodic models

In this section, we analyze the inflation by the ARFIMA models to show the long memory properties. In order to compare the ARFIMA model with conventional approaches, we also used information criteria to select the most appropriate (ARMA) model for the inflation rate in each country.

5.1 ARMA models

In the ARMA models used to modelling inflation rates, we take account of the seasonal properties of the inflation rates by including seasonal dummies variables. We also take

Table 3: Long memory tests results for monthly inflation rates

R/S test								
Countries	USA		Canada		Tunisia		South Africa	
test-statistic	4.7056**		4.3832**		3.5878**		3.7205**	
Modified R/S test								
Countries	USA		Canada		Tunisia		South Africa	
test-statistic	2.322**		2.4819**		2.5325**		2.6023**	
GPH test								
Countries	USA		Canada		Tunisia		South Africa	
$\hat{d}$	$\hat{d}$	t-stat	$\hat{d}$	t-stat	$\hat{d}$	t-stat	$\hat{d}$	t-stat
$\alpha = 0.55$	0.8194	4.8007**	1.0003	5.8589**	0.5888	3.4495**	0.3356	1.9654*
$\alpha = 0.65$	0.7442	6.2771**	0.8320	6.9101**	0.4448	3.7517**	0.4971	4.1283**
$\alpha = 0.75$	0.6802	7.8841**	0.5154	5.8743**	0.2369	2.7462**	0.4361	4.9702**
$\alpha = 0.85$	0.7669	11.6579**	0.4249	6.3168**	0.3982	6.053**	0.4785	7.1126**
$\nu = [T/2]$	0.7743	12.4873**	0.3871	6.1047**	0.4315	6.9592**	0.4375	6.8990**

* : significant at 5% level, ** : significant at 1% level

into account the existing of additive outliers in the data. Therefore, the ARMA models estimated for the inflation rates are

$$y_t = \mu_0 + \sum_{s=1}^{11} \mu_{0,s} D_{s,t} + \sum_{j=1}^{m} \delta_j D(T_{AOj}) + \sum_{i=1}^{p} \Phi_i y_{t-i} + \sum_{i=0}^{q} \theta_i \varepsilon_{t-i} \tag{9}$$

where $\theta_0 = 1$ and m is the number of additive outliers in the data.

The best ARMA model that has significant parameters is selected by the AIC and BIC. The results from the ARMA models estimations, for both cases with and without outlier correction are reported in Table 6, for USA and Canada, and in Table 7, for Tunisia and South Africa. The appropriate ARMA models selected for the inflation rates in United States, Canada, Tunisia and South Africa are AR(2), AR(4), AR(1), and AR(3) respectively.

For United States, the estimation results from AR(2) reveal that there is no need to include dummies variables on additive outliers because the estimated dummies variables are not significant and the AIC and BIC criteria are too lower in the case without outlier correction than in the case with outlier correction. The estimation results from ARMA (2, 0) model specification, show the highly significance of the two autoregressive parameters, where the estimates values of the parameters are $\hat{\Phi}_1 = 0.6505$, and $\hat{\Phi}_2 = 0.1252$. It is to be noticed that not all the seasonal dummies variables are significant.

In contrast to USA inflation case, including dummies variables on additive outliers in the ARMA model specifications selected for Canada, Tunisia, and South Africa lead to a good improvement of the normality tests for the estimated residuals and the coefficients of these dummies variables appear to be significantly different from zero. Moreover, the AIC and BIC criteria values became too lower in this case. The results from AR(4) model applied to Canadian inflation rate show the highly significance of the parameters, where $\hat{\Phi}_1 = 0.123$,

$\hat{\Phi}_2 = 0.176$, $\hat{\Phi}_3 = 0.205$ and $\hat{\Phi}_4 = 0.163$. The constant and the seasonal dummies variables are highly significant. The AR(1) estimated model for Tunisian inflation rate gives a value of $\hat{\Phi}_1 = 0.343$, which is significantly different from zero at 1% level. The mean and some number of the seasonal dummies variables are significant but the rest of the parameters are not significantly different from zero. For South African inflation, the estimation results from AR(3) representation show that $\hat{\Phi}_1 = 0,095$ is significantly different from zero at 10% level, $\hat{\Phi}_2 = 0,217$, and $\hat{\Phi}_3 = 0,159$ are significantly different from zero at 1% level.

5.2 ARFIMA models with seasonal dummies variables

5.2.1 Notation

In the ARFIMA models, we take account for the seasonal behaviour of the inflation rates by including seasonal dummies variables in the equation of the model. Because there are some additive outliers in the inflation data, we also included dummies variables on these outlying observations in the ARFIMA models. Thus, the ARFIMA model that we estimated in the analysis of the inflation series is writen as

$$\Phi_p(L)(1-L)^d(y_t - x_t'\beta) = \theta_q(L)\varepsilon_t \tag{10}$$

Where

$$x_t'\beta = \beta_0 + \beta_1 D_{1t} + \beta_2 D_{2t} + ... + \beta_{11} D_{11t} + \delta_1 D(T_{AO1}) + ... + \delta_m D(T_{AOm})$$

Where m is the number of additive outliers in the data.

Where $\Phi_p(L)$ and $\theta_q(L)$ are polynomials in the lag operator L of degree p and q, respectively, with roots outside the unit circle. $\varepsilon_t \sim i.i.d.(0, \sigma^2)$. $D_{1,t}, D_{2,t}, ...D_{S-1,t}$ are seasonal dummies. $D(T_{AOj})$ is a dummy variables on additive outlier j.

Now, if all roots of $\Phi_p(L)$ and $\theta_q(L)$ lie outside the unit circle and $-0.5 < d < 0.5$; then y_t is stationary and invertible. To estimate this ARFIMA model we need an estimation method that estimate all parameters simultaneously, which is the Exact Maximum likelihood estimation. Sowell (1992) derives the unconditional exact likelihood function for a normally distributed stationary fractionally integrated time series and gives recursive procedures that allow efficient evaluation of the likelihood function.

Based on the normality assumption and with a Sowell's (1992) procedure used to compute the autocovariance function in the covariance matrix $\Sigma = \sigma_\varepsilon^2 R$ of $Y_T = [y_1 y_2 .....y_T]'$, the ML estimates of the ARFIMA model are obtained by maximizing the function

$$-\frac{1}{2}\ln|R| - \frac{T}{2}\ln(\frac{\hat{Z}_T' R^{-1}\hat{Z}_T}{T}) \tag{11}$$

Where $\hat{Z}_T = Y_T - X\hat{\beta}$,and $\hat{\beta} = (X'R^{-1}X)^{-1}X'R^{-1}Y_T$.

5.2.2 Empirical estimates for the inflation rates

For the different inflation series analyzed in this research, we estimate ARFIMA(p, d, q) models with $p, q \leqslant 1$. Thus, five models are estimated for each series, and we select the model that has the minimum AIC and BIC as the most appropriate model.

For the estimation of this ARFIMA model, we wrote a GAUSS program that estimate the long memory parameter d, the autoregressive and the moving average parameters and the parameters of the dummies variables by the Sowell's maximum likelihood estimation method.

According to the AIC, the BIC and the Loglikelihood based criteria, the most appropriate model specifications for the inflation rates are an ARFIMA (0, d, 1) for USA and Canada. However, an ARFIMA (0, d, 0) and an ARFIMA (1, d, 0) appear to be most adequate for Tunisia and South Africa, respectively.

The estimation results of the selected ARFIMA model representations for USA and Canada, and for both cases with dummies variables and without dummies variables on additive outliers, are reported in tables 8. For United States inflation, as a comparison between the two cases with outlier correction and without outlier correction, it is clear that the dummies variables on additive outliers are not significantly different from zero and they did not improve the normality tests of the residuals. In addition, the AIC and the BIC values are at the minimum where the dummies variables are not included. Thus, it is preferred not to include the dummies variables on additive outliers. The estimate value of the long memory parameter d is 0.4113 with standard error value of 0.05 and 95% confidence interval of [0.3129, 0.5096]. The estimation of the moving average parameter θ is 0.1692 with standard error of 0.0784. Thus, both long memory and short memory parameters are significantly different from zero. Moreover, the seasonal dummies variables appear to be highly significant. The estimated residuals standard deviation $\hat{\sigma}_{\varepsilon} = 0.1981$. Note also that looking for the ACF and PACF plots for the residuals, in figure 5, obtained after fitting a fractional ARIMA(0, 0.4113, 1) model to United States inflation, there is no serious indication of dependence. This confirms the results of the residuals autocorrelation test based on the Ljung-Box statistic.

For the inflation series in Canada, Tunisia and South Africa, it is preferred to take into account the existence of additive outliers in the inflation series by including dummies variables in the ARFIMA models. In fact, the results from the normality tests based on Skewness, Kurtosis and Jarque-Bera statistic, show that the null hypothesis of the residuals normality is rejected in the case without outlier correction but the residuals become normally distributed when the dummies variables on additive outliers are included. Moreover, the AIC and the BIC criteria values are too lower in the case with outlier correction than in the case without outlier correction.

For Canadian inflation, the estimate value of the fractional differencing parameter d from an ARFIMA(0, d, 1) is equal to 0.4814 with standard error of 0.0249 and 95% confidence interval of $[0.4323, 0.5306]$. The estimate value of the moving average parameter θ is also significantly different from zero with value of -0.3956 and standard error of 0.0645. As for

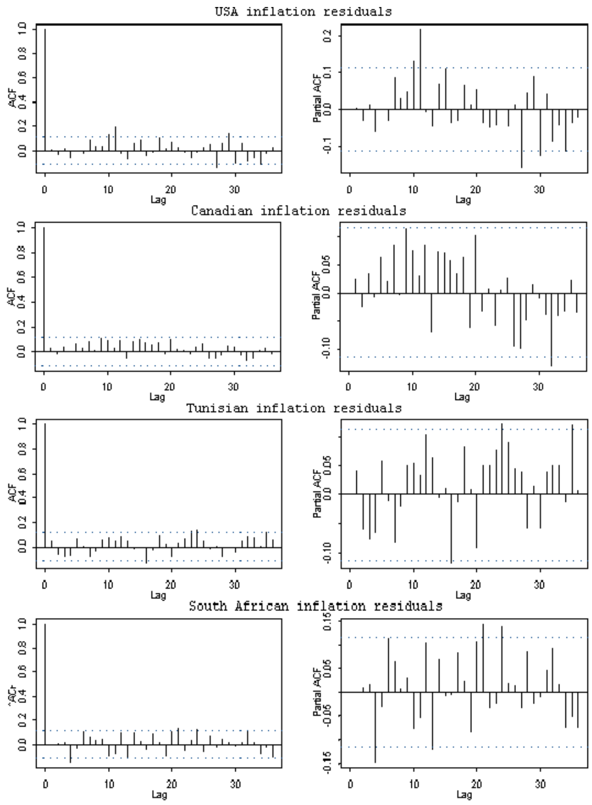

Figure 5: ACF and PACF for the estimated residuals after fitting an ARFIMA (p, d, q) model to the Inflation Rates.

United States inflation, the seasonal dummies variables seem to be significantly different from zero. The residuals standard deviation is estimated to be 0.2431. The Ljung-Box test statistic at lag 20 is equal to 24.737, which is not significant at 10% level. This indicates that there is no residuals correlation. Moreover, The ACF and PACF of the estimated residuals after fitting an ARFIMA (0, 0.4814, 1) on Canadian inflation, in figure 5, reveal that there is no correlation and the residuals seem to have white noise properties, which indicate that the ARFIMA (0, 0.4814, 1) is an appropriate model for this inflation series.

The results from estimating an ARFIMA(0, d, 0) and an ARFIMA(1, d, 0) for Tunisian inflation and South African inflation respectively, are reported in table 9. For Tunisian inflation, the estimate of d corresponding to ARFIMA $(0, d, 0)$ is equal to 0.248 with standard error of 0.045 and with 95% confidence interval of [0.1610, 0.3362]. Thus, the estimated differencing parameter d is significantly different from zero, which suggests long memory behaviour of Tunisian inflation rate. The estimated mean is equal to 0.687 and is significantly different from zero as well as the dummies variables on additive outliers. However, the estimation of the seasonal dummies variables reveals that only some coefficients are significant. The estimated standard deviation of residuals is about 0.4001. The residuals autocorrelation test gives a value of Q statistic at lag 20 equal to 24.885, which is not significant at 10% level and we cannot reject the hypothesis of no residuals correlation. Moreover, the ACF and PACF of the estimated residuals are plotted in figure 5 and reveal that the estimated residuals for Tunisian inflation rates have to be white noise.

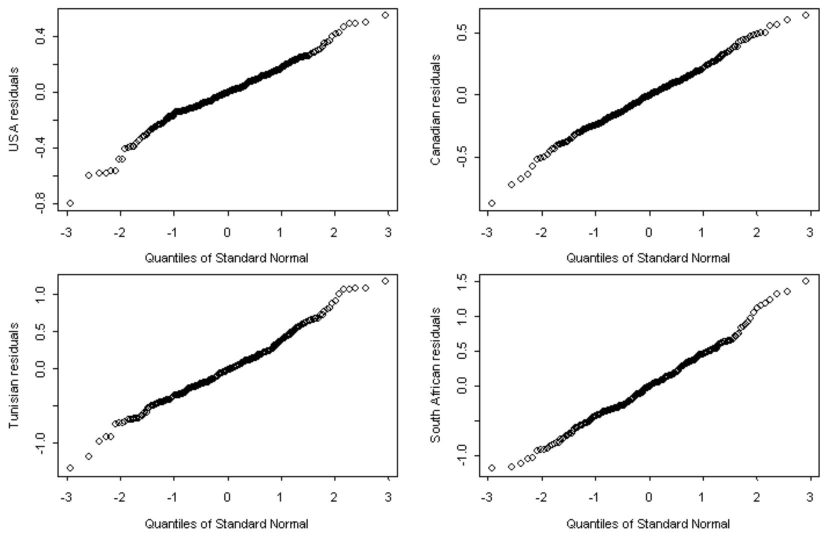

Figure 6: ARFIMA residuals QQ-plots of the inflation rates.

The estimated value of the long memory parameter d from ARFIMA (1, d, 0) for South African inflation is equal to 0.3216 with standard error of 0.057 and with 95% confidence interval of [0.2088, 0.4344]. The highly significance of this parameter provides strong evidence of long memory behaviour in South African inflation rate. The estimate value of the autoregressive parameter Φ is equal to -0.2153 with standard error of 0.0788 is also significantly different from zero at 1% level. The mean and the the dummies variables on additive outliers are highly significant. The residuals standard deviation is estimated to be 0.476. It is to be noticed that not all the seasonal dummies variables are significant. The value of the Ljung-Box test statistic Q at lag 20 is equal to 30.855 is not significant at 5% level. Thus, the null hypothesis of residuals dependence cannot be rejected. As we did for other inflation series, we evaluate the autocorrelation and partial autocorrelation of the estimated residuals shown in figure 5. It is clear that there is no serious indication of dependence. This confirms the hypothesis of no residuals correlation shown by the Ljung-Box test. The QQ-plots of the residuals are shown in figure 6. A visual examination reveals that, for Canadian, South African, and Tunisian residuals, the QQ-plots lie on a straight line, which indicates that the residuals are normally distributed. For United States, the residuals are not normally distributed. This confirms the results of the normality tests presented above.

In summary, for all countries, there is evidence of significant long memory in the inflation rates. In addition, in all series, the long memory parameters estimates are below 0,5 implying stationarity in the inflation series.

6 Seasonal fractionally integrated processes

The fractional differencing filter $(1-L)^d$, for $-0.5 < d < 0.5$, was proposed in the econometric literature by Hosking (1981) and Granger and Joyeux (1980). It has been generalized to seasonal fractional differencing $(1 - L^s)^d$, where s denotes the number of seasons, by Andél (1986), Porter-Hudak (1990), Hassler (1994), Ooms (1995), and Arteche and Robinson (1998).

6.1 Notation and properties

The seasonal fractionally integrated model with zero mean as in Porter-Hudak (1990) is defined as

$$(1 - L^s)^d y_t = \varepsilon_t \tag{12}$$

Where d is the fractionally differenced component and lie inside the interval $(-\frac{1}{2}, \frac{1}{2})$, ε_t are assumed to be independently and identically distributed (i.i.d) with zero mean and variance σ^2, and s is the seasonal periodicity ($s = 12$ for monthly series).

The seasonal fractional filter is defined by a binomial expansion:

$$(1 - L^s)^d = 1 - dL^s - \frac{d(1-d)}{2!}L^{2s} - ... - \frac{d(1-d)...(j-1-d)}{j!}L^{js} - ... = \sum_{j=0}^{\infty} d_j L^{js} \tag{13}$$

$$(1 - L^s)^{-d} = 1 + dL^s + \frac{d(1+d)}{2!}L^{2s} + ... + \frac{d(1+d)...(j-1+d)}{j!}L^{js} + ... = \sum_{j=0}^{\infty} c_j L^{js} \tag{14}$$

If a seasonal fractionally differenced model is appropriate, then, the ACF of the processes displays a hyperbolic decay at the seasonal lags, rather than the slow linear decay characteristic of the conventional seasonal differencing model. The generalization of (12) to an autoregressive fractionally integrated seasonal moving average model with zero mean, ARFISMA (p,d,q) is thus

$$\Phi_p(L)(1 - L^s)^d y_t = \theta_q(L)\varepsilon_t \tag{15}$$

Where $\Phi_p(L)$ and $\theta_q(L)$ are the autoregressive and the moving average polynomials, respectively, and the roots of theses polynomials are assumed to be outside the unit circle. The ARFISMA process is stationary if $d < 0.5$, $\sum_{j=0}^{\infty} c_j < \infty$, and invertible if $d > -0.5$. When the mean of the process is not zero, ARFISMA (p, d, q) model in (15) becomes

$$\Phi_p(L)(1 - L^s)^d (y_t - x_t'\beta) = \theta_q(L)\varepsilon_t \tag{16}$$

Where β is a $k \times 1$ vector of parameters and x_t is a vector of predetermined variables (dummies variables, for example). To estimate all parameters simultaneously we propose a procedure to obtain the Maximum Likelihood estimate of the ARFISMA model.

6.2 Maximum likelihood estimation procedure:

Porter-Hudak (1990) extend the nonseasonal semiparametric estimation technique developed in Geweke and Porter-Hudak (1983) (GPH) to the fractionally differenced seasonal model. Unfortunately, this semiparametric method cannot estimate all parameters in (16) simultaneously. Therefore, we propose a procedure to obtain the maximum likelihood estimate of the ARFISMA model. This maximum likelihood technique allows the estimation of all parameters in one step.

6.2.1 Evaluation of the autocovariance function

Chan and Palma (1998) used a simple procedure to compute the autocovariance of the ARFIMA processes from the MA representation

$$z_t = \Phi_p(L)^{-1}(1-L)^{-d}\theta_q(L)\varepsilon_t = \sum_{j=0}^{\infty} \psi_j L^j \varepsilon_t \tag{17}$$

With $\psi_0 = 1$.Then the autocovariance function is defined as

$$\gamma_k = \sum_{j=0}^{\infty} \psi_j \psi_{j+|k|} \sigma_\varepsilon^2. \tag{18}$$

To compute the autocovariance function of the autoregressive moving average (ARMA) model with a fractionally differenced seasonal component in (16), we extend the procedure proposed by Chan and Palma (1998) for the nonseasonal ARFIMA model to the ARFISMA model. Then the MA representation of the processes is

$$z_t = \Phi_p(L)^{-1}(1-L^s)^{-d}\theta_q(L)\varepsilon_t = \sum_{j=0}^{\infty} \psi_j L^j \varepsilon_t$$

Where the seasonal fractional filter $(1-L^s)^{-d}$ is expanded as in (14). Then, the autocovariance function of the ARFISMA model can be defined as

$$\gamma_k = \sum_{j=0}^{\infty} \psi_j \psi_{j+|k|} \sigma_\varepsilon^2. \tag{19}$$

6.2.2 Evaluation of the loglikelihood function:

Let Y_T be a sample of T observations such that $Y_T = [y_1\ y_2........y_T]'$. We assume that y_t is a stationary normally distributed fractionally integrated time series. Then, $Y_T \sim N(X\beta,$ $\Sigma)$.

Stationarity implies that the covariance matrix is a Toeplitz form:

$$V[Y] = \begin{bmatrix} \gamma_0 & \gamma_1 & \cdots & \cdots & \gamma_{T-1} \\ \gamma_1 & \gamma_0 & \cdots & \cdots & \gamma_{T-2} \\ . & . & . & . & . \\ . & . & . & . & . \\ \gamma_{T-2} & \cdots & \cdots & \gamma_0 & \gamma_1 \\ \gamma_{T-1} & \cdots & \cdots & \gamma_1 & \gamma_0 \end{bmatrix} = \Sigma$$

Where γ_k is the autocovariance function evaluated using the procedure presented above.

Let $z_t = y_{t-}x_t'\beta$, Then $Z_T = [z_1, z_2, ... z_T]' \sim N(0, \Sigma)$ with probability density function:

$$f(Z_T, \Sigma) = (2\pi)^{-T/2} \left|\Sigma\right|^{-1/2} \exp\{-\frac{1}{2} Z_T' \Sigma^{-1} Z_T\}. \tag{20}$$

Given the equation (20), the loglikelihood function is :

$$\ln L(d, \Phi, \theta, \beta, \sigma_\varepsilon^2) = \ln(f(Z_T, \Sigma)) = -\frac{T}{2} \ln(2\pi) - \frac{1}{2} \ln |\Sigma| - \frac{1}{2} Z_T' \Sigma^{-1} Z_T. \tag{21}$$

It is beneficial to concentrate σ_ε^2 out of the likelihood. In fact, it is possible to write $\Sigma = \sigma_\varepsilon^2 R$. Then, the loglikelihood function become:

$$\ln L(d, \Phi, \theta, \beta, \sigma_\varepsilon^2) = -\frac{T}{2} \ln(2\pi) - \frac{1}{2} \ln |R| - \frac{T}{2} \ln(\sigma_\varepsilon^2) - \frac{1}{2\sigma_\varepsilon^2} Z_T' R^{-1} Z_T.$$

Then, differentiating with respect to σ_ε^2 gives:

$$\frac{\partial \ln L}{\partial \sigma_\varepsilon^2} = -\frac{T}{2\sigma_\varepsilon^2} + \frac{1}{2\sigma_\varepsilon^4} Z_T' R^{-1} Z_T$$

Solving this differentiating yields

$$\hat{\sigma}_\varepsilon^2 = T^{-1} Z_T' R^{-1} Z_T \tag{22}$$

Then, the concentrated likelihood function (CLF) is:

$$l_c(d, \Phi, \theta, \beta) = -\frac{T}{2} \ln(2\pi) - \frac{T}{2} - \frac{1}{2} \ln |R| - \frac{T}{2} \ln(T^{-1} Z_T' R^{-1} Z_T)$$

It is also beneficial to concentrate β out of the likelihood. Then, the loglikelihood function, concentrated with respect to $\hat{\beta} = (X'R^{-1}X)^{-1} X'R^{-1} Y_T$

$$l_c(d, \Phi, \theta) = -\frac{T}{2}(1 + \ln(2\pi)) - \frac{1}{2} \ln |R| - \frac{T}{2} \ln(T^{-1} \hat{Z}_T' R^{-1} \hat{Z}_T) \tag{23}$$

Where $\hat{Z}_T = Y_T - X\hat{\beta}$,

Finally, the function to be used in the maximization procedure is:

$$-\frac{1}{2} \ln |R| - \frac{T}{2} \ln(\frac{\hat{Z}_T' R^{-1} \hat{Z}_T}{T}) \tag{24}$$

This function must be maximized with respect to the elements of R, which included d and the parameters of the autoregressive polynomial $\Phi_p(L)$ and the parameters of the moving average polynomial $\theta_q(L)$.

$$x_t' \beta = \mu + \sum_{j=1}^{m} \delta_j D(T_{AOj})$$

Where d is the fractionally differenced parameter,

m is the number of additive outliers in the data, $D(T_{AOj})$ is a dummy variable on the additive outlier j.

6.3 Simulation evidence

Some simulations evidence are reported in table 3 that supports estimation of the seasonal fractionally integrated model by the exact maximum likelihood method. For each of the d-values -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3 and 0.4, 100 series of size $T = 200$, 300 and 500 of the seasonal fractionally integrated model were simulated. The simulation results in table 3 shows the simulated mean and standard errors of d. The estimation method provides a reasonable approximation, for each d, the sample mean and the standard deviation of the estimated parameter d are reasonably close to the theoretical values. As expected, standard errors and estimated values of the differencing parameter d become better as T increases. The results from estimating seasonal fractionally integrated model are carried out using GAUSS program written by the author. the GAUSS program is available from the author on request.

Table 4: Simulation results of estimating the seasonal fractionally integrated model

True d		-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4
	T=200	-0.4135	-0.3024	-0.2060	-0.0980	-0.0052	0.1036	0.1915	0.3061	0.3952
$\hat{d}$	T=300	-0.4057	-0.2994	-0.2025	-0.1055	0.0037	0.0930	0.1985	0.2937	0.3943
	T=500	-0.4051	-0.3042	-0.2009	-0.1016	-0.0025	0.1001	0.1942	0.2952	0.3956
	T=200	0.0742	0.0699	0.0667	0.0633	0.0610	0.0579	0.0564	0.0535	0.0532
Std.err.	T=300	0.0558	0.0543	0.0521	0.0498	0.0480	0.0467	0.0452	0.0437	0.0427
	T=500	0.0408	0.0397	0.0386	0.0375	0.0366	0.0356	0.0348	0.0336	0.0332

6.4 Empirical estimates for the inflation rates

For the inflation series in different countries, we estimate ARFISMA(p, d, q) models in (16) where $x_t'\beta = \mu + \sum_{j=1}^{m} \delta_j D(T_{AOj})$. With m is the number of additive outliers in the data and $D(T_{AOj})$ is a dummy variable on the additive outlier j.

The ARFISMA(p, d, q) model is estimated for $p, q \leqslant 1$ and the appropriate model is selected based on AIC and BIC criteria. The results of the selected models representation based on AIC and BIC criteria, for both cases with and without dummies variables on additives outliers, are reported in table 10 for USA and Canada and in table A.11 for Tunisia and South Africa. Then, the appropriate ARFISMA(p, d, q) model representations are ARFISMA$(1, d, 0)$ for USA and Tunisia and ARFISMA$(1, d, 1)$ for Canada and South Africa.

For United States inflation series, it is not preferred to include the dummies variables on additive outliers in the (16) because only one coefficient of the dummies variables is significant and the normality tests on the estimated residuals are too better in the case without outlier dummies than in the alternative one. In addition, the BIC value, in the case without dummies variables on additive outliers, is is too lower than the value in the case with dummies variables. The estimate value of the seasonal fractionally differencing parameter d is 0.2661 with standard error value of 0.041, which indicates the highly significance of d.

The 95% confidence interval of d is [0.1852, 0.3469]. In particular, $d = 1$ is not included in the interval. Thus, taking the seasonal difference appears to be too strong. The estimate of the autoregressive parameter Φ is 0.6223 with standard error value of 0.047 is significant at 1% level and the residuals standard deviation is estimated to be 0.2207.

In contrast to United States, it is preferred for Canada, Tunisia and South Africa, to take into account the existence of additive outliers in the inflation series by means of including dummies variables in the ARFISMA models. In fact, the results from the normality tests based on Skewness, Kurtosis and Jarque-Bera statistic, reveal that the null hypothesis of the residuals normality is rejected in the case without dummies variables on additive outliers, for all countries, but the residuals become normally distributed when the dummies variables on additive outliers are included, especially for Canada and South Africa. For Tunisian inflation, The inclusion of dummies variables on additive outliers makes the estimated residuals close to the normality. Moreover, the AIC and the BIC criteria values are too lower in the case with outlier correction than in the case without outlier correction.

For Canadian inflation, the estimate value of d is 0.2449 with standard error of 0.044 and with 95% confidence interval of [0.1586 0.3313]. The estimates values of the short memory parameters are $\hat{\Phi} = 0.9832$ with standard error of 0.013 and $\hat{\theta} = -0.8769$ with standard error of 0.036. In addition, the mean and the coefficients of the dummies variables on additive outliers are highly significant. The estimated standard deviation of residuals is equal to 0.2552.

For Tunisian inflation rate, an ARFISMA(1, d, 0) was found to provide an adequate representation of Tunisian inflation series with the estimate of d being 0.2939 and a robust standard error of 0.044 and with 95% confidence interval of [0.2078, 0.3799]. The estimated residuals standard deviation is 0.4213. The estimate value of the autoregressive parameter Φ is 0.3093 with standard error of 0.058. Thus, both the long memory and the short memory parameters are highly significant.

Finally, for South African inflation rate, the estimate value of the seasonal fractionally differencing parameter d is 0.2124 with standard error of 0.052 and 95% confidence interval of [0.1091, 0.3156]. The estimates values of the short memory parameters are $\Phi = 0.8891$ and $\theta = -0.7496$ with standard error of 0.092 and 0.137 respectively. Thus, both short memory and long memory parameters are significantly different from zero. The estimation results shows also the highly significance of the estimated mean mu and the coefficients of the dummies variables on additives outliers. The residuals standard deviation is estimated to be 0.4705.

In figure 7, we plot, for the different countries, the autocorrelations and partial autocorrelations of the residuals after fitting the selected appropriate representation for the Autoregressive moving average (ARMA) with a fractionally differenced seasonal component model, for each inflation series. It is clear that, for United States inflation, some autocorrelations still exist. This confirms the Ljung-Box test statistic given in table 10 with a highly significant value of Q statistic at lag 20 equal to 60.381. For Canada, Tunisia, and South Africa, the ACF and PACF plots show that there is no serious correlation. The Ljung-Box test statistic shown in table 10 for Canada and in table 11 for Tunisia and South Africa

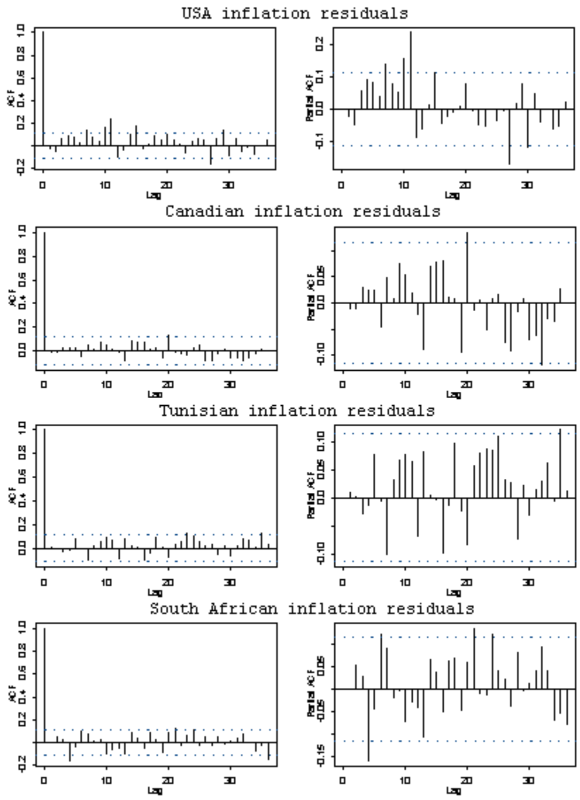

Figure 7: ACF and PACF of the residuals after fitting an ARFISMA(1, d, 0), for United States and Tunisian inflations, and an ARFISMA(1, d, 1) model for canadian and South African inflations

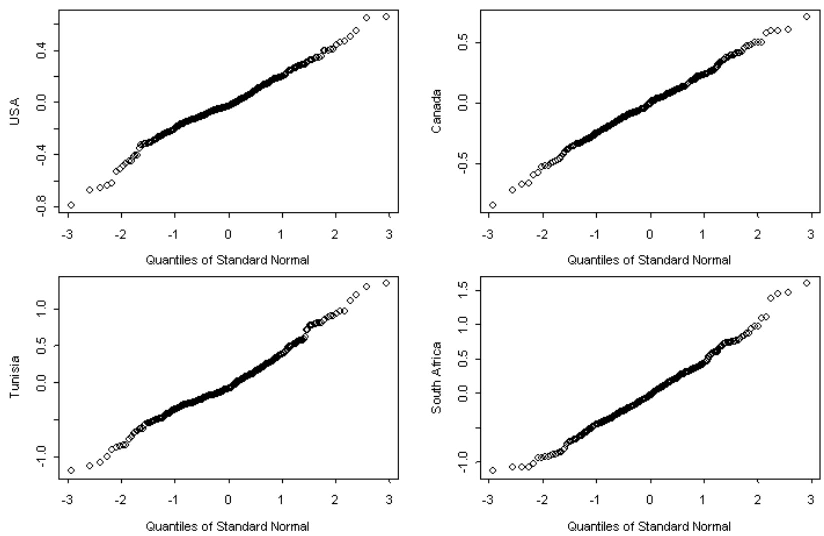

Figure 8: ARFISMA residuals QQ-plots of the inflation rates.

gives values of Q at lag 20 equal to 18.221, 22.441 and 31.223 respectively. They are not significant at 5% level and confirm the hypothesis that the residuals are generated by an uncorrelated process.

The QQ-plots of the residuals are shown in figure 8. If the residuals are normally distributed, the QQ-plots should lie on a straight line. A visual examination of the QQ-plots reveals that the residuals seem to be normally distributed, especially for Canadian and South African inflation rates, which confirm the results of the normality tests presented above.

7 Periodic models:

In this section, we investigate whether it is better to modelling the inflation rates by periodic long memory models, this means that the long memory parameter d varies with the season. For comparison, we also estimated a periodic autoregressive, PAR(1) model for each inflation series.

7.1 PAR model estimation

In the Periodic Autoregressive model, the parameters of the autoregressive polynomial vary with the seasons. As in Franses and Ooms (1997) who fit a PAR (1) model for the quarterly UK inflation rates, we fit a PAR (1) model for our monthly inflation rates series. Then the PAR (1) representation is

$$y_t = \sum_{s=1}^{12} \mu_s D_{s,t} + \sum_{j=1}^{m} \delta_j D(T_{AOj}) + \sum_{s=1}^{12} \Phi_{1,s} D_{s,t} y_{t-1} + \varepsilon_t \tag{25}$$

Where m is the number of additive outliers in the data.

The estimation results from the PAR (1) model for the inflation series in different countries and for both cases with and without additive outliers correction, are reported in table 12 and table 13 in the appendix. For United States, the results from estimating the PAR(1) model with outlier correction show that the dummies variables on additive outliers are not significantly different from zero at 5% level. Therefore, the PAR(1) model without outlier correction seems to be most appropriate. The estimation results show also the highly significance of all periodic autoregressive parameters for United States inflation. The estimated values of $\Phi_{1,i}$, $i = 1, \dots, 12$, range from 0.502 through 1.121. The smallest estimate is obtained for periodic autoregressive parameter $\Phi_{1,10}$ corresponding to season $s = 10$ and the largest is obtained for $\Phi_{1,12}$ corresponding to $s = 12$. For other countries, we observe from table 12 for Canada and table 13 for Tunisia and South Africa that the dummies variables on additive outliers are highly significant and that the PAR (1) model with outliers is best supported by both of criteria AIC and BIC. For Canadian inflation, only one parameter among the periodic autoregressive parameters $\Phi_{1,i}$, $i = 1, \dots, 12$, is not significantly different from zero. In contrast, for Tunisia and South Africa, the major parameters are not significant. Thus, there is significant evidence of periodicity of the autoregressive parameters only for USA and Canadian inflation rates.

7.2 Periodic long memory model:

Periodic long memory modelling is an alternative seasonal modelling technique of seasonal time series with long memory behaviour. The notion of periodic long memory was initially suggested by Franses and Ooms (1997). They raised issue related to the extension of the ARFIMA (0, d, 0) model, in the sense that the novel model allows for periodic variation in differencing parameters d. They analysed the usefulness of a so-called periodic ARFIMA $(0, d_s, 0)$ (PARFIMA) model for quarterly UK inflation, where d_s indicate that the value of d can vary with the season. Possible economic motivations for time varying parameters models are that economic agents may have different behaviour in different seasons due to time dependent utility function, preferences, productions, etc.

7.2.1 PARFIMA models:

The PARFIMA $(0, d_s, 0)$ model is defined as

$$(1-L)^{d_s} y_t = \varepsilon_t \tag{26}$$

Where d_s is the periodic long memory parameter which varies with the season. This model is similar to ARFIMA (0, d, 0) where both the mean of y_t and the value of d can vary with the season. Franses and Ooms (1997) extend the approximate Beran's (1995) maximum likelihood method for the estimation of the ARFIMA model, to estimate PARFIMA $(0, d_s, 0)$ model.

7.2.2 Approximate maximum likelihood estimation:

To estimate the periodic ARFIMA$(0, d_s, 0)$ model in (26). Franses and Ooms (1997) extended the Beran's (1995) approximate maximum likelihood estimation, for ARFIMA model, to PARFIMA $(0, d_s, 0)$ model. The estimate for d_s is obtained through minimizing the sum of squared residuals $\sum_{i=2}^{n} e_t^2(\eta)$ Where

$$e_t(\eta) = \sum_{j=0}^{t-1} a_{j,s}(\eta)(y_{t-j} - \bar{y}_{t,s}) \tag{27}$$

This correspond to allowing for periodic autoregressive parameters $a_{j,s}$ and we subtract seasonal means $\bar{y}_{t,s}$. $a_{j,s}$ can be obtained from the AR(∞) representation of (26).

7.2.3 Empirical estimation for the inflation rates

To take account of periodicity in differencing parameter d, we fitted the PARFIMA $(0, d_s, 0)$ model to monthly inflation rates. Because there are additive outliers in the data, we make the estimates of the seasonal means $\bar{y}_{t,s}$ in (27) more robust by replacing outliers $I_{s,T}$ by $(I_{s,T-1} + I_{s,T+1})/2$.

The approximate maximum likelihood estimation of PARFIMA $(0, d_s, 0)$ model was performed using GAUSS code linked to "nroptmum library" written by Marius Ooms (1997).

The results from estimating a PARFIMA $(0, d_s, 0)$ model, for both cases with and without outlier correction, are reported in table 14 for United States and Canada, and in table 15 for Tunisia and South Africa. For United States inflation, it is found from the results in table 14 that the model without outlier correction is best supported by both of the information criteria ($AIC = -942.24$ and $BIC = -853.35$), whereas the model with outlier correction is not supported at all ($AIC = -936.35$ and $BIC = -836.35$). Moreover, the normality tests checked for the estimated residuals, in the two cases with and without outlier correction, indicate that they are not normally distributed. The estimation results of PARFIMA $(0, d_s, 0)$, for United States inflation, reveal that all periodic differencing parameters are significantly different from zero at 1% level, where the values of d_s range from 0.2722 through 1.1330. The smallest estimate value (0.2722) is obtained for periodic differencing parameter d_{10}, and the largest one is obtained for d_3. Thus, there is strong evidence for periodicity of long memory parameter d for United States inflation rate. The residuals standard deviation is estimated to be 0.191. The Ljung-Box test statistic Q at lag 24 testing for residuals autocorrelation, gives a value of 34.134, which is not significant at 5% level and supports the hypothesis of no residual dependence. For Canadian inflation, the reported estimation results in table 14 reveal that in term of AIC and BIC, the best fitting PARFIMA $(0, d_s, 0)$ model is obtained when the data are corrected from additive outliers. In addition, the normality tests based on Skewness, Kurtosis, and Jarque-Bera statistic show that the estimated residuals are not normally distributed, in the case without outlier correction. However, in the case with outlier correction, the normality tests reveal that the estimated residuals are normality distributed.The estimation results show also that all periodic differencing parameters d_s are significant at 5% level except d_1, which is significant at 10% level. The estimated values of d_s range from 0.2138, for $s = 4$, through 0.6581, for $s = 3$. The estimated residuals standard deviation is 0.254. The $Q(24)$ statistic of the Ljung-Box test for residuals autocorrelation, present a value of 54.927, which is significant at 5% level. Thus, we cannot reject the null hypothesis of no residual autocorrelation.

For Tunisian inflation, the estimation results of PARFIMA $(0, d_s, 0)$ reported in table 15 reveal that the model with outlier correction is best supported by both criteria AIC and BIC. As a comparison between the two cases with and without outlier correction, the residuals normality tests based on Skewness, Kurtosis and Jarque-Bera statistic indicate that the residuals are not normally distributed in both cases. But they become closer to normality in the case with outlier correction. The approximate maximum likelihood estimation of the periodic differencing parameters d_s, for $s = 1, \ldots 12$, shows that there is no evidence for periodicity since only four differencing parameters among 12 are significantly different from zero at 5% level. The residuals standard deviation is estimated to be 0.395. The Ljung-Box test statistic gives a value of Q at lag 24 equal to 35.323, which is not significant at 5% level. This reveals that the residuals are not correlated. For South African inflation, the best fitting PARFIMA (0,d$_s$,0) model, in term of AIC and BIC, is obtained in the case with outlier correction. Moreover, the normality tests on the estimated residuals,

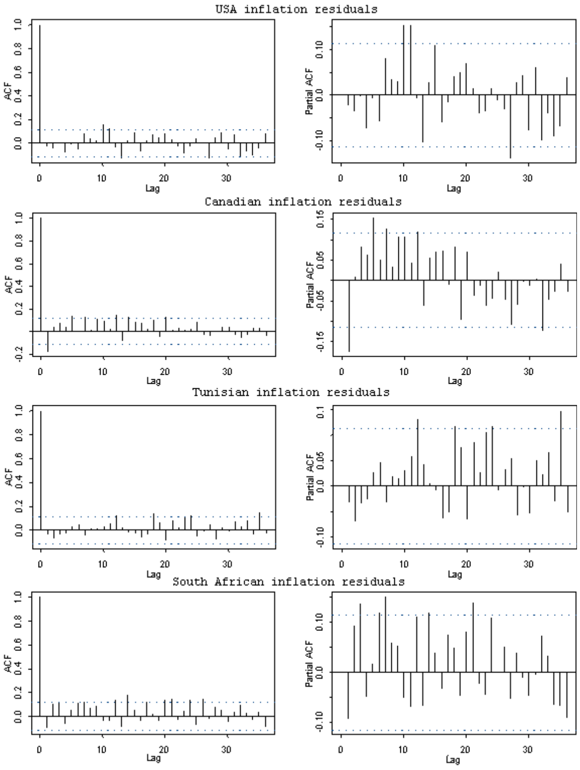

Figure 9: ACF and PACF of the estimated residuals after fitting a PARFIMA (0, d_s, 0) model to the inflation rates

when neglecting the existence of additive outliers, reveal that the residuals are not normally distributed. However, when taking account of the additive outliers, the normality tests statistics indicate that the residuals are normally distributed. As, for Tunisian inflation, the major of the estimated values of the periodic differencing parameters d_s are not significantly different from zero. The estimated residuals standard deviation is 0.470. The Ljung-Box test statistic Q at lag 24 testing for residuals autocorrelation gives a value of 69.215, which is highly significant and provides evidence of residuals autocorrelation.

Figure 9 shows the ACF and PACF of the estimated residuals after fitting a PARFIMA (0,d_s,0) model to the inflation rates in United States, Canada, Tunisia, and South Africa. It is clear that there is no serious autocorrelation especially for United States inflation rate. This confirms the results of the Ljung-Box test for residuals correlation presented above.

8 Model selection for the inflation rates

The seasonality in the inflation rates with long memory behaviour, analysed in this study, was removed using various models. Namely, the ARFIMA models with seasonal dummies variables, ARFISMA and the periodic ARFIMA (PARFIMA) model. For practical purpose, it seems sensible to evaluate the various models and to select a favourite model based on the Akaike Information Criterion (AIC) and the Schwarz information criterion (BIC). Table 5 reports the AIC and BIC criteria values for the different models applied on the inflation rates in different countries. For United States inflation rates, the results from this table reveal that the AIC is at minimum for PARFIMA(0, ds, 0) model. However, the BIC is at minimum for ARFIMA(0, 0.411, 1). For Canadian inflation, and based on the AIC criterion, the best fitting model among these reported in table 5 is an ARFIMA(0, 0.481, 1) model. However, when using the BIC criterion, the appropriate model seems to be an ARFISMA(1, 0.245, 1) model. For Tunisian inflation rate, the AIC is at minimum for the PAR(1) model. However, the periodic autoregressive parameters estimates of this model are not significant. Therefore, the PAR(1) model is not appropriate. The next model selected, in term of AIC criterion, is an ARFIMA(0, 0.249, 0). Based on BIC criterion, the best fitting model is an ARFISMA (1, 0.294, 0). This confirms the visual examination of the correlogram in figure 2, which suggests that Tunisian inflation displays a hyperbolic decay at the seasonal lags, which is characteristic of seasonal fractionally integrated processes.

According to the AIC and BIC criteria, the best fitting model for South African inflation rate seems to be an ARFISMA(1, 0.212, 1) model.

Table 5: Model selection based on AIC and BIC criteria

Models		USA	Canada	Tunisia	South Africa
ARMA	AIC	-915.171	-731.821	-511.277	-371.409
	BIC	-863.458	-655.266	-448.425	-298.430
PAR(1)	AIC	-913.996	-681.680	-518.720	-353.386
	BIC	-814.174	-575.656	-415.201	-247.362
ARFIMA	AIC	-940.511	-773.899	-514,706	-389,167
	BIC	-888.705	-704.369	-451,799	-319,637
ARFISMA	AIC	-897.988	-766.419	-503.698	-393.704
	BIC	-886.886	-733.484	-477.795	-360.769
PARFIMA	AIC	-943.761	-729.623	-500.802	-375.789
	BIC	-854.950	-623.498	-397.189	-269.664

9 Conclusion

This paper considers the application of long memory processes to describe inflation rates time series with seasonal behaviour. Thus various models were estimated for the inflation rates in four different countries; USA, Canada, Tunisia, and South Africa. The ADF test indicates the stationarity of the inflation for all countries . However, the long memory tests indicate that there is long-range dependence in inflation series for all countries. It is to be noticed that the existence of additive outliers in the inflation data was taken into account in these seasonal and periodic long memory models. The analysis was carried out using the Sowell's (1992) maximum likelihood estimation of ARFIMA (p, d, q) model and using the approximate maximum likelihood method for the estimation of PARFIMA model. We implement a procedure to obtain the maximum likelihood estimates of ARFISMA(p, d, q) model, in which dummies variables on additive outliers are included. The advantage of this parametric estimation method is that all parameters are estimated simultaneously in the time domain. We also examined the effect of additive outliers on the estimation results. Neglecting the existence of additive outliers may possibly biased estimates of the parameters. For all countries, the estimates of differencing parameters, in the ARFIMA model, are significantly different from zero. This suggests that the model is significantly different from assuming I(0) or I(1) behaviour. Instead, the inflationary dynamics display long memory. One interesting interpretation of these models is that an inflationary shock will have long memory and persistence, but that ultimately will be mean reverting. Among different models used in this study, the ARFIMA (0, d, 1) model seems to be the most appropriate one for Canadian inflation. However, Periodic ARFIMA estimates indicate evidence of periodicity of the parameter d, especially for USA inflation rates. We have accumulated evidence of the usefulness of seasonal fractional models for characterizing the inflation series. Specifically, we found that for Tunisian and South African inflation, the ARFISMA model is outperformed by the information criteria and produces reasonably clean residuals.

In summary, there is strong evidence of long memory in inflation rates with seasonal behaviour. The robustness of the long memory evidence for the inflation series in all countries suggests that persistence is a common feature of these data and that ARMA and PAR representations will generally be inadequate to capture their dynamic properties. This evidence implies that policy makers may use fractionally integrated models of inflation to good advantage in modelling inflation and to make more accurate short and long term forecasts of the future path of inflation rates. This is instrumental to the successful implementation of deflationary policies based on inflation targeting. Moreover, the empirical regularities of persistence in inflation across countries raise interesting questions as to the type of monetary policy rules and price transmission mechanism that would be consistent with this form of behaviour. Baum et all (1999) have shown that the long memory property of monetary aggregates will be transmitted to inflation, given the dependence of long-run inflation on the growth rate of money.

An interesting issue for future research will focus on analyzing the monetary policy mechanism that gives rise to this persistence in the monetary aggregates, and thus in inflation rates. Another different direction for future research concerns the analysis of possibility of structural instability caused by changing regimes. One could develop a long memory Markov switching model that explains the changing time series behaviour of inflation.

Acknowledgement

We think all members of BESTMOD laboratory at Institut Supérieur de Gestion de Tunis. We also gratefully acknowledge the contribution of the Ministry of Higher Education, Scientific Research and Technology.

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Table 6: Parameter estimates of ARMA models for the inflation rates

Countries	USA				Canada			
Parameters	With AO		Without AO		With AO		Without AO	
	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat
$\hat{\Phi}_1$	0.6307	10.593	0.6505	11.027	0.123	2.362	0.129	2.144
$\hat{\Phi}_2$	0.1131	1.915	0.1252	2.123	0.176	3.474	0.189	3.212
$\hat{\Phi}_3$	–	–	–	–	0.205	4.038	0.255	4.347
$\hat{\Phi}_4$	–	–	–	–	0.163	3.157	0.175	2.913
$\hat{\mu}_0$	-0.0625	-1.427	-0.0698	-1.593	-0.116	-2.040	-0.134	-2.016
$\hat{\mu}_1$	0.4496	7.409	0.4641	7.725	0.309	3.848	0.395	4.249
$\hat{\mu}_2$	0.1529	2.288	0.1602	2.410	0.361	4.508	0.305	3.290
$\hat{\mu}_3$	0.1414	2.324	0.1484	2.450	0.378	4.810	0.379	4.129
$\hat{\mu}_4$	0.1855	3.090	0.1794	2.981	0.138	1.751	0.125	1.357
$\hat{\mu}_5$	0.1237	2.054	0.1173	1.944	0.278	3.464	0.292	3.141
$\hat{\mu}_6$	0.2030	3.402	0.1975	3.301	0.239	3.035	0.256	2.797
$\hat{\mu}_7$	0.0314	0.520	0.0255	0.422	0.214	2.760	0.205	2.250
$\hat{\mu}_8$	0.1923	3.247	0.1888	3.178	0.050	0.645	0.034	0.372
$\hat{\mu}_9$	0.2708	4.493	0.2676	4.426	0.056	0.705	0.041	0.444
$\hat{\mu}_{10}$	0.0448	0.734	0.0386	0.631	0.216	2.721	0.207	2.228
$\hat{\mu}_{11}$	-0.0136	-0.230	-0.0182	-0.306	0.362	4.612	0.318	3.512
$\hat{\delta}_1$	0.2714	1.250	–	–	0.822	3.009	–	–
$\hat{\delta}_2$	0.2549	1.171	–	–	0.655	2.380	–	–
$\hat{\delta}_3$	0.3282	1.508	–	–	2.099	7.699	–	–
$\hat{\delta}_4$	–	–	–	–	-1.102	-4.050	–	–
$\hat{\delta}_5$	–	–	–	–	-1.031	-3.752	–	–
Kurtosis	4.0740		4.0568		3.512		9.663	
Skewness	-0.1216		-0.16938		-0.128		0.647	
JB	15.0062		15.2427		3.855		543.264	
AIC	-914.2667		-915.1707		-731.821		-647.316	
BIC	-851.4733		-863.4585		-655.266		-588.989	

*: $\hat{\delta}_j$, for $j = 1, 2, ..$ are the estimated coefficients of the dummies variables on Additive Outliers occuring at times 1980.01, 1980.02, and 1980.03, respectively, for USA inflation. And occuring at times 1981.06, 1982.05, 1991.01, 1994.02, and 2001.11, respectively, for Canadian inflation.

Table 7: Parameter estimates of ARMA models for the inflation rates

Countries	Tunisia				SouthAfrica			
Parameters	With AO		With AO		With AO		Without AO	
	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat
$\hat{\Phi}_1$	0.343	6.783	0,360	6,512	0,095	1,810	0,116	1,939
$\hat{\Phi}_2$	–	–	–	–	0,217	4,224	0,193	3,264
$\hat{\Phi}_3$	–	–	–	–	0,159	3,037	0,191	3,168
$\hat{\mu}_0$	0.486	5.506	0,475	4,905	0,245	2,020	0,217	1,542
$\hat{\mu}_1$	-0.183	-1.514	-0,055	-0,424	0,360	2,412	0,442	2,582
$\hat{\mu}_2$	-0.361	-3.064	-0,362	-2,795	0,182	1,200	0,304	1,749
$\hat{\mu}_3$	-0.477	-4.070	-0,473	-3,672	0,292	1,952	0,304	1,752
$\hat{\mu}_4$	-0.515	-4.325	-0,507	-3,874	0,453	3,040	0,520	3,038
$\hat{\mu}_5$	-0.452	-3.712	-0,358	-2,698	-0,064	-0,430	-0,067	-0,390
$\hat{\mu}_6$	-0.236	-1.979	-0,228	-1,738	0,000	0,001	0,013	0,079
$\hat{\mu}_7$	0.057	0.484	0,063	0,483	0,435	2,958	0,561	3,318
$\hat{\mu}_8$	0.179	1.516	0,235	1,833	0,260	1,725	0,259	1,484
$\hat{\mu}_9$	-0.111	-0.946	-0,117	-0,903	0,319	2,119	0,395	2,294
$\hat{\mu}_{10}$	-0.008	-0.065	-0,009	-0,070	0,120	0,821	0,108	0,636
$\hat{\mu}_{11}$	-0.114	-0.975	-0,116	-0,900	-0,019	-0,130	-0,011	-0,062
$\hat{\delta}_1$	2.120	5.033	–	–	3,254	6,314	–	–
$\hat{\delta}_2$	1.820	4.314	–	–	1,449	2,775	–	–
$\hat{\delta}_3$	1.286	3.035	–	–	2,786	5,416	–	–
$\hat{\delta}_4$	1.421	3.371	–	–	1,781	3,439	–	–
$\hat{\delta}_5$	–	–	–	–	1,600	3,103	–	–
Kurtosis	3.394		4.870		3.105		7.001	
Skewness	0.397		0.793		0.181		1.067	
JB	9.758		74.686		1.680		243.340	
AIC	-511.277		-458.596		-371.409		-291.403	
BIC	-448.425		-410.534		-298.430		-236.669	

*: $\hat{\delta}_j$, for $j = 1, 2, ..$ are the estimated coefficients of the dummies variables on Additive Outliers occuring at times 1979.05, 1980.01, 1981.01, and 1981.08, respectively, for Tunisian inflation. And occuring at times 1979.07, 1980.09, 1985.02, 1986.01, and 1993.04, respectively, for South African inflation.

Table 8: Best model specification, among the ARFIMA model, for the USA and Canadian inflation series according to the AIC and BIC criteria.

	USA				Canada			
ARFIMA	With AO		Without AO		With AO		Without AO	
Parameters	Est.	Std.err	Est.	Std.err	Est.	Std.err	Est.	Std.err
$\hat{d}$	0.4025	0.0507	0.4113	0.0500	0.4814	0.0249	0.4702	0.0375
$\hat{\phi}$	–	–	–	–	–	–	–	–
$\hat{\theta}$	0.1635	0.0789	0.1692	0.0784	-0.3956	0.0646	-0.3874	0.0699
$\hat{\mu}_0$	0,1197	0,1853	0.1206	0.2043	0.1192	0.3907	0.1156	0.3668
$\hat{\mu}_1$	0,3784	0,0449	0.3870	0.0443	0.3000	0.0680	0.3971	0.0829
$\hat{\mu}_2$	0,3099	0,0540	0.3225	0.0538	0.3832	0.0648	0.3408	0.0790
$\hat{\mu}_3$	0,3068	0,0570	0.3226	0.0571	0.4239	0.0647	0.4235	0.0799
$\hat{\mu}_4$	0,3395	0,0582	0.3395	0.0588	0.2013	0.0653	0.2009	0.0805
$\hat{\mu}_5$	0,2887	0,0590	0.2888	0.0597	0.3826	0.0663	0.4090	0.0809
$\hat{\mu}_6$	0,3383	0,0592	0.3384	0.0600	0.3383	0.0664	0.3658	0.0810
$\hat{\mu}_7$	0,1926	0,0589	0.1927	0.0596	0.3067	0.0655	0.3064	0.0809
$\hat{\mu}_8$	0,2676	0,0580	0.2677	0.0587	0.1346	0.0652	0.1343	0.0804
$\hat{\mu}_9$	0,3774	0,0563	0.3775	0.0569	0.1345	0.0646	0.1343	0.0797
$\hat{\mu}_{10}$	0,2295	0,0529	0.2295	0.0534	0.2451	0.0639	0.2450	0.0788
$\hat{\mu}_{11}$	0,0905	0,0435	0.0905	0.0436	0.3685	0.0671	0.3176	0.0820
$\hat{\delta}_1^*$	0,3944	0,1932	–	–	2.2433	0.2373	–	–
$\hat{\delta}_2$	0,2138	0,1934	–	–	0.6672	0.2371	–	–
$\hat{\delta}_3$	0,3160	0,2118	–	–	-1.2207	0.2372	–	–
$\hat{\delta}_4$	–	–	–	–	0.6391	0.2371	–	–
$\hat{\delta}_5$	–	–	–	–	-1.009	0.2371	–	–
Kurtosis	4.2675		4.2901		3.4412		10.6607	
Skewness	-0.3470		-0.3683		-0.1201		0.7355	
JB	26.0160		27.4926		3.0174		727.6764	
$\hat{\sigma}_{\varepsilon}^{**}$	0.1967		0.1981		0.2431		0.2986	
LB(20)***	31.041 (0.055)		32.120 (0.042)		24.737 (0.212)		16.015 (0.716)	
AIC	-938.9070		-940.5111		-773.8990		-666.1057	
BIC	-875.9994		-888.7049		-704.3688		-614.8730	
log-likelihood	63.0940		60.737066		0.8680		-58.9352	

*: $\hat{\delta}_j$, for $j = 1, 2, ..$ are the estimated coefficients of the dummies variables on Additive Outliers occuring at times 1980.03, 1980.01, and 1980.02 ,respectively, for USA inflation. And occuring at times 1991.01, 1981.06, 2001.11, 1982.05, and 1994.02, respectively, for Canadian inflation.

** : Estimated residuals standard deviation.

*** : Probability in parentheses

Table 9: Best model specification, among the ARFIMA model, for the Tunisian and South African inflation series according to the AIC and BIC criteria.

	Tunisia				South Africa			
ARFIMA	With AO		Without AO		With AO		Without AO	
Parameters	Est.	Std.err	Est.	Std.err	Est.	Std.err	Est.	Std.err
$\hat{d}$	0.2486	0.0445	0.2483	0.0446	0,3216	0,0573	0.2893	0.0556
$\hat{\phi}$	–	–	–	–	-0,2153	0,0788	-0.2050	0.0757
$\hat{\theta}$	–	–	–	–	–	–	–	–
$\hat{\mu}_0$	0,6871	0,1218	0.6907	0.1336	0,6611	0,1819	0.6694	0.1827
$\hat{\mu}_1$	-0,1465	0,1042	-0.0265	0.1114	0,2803	0,1335	0.3578	0.1569
$\hat{\mu}_2$	-0,3268	0,1073	-0.3276	0.1178	0,1334	0,1273	0.2369	0.1492
$\hat{\mu}_3$	-0,5677	0,1099	-0.5684	0.1206	0,2871	0,1308	0.2852	0.1546
$\hat{\mu}_4$	-0,6877	0,1112	-0.6883	0.1221	0,4681	0,1333	0.5379	0.1557
$\hat{\mu}_5$	-0,6655	0,1130	-0.5810	0.1228	-0,0227	0,1327	-0.0240	0.1565
$\hat{\mu}_6$	-0,4116	0,1121	-0.4120	0.1230	0,0506	0,1328	0.0495	0.1567
$\hat{\mu}_7$	-0,0598	0,1118	-0.0602	0.1227	0,4147	0,1340	0.5458	0.1565
$\hat{\mu}_8$	0,2005	0,1122	0.2398	0.1219	0,2098	0,1316	0.2093	0.1555
$\hat{\mu}_9$	-0,0035	0,1097	-0.0037	0.1204	0,3578	0,1320	0.4163	0.1544
$\hat{\mu}_{10}$	0,0173	0,1069	0.0172	0.1174	0,1836	0,1253	0.1835	0.1487
$\hat{\mu}_{11}$	-0,0811	0,1002	-0.0811	0.1100	0,0139	0,1304	0.0141	0.1551
$\hat{\delta}_1^*$	1,5925	0,3933	–	–	3,1681	0,4723	–	–
$\hat{\delta}_2$	2,1269	0,3925	–	–	2,5314	0,4715	–	–
$\hat{\delta}_3$	0,9888	0,3925	–	–	1,7640	0,4719	–	–
$\hat{\delta}_4$	1.2938	0,3933	–	–	1,4092	0,4716	–	–
$\hat{\delta}_5$	–	–	–	–	1,7151	0,4715	–	–
Kurtosis	3,5630		5.2255		3,2448		7.2570	
Skewness	0,2020		0.6665		0,2491		1.1232	
JB	5,9821		83.8417		3,6852		277.0614	
$\hat{\sigma}_\varepsilon^{**}$	0.4001		0.4392		0.4759		0.5601	
LB(20)***	24.885 (0.206)		28.122 (0.107)		36.755 (0.013)		30.855 (0.057)	
AIC	-514,7063		-466.9400		-389,1672		-305.5176	
BIC	-451,7988		-418.8341		-319,6370		-254.2850	
log-likelihood	-150.0663		-177.9193		-194,0005		-240.6370	

*: $\hat{\delta}_j$, for $j = 1, 2, ..$ are the estimated coefficients of the dummies variables on Additive Outliers occuring at times 1980.01, 1979.05, 1981.08, and 1981.01 ,respectively, for Tunisian inflation. And occuring at times 1979.07, 1985.02, 1986.01, 1980.09, and 1993.04, respectively, for South African inflation.

** : Estimated residuals standard deviation.

*** : Probability in parentheses

Table 10: Best model specification, among the Autoregressive Moving Average (ARMA) model whith a fractionally differenced seasonal component, for the USA and Canadian inflation series according to the AIC and BIC criteria.

Countries	USA						Canada					
Parameters	(1, d, 0)						(1, d, 1)					
	With AO dummies			Without AO dummies			With AO dummies			Without AO dummies		
	Est.	Std.er	t-stat	Est.	Std.er	t-stat	Est.	Std.er	t-stat	Est.	Std.er	t-stat
$\hat{d}$	0.2734	0.042	6.583	0.2661	0.041	6.478	0.2449	0.044	5.585	0.2182	0.045	4.877
$\hat{\Phi}$	0.5977	0.050	12.03	0.6223	0.047	13.15	0.9832	0.013	74.52	0.9768	0.018	55.28
$\hat{\theta}$	–	–	–	–	–	–	-0.8769	0.036	-24.45	-0.8667	0.043	-20.17
$\hat{\mu}$	0.3713	0.069	5.358	0.3748	0.073	5.126	0.4294	0.191	2.252	0.4061	0.177	2.290
$\hat{\delta}_1$	0.5186	0.204	2.544	–	–	–	2.1403	0.235	9.114	–	–	–
$\hat{\delta}_2$	0.3900	0.205	1.906	–	–	–	0.6227	0.236	2.645	–	–	–
$\hat{\delta}_3$	0.3228	0.227	1.421	–	–	–	-1.1064	0.236	-4.680	–	–	–
$\hat{\delta}_4$	–	–	–	–	–	–	0.7974	0.235	3.391	–	–	–
$\hat{\delta}_5$	–	–	–	–	–	–	-0.9866	0.235	-4.201	–	–	–
Kurtosis	3.899			3.879			3.263			10.162		
Skewness	-0.175			-0.148			-0.111			0.851		
JB	11.599			10.722			1.416			648.011		
$\hat{\sigma}_{\varepsilon}^{**}$	0.2177			0.2207			0.2552			0.3099		
LB(20)***	62.545 (0.000)			60.381			18.221(0.573)			13.806(0.840)		
AIC	-899.926			-897.988			-766.419			-665.154		
BIC	-877.723			-886.886			-733.484			-650.516		
log-likelihood	32.788			28.761			-11.637			-68.604		

*: $\hat{\delta}_j$, for $j = 1, 2, ..$ are the estimated coefficients of the dummies variables on Additive Outliers occuring at times 1980.03, 1980.01, and 1980.02 ,respectively, for USA inflation. And occuring at times 1991.01, 1981.06, 2001.11, 1982.05, and 1994.02, respectively, for Canadian inflation.

** : Estimated residuals standard deviation.

*** : Probability in parentheses.

Table 11: Best model specification, among the Autoregressive Moving Average (ARMA) model whith a fractionally differenced seasonal component, for the Tunisian and South African inflation series according to the AIC and BIC criteria.

Countries	Tunisia						South Africa					
Parameters	(1, d, 0)						(1, d, 1)					
	With AO dummies			Without AO dummies			With AO dummies			Without AO dummies		
	Est.	Std.er	t-stat	Est.	Std.er	t-stat	Est.	Std.er	t-stat	Est.	Std.er	t-stat
$\hat{d}$	0.2939	0.044	6.719	0.2759	0.043	6.464	0.2124	0.052	4.049	0.1914	0.049	3.883
Φ	0.3093	0.058	5.307	0.3188	0.057	5.614	0.8891	0.092	9.706	0.9769	0.026	36.86
θ	–	–	–	–	–	–	-0.7496	0.137	-5.452	-0.9246	0.053	-17.46
μ	0.4598	0.082	5.559	0.4836	0.086	5.592	0.8636	0.118	7.299	0.9182	0.177	5.186
δ_1	1.1341	0.391	2.899	–	–	–	3.0554	0.476	6.415	–	–	–
δ_2	2.1899	0.379	5.770	–	–	–	2.5155	0.462	5.442	–	–	–
δ_3	1.2671	0.377	3.361	–	–	–	1.9591	0.462	4.238	–	–	–
δ_4	0.9149	0.389	2.351	–	–	–	1.3693	0.465	2.946	–	–	–
δ_5	–	–	–	–	–	–	1.8213	0.462	3.942	–	–	–
Kurtosis	3.5974			5.204			3.2382			7.678		
Skewness	0.3870			0.788			0.2759			1.253		
JB	11.9083			91.466			4.3202			336.817		
$\hat{\sigma}_{\varepsilon}^{**}$	0.4213			0.4583			0.4889			0.5773		
LB(20)***	22.441(0.317)			24.107(0.238)			31.223(0.052)			27.496(0.122)		
AIC	-503.6983			-461.358			-393.7042			-308.320		
BIC	-477.7952			-450.257			-360.7689			-293.682		
log-likelihood	-162.5155			-188.02316			-201.2421			-248.021		

*: $\hat{\delta}_j$, for $j = 1, 2, ..$ are the estimated coefficients of the dummies variables on Additive Outliers occuring at times 1980.01, 1979.05, 1981.08, and 1981.01 ,respectively, for Tunisian inflation. And occuring at times 1979.07, 1985.02, 1986.01, 1980.09, and 1993.04, respectively, for South African inflation.

** : Estimated residuals standard deviation.

*** : Probability in parentheses.

Table 12: PAR (1) estimation results for USA and Canadian inflation rates.

Countries	USA				Canada			
	With AO		Without AO		With AO		Without AO	
Parameters	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat
$\hat{\phi}_{1,1}$	0,517	3,00	0,705	5,25	0,483	2,12	0,262	0,99
$\hat{\phi}_{1,2}$	0,691	3,76	0,824	5,89	-0,017	-0,16	0,057	0,46
$\hat{\phi}_{1,3}$	0,876	5,85	0,940	7,61	0,707	4,80	0,707	4,09
$\hat{\phi}_{1,4}$	0,565	4,87	0,565	4,86	0,360	2,40	0,360	2,04
$\hat{\phi}_{1,5}$	0,736	5,69	0,736	5,68	0,494	2,41	0,589	2,48
$\hat{\phi}_{1,6}$	0,903	7,48	0,903	7,45	0,374	2,44	0,472	2,67
$\hat{\phi}_{1,7}$	0,568	4,84	0,568	4,83	0,356	2,38	0,356	2,03
$\hat{\phi}_{1,8}$	0,560	4,15	0,560	4,14	0,511	2,54	0,511	2,16
$\hat{\phi}_{1,9}$	0,807	4,91	0,807	4,89	0,638	2,94	0,638	2,50
$\hat{\phi}_{1,10}$	0,502	3,18	0,502	3,17	0,571	2,94	0,571	2,5
$\hat{\phi}_{1,11}$	0,824	5,12	0,824	5,11	0,566	2,82	0,804	3,94
$\hat{\phi}_{1,12}$	1,121	7,26	1,121	7,240	0,275	1,89	0,275	1,61
$\hat{\mu}_1$	0,411	9,30	0,412	9,30	0,320	4,97	0,437	5,94
$\hat{\mu}_2$	0,070	0,76	0,019	0,24	0,456	5,69	0,369	4,07
$\hat{\mu}_3$	0,043	0,60	0,024	0,35	0,203	2,38	0,203	2,03
$\hat{\mu}_4$	0,199	3,10	0,199	3,09	0,094	0,99	0,094	0,84
$\hat{\mu}_5$	0,063	0,90	0,063	0,90	0,307	3,79	0,319	3,35
$\hat{\mu}_6$	0,085	1,37	0,085	1,36	0,210	2,25	0,209	1,90
$\hat{\mu}_7$	0,039	0,60	0,039	0,60	0,219	2,50	0,219	2,12
$\hat{\mu}_8$	0,199	3,52	0,199	3,51	0,009	0,10	0,009	0,08
$\hat{\mu}_9$	0,177	2,46	0,177	2,45	0,071	0,96	0,071	0,82
$\hat{\mu}_{10}$	0,083	0,98	0,083	0,98	0,194	2,75	0,194	2,34
$\hat{\mu}_{11}$	-0,085	-1,29	-0,085	-1,28	0,238	2,60	0,131	1,40
$\hat{\mu}_{12}$	-0,113	-2,29	-0,113	-2,28	-0,046	-0,57	-0,046	-0,48
$\hat{\delta}_1^*$	0,472	1,74	–	–	1,088	3,62	–	–
$\hat{\delta}_2$	0,307	1,10	–	–	0,904	3,02	–	–
$\hat{\delta}_3$	0,191	0,74	–	–	2,3127	7,752	–	–
$\hat{\delta}_4$	–	–	–	–	-1,2207	-4,064	–	–
$\hat{\delta}_5$	–	–	–	–	-0,8067	-2,360	–	–
JB	13.761		9.753859		0.726		596.753	
AIC	-914.765		-913.996		-681.680		-594.152	
BIC	-826.035		-814.174		-575.656		-506.409	

*: $\hat{\delta}_j$, for $j = 1, 2, ..$ are the estimated coefficients of the dummies variables on Additive Outliers occuring at times 1980.01, 1980.02, and 1980.03, respectively, for USA inflation. And occuring at times 1981.06, 1982.05, 1991.01, 1994.02, and 2001.11, respectively, for Canadian inflation.

Table 13: PAR (1) estimation results for Tunisian and South African inflation rates.

Countries	Tunisia				South Africa			
	With AO		Without AO		With AO		Without AO	
Parameters	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat
$\hat{\phi}_{1,1}$	0,267	1,42	0,503	2,590	-0,177	-0,78	0,226	0,954
$\hat{\phi}_{1,2}$	0,381	3,03	0,381	2,749	-0,254	-1,48	-0,247	-1,232
$\hat{\phi}_{1,3}$	0,056	0,33	0,056	0,296	-0,110	-0,83	-0,110	-0,705
$\hat{\phi}_{1,4}$	0,173	0,67	0,173	0,606	0,206	1,03	0,291	1,253
$\hat{\phi}_{1,5}$	0,171	0,68	0,381	1,396	0,164	0,90	0,164	0,765
$\hat{\phi}_{1,6}$	0,391	2,80	0,391	2,535	0,225	0,84	0,225	0,72
$\hat{\phi}_{1,7}$	0,186	0,98	0,186	0,89	0,246	1,03	0,474	1,72
$\hat{\phi}_{1,8}$	-0,169	-1,02	-0,251	-1,40	0,217	1,80	0,217	1,54
$\hat{\phi}_{1,9}$	0,377	2,71	0,377	2,45	0,683	3,72	0,446	2,18
$\hat{\phi}_{1,10}$	0,711	3,71	0,711	3,36	0,417	2,58	0,417	2,21
$\hat{\phi}_{1,11}$	0,615	3,56	0,615	3,23	0,452	2,25	0,452	1,92
$\hat{\phi}_{1,12}$	0,848	4,58	0,848	4,15	0,331	1,74	0,331	1,49
$\hat{\mu}_1$	0,356	2,31	0,317	1,89	1,052	5,80	0,875	4,25
$\hat{\mu}_2$	0,099	0,84	0,099	0,76	1,047	5,04	1,163	4,80
$\hat{\mu}_3$	0,118	1,14	0,118	1,03	1,064	6,60	1,064	5,64
$\hat{\mu}_4$	-0,006	-0,06	-0,006	-0,06	0,957	4,39	0,935	3,67
$\hat{\mu}_5$	0,035	0,43	0,118	1,33	0,453	1,84	0,453	1,57
$\hat{\mu}_6$	0,244	2,97	0,244	2,69	0,578	2,84	0,578	2,43
$\hat{\mu}_7$	0,589	6,04	0,589	5,48	0,915	4,56	0,876	3,73
$\hat{\mu}_8$	1,003	7,36	1,104	7,60	0,618	3,43	0,618	2,93
$\hat{\mu}_9$	0,343	2,23	0,343	2,02	0,389	1,94	0,695	3,18
$\hat{\mu}_{10}$	0,221	1,42	0,221	1,29	0,401	1,96	0,401	1,67
$\hat{\mu}_{11}$	0,177	1,20	0,177	1,09	0,298	1,48	0,298	1,26
$\hat{\mu}_{12}$	0,174	1,24	0,174	1,13	0,443	2,65	0,443	2,26
$\hat{\delta}_1$	2,166	5,23	–	–	3,030	5,69	–	–
$\hat{\delta}_2$	1,832	4,46	–	–	2,326	4,22	–	–
$\hat{\delta}_3$	1,346	3,09	–	–	2,819	5,37	–	–
$\hat{\delta}_4$	1,199	2,89	–	–	2,344	3,99	–	–
$\hat{\delta}_5$	–	–	–	–	1,430	2,69	–	–
JB	5.276		80.957		3.306		192.491	
AIC	-518.720		-463.794		-353.386		-267.690	
BIC	-415.201		-375.064		-247.362		-179.946	

*: $\hat{\delta}_j$, for $j = 1, 2, ..$ are the estimated coefficients of the dummies variables on Additive Outliers occuring at times 1979.05, 1980.01, 1981.01, and 1981.08, respectively, for Tunisian inflation. And occuring at times 1979.07, 1980.09, 1985.02, 1986.01, and 1993.04, respectively, for South African inflation.

Table 14: Parameter estimates of PARFIMA models for the inflation rates

Countries	USA				Canada			
Estimates	With A.O		Without A.O		With A.O		Without A.O	
	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat
$\hat{d}_1$	0.277	1.93	0.498	3.06	0.240	1.86	0.212	1.79
$\hat{d}_2$	0.429	3.12	0.501	4.24	0.240	2.63	0.142	2.07
$\hat{d}_3$	1.128	7.27	1.133	7.37	0.658	2.79	0.652	3.18
$\hat{d}_4$	0.419	3.89	0.377	3.86	0.214	1.98	0.175	1.49
$\hat{d}_5$	0.443	4.10	0.420	4.00	0.392	2.89	0.423	3.11
$\hat{d}_6$	0.960	4.32	0.948	4.24	0.252	2.74	0.309	3.03
$\hat{d}_7$	0.331	3.46	0.316	3.27	0.235	2.32	0.191	1.96
$\hat{d}_8$	0.319	2.89	0.318	2.81	0.344	2.50	0.341	2.02
$\hat{d}_9$	0.523	2.88	0.518	2.86	0.398	2.41	0.397	1.95
$\hat{d}_{10}$	0.276	2.74	0.272	2.71	0.357	3.00	0.355	2.47
$\hat{d}_{11}$	0.582	3.58	0.576	3.53	0.341	2.52	0.605	2.63
$\hat{d}_{12}$	1.019	3.97	1.017	3.91	0.282	2.27	0.185	1.94
K	4.203		4.221		3.130		11.248	
SK	-0.155		-0.176		-0.107		0.983	
JB	19.235		20.122		0.747		859.648	
$\hat{\sigma}_\varepsilon^*$	0.191		0.191		0.254		0.303	
LB(24)**	35.585 (0.060)		34.134 (0.082)		54.927 (0.000)		41.338 (0.015)	
AIC	-937.294		-943.761		-729.623		-638.765	
BIC	-837.382		-854.950		-623.498		-550.938	

* : Estimated residuals standard deviation

** : Probability in parentheses

Table 15: Parameter estimates of PARFIMA models for the inflation rates

Countries	Tunisia				South Africa			
Estimates	WithA.O		Without A.O		With A.O		Without A.O	
	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat
$\hat{d}_1$	0.159	1.34	0.334	1.95	-0.073	-1.29	0.005	0.06
$\hat{d}_2$	0.169	1.36	0.187	1.46	0.137	1.42	0.126	1.35
$\hat{d}_3$	0.082	0.90	0.071	0.76	0.120	1.13	0.030	0.39
$\hat{d}_4$	-0.002	-0.03	-0.011	-0.12	0.221	1.67	0.354	2.01
$\hat{d}_5$	0.160	0.95	0.317	0.86	0.228	1.56	0.138	1.14
$\hat{d}_6$	0.941	3.94	0.322	2.06	0.143	0.97	0.161	0.93
$\hat{d}_7$	0.195	1.61	0.179	1.42	0.061	0.51	0.112	0.50
$\hat{d}_8$	0.107	1.40	0.084	1.09	0.332	2.89	0.238	2.41
$\hat{d}_9$	0.169	1.43	0.305	1.81	0.398	2.65	0.365	2.36
$\hat{d}_{10}$	0.726	3.02	0.598	2.30	0.272	2.19	0.289	2.02
$\hat{d}_{11}$	0.490	3.20	0.466	2.70	0.310	2.44	0.282	1.91
$\hat{d}_{12}$	0.779	2.82	0.752	2.35	0.229	1.81	0.241	1.62
Kurtosis	3.837		5.500		3.263		7.46	
Skewness	0.290		0.629		0.236		1.236	
JB	12.902		97.604		3.488		310.557	
$\hat{\sigma}_{\varepsilon}^{*}$	0.395		0.426		0.470		0.558	
LB (24)**	35.32 (0.06)		27.46 (0.28)		69.21 (0.00)		60.63 (0.00)	
AIC	-500.802		-462.520		-375.789		-287.845	
BIC	-397.189		-373.710		-269.664		-200.018	

* : Estimated residuals standard deviation.

** : Probability in parentheses