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February 2009

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First draft: September 2006
This draft: May 2010

Abstract

This paper investigates the business cycle fluctuations of the tradeable and nontradeable sectors of the US economy. Then, it evaluates whether a "New Open Economy" model can reproduce the observed fluctuations qualitatively. The answer is positive: both in the model and in the data the standard deviations of tradeable inflation, output and employment are significantly higher than the standard deviations of the corresponding nontradeable sector variables. A key role in generating this result is played by the greater responsiveness of tradeable sector variables to monetary shocks.

JEL classification: F41; E32

Keywords: New Open Economy Macroeconomics; Tradeable and Nontradeable Sectors; Business Cycles.

*I would like to thank Luca Benati, Gianluca Benigno, Fabio Canova, Wendy Carlin, Huw Dixon, Vincent Labhard, Kerry Patterson, Christoph Schleicher and Gianluca Violante for useful comments and advice. I also thank seminar participants at the Bank of England, Durham University, University of Loughborough, Universidad del País Vasco, University of Sydney, Uppsala University, CDMA conference in St. Andrews, 2nd conference Recent Developments in Macroeconomics and the EEA for their comments and discussions. Any errors or shortcomings are my sole responsibility.

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1 Introduction

In the field of international macroeconomics there are now many models that explicitly consider two sectors, one producing tradeable and the other producing nontradeable goods. The explicit modelling of the tradeable and nontradeable sectors has often been done solely in order to explain certain features of the aggregate economy (for example, the observed deviations from purchasing power parity), rather than to understand the properties of the sectors themselves.

However, the strategy of adding a tradeable and a nontradeable sector to an open economy model is not exempt from its own challenges. For example, it is interesting to see whether the implications of these models for the two sectors are matched by real-world observations.

The purpose of this paper is to develop an open economy model with tradeables and nontradeables, estimate it by the Generalised Method of Moments (GMM), and then check whether its implications for the tradeable and nontradeable sectors are reflected in the US data. The model presented in this paper follows the "New Open Economy Macroeconomics" (NOEM) paradigm, and the comparison between the data and the model is restricted to second-order moments. The NOEM paradigm is chosen because of its importance in the literature. The decision to restrict the comparison to second-order moments is motivated by the existence of measurement problems,[1] and by the relatively stylised nature of the model.

From the point of view of the empirical researcher, large-scale estimated models, such as, for example, Smets and Wouters (2003), are clearly superior. On the other hand, the more complexity is added into a model, the more it becomes difficult to isolate (among shocks, ad-hoc frictions and theoretical underpinnings) the exact causes of certain facts. The choice made in this paper is to include, whenever possible, many modelling assumptions already present in the NOEM literature, but with the aim of offering a comprehensive yet parsimonious framework,[2] rather than searching for an *ad hoc* specification

[1] This approach in dealing with measurement problems originates from Kydland and Prescott (1982).

[2] The closest model to the one presented in this paper is Benigno and Thoenissen (2003). They construct a comprehensive framework, encompassing several modelling assumptions that had been analysed individually in the previous literature. The model presented in this paper is different from their model because it includes government expenditure shocks, it specifies monetary policy in terms of the growth rate of money rather than an interest rate feedback rule, and it does not restrict the elasticities of substitution (between tradeables

that fits the data.

After the initial contributions of Ghironi (2000), Bergin (2003), and Lubik and Schorfheide (2005), the literature on estimating NOEM models has grown considerably in recent years. This paper differs from other contributions not just because of the estimation methodology,[3] but because of the goal of the investigation, which is to compare the properties of the tradeable and nontradeable sectors in the model and in the US data. To this purpose, the paper also derives a system of three equations in three unknowns that illustrates why the shocks in the NOEM affect the two sectors differently. In this way it is possible isolate the exact causes of the model's implications.

Earlier on, it was hinted that this sort of analysis is hampered by a measurement problem. In a nutshell, the properties of the tradeable and nontradeable sectors can only be imperfectly measured, since virtually all sectors (as measured in the official statistics) have both tradeable and nontradeable goods. The strategy adopted here to deal with this problem is to find robust features of the data by comparing the statistics among several sectors, and to restrict ourselves to qualitative, rather than quantitative, comparisons.

In spite of this measurement problem in the data, there is sufficient evidence to suggest that in the US economy business cycle fluctuations are more pronounced in the tradeable than in the nontradeable sector. When the NOEM model is fed with the estimated values, it is successful in generating standard deviations of tradeable inflation, output and employment that are significantly higher than the standard deviations of the corresponding nontradeable sector variables. This occurs because of the high responsiveness of tradeable sector variables to domestic monetary shocks, which are the most important source of fluctuations in the model (although technology shocks also have a role in explaining sectoral employment fluctuations).

One of the contributions of this paper is to derive a system of three equations that illustrates the key variables or channels of transmission of the exogenous shocks to the ratios of tradeable to nontradeable prices, output and employment. This system shows that the same channels which ensure the international transmission of shocks (the nominal exchange rate, the terms of trade and the asset market) also affect the responses of tradeable sector variables to a domestic monetary shock. In particular, the responses of tradeable

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and nontradeables, and between Home and Foreign tradeables) to being equal to one.

[3] For example, Ghironi estimates a NOEM by nonlinear least squares at the single-equation level and FIML system-wide regressions. Bergin uses maximum likelihood techniques, and Lubik and Schorfheide put forward a Bayesian approach.

sector variables to domestic monetary shocks are amplified through the nominal exchange rate and the asset market channels, while the terms of trade channel is comparatively weaker.

The outline of the remainder of the paper is as follows. Section 2 considers the measurement problem and presents some statistics for several sectors of the US economy. Section 3 explains the model and its numerical solution. Section 4 puts forward a system of log-linearised equations that illustrate why the shocks have different effects in the two sectors. The estimation and calibration of the model is explained in Section 5. Using the equations of Section 4, we can understand the model-implied statistics, which are presented in Section 6. By checking whether the results are sensitive to some of the parametrized values, we can further investigate the properties of the NOEM model. These sensitivity checks are discussed in Section 7. Finally, Section 8 concludes.

2 The evidence

It is often problematic to find data series disaggregated by sector, for example, the US' Bureau of Economic Analysis produces only annual, not quarterly, estimates of its GDP-by-industry accounts. Moreover, it is difficult to isolate in the data the tradeable and the nontradeable sectors explicitly, since virtually in any sector there are goods that are actually traded and goods that are not traded.[4] However, the proportion of output that is traded is not the same in all sectors, so it is possible to decide an approximation, in order to translate the abstract notion of tradability into an operational concept, but only at the cost of accepting a measurement error.

With these considerations in mind, we can start to investigate the cyclical properties of the tradeable and nontradeable sectors by looking at the standard deviation of output and inflation in all US industries, and see whether we can identify any visible pattern. The industry classification is the one adopted by the Bureau of Economic Analysis. As noted above, this data is at the annual frequency and unfortunately there is no data on employment, imports and exports in the same industries. To facilitate the analysis, the industries in Table 1 are divided into two groups, tradeables and nontradeables, following a common classification in the literature.[5] In order to establish some

[4] Conceptually it is possible to divide goods into tradeables and nontradeables, but disaggregated macroeconomic data, if available, is only for sectors as defined in the statistics.

[5] See, among others, Betts and Kehoe (2006). Agriculture, mining and manufacturing are

proportions and facilitate the analysis, the industries in Table 1 are listed by their contribution to total GDP, with the largest contributors coming first.

TABLE 1 HERE

By looking at Table 1, it is evident that, overall, the tradeable sector is characterised by more volatility than the nontradeable sector. As far as output is concerned, only one nontradeable industry, construction, has more volatile output than manufacturing, the largest tradeable industry. But construction only accounts for 4.4% of US GDP, and all the three larger nontradeable industries (Finance, Government and Professional services), much bigger in size than construction, have less volatile output than manufacturing.[6]

As far as inflation is concerned, the evidence is somehow less strong, but it still points to more volatility in the tradeable sector. As much as 5 nontradeable industries (Utilities, Wholesale trade, Transportation and warehousing, Retail trade and Construction) have more volatile inflation rates than manufacturing. However, overall these 5 industries contribute to total GDP by significantly less than the three larger nontradeable industries, which all have less volatile inflation than manufacturing.

Additional evidence, obtained from quarterly data on manufacturing and services only, will be presented in Section 6, but Table 1 remains useful for comparison purposes. By comparing the data at different frequencies and sectoral classifications, we can identify which findings are not robust, and therefore may have been induced by the choice of tradeable-nontradeable approximation.

3 The model

The building blocks of the model are illustrated in this section. Most of the assumptions and functional forms are already present in the NOEM literature, so as to facilitate comparisons.

However, the model possesses one feature that is not common in the literature, namely the assumption that individuals cannot contemporaneously supply their labour to the production of both tradeable and nontradeable goods, but they can work only in one sector at a time. This assumption is

commonly classified as tradeable, and services, utilities, and construction as nontradeable.

[6] Moreover, the two other tradeable industries, agriculture and mining, have even more volatile output than manufacturing.

often true in practice, and, from a modelling point of view, it is also sufficient to ensure that all the labour adjustment takes place along the extensive margin.[7] This result is useful for the estimation of the model, since it is possible to find quarterly data on persons employed, but not on hours worked, in each sector.

3.1 Building blocks of the model

The world economy consists of two countries of equal size, named Home and Foreign, that engage in the production and trade of differentiated goods (or differentiated brands of the same good) for final consumption. Each country has two sectors, one producing a continuum of tradeables and the other a continuum of nontradeables.

In each country and in each sector there exists a continuum of monopolistic firms, each of them producing a single differentiated product, or brand. The firms and the goods they produce are indexed by $f_{TH} \in [0, 1]$ for the Home tradeable sector and $f_N \in [0, 1]$ for the Home nontradeable sector. In the Foreign country, they are indexed by $f^*_{TF} \in [0, 1]$ and $f^*_N \in [0, 1]$ respectively (Foreign variables and indexes are denoted with stars). Moreover, both the Home and the Foreign countries are populated by a continuum of identical individuals of measure one.

3.1.1 Individual preferences and budget constraints

There is no possibility of migration across countries, but individuals can move costlessly from one sector to the other within each country. As in Burnside, Eichenbaum and Rebelo (1993), any individual who works incurs a fixed participation cost, measured in units of foregone leisure.

Labour services cannot be contemporaneously supplied to both the tradeable and nontradeable goods sector, but since sectors could pay different wages, this restriction introduces individual heterogeneity in the model.

Nonetheless, this problem can be easily dealt with by applying Rogerson's (1988) result for sectoral economies. It basically states that, under the assumption of separable utility, if individuals can choose the probabilities of working in sectors and buy insurance against the resulting income risk, then the decentralized equilibrium reproduces the socially optimal allocation. Moreover, the socially optimal allocation for initially identical individ-

[7] This point will be clarified on page 13.

uals specifies that the marginal utility of consumption must be equal for all individuals. If utility is separable, then this implies that consumption levels must be equal for all individuals in each period. As a result, ex-ante identical individuals will be also identical ex-post.

Following Rogerson, the probabilities of working in each sector are added to the individual maximization problem, and individuals are allowed to vary their labour supply along both the extensive and the intensive margins. That is, the utility of a representative individual in the Home country is written as follows:

$$U_0 = E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \frac{C_t^{1-\sigma}-1}{1-\sigma} + \frac{\chi}{1-\varepsilon}\left(\frac{M_t}{P_t}\right)^{1-\varepsilon} + n_{TH,t}\cdot\kappa\left(\Gamma-\psi-\mathbf{h}_{TH,t}\right) \\ +n_{N,t}\cdot\kappa\left(\Gamma-\psi-\mathbf{h}_{N,t}\right) \\ +\left(1-n_{TH,t}-n_{N,t}\right)\cdot\kappa\left(\tau\right) \end{array} \right], \tag{1}$$

where C is the aggregate consumption index, $\frac{M}{P}$ are real money balances, n_{TH}, n_N are the probabilities of working in the tradeable and nontradeable sector respectively, ψ is a fixed cost of participation, the same for all individuals,[8] and $\mathbf{h}_{TH} = \int_0^1 h_{TH}\left(f_{TH}\right) df_{TH}$ and $\mathbf{h}_N = \int_0^1 h_N\left(f_N\right) df_N$ are the total hours that the individual supplies to the sectors TH and N respectively. Foreign preferences are similarly written, with the same parameters σ, χ, ε, Γ, τ and ψ and functional form κ.

At the international level, markets are incomplete: individuals trade in a one-period non-contingent real bond, denominated in units of the Home tradeable goods consumption index, sold at the price P_T. Interest is decided at the beginning of the period and paid at the end. Similarly to Benigno (2001), individuals must pay a small cost in order to undertake a position in the international asset market.[9] This cost is assumed to be a payment in exchange for intermediation services, offered by financial firms located in both the Home and the Foreign country. Individuals pay this cost only to firms located in their own country.

The period-t budget constraint of the representative individual in the Home country is as follows:

[8] Total time available is different for the employed (Γ) and the unemployed (τ). By assuming that τ is sufficiently small, it is possible to ensure that the unemployed do not enjoy greater utility ex-post than the employed.

[9] This assumption ensures stationarity of the model and a well-defined steady state, as demonstrated by Schmitt-Grohe and Uribe (2003).

$$B_t P_{T,t} + \frac{\nu}{C_0} B_t^2 P_{T,t} + M_t \leq (1 + r_{t-1})\, B_{t-1} P_{T,t} + M_{t-1}$$
$$+TR_t - P_t C_t + n_{TH,t} W_{TH,t} \mathbf{h}_{TH,t} + n_{N,t} W_{N,t} \mathbf{h}_{N,t}$$
$$+ \int_0^1 \Pi_{TH,t}\,(f_{TH})\, df_{TH} + \int_0^1 \Pi_{N,t}\,(f_N)\, df_N + R_t \ , \tag{2}$$

where B is the internationally traded bond, $\frac{\nu}{C_0} B$ is the cost of holding one unit of the bond,[10] which depends on the positive parameter ν, M are nominal money balances, r is the real interest rate, TR are government transfers, W_{TH} and W_N are the wages paid in the tradeable and nontradeable sector respectively, $\Pi_{TH}\,(f_{TH})$ and $\Pi_N\,(f_N)$ are the profits that the individual receives from firms f_{TH} (tradeable sector) and f_N (nontradeable sector), and R represents the rents generated by the financial intermediaries.[11]

The Foreign budget constraint is entirely similar, with the same parameter ν. The internationally traded bond B is in zero net supply worldwide.

3.1.2 Consumption indexes

The preferences over tradeable and nontradeable goods in the Home country are specified as follows:

$$C_t = \left[(1-\gamma)^{\frac{1}{\phi}} \left(C_{T,t}\right)^{\frac{\phi-1}{\phi}} + \gamma^{\frac{1}{\phi}} \left(C_{N,t}\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \ ,$$

where $(1-\gamma)$ and γ are preference weights, and ϕ is the substitution elasticity. Preferences in the Foreign country are described by an equivalent aggregator, with the same parameters γ and ϕ.

The aggregators for tradeable goods consumption in the Home and Foreign countries at date t are, respectively:

$$C_{T,t} = \left[(1-\delta)^{\frac{1}{\theta}} \left(C_{TH,t}\right)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}} \left(C_{TF,t}\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \ ,$$

[10] C_0 denotes the steady-state value of Home consumption.

[11] Individuals are allocated to the sectors randomly, but they can perfectly share the income risk resulting from the lottery. All individuals then receive the average wage, given their chosen n_{TH} and n_N, as demonstrated by Rogerson (1988). Hence probabilities appear in the budget constraint (2).

$$C^*_{T,t} = \left[(1-\delta^*)^{\frac{1}{\theta}} \left(C^*_{TH,t} \right)^{\frac{\theta-1}{\theta}} + (\delta^*)^{\frac{1}{\theta}} \left(C^*_{TF,t} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} .$$

The elasticity of substitution θ between type-TH and type-TF goods is the same in both countries, but the weights δ and δ^* can differ.

The preferences for the individual goods or varieties are also represented by CES aggregators, for example, in the Home country the preferences for the domestic tradeable varieties are given by:

$$C_{TH,t} = \left[\int_0^1 c_{TH,t} \left(f_{TH} \right)^{\frac{\eta_T - 1}{\eta_T}} df_{TH} \right]^{\frac{\eta_T}{\eta_T - 1}} .$$

The elasticities of substitution among differentiated varieties, tradeables and nontradeables, may be different. However these two parameters, which are inversely related to the degree of monopolistic competition, are assumed to be the same in both countries.

3.1.3 Government budget constraint and money supply

The Home and Foreign governments purchase only nontradeable goods[12] produced in their own country. As in Chari, Kehoe and McGrattan's (2002) model, money growth rates follow AR(1) processes, having zero unconditional mean. The budget constraint of the Home government at date t is given by:

$$M_t - M_{t-1} = P_{N,t} G_t + TR_t \ , \tag{3}$$

where G is a public expenditure aggregator or production function:

$$G_t = \left[\int_0^1 g_t \left(f_N \right)^{\frac{\eta_N - 1}{\eta_N}} df_N \right]^{\frac{\eta_N}{\eta_N - 1}} .$$

The Foreign government budget constraint and the public expenditure aggregator are entirely analogous. Government expenditures in both countries follow AR(1) processes with zero unconditional mean.

[12] According to the Bureau of Economic Analysis' "Guide to the National Income and Product Accounts of the United States", government expenditure essentially consists of services provided to the public free of charge. Goods (and services) that are sold by the government are instead classified as personal consumption expenditure (if purchased by individuals), or intermediate inputs (if purchased by businesses).

3.1.4 Firms

Nominal rigidities are introduced à la Calvo (1983), by assuming that each firm has a fixed probability of changing her price at date t. All prices are set in the currency of the buyer, thus tradeable goods firms in both countries set two different prices, one for the Home market and one for the Foreign market, denominated in the respective local currencies. However, the degree of exchange rate pass-through is not necessarily zero, since export prices can adjust to changes in the nominal exchange rate.

More formally, I follow the approach of Corsetti and Pesenti (2005), and assume that the local currency prices[13] of exports of Home and Foreign tradeable varieties f_{TH} and f^*_{TF} are given, respectively, by:

$$p^*_{TH,t}\left(f_{TH}\right) = \frac{\widetilde{p}_{TH,t}\left(f_{TH}\right)}{e_t^{\zeta^*}} \ , \qquad p_{TF,t}\left(f^*_{TF}\right) = e_t^{\zeta} \widetilde{p}^*_{TF,t}\left(f^*_{TF}\right) \ ,$$

where e is the nominal exchange rate (price of the Home currency in terms of the Foreign currency), ζ^* and ζ are the pass-through elasticities, constant by assumption, and $\widetilde{p}_{TH}\left(f_{TH}\right)$ and $\widetilde{p}^*_{TF}\left(f^*_{TF}\right)$ are predetermined components that are not adjusted to variations in the exchange rate during period t.[14]

The Home tradeable sector firm f_{TH} chooses the price $p_{TH,t}\left(f_{TH}\right)$ of domestic sales, and the predetermined component $\widetilde{p}_{TH,t}\left(f_{TH}\right)$ of the export price, by solving the following problem:[15]

[13] Prices of individual varieties are denoted with lower cases, price indexes (the prices of the consumption aggregators) are denoted with upper cases. Price indexes are defined in the standard way, as the minimal expenditures needed to buy one unit of the corresponding consumption aggregators.

[14] Thus, if ζ^* and ζ are equal to one exchange rate pass-through is complete, and if ζ^* and ζ are equal to zero the exchange rate pass-through is zero.

[15] In this model firms take into account the demand for their product when maximizing profits, but they take the individuals' allocative choices and supply of hours as given. The assumptions on the functional forms and the requirement that $\alpha \leq 1$ ensure that profits are a concave function of prices.

$$\max \quad E_t \sum_{j=0}^{\infty} (\varphi_T \beta)^j Q_{t,t+j} \left[\begin{array}{c} \frac{p_{TH,t}(f_{TH})}{P_{t+j}} \cdot y_{TH,t+j|t}(f_{TH}) \\ +e_{t+j} \frac{p^*_{TH,t+j}(f_{TH})}{P_{t+j}} y^*_{TH,t+j|t}(f_{TH}) \\ -\frac{W_{TH,t+j}}{P_{t+j}} \cdot \widetilde{h}_{TH,t+j|t}(f_{TH}) \end{array} \right] ,$$

$$\begin{aligned} \text{s.t.} \quad y_{TH,t+j|t}(f_{TH}) &= \left(\frac{p_{TH,t}(f_{TH})}{P_{TH,t+j}} \right)^{-\eta_T} C_{TH,t+j} \ , \\ y^*_{TH,t+j|t}(f_{TH}) &= \left(\frac{p^*_{TH,t+j|t}(f_{TH})}{P^*_{TH,t+j}} \right)^{-\eta_T} C^*_{TH,t+j} \ , \\ p^*_{TH,t+j|t}(f_{TH}) &= \widetilde{p}_{TH,t}(f_{TH}) \, e_{t+j}^{-\zeta^*} \ , \end{aligned}$$

where $Q_{t,t+j} = \frac{u'(C_{t+j})}{u'(C_t)}$, and $(\varphi_T)^j$ is the probability that $p_{TH,t}(f_{TH})$ and $\widetilde{p}_{TH,t}(f_{TH})$ still apply at the future date $t+j$. The variables $y_{TH,t+j|t}(f_{TH})$, $y^*_{TH,t+j|t}(f_{TH})$ and $\widetilde{h}_{TH,t+j|t}(f_{TH})$ denote the demands for the good and the total labour input used by the firm, if the prices decided at t still apply at date $t+j$.

Output sold at Home and abroad is produced using a common plant or production function:[16]

$$y_{TH,t}(f_{TH}) + y^*_{TH,t}(f_{TH}) = z_{TH,t} \cdot \widetilde{h}_{TH,t}(f_{TH})^{\alpha_T} \ , \tag{4}$$

where α_T is a sector-specific parameter that allows for decreasing returns to labour, and z_{TH} represents technology, which affects the productivity of labour. Wages are flexible. The aggregate of all labour inputs used by firm f_{TH} is given by:[17]

$$\widetilde{h}_{TH,t}(f_{TH}) = n_{TH,t} \cdot h_{TH,t}(f_{TH}) \ .$$

Tradeable and nontradeable goods differ not only with respect to consumption, but also from the point of view of production, as the key parameters are

[16] The assumption of no investment in physical capital is still very common in new open economy models, therefore it is also made here. The inclusion of capital may or may not alter the transmission of shocks in these models, at least along some dimensions. For example, Chari, Kehoe and McGrattan (2002) found that almost all of the movements in output come from variations in labour, with little or no impact from physical investment.

[17] The aggregate labour input is given by the number of hours worked in the sector by each individual, times the measure of individuals working in that sector. Because of the law of large numbers, the probabilities chosen at the individual level and the fraction of individuals at the aggregate level that work in a given sector coincide.

allowed to be different. The parameters that are specific to nontradeable sector firms are denoted with φ_N, η_N and α_N. The production functions and maximization problems of Foreign firms f^*_{TF} and f^*_N are the same as in the Home country.

Finally, the growth rate of technology for each country and sector follows an AR(1) process with zero unconditional mean.

3.2 The solution of the model

The rest of the paper focuses on a symmetric equilibrium, so all firms that can modify their price at date t set the same price.

The model cannot be solved in closed form, and a numerical approximated solution must be found instead. This is obtained by log-linearising the equations around a deterministic equilibrium or steady state[18] in which all the exogenous stochastic processes are set equal to their unconditional means, their variances are set to zero, and net foreign asset positions are normalised at zero.[19] The resulting system is then solved using Uhlig's "Toolkit" algorithm (1999).[20] The shocks to the exogenous stochastic processes are all assumed to be temporary.

Importantly, the steady-state terms of trade is not normalised but it is computed explicitly.[21] A close inspection of the steady-state equations reveals that the steady-state terms of trade depends not only on the preference parameters but also on real factors, such as the unconditional means of the productivity processes. In particular, three of these unconditional means are free parameters, which are calibrated so as to ensure that the steady state of the model reproduces three facts in the data: the ratios of tradeable to nontradeable output in the two countries, and the ratio of Home to Foreign

[18] We can think of the steady state as the deterministic equilibrium that is attained in the limit, as $t \to \infty$, when there is no money growth and all the exogenous processes are constant and equal to their expected level. Because of the assumptions made earlier on the international asset market, this equilibrium is stationary. Moreover, it coincides with the flexible price equilibrium. In fact, as $t \to \infty$, everybody has been given the chance to adjust the price. If there are no shocks, then at each date all firms that adjust the price set the same price, thus the economy approaches the flexible price equilibrium as $t \to \infty$.

[19] No country is a net borrower or lender in the steady state, but international borrowing and lending occur in the short-run or transitional equilibrium path.

[20] The computer code is available from the author on request.

[21] The method used in the computation of the steady state is adapted from Obstfeld and Rogoff (1995). The calculations are available from the author on request.

tradeable output. These ratios are computed using year-2000 data from the Groningen 60-Industry Database.[22]

An important feature of the solution is that hours are always endogenously constant. As a result, all the adjustment in the labour inputs takes place through the extensive margin, i.e. the participation rates or probabilities.[23]

4 The transmission of shocks to the tradeable and nontradeable sectors

4.1 Introduction and methodology

In a general equilibrium model, the channels through which the exogenous shocks are propagated to the economy can be many. While the effects of the shocks can be seen in the impulse responses, it is possible to identify analytically the channels through which the shocks are transmitted to the sectors only with a closed form solution. A closed form solution is not available, but we can proceed by aggregating as many optimality and equilibrium conditions as it is possible without losing analytical tractability.

The purpose of this Section is to present a system of three equations, (6), (8) and (9), which illustrates the key variables or channels of transmission through which the shocks are propagated to sectoral output, employment and prices.[24] Since some key explanatory variables are endogenous, the system

[22] Groningen Growth and Development Centre, 60-Industry Database, February 2005, http://www.ggdc.net. The database is comparable with the OECD STAN Database. Since the year 2000 is the base year of the Groningen dataset, the data for the year 2000 does not depend on the computation of output deflators.

[23] This happens for the following reason. From the Home individual maximization problem, by combining the first order condition with respect to $\mathbf{h}_{TH,t}$ with the first-order condition with respect to $n_{TH,t}$, we obtain:

$\kappa\left(\Gamma-\psi-\mathbf{h}_{TH,t}\right)-\kappa\left(\tau\right)=-\kappa'\left(\Gamma-\psi-\mathbf{h}_{TH,t}\right)\mathbf{h}_{TH,t}$

Analogously, by combining the first order condition with respect to $\mathbf{h}_{N,t}$ with the first-order condition with respect to $n_{N,t}$, we obtain:

$\kappa\left(\Gamma-\psi-\mathbf{h}_{N,t}\right)-\kappa\left(\tau\right)=-\kappa'\left(\Gamma-\psi-\mathbf{h}_{N,t}\right)\mathbf{h}_{N,t}$

It is then immediate to see that, at least for most commonly used functional forms, both the above two equations are satisfied when hours worked in the two sectors are constant and equal to each other, in the steady state and at each date t.

[24] The idea is to understand why the shocks affect the two sectors differently. For example, if the ratio $Y_{TH,t}^{Tot}/Y_N$ remains constant after a given shock occurs, then the responses of tradeable and nontradeable output to the shock are identical. If, for example, $Y_{TH,t}^{Tot}/Y_N$ increases and both responses have positive sign, then the response of $\widehat{Y}_{TH,t}^{Tot}$ is larger than the

provides a "partial equilibrium" analysis, therefore, some knowledge of how the shocks affect the explanatory variables is required.[25]

Equations (6) to (9) are derived under the assumption that the probability of changing prices (φ), the elasticity of output with respect to hours (α) and the elasticity of substitution among varieties (η) are the same in both sectors[26]. Moreover, in this Section we also assume $\theta = 1$, as this also simplifies the equations without affecting our understanding.[27]

All the equations presented in this section describe the short-run equilibrium after a shock occurs at date t, under the assumption that in period $t-1$ the economy is at its steady state.

4.2 Definitions

Since the equations of the system are all derived from the log-linearised solution, it is necessary to introduce first some notation. For any variable X, let X_0 denote the value of the variable at the deterministic equilibrium or steady state. Let $\widehat{X}_t \equiv \log(X_t/X_0) \simeq (X_t - X_0)/X_0$ denote the approximate short-run log-deviation from the initial steady state, and let $dX_t \equiv (X_t - X_0)/C_0$ denote instead the linear deviation, normalised with respect to steady-state consumption.

Total tradeable output is the sum of output sold at home and abroad:

$$Y_{TH,t}^{Tot} \equiv Y_{TH,t} + Y_{TH,t}^{*} = C_{TH,t} + C_{TH,t}^{*} \ .$$

Tradeable sector firms set two different prices, one for domestic sales and one for exports. I define the price index for all Home tradeable goods as a weighted average, with weights taken from the steady state:

response of $\widehat{Y}_N$. If $\widehat{Y}_{TH,t}^{Tot}$ responds more than $\widehat{Y}_N$ after all shocks (or the most significant ones), then the model predicts that tradeable output is more volatile than nontradeable output.

[25] In a closed form solution, endogenous variables are functions of only exogenous shocks and parameters. This approach is used for explanatory purposes only, the statistics and the impulse responses of Section 6 result from the full DSGE model.

[26] These simplifying assumptions actually aid our understanding of the transmission mechanism, as we can see why the shocks can have a fundamentally different impact in the two sectors, even if the tradeable and nontradeable sector do not possess any distinguishing feature apart from the use of output in consumption.

[27] If θ is different from one then the parameter δ in equations (6) and (8) is replaced by the steady-state export share, which is increasing in δ.

$$P_{TH,t}^{Tot} \equiv \frac{P_{TH,t} \cdot Y_{TH0} + e_t P_{TH,t}^{*} \cdot Y_{TH0}^{*}}{P_{TH0} \cdot Y_{TH0} + e_0 P_{TH0}^{*} \cdot Y_{TH0}^{*}} \ . \tag{5}$$

The terms of trade plays a crucial role in the transmission of shocks. It is defined as the price of Home imports over the price of Home exports:

$$T_t \equiv \frac{P_{TF,t}}{e_t \cdot P_{TH,t}^{*}} \ .$$

4.3 Prices

In the model, prices are determined by the firms' price setting behaviour. From the first-order condition of the firm maximization problem, it is possible to derive an expression describing the evolution of inflation in the Home tradeable sector. By subtracting from that expression its counterpart for the Home nontradeable sector,[28] we obtain:

$$\begin{aligned} \widehat{P}_{TH,t}^{Tot} - \widehat{P}_{N,t} &= \delta\left(1-\zeta^{*}\right)\widehat{e}_t \\ +\beta E_t &\left[\pi_{TH,t+1}^{Tot} - \pi_{N,t+1} - \delta\left(1-\zeta^{*}\right)\left(\widehat{e}_{t+1} - \widehat{e}_t\right)\right] \\ + &\left(\frac{1-\varphi\beta}{1+\eta\frac{1-\alpha}{\alpha}}\frac{1-\varphi}{\varphi}\right)\left(\widehat{MC}_{TH,t} - \widehat{MC}_{N,t}\right) \ , \end{aligned} \tag{6}$$

where $\pi_{TH,t+1}^{Tot} \equiv \widehat{P}_{TH,t+1}^{Tot} - \widehat{P}_{TH,t}^{Tot}$ denotes inflation in the tradeable sector, $\pi_{N,t+1} \equiv \widehat{P}_{N,t+1} - \widehat{P}_{N,t}$ denotes inflation in the nontradeable sector, MC_{TH} denotes real marginal cost in the tradeable sector:

$$\widehat{MC}_{TH,t} \equiv \widehat{W}_{TH,t} - \widehat{P}_{TH,t}^{Tot} - \frac{1}{\alpha}\widehat{z}_{TH,t} + \frac{1-\alpha}{\alpha}\widehat{Y}_{TH,t}^{Tot} \ , \tag{7}$$

and MC_N is analogously defined.

From Equation (6), we can infer that short-run movements in the relative price depend on changes in the current and future nominal exchange rate, expectations of future inflation and real marginal cost differentials.

Monetary shocks are transmitted to the relative price equation (i.e. the supply of relative output) via changes in the nominal exchange rate and expected inflation differentials. The response of the relative price $P_{TH,t}^{Tot}/P_{N,t}$

[28] Detailed derivations of all the equations are available from the author on request.

to changes in the nominal exchange rate crucially depends on the degree of pass-through. If the pass-through is incomplete ($\zeta^* < 1$), then a depreciation of the Home currency in the current period (a positive $\widehat{e}_t$) has a positive effect on the tradeable goods price index. This happens because export prices are set in Foreign currency, so after a depreciation Home tradeable sector firms receive more Home currency for each unit of output sold abroad (Equation 5). However, an expected depreciation in the next period will have, *ceteris paribus*, an opposite effect on today's relative tradeable goods price index. In this case, Home tradeable sector firms know that in the next period they will automatically receive more Home currency for each unit of exports, so today they increase their prices less.

Productivity shocks are transmitted to Equation (6) via changes in marginal costs. A positive productivity shock, for example, lowers firms' real marginal costs, and induces them to lower their prices. If the productivity shock and the resulting fall in the marginal cost are persistent, then expected future inflation, which appears on the right-hand side of Equation (6), also falls. Therefore, under a positive productivity shock in the tradeable sector the relative price falls, while the opposite happens under a positive productivity shock in the nontradeable sector.[29]

4.4 Output and employment

In the short-run output is demand-determined. By manipulating the demands for tradeable and nontradeable goods, and using the Foreign resource constraint to substitute out the demand for Home exports, we obtain:

$$\widehat{Y}_{TH,t}^{Tot} - \widehat{Y}_{N,t} = -\phi \left(\widehat{P}_{TH,t}^{Tot} - \widehat{P}_{N,t} \right) + \delta \left(1 - \phi\right) \widehat{T}_t + \delta k_4 dB_t - k_7 dG_t \ . \tag{8}$$

The coefficients k_4 and k_7 are computed from the steady state equations, and they are both positive. Notice that if there were no imports ($\delta = 0$), and thus the economy was closed, then only the relative price and government expenditure would affect relative output demand.

Equation (8) shows that, keeping everything else unchanged, when the relative price $P_{TH,t}^{Tot}/P_{N,t}$ increases the demand for relative output decreases.

[29] Notice that $\widehat{P}_{TH,t}^{Tot}$ and $\widehat{P}_{N,t}$ appear both on the left and on the right-hand side of equation (6), since they affect the two marginal costs. It is possible re-write equation (6) so that the price indexes are all on the left-hand side, but the analysis would stay unchanged.

Ceteris paribus, when the terms of trade increases relative output increases (provided ϕ is less than one). An increase in (Home) bond holdings relative to the steady state implies that the Foreign country is increasing consumption through debt, so there is more demand for Home exports and relative output goes up. Finally, when government expenditure increases relative output decreases, as there is more demand for nontradeable goods.

The terms of trade and bond holdings, which are on the left-hand side of Equation (8), are affected by Home monetary shocks and by all Foreign shocks. Thus, Home monetary shocks and Foreign shocks are transmitted to the relative output demand through changes in the terms of trade and, by means of the interest rate, changes in bond holdings.

Equations (6) and (8) can be described as supply and demand respectively, which jointly determine relative output and the relative price (Figure 1).[30] It is worth pointing out that the Home and Foreign money demand and the Euler equations for consumption are the only equations that were left out in the derivation of Equations (6) and (8). However, by adding them we would not recover another channel of transmission, because the transmission of shocks through intertemporal substitution and the interest rate is already represented by the change in bonds in Equation (8). Therefore, the system is sufficient to capture all the channels through which Home monetary shocks and Foreign shocks are transmitted to the demand and supply of relative output: the nominal exchange rate and expected inflation differentials (Equation 6), and the terms of trade and the asset market (Equation 8).[31]

Finally, a simple manipulation of the production functions in the two sectors:

$$\widehat{n}_{TH,t} - \widehat{n}_{N,t} = \frac{1}{\alpha}\left(\widehat{Y}^{Tot}_{TH,t} - \widehat{Y}_{N,t}\right) - \frac{1}{\alpha}\left(\widehat{z}_{TH,t} - \widehat{z}_{N,t}\right) \ , \tag{9}$$

shows that the changes in relative employment depend only on changes in relative output and on the productivity shocks.

[30] The slope of the relative supply curve depends on $(1-\alpha)/\alpha$, the coefficient on output in Equation (7). If $\alpha < 1$, the slope is positive because the marginal productivity falls with production, so firms charge higher prices to compensate for the fall in productivity.

[31] Canova (2005) identifies two channels of international transmission, one operating through the terms of trade and the other through the interest rate. Both of them feature on the right-hand side of Equation (8).

5 Estimation

This section begins with some background information on the sample period and presents some applied choices.[32] Then, it illustrates some parameter choices prior to the GMM estimation, describes the choice of moment conditions, and finally concludes with a brief comment on the estimated parameters values.

The sample period is 1980:1 to 2007:4. The Home country is represented by the US, and the Foreign country by an aggregate of its major trading partners. The latter is comprised by Canada, France, Germany,[33] Japan, Mexico and the UK, which together represented 46% of the US total trade in goods in 2007.[34] The combined GDP of these six countries was 104% of the US GDP in the last quarter of 2007.

The tradeable sector is represented by manufacturing, and the nontradeable sector by services. This approximation is advantageous because quarterly observations on output, prices and employment levels are available, and it is consistent with standard assumptions in the literature.

Not all of the model parameters could be estimated by GMM, as in some cases identification problems occurred during estimation. Table 2 shows the parameters that have not been estimated by GMM but instead have been chosen according to suggestions made in the literature.[35] I check the robustness of the results of Section 6 to changes in all the parameters of Table 2. The most interesting of these sensitivity checks are presented in Section 7.[36]

TABLE 2 HERE

The intermediation cost parameter ν is chosen so that the spread in the nominal interest rates approximates the benchmark value suggested by Benigno (2001). The preference weights γ and δ are calibrated so that the steady-state import and service shares in consumption are consistent with the

[32] Detailed appendices illustrating the construction of the data variables and the derivation of the moment conditions are available from the author on request.

[33] East Germany is not included in the time series up until 1990:4.

[34] Author's calculations based on Bureau of Economic Analysis data. China has recently emerged as another top US trading partner, but it was not included in the aggregate of Foreign countries because of the limited availability of data on the Chinese economy.

[35] In doing so, I do not take into account parameter uncertainty in the GMM estimation of the other parameters.

[36] The specification of the functional form κ and the calibration of the parameters χ, Γ, τ and ψ are irrelevant for the solution.

US data,[37] while δ^* is set equal to $1-\delta$ for symmetry. The benchmark value for the elasticity of substitution θ between Home and Foreign tradeables is taken from Obstfeld and Rogoff (2005). The values for η_T and η_N are those suggested by Faruqee, Laxton, Muir and Pesenti (2005) for the US economy. I use the short-run elasticities of exchange rate pass-through into import prices estimated by Campa and Goldberg (2005) to parameterize ζ and ζ^*.[38] The probabilities of not changing prices are set equal in both countries and sectors, and their value implies an average price duration of one year. Finally, α_T and α_N are chosen so as to match the labour shares in value added in the US manufacturing and service sectors.[39]

Since the parametrized values of β, α_T and α_N enter the moment conditions, they might affect the GMM estimates. I have found that if β is in the range $[0.97, 0.99]$ and both α_T and α_N are between their calibrated values and 0.65, the parameter estimates of Table 3 are not very much affected.[40]

The estimated parameters and the moment conditions are presented in Tables 3 and 4 respectively. The choice of an exactly identified system is motivated by the small size of the sample. The optimal weighting matrix is computed using the Newey and West (1987) estimator with a Bartlett kernel.[41]

TABLE 3 HERE
TABLE 4 HERE

The moment conditions are derived from the log-linearised solution (as in Ghironi 2000), and have been estimated using logged, seasonally adjusted and Hodrick-Prescott (HP) filtered data,[42] with $\lambda = 1,600$.

The first and second moment conditions are obtained by combining the Home and Foreign consumption Euler equations, the first-order conditions

[37] That is, the ratio of imports of goods over total expenditure for goods (equal to 0.35), and the share of services in total (tradeable and nontradeable) consumption (equal to 0.56). The calibrated values for γ, δ and δ^* are broadly consistent with the literature: see, among others, Benigno and Thoenissen (2003), and Obstfeld and Rogoff (2004).

[38] Specifically, ζ is their estimated value for the US, and ζ^* is a weighted average of their estimates for Canada, France, Germany, Japan and the UK.

[39] These are equal to 0.64 and 0.56 respectively.

[40] All sensitivity checks are available on request.

[41] I have also verified that the estimates are not significantly affected by the choice of kernel or lag length.

[42] Variables must be detrended because they enter the log-linearised equations as percentage deviations from the steady state. In Ghironi (2000), the steady state is a constant trend, while in the real business cycle literature it is common to detrend the variables using the HP filter instead. I prefer to use the HP filter to allow for nonlinear trends in the data.

for money balances and the definitions of the nominal interest rates, using contemporaneous real money balances and consumption differentials as instruments.[43]

The third moment condition is obtained from the log-linearised nontradeables expenditure share, using the contemporaneous price ratio as the instrument.

Finally, the remaining moment conditions result from the properties of the exogenous stochastic processes $\widehat{x}_j$. In order to reduce the computational cost, I do not estimate all the covariances among shocks. Instead, I proceed as follows. First, I run a separate estimate having the full variance-covariance matrix, and compute all the correlation coefficients. Then, I keep in the final system only the covariances associated with correlation coefficients not lower than than 0.15, and I fix all the other covariances at zero.[44]

On the whole, the estimated parameter values agree with the suggestions made in the literature.[45] The estimated risk aversion for consumption σ is very close to the value suggested by Chari, Kehoe and McGrattan (2002). Obstfeld and Rogoff (2005) noted that the elasticity of substitution between tradeables and nontradeables was found to be lower than one in some empirical studies. Finally, a quick calculation shows that the estimated standard deviation of US tradeable productivity shocks is equal to 0.82%, thus broadly consistent with the values found in the real business cycle literature.[46]

[43] Chari, Kehoe and McGrattan (2002) estimate the utility parameters from a single-country money demand equation, estimated using US data. I prefer to use a relative money demand equation in order to make use of both US and Foreign data (the model restricts ε and σ to be the same in the two countries), with a parsimonious instrument set.

[44] All the covariances fixed at zero were not statistically significant.

[45] The GMM estimation of DSGE models is often barred by convergence problems, but not in our case. This is because most of the moment conditions of Table 4 are almost derived from the definitions of the parameters, and in practice describe the data quite well. In general, it is more difficult to obtain estimates from a model's optimality conditions, since small-scale models may not fit the data well. In our model, the only two parameters that are estimated from optimality conditions are σ and ε: unfortunately both estimates have a relatively high standard error, but, on a more positive note, they are both economically acceptable.

[46] For example, Prescott's (1986) estimate of the standard deviation of US aggregate (not sectoral) productivity shocks is 0.763%.

6 Results

6.1 Identifying the properties of the data

As explained in Section 2, the compilation of statistics on the tradeable and nontradeable sectors is affected by a measurement problem. The measurement problem affects also the GMM estimates, since these were based on the approximation of tradeables with manufacturing, and nontradeables with services. However, by identifying the tradeable sector with manufacturing we neglect agriculture or mining, and by identifying the nontradeable sector with the service sector we include also services that are actually traded. As far as the estimates are concerned, this measurement problem is unfortunately unavoidable.[47]

In order to take into account the measurement problem in the comparison of the model with the data, this paper adopts a specific approach, outlined as follows. First, only second-order moments are considered, obtained from the same data set that was used to estimate the model. These data moments are presented in Table 5, and they are chosen so as to characterise the cyclical properties of the US tradeable and nontradeable sectors.

TABLE 5 HERE

Secondly, wherever possible the findings of Table 5 are validated by seeing whether they are also reproduced in Table 1, which includes more sectors.[48] Finally, the comparison between the data and the model's statistics is qualitative in nature rather than quantitative. This is reasonable since in practice there is no dichotomy between the tradeable and the nontradeable sectors.

We can now concentrate on the properties of the data as illustrated by Table 5. We will first check whether they are compatible with the findings of Section 2, and then we will turn our attention to the model-generated statistics.

According to Table 5, the time series volatility is remarkably higher in the tradeable sector, which confirms all the findings of Table 1. And although

[47] However, this measurement problem does not affect equally all the estimated values, for example, it does not affect the variance of the monetary shocks. This consideration confirms that the comparison between the data and the model-generated statistics cannot be strictly quantitative.

[48] The actual numbers cannot be compared since Table 5 is based on quarterly data and Table 1 on annual data.

Table 1 does not report any statistics regarding employment, if we postulate that tradeable output is more volatile than nontradeable output, then it is reasonable to assume that the labour input is more volatile too.

Therefore, in order to match the data the model must generate standard deviations of inflation, output and employment in the tradeable sector that are significantly higher than the analogous standard deviations in the nontradeable sector.

6.2 The model-implied statistics

The statistics obtained from the estimated model are presented in Table 6, while the impulse responses to all shocks are presented in Figures 2 to 4.

TABLE 6 HERE
FIGURES 2 TO 4 HERE

The impulse responses are ordered according to the estimated standard deviation of the shocks, with the responses to the shocks having the higher standard deviation coming first. There exist a clear demarcation among shocks, since the standard deviation of the first four is considerably higher than the standard deviation of the last four shocks.

Overall, the estimated model generates standard deviations of tradeable inflation, output and employment that are significantly higher than the standard deviations of the corresponding nontradeable sector variables. Moreover, the cross correlations are all positive, as in Table 5.

In order to assess the contribution of each shock to the volatility of each variable, I perform a variance decomposition exercise. I orthogonalise the shocks using the Cholesky method, but since this method gives a different answer depending on the ordering of the shocks, I compute variance decompositions for each possible ordering of the 8 shocks (40,320), and then calculate the averages. Table 7 reveals that Home monetary shocks are the most important source of fluctuations of sector-specific inflation rates and output levels, while the other shocks have a considerably smaller influence. Home monetary shocks also explain a considerable share of the total variance of employment in the tradeable and nontradeable sectors, but employment levels are also significantly influenced by Home technology shocks.

TABLE 7 HERE

The increase in prices, output and employment levels after a positive Home monetary shock (Figures 2 to 4) is a standard result, common to both the producer currency pricing model of Obstfeld and Rogoff (1995) and the local currency pricing model of Betts and Devereaux (2000). However, the responses are not the same in the two sectors, since tradeable inflation, output and employment levels react more to a domestic monetary shock than the corresponding nontradeable variables. The sensitivity or responsiveness of tradeable sector variables to Home monetary shocks is the main cause of the higher volatility in the tradeable sector, while Home technology shocks play a more important role in explaining the volatility of sectoral employment levels.

How can Section 4's equations be used to explain the sensitivity of tradeable sector variables to Home monetary shocks? Consider, for example, Equation (8). If tradeable output reacts more to a Home monetary shock than nontradeable output, then it must be true that the monetary shock affects the right-hand side of Equation (8), causing $\widehat{Y}_{TH,t}^{Tot} - \widehat{Y}_{N,t}$ to become positive. This can be explained by considering separately two channels of transmission of Home monetary shocks, the asset market and the terms of trade.

The transmission of a Home monetary shock through the asset market can be explained as follows. A positive Home monetary shock causes a fall in the real interest rate and an increase in Home bond holdings.[49] Since dB_t becomes positive, the demand for relative output (8) shifts to the right, causing $\widehat{Y}_{TH,t}^{Tot} - \widehat{Y}_{N,t}$ to become positive. This shift to the right has the following economic motivation. The asset market allows the Foreign country to increase its consumption via borrowing. As a result, there is more demand for Home exports, so Home tradeable output increases more than nontradeable output.

The transmission through the terms of trade can be explained as follows. A positive Home monetary shock causes a nominal depreciation, which results in a terms of trade deterioration. Given that ϕ is lower than one, if nothing else happened, a decrease in the terms of trade would cause the demand for relative output (8) to shift to the left, and tradeable output to increase less than nontradeable output. Since the opposite happens instead (Figure 3), then it must be true that the transmission to the relative output demand through the terms of trade is "weaker" than the transmission through the asset market.

[49] Impulse responses of all variables are available from the author on request.

The equations of Section 4 can also be used to explain the higher response of tradeable inflation after a Home monetary shock (Figure 2). With the same price stickiness parameter for both sectors, expected future inflation rate differentials are small; moreover, with this parametrization there is only a mild exchange rate overshooting. Therefore, the most significant change on the right-hand side of the relative supply curve (6) is the exchange rate depreciation at time t, which causes it to shift up and $\widehat{P}^{Tot}_{TH,t} - \widehat{P}_{N,t}$ to become positive. This occurs because, with imperfect pass-through, the Foreign currency revenues of Home firms increase, so the tradeable price index (5) increases.

Since the nominal exchange rate, the terms of trade and bond holdings are affected not only by Home monetary shocks, but by Foreign shocks too, these open economy channels also amplify the responses of tradeable sector variables to Foreign shocks, a fact that can be easily verified by looking at Figures 2 to 4. In other words, the same channels which ensure the international transmission of shocks are also key to understand the stronger responses of tradeable sector variables after a Home monetary shock.

The higher volatility of employment in the tradeable sector than in the nontradeable sector can be understood by looking at Equation (9). Since tradeable output responds more to Home monetary shocks, then the firms' demand for the labour input has to respond more too. Moreover, Home productivity shocks, which directly affect relative employment (9), are significantly more volatile in the tradeable sector than in the nontradeable sector.

Finally, the cross-correlations in the model are all positive because of the importance of the US monetary shocks, which cause Home inflation rates, output and employment in the two sectors to move all in the same direction, and thereby induce a positive correlation among these variables.

7 Sensitivity analysis

The parameter values of Table 2 were not estimated but were instead taken from the literature. However, for these parameters the range of acceptable values is rather limited in practice, so, provided that the parametrized values stay in that range, the qualitative findings of the previous Section do not change.[50] Therefore, only the most interesting sensitivity checks are reported

[50] In addition to the parameter values shown in Table 8, I have considered the following extremes: $(.5, .9)$ for γ, $(.2, .5)$ for δ and $1 - \delta^*$, $(3, 15)$ for η_T and η_N, $(.6, .9)$ for the Home and Foreign sector-specific probabilities of not changing prices.

here.[51]

TABLE 8 HERE

The elasticity of substitution between Home and Foreign tradeable goods could not be estimated from this dataset. In the baseline parametrization θ is set equal to 2, but other studies in this literature choose a lower value. The second column of Table 8 shows the model-implied standard deviations when θ is equal to one, as in Benigno and Thoenissen (2003). We can notice that all tradeable sector variables remain more volatile than the corresponding nontradeable sector variables, but the standard deviations of tradeable output and employment are considerably reduced.

The reason why tradeable output and employment are less volatile when θ is equal to one is as follows. The impulse responses[52] show that, after a Home monetary shock or a Foreign shock, tradeable output and employment react less under this scenario; moreover, among the transmission channels identified in Section 4, it is the asset market which is the most affected. Specifically, if θ is equal to one the response of bond holdings after a Home monetary shock or a Foreign shock is muted. Now consider, for example, a positive Home monetary shock. If θ is lower, Foreign individuals have less desire to substitute Foreign for Home-produced tradeables,[53] hence less desire to increase their consumption of Home exports by means of borrowing. As a result, (Home) bond holdings and tradeable output increase less after a positive Home monetary shocks, and the same happens for the demand for labour input. Given that Home monetary shocks are the most important source of fluctuations, this explains why the standard deviations of tradeable output and employment are reduced.

Another interesting scenario is the increase in the weight of foreign-produced goods in consumer preferences, caused, for example, by the ongoing process of trade integration. Table 8 shows what happens to the standard deviations of inflation, output and employment in the two sectors if US and Foreign individuals increase the share assigned to each other's goods in the tradeable

[51] I have also experimented with linear detrending and band-pass filtering, with single-country money demand equations, with lagged instruments and with single-equation estimates. In all these cases the NOEM model generates standard deviations that are compatible, from a qualitative point of view, with the pattern in the data.

[52] Available on request.

[53] After a positive Home monetary shock, the local currency price of Home exports decreases.

consumption basket. All tradeable sector variables are now more volatile compared to Table 6, but the increase in the volatility is more marked for output and inflation.

This result can be easily explained by noting that the coefficients multiplying changes in the period t nominal exchange rate on the right-hand side of Equation (6), and the terms of trade and bond holdings on the right-hand side of Equation (8) increase if δ increases. Therefore, if δ increases both curves shift more after a Home monetary shock and all Foreign shocks. In other words, if both countries become more open then tradeable sector variables are more volatile because they become more responsive to Home monetary and Foreign shocks.

Finally, we may want to analyse what happens if we assume that prices are more flexible in the tradeable sector; for example, Leith and Wren-Lewis (2007) consider $\varphi_T = 0.6$. Table 8 shows that, since tradeable sector firms are allowed to adjust their prices more frequently, the standard deviation of tradeable inflation increases. Because changes in demand are curbed by stronger price responses, the standard deviation of tradeable output falls. The standard deviation of nontradeable output increases because larger price differentials (caused by $\varphi_T \neq \varphi_N$) between tradeable and nontradeables induce individuals to substitute much more to, or away from, nontradeable goods.

8 Conclusion

This paper has developed and estimated by GMM a new open economy model, with the purpose of analysing the fluctuations of the tradeable and nontradeable sectors.

The estimated model generates standard deviations that are compatible, from a qualitative point of view, with the pattern observed in the data. The data suggests that the standard deviations of inflation, output and employment are higher in the tradeable sector than in the nontradeable sector. All these facts are reproduced by the model.

Finally, the model-implied responses of tradeable and nontradeable output levels to monetary shocks are broadly consistent with the VAR-based investigations of Doyle, Erceg, and Levin (unpublished), Ganley and Salmon (1997), and Llaudes (2007), who have found that tradeable or manufacturing output is more responsive to monetary policy shocks than nontradeable or service output.

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Table 1: Sectoral statistics

Sectors	% Std deviation		% value
	inflation	output	added
Tradeable:			
Manufacturing	2.04	4.80	14.5
Mining	11.57	4.00	1.2
Agriculture, forestry, fishing and hunting	10.11	6.07	1.0
Nontradeable:			
Finance, insurance, real estate, rental and leasing	1.06	1.10	19.7
Government	1.14	2.67	12.3
Professional and business services	1.36	2.95	11.6
Educational services, health care, etc.	1.91	1.49	6.9
Retail trade	2.36	3.00	6.7
Wholesale trade	3.13	3.01	6.0
Information	1.39	2.53	4.7
Construction	2.36	5.27	4.4
Arts, entertainment, recreation, etc.	1.73	2.09	3.6
Transportation and warehousing	2.49	3.77	3.1
Other services, except government	1.21	2.33	2.3
Utilities	3.61	4.58	1.9

Note: Calculations based on chain-type price and quantity indexes for value added by industry. Source: Bureau of Economic Analysis. The last column reports the value added by the sector as a percentage of aggregate GDP. Statistics were computed using logged and HP-filtered annual data. The sample is 1947 to 2005.

Table 2: Parametrization

	Description	*Value*
β	Discount factor	0.99
ν	Intermediation cost	0.0005
γ	Weight of nontradeable goods in total consumption	0.665
θ	Elasticity of substitution Home-Foreign tradeables	2
δ	Weight of Foreign goods in Home tradeable consumption	0.33
δ^*	Weight of Foreign goods in Foreign tradeable consumption	0.67
η_T	Elasticity of substitution among tradeable goods	7.67
η_N	Elasticity of substitution among nontradeable goods	4.58
ζ	Pass-through elasticity for Home imports	0.23
ζ^*	Pass-through elasticity for Foreign imports	0.4787
φ_T , φ_N	Probabilities of not changing prices (Home and Foreign)	0.75
α_T	Elasticity of output with respect to hours (tradeables)	0.7364
α_N	Elasticity of output with respect to hours (nontradeables)	0.7218

Table 3: GMM estimates

	Description	*Estimate*[a]
ε	Elasticity of marginal utility of real money balances	2.3044 (0.9882)
σ	Risk aversion for consumption	6.3679 (2.9847)
ϕ	Elasticity of substitution tradeable-nontradeables	0.6648 (0.0981)
Exogenous processes: $\widehat{x}_{j,t} = \rho_j \cdot \widehat{x}_{j,t-1} + \epsilon_j$		
ρ_j	AR coefficient Home nominal money growth	0.4441 (0.1030)
	AR coefficient Home tradeable technology	0.8321 (0.0592)
	AR coefficient Home nontradeable technology	0.8045 (0.0498)
	AR coefficient Home government expenditure	0.6774 (0.0590)
	AR coefficient Foreign nominal money growth	0.3494 (0.0839)
	AR coefficient Foreign tradeable technology	0.8374 (0.0499)
	AR coefficient Foreign nontradeable technology	0.5852 (0.0721)
	AR coefficient Foreign government expenditure	0.6462 (0.0992)
$Var(\epsilon_j)$	Variance Home nominal money growth	$8.50 \cdot 10^{-5}$ $(1.54 \cdot 10^{-5})$
	Variance Home tradeable technology	$6.52 \cdot 10^{-5}$ $(1.73 \cdot 10^{-5})$
	Variance Home nontradeable technology	$1.17 \cdot 10^{-5}$ $(2.75 \cdot 10^{-6})$
	Variance Home government expenditure	$1.55 \cdot 10^{-6}$ $(2.87 \cdot 10^{-7})$
	Variance Foreign nominal money growth	$6.36 \cdot 10^{-5}$ $(1.78 \cdot 10^{-5})$
	Variance Foreign tradeable technology	$9.24 \cdot 10^{-5}$ $(1.45 \cdot 10^{-5})$
	Variance Foreign nontradeable technology	$2.14 \cdot 10^{-5}$ $(3.86 \cdot 10^{-6})$
	Variance Foreign government expenditure	$2.20 \cdot 10^{-6}$ $(6.24 \cdot 10^{-7})$

Table 3 (continues): GMM estimates

	Description	*Estimate*[a]
$Cov\left(\epsilon_j, \epsilon_j'\right)$	Cov(Home nom. money growth, Home nontrad. prod.)	$1.21 \cdot 10^{-5}$ $\left(4.23 \cdot 10^{-6}\right)$
	Cov(Home nom. money growth, Home gov. exp.)	$2.29 \cdot 10^{-6}$ $\left(1.14 \cdot 10^{-6}\right)$
	Cov(Home nom. money growth, Foreign trad. prod.)	$-3.25 \cdot 10^{-5}$ $\left(7.71 \cdot 10^{-6}\right)$
	Cov(Home nom. money growth, Foreign gov. exp.)	$-2.19 \cdot 10^{-6}$ $\left(1.77 \cdot 10^{-6}\right)$
	Cov(Home trad. prod., Foreign trad. prod.)	$3.14 \cdot 10^{-5}$ $\left(8.57 \cdot 10^{-6}\right)$
	Cov(Home nontrad. prod., Home gov. exp.)	$2.52 \cdot 10^{-6}$ $\left(6.35 \cdot 10^{-7}\right)$
	Cov(Home nontrad. prod., Foreign trad. prod.)	$-8.59 \cdot 10^{-6}$ $\left(3.22 \cdot 10^{-6}\right)$
	Cov(Home gov. exp., Foreign trad. prod.)	$-2.46 \cdot 10^{-6}$ $\left(1.14 \cdot 10^{-6}\right)$
	Cov(Foreign trad. prod., Foreign nontrad. prod.)	$1.76 \cdot 10^{-5}$ $\left(3.96 \cdot 10^{-6}\right)$
	Cov(Foreign nontrad. prod., Foreign gov. exp.)	$-1.12 \cdot 10^{-6}$ $\left(1.31 \cdot 10^{-6}\right)$

[a] Numbers in parenthesis are standard errors.

Table 4: List of moment conditions

(1):	$E\left\{\left[\begin{array}{c}\varepsilon\left(\widehat{M}_t-\widehat{P}_t-\widehat{M}_t^*+\widehat{P}_t^*\right)\\-\sigma\left(\widehat{C}_t-\widehat{C}_t^*\right)+\beta\left(\widehat{i}_t-\widehat{i}_t^*\right)\end{array}\right]\cdot\left[\widehat{M}_t-\widehat{P}_t-\widehat{M}_t^*+\widehat{P}_t^*\right]\right\}=0$
(2):	$E\left\{\left[\begin{array}{c}\varepsilon\left(\widehat{M}_t-\widehat{P}_t-\widehat{M}_t^*+\widehat{P}_t^*\right)\\-\sigma\left(\widehat{C}_t-\widehat{C}_t^*\right)+\beta\left(\widehat{i}_t-\widehat{i}_t^*\right)\end{array}\right]\cdot\left[\widehat{C}_t-\widehat{C}_t^*\right]\right\}=0$
(3):	$E\left\{\left[\widehat{\frac{P_{N,t}C_{N,t}}{P_tC_t}}-(1-\phi)\left(\widehat{P}_{N,t}-\widehat{P}_t\right)\right]\cdot\left[\widehat{P}_{N,t}-\widehat{P}_t\right]\right\}=0$
(4) to (11):	$E\left[\widehat{x}_{j,t}\cdot\widehat{x}_{j,t-1}-\rho_j\cdot\widehat{x}_{j,t-1}^2\right]=0$
(12) to (19):	$E\left[\left(\widehat{x}_{j,t}-\rho_j\cdot\widehat{x}_{j,t-1}\right)^2-Var\left(\epsilon_j\right)\right]=0$
(20) to (29):	$E\left[\left(\widehat{x}_{j,t}-\rho_j\cdot\widehat{x}_{j,t-1}\right)\left(\widehat{x}'_{j,t}-\rho'_j\cdot\widehat{x}'_{j,t-1}\right)-Cov\left(\epsilon_j,\epsilon'_j\right)\right]=0$

Note: the estimated exogenous processes $\widehat{x}_j$ in the Home country are defined as follows: $\mu =$ nominal money growth rate; $\widehat{z}_{TH}=\widehat{Y}_{TH}^{Tot}-\alpha\widehat{n}_{TH}$ tradeable technology; $\widehat{z}_N=\widehat{Y}_N-\alpha\widehat{n}_N=$ nontradeable technology; $dG =$ government expenditure. The estimated exogenous processes in the Foreign country are similarly defined.

Equations 1 and 2 are derived using the following definitions: $i_t\equiv E_t\left[\frac{P_{T,t+1}}{P_{T,t}}\left(1+r_t\right)\right]-1$ and $i_t^*\equiv E_t\left[\frac{P_{T,t+1}}{P_{T,t}}\frac{e_t}{e_{t+1}}\left(1+r_t\right)\right]-1$.

Table 5: Data moments

	% st dev	1-st AC	Correlogram π_{TH}^{Tot}	π_N	$\widehat{Y}_{TH}^{Tot}$	$\widehat{Y}_N$	$\widehat{n}_{TH}$	$\widehat{n}_N$
π_{TH}^{Tot} - Home tradeable inflation	0.83	0.14	1.00					
π_N - Home nontradeable inflation	0.45	0.32	0.14	1.00				
$\widehat{Y}_{TH}^{Tot}$ - Home tradeable output	2.50	0.86	0.32	0.44	1.00			
$\widehat{Y}_N$ - Home nontradeable output	0.50	0.80	0.14	0.15	0.34	1.00		
$\widehat{n}_{TH}$ - Home tradeable employment	1.98	0.91	0.20	0.55	0.85	0.29	1.00	
$\widehat{n}_N$ - Home nontradeable employment	0.89	0.94	0.27	0.53	0.69	0.49	0.87	1.00

Note: Data sources and definitions are available from the author on request. Statistics were computed using logged and HP-filtered prices, output and employment levels.

Table 6: Model moments

	% st dev	1-st AC	Correlogram π_{TH}^{Tot}	π_N	$\widehat{Y}_{TH}^{Tot}$	$\widehat{Y}_N$	$\widehat{n}_{TH}$	$\widehat{n}_N$
π_{TH}^{Tot} - Home tradeable inflation	0.52	0.18	1.00					
π_N - Home nontradeable inflation	0.32	0.64	0.79	1.00				
$\widehat{Y}_{TH}^{Tot}$ - Home tradeable output	0.88	0.67	0.75	0.89	1.00			
$\widehat{Y}_N$ - Home nontradeable output	0.39	0.62	0.73	0.88	0.85	1.00		
$\widehat{n}_{TH}$ - Home tradeable employment	1.68	0.63	0.66	0.64	0.65	0.65	1.00	
$\widehat{n}_N$ - Home nontradeable employment	0.50	0.63	0.42	0.63	0.51	0.43	0.41	1.00

Note: Statistics are averages over 100 simulations, each of length 111, after the first 1,000 observations were discarded. Statistics were computed using logged and HP-filtered variables. The model parameters are those of Tables 2 and 3.

Table 7: Variance decompositions

Shocks:	π_{TH}^{Tot}	π_N	$\widehat{Y}_{TH}^{Tot}$	$\widehat{Y}_N$	$\widehat{n}_{TH}$	$\widehat{n}_N$
US money growth	73.71	86.11	60.07	57.92	37.86	47.92
Foreign money growth	9.87	0.00	14.59	2.12	9.69	2.73
US tradeable technology	1.90	0.42	9.32	2.21	30.73	2.88
Foreign tradeable technology	6.94	6.19	8.94	9.42	16.89	5.43
US nontradeable technology	5.14	4.76	4.51	14.55	2.83	35.65
Foreign nontradeable technology	0.37	0.35	0.74	0.41	0.81	0.30
US government expenditure	1.00	0.89	1.04	12.64	0.67	4.10
Foreign government expenditure	1.06	1.27	0.79	0.74	0.52	1.00

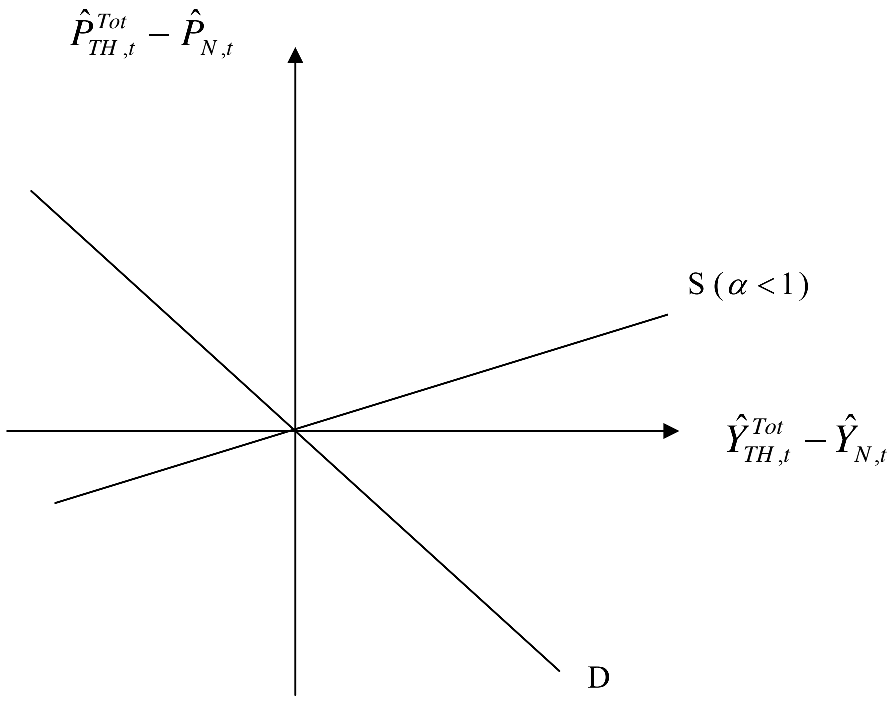
Note: Shocks are orthogonalised using the Cholesky method, and the horizon is set at 200 quarters. Each column reports, for each variable, the share of the total variance explained by every shock, measured in per cent. The numbers are averages across all possible variance decompositions, given by the number of different orderings of the 8 shocks (40,320).

Table 8: Sensitivity analysis

	Percent standard deviations		
	$\theta = 1$	$\delta = 0.40$ & $\delta^* = 0.60$	$\varphi_T = 0.6$
π_{TH}^{Tot} - Home tradeable inflation	0.50	0.58	0.69
π_N - Home nontradeable inflation	0.32	0.33	0.33
$\widehat{Y}_{TH}^{Tot}$ - Home tradeable output	0.53	0.96	0.82
$\widehat{Y}_N$ - Home nontradeable output	0.38	0.41	0.48
$\widehat{n}_{TH}$ - Home tradeable employment	1.42	1.75	1.49
$\widehat{n}_N$ - Home nontradeable employment	0.49	0.52	0.58

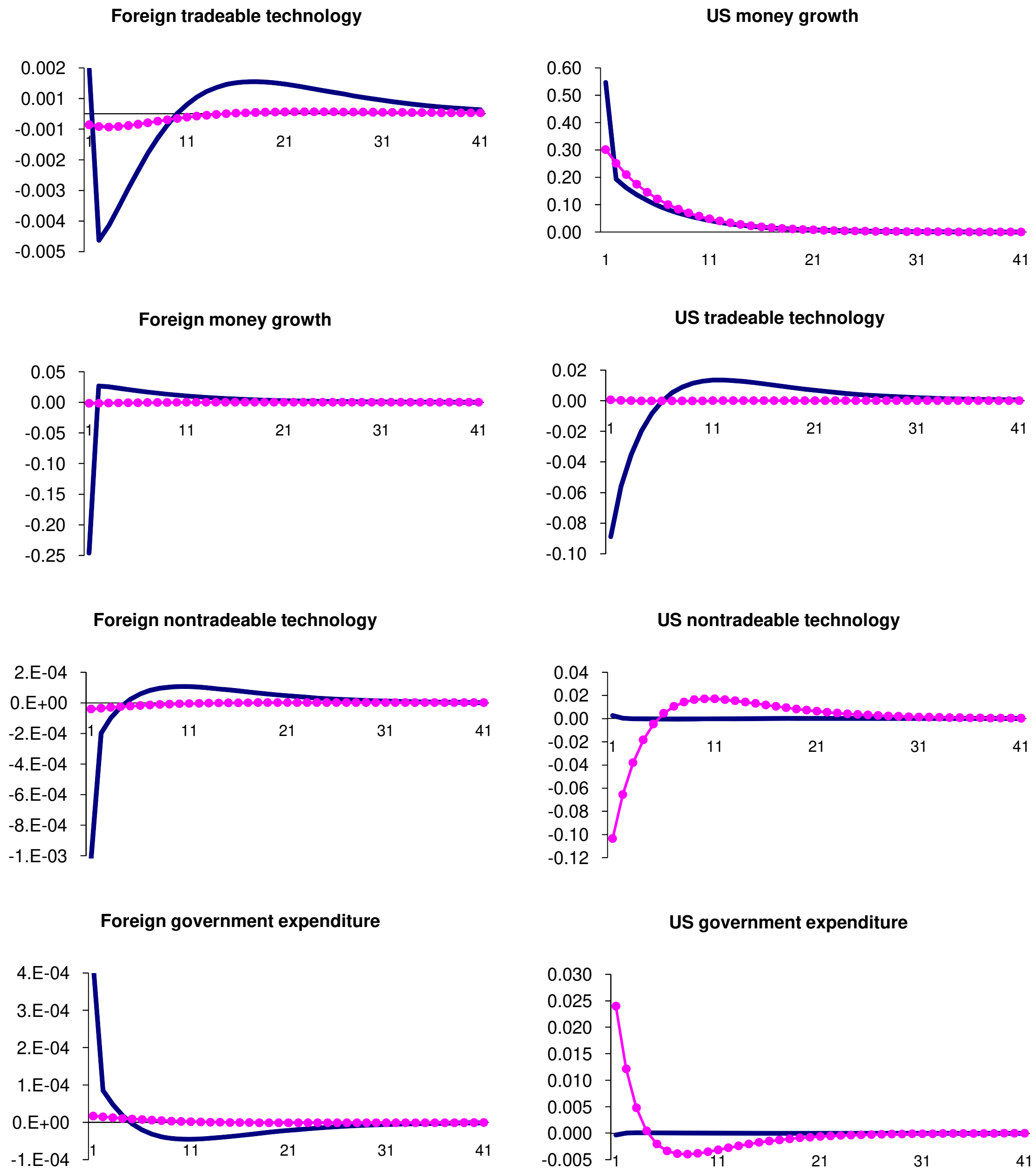
Note: The calibration of the model differs from Table 6 only with respect to the parameters indicated at the top of each column. Statistics are computed as averages over simulations.

Figure 1: The short-run demand and supply for relative output



The supply (S) and demand (D) schedules are given by equations (6) and (8). The supply schedule is upward-sloping if α < 1; in the particular case of constant returns to labour, α = 1, the supply relationship is horizontal.

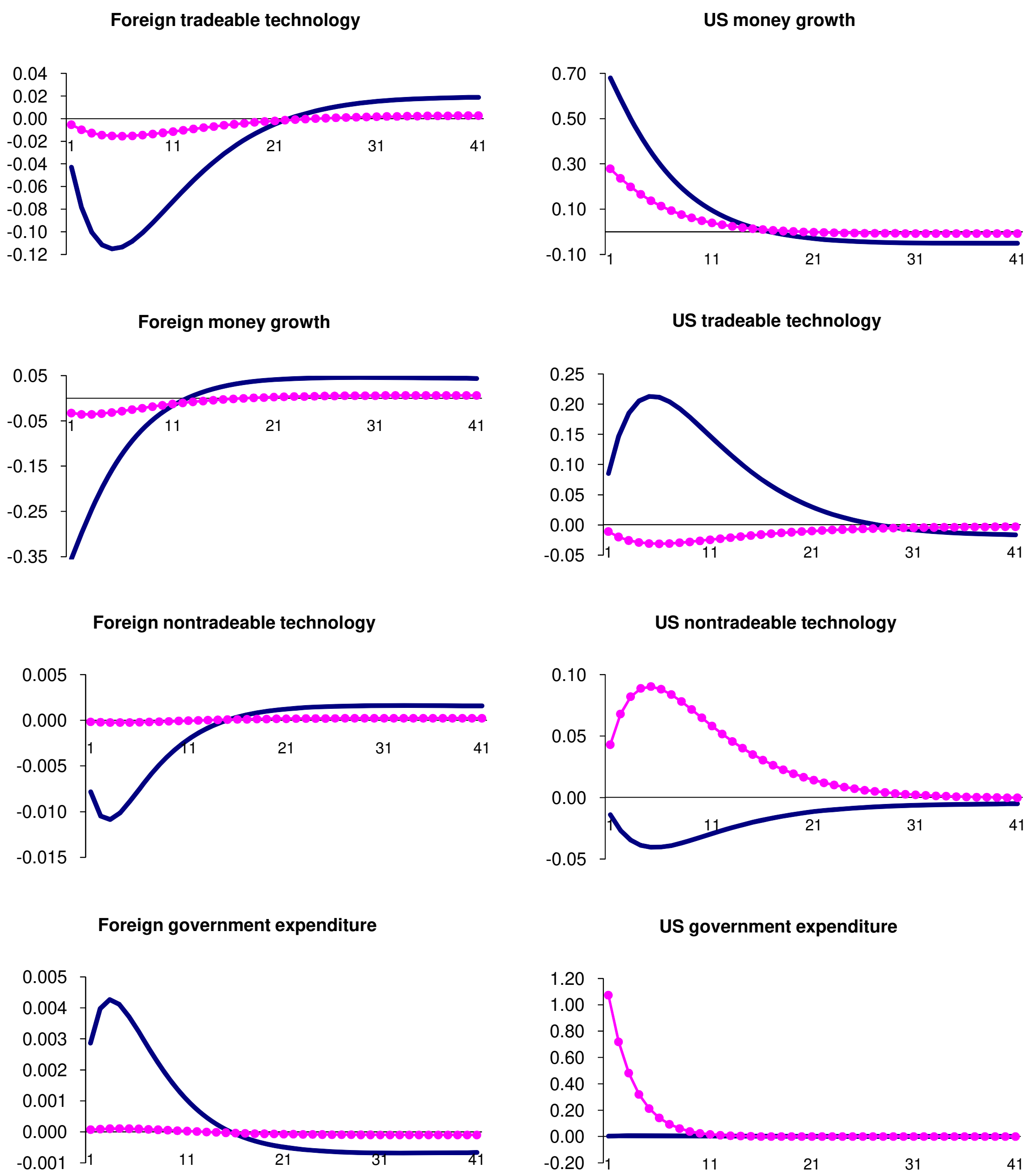
Figure 2: Impulse responses of inflation rates



The solid line indicates tradeable inflation, the dotted line nontradeable inflation. Time is in quarters.

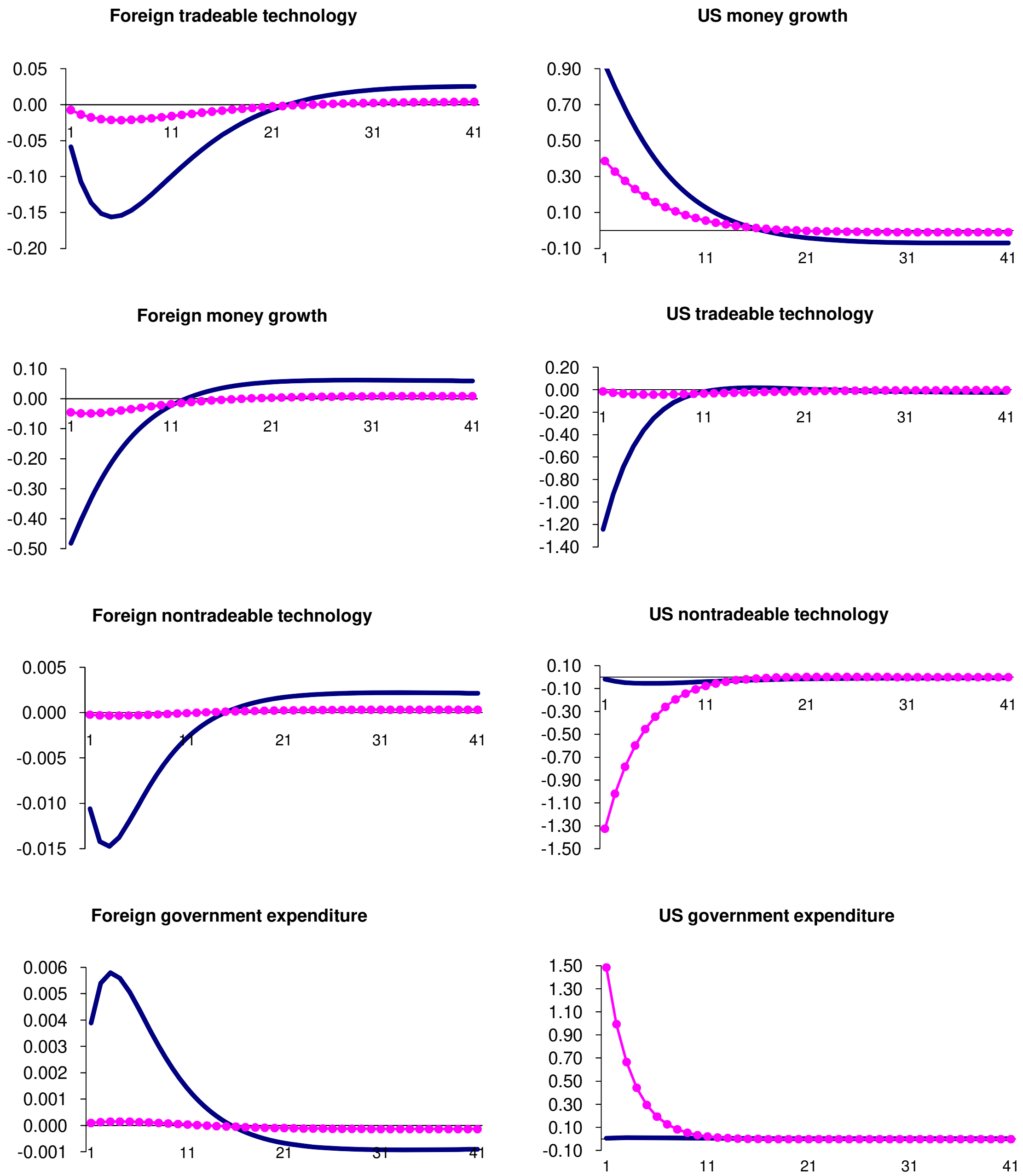
Estimated standard deviations (percent): Foreign tradeable productivity 0.96, US money growth 0.93, Foreign money growth 0.81, US tradeable productivity 0.81, Foreign nontradeable productivity 0.46, US nontradeable productivity 0.33, Foreign gov. expenditure 0.15, US gov. expenditure 0.12.

Figure 3: Impulse responses of output



The solid line indicates tradeable inflation, the dotted line nontradeable inflation. Time is in quarters.

Figure 4: Impulse responses of employment



The solid line indicates tradeable inflation, the dotted line nontradeable inflation. Time is in quarters.

Appendices

These Appendices describe the data used in the estimation, and explain the derivation of the equations presented in the paper.

A brief overview is as follows:

Appendix A: Description of data used in the estimation

Several statistical sources have been used in the construction of the dataset. Table A.1 provides a list of all the raw data series and their respective sources, and Table A.2 illustrates the construction of the data variables. Foreign variables are obtained as either geometric or arithmetic weighted averages of individual country variables. The weights are time-varying, and are given by each country's share of total real GDP, measured in a common currency. For consistency, all aggregates are constructed using the same GDP weights. Moreover, real variables are obtained using constant 2000 prices and nominal exchange rates.

The definition of total tradeable output $Y^{Tot}_{TH,t}$ (page 13) includes both goods sold domestically and goods sold abroad. This variable is mapped to the Index of production in total manufacturing, which includes both domestic sales and exports.

Data series on consumer price indexes of tradeable or manufacturing goods include both domestically and foreign-produced goods. This is not true of producer price indexes, which include only domestically-produced goods. This consideration motivates my choice of the manufacturing PPI index as the empirical correspondent of the price of tradeables P^{Tot}_{TH} . On the other hand, CPI indexes for services (nontradeables) are likely to contain only a small proportion of foreign-produced services, moreover, there is no distinction in the model between producer and consumer prices. I was unable to find PPI indexes for services, so I use the CPI index for services as the empirical correspondent of the price of nontradeable goods P_N.

In the model, the price of Home tradeable goods is a weighted average of the domestic-currency prices of goods sold locally and exported, as a result, it must inevitably be affected by the nominal exchange rate (see Equation 5 on page 13). I have contacted the Bureau of Labor Statistics (who collects the data for the OECD) and asked whether the manufacturing PPI index (which includes exports and government purchases) is likely to be sensitive to currency fluctuations, and I have obtained a positive answer.[1] Although the prices collected for the US manufacturing PPI index are always in dollars, producers may adjust prices to currency fluctuations to accommodate the buyer (consistently with imperfect pass-through).

[1] Emails are available upon request.

Table A.1: Raw data

Alias	*Description*	*Sources*[a]
$Cons$	Private final consumption expenditure	OECD QNA
$Cons_N$	Expenditure on services, National Income and Product Accounts *(US only)*	BEA
CPI	Consumer Price Index for all items	OECD MEI
CPI_N	Consumer Price Index for services *(US only)*	BLS
$EmpMan$	Employees in manufacturing	OECD MEI UK: ONS
$EmpSer$	Employees/Employment in the Service sector. Not including Mexico	OECD MEI BLS Eurostat
Exp	Personal consumption expenditure, National Income and Product Accounts *(US only)*	BEA
GDP	Gross Domestic Product	OECD QNA
$GExp$	Government final consumption expenditure	OECD QNA
IR	Short-term nominal interest rates - US: 3-month Treasury bill rate, bond equivalent - Canada: 3-month Treasury bill rate - France: 3-month Treasury Bill Rate - Germany: Call money rate - Japan: Call money rate - Mexico: rate on 91-day treasury certificates - UK: 3-month Treasury bill rate, bond equivalent	IMF IFS Bank of France Mexico: OECD MEI
Mon	Monetary aggregate M1. Except UK: M2; Canada: M1+; and Mexico: M1a	OECD MEI & IMF IFS
$PrMan$	Index of production in total manufacturing	OECD MEI
$PrSer$	Gross Domestic Product in the Service Sector, National Accounts. Except US: Services Production, National Income and Product Accounts; and Japan: Index of Production in Total Services Sectors. Not including Mexico	OECD QNA US: BEA Jap: OECD MEI
PPI_M	Producer Price Index in manufacturing *(US only)*	OECD MEI

[a] Legend: BEA = Bureau of Economic Analysis, US; BLS = Bureau of Labor Statistics, US; ILO = International Labour Organization; IMF IFS = IMF International Financial Statistics; OECD MEI = OECD Main Economic Indicators; OECD QNA = OECD Quarterly National Accounts; ONS = Office for National Statistics, UK.

Table A.2: Constructed data variables

Series	*Description*
C	Home consumption: $C_t = Cons_t^{US}$
C^*	Foreign consumption: $C_t^* = \sum_j Cons_t^j \cdot ER_0^j$
G	Home government expenditure relative to consumption: $G_t = \frac{GExp_t^{US}}{Cons_0^{US}}$
G^*	Foreign government exp. relative to consumption: $G_t = \prod_j \left(\frac{GExp_t^j}{Cons_0^j}\right)^{w_t^j}$
i	Home nominal interest rate: $i_t = IR_t^{US}$
i^*	Foreign nominal interest rate: $i_t^* = \sum_j w_t^j \cdot IR_t^j$
μ	Home nominal money growth rate: $\mu_t = \frac{Mon_t^{US} - Mon_{t-1}^{US}}{Mon_{t-1}^{US}}$
μ^*	Foreign nominal money growth rate: $\mu_t^* = \sum_j w_t^j \cdot \frac{Mon_t^j - Mon_{t-1}^j}{Mon_{t-1}^j}$ From 1999:1, the nominal money growth rates for France and Germany are equal to the euro-area money growth rate
M	Home nominal money balances: $M_t = Mon_t^{US}$
M^*	Foreign nominal money balances: $M_t^* = M_{t-1}^* \cdot (1 + \mu_t^*)$
n_N	Employment Home nontradeable sector: $n_{N,t} = EmpSer_t^{US}$
n_N^*	Employment Foreign nontradeable sector: $n_{N,t}^* = \sum_j EmpSer_t^j$ The individual country series are normalised to ensure that the number of persons engaged in the service sector in 2000 is the same as in the Groningen 60-Industry Database
n_{TH}	Employment in the Home tradeable sector: $n_{TH,t} = EmpMan_t^{US}$
n_{TF}^*	Employment in the Foreign tradeable sector: $n_{TF,t}^* = \sum_j EmpMan_t^j$ The individual country series are normalised to ensure that the number of opersons engaged in manufacturing in 2000 is the same as in the Groningen 60-Industry Database (Mexico: ILO)
P	Home price level: $P_t = CPI_t^{US}$
P^*	Foreign price level: $P_t^* = \prod_j \left(CPI_t^j\right)^{w_t^j}$
P_N	Home nontradeable prices: $P_{N,t} = CPI_{N,t}$
P_{TH}^{Tot}	Price of Home tradeable goods: $P_{TH,t}^{Tot} = PPI_{M,t}$
$\frac{P_N \cdot C_N}{P \cdot C}$	Home nontradeable expenditure share: $\frac{P_{N,t} \cdot C_{N,t}}{P_t \cdot C_t} = \frac{Cons_{N,t}}{Exp_t}$
w^j	Country weights: $w_t^j = \frac{GDP_t^j \cdot ER_0^j}{Y_t^*}$

Table A.2 (continues): Constructed data variables

Series	*Description*
Y^*	Foreign output: $Y_t^* = \sum_j GDP_t^j \cdot ER_0^j$
Y_N	Home nontradeable output: $Y_{N,t} = PrSer_t^{US}$
Y_N^*	Foreign nontradeable output: $Y_{N,t}^* = \sum_j PrSer_t^j \cdot ER_0^j$ The individual country series are normalised to ensure that the value of output in the service sector in 2000 is the same as the value added in services according to the Groningen 60-Industry Database
Y_{TH}^{Tot}	Home tradeable output: $Y_{TH,t}^{Tot} = PrMan_t^{US}$
Y_{TF}^{*Tot}	Foreign tradeable output $Y_{TF,t}^{*Tot} = \sum_j PrMan_t^j \cdot ER_0^j$ The individual country series are normalised to ensure that the value of output in manufacturing in 2000 is the same as the value added in manufacturing according to the Groningen 60-Industry Database (Mexico: OECD QNA)

Notes: Data variables were constructed with seasonally adjusted data, converted to constant (2000) prices and quarterly frequency. Superscripts are used to denote the country: US denotes the United States, j any of the 6 countries that constitute the Foreign aggregate. Subscripts are used to denote time, with 0 denoting the year 2000. The Groningen 60-Industry Database is constructed by the Groningen Growth and Development Centre.

Appendix B: Deriving the equations of Section 4

This Appendix describes the derivation of the equations presented in Section 4. Variables with a 'hat' denote percentage or log-deviations from the steady state, while the operator 'd' denotes linear deviations, calculated in proportion to the steady state level of consumption. That is, for any variable X, let X_0 denote the value of the variable at the steady state. Then, $\widehat{X}_t \equiv \frac{X_t - X_0}{X_0} \simeq \log\left(\frac{X_t}{X_0}\right)$, while $dX_t \equiv \frac{X_t}{C_0}$. Money growth rates, government expenditures and bond holdings are all normalised at zero in the steady state.

Profit maximisation implies that the law of one price holds in the steady state: $p_{TH,0}(f_{TH}) = e_0 \cdot p^*_{TH,0}(f_{TH})$.

The short-run demand for relative output

The derivation of the short-run demand for relative output is divided into the following steps:

1. First, find the expressions for the aggregate Home tradeable and nontradeable output demands.
2. Find the log-linearised demands for aggregate Home tradeable and nontradeable output and for Foreign tradeable output.
3. Using the Home and Foreign aggregate resource constraints, substitute out from the demand for Y_{TH} the share that comes from the Foreign country.
4. Using the formulas for the CES aggregators, substitute out the consumption indexes, then find the short-run demand for relative output.

Step 1

The domestic demand for output produced by the individual firm f_{TH} is given by:

$$y_{TH,t}(f_{TH}) = \left(\frac{p_{TH,t}(f_{TH})}{P_{TH,t}}\right)^{-\eta_T} C_{TH,t} \ ,$$

and the export demand is given by:

$$y^*_{TH,t}(f_{TH}) = \left(\frac{p^*_{TH,t}(f_{TH})}{P^*_{TH,t}}\right)^{-\eta_T} C^*_{TH,t} \ .$$

The aggregate price indexes are:

$$P_{TH,t} = \left(\int_0^1 p_{TH,t}(f_{TH})^{1-\eta_T} df_{TH}\right)^{\frac{1}{1-\eta_T}} \ ,$$

$$P^*_{TH,t} = \left(\int_0^1 p^*_{TH,t}(f_{TH})^{1-\eta_T} df_{TH}\right)^{\frac{1}{1-\eta_T}} \ .$$

Using the following definitions:

$$Y_{TH,t} \equiv \left[\int_0^1 y_{TH,t} \left(f_{TH} \right)^{\frac{\eta_T - 1}{\eta_T}} df_{TH} \right]^{\frac{\eta_T}{\eta_T - 1}} ,$$

$$Y^*_{TH,t} \equiv \left[\int_0^1 y^*_{TH,t} \left(f_{TH} \right)^{\frac{\eta_T - 1}{\eta_T}} df_{TH} \right]^{\frac{\eta_T}{\eta_T - 1}} ,$$

we obtain:

$$Y_{TH,t} = C_{TH,t} , \qquad Y^*_{TH,t} = C^*_{TH,t} .$$

Moreover:

$$Y^{Tot}_{TH,t} \equiv Y_{TH,t} + Y^*_{TH,t} = C_{TH,t} + C^*_{TH,t} , \tag{1}$$

thus log-linearising (1):

$$\widehat{Y}^{Tot}_{TH,t} = k_1 \widehat{Y}_{TH,t} + (1 - k_1) \widehat{Y}^*_{TH,t} , \tag{2}$$

where $k_1 = \frac{C_{TH0}}{Y^{Tot}_{TH0}} = (1 - \delta) \left(\frac{P_{TH0}}{P_{T0}} \right)^{1-\theta}$. The demand for aggregate Home nontradeable output is similarly obtained, and it includes government expenditure:

$$Y_{N,t} = C_{N,t} + G_t .$$

Step 2

The price indexes in the tradeable sector are defined as arithmetic weighted averages, with weights taken from the steady state:

$$P^{Tot}_{TH,t} \equiv \frac{P_{TH,t} \cdot Y_{TH0} + e_t P^*_{TH,t} \cdot Y^*_{TH0}}{P_{TH0} \cdot Y_{TH0} + e_0 P^*_{TH0} \cdot Y^*_{TH0}} , \tag{3}$$

$$P^{*Tot}_{TF,t} \equiv \frac{\frac{P_{TF,t}}{e_t} \cdot Y_{TF0} + P^*_{TF,t} \cdot Y^*_{TF0}}{\frac{P_{TF0}}{e_0} \cdot Y_{TF0} + P^*_{TF0} \cdot Y^*_{TF0}} .$$

Log-linearising:

$$\widehat{P}^{Tot}_{TH,t} = k_1 \widehat{P}_{TH,t} + (1 - k_1) \left(\widehat{e}_t + \widehat{P}^*_{TH,t} \right) , \tag{4}$$

$$\widehat{P}^{*Tot}_{TF,t} = k^*_1 \left(\widehat{P}_{TF,t} - \widehat{e}_t \right) + (1 - k^*_1) \widehat{P}^*_{TF,t} . \tag{5}$$

Substituting into the total demand for aggregate Home tradeable output (1) the following expressions:

$$C_{TH,t} = (1 - \delta) \left(\frac{P_{TH,t}}{P_{T,t}} \right)^{-\theta} C_{T,t} ,$$

$$C^*_{TH,t} = (1-\delta^*)\left(\frac{P^*_{TH,t}}{P^*_{T,t}}\right)^{-\theta} C^*_{T,t} \ ,$$

$$\left(\frac{P_{TH,t}}{P_{T,t}}\right)^{-\theta} = \left[(1-\delta)+\delta\left(\frac{P_{TF,t}}{P_{TH,t}}\right)^{1-\theta}\right]^{\frac{\theta}{1-\theta}} \ ,$$

$$\left(\frac{P^*_{TH,t}}{P^*_{T,t}}\right)^{-\theta} = \left[(1-\delta^*)+\delta^*\left(\frac{P^*_{TH,t}}{P^*_{TF,t}}\right)^{\theta-1}\right]^{\frac{\theta}{1-\theta}} \ ,$$

and log-linearising, we get:

$$\begin{aligned} \widehat{Y}^{Tot}_{TH,t} &= k_1\widehat{C}_{T,t} + (1-k_1)\,\widehat{C}^*_{T,t} \\ +\theta\,(1-k_1)&\left[k_1\left(\widehat{P}_{TF,t}-\widehat{P}_{TH,t}\right)+(1-k_1^*)\left(\widehat{P}^*_{TF,t}-\widehat{P}^*_{TH,t}\right)\right] \ , \end{aligned} \tag{6}$$

where the coefficient $k_1^* = \frac{C_{TF0}}{Y^{*Tot}_{TF,t}} = (1-\delta^*)\left(\frac{P^*_{TH0}}{P^*_{T0}}\right)^{1-\theta}$ can be computed from the steady state equations. Using the same procedure for Home non-tradeable output and Foreign tradeable output we get:

$$\widehat{Y}_{N,t} = \widehat{C}_{N,t} + k_7 dG_t \ , \tag{7}$$

$$\widehat{Y}^{*Tot}_{TF,t} = k_1^*\widehat{C}_{T,t} + (1-k_1^*)\,\widehat{C}^*_{T,t} - \theta k_1^*\left[k_1\left(\widehat{P}_{TF,t}-\widehat{P}_{TH,t}\right)+(1-k_1^*)\left(\widehat{P}^*_{TF,t}-\widehat{P}^*_{TH,t}\right)\right] \ , \tag{8}$$

where $k_7 = \frac{C_0}{C_{N,0}}$ is a coefficient from the steady state.

Step 3

Equations (6) and (8) together imply:

$$\begin{aligned} \widehat{Y}^{Tot}_{TH,t} - \widehat{Y}^{*Tot}_{TF,t} &= (k_1-k_1^*)\left(\widehat{C}_{T,t}-\widehat{C}^*_{T,t}\right) \\ +\theta\,(1-k_1+k_1^*)&\left[k_1\left(\widehat{P}_{TF,t}-\widehat{P}_{TH,t}\right)+(1-k_1^*)\left(\widehat{P}^*_{TF,t}-\widehat{P}^*_{TH,t}\right)\right] \ . \end{aligned} \tag{9}$$

Equation (9) is the log-linearised demand for $\frac{Y^{Tot}_{TH}}{Y^{*Tot}_{TF}}$ obtained from the individual demand equations.

The Home and Foreign aggregate resource constraints are:

$$B_t P_{T,t} = (1+r_{t-1})\,B_{t-1}P_{T,t} + P_{TH,t}\cdot Y_{TH,t} + e_t P^*_{TH,t}\cdot Y^*_{TH,t} - P_{T,t}C_{T,t} \ ,$$

$$B^*_t\frac{P_{T,t}}{e_t} = (1+r_{t-1})\,B^*_{t-1}\frac{P_{T,t}}{e_t} + P^*_{TF,t}\cdot Y^*_{TF,t} + \frac{P_{TF,t}}{e_t}\cdot Y_{TF,t} - P^*_{T,t}\cdot C^*_{T,t} \ .$$

After log-linearising around a steady state with $B_0 = 0$ and government expenditures equal to zero, and substituting prices out, we obtain:

$$dB_t = \frac{1}{\beta} dB_{t-1} - (1 - k_1) k_2 k_3 \left(\widehat{P}_{TF,t} - \widehat{P}^*_{TH,t} - \widehat{e}_t \right) + k_2 k_3 \widehat{Y}^{Tot}_{TH,t} - k_2 k_3 \widehat{C}_{T,t} \ ,$$

$$\frac{P_{T0}}{e_0 P^*_{T0}} dB^*_t = \frac{P_{T0}}{e_0 P^*_{T0}} \frac{1}{\beta} dB^*_{t-1} + k^*_1 k^*_2 k^*_3 \left(\widehat{P}_{TF,t} - \widehat{P}^*_{TH,t} - \widehat{e}_t \right) + k^*_2 k^*_3 \widehat{Y}^{*Tot}_{TF,t} - k^*_2 k^*_3 \widehat{C}^*_{T,t} \ ,$$

where $k_2 = \frac{P_{TH0} Y^{Tot}_{TH0}}{P_0 C_0} = \frac{P_{T0} C_{T0}}{P_0 C_0} = (1 - \gamma) \left(\frac{P_{T0}}{P_0} \right)^{1-\phi}$, $k^*_2 = \frac{P^*_{TF0} Y^{*Tot}_{TF0}}{P^*_0 C^*_0} = \frac{P^*_{T0} C^*_{T0}}{P^*_0 C^*_0} = (1 - \gamma) \left(\frac{P^*_{T0}}{P^*_0} \right)^{1-\phi}$, $k_3 = \frac{P_0}{P_{T0}}$ and $k^*_3 = \frac{P^*_0}{P^*_{T0}}$ are coefficients from the steady state. Since $dB^*_t = -\frac{C_0}{C^*_0} dB_t$, we obtain:

$$\widehat{Y}^{Tot}_{TH,t} = \frac{1}{k_2 k_3} \left(dB_t - \frac{1}{\beta} dB_{t-1} \right) + (1 - k_1) \left(\widehat{P}_{TF,t} - \widehat{P}^*_{TH,t} - \widehat{e}_t \right) + \widehat{C}_{T,t} \ ,$$

$$\widehat{Y}^{*Tot}_{TF,t} = -\frac{1}{k^*_2 k^*_3} \frac{P_{T0}}{e_0 P^*_{T0}} \frac{C_0}{C^*_0} \left(dB_t - \frac{1}{\beta} dB_{t-1} \right) - k^*_1 \left(\widehat{P}_{TF,t} - \widehat{P}^*_{TH,t} - \widehat{e}_t \right) + \widehat{C}^*_{T,t} \ .$$

Therefore:

$$\begin{aligned} \widehat{Y}^{Tot}_{TH,t} - \widehat{Y}^{*Tot}_{TF,t} &= (1 - k_1 + k^*_1) k_4 \left(dB_t - \frac{1}{\beta} dB_{t-1} \right) + \widehat{C}_{T,t} - \widehat{C}^*_{T,t} \\ &\quad + (1 - k_1 + k^*_1) \left(\widehat{P}_{TF,t} - \widehat{P}^*_{TH,t} - \widehat{e}_t \right) \ , \end{aligned} \tag{10}$$

where $k_4 = \frac{1}{1 - k_1 + k^*_1} \left(\frac{1}{k_2 k_3} + \frac{1}{k^*_2 k^*_3} \frac{P_{T0}}{e_0 P^*_{T0}} \frac{C_0}{C^*_0} \right)$. Equation (10) is the log-linearised demand for $\frac{Y^{Tot}_{TH,t}}{Y^{*Tot}_{TF,t}}$ obtained from the Home and Foreign aggregate resource constraints. Equations (9) and (10) together imply:

$$\begin{aligned} \widehat{C}^*_{T,t} &= \widehat{C}_{T,t} + k_4 \left(dB_t - \frac{1}{\beta} dB_{t-1} \right) + \left(\widehat{P}_{TF,t} - \widehat{P}^*_{TH,t} - \widehat{e}_t \right) \\ &\quad - \theta \left[k_1 \left(\widehat{P}_{TF,t} - \widehat{P}_{TH,t} \right) + (1 - k^*_1) \left(\widehat{P}^*_{TF,t} - \widehat{P}^*_{TH,t} \right) \right] \ . \end{aligned} \tag{11}$$

Substituting (11) into (6) we obtain:

$$\widehat{Y}^{Tot}_{TH,t} = \widehat{C}_{T,t} + (1 - k_1) \left(\widehat{P}_{TF,t} - \widehat{P}^*_{TH,t} - \widehat{e}_t \right) + (1 - k_1) k_4 \left(dB_t - \frac{1}{\beta} dB_{t-1} \right) \ . \tag{12}$$

Step 4

From the equations:

$$C_{T,t} = (1-\gamma)\left(\frac{P_{T,t}}{P_t}\right)^{-\phi} C_t \ ,$$

$$C_{N,t} = \gamma\left(\frac{P_{N,t}}{P_t}\right)^{-\phi} C_t \ ,$$

and substituting out the price indexes, we get the log-linearised demands for C_T and C_N:

$$\widehat{C}_{T,t} = -\phi\,(1-k_2)\left[k_1\widehat{P}_{TH,t} + (1-k_1)\,\widehat{P}_{TF,t} - \widehat{P}_{N,t}\right] + \widehat{C}_t \ , \tag{13}$$

$$\widehat{C}_{N,t} = \phi k_2\left[k_1\widehat{P}_{TH,t} + (1-k_1)\,\widehat{P}_{TF,t} - \widehat{P}_{N,t}\right] + \widehat{C}_t \ . \tag{14}$$

By substituting (13) into (12) we obtain:

$$\begin{aligned}\widehat{Y}^{Tot}_{TH,t} \ &= \ -\phi\,(1-k_2)\left[k_1\widehat{P}_{TH,t} + (1-k_1)\,\widehat{P}_{TF,t} - \widehat{P}_{N,t}\right] \\ &\quad + (1-k_1)\left(\widehat{P}_{TF,t} - \widehat{P}^{*}_{TH,t} - \widehat{e}_t\right) + \widehat{C}_t + (1-k_1)\,k_4\left(dB_t - \frac{1}{\beta}dB_{t-1}\right)\end{aligned} \tag{15}$$

And by substituting (14) into (7) we obtain:

$$\widehat{Y}_{N,t} = \phi k_2\left[k_1\widehat{P}_{TH,t} + (1-k_1)\,\widehat{P}_{TF,t} - \widehat{P}_{N,t}\right] + \widehat{C}_t + k_7 dG_t \ . \tag{16}$$

Finally, by subtracting (16) from (15) and after some substitutions we obtain the short-run demand for relative output:

$$\widehat{Y}^{Tot}_{TH,t} - \widehat{Y}_{N,t} = -\phi\left(\widehat{P}^{Tot}_{TH,t} - \widehat{P}_{N,t}\right) + (1-\phi)\,(1-k_1)\,\widehat{T}_t + (1-k_1)\,k_4\left(dB_t - \frac{1}{\beta}dB_{t-1}\right) - k_7 dG_t \ .$$

Under the assumption that in period $t-1$ the economy is at its steady state, $dB_{t-1} = 0$. Notice that, in the special case $\theta = 1$, $k_1 = 1-\delta$, so the demand is:

$$\widehat{Y}^{Tot}_{TH,t} - \widehat{Y}_{N,t} = -\phi\left(\widehat{P}^{Tot}_{TH,t} - \widehat{P}_{N,t}\right) + \delta\,(1-\phi)\,\widehat{T}_t + \delta k_4 dB_t - k_7 dG_t \ .$$

The short-run supply for relative output

The maximisation problem faced by firm f_{TH} in the Home tradeable sector changing prices at time t is:

$$\max \quad E_t \sum\nolimits_{j=0}^{\infty} (\varphi_{TH}\beta)^j \, Q_{t,t+j} \left[\begin{array}{c} \frac{p_{TH,t}(f_{TH})}{P_{t+j}} \cdot y_{TH,t+j|t}\left(f_{TH}\right) \\ +e_{t+j} \frac{p^*_{TH,t+j}(f_{TH})}{P_{t+j}} y^*_{TH,t+j|t}\left(f_{TH}\right) \\ -\frac{W_{TH,t+j}}{P_{t+j}} \cdot \widetilde{h}_{TH,t+j|t}\left(f_{TH}\right) \end{array} \right] ,$$

$$\begin{aligned} \text{s.t.} \quad & y_{TH,t+j|t}\left(f_{TH}\right) = \left(\frac{p_{TH,t}(f_{TH})}{P_{TH,t+j}}\right)^{-\eta_T} C_{TH,t+j} \ , \\ & y^*_{TH,t+j|t}\left(f_{TH}\right) = \left(\frac{p^*_{TH,t+j|t}(f_{TH})}{P^*_{TH,t+j}}\right)^{-\eta_T} C^*_{TH,t+j} \ , \\ & p^*_{TH,t+j|t}\left(f_{TH}\right) = \widetilde{p}_{TH,t}\left(f_{TH}\right) e_{t+j}^{-\zeta^*} \ . \end{aligned}$$

The first-order conditions describing optimal price setting are as follows:

$$E_t \sum\nolimits_{j=0}^{\infty} (\varphi_{TH}\beta)^j \, Q_{t,t+j} \left[\begin{array}{c} \frac{1}{P_{t+j}} \cdot y_{TH,t+j|t}\left(f_{TH}\right)(1-\eta_T) \\ +\eta_T \cdot \frac{W_{TH,t+j}}{P_{t+j}} \cdot \frac{\vartheta \widetilde{h}_{TH,t+j|t}(f_{TH})}{\vartheta y_{TH,t+j|t}(f_{TH})} \cdot \frac{y_{TH,t+j|t}(f_{TH})}{p_{TH,t}(f_{TH})} \end{array} \right] = 0 \tag{17}$$

$$E_t \sum\nolimits_{j=0}^{\infty} (\varphi_{TH}\beta)^j \, Q_{t,t+j} \left[\begin{array}{c} \frac{e_{t+j}^{1-\zeta^*}}{P_{t+j}} \cdot y^*_{TH,t+j|t}\left(f_{TH}\right)(1-\eta_T) \\ +\eta_T \cdot \frac{W_{TH,t+j}}{P_{t+j}} \cdot \frac{\vartheta \widetilde{h}_{TH,t+j|t}(f_{TH})}{\vartheta y^*_{TH,t+j|t}(f_{TH})} \cdot \frac{y^*_{TH,t+j|t}(f_{TH})}{\widetilde{p}_{TH,t}(f_{TH})} \end{array} \right] = 0 \tag{18}$$

Given the sequences $\{C_t\}$, $\{P_t\}$, $\{e_t\}$, $\{W_{TH,t}\}$, $\{P_{TH,t}\}$, $\left\{P^*_{TH,t}\right\}$, $\{C_{TH,t}\}$ and $\left\{C^*_{TH,t}\right\}$, the sequences of shocks and the initial conditions, each producer that chooses new prices in period t will choose the same $p_{TH,t}\left(f_{TH}\right)$ and $\widetilde{p}_{TH,t}\left(f_{TH}\right)$, and the same output levels $y_{TH,t+j|t}\left(f_{TH}\right)$ and $y^*_{TH,t+j|t}\left(f_{TH}\right)$. Then the optimal prices $\{p_{TH,t}\left(f_{TH}\right), P_{TH,t}\}$ $\left\{\widetilde{p}_{TH,t}\left(f_{TH}\right), \widetilde{P}_{TH,t}\right\}$ must satisfy the first-order conditions above and the following laws of motion:

$$P_{TH,t} = \left[\varphi_{TH} P_{TH,t-1}^{1-\eta_T} + (1-\varphi_{TH})\, p_{TH,t}\left(f_{TH}\right)^{1-\eta_T}\right]^{\frac{1}{1-\eta_T}} \quad ,$$

$$\widetilde{P}_{TH,t} = \left[\varphi_{TH} \widetilde{P}_{TH,t-1}^{1-\eta_T} + (1-\varphi_{TH})\, \widetilde{p}_{TH,t}\left(f_{TH}\right)^{1-\eta_T}\right]^{\frac{1}{1-\eta_T}} \quad .$$

By log-linearising the laws of motion above we get:

$$\widehat{X}_t = \frac{\varphi_{TH}}{1-\varphi_{TH}} \pi_{TH,t} \ ,$$

$$\widehat{\widetilde{X}}_t = \frac{\varphi_{TH}}{1-\varphi_{TH}} \widetilde{\pi}_{TH,t} \ ,$$

where $X_t \equiv \frac{p_{TH,t}(f_{TH})}{P_{TH,t}}$, $\widetilde{X}_t \equiv \frac{\widetilde{p}_{TH,t}(f_{TH})}{\widetilde{P}_{TH,t}}$, $\pi_{TH,t} \equiv \log \frac{P_{TH,t}}{P_{TH,t-1}}$, and $\widetilde{\pi}_{TH,t} \equiv \log \frac{\widetilde{P}_{TH,t}}{\widetilde{P}_{TH,t-1}}$. Notice that:

$$\widehat{X}_{t+j} = \widehat{X}_t - \sum\nolimits_{s=1}^{j} \pi_{TH,t+s} = \frac{\varphi_{TH}}{1-\varphi_{TH}} \pi_{TH,t} - \sum\nolimits_{s=1}^{j} \pi_{TH,t+s} \ ,$$

$$\widehat{\widetilde{X}}_{t+j} = \widehat{\widetilde{X}}_t - \sum\nolimits_{s=1}^{j} \widetilde{\pi}_{TH,t+s} = \frac{\varphi_{TH}}{1-\varphi_{TH}} \widetilde{\pi}_{TH,t} - \sum\nolimits_{s=1}^{j} \widetilde{\pi}_{TH,t+s} \ ,$$

where $X_{t+j} \equiv \frac{p_{TH,t}(f_{TH})}{P_{TH,t+j}}$ and $\widetilde{X}_{t+j} \equiv \frac{\widetilde{p}_{TH,t}(f_{TH})}{\widetilde{P}_{TH,t+j}}$. From the individual firm's production function:

$$y_{TH,t}(f_{TH}) + y^*_{TH,t}(f_{TH}) = z_{TH,t} \cdot \widetilde{h}_{TH,t}(f_{TH})^{\alpha_T} \ ,$$

we compute the derivatives in the following way:

$$\frac{\vartheta \widetilde{h}_{TH,t+j|t}(f_{TH})}{\vartheta y_{TH,t+j|t}(f_{TH})} = \frac{\vartheta \widetilde{h}_{TH,t+j|t}(f_{TH})}{\vartheta y^*_{TH,t+j|t}(f_{TH})} = \frac{1}{\alpha_T} \cdot (z_{TH,t+j})^{-\frac{1}{\alpha_T}} \cdot \left(y_{TH,t+j|t}(f_{TH}) + y^*_{TH,t+j|t}(f_{TH}) \right)^{\frac{1}{\alpha_T}-1}$$

Substituting the above expression into the first-order condition (17) and multiplying by $p_{TH,t}(f_{TH})$ we obtain:

$$E_t \sum\nolimits_{j=0}^{\infty} (\varphi_{TH}\beta)^j Q_{t,t+j} \left[\begin{array}{c} \frac{p_{TH,t}(f_{TH})}{P_{TH,t+j}} \frac{P_{TH,t+j}}{P_{t+j}} \cdot y_{TH,t+j|t}(f_{TH})(1-\eta_T) \\ +\frac{\eta_T}{\alpha_T} \cdot (z_{TH,t+j})^{-\frac{1}{\alpha_T}} \cdot \frac{W_{TH,t+j}}{P_{t+j}} \cdot \\ \cdot \left(y_{TH,t+j|t}(f_{TH}) + y^*_{TH,t+j|t}(f_{TH}) \right)^{\frac{1}{\alpha_T}-1} \cdot y_{TH,t+j|t}(f_{TH}) \end{array} \right] = 0 \ ,$$

analogously:

$$E_t \sum\nolimits_{j=0}^{\infty} (\varphi_{TH}\beta)^j Q_{t,t+j} \left[\begin{array}{c} e_{t+j}^{1-\zeta^*} \frac{\widetilde{p}_{TH,t}(f_{TH})}{\widetilde{P}_{TH,t+j}} \frac{\widetilde{P}_{TH,t+j}}{P_{t+j}} \cdot y^*_{TH,t+j|t}(f_{TH})(1-\eta_T) \\ +\frac{\eta_T}{\alpha_T} \cdot (z_{TH,t+j})^{-\frac{1}{\alpha_T}} \cdot \frac{W_{TH,t+j}}{P_{t+j}} \cdot \\ \cdot \left(y_{TH,t+j|t}(f_{TH}) + y^*_{TH,t+j|t}(f_{TH}) \right)^{\frac{1}{\alpha_T}-1} \cdot y^*_{TH,t+j|t}(f_{TH}) \end{array} \right] = 0 \ .$$

Notice that the two first-order conditions imply that the law of one price is recovered in the steady state, as stated earlier.

Now we log-linearise around a deterministic equilibrium or steady state in which all the exogenous stochastic processes are set equal to their unconditional means, their variances are set to zero, and individuals hold no internationally traded bond. In this deterministic equilibrium $p_{TH,0}(f_{TH}) = P_{TH,0}$ and $\widetilde{p}_{TH,0}(f_{TH}) = \widetilde{P}_{TH,0}$. We obtain:

$$E_t \sum\nolimits_{j=0}^{\infty} (\varphi_{TH}\beta)^j \left[\begin{array}{c} \widehat{X}_{t+j} + \widehat{P}_{TH,t+j} + \frac{1}{\alpha_T} \cdot \widehat{z}_{TH,t+j} - \widehat{W}_{TH,t+j} \\ -\frac{1-\alpha_T}{\alpha_T} k_1 \widehat{y}_{TH,t+j|t}(f_{TH}) - \frac{1-\alpha_T}{\alpha_T}(1-k_1)\widehat{y}^*_{TH,t+j|t}(f_{TH}) \end{array} \right] = 0 \ ,$$

$$E_t \sum_{j=0}^{\infty} (\varphi_{TH}\beta)^j \left[\begin{array}{c} (1-\zeta^*)\widehat{e}_{t+j} + \widehat{\widetilde{X}}_{t+j} + \widehat{\widetilde{P}}_{TH,t+j} + \frac{1}{\alpha_T} \cdot \widehat{z}_{TH,t+j} - \widehat{W}_{TH,t+j} \\ -\frac{1-\alpha_T}{\alpha_T} k_1 \widehat{y}_{TH,t+j|t}(f_{TH}) - \frac{1-\alpha_T}{\alpha_T}(1-k_1)\widehat{y}^*_{TH,t+j|t}(f_{TH}) \end{array} \right] = 0 \ ,$$

where $k_1 \equiv \frac{C_{TH,0}}{C_{TH,0}+C^*_{TH,0}} = \frac{Y_{TH,0}}{Y_{TH,0}+Y^*_{TH,0}}$.

By log-linearising the demands for output:

$$\widehat{y}_{TH,t+j|t}\left(f_{TH}\right) = -\eta_T \cdot \widehat{X}_{t+j} + \widehat{Y}_{TH,t+j} \ ,$$

$$\widehat{y}^*_{TH,t+j|t}\left(f_{TH}\right) = -\eta_T \cdot \widehat{\widehat{X}}_{t+j} + \widehat{Y}^*_{TH,t+j} \ ,$$

since $\frac{\widetilde{p}_{TH,t}(f_{TH})}{\widetilde{P}_{TH,t+j}} = \frac{p^*_{TH,t+j|t}(f_{TH})}{P^*_{TH,t+j}}$.

We can substitute into the log-linearised first-order conditions the expressions for $\widehat{X}_{t+j}$, $\widehat{\widehat{X}}_{t+j}$ and $\widehat{y}_{TH,t+j|t}\left(f_{TH}\right)$, $\widehat{y}^*_{TH,t+j|t}\left(f_{TH}\right)$, and after some simplifications we obtain:

$$E_t \sum\nolimits_{j=0}^{\infty} \left(\varphi_{TH}\beta\right)^j \left[\begin{array}{c} \left(1 + \eta_T \frac{1-\alpha_T}{\alpha_T} k_1\right) \cdot \left(\frac{\varphi_{TH}}{1-\varphi_{TH}} \pi_{TH,t} - \sum_{s=1}^{j} \pi_{TH,t+s}\right) \\ +\eta_T \frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right) \cdot \left(\frac{\varphi_{TH}}{1-\varphi_{TH}} \widetilde{\pi}_{TH,t} - \sum_{s=1}^{j} \widetilde{\pi}_{TH,t+s}\right) \\ +\widehat{P}_{TH,t+j} - \widehat{W}_{TH,t+j} + \frac{1}{\alpha_T} \cdot \widehat{z}_{TH,t+j} \\ -\frac{1-\alpha_T}{\alpha_T} k_1 \widehat{Y}_{TH,t+j} - \frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right) \widehat{Y}^*_{TH,t+j} \end{array} \right] = 0 \ ,$$

$$E_t \sum_{j=0}^{\infty} \left(\varphi_{TH}\beta\right)^j \left[\begin{array}{c} \eta_T \frac{1-\alpha_T}{\alpha_T} k_1 \cdot \left(\frac{\varphi_{TH}}{1-\varphi_{TH}} \pi_{TH,t} - \sum_{s=1}^{j} \pi_{TH,t+s}\right) \\ +\left(1 + \eta_T \frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right)\right) \cdot \left(\frac{\varphi_{TH}}{1-\varphi_{TH}} \widetilde{\pi}_{TH,t} - \sum_{s=1}^{j} \widetilde{\pi}_{TH,t+s}\right) \\ +\left(1-\zeta^*\right)\widehat{e}_{t+j} + \widehat{\widehat{P}}_{TH,t+j} - \widehat{\widehat{W}}_{TH,t+j} + \frac{1}{\alpha_T} \cdot \widehat{z}_{TH,t+j} \\ -\frac{1-\alpha_T}{\alpha_T} k_1 \widehat{Y}_{TH,t+j} - \frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right) \widehat{Y}^*_{TH,t+j} \end{array} \right] = 0 \ ,$$

which can be further simplified as follows:

$$\begin{array}{l} \frac{1}{1-\varphi_{TH}\beta}\left(1 + \eta_T \frac{1-\alpha_T}{\alpha_T} k_1\right) \frac{\varphi_{TH}}{1-\varphi_{TH}} \pi_{TH,t} + \frac{1}{1-\varphi_{TH}\beta} \eta_T \frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right) \frac{\varphi_{TH}}{1-\varphi_{TH}} \widetilde{\pi}_{TH,t} \\ = \frac{1}{1-\varphi_{TH}\beta}\left(1 + \eta_T \frac{1-\alpha_T}{\alpha_T} k_1\right) E_t \sum_{j=1}^{\infty} \left(\varphi_{TH}\beta\right)^j \pi_{TH,t+j} \\ +\frac{1}{1-\varphi_{TH}\beta} \eta_T \frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right) E_t \sum_{j=1}^{\infty} \left(\varphi_{TH}\beta\right)^j \widetilde{\pi}_{TH,t+j} \\ -E_t \sum_{j=0}^{\infty} \left(\varphi_{TH}\beta\right)^j \left[\begin{array}{c} +\widehat{P}_{TH,t+j} - \widehat{W}_{TH,t+j} + \frac{1}{\alpha_T} \cdot \widehat{z}_{TH,t+j} \\ -\frac{1-\alpha_T}{\alpha_T} k_1 \widehat{Y}_{TH,t+j} - \frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right) \widehat{Y}^*_{TH,t+j} \end{array} \right] \ , \end{array}$$

$$\begin{array}{l} \frac{1}{1-\varphi_{TH}\beta} \eta_T \frac{1-\alpha_T}{\alpha_T} k_1 \frac{\varphi_{TH}}{1-\varphi_{TH}} \pi_{TH,t} + \frac{1}{1-\varphi_{TH}\beta}\left(1 + \eta_T \frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right)\right) \frac{\varphi_{TH}}{1-\varphi_{TH}} \widetilde{\pi}_{TH,t} \\ = \frac{1}{1-\varphi_{TH}\beta} \eta_T \frac{1-\alpha_T}{\alpha_T} k_1 E_t \sum_{j=1}^{\infty} \left(\varphi_{TH}\beta\right)^j \pi_{TH,t+j} \\ +\frac{1}{1-\varphi_{TH}\beta}\left(1 + \eta_T \frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right)\right) E_t \sum_{j=1}^{\infty} \left(\varphi_{TH}\beta\right)^j \widetilde{\pi}_{TH,t+j} \\ -E_t \sum_{j=0}^{\infty} \left(\varphi_{TH}\beta\right)^j \left[\begin{array}{c} +\left(1-\zeta^*\right)\widehat{e}_{t+j} + \widehat{\widehat{P}}_{TH,t+j} - \widehat{\widehat{W}}_{TH,t+j} + \frac{1}{\alpha_T} \cdot \widehat{z}_{TH,t+j} \\ -\frac{1-\alpha_T}{\alpha_T} k_1 \widehat{Y}_{TH,t+j} - \frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right) \widehat{Y}^*_{TH,t+j} \end{array} \right] \ . \end{array}$$

Finally, simplifying and using the law of iterated expectations, we can write:

$$\left(1+\eta_T\tfrac{1-\alpha_T}{\alpha_T}k_1\right)\pi_{TH,t}+\eta_T\tfrac{1-\alpha_T}{\alpha_T}\left(1-k_1\right)\widetilde{\pi}_{TH,t}$$
$$=\left(1+\eta_T\tfrac{1-\alpha_T}{\alpha_T}k_1\right)\beta E_t\pi_{TH,t+1}+\eta_T\tfrac{1-\alpha_T}{\alpha_T}\left(1-k_1\right)\beta E_t\widetilde{\pi}_{TH,t+1}$$
$$+\left(1-\varphi_{TH}\beta\right)\tfrac{1-\varphi_{TH}}{\varphi_{TH}}\left[\begin{array}{c}\widehat{W}_{TH,t}-\widehat{P}_{TH,t}-\frac{1}{\alpha_T}\cdot\widehat{z}_{TH,t}\\ +\frac{1-\alpha_T}{\alpha_T}k_1\widehat{Y}_{TH,t}+\frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right)\widehat{Y}^*_{TH,t}\end{array}\right], \tag{19}$$

$$\eta_T\tfrac{1-\alpha_T}{\alpha_T}k_1\pi_{TH,t}+\left(1+\eta_T\tfrac{1-\alpha_T}{\alpha_T}\left(1-k_1\right)\right)\widetilde{\pi}_{TH,t}$$
$$=\eta_T\tfrac{1-\alpha_T}{\alpha_T}k_1\beta E_t\pi_{TH,t+1}+\left(1+\eta_T\tfrac{1-\alpha_T}{\alpha_T}\left(1-k_1\right)\right)\beta E_t\widetilde{\pi}_{TH,t+1}$$
$$+\left(1-\varphi_{TH}\beta\right)\tfrac{1-\varphi_{TH}}{\varphi_{TH}}\left[\begin{array}{c}-\left(1-\zeta^*\right)\widehat{e}_t+\widehat{W}_{TH,t}-\widehat{\widetilde{P}}_{TH,t}-\frac{1}{\alpha_T}\cdot\widehat{z}_{TH,t}\\ +\frac{1-\alpha_T}{\alpha_T}k_1\widehat{Y}_{TH,t}+\frac{1-\alpha_T}{\alpha_T}\left(1-k_1\right)\widehat{Y}^*_{TH,t}\end{array}\right]. \tag{20}$$

Log-linearising (3) we obtain:

$$\pi^{Tot}_{TH,t}=k_1\pi_{TH,t}+\left(1-k_1\right)\left(\widehat{e}_t-\widehat{e}_{t-1}+\pi^*_{TH,t}\right). \tag{21}$$

Using $P^*_{TH,t}=\frac{\widetilde{P}_{TH,t}}{e_t^{\zeta^*}}$, it is easy to show that:

$$\widetilde{\pi}_{TH,t}=\pi^*_{TH,t}+\zeta^*\left(\widehat{e}_t-\widehat{e}_{t-1}\right). \tag{22}$$

By substituting Equations (2), (21) and (22) into Equation (19) we obtain:

$$\pi_{TH,t}+\eta_T\tfrac{1-\alpha_T}{\alpha_T}\left[\pi^{Tot}_{TH,t}-\left(1-k_1\right)\left(1-\zeta^*\right)\left(\widehat{e}_t-\widehat{e}_{t-1}\right)\right]$$
$$=\beta E_t\pi_{TH,t+1}+\eta_T\tfrac{1-\alpha_T}{\alpha_T}\beta E_t\left[\pi^{Tot}_{TH,t+1}-\left(1-k_1\right)\left(1-\zeta^*\right)\left(\widehat{e}_{t+1}-\widehat{e}_t\right)\right]$$
$$+\left(1-\varphi_{TH}\beta\right)\tfrac{1-\varphi_{TH}}{\varphi_{TH}}\left[\widehat{W}_{TH,t}-\widehat{P}_{TH,t}-\tfrac{1}{\alpha_T}\cdot\widehat{z}_{TH,t}+\tfrac{1-\alpha_T}{\alpha_T}\widehat{Y}^{Tot}_{TH,t}\right], \tag{23}$$

and by substituting Equations (2), (21) and (22) into Equation (20) we obtain:

$$\eta_T\tfrac{1-\alpha_T}{\alpha_T}\left[\pi^{Tot}_{TH,t}-\left(1-k_1\right)\left(1-\zeta^*\right)\left(\widehat{e}_t-\widehat{e}_{t-1}\right)\right]+\pi^*_{TH,t}+\zeta^*\left(\widehat{e}_t-\widehat{e}_{t-1}\right)$$
$$=\eta_T\tfrac{1-\alpha_T}{\alpha_T}\beta E_t\left[\pi^{Tot}_{TH,t+1}-\left(1-k_1\right)\left(1-\zeta^*\right)\left(\widehat{e}_{t+1}-\widehat{e}_t\right)\right]+\beta E_t\left[\pi^*_{TH,t+1}+\zeta^*\left(\widehat{e}_{t+1}-\widehat{e}_t\right)\right]$$
$$+\left(1-\varphi_{TH}\beta\right)\tfrac{1-\varphi_{TH}}{\varphi_{TH}}\left[-\left(1-\zeta^*\right)\widehat{e}_t+\widehat{W}_{TH,t}-\widehat{P}^*_{TH,t}-\zeta^*\widehat{e}_t-\tfrac{1}{\alpha_T}\cdot\widehat{z}_{TH,t}+\tfrac{1-\alpha_T}{\alpha_T}\widehat{Y}^{Tot}_{TH,t}\right]. \tag{24}$$

Next, we can multiply (23) by k_1 and (24) by $(1-k_1)$, sum the two equations and after some simplifications we arrive at the forward-looking equation for total inflation in the Home tradeable goods sector:

$$\begin{aligned}
& \pi_{TH,t}^{Tot} - (1-\zeta^*)(1-k_1)(\widehat{e}_t - \widehat{e}_{t-1}) \\
= \; & \beta E_t \left[\pi_{TH,t+1}^{Tot} - (1-\zeta^*)(1-k_1)(\widehat{e}_{t+1} - \widehat{e}_t)\right] \\
& + \left(\frac{1-\varphi_{TH}\beta}{1+\eta_T \frac{1-\alpha_T}{\alpha_T}} \frac{1-\varphi_{TH}}{\varphi_{TH}}\right) \left[\widehat{W}_{TH,t} - \widehat{P}_{TH,t}^{Tot} - \frac{1}{\alpha_T} \cdot \widehat{z}_{TH,t} + \frac{1-\alpha_T}{\alpha_T} \widehat{Y}_{TH,t}^{Tot}\right] .
\end{aligned}$$

We can write variations in the total real marginal cost (MC_{TH}^{Tot}) in sector TH as:

$$\widehat{MC}_{TH,t}^{Tot} = \widehat{W}_{TH,t} - \widehat{P}_{TH,t}^{Tot} - \frac{1}{\alpha_T} \cdot \widehat{z}_{TH,t} + \frac{1-\alpha_T}{\alpha_T} \widehat{Y}_{TH,t}^{Tot} .$$

In the particular case of constant returns to labour ($\alpha_T = 1$), the level of output does not affect real marginal costs.

Following analogous steps, we can derive also the forward-looking equation for inflation in the Home nontradeable sector:

$$\pi_{N,t} = \beta E_t \pi_{N,t+1} + \left(\frac{1-\varphi_N\beta}{1+\eta_N \frac{1-\alpha_N}{\alpha_N}} \frac{1-\varphi_N}{\varphi_N}\right) \left(\widehat{W}_{N,t} - \widehat{P}_{N,t} - \frac{1}{\alpha_N} \cdot \widehat{z}_{N,t} + \frac{1-\alpha_N}{\alpha_N} \widehat{Y}_{N,t}\right) .$$

If we make use of the simplifying assumptions $\theta = 1$, $\varphi_{TH} = \varphi_N = \varphi$, $\eta_{TH} = \eta_N = \eta$, and $\alpha_{TH} = \alpha_N = \alpha$ then the following relationship holds:

$$\begin{aligned}
& \pi_{TH,t}^{Tot} - \pi_{N,t} - \delta(1-\zeta^*)(\widehat{e}_t - \widehat{e}_{t-1}) \\
= \; & \beta E_t \left[\pi_{TH,t+1}^{Tot} - \pi_{N,t+1} - \delta(1-\zeta^*)(\widehat{e}_{t+1} - \widehat{e}_t)\right] \\
& + \left(\frac{1-\varphi\beta}{1+\eta\frac{1-\alpha}{\alpha}} \frac{1-\varphi}{\varphi}\right) \left[\begin{array}{c} \widehat{W}_{TH,t} - \widehat{P}_{TH,t}^{Tot} - \frac{1}{\alpha} \cdot \widehat{z}_{TH,t} + \frac{1-\alpha}{\alpha} \widehat{Y}_{TH,t}^{Tot} \\ - \left(\widehat{W}_{N,t} - \widehat{P}_{N,t} - \frac{1}{\alpha} \cdot \widehat{z}_{N,t} + \frac{1-\alpha}{\alpha} \widehat{Y}_{N,t}\right) \end{array}\right] .
\end{aligned}$$

Moreover, if we assume that the economy is at the steady state in period $t-1$, then $\pi_{TH,t}^{Tot} = \widehat{P}_{TH,t}^{Tot}$ and $\pi_{N,t} = \widehat{P}_{N,t}$, therefore we can write:

$$\begin{aligned}
& \widehat{P}_{TH,t}^{Tot} - \widehat{P}_{N,t} - \delta(1-\zeta^*)\widehat{e}_t \\
= \; & \beta E_t \left[\pi_{TH,t+1}^{Tot} - \pi_{N,t+1} - \delta(1-\zeta^*)(\widehat{e}_{t+1} - \widehat{e}_t)\right] \\
& + \left(\frac{1-\varphi\beta}{1+\eta\frac{1-\alpha}{\alpha}} \frac{1-\varphi}{\varphi}\right) \left[\begin{array}{c} \widehat{W}_{TH,t} - \widehat{P}_{TH,t}^{Tot} - \frac{1}{\alpha} \cdot \widehat{z}_{TH,t} + \frac{1-\alpha}{\alpha} \widehat{Y}_{TH,t}^{Tot} \\ - \left(\widehat{W}_{N,t} - \widehat{P}_{N,t} - \frac{1}{\alpha} \cdot \widehat{z}_{N,t} + \frac{1-\alpha}{\alpha} \widehat{Y}_{N,t}\right) \end{array}\right] .
\end{aligned}$$

Appendix C: Derivation of the moment conditions

This Appendix illustrates the derivation of the moment conditions presented in Table 4.

Moment conditions # 1 and 2:

The Home and Foreign Euler equations for consumption are given by:

$C_t^{-\sigma}\left\{1+\frac{\nu}{C_0}B_t\right\}\frac{P_{T,t}}{P_t}=\beta E_t\left[\left(1+r_t\right)C_{t+1}^{-\sigma}\frac{P_{T,t+1}}{P_{t+1}}\right]$

$\left(C_t^*\right)^{-\sigma}\left[1+\frac{\nu}{C_0}B_t^*\right]\frac{P_{T,t}}{e_tP_t^*}=\beta E_t\left[\left(1+r_t\right)\left(C_{t+1}^*\right)^{-\sigma}\frac{P_{T,t+1}}{e_{t+1}P_{t+1}^*}\right]$

The cost parameter ν is the same for the Home and Foreign countries and $B_t+B_t^*=0$ at any date t. Log-linearising and linearising around the steady state and substituting out $dB_t^*=-dB_t$:

$$\sigma E_t\widehat{C}_{t+1}-\sigma\widehat{C}_t+\nu dB_t=\left(1-\beta\right)\widehat{r}_t+E_t\widehat{P}_{T,t+1}-\widehat{P}_{T,t}-E_t\widehat{P}_{t+1}+\widehat{P}_t \quad (25)$$

$$\sigma E_t\widehat{C}_{t+1}^*-\sigma\widehat{C}_t^*-\nu dB_t=\left(1-\beta\right)\widehat{r}_t+E_t\widehat{P}_{T,t+1}-E_t\widehat{e}_{t+1}-\widehat{P}_{T,t}+\widehat{e}_t-E_t\widehat{P}_{t+1}^*+\widehat{P}_t^* \quad (26)$$

If we define the nominal interest rate as the opportunity cost of holding money with respect to bonds, then we need to adjust the standard Fisher parity condition, to adapt it to the presence of the adjustment cost on bonds.

Home:

$\left(1+i_t\right)\left(1+\frac{\nu}{C_0}B_t\right)=\left(1+r_t\right)E_t\left[\frac{P_{T,t+1}}{P_{T,t}}\right]$

Foreign:

$\left(1+i_t^*\right)\left(1-\frac{\nu}{C_0}B_t\right)=\left(1+r_t\right)E_t\left[\frac{P_{T,t+1}}{P_{T,t}}\frac{e_t}{e_{t+1}}\right]$

Log-linearisation:[2]

$$\widehat{i}_t=\left(\frac{1}{1-\beta}\right)\left(E_t\widehat{P}_{T,t+1}-\widehat{P}_{T,t}\right)+\widehat{r}_t-\frac{1}{1-\beta}\nu dB_t \quad (27)$$

$$\widehat{i}_t^*=\left(\frac{1}{1-\beta}\right)\left(E_t\widehat{P}_{T,t+1}-E_t\widehat{e}_{t+1}\right)-\left(\frac{1}{1-\beta}\right)\left(\widehat{P}_{T,t}-\widehat{e}_t\right)+\widehat{r}_t+\frac{1}{1-\beta}\nu dB_t \quad (28)$$

Finally, the Home and Foreign first-order conditions with respect to money holdings are given by:

$\chi\left(\frac{M_t}{P_t}\right)^{-\varepsilon}=C_t^{-\sigma}-\beta E_t\left[C_{t+1}^{-\sigma}\frac{P_t}{P_{t+1}}\right]$

$\chi\left(\frac{M_t^*}{P_t^*}\right)^{-\varepsilon}=\left(C_t^*\right)^{-\sigma}-\beta E_t\left[\left(C_{t+1}^*\right)^{-\sigma}\frac{P_t^*}{P_{t+1}^*}\right]$

Log-linearising:

[2] As in Benigno (2001), uncovered interest parity does not hold. The spread in the nominal interest rates rates reflects a premium on top of the expected exchange rate depreciation:

$$\widehat{i}_t-\widehat{i^*}_t=\left(\frac{1}{1-\beta}\right)\left(E_t\widehat{e}_{t+1}-\widehat{e}_t\right)-2\frac{\nu C_0}{1-\beta}dB_t$$

$$-\varepsilon\widehat{M}_t+\varepsilon\widehat{P}_t=\frac{1}{1-\beta}\left[-\sigma\widehat{C}_t+\sigma\beta E_t\widehat{C}_{t+1}-\beta\widehat{P}_t+\beta E_t\widehat{P}_{t+1}\right] \qquad (29)$$

$$-\varepsilon\widehat{M}_t^*+\varepsilon\widehat{P}_t^*=\frac{1}{1-\beta}\left[-\sigma\widehat{C}_t^*+\sigma\beta E_t\widehat{C}_{t+1}^*-\beta\widehat{P}_t^*+\beta E_t\widehat{P}_{t+1}^*\right] \qquad (30)$$

Chari, Kehoe and McGrattan (Review of Economic Studies 2002, page 547), estimate the utility parameters from the US money demand equation with consumption and interest rates. An analogous money demand equation is obtained by using (25) to substitute out $\widehat{C}_{t+1}$ from Equation (29):

$$-\varepsilon\widehat{M}_t+\varepsilon\widehat{P}_t=\frac{1}{1-\beta}\left[\begin{array}{c}-\sigma\widehat{C}_t-\beta\widehat{P}_t+\beta E_t\widehat{P}_{t+1}\\ +\beta\left((1-\beta)\,\widehat{r}_t+E_t\widehat{P}_{T,t+1}-\widehat{P}_{T,t}-E_t\widehat{P}_{t+1}+\widehat{P}_t+\sigma\widehat{C}_t-\nu dB_t\right)\end{array}\right]$$

$$-\varepsilon\widehat{M}_t+\varepsilon\widehat{P}_t=\frac{1}{1-\beta}\left[\begin{array}{c}-(1-\beta)\,\sigma\widehat{C}_t\\ +\beta\,(1-\beta)\,\widehat{r}_t+\beta\left(E_t\widehat{P}_{T,t+1}-\widehat{P}_{T,t}\right)-\beta\nu dB_t\end{array}\right]$$

$$-\varepsilon\widehat{M}_t+\varepsilon\widehat{P}_t=-\sigma\widehat{C}_t+\beta\widehat{r}_t-\tfrac{\beta}{1-\beta}\nu dB_t+\tfrac{\beta}{1-\beta}\left(E_t\widehat{P}_{T,t+1}-\widehat{P}_{T,t}\right)$$

However, the problem with estimating the equation above is the need to have observations on the real interest rate and bond holdings, which may be imperfectly measured. Therefore, I use Equation (27) to substitute out $\widehat{r}_t$:

$$-\varepsilon\widehat{M}_t+\varepsilon\widehat{P}_t = -\sigma\widehat{C}_t+\beta\left[\widehat{i}_t-\left(\tfrac{1}{1-\beta}\right)\left(E_t\widehat{P}_{T,t+1}-\widehat{P}_{T,t}\right)+\tfrac{1}{1-\beta}\nu dB_t\right]-\tfrac{\beta}{1-\beta}\nu dB_t+\tfrac{\beta}{1-\beta}\left(E_t\widehat{P}_{T,t+1}-\widehat{P}_{T,t}\right)$$

Thus, in this setup the money demand equation in the Home country is given by:

$$\varepsilon\left(\widehat{M}_t-\widehat{P}_t\right)-\sigma\widehat{C}_t+\beta\widehat{i}_t=0$$

and in the Foreign country:

$$\varepsilon\left(\widehat{M}_t^*-\widehat{P}_t^*\right)-\sigma\widehat{C}_t^*+\beta\widehat{i}_t^*=0$$

Instead of estimating σ and ε from either the Home or the Foreign money demands, I prefer to use a linear combination of the two:

$$\varepsilon\left[\widehat{M}_t-\widehat{P}_t-\left(\widehat{M}_t^*-\widehat{P}_t^*\right)\right]-\sigma\left(\widehat{C}_t-\widehat{C}_t^*\right)+\beta\left(\widehat{i}_t-\widehat{i}_t^*\right)=0 \qquad (31)$$

Equation (31) enables me to use both US and Foreign data with a parsimonious instrument set. It is a "relative" money demand equation, linking changes in $\frac{M/P}{M^*/P^*}$ to: a) changes in relative consumption, and b) changes the interest rate differential.

Moment condition # 3:

The demand for Home nontradeables is given by:

$C_{N,t}=\gamma\left(\frac{P_{N,t}}{P_t}\right)^{-\phi}C_t$

Therefore:

$\frac{P_{N,t}C_{N,t}}{P_tC_t}=\gamma\left(\frac{P_{N,t}}{P_t}\right)^{1-\phi}$

Log-linearising:

$$\frac{P_{N,t}\widehat{C}_{N,t}}{P_t C_t} - (1-\phi)\left(\widehat{P}_{N,t} - \widehat{P}_t\right) = 0$$

Finally, the remaining moment conditions **# 4 to 29** result from the properties of the exogenous stochastic processes $\widehat{x}_i$.

End