Are there gains from including monetary aggregates and stock market indices in the monetary policy reaction function? A simulation study of recent U.S. monetary policy

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Abstract

We study how the inclusion of growth rates of monetary aggregates or changes in stock market indices affects the stabilization performance of optimal monetary policy rules when there is uncertainty about the structure of the economy. With a simulation model of the U.S. economy we show that the performance of monetary policy rules that include these variables deteriorates much stronger than that of rules without them if the true economic structure deviates from the one used to derive the rule. We also investigate whether money growth and changes in stock market indices help explaining the Fed's recent monetary policy.

Keywords: optimal monetary policy, monetary policy reaction function, robust monetary policy
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1 Introduction

For more than a decade there has been a resurgent interest in the theoretical analysis, estimation and empirical modeling of monetary policy reaction functions. Since most of the important central banks implement monetary policy by more or less directly manipulating short-term interest rates theoretical and empirical research on monetary policy reaction functions has focused on interest rate rules that express the (desired) value of the central bank's policy rate as a function of various state variables describing current, past or future expected conditions of the economy.

Among the empirical literature, numerous studies have fitted simple interest rate rules, like the famous Taylor rule (Taylor (1993)) and variants thereof, to the observed path of the central bank's interest rate instrument (e.g. Clarida, Galí, and Gertler (1998, 2002), Gordon (2005), Judd and Rudebusch (1998)).[1] VAR studies (e.g. Christiano, Eichenbaum, and Evans (1999) and the literature cited therein), although focussing on monetary policy "shocks", i.e. the unsystematic part of monetary policy, provide estimates of richly parameterized reaction functions.
In the theoretical literature the issue of the optimal design of monetary policy rules has been discussed intensively. Here, two important questions have to be considered: 1) which variables should be included in the reaction function, that is which variables should the central bank react to, and 2) how should the central bank react to these variables? Simple Taylor type rules have the policy rate only react to (forecasts of) the inflation rate and the output gap/the deviation of the unemployment rate from its natural level. However, it has also been discussed whether the central bank should react to the exchange rate as well (e.g. Ball (2000) and Leitemo and Söderström (2005)).[2] There has also been a discussion whether growth rates of monetary aggregates should be included in simple monetary policy rules. In the now standard New Keynesian models there is no special role for monetary aggregates in the conduct of monetary

[1] See also the rather critical discussion in Carare and Tchaidze (2005).

[2] Taylor type rules that include exchange rates are estimated by Chadha, Sarno, and Valente (2004) and Lubik and Schorfheide (2003) for various open economies.

policy. Money just reacts endogenously to the interest rate set by the central bank.[3] However, it has also been argued that even in this type of models monetary aggregates may contain information that is useful for monetary policy making.[4] As far as monetary policy practice is concerned monetary aggregates featured prominently in the monetary policy strategy of the German Bundesbank and are an important element of the strategy of the European Central Bank (ECB).[5] In contrast, the Federal Reserve System officially abandoned monetary targets in 1993.

Asset prices, particularly stock prices, are a second group of variables for which it has been intensively discussed whether or not they should be included in the monetary policy reaction function. One argument in favor of monetary policy reacting to changes in stock prices refers to the predictive content of asset prices for future output and inflation. These studies recommend that monetary policy should not care about asset prices per se but should respond to changes in asset prices to the extent that these signal future deviations of unemployment, output and inflation from its targets (e.g. Bernanke and Gertler (2000, 2001), Gilchrist and Leahy (2002)). Other authors argue that asset prices should be targeted by monetary policy in their own right because drastic changes in asset prices – in particular stock market or housing market crashes – have strong negative effects on output and employment (eg. Cecchetti et. al. (2000), Bordo and Jeanne (2002)). However, there are also warnings that including stock market variables into monetary policy reaction functions might at best be irrelevant for the overall economic outcome but might do considerable harm in some circumstances (Bullard and Schaling (2002)).[6]

[3] For an extended discussion, see Nelson (2003).

[4] For example, Coenen, Levin, and Wieland (2005) argue that monetary aggregates convey information on the mismeasurement of the output gap. Söderström (2005) shows that assigning a monetary growth target to the central bank makes discretionary monetary policy more inertial and improves social welfare relative to standard discretionary policy. Empirical models of the monetary policy reaction function including monetary aggregates have been estimated for the Euro Area by Gerdesmeier and Roffia (2004).

[5] In contrast, Bernanke and Mihov (1997) argue that the Bundesbank did, in fact, not react to deviations of money growth from its target values.

[6] Some empirical studies have investigated the effects of the stock market on monetary policy. Evidence that the Fed reacts to the stock market is presented e.g. in Bjørnland and Leitmo (2005), Chadha, Sarno, and Valente (2004), D'Agostino, Sala, and Surico (2005), D'Amico and Farka (2003),

The questions which variables should be included in a monetary policy rule and how the interest rate should be set in response to them are answered analytically in the literature on optimal monetary policy rules. Optimal monetary policy rules are reaction functions that minimize a social/central bank loss function (e.g. Ball (1999), Clarida, Galí, and Gertler (2001), Giannoni and Woodford (2003a,b), Schmitt-Grohe and Uribe (2004), Svensson (2003)). In models like these the optimal reaction function is determined by assumptions about the structural relationships within the economy and by the parameters of the loss function. The central bank reacts to all economic variables relative to their predictive content with respect to future values of the central bank's goal variables.

These theoretically derived optimal policy rules assume that the central bank knows the structural relationships within the economy. In practice however, central banks have to rely on estimated models of the monetary transmission mechanism and only have a rough understanding of the functioning of the economy. This leads to an important caveat concerning the practical implementation of optimal monetary policy rules: For many models optimal reaction functions tend to be quite complex and are very sensitive to changes in the structural assumptions of the model. This sensitivity of optimal policy rules in combination with the fact that the central bank faces uncertainty about the structure of the economy has led to a number of studies on "robust" monetary policy rules. A monetary policy reaction function is robust if its performance as measured by a loss function is not very sensitive to changes in the structural equations of the economic model, that is the value of the loss function does not deteriorate dramatically if the structural equations or coefficients of the model are changed. Typically, reaction functions considered in these studies are of a "simple" structure, i.e. they contain only a small number of variables that the central bank reacts to. Examples are modified versions of the Taylor rule or difference rules that relate the change in the policy rate to the deviations in unemployment/output and inflation from their target values (e.g. Orphanides and Williams (2002), Walsh (2003)). The parameters of the reaction function are chosen to minimize the central bank's loss function under the assumption of a particular structural economic model. Then, the performance of the optimized simple reaction function under alternative structural models, i.e. structures different from

and Rigobon and Sack (2003).

the one it was optimized for, is studied by simulating the changed structural model with the unchanged reaction function and comparing the resulting values of the loss function or the resulting variances of the central bank's goal variables (e.g. Walsh (2003)). This exercise captures the inherent uncertainty of monetary policy makers about the true structure of the economy. While relatively simple monetary policy reaction functions generally perform worse than complex ones in the model they were optimized for, many studies have shown their performance to deteriorate less under different economic structures (e.g. Levin and Williams (1998), Levin, Wieland, and Williams (1998), Williams (2003)).

From this research arises a third question. Considering the different variables that might possibly be included in the policy rule how do these variant policy rules compare in their robustness with respect to changes in the economic structure? In particular, how is the relative performance of these rules if central assumptions under which they were derived turn out to be erroneous?

Our task in this paper is to contribute to this discussion in two ways: First, from a purely descriptive point of view we ask whether the inclusion of monetary aggregates or stock market variables in an optimal monetary policy reaction function improves its ability to explain actually observed monetary policy in the U.S. in the recent period. This is not meant to conclude whether the Fed did or did not use these variables in its monetary policy deliberations. Nevertheless, the results can tell us whether, for example, monetary aggregates help with explaining the Fed's behavior, perhaps because these variables are correlated with information the Fed actually uses when deciding about the target Federal Funds rate.

Second, we study the questions concerning the design of the monetary policy rule discussed above by comparing the performance of optimal policy rules containing alternative sets of economic variables. In particular, we focus on the performance of alternative optimal policy rules when the policymaker is uncertain about the true structural relationships in the economy.

Our starting point is an empirical model suggested by Sack (2000) who shows that U.S. monetary policy can be approximated fairly well by a policy rule derived from an optimal control model in which the structure of the economy is assumed to be given by an estimated vector autoregressive (VAR) model. This VAR is a relatively

unrestricted empirical model of the structural relationships in the economy. Using this dynamic structure we solve the dynamic programming problem of the central bank and derive an optimal monetary policy reaction function. In this we follow the literature on optimal policy rules but impose relatively few restrictions on the economic model. By including different sets of variables in our model (monetary aggregates or stock market indices) we can derive alternative monetary policy rules. We compare how the interest rate generated by the rule in question "fits" the observed time series of the Federal Funds rate. If the fit of the policy rule improves by adding a specific economic variable this might indicate that this variable represents information the Fed looked at in setting the Federal Funds rate.
Then, we study the robustness of the extended policy rules, i.e. the ones including money growth or stock market indices, under changing assumptions about the structure of the economy. We compare them to a restricted rule in which the policy rate does not respond to these variables. Uncertainty about the structure of the economy is considered in two different ways: 1) we look at relatively small perturbances of the economic structure from the one the rule was optimized for by allowing for small random variations in the structural coefficients of the model, and 2) we drastically alter the predictive power of monetary aggregates and the stock market index for the other economic variables by adding or removing zero restrictions from our structural model. By these experiments we can show if anything and how much might be gained for stabilization policy by including money growth or stock market variables in the policy rule. Fair (2005) performs a similar simulation exercise with optimal policy rules within an estimated structural multi-country model. However, he does not study the robustness of different policy rules and does not focus on monetary aggregates or on the stock market. In addition, our empirical model imposes much fewer restrictions on the economic structure.

Our results show that the inclusion of monetary aggregates in the policy reaction function improves its fit. However, the extended policy rules are much more sensitive with respect to uncertainty about the economy's structure than the simple rule which includes neither money growth nor the stock market index.

The paper is structured as follows. In section 2.1 we describe the empirical model of the

U.S. economy and derive the optimal monetary policy rules if only additive uncertainty is present, that is, if the structural coefficients of the economy are known with certainty. In section 2.2 we estimate the free parameters of the model and compare the different policy rules with respect to their ability to track the actually observed Federal Funds rate and the values for the central bank's loss function they generate. The optimal reaction functions under additive uncertainty respond much more aggressive to changes in economic conditions than it is actually observed. In section 3 we therefore use an approximation technique from Sack (2000) to derive optimal reaction functions assuming that the central bank faces uncertainty about the parameters in the structural economic model (section 3.1). As before, we compare the fit of the simple and the extended rules (section 3.2). We then study their relative stabilization performance in the presence of random perturbations to the structural economic model. In addition, we consider modified versions of our models in which we dramatically alter the information content of monetary aggregates or stock market variables and compare how the various rules perform under averse conditions (section 3.2). Finally, section 4 concludes.

2 Optimal policy under additive uncertainty

In this section we derive the optimal monetary policy reaction function for the Fed using a standard linear-quadratic optimal control approach as described in Sack (2000).[7] We assume that the structure of the economy can be described by a linear structural model that is based on an estimated vector autoregression. The Fed is assumed to set its policy rate (the Federal Funds rate) according to a linear policy reaction function. This reaction function is chosen to minimize a quadratic loss function for unemployment and inflation.

2.1 Deriving the optimal policy rule

Our analysis starts with the estimation of a structural VAR on monthly data

[7] For a general discussion of this approach, see Ljunqvist and Sargent (2004).

$$
\begin{aligned}
Z_t &= \sum_{i=0}^{q} \mathbf{A}_i Z_{t-i} + \sum_{i=0}^{q} \mathbf{b}_i i_{t-1} + \nu_t^Z &(1)\\
i_t &= \sum_{i=0}^{q} \mathbf{c}_i' Z_{t-i} + \sum_{i=1}^{q} d_i i_{t-1} + \nu_t^i. &(2)
\end{aligned}
$$

Z_t is an $(n \times 1)$-vector of non-policy variables, i_t is the policy variable (the Federal Funds rate), q is the number of lags in the VAR, ν_t^Z is a $(n \times 1)$-vector of uncorrelated structural shocks that is also uncorrelated with the structural policy disturbance ν_t^i. $\mathbf{A}_i$ are $(n \times n)$ matrices and $\mathbf{b}_i$ and $\mathbf{c}_i$ are $(n \times 1)$-vectors. Z_t contains the deviation of the unemployment rate from the natural rate u, the growth rate of industrial production y, the inflation rate π, a rate of inflation for commodity prices c and, alternatively, the annual growth rate of M1, the annual growth rate of M3, or the annual growth rate of the S&P 500 stock market index.

We estimate the natural rate of unemployment as in Staiger, Stock and Watson (1997) from a Phillips curve equation as an unobserved component that follows a random walk. The inflation rate is the annual growth rate of the Consumer Price Index. Our measure of commodity price inflation is the annual growth rate of the spot price index of all commodities from the Commodity Research Bureau.[8]

Equation (2) represents the estimated (actual) policy reaction function. In our simulations we will replace this equation with the optimal policy reaction function while we will retain (1) as the description of the structural relations within the economy.

In order to derive the optimal policy reaction function we have to transform the structural model (1) and (2) into a linear-quadratic dynamic programming problem (see Sack (2000)). We assume that the Fed maximizes a quadratic objective function

$$
-\frac{1}{2} E_t \left\{ \sum_{i=1}^{\infty} \beta^i \left[(\pi_{t+i} - \pi^*)^2 + \lambda_u (u_{t+i} - u^*)^2 \right] \right\}, \tag{3}
$$

[8] In the VAR literature commodity prices are often used to avoid the "price puzzle", i.e. an observed increase in inflation following a contractionary monetary policy shock. Introducing commodity prices often alleviates this problem since they control for expected future inflation. For an extensive discussion of this issue, see Hanson (2004). For our estimated model commodity price inflation does not reduce the price puzzle in the impulse response functions. We decided to include this variable nevertheless in order to conform to the standard VAR literature.

with λ_u as the weight on the Fed's employment objective relative to its inflation objective. Since u is defined as the deviation of unemployment from the natural rate we set u^* equal to zero, i.e. we assume that the Fed's unemployment target is equal to the natural rate of unemployment.

The structural economic relations (1) are written in state space form as

$$X_{t+1} = \mathbf{F} \cdot X_t + \mathbf{H} i_t + \mathbf{J} + \mu_{t+1}. \tag{4}$$

The coefficients in $\mathbf{F}, \mathbf{H}, \mathbf{J}$ are derived from (1) as will be described shortly. The state vector X_t contains current and lagged values of the variables in Z_t and its structure will depend on the assumptions made about the Fed's information set.

The optimal policy reaction function relates the policy instrument i_t to the vector of state variables X_t and solves the Bellman equation

$$V(X_t) = \max_{i_t} \left\{ -\left(X_t - X^*\right)' \mathbf{G} \left(X_t - X^*\right) + \beta E_t \left[V(X_{t+1}) \right] \right\} \tag{5}$$

subject to (4). The only non-zero elements in $\mathbf{G}$ are the diagonal entries corresponding to contemporaneous unemployment and inflation which are equal to λ_u and 1 respectively. The vector X^* consists of the target values for the state vector. Its only non-zero elements are the contemporaneous values of the unemployment gap and inflation and are equal to 0 and π^*.

For this linear quadratic dynamic programming problem the value function will be of the form

$$V(X) = X' \mathbf{\Lambda} X + 2X'\omega + \rho. \tag{6}$$

The solution for the optimal policy reaction function is

$$i_t^* = -\left(\mathbf{H}' \mathbf{\Lambda} \mathbf{H}\right)^{-1} \left(\mathbf{H}' \mathbf{\Lambda} \mathbf{F} X_t + \mathbf{H}' \mathbf{\Lambda} \mathbf{J} + \mathbf{H}'\omega\right), \tag{7}$$

where the symmetric matrix $\mathbf{\Lambda}$ is defined implicitly by the Riccati equation

$$\mathbf{\Lambda} = -\mathbf{G} + \beta \mathbf{F}'\mathbf{\Lambda}\mathbf{F} - \beta \mathbf{F}'\mathbf{\Lambda}\mathbf{H}\left(\mathbf{H}'\mathbf{\Lambda}\mathbf{H}\right)^{-1}\mathbf{H}'\mathbf{\Lambda}\mathbf{F}. \tag{8}$$

The vector ω is given by

$$\begin{aligned} \omega &= \left(\mathbf{I} - \beta \mathbf{F}'\left(\mathbf{I} - \mathbf{\Lambda}\mathbf{H}\left(\mathbf{H}'\mathbf{\Lambda}\mathbf{H}\right)^{-1}\mathbf{H}\right)\right)^{-1} \\ &\quad \times \left(\mathbf{G}X^{*} + \beta \mathbf{F}'\mathbf{\Lambda}\left(\mathbf{I} - \mathbf{H}\left(\mathbf{H}'\mathbf{\Lambda}\mathbf{H}\right)^{-1}\mathbf{H}'\mathbf{\Lambda}\right)\mathbf{J}\right). \end{aligned} \tag{9}$$

The dynamics of the economy under the optimal policy reaction function are given by (4) and (7) with (8) and (9) and imply a recursive structure: The monetary policy instrument i_t depends on the current state of the economy X_t and both determine X_{t+1}, the state of the economy in the next period. The only source of uncertainty are the shocks in μ_{t+1} which are not observed by the Fed when it sets i_t. The optimal policy reaction function (7) is much less restrictive than a Taylor-type rule and allows the policy rate to react to current and lagged values of all of the non-policy variables and to the lagged policy rate itself.
In deriving the optimal policy reaction function we treat the structural economic parameters in (4) as fixed. In particular, we assume them to be invariant with respect to the policy reaction function. This assumption is vulnerable to the Lucas critique (Lucas (1976)) because the parameters in (4) are constructed from the reduced form coefficients of the VAR in (1) and (2) that were estimated under a particular policy regime. However, as explained in Sack (2000), the optimal policy rule turns out to be sufficiently close to the observed policy so that much of the impact of the Lucas critique is alleviated. In addition, Rudebusch (2005) shows that for empirically reasonable policy shifts the reduced form coefficients of VARs are largely unaffected.

Since the monetary policy maker observes X_t when setting his policy instrument the variables to be included in Z_t depend on the assumptions about the central bank's information set. Sack (2000) assumes that the Fed observes and reacts to the contemporaneous values of all variables in Z_t so that X_t contains current and lagged values of all variables in Z_t as well as lagged values of the policy rate

$$
\begin{aligned}
X_t &= \{u_t, u_{t-1}, \cdots, u_{t-q}, y_t, y_{t-1}, \cdots, y_{t-q}, \pi_t, \pi_{t-1}, \cdots, \pi_{t-q}, \\
&\quad c_t, c_{t-1}, \cdots, c_{t-q}, i_{t-1}, \cdots, i_{t-q}\}.
\end{aligned} \tag{10}
$$

This assumption implies a recursive structure of the VAR in (1) and (2) in which i_t is ordered last, i.e. the variables in Z_t do not contemporaneously react to i_t and $b_0 = 0$. In this case $\mathbf{F}, \mathbf{H}$ and $\mathbf{J}$ can be constructed from the estimated reduced form coefficients of the VAR. Note that under this assumption it is not necessary to impose additional identifying assumptions on A_0, the matrix of contemporaneous coefficients, in order to retrieve $\mathbf{F}, \mathbf{H}$ and $\mathbf{J}$, that is the ordering of the variables in Z_t is irrelevant for the coefficients in (4).[9]

However, as our goal is to include additional financial markets variables in the system (4) that are likely to respond contemporaneously to monetary policy actions we also propose a version of (4) and (10) that puts one variable from Z_t (the financial markets variable) *after* the policy rate. This assumes that monetary policy does not contemporaneously react to the financial markets variable whereas the financial markets variable is contemporaneously affected by the policy rate. Hence, the current value of the financial markets variable $s_t = m1_t, m3_t, sp500_t$ is not part of the central bank's information set and the state vector is given by

$$
\begin{aligned}
X_t &= \{u_t, u_{t-1}, \cdots, u_{t-q}, y_t, y_{t-1}, \cdots, y_{t-q}, \pi_t, \pi_{t-1q}, \cdots, \pi_{t-q}, \\
&\quad c_t, c_{t-1}, \cdots, c_{t-q}, i_{t-1}, \cdots, i_{t-q}, s_{t-1}, \cdots, s_{t-q}\}.
\end{aligned} \tag{11}
$$

The construction of (4) for this case is a bit more complicated. The variables ordered in front of i in X_{t+1} are affected by s_t but s_t does not appear in X_t on the right-hand side in (4) as it is not a state variable monetary policy can react to. We rewrite (4) as

[9] As explained in Christiano, Eichenbaum, and Evans (1999) the behaviour of the VAR in response to a monetary policy shock ν^i is independent of the ordering of the variables in front of the policy rate. However, the exact VAR ordering becomes relevant if we want to simulate the dynamic effects of the structural shocks in ν^Z.

$$\mathbf{\Phi}_0 X_{t+1} = \mathbf{\Phi}_1 X_t + \mathbf{\Theta} i_t + \mathbf{\Psi} + \xi_{t+1}. \tag{12}$$

The matrices $\mathbf{\Phi}_0$ and $\mathbf{\Phi}_1$ and the vectors $\mathbf{\Theta}$ and $\mathbf{\Psi}$ can be constructed from the VAR estimates. However, while the coefficients in the rows in front of the policy variable still contain the reduced form coefficients we use the coefficients from a *structural* identification for the variables ordered after the policy rate. The structural coefficients are obtained assuming the recursive ordering (u, y, π, c, i, s).[10] The coefficients in $\mathbf{\Phi}_0$ represent the contemporaneous reactions of u, y, π and c to each other and their dependence on the *lagged* value of s. Note that the structural coefficients of the policy reaction function (2) are not used for the construction of (4) since the reaction function will be replaced by (7). The corresponding system (4) is then obtained by premultiplying with $(\mathbf{\Phi}_0)^{-1}$. This will have an effect on the shock vector μ_{t+1}. Since ξ_{t+1} in (12) is equal to

$$\begin{aligned}
&\{\nu^Z_{u,t+1}, 0, \cdots, \nu^Z_{u,t-q+1}, \nu^Z_{y,t+1}, 0, \cdots, \nu^Z_{y,t-q+1}, \nu^Z_{\pi,t+1}, 0, \cdots, \nu^Z_{\pi,t-q+1}, \\
&\nu^Z_{c,t+1}, 0, \cdots, \nu^Z_{c,t-q+1}, 0, \cdots, \nu^Z_{s,t}, 0, \cdots, \nu^Z_{s,t-q}\}
\end{aligned} \tag{13}$$

premultiplying with $(\mathbf{\Phi}_0)^{-1}$ results in μ_{t+1} being linear combinations of the period-$t+1$ structural shocks to u, y, π, c and the period-t structural shock to s. However, since μ_{t+1} remains serially uncorrelated and the period-t shock to the financial variable is not known by the central bank the optimal policy rule remains to be given by (7) with (8) and (9).

2.2 Estimation and Results

We obtain our structural model from a VAR estimated on monthly data from 1992:1 to 2005:3. A stability test as proposed in Sims (1980) indicates stability of the estimated VAR coefficients over this period. The VAR is estimated with 12 lags which yields the

[10] Using the structural coefficients for all variables leads to the same result for (4) after premultiplying with $\mathbf{\Phi}_0^{-1}$.

best fit of the optimal to the actual Federal Funds rate.[11]

The policy rule (7) contains three free parameters: the discount factor β, the relative weight of unemployment in the central bank's loss function λ_u, and the inflation target π^*. As in Sack (2000) we impose $\beta = 0.996$ and estimate λ_u and π^* by minimizing the sum of squared deviations of the optimal from the actually observed Federal Funds rate. Thus, we impose the identifying assumption that the Fed's policy can be adequately described by our dynamic optimization model (3)-(9).[12] For any values of λ_u and π^* we compute the optimal Federal Funds rate using the optimal policy reaction function (7) and the observed values for the state variables in X_t. We choose values for λ_u and π^* such as to minimize the sum of squared deviations of the optimal from the observed Federal Funds rate.[13] This estimation procedure yields values $\lambda_u = 0.15$ and $\pi^* = 2.5$ for the five variable system (u, y, π, c, i). The estimated weight on unemployment is considerably lower than in Sack (2000) who finds a value of 0.79 for the period 1984:1-1998:6.[14]

Figure 1 shows the observed path of the Federal Funds rate together with the optimal Funds rate for various model specifications. The top left panel shows the optimal Federal Funds rate for the benchmark model with u, y, π, c, i. The optimal policy rule broadly tracks the actual path of the Funds rate but leads to a much lower interest rate in the early 90s and a much higher interest rate in 1997 and 1998. It captures the strong decline of the Funds rate in 2001 but the interest rate starts to increase again early in 2003. As shown in Table 1 the optimal Funds rate is more than 2.5 times as volatile as the actual Funds rate.

The other three panels show the optimal Funds rate path for different structural models

[11]Sack (2000) uses eight lags. Formal lag-length selection criteria suggest a much lower lag order. With such a low number of lags the optimal policy rate becomes very volatile and the fit deteriorates considerably.

[12]Matching an optimal monetary policy rule to the historical Fed policy is also done in Rudebusch (2001) with considerable success.

[13]This search procedure is necessary because the matrix $\mathbf{\Lambda}$ in (8) cannot be solved for explicitly. As a consequence we cannot provide information on the precision of the estimates.

[14]However, very low weights on output or employment targets in the Fed's loss function also have been found in some other studies, e.g. Favero and Rovelli (2003) who find a weight of 0.00125 on the output gap for the period 1980:3-1998:3. Furthermore, λ_u turned out to be very sensitive with respect to the number of lags in the VAR.

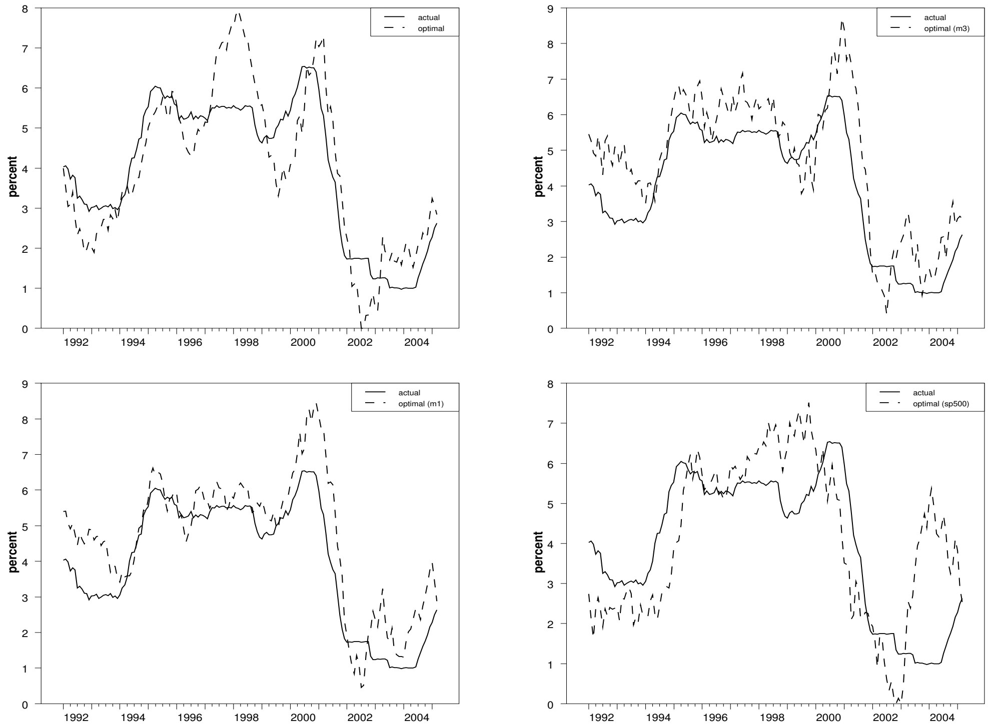

Figure 1: Actual and optimal Federal Funds rate for various model specifications (fixed preference parameters)

(4) holding the parameters λ_u and π^* constant. Including the growth rate of $M1$ (bottom left panel) moves the optimal Funds rate closer to the actual Funds rate in the mid-90s but produces a stronger interest rate hike in 2000. In addition, the fit at the beginning and at the end of the sample is worse. Table 1 shows that the overall fit of the optimal policy rule deteriorates slightly after the inclusion of $M1$ and that the optimal Funds rate becomes more volatile relative to the benchmark model.

Including the growth rate of $M3$ leads to similar results as shown in the top right panel in Figure 1. Here, the fit of the optimal policy reaction function worsens considerably and the volatility of the optimal Federal Funds rate increases to 0.64 (see Table 1). Finally, augmenting our basic model with the growth rate of the S&P500 Index leads to an even worse fit. Note however, that while the other three specifications have the optimal Funds rate lagging the steep decline in the actual Funds rate in 2001 the optimal Funds rate for the model with the stock market index leads the observed Funds rate over this episode. Predicting the expansionary monetary policy of the Fed in 2000/2001 appears to be conditional on including information on the stock market.

variables	parameter values	SSD	$\sigma_{\Delta i}$
u, y, π, c, i	$\lambda_u = 0.15,\ \pi^* = 2.5$	158.47	0.4803
$u, y, \pi, c, i, m1$	$\lambda_u = 0.15,\ \pi^* = 2.5$	164.46	0.5590
$u, y, \pi, c, i, m3$	$\lambda_u = 0.15,\ \pi^* = 2.5$	203.50	0.6364
$u, y, \pi, c, i, sp500$	$\lambda_u = 0.15,\ \pi^* = 2.5$	383.69	0.4694
$u, y, \pi, c, i, m1$	$\lambda_u = 0.20,\ \pi^* = 1.55$	85.76	0.5132
$u, y, \pi, c, i, m3$	$\lambda_u = 0.20,\ \pi^* = 1.40$	104.78	0.5910
$u, y, \pi, c, i, sp500$	$\lambda_u = 0.475,\ \pi^* = 2.45$	360.91	0.4044

SSD: Sum of squared deviations of optimal from observed Federal Funds rate. $\sigma_{\Delta i}$: Standard deviation of change in optimal Federal Funds rate (standard deviation of change in observed Federal Funds rate is 0.1821). The optimal Federal Funds rate is calculated using (7) and the actual values of the variables in X_t.

Table 1: Optimal Federal Funds rate under additive uncertainty - fit and volatility

Table 1 shows the fit of the various model specifications. In the upper panel we imposed the estimated preference parameters of our benchmark model on all model variants. In the lower panel we re-estimated λ_u and π^* for each model separately. Figure 2 presents the resulting paths for the optimal policy rate.

The estimation results for the models with $M1$ and $M3$ are similar. Both models produce a small increase in λ_u and a decline in the inflation target value π^*. In both cases the fit of the policy reaction function improves considerably and far surpasses that of the benchmark model. The volatility of the policy rate declines but is still higher than that of the benchmark specification. Figure 2 shows that the behavior of the optimal Funds rate remains very similar to that shown in Figure 1. For the model including the S&P500 Index we find only a relatively small improvement in the fit compared to the upper panel in Table 1 and the sum of squared deviations is still more than twice as high as that of the benchmark specification. However, the table shows a drastic change in the estimated weight on the unemployment deviation. Since only the fit of the reaction functions that include money growth improves relative to the standard specification, our results so far suggest that monetary aggregates help in explaining the evolution of the Federal Funds rate over the sample period while the stock market index does not.

Figures 3-6 show simulated paths for the policy rate, unemployment (deviation from

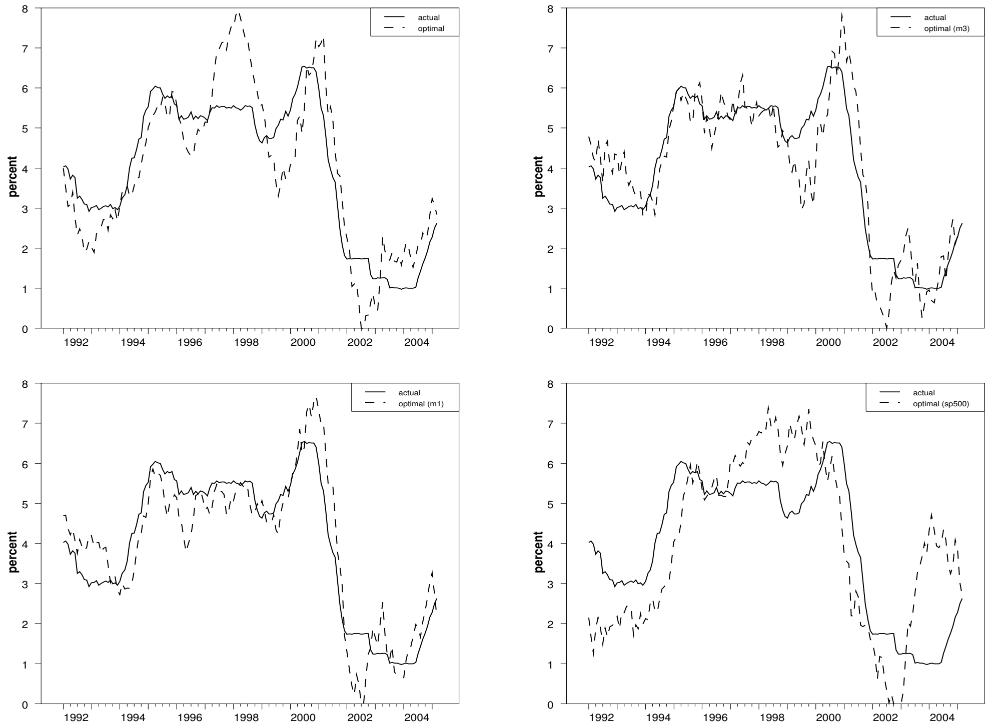

Figure 2: Actual and optimal Federal Funds rate for various model specifications (re-estimated preference parameters)

natural rate) and inflation that result from the optimal policy reaction functions. (For comparison the observed paths of these variables are plotted as well.) The paths are derived by simulating (4) together with the optimal policy reaction function and using the same series for the non-policy shocks as historically observed.[15] Figure 3 shows that in the benchmark model the policy rate increases strongly in the mid-90s, comes down again in the late 90s and runs through a new cycle up to 2005. It is evident that unemployment and inflation are better stabilized around their respective target values compared to their actually observed paths.

The optimal policy path for the $m1$-model starts out with a higher policy rate that exhibits high volatility for the mid-90s. The Federal Funds rate declines in 1998/99 and rises again in 2000. In 2001 it falls strongly, lagging the observed funds rate, and

[15]The starting values for the state vector X_t are the observed values of the variables at the beginning of the sample period. The shock series are constructed from the reduced form shocks of the VAR for the variables u, y, π, c and the structural shocks for s using the transformation (12). While Figures 1-2 set the value of X_t in each period to its actually observed value the experiment in Figures 3-6 allows the system to evolve according to its dynamics in (4) and the policy reaction function.

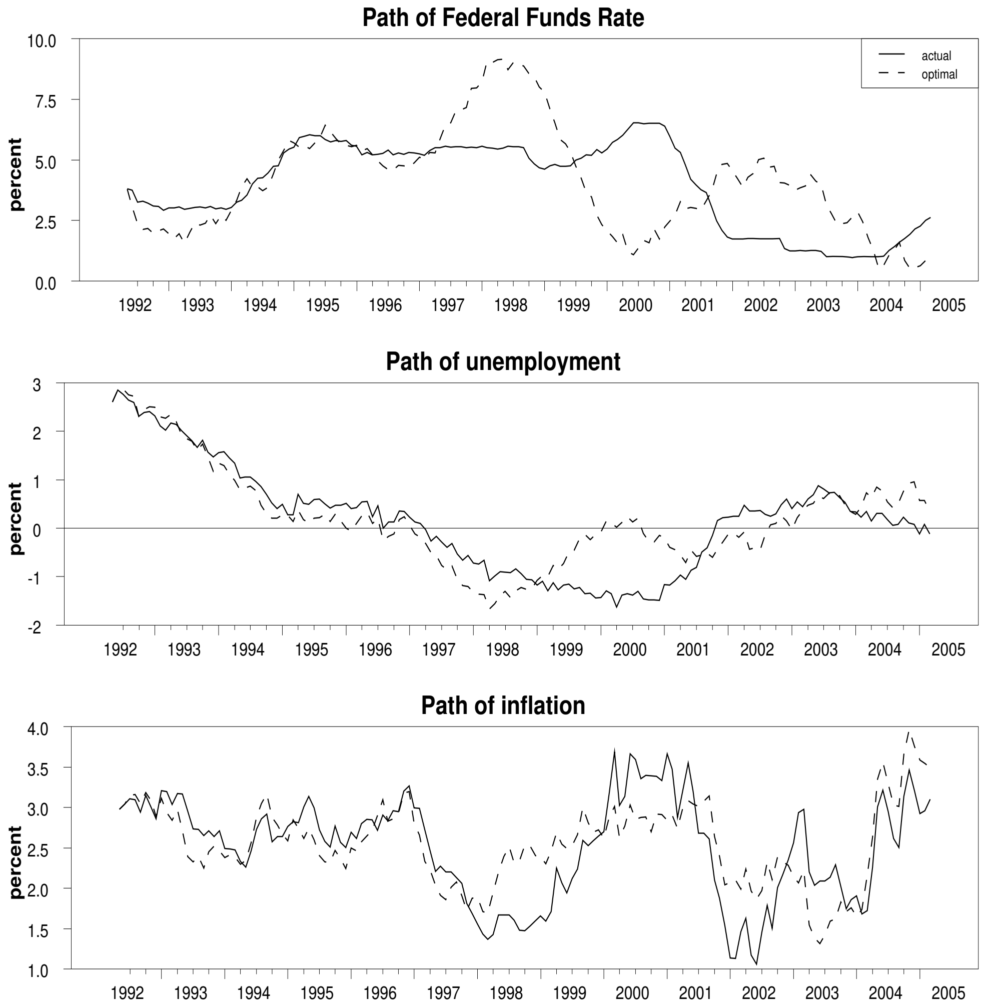

Figure 3: Actual and optimal policy under additive uncertainty - standard model

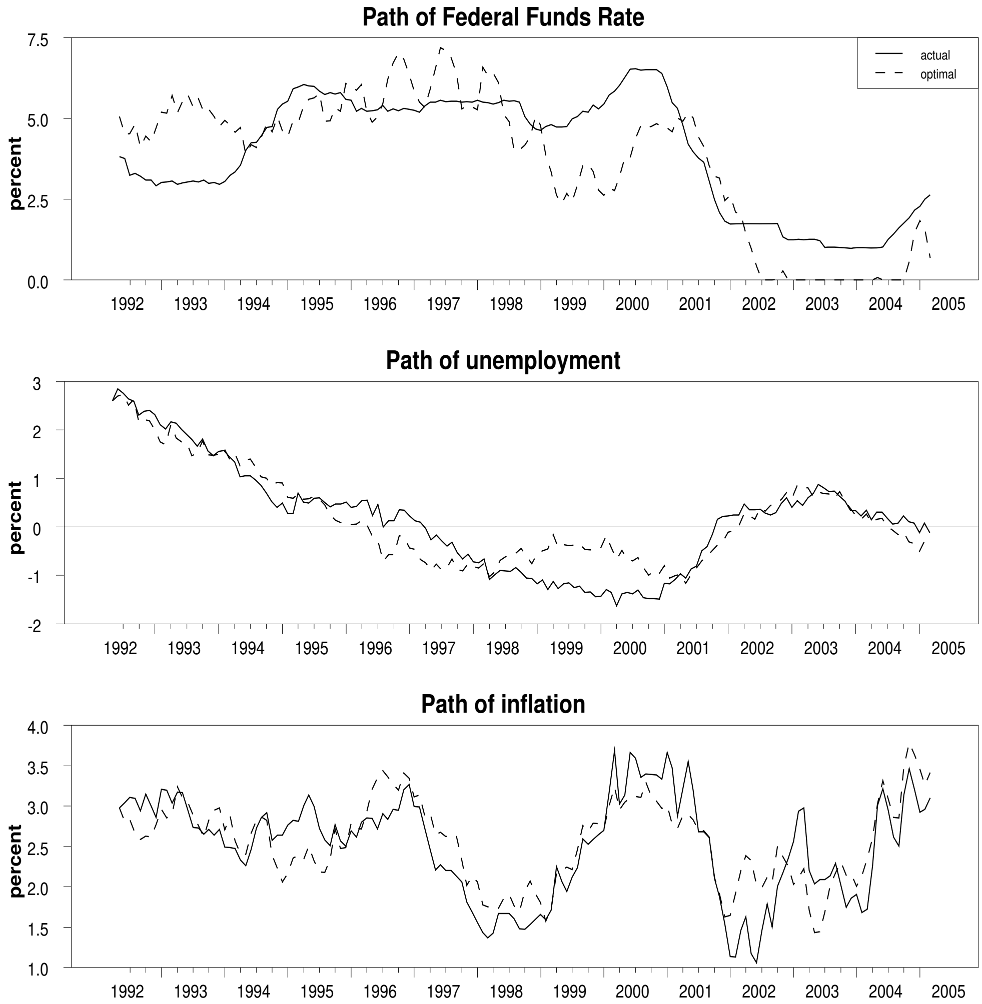

Figure 4: Actual and optimal policy under additive uncertainty - extended model (m1)

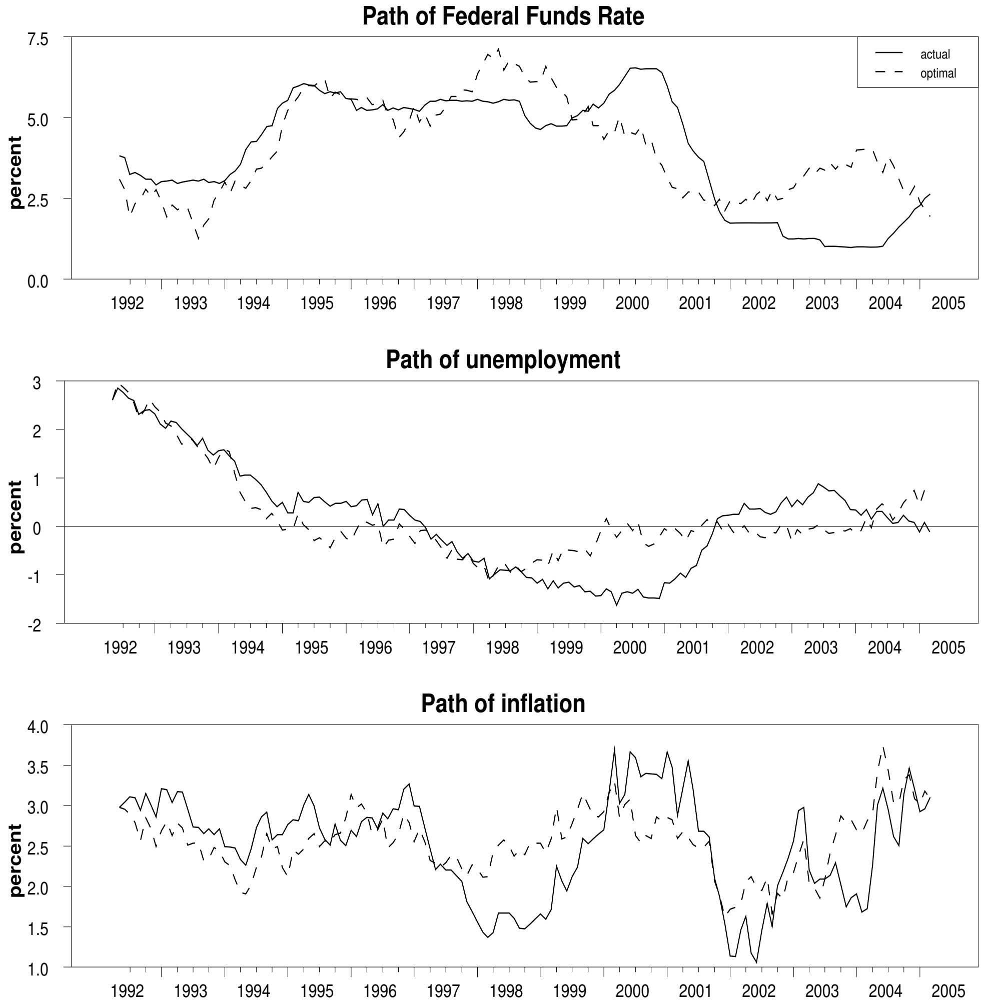

Figure 5: Actual and optimal policy under additive uncertainty - extended model (m3)

hits the zero bound.[16] The unemployment and inflation paths seem to be somewhat smoother but the stabilization gains appear to be limited.

The optimal path for the Funds rate in the $m3$-model looks broadly similar to the one in the benchmark model. The the interest rate hike in the mid-to-late 90s is less pronounced and the Federal Funds Rate declines more slowly beginning in the late 1990s. Similar to the benchmark case inflation and unemployment deviations from their target values are much lower than for the observed policy.

In the final model the optimal Funds rate tracks the observed Funds rate up to the mid-

[16]Note that the model used to derive the optimal policy reaction function does not constrain the policy instrument to positive values.

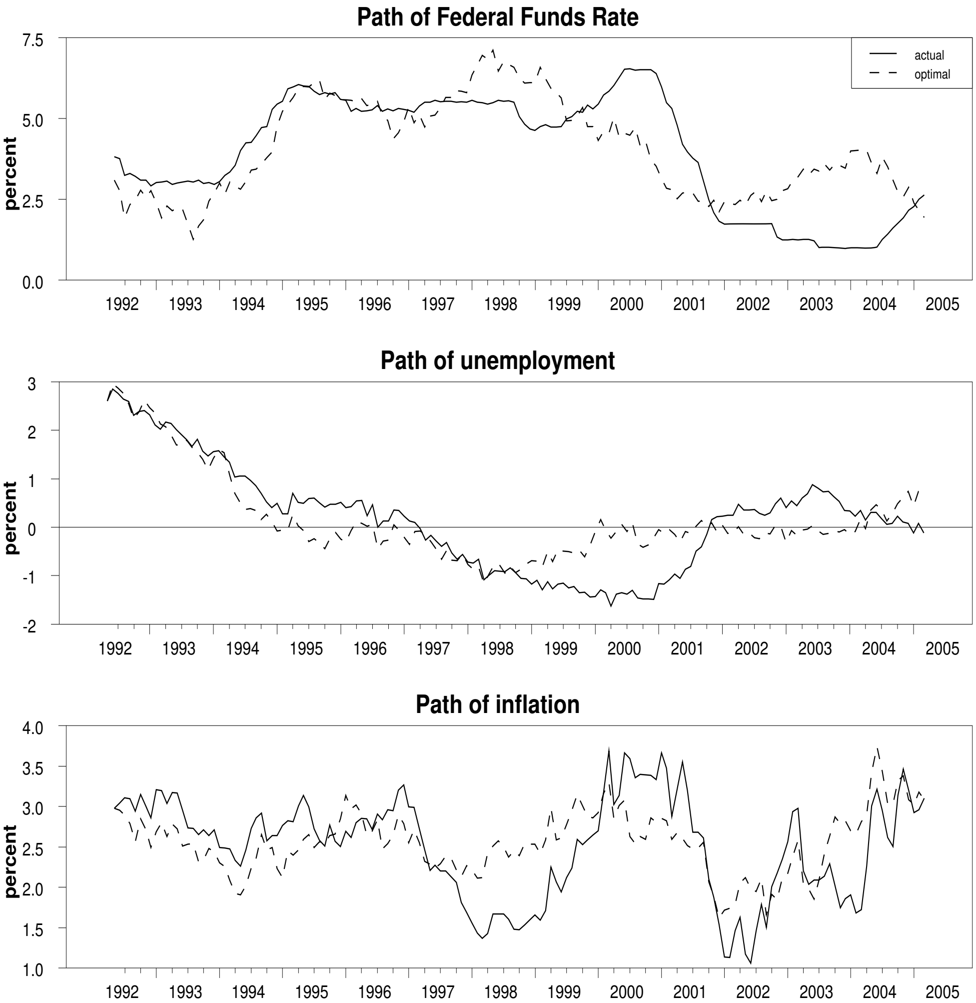

Figure 6: Actual and optimal policy under additive uncertainty - extended model (sp500)

	loss function (σ_u/σ_π)			
	benchmark	$m1$	$m3$	$sp500$
$\lambda_u = 0.15$, $\pi^* = 2.5$	-25.98	-28.84	-35.18	-39.23
	0.544/0.398	0.527/0.424	0.528/0.465	0.452/0.526
$\lambda_u = 0.20$, $\pi^* = 1.55$	-77.27	-84.14	-98.26	-56.54
	0.515/0.403	0.514/0.422	0.522/0.465	0.450/0.524
$\lambda_u = 0.475$, $\pi^* = 2.45$	-38.51	-40.75	- 47.01	-48.63
	0.478/0.429	0.488/0.436	0.464/0.470	0.407/0.547
estimated rules	0.617/0.570	0.554/0.548	0.587/0.590	0.604/0.643
$\lambda_u = 0.15$, $\pi^* = 2.5$	-82.62	-67.04	-67.76	-88.63
$\lambda_u = 0.20$, $\pi^* = 1.55$	-102.88	-115.08	-109.81	-102.88
$\lambda_u = 0.475$, $\pi^* = 2.45$	-167.19	-133.16	-130.54	-79.02

σ_u, σ_π: Standard deviation of unemployment and inflation

Table 2: Performance of different optimal reaction functions under additive uncertainty

90s. It follows a path similar to the one in the benchmark model but less pronounced and more gradual. The policy rate does not rise in the late 90s as it was historically the case.

It is obvious that the optimal policy reaction functions prescribe a more aggressive behavior of the policy rate than actually observed. However, they also imply a strong persistence of the Federal Funds rate. As already pointed out by Sack (2000), this persistence is caused by the high degree of serial correlation in the variables the central bank reacts to.

Table 2 shows the average values of the loss function under the different optimal policy rules obtained from simulating (4) and (7) and using 500 draws of random shock series μ for the non-policy variables. Since we found some variation in the estimated preference parameters for the different models the table shows the values of the loss function for these various parameter combinations. Below the values of the loss function are the average standard deviations of the unemployment gap and the inflation rate obtained from the 500 simulated time series for inflation and unemployment.

In the first row are the values of the loss function obtained with the parameter estimates from the benchmark model. Including additional variables in the central bank's information set and policy reaction function does not improve the loss function. We get a similar result for the parameter estimates from the model with the stock market index (third row) which imply a much higher weight on unemployment in the loss function. Again, the benchmark model performs better than the three extended models. A different result can be found for the model in the second row which uses the parameter estimates from the model with $M1$.[17] Here, the benchmark model is superior to the $M1$ model which in turn is better than the $M3$ model but the smallest loss results for the model including the stock market index. However, the inflation target π^* is unreasonably small so that this result should be taken with caution.[18]. In the lower panel we show the values of the loss function that result from using the estimated policy reaction functions, that is the estimated equation (2) for each model. It is obvious that the more aggressive optimal reaction functions offer substantial stabilization gains. This is also evident by looking at the standard deviations of the unemployment gap and inflation which are always lower for the optimal reaction functions.[19]

Overall, Table 2 shows that there is not much evidence that including monetary aggregates or stock market variables in the monetary policy reaction function offers substantial stabilization gains. However, in Table 1 we found some evidence for the ability of monetary aggregates to contribute to explaining the actually observed behavior of the Federal Funds rate. A possible explanation for money growth being helpful in explaining the Funds rate in this model is that in fitting the optimal to the observed Funds rate our model has to try to mimic its high degree of autocorrelation. Since both money growth rates exhibit a high degree of persistence themselves, reacting to these variables strengthens the persistence of the optimal Funds rate and improves the fit.[20] In the next section, we will modify our model by introducing uncertainty about the structure of the economy. This will make the optimal reaction function respond less aggressively to changes in economic conditions and introduce a higher degree of persistence into the optimal Federal Funds rate path. This more persistent behavior of the optimal Funds rate might be able to explain the observed Funds rate without a need for monetary aggregates.

[17]Since the estimates for the model with $M3$ only differed slightly for π^*, we did not consider this parameter combination on its own.

[18]The ECB aims for an inflation rate of 2% or slightly below while the Fed is generally believed to have a higher implied inflation target

[19]The price for this enhanced stabilization is an about 2-3 times higher standard deviation of the changes in the Federal Funds rate (not shown) under the optimal reaction functions.

[20]The autocorrelations for the growth rate of $M1$ ($M3$) are above 0.9 for four (five) months.

3 Optimal policy under parameter uncertainty

In the last section we assumed that the central bank knew the true dynamic structure of the economy as represented in (4) and that all uncertainty was due to the stochastic disturbances in μ. With a quadratic loss function and linear constraints certainty equivalence holds, that is additive uncertainty does not affect the optimal policy reaction function. However, in the real world, central banks can only rely on an estimated model of the structural relations within the economy and this estimated model must not necessarily be correct. In our study we follow Sack (2000) and focus on uncertainty about the true values of the coefficients in (4). Brainard (1967) shows that parameter uncertainty about the effects of policy on the economy can lead to a less aggressive policy compared to that under certainty equivalence. However, other studies have concluded that parameter uncertainty must not necessarily make monetary policy more cautious but may actually lead to a more aggressive monetary policy (e.g. Söderström (2002)).[21]

3.1 Deriving the optimal policy reaction function

To account for parameter uncertainty Sack (2000) proposes to rewrite (4) as

$$\hat{X}_{t+1} = \mathbf{F} \cdot \hat{X}_t + \mathbf{H} i_t + \mathbf{J} + \mu_{t+1}, \tag{14}$$

where $\hat{X}_t = \mathbf{E}_{t-1}\hat{X}_t$ is the forecast of X_t based on time $t-1$ information. The optimal policy sets the interest rate as a function of $\hat{X}_t$. This implies that the central bank reacts to shocks to the elements of X_t with a lag of one period.[22]

Sack (2000) shows that the solution to the optimization problem (5) can be approximated by the Bellman equation

[21]We assume the economic structure to be constant over the sample period and thus take no account of possible structural change. We also do not consider learning models in which the policymaker gradually improves his estimate of the structure of the economy as more and more data become available – possibly gaining additional data by variations in policy – as, for example, in Orphanides and Williams (2005), Sack (1998) and Wieland (1999, 2000).

[22]For the variable s this implies a lag of two periods since even in the model with additive uncertainty the central bank can only react to s with a one period lag.

$$
\begin{aligned}
V(\hat{X}_t) &= \max_{i_t}\Big\{-\left(\hat{X}_t - X^*\right)'\mathbf{G}\left(\hat{X}_t - X^*\right) - \left(\hat{X}_t'\mathbf{K}\hat{X}_t + 2\hat{X}_t\mathbf{L}\right) \\
&\quad +\beta E_t\left[V(\hat{X}_{t+1})\right]\Big\}. \qquad (15)
\end{aligned}
$$

subject to (14). This reformulation of the problem leaves the dynamic structure of equation (4) unchanged – the coefficients are still given by those derived from the VAR – and incorporates the effects of parameter uncertainty into the loss function. The matrix $\mathbf{K}$ and the vector $\mathbf{L}$ are weighted sums of the variance-covariance matrices of the parameters describing the dynamic behaviour of the variables in the central bank's loss function. $\mathbf{K} = \Sigma_{\beta(\pi)} + \lambda_u \Sigma_{\beta(u)}$, where $\Sigma_{\beta(n)}$, $n = u, y$, is the covariance matrix between the coefficients within the equation of current unemployment and inflation in (4) and $\mathbf{L}$ is a similar construction containing the covariance of the n-th equation's elements in $\mathbf{F}$ with the n-th element of the vector $\mathbf{J}$. The solution in (16) is based on the assumption that $\mathbf{K}$ and $\mathbf{L}$ are constant.[23]

One problem with this approach is that $\hat{X}_{t+1} = \mathbf{F} \cdot X_t + \mathbf{H}i_t + \mathbf{J}$ actually evolves from X_t and not from $\hat{X}_t$ so that μ_{t+1} in (14) picks up a term related to $X_t - \hat{X}_t$. μ is still uncorrelated with $\hat{X}_t$ and the dynamics in (14) remain unbiased. However, the accumulation of the effects of $X_t - \hat{X}_t$ through time leads to an increasing variance of μ. Hence, the solution (15) is only an approximation to the solution of the dynamic programming problem under parameter uncertainty. Sack (2000) shows that the optimal solution can be retrieved by assigning different weights to the first and the second term in (15), that is by replacing $\mathbf{G}$ with $\hat{G} = (1-\rho)\mathbf{G}$, $\mathbf{K}$ with $\hat{K} = \rho\mathbf{K}$, and $\mathbf{L}$ with $\hat{L} = \rho\mathbf{L}$. ρ is chosen to minimize the loss function.[24]

[23] The values of these matrices are derived from the variance-covariance matrix estimated over the complete sample period. This probably leads to an underestimation of the actual degree of uncertainty the central bank is facing.

[24] Specifically, for a given value of ρ we compute from (16) the optimal policy reaction function under parameter uncertainty. We then construct a hypothetical path for the policy rate using the actually observed values of X_t at each point in time and search for the combination of λ_u and π^* that minimizes the sum of squared deviations of the optimal from the actually observed policy rate. This gives us a set of ρ and associated combinations λ_u, π^*. For each combination ρ, λ_u, π^* we then draw 500 sets of coefficients in $\mathbf{F}$, $\mathbf{H}$ and $\mathbf{J}$, based on the variance-covariance matrix of their VAR estimates and then simulate (4) with these new coefficients but using the optimal policy rule. Finally, we select for ρ, λ_u and π^* that combination among the ones available that yields the lowest average value of the loss function.

The optimal policy reaction function is similar to that under additive uncertainty with X_t being replaced by $\hat{X}_t$

$$i_t^* = -\left(\mathbf{H}'\mathbf{\Lambda}\mathbf{H}\right)^{-1}\left(\mathbf{H}'\mathbf{\Lambda}\mathbf{F}\hat{X}_t + \mathbf{H}'\mathbf{\Lambda}\mathbf{J} + \mathbf{H}'\omega\right). \tag{16}$$

The Riccati equation becomes

$$\mathbf{\Lambda} = -\hat{\mathbf{G}} - \hat{\mathbf{K}} + \beta\mathbf{F}'\mathbf{\Lambda}\mathbf{F} - \beta\mathbf{F}'\mathbf{\Lambda}\mathbf{H}\left(\mathbf{H}'\mathbf{\Lambda}\mathbf{H}\right)^{-1}\mathbf{H}'\mathbf{\Lambda}\mathbf{F}, \tag{17}$$

and

$$\begin{aligned} \omega &= \left(\mathbf{I} - \beta\mathbf{F}'\left(\mathbf{I} - \mathbf{\Lambda}\mathbf{H}\left(\mathbf{H}'\mathbf{\Lambda}\mathbf{H}\right)^{-1}\mathbf{H}\right)\right)^{-1} \\ &\quad \times \left(\hat{\mathbf{G}}X^* - \hat{\mathbf{L}} + \beta\mathbf{F}'\mathbf{\Lambda}\left(\mathbf{I} - \mathbf{H}\left(\mathbf{H}'\mathbf{\Lambda}\mathbf{H}\right)^{-1}\mathbf{H}'\mathbf{\Lambda}\right)\mathbf{J}\right). \end{aligned} \tag{18}$$

3.2 Estimation and Results

As Table 3 shows, the volatility of the optimal policy rate decreases relative to the situation under additive uncertainty (Table 1). In the upper panel we find the results for holding the preference parameters fixed at the values from the upper panel of Table 1 and only estimating ρ. The volatility of the optimal Federal Funds rate is much lower than in Table 1 but is still higher than the actually observed one. The fit of the optimal policy rule improves strongly

The lower panel shows the results after re-estimating the preference parameters together with ρ. For the first model the volatility of the optimal policy increases by a small amount while it declines for the other models. The fit improves strongly for the standard model and the models with $m1$ and the stock market index but only slightly for the $m3$-model. The estimated preference parameter λ_u is lower than in Table 1 for all specifications and the estimated target value for inflation π^* falls into a relatively close range for all models. In terms of fit both models with growth rates of monetary aggregates turn out to be superior to the benchmark model. These results indicate again that monetary aggregates appear to have some explanatory power for the setting

variables	parameter values	SSD	$\sigma_{\Delta i}$
u, y, π, c, i	$\lambda_u = 0.15,\ \pi^* = 2.5,\ \rho = 0.20$	92.63	0.4029
$u, y, \pi, c, i, m1$	$\lambda_u = 0.15,\ \pi^* = 2.5,\ \rho = 0.20$	80.94	0.4890
$u, y, \pi, c, i, m3$	$\lambda_u = 0.15,\ \pi^* = 2.5,\ \rho = 0.20$	59.38	0.4832
$u, y, \pi, c, i, sp500$	$\lambda_u = 0.15,\ \pi^* = 2.5,\ \rho = 0.20$	254.30	0.3896
u, y, π, c, i	$\lambda_u = 0.05,\ \pi^* = 2.60,\ \rho = 0.30$	72.80	0.4111
$u, y, \pi, c, i, m1$	$\lambda_u = 0.05,\ \pi^* = 2.55,\ \rho = 0.40$	46.00	0.4400
$u, y, \pi, c, i, m3$	$\lambda_u = 0.10,\ \pi^* = 2.40,\ \rho = 0.20$	57.29	0.4776
$u, y, \pi, c, i, sp500$	$\lambda_u = 0.05,\ \pi^* = 2.60,\ \rho = 0.60$	124.17	0.3377

SSD: Sum of squared deviations of optimal from observed Federal Funds rate. $\sigma_{\Delta i}$: Standard deviation of change in optimal Federal Funds rate (standard deviation of change in observed Federal Funds rate is 0.1821). The optimal Federal Funds rate is calculated using (16) and the actual values of the variables in X_t.

Table 3: Optimal Federal Funds rate under parameter uncertainty - fit and volatility

of the Federal Funds rate.

Figure 7 shows the optimal paths of the policy rate based on the parameter estimates in the lower panel of Table 3. As in Figure 1 the time series are those that would be obtained if the variables in X_t were at their historically observed values at each point in time. All models do fairly well in capturing the behaviour of the Federal Funds rate through our sample period. Compared to Figures 1 and 2 the optimal Funds rate evolves more smoothly. All models mimic the strong interest rate decline in 2001 but suggest an even stronger lowering of the interest rate. The first three models also capture the interest rate increase in 1999/2000 but the timing is only approximately correct for the $m3$-model. Interestingly, the $sp500$-model suggests a monetary tightening beginning in 2002 well before the Fed actually began raising interest rates again in 2004. In contrast to the extended models, the benchmark model prescribes an additional peak in the Funds rate in 1997/1998 that is much more pronounced than actually observed.

In Figure 8 we present the simulated paths of the policy rate, unemployment (deviation from natural rate) together with the actually observed path of the respective variables. As in Figures 3-6 the simulated paths are computed by iterating on (4) together with

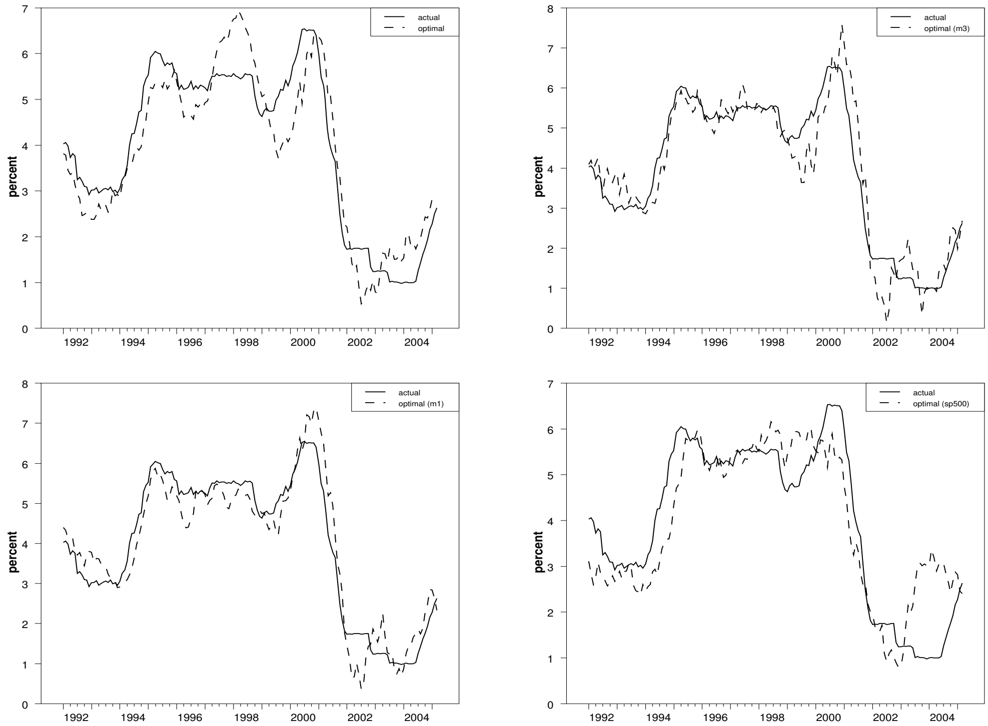

Figure 7: Actual and optimal Federal Funds rate for various model specifications under parameter uncertainty

the optimal policy reaction function under parameter uncertainty (16) and using the same series for the non-policy shocks as observed historically.
The path for the optimal policy rate for the standard model u, y, π, c, i looks similar to that in Figure 3. For all the simulated paths the volatility of the Federal Funds rate is similar to its actually observed volatility (0.1821) (benchmark model: 0.2502, $m1$-model: 0.2618 (not shown), $m3$-model: 0.2719 (not shown) , $sp500$-model: 0.1913 (not shown)).

In Table 4 we compare the stabilization performance of the different optimal rules in the presence of uncertainty about the economy's structural relationships. Specifically, we compute the average values of the central bank's loss function and the standard deviations of unemployment and inflation for each of the augmented rules and compare them to those of the simple rule. In each comparison one aspect of uncertainty is captured by combining the policy rule with a structural model (4) that differs from the originally estimated model by drawing from the distribution of the estimated coefficients (see the description in footnote 24). For each model we run 500 simulations

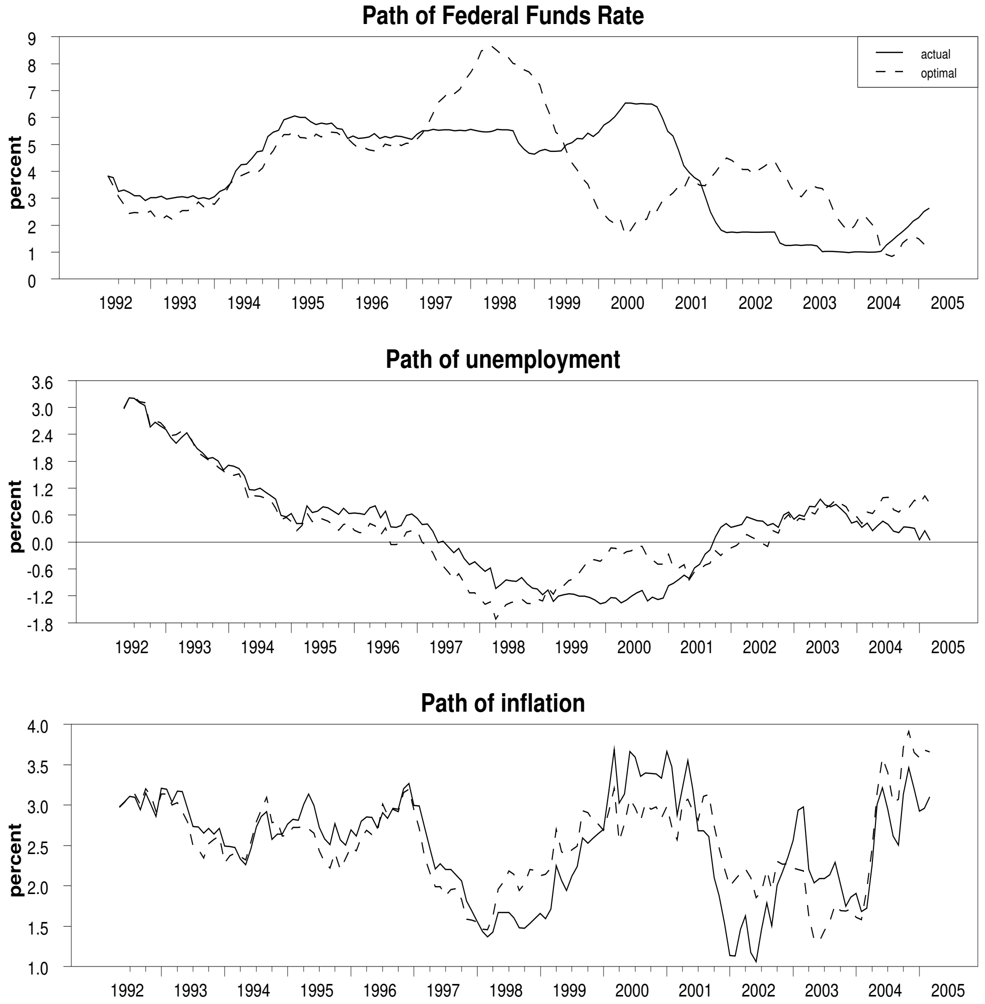

Figure 8: Actual and optimal policy under parameter uncertainty - standard model

true model preference parameters	loss function (σ_u/σ_π) benchmark	 $m1$	 $m3$	 $sp500$
u, y, π, c, i	-106.56	-450.39	-544.57	n.a.
$\lambda_u = 0.05$, $\pi^* = 2.60$, $\rho = 0.30$	1.29/0.75	1.69/1.25	1.51/1.03	n.a.
$u, y, \pi, c, i, m1$	-127.19	-135.44	–	–
$\lambda_u = 0.05$, $\pi^* = 2.60$, $\rho = 0.30$	1.41/0.80	1.36/0.86	–	–
$u, y, \pi, c, i, m3$	-223.97	–	-224.57	–
$\lambda_u = 0.05$, $\pi^* = 2.60$, $\rho = 0.30$	1.53/1.09	–	1.52/1.03	–
$u, y, \pi, c, i, sp500$	-116.98	–	–	-168.30
$\lambda_u = 0.05$, $\pi^* = 2.60$, $\rho = 0.30$	1.41/0.78	–	–	1.26/1.00
$u, y, \pi, c, i, m1$	-251.97	-141.02	–	–
$\lambda_u = 0.05$, $\pi^* = 2.55$, $\rho = 0.40$	1.53/1.19	1.40/0.89	–	–
$u, y, \pi, c, i,$	-110.36	-537.83	–	–
$\lambda_u = 0.05$, $\pi^* = 2.55$, $\rho = 0.40$	1.34/0.76	1.75/1.18	–	–
$u, y, \pi, c, i, m3$	-260.51	–	-205.30	–
$\lambda_u = 0.10$, $\pi^* = 2.40$, $\rho = 0.20$	1.56/1.11	–	1.40/0.99	–
u, y, π, c, i	-124.91	–	-373.99	–
$\lambda_u = 0.10$, $\pi^* = 2.40$, $\rho = 0.20$	1.35/0.76	–	1.77/1.10	–
$u, y, \pi, c, i, sp500$	-122.61	–	–	-161.33
$\lambda_u = 0.05$, $\pi^* = 2.60$, $\rho = 0.60$	1.17/0.65	–	–	1.30/0.97
u, y, π, c, i	-126.84	–	–	n.a.
$\lambda_u = 0.05$, $\pi^* = 2.60$, $\rho = 0.60$	1.42/0.79	–	–	n.a.

σ_u, σ_π: Standard deviation of unemployment and inflation

Table 4: Performance of different optimal reaction functions under parameter uncertainty

with random coefficient draws and for each draw we simulate the model 500 times using random series for the shocks in μ drawn using the variance-covariance matrix of the residuals obtained from imposing the drawn coefficients on the model. In addition we consider a more serious element of uncertainty: We compute values for the loss function of the augmented rules assuming that the additionally included variable (M1, M3, or the stock market index) has no predictive power for future values of unemployment and inflation, that is we assume that the structural assumptions made for the standard model are in fact the correct ones. In addition to the always applied random changes to the parameters this implies setting some of the coefficients on the money or stock market variable in the structural model to zero.[25] In turn, we will perform a similar simulation for the simple rule by assuming that the variables ignored in it (M1, M3, or the stock market index) in fact help to predict future values of unemployment and inflation, that is we assume the structural models used to derive the augmented rules to be correct (up to some small random changes in the parameters) but combine them with the simple rule. Naturally, the performance of each rule will deteriorate if it is combined with a structurally different model. The robustness of each rule therefore can be evaluated from how much worse each rule performs under these different circumstances.

The top panel in Table 4 presents the results from using the preference parameter values originally estimated for the standard system (u, y, π, c, i), that is from the first row in the second panel of Table 3.[26] The first two rows show the average values of the loss function and the standard deviations of unemployment and inflation for all rules assuming that monetary aggregates and the stock market index do not help in forecasting future inflation and unemployment. Naturally, the rule derived from the standard model performs best while the loss function worsens strongly for the two rules that include monetary growth rates. If we use the rule with the stock market index and impose the restrictions that changes in the $S\&P500$ have to predictive content for the other variables in the system almost all randomly drawn coefficient matrices result

[25]We assume that s reacts to all other variables in the system but that these variables do neither react to current nor lagged values of s

[26]Since λ_u and π^* are elements of the loss function they have an important direct impact on the ranking of the alternative rules in addition to their indirect impact through the coefficients of the policy rules.

in unstable systems.

The third and fourth rows in the top panel compare the optimal rule for the standard system to the rule including the growth rate of M1 assuming that M1 helps in forecasting the other variables, that is using the structural model which includes M1. The next two rows perform the same exercise with the standard and the augmented rule for the system including M3 and the final two rows show the results for the comparison of the standard to the augmented ($sp500$) rule assuming that the stock market index has predictive content. The standard rule always has a lower value of the loss function both in the standard model (top row) and in the other models. That implies that even if one of the extended models is assumed to be the correct model the standard rule is superior to the augmented rules because the latter ones are very vulnerable to uncertainty about the coefficients of the structural model.

The following panels show the results of analogous exercises using the estimated preference parameters from the $m1$, $m3$, and $sp500$ models, respectively. In the $m1$ and $m3$ case the standard rule is inferior to the extended rule if the extended model is assumed to be the correct one. However, the loss function for the $m1$ augmented rule deteriorates much stronger (from -141.02 to -537.83, i.e. by 280%) than the standard rule (from -110.36 to -251.97, i.e. by 130%) if combined with the "wrong" model. Clearly for this set of preference parameters the standard rule is more robust than the $m1$ augmented rule. For the preference parameters estimated from the $m3$ model (next to last panel in Table 3) the comparison between the $m3$ augmented rule and the standard rule is less clear cut. Here the average loss function value of the $m3$ augmented rule increases by 82% (from -205.30 to -373.99) if M3 becomes non-informative while the average loss under the standard rule increases by 108% if the growth rate of M3 in fact contains important forecasting information. For these parameter values the $m3$ augmented rule appears to be slightly more robust than the standard rule. In the bottom panel we impose the preference parameters estimated from the model that includes the stock market index (last row of Table 3). Here again we encounter the problem that combining the $sp500$ augmented rule with a model in which the stock market index has no predictive power leads to an unstable dynamic system. However, as the standard rule already dominates the $sp500$ augmented rule even if we take the extended model as the reference model it is very likely that this result will not be reversed for the standard model.

To sum up, the results in Table 4 strongly indicate that a rule without monetary growth rates or a stock market index is more robust than the augmented optimal rules. If we apply the minimax criterion, that is if we select the policy rule which minimizes the maximum possible loss, the standard reaction function beats all other contenders handsomely. In particular, including the stock market index in the information set of the central bank leads to worse results than the standard rule. Obviously, if we assume uncertainty about the structural relationships in the economy the stock market index must introduce a lot of noise into the model. Responding to such a noisy signal leads to adverse economic results.

4 Conclusion

We have shown that the recent path of the Federal Funds rate can be approximated by an optimal monetary policy reaction function derived from a relatively simple structural model. Having the Federal Funds rate also responding optimally to growth rates of M1 or M3 improved the ability of the optimal rule to explain the actual time series of the Federal Funds rate. In contrast, including a stock market index did not improve the fit of the optimal policy rule. These results show that monetary aggregates contain some information that helps with explaining Fed behavior. This does not imply that the Fed in fact reacts to changes in the growth rates of monetary aggregates but that money growth might be correlated with some variables that the Fed looks at in setting interest rates, for example credit growth, financial market conditions etc. While the optimal reaction functions reproduce the general path of the Funds rate quite well, the simulated interest rate is much more volatile than was actually observed.

Taking account of uncertainty about the true economic structure affects the comparisons between different optimal policy rules in various ways. It was shown that the volatility of the optimal Federal Funds rate declined for all the policy rules under consideration. The policy rules that included monetary growth rates still tracked the actually observed Funds rate more closely than the policy rule without them. Interestingly, all our optimized policy rules were able to explain the large decrease in the

Federal Funds rate in 2001. Even those rules that do not react to changes in stock prices recommended a strong decline in the Federal Funds rate as an optimal response.

We then studied the robustness of the different policy rules and showed that the stabilization performance of the simple rule (that did not include monetary growth rates or a stock market index) worsened less than that of the other optimal rules if the model under which it was derived is assumed to be wrong. The simple rule is less sensitive to changes in the structure of the economy or errors in the estimation of the structural relationships. While money growth or stock market data might have some predictive power for the other economic variables, the possible gains for stabilization policy from this, however, are often compensated by the loss in stabilization performance that results from unstable and noisy relationships between these financial variables and the rest of the economic system.

Overall, our model shows that assigning a prominent role to monetary aggregates in the formulation of monetary policy might have some benefits but also carries high risks. Such a strategy is only attractive if the policymaker has high confidence in his knowledge of the structural relationships within the economy. The stock market index, however, has such an unreliable relationship to the other economic variables that including it consistently in the the policy rule will lead to a severe deterioration of the stabilization performance.

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