Modeling Electricity Markets as Two-Stage Capacity Constrained Price Competition Games under Uncertainty

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***Abstract*— The last decade has seen an increasing application of game theoretic tools in the analysis of electricity markets and the strategic behavior of market players. This paper focuses on the model examined by Fabra *et al.* (2008), where the market is described by a two-stage game with the firms choosing their capacity in the first stage and then competing in prices in the second stage. By allowing the firms to endogenously determine their capacity, through the capacity investment stage of the game, they can greatly affect competition in the subsequent pricing stage. Extending this model to the demand uncertainty case gives a very good candidate for modeling the strategic aspect of the investment decisions in an electricity market. After investigating the required assumptions for applying the model in electricity markets, we present some numerical examples of the model on the resulting equilibrium capacities, prices and profits of the firms. We then proceed with two results on the minimum value of price caps and the minimum required revenue from capacity mechanisms in order to induce adequate investments.**

***Keywords* — Capacity Constraints, Electricity Markets, Regulatory Policy, Strategic Behaviour.**

I. Introduction

The development of electricity markets all around the world has been accompanied by the announcement of a significant number of investments by market participants, either aiming to strengthen their position in the market or reflecting their desire to enter the market. In reality though, only a small percentage of these projects was actually completed or is under construction, while the majority of the announced investments will likely be cancelled. A clear and widely accepted explanation of this phenomenon doesn't exist.

Moreover, the investments in generation capacity are long term investments characterized by high fixed costs associated with significant risks. Generation units, and especially mid-merit and peaking plants, can recover these costs during hours of high prices. Therefore, as prices result from the intersection of the supply and demand curves, producers have strong incentives to influence the supply curve by making it steeper.

The strategic behavior of market participants has been examined extensively in the literature through the use of game theoretic tools. Still, the focus of most efforts was in the strategic bidding of the participants in the spot market. A comparatively unexplored area in the literature is related to the strategic investments in electricity markets, affecting directly and in a more consistent way the supply curve in the spot market than, for example, economic or physical withholding.

In this paper we examine how a variation of the game-theoretic model presented in [5] can be practically applied to electricity markets. More specifically, in Section II we briefly review the literature on modeling strategic behavior in electricity markets. In Section III we give the theoretical background of the model. In Section IV we discuss the model and how it can be applied in the context of electricity markets, giving at the same time some numerical results. Finally, in Section V we provide two applications with regulatory interest, on the minimum values for price caps and capacity mechanism revenues required to attract sufficient investments.

II. Literature Survey

A. General

The prediction and analysis of the strategic behavior of electricity market participants has been modeled using various approaches. A survey can be found in [1], where it is shown that there are three main lines of modeling trends: optimization, equilibrium and simulation models. Our focus in this paper falls in the equilibrium modeling of the market, both in the short term (corresponding to the second stage of our model), concerning spot market competition, as well as in the medium to long term (first stage of our model), representing investment decisions in imperfect electricity markets.

B. Spot Market Competition

The main models used in the electricity market literature are based on the competition models of Cournot, Stackelberg and Supply Function Equilibrium (SFE). On the contrary, the capacity constrained price competition models[1], often used in economic literature, have not received much attention.

The difference in the above models is the strategic variable of the players: in the Cournot and Stackelberg models firms compete in quantities, in the SFE in supply curves, and in the capacity constrained price competition model they compete in prices. The solution of all these games is based on the concept of Nash equilibrium.

Most models in the literature apply the Cournot competition model, mainly due to its simplicity and ease in extending it. The main criticism against it is related to the use of quantities as strategic variables, when in reality firms submit supply curves in the form of stepwise increasing price-quantity functions. This is the main advantage of the SFE approach,

[1] In the capacity constrained price competition models we include both the Bertrand-Edgeworth type models, typically corresponding to a discriminatory auction, as well as the multi-unit auction models, where bids are offer prices corresponding to given capacities.

which on the other hand is characterized by complexity and multiplicity of equilibria. More details in the advantages and disadvantages of these models can be found in [1] and references therein. A detailed comparison of the SFE and Cournot approach can also be found in [2].

The capacity constrained price competition model provides an interesting alternative to the Cournot and SFE ones, especially in the form of the multi-unit auction model, first proposed in [3][2]. Although it has seen less application than the first two, there has been an increasing number of papers applying it the last few years, especially in the context of the strategic investment models discussed below ([4]-[7]).

C. *Strategic Investment Models*

There are two strands of literature examining strategic investment in an imperfect electricity market. One strand examines the dynamic aspect of investments using simulation modeling (see for example [8]-[12]). These models examine how a sequence of capacity investments decisions are made under uncertain and evolving market demand. The strategic decisions of the firms are modeled using either the Cournot or the Stackelberg competition framework.

The second strand of literature, related to the present paper, involves game-theoretic models that study the strategic behavior of firms under a two-stage framework ([5]-[6] and [13]-[17]). The first stage of the model corresponds to the investment stage, where firms decide on how much capacity to build, while on the second stage firms compete in the spot market. The two-stage game is solved by backward induction in order to find its subgame perfect equilibrium. These models examine a specific investment period (or cycle) of the market under a static environment, with the scope to "isolate" strategic behavior from exogenous parameters and thus better investigate it. Therefore, there seem to be well suited for studies on regulatory issues, as in [5] on market rules.

The work in [14]-[17] assumes Cournot competition in the second stage, while [5], [6] and [13] assume price competition under capacity constraints. All models, apart from [14], assume uncertain demand with a continuous demand distribution, while demand is inelastic (i.e. vertical) only in [5] and [6]. Asymmetric firms (i.e. firms with different marginal and/or capacity costs) are considered only in [14] and [17]. Finally, no model assumes firms having initial capacities.

The model presented in this paper is a variation of the one in [5], involving a discrete demand function and non-zero marginal production cost. As we are mainly investigating how this model can be applied to electricity markets, we rely on [5], [18] and [19] for the theoretical background of the model.

III. THE THEORETICAL MODEL

A. *General Description*

In this section we present a general framework under which someone can model an electricity wholesale market in a medium term horizon. The two-stage model presented is useful in investigating the strategic character of the investment decisions of firms in an electricity market and how these may affect aggregate investments and spot market prices. A number of simplified assumptions have been made in order to make the model tractable, but the presented framework can be easily extended in numerous ways in order to account for all the peculiarities of electricity markets.

More specifically, the electricity market is modeled as a two stage game under uncertainty. In the first stage firms choose their investments in generation capacity, while in the second stage firms compete in prices under capacity constraints. During the investment stage there is uncertainty about the future demand, which is resolved right before the pricing stage. We want to find the subgame perfect equilibria of the game, thus the game is solved by backward induction: we first solve the pricing stage and then, taking this solution as given, we proceed to the solution of the investment stage. The timing of the game is illustrated in Fig.1.

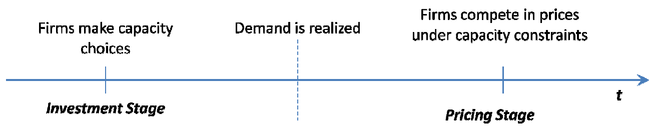

Fig.1. The timing of the game.

Investment Stage. At the investment stage, firms choose simultaneously the amount of capacity they want to build. Their decisions are irreversible. In order to have tractable results, we assume that the two firms choose the same technology, with capacity cost $c > 0$. We do not consider the generally lumpy character of these investments.

Demand Realization. We assume demand is price-inelastic[3], and can be approximated by a binomial distribution function. Hence, demand can take two values, either the low demand value ϑ^L, with probability $p > 0$, or the high demand value ϑ^H, with probability $1 - p$. We assume that $0 < \vartheta^L < \vartheta^H$. Both firms have the same beliefs for the demand distribution function. The value of demand is realized and revealed to both players between the investment and the pricing stage. Parallel to the demand realization, we also assume that the investment decisions of each firm become common knowledge.

Pricing Stage. During the pricing stage, firms choose simultaneously prices and compete under capacity constraints. Bids are subject to a price cap r, for which it holds $r - mc > c$, thus always allowing not only the recovery of the marginal cost but also of the capital cost. Both firms have the same marginal cost mc. We assume that the consumers first buy from the lowest priced firm and only if its capacity is fully utilized they continue buying from the next firm, while firms are paid based on their offers (i.e. we assume discriminate pricing[4]). Note that the competition held at the pricing stage involves only the newly installed capacity, as no initial capacity for the firms is considered.

In the following we will examine the model under a duopoly. It will be solved by backwards induction, thus we will first present the solution of the pricing stage, based on

[2] For reasons of tractability, usually a single price offer is assumed to be submitted, instead of multiple. It is shown though in [4] that the outcome of the auction "*is independent of the number of admissible steps in the offer price-functions, so long as this number is finite*".

[3] This is a common assumption in electricity market literature (see [5], [6]).

[4] Alternatively one could assume uniform pricing, like in [3]-[7] and [13], without significantly changing the results of the capacity stage. A comparison of the two designs and how they affect investments can be found in [5].

Lemma 2 of [18] and Proposition 1 of [19], and then the solution of the capacity stage, based on Proposition 3 of [19].

B. Pricing Stage

In the pricing stage the two firms have capacities k_i, $i = \{1,2\}$ with $0 \leq k^- = \min\{k_1, k_2\} \leq \max\{k_1, k_2\} = k^+$, where with k^- we refer to the capacity of the small firm and with k^+ to the capacity of the large firm. Then the solution of the pricing stage is given by Proposition 1.

Proposition 1 *Suppose that the demand is ϑ. Then there is a unique equilibrium which satisfies the following:*

(i) If $\vartheta \leq k^-$, there exists a unique pure-strategy equilibrium where both firms set prices equal to marginal cost and make zero expected profits.

(ii) If $k^- < \vartheta < k^- + k^+$, a pure strategy equilibrium fails to exist. There is a unique mixed strategy equilibrium, where the large firm's profit is $(r - mc)(\vartheta - k^-)$ and the small seller's profit is $(r - mc)(\vartheta - k^-)\frac{k^-}{\min\{k^+,\vartheta\}}$. Moreover, the support of the prices for both firms is the interval $\left[mc + (r - mc)\frac{\vartheta - k^-}{\min\{k^+,\vartheta\}}, r\right]$, with equilibrium price distributions for the small firm $F^-[p] = \frac{\min\{k^+,\vartheta\}}{\Delta\vartheta} - \frac{\vartheta - k^-}{\Delta\vartheta}\frac{r - mc}{p - mc}$ and for the large firm $F^+[p] = \frac{k^-}{\Delta\vartheta} - \frac{k^-}{\Delta\vartheta}\frac{\vartheta - k^-}{\min\{k^+,\vartheta\}}\frac{r - mc}{p - mc}$ for $p < r$ with a mass point of $\left(1 - {k^-}/{k^+}\right)$ at $p = r$, where $\Delta\vartheta = \min\{k^+, \vartheta\} + k^- - \vartheta$.

(iii) If $\vartheta \geq k^+ + k^-$, in the unique equilibrium both firms set prices equal to the price cap and sell at their capacities.

Proof The proof for (i) and (iii) is immediate, as (i) corresponds to the classical Bertrand competition result, while in (iii) capacity does not suffice so the price goes to the price cap. For the proof of (ii) the reader is directed to [18], as it is a slight generalization of Lemma 2.

C. Capacity Stage with Demand Uncertainty

Under demand uncertainty five regions need to be examined. Note that it will always hold $k^+ \leq \vartheta^H$ for the large firm, as it cannot sell more quantity than the maximum demanded and will avoid having excess capacity as it is costly.

1. $k^- + k^+ \leq \vartheta^L < \vartheta^H$
2. $k^- \leq \vartheta^L \leq k^- + k^+ \leq \vartheta^H$
3. $k^- \leq \vartheta^L \leq \vartheta^H \leq k^- + k^+$
4. $\vartheta^L \leq k^- \leq k^- + k^+ \leq \vartheta^H$
5. $\vartheta^L \leq k^- \leq \vartheta^H \leq k^- + k^+$

In each of those regions the expected profits of the two firms will be a linear combination of the profits derived in Proposition 1 for the corresponding demand value, weighted by its respective probability of realization. For example, assume we are in region 2. Then if the demand is equal to $\vartheta = \vartheta^L$ the profits of the firms correspond to the ones of region (ii), while for $\vartheta = \vartheta^H$ they correspond to region (i). All the profit functions corresponding to the above five regions can be found in the Appendix. The equilibrium capacities[5] then are characterized by the following proposition, based on Proposition 3 part (ii) in [19][6].

Proposition 2 *Suppose that the demand can take either the value ϑ^L, with probability $p > 0$, or the value ϑ^H, with probability $1 - p$. Moreover let $\tilde{c} = \frac{c}{r - mc}$. Then in any subgame perfect pure-strategy equilibrium, aggregate capacity is ϑ^L if $p \in (1 - \tilde{c}, 1)$ and ϑ^H if $p \in (0, 1 - \tilde{c})$. For $p = 1 - \tilde{c}$ any aggregate capacity in the interval $[\vartheta^L, \vartheta^H]$ can be sustained as an equilibrium.*

Proof We refer the interested reader to [19].

Corollary 1 *Let $p < 1 - \tilde{c}$. Then in equilibrium both firms make positive profits, while the profits per unit of capacity of the small firm are larger or equal to the ones of the large firm.*

Proof See the Appendix.

In general the capacity stage is characterized by a multiplicity of equilibria which are proven to be very dependent on the parameter values, as it is can be seen in [19]. In order to present some numerical results of the model, we will characterize capacity equilibria under a specific set of parameter values[7], as defined in Lemma 1.

Lemma 1 *Let $\vartheta^L < \vartheta^H \leq \frac{2p}{3p + \tilde{c} - 1}\vartheta^L$ and $3p > 1 - \tilde{c} > p$. Then for the capacity stage there is a continuum of subgame perfect pure strategy equilibria, with equilibrium capacities being all pairs (k^+, k^-) with $k^+ \in \left[\frac{\vartheta^H}{2}, \frac{(1+p)\vartheta^H - p\vartheta^L}{2 - \tilde{c}}\right]$ and $k^- = \vartheta - k^+$.*

IV. Modeling the Electricity Market

A. Applying the Model to Electricity Markets

The proposed two-stage framework conceptually matches the decision stages of the wholesale electricity markets and the long run character of the investment decisions prior to the realization of uncertain demand. Moreover it accurately depicts the strategic complementarities of capacity decisions during the investment stage, which in turn are a crucial parameter in the results of the subsequent competition stage.

The major drawback of the model, when compared to electricity markets, is the assumption of a single pricing stage period. Although this is the usual approach in the literature (for example in [5], [6]), in reality firms compete repeatedly during the life of their investment under a continuously evolving game, similar to the one examined in [8]. As the scope of the paper is to give some intuitive results that could be used as a benchmark, modeling in more detail the aforementioned stochastic game is left for another instance. Thus the pricing stage will be assumed to correspond to a representative trading period for the realized demand state. Still one can see that as long as all parameters[8] of the game stay constant, the pricing stage will always give the same equilibrium[9]. This can be "exploited" in order to make the application of the model more realistic.

A second aspect of the model that must be discussed is the interpretation of the demand and its distribution. Based on the formulation of the model, the demand in the pricing stage is

[5] Like in [5] and [19], we focus on pure strategy equilibria in the investment stage.

[6] The result for $p = 1 - \tilde{c}$ can also be found in a revised version of [19] that recently came into our attention.

[7] Note, in relation to [19], that $\vartheta^H \leq \frac{2p}{3p + \tilde{c} - 1}\vartheta^L \leftrightarrow p \leq \frac{(1 - \tilde{c})\vartheta^H}{3\vartheta^H - 2\vartheta^L} < 1 - \tilde{c}$.

[8] These are: the player's capacities, the demand values and probabilities and the costs (marginal and capacity).

[9] We are solving for the non-cooperative equilibrium of the game, without considering issues related to repeated games like learning and discounting.

covered only by the newly installed generation capacity[10]. Provided that the range of the derived equilibrium capacities is not large and considering the generally increasing trend of demand and the retirement of older units, this underlying assumption of the model can be ignored. The demand value can either refer to the average expected demand (thus being high or low[11]), or it can correspond to a high and low demand state on the yearly load duration curve. In the latter case the probability p would refer to the relative weight of these periods. Since firms maximize expected profits over the demand states, both interpretations are equivalent. For the demand distribution, although a two point distribution may seem simplistic, in many cases the decisions of public authorities (ministries / regulators / TSOs) are based on high/low or high/medium/low demand scenarios, hence, at least for the scope of this paper, it is considered sufficient.

As far as the predictions of the model are concerned, the resulting continuum of equilibria is another drawback of the model, as it is not clear which of them will prevail. This gives little predictive value in the model and requires the application of an equilibrium selection method. Still though, the multiplicity of equilibria doesn't prevent us from drawing some useful results from the model, presented in Section V.

Finally, the presented model does not take into account the initial capacities of firms and a possible choice over different technologies. All these constitute possible extensions of the model, which will add some complexity to it, but may also lead to the reduction of the number of equilibria.

B. Conventions and Parameter Specification

In order to proceed to the application of the model we first need to make some conventions and further assumptions:

I. The capacity stage refers to an annual period, after the investments have been completed. The pricing stage refers to one representative trading period (one hour). During this period all parameter values remain constant.
II. There are *N* pricing stages, corresponding to the number of hours the generation plant is expected to run. The model examines only one representative stage.
III. The capacity cost *c* corresponds to the annualized investment cost of the plant, spread equally among the *N* pricing stages[12].

Then we will apply the model for two technologies, a mid-merit CCGT operating 6000 hours and a peaking OCGT operating 200 hours. The hypothetical annualized capacity costs for the two technologies are 100,000 €/MW-year for the CCGT and 50,000 €/MW-year for the OCGT, while their variable costs are 60 €/MWh and 100 €/MWh, respectively[13].

For the demand we will investigate two scenarios, based on the two possible interpretations of demand described above:

(A1) There are two equally probable ($p = 0.5$) values for demand, ϑ^H and ϑ^L, with $\vartheta^H = 1.2\vartheta^L$. This is closer to the interpretation of the demand as the average yearly demand.

(A2) The expected load duration curve is split in three parts, first by defining the hours the expected technology is going to be operating and then by splitting this interval so that the higher demand hours will be twice the lower demand hours. Then we set the value of ϑ^H equal to the average demand of the higher hourly demand interval, while ϑ^L is calculated in a similar way for the other interval. Following this procedure, p will equal $^2/_3$, while at the same time we assume that the calculations lead to $\vartheta^H = 1.5\vartheta^L$. As an example, the above calculation for the case of the CCGT is illustrated in Fig. 2. In both demand scenarios we assume $\vartheta^L = 5{,}000$ MWh.

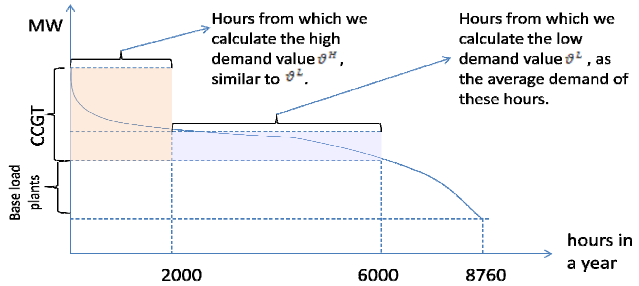

Fig.2. Load duration curve approach for CCGT

C. Numerical Results of the Model

We now proceed to a numerical application of the model, based on the above comments. We investigate only the case of the CCGT plant, assuming that $\vartheta^H = 1.2\vartheta^L = 6{,}000$ MWh, in order to be able to satisfy the conditions of Lemma 1.

In Table I one can see the equilibrium capacities of the two firms, calculated for different values of the price cap *r* and the probability *p* in order to assess the robustness of our results.

TABLE I
EQUILIBRIUM CAPACITIES

Equilibrium capacities of the large firm[14]			
p\r	150 €/MWh	300 €/MWh	600 €/MWh
0.33	[3000 , 3489.8]	[3000 , 3280.58]	[3000 , 3227.8]
0.50	[3000 , 3581.63]	[3000 , 3366.91]	[3000 , 3312.74]
0.67	-	[3000 , 3453.24]	[3000 , 3397.68]
Equilibrium capacities of the small firm			
0.33	[2510.2 , 3000]	[2719.42 , 3000]	[2772.2 , 3000]
0.50	[2418.37 , 3000]	[2633.09 , 3000]	[2687.26 , 3000]
0.67	-	[2546.76 , 3000]	[2602.32 , 3000]

It is interesting to note that although the two firms initially are symmetric, facing the same costs, in equilibrium they will have asymmetric capacities. Therefore it is expected that one firm will choose to be the small firm and one firm will choose to be the large firm. Why would a firm prefer to be small? By being small it will have a greater return on investment than the large firm (see Corollary 1). Note also, in this example, the small range of equilibrium capacities and how low the respective values are even compared to the low demand value.

[10] Because only new capacity participates in the pricing stage. Therefore it could be considered as the contestable demand for new generation units.

[11] In order to be exact, the states should be named high and medium, as our model implies that a third low demand state exists, corresponding to all periods where the market is not contestable by the examined technology. This would be the case for example when nuclear units operate in the market, usually having contracted the total of their capacity at low or regulated prices.

[12] Since all pricing stages yield the same equilibrium profits.

[13] The assumptions on the parameter values were made purely for illustrative purposes and have not been the result of an analysis.

[14] The case $p = 0.67$ and $r = 150$ €/MWh doesn't fall under Lemma 1 so it will be omitted. The corresponding equilibria can be found in [19].

Now in the pricing stage we will examine separately the two demand cases. In the high demand case we will always have $k^+ + k^- = \vartheta$, with both firms offering their energy at the price cap. Assuming $r = 150$ €/MWh, the firms' profits will belong to the intervals presented in Table II.

TABLE II
FIRMS' PROFITS IN HIGH DEMAND CASE

	Demand (A1)	Demand (A2)
Large Firm Profits (€)	[270,000 , 314,082]	[270,000 , 322,347]
Small Firm Profits (€)	[225,918 , 270,000]	[217,653 , 270,000]

The low demand case is not as straightforward, as the pricing stage equilibrium is in mixed strategies. We assume again $r = 150$ €/MWh. For illustration purposes, we will not restrict ourselves just to the equilibrium capacities, but will present the results of this case for various firm capacities. Then, from Proposition 1, we can calculate the support of the prices as shown in Table III. Note that the last line of Table III depicts the equilibrium price supports, when the respective pair of capacities constitutes an equilibrium in the capacity stage.

TABLE III
SUPPORT OF PRICES IN LOW DEMAND CASE

Price Support based on Proposition 1 for various firms' capacities					
k- \ k+	3250 MW	3500 MW	3750 MW	4500 MW	6000 MW
0.6 k+	[144,150]	[135,150]	[126,150]	[106,150]	[85,150]
0.7 k+	[135,150]	[126,150]	[117,150]	[97,150]	[74,150]
0.8 k+	[126,150]	[117,150]	[108,150]	[88,150]	[64,150]
0.9 k+	[117,150]	[108,150]	[99,150]	[79,150]	60
k+	[108,150]	[99,150]	[90,150]	[70,150]	60
ϑ^H -k+	**[122,150]**	**[124,150]**	**[126,150]**	**[130,150]**	**150**

Another interesting result, which can be seen in Fig. 3, is that the price distribution of the large firm stochastically dominates the one of the small firm. This means it is more likely for the price of the small firm to be lower than the one of the large firm. Therefore the small firm is more likely to sell at capacity.

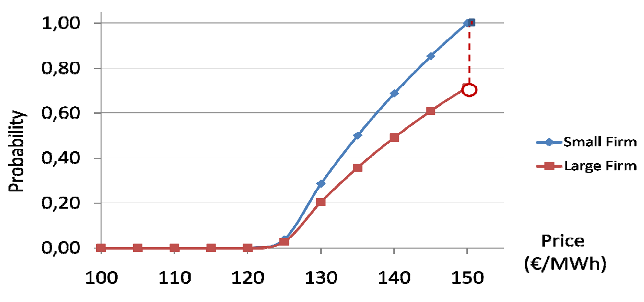

Fig.3. Equilibrium Price Distributions for $r = 150$ €/MWh, $k^+ = 3500$ MW and $k^- = 2500$ MW *in the low demand case.*

It is also interesting to note that the profits of the large firm, when in region (ii) of Proposition 1, are independent (directly) of the capacity of the large firm. Looking at its profit function, this could be interpreted as if the large firm always chose to serve the residual demand, after the small firm has sold all its capacity. On the contrary the total profits of the small firm are always a specific percentage of the large firms' profits, equal to $k^-/\min\{k^+, \vartheta\}$.

TABLE IV
FIRMS' PROFITS IN LOW DEMAND CASE

Large firm's profits for various firms' capacities					
k- \ k+	3250 MW	3500 MW	3750 MW	4500 MW	6000 MW
0.6 k+	274,500	261,000	247,500	207,000	126,000
0.7 k+	245,250	229,500	213,750	166,500	72,000
0.8 k+	216,000	198,000	180,000	126,000	18,000
0.9 k+	186,750	166,500	146,250	85,500	0
k+	157,500	135,000	112,500	45,000	0
ϑ^H -k+	**202,500**	**225,000**	**247,500**	**315,000**	**450,000**
Small firm's profits for various firms' capacities					
0.6 k+	164,700	156,600	148,500	124,200	90,720
0.7 k+	171,675	160,650	149,625	116,550	60,480
0.8 k+	172,800	158,400	144,000	100,800	17,280
0.9 k+	168,075	149,850	131,625	76,950	0
k+	157,500	135,000	112,500	45,000	0
ϑ^H -k+	**171,346**	**160,714**	**148,500**	**105,000**	**0**

V. APPLICATIONS

A. *Defining the Minimum Price Cap*

The most common measure for mitigating market power, especially in systems with tight capacity reserve margins, is the use of price caps either on the offers of the generators or on the electricity spot price. Although price caps have been shown to effectively reduce the incentives of firms to manipulate market prices, they also have an important effect on the investment decisions of firms that should not be overlooked. This effect has been examined in a number of papers (see for example [20] and [21]), all stressing how price caps may deter investments, especially of peaking capacity, if not appropriately chosen.

One can see from Proposition 1 that, in our model, price caps significantly affect the pricing strategies of the firms and thus their profits for each expected level of demand. This in turn directly affects the resulting equilibrium of the capacity stage, which can have serious implications on the security of supply of the electricity market: a "low" price cap may lead to inadequate investments on behalf of the participants and thus the market may run into the risk of power curtailments.

The strategic model we presented can give an easy way to define a benchmark for what may be considered as a "low" price cap. More specifically Proposition 2 states that the firms will invest enough to cover the high demand scenario only if $p < 1 - \tilde{c} = 1 - \frac{c}{r - mc}$, which is equivalent to $r > mc + \frac{c}{1-p}$.

According to the above inequality, in order for the market to converge to the high aggregate capacity equilibrium, the price cap should be at least as high as the marginal cost of the examined technology plus its capital cost divided by the probability of appearance of the high demand. Moreover it should be noted that the price cap is not dependant on the assumed values of the demand, but only on their probabilities, and that all parameters on the right hand side are exogenous.

Applying the model to the parameter values described in Subsection IV.B we obtain the results presented in Table V.

TABLE V
MINIMUM PRICE CAP VALUES

	Demand (A1)	Demand (A2)
Mid-merit (CCGT)	93.33 €/MWh	110 €/MWh
Peaking (OCTT)	600 €/MWh	850 €/MWh

Thus a relatively small price cap is sufficient for the CCGT's, while a much larger price cap is required for the OCGT's. More generally, if p takes values in the interval $\left[\frac{1}{4},\frac{3}{4}\right]$ then the corresponding minimum price cap intervals for the CCGT and OCGT are $[82.22\,,126.66]$ and $[350\,,1100]$ respectively. The big difference between the range of values of the two price caps implies that the best policy, under the examined pay-as-bid framework, would be to implement different price caps on the offers of each generation unit technology, instead of a uniform market price cap[15].

B. *Capacity Mechanisms*

Assume now that apart from a price-cap, a capacity mechanism is also available in order to solve the "missing money" problem of the more expensive units like OCGT. Practically, the main purpose of the capacity mechanism is to "push" the market to the high capacity equilibrium by reducing the investment cost of the firms. Then, if the firms receive an annual income of c_M, it must hold[16] $c_M > c - (r - mc)(1 - p)N$. Applying the formula to various levels of price caps, for the case of the OCGT, we get the results presented in Table VI.

TABLE VI
MINIMUM REVENUE FROM CAPACITY MECHANISM FOR OCGT IN €/MW-YEAR

	r=150 €/MWh	*r*=300 €/MWh	*r*=500 €/MWh
Demand (A1)	41,000	26,000	6,000
Demand (A2)	44,000	34,000	20,667

The above exercise doesn't necessarily have to be applied with r equal to the price cap. Instead one can use an even lower value which statistically the market rarely exceeds, depicting the empirical observation that prices rarely reach the price cap (see [21]).

VI. CONCLUSIONS

This paper examines the application of a game-theoretic model, described in [5], meant to capture the strategic element of the investment decisions in electricity markets. The two stages of the model closely resemble the firm decision process, when determining their level of investments.

Due to the stylized nature of the model, in order to apply it to real-world data, a series of assumptions and conventions need to be made. These involve mainly the period the model is examining, assumed here to be annual, and the representation of the demand uncertainty. As this paper is a first effort in investigating the applicability of the model in a realistic context, we have followed a static approach, closer to the spirit of the theoretical model. Alternatively, one could apply the model in a dynamic context, more accurately describing the stochastic demand and the spot market competition, or extend it, to account for initial capacities or asymmetric costs.

Despite the simplicity of the applied model, it manages to give some straightforward results, especially important in a regulatory context. For example in Greece, where both a price cap and a capacity mechanism are in place, the relevant values have been set to *r*=150 €/MWh and $c_M = 35{,}000$€/MW-year. Although three new CCGT plants are expected to come online in the next year, no OCGT plant is planned to be constructed, despite the official call for such investments. The model offers an explanation for this, as well as how it can be resolved, by the proper re-evaluation of the above values.

APPENDIX: PROFIT FUNCTIONS AND PROOF OF COROLLARY 1

As it was discussed in subsection III.C, in the demand uncertainty case there will be five regions. The (expected) profit functions in these regions are:

$$(1)\ \begin{cases}\text{Profits}^{1-} = (r - mc - c)k^- \\ \text{Profits}^{1+} = (r - mc - c)k^+\end{cases}$$

$$(2)\ \begin{cases}\text{Profits}^{2-} = (r - mc)\left[p(\vartheta^L - k^-)\frac{k^-}{\min\{\vartheta^L.k^+\}} + (1-p)k^-\right] - ck^- \\ \text{Profits}^{2+} = (r - mc)[p(\vartheta^L - k^-) + (1-p)k^+] - ck^+\end{cases}$$

$$(3)\ \begin{cases}\text{Profits}^{3-} = (r - mc)\left[p(\vartheta^L - k^-)\frac{k^-}{\min\{\vartheta^L.k^+\}} + (1-p)(\vartheta^H - k^-)\frac{k^-}{k^+}\right] - ck^- \\ \text{Profits}^{3+} = (r - mc)[p\vartheta^L + (1-p)\vartheta^H - k^-] - ck^+\end{cases}$$

$$(4)\ \begin{cases}\text{Profits}^{4-} = (r - mc)(1-p)k^- - ck^- \\ \text{Profits}^{4+} = (r - mc)(1-p)k^+ - ck^+\end{cases}$$

$$(5)\ \begin{cases}\text{Profits}^{5-} = (r - mc)(1-p)(\vartheta^H - k^-)\frac{k^-}{k^+} - ck^- \\ \text{Profits}^{5+} = (r - mc)(1-p)(\vartheta^H - k^-) - ck^+\end{cases}$$

Since $p < 1 - \tilde{c}$, from Proposition 2 in equilibrium it will hold $k^- = \vartheta^H - k^+$. Therefore by diving the profit functions by $r - mc$, denoting the scaled profit functions by $\widetilde{Profits}$ and replacing k^- by $\vartheta^H - k^+$ wherever needed, we get:

$$(1)\ \begin{cases}\widetilde{Profits}^{1-} = (1 - p - \tilde{c})k^- + pk^- \\ \widetilde{Profits}^{1+} = (1 - p - \tilde{c})k^+ + pk^+\end{cases}$$

$$(2)\ \begin{cases}\widetilde{Profits}^{2-} = (1 - p - \tilde{c})k^- + p\frac{k^+}{\min\{\vartheta^L.k^+\}}k^- \\ \widetilde{Profits}^{2+} = (1 - p - \tilde{c})k^+ + p(k^+ + \vartheta^L - \vartheta^H)\end{cases}$$

$$(3)\ \begin{cases}\widetilde{Profits}^{3-} = (1 - p - \tilde{c})k^- + p\frac{k^+}{\min\{\vartheta^L.k^+\}}k^- \\ \widetilde{Profits}^{3+} = (1 - p - \tilde{c})k^+ + p(k^+ + \vartheta^L - \vartheta^H)\end{cases}$$

$$(4)\ \begin{cases}\widetilde{Profits}^{4-} = (1 - p - \tilde{c})k^- \\ \widetilde{Profits}^{4+} = (1 - p - \tilde{c})k^+\end{cases}$$

$$(5)\ \begin{cases}\widetilde{Profits}^{5-} = (1 - p - \tilde{c})k^- \\ \widetilde{Profits}^{5+} = (1 - p - \tilde{c})k^+\end{cases}$$

The only profit function that isn't clear if it is positive is $\widetilde{Profits}^{3+}$. It will be so if $k^+ \geq \vartheta^H - \vartheta^L$, which always holds in region (3), as $k^+ \geq \vartheta^H - k^- > \vartheta^H - \vartheta^L$.

Moreover, dividing each profit function by the corresponding firm's capacity we notice that the profit per unit of capacity invested is equal for both the small and the large firm in Regions 1, 4 and 5, while it is larger for the small firm in Regions 2 and 3, as $\frac{k^+}{\min\{\vartheta^L.k^+\}} \geq 1 = \frac{k^+}{k^+} \geq \frac{k^+ + \vartheta^L - \vartheta^H}{k^+}$.

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[15] It is interesting to note that if there were two technologies i, j with $mc_i < mc_j$ and a regulator set different price caps on offers, so that $mc_i < r_i < mc_j$, then the market could be treated as two separate markets, where the demand of the "high marginal cost market" would correspond to the demand exceeding the aggregate capacity of technology i.

[16] The cost *c* here refers to the annual cost.

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