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When an inefficient firm makes higher profit than its efficient rival*

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Abstract

This paper considers a Cournot duopoly game with endogenous organization structures. There are two firms A and B who compete in the retail market, where A is more efficient than B. Prior to competition in the retail stage, firms simultaneously choose their organization structures which can be either 'centralized' (one central unit chooses quantity to maximize firm's profit) or 'decentralized' (the retail unit chooses quantity to maximize firm's revenue while the production unit supplies the required quantity). Identifying the (unique) Nash Equilibrium for every retail-stage subgame, we show that the reduced form game of organization choices is a potential game. The main result is that with endogenous organization structures, situations could arise where the less efficient firm B obtains a higher profit than its more efficient rival A.

Keywords: Centralized structure; decentralized structure; potential games

JEL Classification: C72, D43, L13, L21

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1 Introduction

Consider the market for a good where two firms with different marginal costs compete in a duopoly. The standard result in economics is that the firm with the lower cost obtains a higher profit in this market. A firm in textbook economics acts as a single decision-making unit. In real world, however, a firm's organization could be more complex and consist of several units such as production, distribution, sales and advertising. This paper shows that when organization structure of competing firms is endogenized, the standard result can be reversed in that situations could arise in a duopoly where the less efficient firm obtains a higher profit than its more efficient rival.

We carry out our analysis in a Cournot duopoly framework. There is a retail market with two firms A and B. Both firms have positive marginal costs, A having the lower cost of the two. The organization structure of any firm can be either *centralized* or *decentralized*. Under a centralized structure, a firm has one *central* unit that chooses its quantity to maximize the firm's profit. Under a decentralized structure, a firm has two units: the *retail* unit, which chooses its quantity to maximize the revenue or sales of the firm, and the *production* unit, which supplies the required quantity to the retail unit. The strategic interaction is modeled as an extensive-form game where in the first stage, firms choose their organization structures simultaneously. Following the choice of structures, in the second stage, the "assigned" units of both firms choose quantities in the retail market, where the assigned unit is a firm's central unit if the firm is centralized and it is the retail unit if the firm is decentralized. Each firm's payoff is its profit from the retail market.

For every profile of organization choices, the subgame played between the assigned units in the retail stage has a unique Nash Equilibrium (NE). Identifying the NE for every such subgame and applying backward induction, we show that the reduced form game of organization choices played between A and B in the first stage is a *potential game*, as introduced by Monderer and Shapley (1996). Using the potential function, we completely characterize NE of the reduced form game and consequently, Subgame Perfect Nash Equilibrium (SPNE) of the original extensive-form game.

Our main result is that, provided firm B is not very inefficient compared to firm A, for intermediate sizes of the market there is an SPNE where (i) A chooses the centralized and B the decentralized structure and (ii) firm B, in spite of being the less efficient firm, obtains a higher profit than firm A.

First we see the intuition for result (ii), given result (i). Observe that for a decentralized firm, its retail unit chooses quantity to maximize the firm's revenue and thus, effectively acts as a firm in the retail market that has zero cost. In contrast, for a centralized firm, its central unit solves the standard problem of maximizing profit with positive cost. Accordingly, if A is centralized and B is decentralized, the quantity that B supplies in the retail market is higher than the quantity of A. Therefore, B would obtain a higher profit than A if firms have the same costs. By continuity, the same result holds even if B's costs are higher, as long as they are not too high.

To see the intuition for (i), observe that for two demand curves parallel to each other, at any price, the elasticity is higher at the demand curve that lies on the right. In other words, for any price, demand becomes more elastic as the market expands, where the expansion is presented by parallel rightward shift of the demand curve. This drives the result (i) as follows.

If in response to B's choice of decentralized structure, A also chooses to be decentralized, then both firms supply a high quantity in the retail market that results in a low price. When the market size is not too large, such a low price corresponds to the inelastic portion of the demand curve. Firm A can then improve its profit by deviating to the centralized structure that results in lower output and higher price. On the other hand, if in response to A's choice of centralized structure, B chooses to be centralized, then the price becomes high. When the market size is not too small, such a high price corresponds to the elastic portion of the demand curve. In that case, firm B can improve its profit by deviating to the decentralized structure that results in lower price. This explains that A's choice to be centralized and B's to be decentralized can be sustained as an equilibrium when the market size is intermediate, i.e., it is not too large or too small. The range of market sizes that are "intermediate" and can sustain this equilibrium depend on the costs of the firms, as we demonstrate in Proposition 1.

As mentioned before, a firm in the real world may consist of many units. For tractability and clarity of presentation, in this paper we focus on two basic units that can be considered to be most crucial for a firm's operation: production and sales. The organization structures that we study naturally follow from these two basic units: a centralized structure has production and sales working as one unit, while for a decentralized structure, they are separate. There is a large literature in management, as well as anecdotal evidence, that suggests that both of these structures are observed in practice. For example, Collins and Porras (1994) point out the contrasting nature of organization of the discount department stores of Wal-Mart and Ames:[1]

> "Walton [founder of Wal-Mart] gave department managers the authority and freedom to run each department as if it were their own business...Ames leaders dictated all changes from above and detailed in a book the precise steps a store manager should take, leaving no room for initiative."

Similarly, Siggenkawl and Levinthal (2003) put forward the difference in organization structures of firms with regard to online commerce:

> "...The Gap, a fashion retailer, considered its website simply as one more store and serviced it using its existing infrastructure. The Vanguard Group and Dell considered the Internet as one more distribution channel to be exploited by the existing organization. In contrast, other firms pursued more decentralized search efforts. For instance, Bank One formed an independent Web subsidiary, Wingspan, to explore Internet-based banking. Likewise, Disney created Go.com, grouping all its Web-based activities into one organizational entity and creating a separate tracking stock for this business."

This paper is closely related to a large economics literature on managerial incentives. The seminal papers of this literature are Vickers (1985), Sklivas (1987) and Fershtman and Judd (1987), all of which study endogenous incentive structure in oligopolies, where each firm designs incentives for its manager by asking the manager to maximize a convex or possibly non-convex combination of profit and revenue of the firm.[2] Our model can be

[1] This is the citation of Collins and Porras (1994) by Chang and Harrington (2000).

[2] In the model of Vickers (1985), the manager of a firm is asked to maximize a weighted sum of profit and quantity of the firm; this sum can be equivalently expressed as a linear combination of profit and revenue.

viewed as a situation where a firm asks its manager to either maximize profit (centralized) or revenue (decentralized). Alternatively, the formulations of Fershtman-Judd and Sklivas can be viewed as a mixed strategy extension over two "pure" organization structures that we study. Understanding these pure organization structures is important as asking a manager to maximize a combination of profit and revenue could be complex and difficult to implement in practice, while the pure organizational objectives of revenue or profit maximization are simple, clearly defined and easily implementable.

Our result that a less efficient firm can obtain higher profit under endogenous organization structures is in contrast with the conclusion of the existing literature, where the less efficient firm always makes lower profit under the incentive equilibrium (Fershtman and Judd). Moreover, our result for the case of cost-symmetric duopoly[3] also differs from the existing literature which concludes that under the incentive equilibrium, profits of both firms are less than their Cournot profits. In contrast, we show that under pure organization structures, it is possible to have two equilibria in a symmetric duopoly: in one equilibrium, firm A obtains a profit that is lower than its Cournot profit while firm B obtains a higher profit and in the other one, the converse is true.[4]

The paper is organized as follows. We present the model in Section 2. The results are stated and proved in Section 3. Finally, we conclude in Section 4.

2 The model

Consider the retail market for a good η that is a Cournot duopoly with two firms A and B. For $i = A, B$, let $q_i \geq 0$ be the quantity produced by firm i, $Q = q_A + q_B$ the market quantity and $\rho(Q)$ the market price. Firms face the inverse demand

$$\rho(Q) = \alpha - Q \text{ if } Q < \alpha \text{ and } \rho(Q) = 0 \text{ if } Q \geq \alpha \tag{1}$$

Firms operate under constant marginal costs. The marginal cost of firm i is c_i. We assume

$$0 < c_A < c_B < \alpha \tag{2}$$

Each firm $i = A, B$ seeks to maximize its profit

$$\pi_i(q_A, q_B) = \rho(q_A + q_B)q_i - c_i q_i \tag{3}$$

This model extends the standard Cournot duopoly by allowing each firm to choose its organization structure prior to the retail stage of quantity competition. A firm can choose one of the following organization structures.

(i) *Centralized*: If firm i operates under this structure, it has one *central unit* which chooses q_i to maximize firm i's profit π_i given in (3).

(ii) *Decentralized*: If firm i operates under this structure, its organization is divided in two units: the *retail unit* and the *production unit*. The retail unit chooses q_i to maximize firm i's revenue

$$R_i(q_A, q_B) = \rho(q_A + q_B)q_i \tag{4}$$

and the production unit simply supplies the required q_i chosen by the retail unit.

[3] Vickers (1985) and Sklivas (1987) only consider situations where firms are cost-symmetric.

[4] See Remark 1 (p.11) after Proposition 2.

For $i = A, B$, define the indicator variable

$$\lambda_i = \begin{cases} 1 & \text{if firm } i \text{ chooses centralized structure} \\ 0 & \text{if firm } i \text{ chooses decentralized structure} \end{cases} \tag{5}$$

Denote

$$i^{\lambda_i} = \begin{cases} \text{the central unit of firm } i \text{ if } \lambda_i = 1 \\ \text{the retail unit of firm } i \text{ if } \lambda_i = 0 \end{cases} \tag{6}$$

2.1 The game Γ

The strategic interaction between the two firms is modeled as the extensive-form game Γ that has the following stages.

Stage 1: Firms A, B simultaneously choose their organization structures $\lambda_A, \lambda_B \in \{0, 1\}$. The chosen (λ_A, λ_B) becomes commonly known in the end of stage 1.

Stage 2: For every $\lambda_A, \lambda_B \in \{0, 1\}$, the simultaneous-move game $G(\lambda_A, \lambda_B)$ is played between A^{λ_A} and B^{λ_B} in the retail market η where i^{λ_i} chooses q_i. By (5) and (6):

(i) if $\lambda_i = 1$, then i^{λ_i} is the central unit of firm i and the payoff of i^{λ_i} is $\pi_i(q_A, q_B)$

(ii) if $\lambda_i = 0$, then i^{λ_i} is the retail unit of firm i and the payoff of i^{λ_i} is $R_i(q_A, q_B)$

By (3) and (4), the payoff of i^{λ_i} in $G(\lambda_A, \lambda_B)$ is

$$\pi_i^{\lambda_i}(q_A, q_B) = \rho(q_A + q_B)q_i - \lambda_i c_i q_i \tag{7}$$

For $i = A, B$, the payoff of firm i is its profit given in (3). Then by (7), the payoff of firm i in Γ is

$$\pi_i(\lambda_A, \lambda_B, q_A, q_B) = \pi_i^{\lambda_i}(q_A, q_B) - (1 - \lambda_i)c_i q_i \tag{8}$$

This completes the description of the game Γ.

We seek to determine Subgame Perfect Nash Equilibrium (SPNE) of Γ. Any SPNE of Γ must generate Nash Equilibrium (NE) of $G(\lambda_A, \lambda_B)$ for all $\lambda_A, \lambda_B \in \{0, 1\}$. First we determine NE of $G(\lambda_A, \lambda_B)$ and then apply backward induction to find SPNE of Γ.

2.1.1 Stage 2 of Γ: NE of $G(\lambda_A, \lambda_B)$

Observe from (7) that $G(\lambda_A, \lambda_B)$ can be viewed as a standard Cournot duopoly game played between A^{λ_A} and B^{λ_B} where i^{λ_i} has marginal cost $\lambda_i c_i$ for $i = A, B$. For this reason, to determine NE of $G(\lambda_A, \lambda_B)$, it will be useful to consider a generic standard Cournot duopoly game played between two firms 1 and 2 under demand (1) where 1 has marginal cost τ_1 and 2 has marginal cost τ_2, with $0 \leq \tau_1 \leq \tau_2 \leq \alpha$. We know that this game has a unique NE where

(i) if $\tau_2 \geq (\alpha + \tau_1)/2$, then 1 becomes a monopolist and

(ii) if $\tau_2 < (\alpha + \tau_1)/2$, then both firms stay in the market and for $i, j = 1, 2$ and $j \neq i$, the NE output of firm i is $q_i = (a - 2\tau_i + \tau_j)/3$ and its NE payoff is $(q_i)^2$.

Therefore the game $G(\lambda_A, \lambda_B)$ has a unique NE for any $\lambda_A, \lambda_B \in \{0,1\}$. For the purpose of this paper, we will be interested only in situations where both A^{λ_A} and B^{λ_B} supply positive output in the retail market. Since $\lambda_i \in \{0,1\}$ and $0 < c_A < c_B$, the maximum possible cost of i^{λ_i} in $G(\lambda_A, \lambda_B)$ is c_B and its minimum possible cost is 0. Taking $\tau_1 = 0$ and $\tau_2 = c_B$, the condition $\tau_2 < (\alpha + \tau_1)/2$ reduces to

$$\alpha > 2c_B \tag{9}$$

Together with assumption (2), we maintain assumption (9) throughout, which ensures that both q_A and q_B are positive in the NE of $G(\lambda_A, \lambda_B)$. Let $i, j = A, B$ and $i \neq j$. At the NE, i^{λ_i} chooses $q_i = q_i(\lambda_A c_A, \lambda_B c_B)$ and obtains payoff $\pi_i^{\lambda_i}(q_A, q_B) = \phi_i(\lambda_A c_A, \lambda_B c_B)$ where

$$q_i(\lambda_A c_A, \lambda_B c_B) = (\alpha - 2\lambda_i c_i + \lambda_j c_j)/3 \text{ and } \phi_i(\lambda_A c_A, \lambda_B c_B) = [q_i(\lambda_A c_A, \lambda_B c_B)]^2 \tag{10}$$

2.1.2 Stage 1 of Γ: The reduced form game Γ^*

Now we move back to stage 1 of Γ. Using the unique NE of $G(\lambda_A, \lambda_B)$ from (10) for all $\lambda_A, \lambda_B \in \{0,1\}$ and taking $\pi_i^{\lambda_i}(q_A, q_B) = \phi_i(\lambda_A c_A, \lambda_B c_B)$ in (8), in any SPNE of Γ, firms A and B play the 2×2 reduced form game Γ^* in stage 1 where each firm $i = A, B$ has two strategies: $\lambda_i = 0$ and $\lambda_i = 1$. The payoff function of firm i in Γ^* is

$$\pi_i(\lambda_A, \lambda_B) = \phi_i(\lambda_A c_A, \lambda_B c_B) - (1 - \lambda_i) c_i q_i(\lambda_A c_A, \lambda_B c_B)$$

Using (10), the game Γ^* is given as follows.

Table 1: The game Γ^*

	$\lambda_B = 0$	$\lambda_B = 1$
$\lambda_A = 0$	$\pi_A(0,0) = \phi_A(0,0) - c_A q_A(0,0)$ $= \alpha^2/9 - c_A\alpha/3$ $\pi_B(0,0) = \phi_B(0,0) - c_B q_B(0,0)$ $= \alpha^2/9 - c_B\alpha/3$	$\pi_A(0,1) = \phi_A(0,c_B) - c_A q_A(0,c_B)$ $= (\alpha + c_B)^2/9 - c_A(\alpha + c_B)/3$ $\pi_B(0,1) = \phi_B(0,c_B) = (\alpha - 2c_B)^2/9$
$\lambda_A = 1$	$\pi_A(1,0) = \phi_A(c_A,0)$ $= (\alpha - 2c_A)^2/9$ $\pi_B(1,0) = \phi_B(c_A,0) - c_B q_B(c_A,0)$ $= (\alpha + c_A)^2/9 - c_B(\alpha + c_A)/3$	$\pi_A(1,1) = \phi_A(c_A,c_B)$ $= (\alpha - 2c_A + c_B)^2/9$ $\pi_B(1,1) = \phi_B(c_A,c_B)$ $= (\alpha - 2c_B + c_A)^2/9$

For $\lambda_A, \lambda_B \in \{0,1\}$, we say that (λ_A, λ_B) *is an SPNE of* Γ if (λ_A, λ_B) is an NE of the reduced form game Γ^*. Denote

$$\widetilde{\alpha} \equiv 4c_A - c_B, \underline{\alpha} \equiv 4c_B - c_A \text{ and } \overline{\alpha} \equiv 4c_A \tag{11}$$

From Table 1, we have

$$\pi_A(0,1) - \pi_A(1,1) = c_A(\alpha - \widetilde{\alpha})/9, \ \pi_B(1,0) - \pi_B(1,1) = c_B(\alpha - \underline{\alpha})/9$$

$$\pi_A(0,0) - \pi_A(1,0) = c_A(\alpha - \overline{\alpha})/9 \text{ and } \pi_B(0,0) - \pi_B(0,1) = c_B(\alpha - 4c_B)/9 \tag{12}$$

Throughout the paper, we consider generic values of the parameters c_A, c_B, α, so we confront only strict inequalities.

3 The results

3.1 The main result

Proposition 1 states the main result of the paper which shows that for certain parameter values of the model, the game Γ^* has an NE in which the less efficient firm B obtains higher profit than the more efficient firm A.

Proposition 1 *The following hold for generic values of the parameters c_A, c_B, α.*

(i) $(1,0)$ *is an NE of* Γ^* *if and only if* $c_B < 5c_A/4$ *and* $\alpha \in (\underline{\alpha}, \overline{\alpha})$. *Under this NE,* $\pi_B(1,0) > \pi_A(1,0)$, *i.e., firm B obtains higher profit than firm A.*

(ii) *In any NE of* Γ^* *other than* $(1,0)$, *firm B obtains lower profit than firm A.*

Proof (i) $(1,0)$ is an NE of Γ^* if and only if (a) $\pi_A(1,0) > \pi_A(0,0)$ and (b) $\pi_B(1,0) > \pi_B(1,1)$. Note from (12) that (a) holds iff $\alpha < \overline{\alpha}$ and (b) holds iff $\alpha > \underline{\alpha}$. By (11), we have

$$\underline{\alpha} \lesseqgtr \overline{\alpha} \Leftrightarrow c_B \lesseqgtr 5c_A/4 \tag{13}$$

If $c_B > 5c_A/4$, then $\overline{\alpha} < \underline{\alpha}$, so $(1,0)$ cannot be an NE of Γ^*. If $c_B > 5c_A/4$, then $\underline{\alpha} < \overline{\alpha}$ and $(1,0)$ is an NE if and only if $\alpha \in (\underline{\alpha}, \overline{\alpha})$, proving the first part of (i).

To prove the last part of (i), suppose $(1,0)$ is an NE of Γ^*. Then $c_B < 5c_A/4$ and $\alpha \in (\underline{\alpha}, \overline{\alpha})$. Observe that

$$\pi_B(1,0) - \pi_A(1,0) = (\alpha + c_A)^2/9 - c_B(\alpha + c_A)/3 - (\alpha - 2c_A)^2/9 = f(\alpha)/3$$

where $f(\alpha) := (2c_A - c_B)\alpha - c_A(c_A + c_B)$. As $c_B < 5c_A/4 < 2c_A$, $f(\alpha)$ is increasing. As $f(\underline{\alpha}) = 2(2c_B - c_A)(3c_A/2 - c_B) > 0$ (since $c_A < c_B < 2c_B$ and $c_B < 5c_A/4 < 3c_A/2$), $f(\alpha) > 0$ for all $\alpha \in (\underline{\alpha}, \overline{\alpha})$, which completes the proof of (i).

(ii) Consider any NE other than $(1,0)$. Such an NE could be either $(0,0)$ or $(1,1)$ or $(0,1)$. As $c_A < c_B$, from Table 1, we have $\pi_B(0,0) < \pi_A(0,0)$ and $\pi_B(1,1) < \pi_A(1,1)$. For the outcome $(0,1)$, observe from (10) that $q_A(0,1) > q_B(0,1)$. Let ρ be the price at the retail market under $(0,1)$. Since $c_B > c_A$, we have

$$\begin{aligned} \pi_A(0,1) - \pi_B(0,1) &= [\rho q_A(0,1) - c_A q_A(0,1)] - [\rho q_B(0,1) - c_B q_B(0,1)] \\ &> (\rho - c_A)[q_A(0,1) - q_B(0,1)] > 0 \end{aligned}$$

This completes the proof. ■

Proposition 1 makes precise the intuition described in the introduction that if firm B is not very inefficient in relation to firm A ($c_B < 5c_A/4$), then for "intermediate" sizes of the market (captured by the condition on the demand intercept α that $\underline{\alpha} < \alpha < \overline{\alpha}$), the outcome $(1,0)$ is an NE of Γ^* where the less efficient firm B obtains higher profit than its more efficient rival A.

3.2 Complete characterization of NE of Γ^*

3.2.1 Γ^* is a potential game

Definition (Monderer and Shapley, 1996) A function $P : \{0,1\} \times \{0,1\} \to \mathbb{R}$ is a *potential function* of Γ^* if for $\lambda_A, \lambda'_A, \lambda_B, \lambda'_B \in \{0,1\}$, the following hold.
(i) $P(\lambda_A, \lambda_B) - P(\lambda'_A, \lambda_B) = \pi_A(\lambda_A, \lambda_B) - \pi_A(\lambda'_A, \lambda_B)$
(ii) $P(\lambda_A, \lambda_B) - P(\lambda_A, \lambda'_B) = \pi_B(\lambda_A, \lambda_B) - \pi_B(\lambda_A, \lambda'_B)$
Γ^* is a *potential game* if it has a potential function.

Note that if P is a potential function of Γ^*, then (λ_A, λ_B) is an NE of Γ^* if and only

$$P(\lambda_A, \lambda_B) \geq P(\lambda'_A, \lambda_B) \text{ and } P(\lambda_A, \lambda_B) \geq P(\lambda_A, \lambda'_B) \text{ for } \lambda'_A, \lambda'_B \in \{0,1\} \tag{14}$$

Define $P : \{0,1\} \times \{0,1\} \to \mathbb{R}$ as

$$P(1,1) = 0, P(0,1) = \pi_A(0,1) - \pi_A(1,1),$$

$$P(1,0) = \pi_B(1,0) - \pi_B(1,1), P(0,0) = P(0,1) + [\pi_B(0,0) - \pi_B(0,1)] \tag{15}$$

Lemma 1 *The function P defined in* (15) *is a potential function of the game* Γ^*. *Consequently* Γ^* *is a potential game.*

Proof Observe from (15) that $P(0,1) - P(1,1) = \pi_A(0,1) - \pi_A(1,1)$, $P(1,0) - P(1,1) = \pi_B(1,0) - \pi_B(1,1)$ and $P(0,0) - P(0,1) = \pi_B(0,0) - \pi_B(0,1)$. These three conditions simply follow from the way the function P is constructed.[5]

To complete the proof, it remains to show that $P(0,0) - P(1,0) = \pi_A(0,0) - \pi_A(1,0)$. Note from (11) that $\overline{\alpha} - \widetilde{\alpha} = c_B$ and $4c_B - \underline{\alpha} = c_A$. Using this in (12), we have

$$[\pi_A(0,1) - \pi_A(1,1)] - [\pi_A(0,0) - \pi_A(1,0)] = [\pi_B(1,0) - \pi_B(1,1)] - [\pi_B(0,0) - \pi_B(0,1)] \tag{16}$$

Observe from (15) that

$$P(0,0) - P(1,0) = P(0,1) + [\pi_B(0,0) - \pi_B(0,1)] - P(1,0)$$

$$= [\pi_A(0,1) - \pi_A(1,1)] + [\pi_B(0,0) - \pi_B(0,1)] - [\pi_B(1,0) - \pi_B(1,1)] = \pi_A(0,0) - \pi_A(1,0)$$

where the last equality follows from (16). This completes the proof. ■

Putting forward the notion of potential games, Monderer and Shapley (1996: 125-126) state:

> "...firms that are jointly trying to maximize the potential function P^*...end up in an equilibrium...This raises the natural question about the economic content (or interpretation) of P^*: What do the firms try to jointly maximize?
>
> We do not have an answer to this question. However, it is clear that the mere existence of a potential function helps us (and the players) to better analyze the game."

[5]Note that for a function P to be a potential function for a 2×2 game, it needs to satisfy four conditions. A function such as the one in (15) can always be constructed for *any* 2×2 game such that three out of these four conditions hold. It will be a potential function only if it satisfies the remaining condition as well.

We attempt to provide an economic interpretation of the potential function for the game Γ^*. Consider the outcome $(1,1)$ where both firms choose the centralized structure and the standard Cournot duopoly game is played in the retail stage. In (15), the value of the potential function at this benchmark outcome is normalized at 0. Suppose starting from this "fully centralized" profile $(1,1)$, we gradually move towards decentralized profiles through individual deviations by firms. We can go to the "partial decentralized" profile $(0,1)$ if A moves to 0. The value of $P(0,1)$ gives the net gain for the deviating firm A from such a move. Similarly, $P(1,0)$ gives the deviating firm B's net gain if B moves to 0. Finally consider the "fully decentralized" profile $(0,0)$. In the spirit of *improvement paths* of Monderer and Shapley, we can move from $(1,1)$ to $(0,0)$ in two alternative paths: (i) $(1,1) \to (0,1) \to (0,0)$ (i.e. first A deviates to 0, we reach $(0,1)$ and then B deviates to 0 and we reach $(0,0)$) or (ii) $(1,1) \to (1,0) \to (0,0)$ (i.e. first B deviates to 0 and then A deviates to 0). For path (i), the sum of the net gains of deviating players is $[\pi_A(0,1) - \pi_A(1,1)] + [\pi_B(0,0) - \pi_B(0,1)] = P(0,0)$ (by (15)). For path (ii), the sum of the net gains of deviating players is $[\pi_B(1,0) - \pi_B(1,1)] + [\pi_A(0,0) - \pi_A(1,0)]$ and by (15)-(16), this sum also equals $P(0,0)$.

Therefore, the potential function presents *the sum of net gains for firms that deviate to the decentralized structure* from the starting point of fully centralized profile $(1,1)$ and this is the sum that firms A, B try to jointly maximize. An implication of Γ^* being a potential game is that this sum does not depend on the order of the deviations.

3.2.2 Complete characterization of NE of Γ^*

Using the function P, now we are in a position to completely characterize all NE of Γ^*. By (12) and (15), we have

$$P(1,1) = 0, P(0,1) = c_A(\alpha - \widetilde{\alpha})/9, P(1,0) = c_B(\alpha - \underline{\alpha})/9 \text{ and}$$

$$P(0,0) = P(0,1) + c_B(\alpha - 4c_B)/9 = P(1,0) + c_A(\alpha - \overline{\alpha})/9 \tag{17}$$

Lemma 2 *The following hold for the game Γ^* for generic values of c_A, c_B, α.*

- (i) (a) $(0,0)$ *is an NE iff* $\alpha > 4c_B$.
  - (b) $(1,1)$ *is an NE iff* $\alpha < \widetilde{\alpha}$.
- (ii) (a) $(0,0)$ *is the unique NE for* $\alpha > 4c_B$.
  - (b) $(1,1)$ *is the unique NE for* $\alpha < \widetilde{\alpha}$.
- (iii) (a) *If* $c_B > 5c_A/4$, *then* $(0,1)$ *is the unique NE for* $\alpha \in (\widetilde{\alpha}, 4c_B)$.
  - (b) *If* $c_B < 5c_A/4$, *then* $\widetilde{\alpha} < \underline{\alpha} < \overline{\alpha}$; *there are two NE* $(0,1)$ *and* $(1,0)$ *for* $\alpha \in (\underline{\alpha}, \overline{\alpha})$ *and* $(0,1)$ *is the unique NE for* $\alpha \in (\widetilde{\alpha}, \underline{\alpha}) \cup (\overline{\alpha}, 4c_B)$.

Proof (i) Since $c_A < c_B$, by (11) we have $\overline{\alpha} < 4c_B$ and $\widetilde{\alpha} < \underline{\alpha}$.

By Lemma 1, $(0,0)$ is an NE of Γ^* iff $P(0,0) > P(1,0)$ and $P(0,0) > P(0,1)$. By (17), these two inequalities hold iff $\alpha > \max\{\overline{\alpha}, 4c_B\} = 4c_B$, proving (a). For (b), note that $(1,1)$ is an NE iff $P(1,1) > P(0,1)$ and $P(1,1) > P(0,1)$ and by (17), these hold iff $\alpha < \min\{\widetilde{\alpha}, \underline{\alpha}\} = \widetilde{\alpha}$, proving (b).

(ii) Since $c_A < c_B$, by (11) we have $\widetilde{\alpha} < 4c_B$.

(a) If $\alpha > 4c_B$, then by (i), $(0,0)$ is an NE and $(1,1)$ is not an NE. As $P(0,0) > P(1,0)$ and $P(0,0) > P(0,1)$, neither $(1,0)$ nor $(0,1)$ is an NE, proving the uniqueness.

(b) If $\alpha < \widetilde{\alpha}$, then by (i), $(1,1)$ is an NE and $(0,0)$ is not an NE. As $P(1,1) > P(0,1)$ and $P(1,1) > P(1,0)$, neither $(0,1)$ nor $(1,0)$ is an NE, proving the uniqueness.

(iii) If $\alpha \in (\widetilde{\alpha}, 4c_B)$, then by (17), $P(0,1) > P(1,1)$ and $P(0,1) > P(0,0)$, proving that $(0,1)$ is an NE and neither $(0,0)$ nor $(1,1)$ is an NE.

$(1,0)$ is an NE iff $P(1,0) > P(0,0)$ and $P(1,0) > P(1,1)$ and by (17), these hold iff $\alpha < \overline{\alpha}$ and $\alpha > \underline{\alpha}$.

(a) If $c_B > 5c_A/4$, then by (13), $\overline{\alpha} < \underline{\alpha}$. Hence we cannot have $\alpha < \overline{\alpha}$ and $\alpha > \underline{\alpha}$, so $(1,0)$ cannot be an NE. For this case, the unique NE is $(0,1)$.

(b) If $c_B < 5c_A/4$, then $\underline{\alpha} < \overline{\alpha}$ (by (13)). Since $\widetilde{\alpha} < \underline{\alpha}$, we have $\widetilde{\alpha} < \underline{\alpha} < \overline{\alpha}$. If $\alpha \in (\underline{\alpha}, \overline{\alpha})$, then $(1,0)$ is an NE, so there are two NE $(0,1)$ and $(1,0)$. If $\alpha \in (\widetilde{\alpha}, \underline{\alpha}) \cup (\overline{\alpha}, 4c_B)$, then $(1,0)$ is not an NE, so $(0,1)$ is the unique NE. ■

Proposition 2 completely characterizes NE of Γ^*.

Proposition 2 *Let $0 < c_A < c_B$ and $\alpha > 2c_B$. The following hold for the game Γ^* for generic values of c_A, c_B, α.*

(i) $(0,0)$ *is the unique NE for* $\alpha > 4c_B$.

(ii) *Let* $c_B < 5c_A/4$. *Then* $2c_B < \widetilde{\alpha} < \underline{\alpha} < \overline{\alpha} < 4c_B$ *and*

  (a) $(1,1)$ *is the unique NE for* $\alpha \in (2c_B, \widetilde{\alpha})$,

  (b) $(0,1)$ *is the unique NE for* $\alpha \in (\widetilde{\alpha}, \underline{\alpha}) \cup (\overline{\alpha}, 4c_B)$,

  (c) *there are two NE* $(0,1)$ *and* $(1,0)$ *for* $\alpha \in (\underline{\alpha}, \overline{\alpha})$.

(iii) *Let* $5c_A/4 < c_B < 4c_A/3$. *Then* $2c_B < \widetilde{\alpha} < 4c_B$ *and*

  (a) $(1,1)$ *is the unique NE for* $\alpha \in (2c_B, \widetilde{\alpha})$,

  (b) $(0,1)$ *is the unique NE for* $\alpha \in (\widetilde{\alpha}, 4c_B)$.

(iv) *Let* $c_B > 4c_A/3$. *Then* $(0,1)$ *is the unique NE for all* $\alpha \in (2c_B, 4c_B)$.

Proof (i) Follows from Lemma 2(ii)(a).

(ii) Since $c_A < c_B$, we have $\overline{\alpha} = 4c_A < 4c_B$. If $c_B < 5c_A/4$, then $\widetilde{\alpha} < \underline{\alpha} < \overline{\alpha}$ (Lemma 2(iii)(b)) and by (11), $\widetilde{\alpha} - 2c_B = 3(4c_A/3 - c_B) > 0$ (since $c_B < 5c_A/4 < 4c_A/3$), proving that $2c_B < \widetilde{\alpha} < \underline{\alpha} < \overline{\alpha} < 4c_B$. Part (a) follows from Lemma 2(ii)(b) and parts (b),(c) from Lemma 2(iii)(b).

(iii) By (11), $\widetilde{\alpha} < 4c_B$ (since $c_A < c_B$) and $\widetilde{\alpha} - 2c_B = 3(4c_A/3 - c_B) > 0$ for $c_B < 4c_A/3$. Hence $2c_B < \widetilde{\alpha} < 4c_B$. Part (a) follows from Lemma 2(ii)(b) and part (b) from Lemma 2(iii)(a).

(iv) By (11), $\widetilde{\alpha} - 2c_B = 3(4c_A/3 - c_B) < 0$ for $c_B > 4c_A/3$. Hence $(2c_B, 4c_B) \subset (\widetilde{\alpha}, 4c_B)$. Since $c_B > 4c_A/3 > 5c_A/4$, it follows from by Lemma 2(iii)(a) that $(0,1)$ is the unique NE for all $\alpha \in (2c_B, 4c_B)$. ■

Remark 1 When $c_A = c_B = c$, we have the special case of a cost-symmetric duopoly. In that case, part (ii) of Prop 2 applies (since $c_B = c < 5c_A/4 = 5c/4$). By (11), it follows that $\underline{\alpha} = 4c_B - c_A = 3c$ and $\overline{\alpha} = 4c_A = 4c$. Then by Prop 2(ii)(c), if $\alpha \in (3c, 4c)$, there are two NE of Γ^*: $(0,1)$ and $(1,0)$. Taking $c_A = c_B = c$ in Table 1 (p.6), under $(0,1)$, firm A obtains $(\alpha+c)^2/9 - c(\alpha+c)/3$, which is higher than its Cournot profit $(\alpha-c)^2/9$ for $\alpha > 3c$, while firm B obtains $(\alpha-2c)^2/9$, which is lower than its Cournot profit $(\alpha-c)^2/9$. The converse is true under the outcome $(1,0)$.

4 Concluding remarks

In this paper we have shown that when organization structures of firms are endogenized, situations could arise where a less efficient firm obtains higher profit than its more efficient competitor. To the best of our knowledge, such a result has not appeared before in the literature of industrial economics.

We have carried out our analysis in a Cournot duopoly framework with linear demand. So far as general demand is concerned, we note that our results depend on the fact that elasticity is decreasing in price for linear demand, and our qualitative conclusions will not be altered under more general demand functions where elasticity is non-increasing in price. Other variations could be in regard to the number of firms in the market or the mode of competition (e.g., price competition instead of quantity competition and/or differentiated products instead of homogenous goods). Studying the extent to which our conclusions are robust to these variations is left for future research.

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