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Abstract
We propose both a monopoly and a duopoly model of a two-sided market. Both settings are fully comparable, as we impose a homogeneous good produced at zero costs without capacity constraints, as well as identical parameterization of market sizes. We determine the duopoly equilibrium and the monopoly optimum in terms of the parameters and obtain solutions with and without subsidization (prices below marginal cost) of one market side. We show that there exists a continuum of economically plausible parameter sets for which duopoly equilibrium prices exceed optimal monopoly prices and one with no observable price effect of competition, i.e. one where optimum and equilibrium prices become equal. Despite the fact that virtually everything except for the number of platform operators is identical in the latter situations, total demand on both market sides in the duopoly market exceeds total demand in the monopoly market. Furthermore, even though there is no observable price effect, there is still a competitive effect in so far that total profits in the duopoly equilibrium are strictly smaller than monopoly profits. The relationship of total welfare is ambiguous in subsidization cases, while it is strictly greater in duopoly, if no subsidization takes place. Our results sharply contradict economic intuition and common economic knowledge from one-sided markets.

Keywords: two-sided markets, platform competition, price-concentration relationship, welfare analysis

JEL Classification: D42, D43, K20, L12, L13, L51

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1. Introduction and Literature Review

The well-known literature on network effects studies markets, in which firms face a group of more or less homogeneous customers with externalities emerging “within” this group. On the other hand, there are many markets, in which firms face two or more distinct and distinguishable customer groups with externalities emerging “in between” these groups. In media economics this setting describes the relation of the media provider, media consumers, and advertisers. There it is termed “circulation industry” (Chaudhri (1998)) or “dual product market” (Picard (1989)). In Industrial Organization, this setting became prominent as “two-sided market” following the seminal analyses by Rochet and Tirole (2003) and Caillaud and Jullien (2003).

Rochet and Tirole (2003) instance credit cards as an example of a two-sided market: Consumers and retailers constitute two distinct and distinguishable groups of customers to a credit card provider. The number of retailers who wish to be connected to a specific credit card network is affected by the number of consumers who wish to pay using this credit card. Vice versa, the number of consumers who apply for a specific credit card is affected by the number of retailers who accept it. The externality is positive for both market sides in this example: Increasing demand by one group increases demand by the other group.

The features of two-sided markets are, however, hard to generalize. For instance, two-sided markets might as well be characterized by negative externalities or externalities might be positive for one side and negative for the other side, as will be discussed for the example of advertising further below.

Furthermore, credit cards are a club good to both market sides, which means that specific consumers and retailers can be excluded from the service, but there is no rivalry in consumption of a credit card service. Obviously, other goods feature different characteristics, that is, they might be either rival or non-rival in consumption, and they might be excludable or non-excludable or their characteristics might depend on the market side. For instance free-to-air radio broadcasts are a public good (non-rival, non-excludable) to the audience, but since advertising slots are naturally limited by the available air time, advertising slots are a private good (rival, excludable) to advertisers.

Third, credit cards are goods that allow for joint consumption also known as “multi-homing”. That is, a consumer might well own more than one credit card at the same time (and even use more than one card for the same transaction by splitting the amount invoiced), and retailers might accept more than one card. Other two-sided markets require one or both market sides to decide for exactly one provider (or, of course, none at all). For instance, a moviegoer has to decide for exactly one cinema on a Saturday night, but a firm can place advertisements in more than one movie theater.

Fourth, credit cards are a homogeneous product. That is, differences in utility from using the one or the other credit card service only stem from price differences and the two-sided market effect, i.e. the diffusion of a specific service on the other market side. Other two-sided markets allow for product differentiation, e.g. in the newspaper market, there might be differentiation according to political views or intellectual level of the target group.

Finally, there are differences in strategic interaction in terms of timing (simultaneous or sequential action) and the strategic variables (price or quantity) used by the players in a two-sided market game.

Because of this, the literature on two-sided markets tailors models around specific examples. Armstrong's (2006) study provides a variety of constellations, including those studied by Caillaud and Jullien (2003) and Rochet and Tirole (2003). Armstrong (2006) derives optimal pricing rules for monopoly and duopoly markets in terms of elasticities and provides some applications, e.g. to advertising. He finds that there is no difference in advertising prices between monopolies and oligopolies, given media consumers must single-home and advertisers can multi-home - a constellation he calls “competetive bottlenecks”. He argues that competition only emerges on the market for media consumers: Media providers compete for consumers, since advertisers’ demand depends on the number of consumers that are exposed to the advertisement, and consumers need to single-home. However, media providers are still monopolists when providing their consumers’ attention to the advertiser, because a specific consumer can ony be reached by advertising with the provider this consumer chose.

Nilssen and Sørgard (2001) and Gabszewicz et al. (1999) study TV advertising as a sequential two-sided market game in which advertising revenues finance programming investments of TV stations. Nilssen and Sørgard (2001) show -in contrast to Armstrong's (2006) result- that in this case the advertising price in a symmetric duopoly is always lower than in a monopoly. Although the price is lower in duopoly, the total amount of advertising might be lower than in monopoly. This is because each duopolist’s programming investment is lower than the monopolist’s one. Lower programming investment implies lower program quality which in turn implies a lower number of viewers. This lower number of viewers might offset the increasing demand for advertising caused by the lower duopoly advertising price, and cause a decrease in the total amount of advertising.

While there seems to be broad consensus in that the externality of a large audience on advertising demand is positive, there is some dissent about the effect in the other direction. Theoretical models, e.g. those of Nilssen and Sørgard (2001) and Gabszewicz et al. (1999) mentioned above, usually assume that media consumers wish to consume the media content only, and are "coerced" to consume advertising as well. Empirical results support this "ad-aversion" assumption for television viewers (see e.g. Danaher (1995), Wilbur (2007)), but reject it for print media, where readers' attitude towards advertising seems to be driven by the informative value of the advertisement (Kaiser and Song (2009), Rysman (2004)). This example supports our earlier statement that it is hard to generalize two-sided market models, and the approach of the literature to tailor models to specific examples.

Chaudhri (1998) and Dewenter (2006) develop two-sided market models of a newspaper monopoly. Comparing the optimal monopoly advertising and consumer prices and the competitive market outcome, Chaudhri (1998) finds that the optimal monopoly consumer price might be even lower than the competitive one, because increased circulation on the monopoly market yields higher advertising revenue. However, Chaudhri (1998) does not model demand explicitly and ignores the feedback effect of circulation on advertising demand. The increase in advertising revenue in his analysis stems from the pricing model (per-contact-pricing) for advertising. Advertising demand itself is not affected by circulation.
Häckner and Nyberg (2008), and similarly Anderson and Gabszewicz (2006), also having newspaper markets in mind, propose a two-sided market model with endogeneous market structure (duopoly or monopoly). In a simultaneous game, they examine conditions for symmetric and asymmetric duopoly equilibria as well as conditions for a natural monopoly. Product differentiation is a crucial feature of their model that drives the resulting market structure. Another basic assumption through most of their study is a positive externality of advertising on consumer demand, i.e. they assume that readers like advertising. In case of a negative effect of advertising on consumers, however, they find that only symmetric equilibria can exist.

The aim of the present paper is to compare prices on two-sided monopoly and duopoly markets in more detail than it was done in previous literature. Unlike previous literature, we neither allow for product differentiation, since it would make us compare apples and oranges, nor do we impose structural differences in terms of costs or capacity constraints that might favor the one or the other market structure. We also sacrifice generality by restricting our model to a “competitive bottleneck” scenario and by introducing explicit utility functions for the agents on both market sides, which enables us to parameterize our model and to obtain results in terms of the parameters. We use these results to study the behavior of the outcomes in monopoly and duopoly depending on the parameters. We are especially interested in the question of whether there are economically plausible parameter sets that equalize prices in monopoly and duopoly or yield even higher prices in duopoly than in monopoly, which would sharply contradict economic intuition and common economic knowledge from one-sided markets.

The question of price changes with regard to market structure becomes relevant when policy makers consider subsidies in order to attract new entrants to given monopoly markets. Reversely, antitrust authorities need to assess the impact of mergers on prices. A positive correlation of price and market concentration is also the fundamental assumption of the empirical price-concentration literature that aims at measuring the price effect of market concentration.

The paper generally contributes to the literature on two-sided markets by explicitly comparing monopoly and duopoly outcomes on a competitive bottleneck two-sided private good market. It is therefore complementary to Nilssen and Sørgard (2001) who study a two-sided market where the good is public to one side, but private to the other side of the market, Weyl (2006), who studies pricing in the Rochet and Tirole (2003) framework, and a refinement of Armstrong's (2006) “competitive bottleneck” scenario.

To help the reader grasp the economic intuition behind the model, we will follow the common habit of the literature and refer from time to time to a specific example, namely to the one of mainstream movie theaters. In contrast to the aforementioned TV broadcasts, movie theaters provide a private good to both advertisers and moviegoers, since individuals on both market sides can easily be excluded. A consumer feels rivalry in consumption at least if her favorite seat is already taken. Advertisers feel rivalry, because consumer attention will decrease the more advertisements are shown. The example fits the competitive bottleneck scenario, because by nature consumers can only visit one cinema at the same time, hence they must single-home, while advertisers can place advertisements in more than one cinema, hence they can multi-home. Mainstream movie theaters are usually large multiplex facilities that feature the latest projection and audio technology and the same portfolio of the latest movies.

Therefore, in the duopoly case, an average consumer will have no intrinsic preference for the one or the other cinema: both facilities are homogeneous.

The paper is constructed as follows: First we develop a monopoly model and derive the monopolist's optimal pricing policy. Second, we suggest a model of duopolistic competition that is founded on the same assumptions as the monopoly model, thus being fully comparable to the monopoly case. In Section 4, we compare the equilibrium outcome in the duopoly case and the monopolist's optimum, and highlight some implications of our findings in Section 5.

2. The Monopoly Model

Consider a two-sided market for a consumption good that is offered in combination with advertising. To foster intuition and readability, we will label customers on the one market side "consumers" and customers on the other market side "advertisers". In this section, we assume that the market is served by a monopolistic platform operator. Similar to Anderson and Gabszewicz (2006), we assume that consumers are homogeneous, except for their preference for the good (e.g. "movie theater experience"). Let the individual net utility function be additive-separable and given by

$$(1)\quad U_c(q_c) = \theta \cdot q_c - e \cdot d_a \cdot q_c - p_c \cdot q_c, \quad q_c \in \{0,1\},$$

where p_c is the price for one unit of the consumption good, while θ is the taste parameter or marginal willingness to pay, q_c is the quantity, and e is a parameter for the influence of advertising on the individual's utility. Heterogeneous preferences for the good are reflected by θ, which is assumed to be continuously distributed on the interval $[0, ms_c]$, where $ms_c > 0$ determines the market size for the considered goods market. We limit q_c to the values 0 and 1, so that the individual's decision problem is reduced to whether or not to consume a single unit of the good (e.g. whether to go to the movies or not). Obviously, an individual demands the good, if its net utility of doing so is greater than the net utility of refraining from consumption, that is if

$$U_c(1) \geq U_c(0) = 0 \Leftrightarrow \theta \geq e \cdot d_a + p_c$$

holds. Therefore, we obtain

$$(2)\quad d_c(p_c, d_a) = \int_{e \cdot d_a + p_c}^{ms_c} 1 \cdot d\theta = ms_c - e \cdot d_a - p_c$$

as the aggregated consumer demand function on the market.

Similar to Armstrong (2006), firms are assumed to generate constant net profits from advertising. For simplicity, we assume advertisements to be standardized, so that firms only decide whether or not to place an advertisement. Therefore a single firm's advertising demand q_a is either 0 or 1. The firm's net profit is given by

$$(3)\ \ \Pi_a(q_a) = \mu \cdot q_a - p_a \cdot d_c \cdot q_a, \quad q_a \in \{0,1\},$$

where p_a is the per-contact advertising price, so that the firm has to pay $p_a \cdot d_c$ to place an advertisement. μ is the parameter that describes the gross benefit of advertising. Firms are assumed to be heterogeneous with respect to μ, and μ is assumed to be continuously distributed on $[0, ms_a \cdot d_c]$. The expression $ms_a \cdot d_c$, with $ms_a > 0$ determines the size of the advertising market. Note that the net profit to be gained from advertising and therefore the size of the advertising market, depends on consumer demand. The economic intuition is straightforward: The higher the demand for the good, the more consumers will be exposed to the advertisement, the more profitable advertising becomes to a firm. Firms are willing to advertise, if their net profit from doing so is positive, that is if

$$\Pi_a(1) \geq \Pi_a(0) = 0 \Leftrightarrow \mu \geq p_a \cdot d_c .$$

Hence, total advertising demand is given by

$$(4)\ \ d_a(p_a, d_c) = \int_{p_a \cdot d_a}^{ms_a \cdot d_c} 1 \cdot d\mu = ms_a \cdot d_c - p_a \cdot d_c .$$

This specific functional form assures that $d_a(p_a,0) = 0$, which implies that there is no demand for advertising if consumer demand is equal to zero.

Solving equations (2) and (4) for p_c and p_a, respectively, yields the inverse demand functions. We assume fixed and variable costs to be zero and capacity constraints to be non-binding. Thus, the monopolist's optimization problem is

$$\max_{d_c, d_a} \Pi_M = d_c \cdot (ms_c - e \cdot d_a - d_c) + d_a \cdot d_c \cdot \left(ms_a - \frac{d_a}{d_c} \right),$$

yielding the first order conditions

$$d_c = \frac{1}{2} \cdot (ms_c - d_a \cdot \eta), \text{ where } \eta = e - ms_a$$

$$d_a = -\frac{1}{2} \cdot d_c \cdot \eta$$

Therefore, the optimal monopoly solution is

$$(5)\ \ d_c^M = \frac{2 \cdot ms_c}{4 - \eta^2} \qquad\qquad (6)\ \ d_a^M = \frac{\eta \cdot ms_c}{\eta^2 - 4}$$

$$(7)\ p_c^M = \frac{(ms_a \cdot \eta + 2)\cdot ms_c}{4-\eta^2} \qquad (8)\ p_a^M = \frac{e + ms_a}{2},$$

and yields a monopoly profit of

$$(9)\ \Pi^M = \frac{ms_c^{\ 2}}{4+\eta^2}.$$

An economically plausible solution requires that demands and profit are nonnegative, i.e. $d_c^M, d_a^M, \Pi^M \geq 0$. Note that optimal pricing on two-sided markets might involve prices below marginal cost ("subsidization") for one market side. To determine economically plausible parameter ranges, we consider three cases:[1]

If consumers are ad-averse (e > 0), $(\mathrm{ms_a}, \mathrm{e}) \in \{(\mathrm{ms_a} > 0, \max(0, \mathrm{ms_a} - 2) < \mathrm{e} \leq \mathrm{ms_a})\}$ yields plausible solutions. For ad-neutral consumers (e = 0), $0 < \mathrm{ms_a} < 2$ is required and in case consumers are ad-likers (e < 0), the monopoly model yields plausible results for $(\mathrm{ms_a}, \mathrm{e}) \in \{(0 < \mathrm{ms_a} < 2, \mathrm{ms_a} - 2 < \mathrm{e} < 0)\}$.

3. A Model of Duopolistic Competition

In this section, we develop a model of duopolistic competition in order to identify competitive effects on the market that has been presented in the former section. Since our paper focuses on the comparison of monopoly and duopoly markets, all assumptions of Section 2 remain, except that we now assume the market to be served by two identical platforms, denoted $i = 1,2$.

The consumption good offered by both platforms is assumed to be perfectly homogenous (e.g. two multiplexes offering the same menu of movies in direct proximity). Just like the monopolist, the duopolists are assumed to produce without variable and fixed costs. Consumers are assumend to be the same utility-maximizing individuals they were in the previous section. Additionally, we assume that consumers are required to single-home, that is, if they buy, they will have to decide for one and only one platform to buy from (e.g. a moviegoer can only be in one cinema at the same time). Obviously, consumers will prefer the platform that offers most net utility. If consumers' net utility is equal on both platforms, aggregate demand is assumed to be equally shared among the two operators. Thus, using

[1] Remember that we restricted $ms_c > 0$. Since throughout (5) - (9) ms_c only appears -if at all- as a factor in the nominator, and therefore only has a scaling function, we will ignore $\mathrm{ms_c}$ in the parameter sets to simplify notation. We will apply this simplification throughout the paper.

equation (1) for $q_c = 1$ and equation (2), the consumer demand function platform operator i faces is

$$(10)\ d_c^i\left(p_c^i, d_a^i\right) = \begin{cases} 0 \ for\ \theta - e \cdot d_a^i - p_c^i < \theta - e \cdot d_a^j - p_c^j \\ \dfrac{ms_c - e \cdot d_a^i - p_c^i}{2} \ for\ \theta - e \cdot d_a^i - p_c^i = \theta - e \cdot d_a^j - p_c^j \\ ms_c - e \cdot d_a^i - p_c^i \ for\ \theta - e \cdot d_a^i - p_c^i > \theta - e \cdot d_a^j - p_c^j \end{cases} \qquad i, j = 1,2\ , i \neq j$$

Unlike consumers, advertisers are allowed to multi-home, which implies that they can place a single unit of advertising on one platform only or on both platforms simultaneously. Therefore, the advertisers' decision problem only depends on the advertising price and the consumer demand of the corresponding platform. In other words, platform i's advertising demand does not directly depend on platform j's behavior.[2] Using equation (4), advertising demand of platform i is therefore given by

$$(11)\ d_a^i\left(p_a^i, d_c^i\right) = ms_a \cdot d_c^i - p_a^i \cdot d_c^i = d_c^i \cdot \left(ms_a - p_a^i\right),$$

which is analog to Section 2.

We assume that both platforms compete in a Bertrand-type pricing game, simultaneously choosing prices p_c^i and p_a^i . Since the platforms are perfectly identical, we focus on symmetric equilibria.[3] Generally, a symmetric solution for $i,j = 1,2$, $i \neq j$ is characterized by

$$p_c^i = p_c^j = p_c^s \text{ and } p_a^i = p_a^j = p_a^s,$$

which implies

$$(12)\ d_a^i = d_a^j = d_a^s = \left(msa - p_a^s\right) \cdot \frac{d_c^s}{2},$$

where d_a^s is the advertising demand faced by one platform operator, while d_c^s is the aggregate consumer demand in the market, equally shared among the operators, given any symmetric solution p_c^s and p_a^s. In order to calculate d_c^s, we have to take into account that single-homing consumers are interested in the amount of advertising on *each platform*, which is d_a^s. The *total number* of ads $d_a^i + d_a^j = 2 \cdot d_a^s$ is not relevant for consumers' decision making. Thus, using equations (2) and (12), aggregate consumer demand is given by

$$(13)\ d_c^s = ms_c - e \cdot d_a^s - p_c^s = ms_c - e \cdot \left(ms_a - p_a^s\right) \cdot \frac{d_c^s}{2} - p_c^s \Leftrightarrow d_c^s = \frac{ms_c - p_c^s}{1 + \frac{1}{2} \cdot e \cdot \left(ms_a - p_a^s\right)},$$

[2] It is, of course, indirectly dependent of j's behavior, because d_a^i depends on d_c^i, and by (10), d_c^i depends on d_c^j.
[3] See Nilssen and Sørgard (2001) for a model with heterogeneous platforms and asymmetric equilibria.

so that d_a^s can be expressed as

$$(14)\ d_a^s = \left(ms_a - p_a^s\right) \cdot \frac{ms_c - p_c^s}{2 + e \cdot \left(ms_a - p_a^s\right)}.$$

Therefore, for any given p_c^s and p_a^s, firm i's profit in the symmetry case is

$$(15)\ \ \Pi_i^s\left(p_c^s, p_a^s\right) = \frac{ms_c - p_c^s}{2 + e \cdot \left(ms_a - p_a^s\right)} \cdot \left(p_c^s + p_a^s\left(msa - p_a^s\right) \cdot \frac{ms_c - p_c^s}{2 + e \cdot \left(ms_a - p_a^s\right)} \right).$$

Suppose that the candidate equilibrium (p_c^*, p_a^*) is characterized by

$$p_c^{i*} = p_c^{j*} = p_c^* \qquad p_a^{i*} = p_a^{j*} = p_a^*.$$

In this case, platform operator i's deviation strategies can be defined as

$$p_c^{i/low}, p_a^{i/low} \ \ for\ \theta - e \cdot d_a^{i/low} - p_c^{i/low} < \theta - e \cdot d_a^{j*}\left(p_c^{j*}, p_a^{j*}\right) - p_c^{j*}$$

$$p_c^{i/equal}, p_a^{i/equal} \ \ for\ \theta - e \cdot d_a^{i/equal} - p_c^{i/equal} = \theta - e \cdot d_a^{j*}\left(p_c^{j*}, p_a^{j*}\right) - p_c^{j*},$$

$$p_c^{i/high}, p_a^{i/high} \ \ for\ \theta - e \cdot d_a^{i/high} - p_c^{i/high} > \theta - e \cdot d_a^{j*}\left(p_c^{j*}, p_a^{j*}\right) - p_c^{j*}$$

where $(p_c^{i/low}, p_a^{i/low})$ is a strategy that implies lower consumer utility, $(p_c^{i/equal}, p_a^{i/lequal})$ is a strategy that implies the same consumer utility, and $(p_c^{i/high}, p_a^{i/high})$ is a strategy that implies higher consumer utility than strategy (p_c^*, p_a^*). Note that platform operator i might deviate by changing one or both prices, and that the operator might alter both price in the same direction or in opposing directions. Therefore the indices *low*, *equal*, and *high* do not imply prices in the deviation strategy being higher, equal or lower than the equilibrium candidate prices.

The well-known condition for a Nash equilibrium is that operator i cannot deviate profitably, which means that (p_c^*, p_a^*) is an equilibrium, if and only if

$$\Pi_i^s\left(\left(p_c^{i*}, p_a^{i*}\right), \left(p_c^{j*}, p_a^{j*}\right)\right) \geq \Pi_i\left(\left(p_c^{i/low}, p_a^{i/low}\right), \left(p_c^{j*}, p_a^{j*}\right)\right)$$

$$\Pi_i^s\left(\left(p_c^{i*}, p_a^{i*}\right), \left(p_c^{j*}, p_a^{j*}\right)\right) \geq \Pi_i\left(\left(p_c^{i/equal}, p_a^{i/equal}\right), \left(p_c^{j*}, p_a^{j*}\right)\right),$$

$$\Pi_i^s\left(\left(p_c^{i*}, p_a^{i*}\right), \left(p_c^{j*}, p_a^{j*}\right)\right) \geq \Pi_i\left(\left(p_c^{i/high}, p_a^{i/high}\right), \left(p_c^{j*}, p_a^{j*}\right)\right)$$

Thus, in order to find equilibria, we will have to analyze these cases separately. We will do this, using the following propositions:

Proposition 1: If platform operator i deviates by choosing any $(p_c^{i/low}, p_a^{i/low})$, the resulting profit is always $\Pi_i^{low}(.) = 0$.

Proof: Equation (10) implies that $d_c^i(.) = 0$, which means that demand for platform i is taking the value zero as all consumers will decide to use the rival platform j. In addition, using equation (11) we obtain $d_a^i(.) = 0$, because advertisers are not willing to place an ad on platform i, when there are no consumers. Thus, for any $(p_c^{i/low}, p_a^{i/low})$ i's profit is zero.

As economic intuition suggests, it is not profitable for a platform operator to deviate by offering less consumer utility than the rival platform. Since any equilibrium with $\Pi_i(.) < 0$ is not economically plausible, there will never be an incentive for operator i to charge $(p_c^{i/low}, p_a^{i/low})$. Therefore, this strategy can be neglected for further analysis of candidate equilbria.

Proposition 2: Suppose platform operator i is maximizing her profit, while offering the same utility as operator j by charging any $(p_c^{i/equal}, p_a^{i/equal})$. Then, it is always profit-maximizing to charge the monopoly advertising price.

Proof: Since both platforms offer equal consumer utility, we know that

$$(16)\ \theta - e\cdot d_a^{i/equal} - p_c^{i/equal} = \theta - e\cdot d_a^{j*} - p_c^{j*} \Leftrightarrow -e\cdot d_a^{i/equal} - p_c^{i/equal} = -e\cdot d_a^{j*} - p_c^{j*} \Leftrightarrow$$

$$p_c^{i/equal} = e\cdot d_a^{j*} - e\cdot d_a^{i/equal} + p_c^{j*}$$

must hold, which also implies that i's consumer demand, denoted by $d_c^{i/equal}$, is fixed. Solving (11) for the advertising price yields the inverse advertising demand as

$$p_a^{i/equal} = ms_a - \frac{d_a^{i/equal}}{d_c^{i/equal}}.$$

Therefore, operator i's (constrained) maximization problem is

$$\max \Pi_i\left(d_a^{i/equal}\right) = \left(e\cdot d_a^{j*} - e\cdot d_a^{i/equal} + p_c^{j*}\right)\cdot d_c^{i/equal} + \left(ms_a - \frac{d_a^{i/equal}}{d_c^{i/equal}}\right)\cdot d_c^{i/equal}\cdot d_a^{i/equal},$$

yielding

$$(17)\ d_a^{i/equal*} = \frac{-d_c^{i/equal}\cdot\eta}{2} \Leftrightarrow p_a^{i/equal*} = \frac{ms_a + e}{2},$$

and the profit-maximizing advertising price is equal to the optimal monopoly advertising price (8).

Proposition 2 implies a very important result: In any symmetric situation, operator i's deviation profit implied by (17) is at least as great as the profit in the symmetric situation (proof: see Appendix 1). Therefore, in any symmetric situation there is an incentive to charge the monopoly advertising price. Since the platforms are identical, we can expect that $d_a^{j*} = d_a^{i/equal*}$, so that (16) simplifies to $p_c^{i/equal} = p_c^{j*}$. Thus, we can tentatively conclude that a symmetric equilibrium requires

$$(18)\ p_a^{i*} = p_a^{j*} = \frac{ms_a + e}{2}$$

for any $p_c^{i*} = p_c^{j*}$. However, at this stage of our analysis the equilibrium level of the consumer price remains unspecified.

Proposition 3: When offering consumers more net utility than her rival firm, thus charging ($p_c^{i/high}$, $p_a^{i/high}$), platform operator i's profit-maximizing strategy is either (i) the monopoly solution or (ii) $d_c^{i/high} = ms_c - p_c^{j*}$.

Proof: In order to attract all consumers, the constraint

$$(19)\ \theta - e\cdot d_a^{i/high} - p_c^{i/high} > \theta - e\cdot d_a^{j*} - p_c^{j*} \quad \Leftrightarrow \quad -e\cdot d_a^{i/high} - p_c^{i/high} > -e\cdot d_a^{j*} - p_c^{j*}$$

must be satisfied. From equation (10) we know that consumer demand for operator i is

$$(20)\ d_c^{i/high} = ms_c - e\cdot d_a^{i/high} - p_c^{i/high} \quad \Leftrightarrow \quad p_c^{i/high} = ms_c - e\cdot d_a^{i/high} - d_c^{i/high}.$$

As long as (19) is satisfied, rival platform j's consumer demand is equal to zero, which implies that j's advertising demand is also zero. Therefore, we assume that consumers anticipate that $d_a^{j*} = 0$, so that (19) simplifies to

$$(21)\ -e\cdot d_a^{i/high} - \left(ms_c - e\cdot d_a^{i/high} - d_c^{i/high}\right) > -p_c^{j*} \quad \Leftrightarrow \quad d_c^{i/high} > ms_c - p_c^{j*}.$$

The corresponding (Kuhn-Tucker-) optimization problem for operator i can be expressed as

$$\max \Pi_i\left(d_c^{i/high}, d_a^{i/high}\right) = d_c^{i/high}\cdot\left(ms_c - e\cdot d_a^{i/high} - d_c^{i/high}\right) + \left(ms_a - \frac{d_a^{i/high}}{d_c^{i/high}}\right) d_c^{i/high}\cdot d_a^{i/high}$$
$$+\lambda\left(d_c^{i/high} - ms_c + p_c^{j*}\right),$$

which yields the monopoly solution of Section 2 if (21) is not binding, i.e. $\lambda = 0$. In case (21) is binding ($\lambda > 0$)[4], operator i will choose the slightest possible $d_c^{i/high}$ without violating the constraint. The resulting solution is approximately

$$(22)\ d_c^{i/high*} = ms_c - p_c^{j*},$$

while $p_a^{i/high*}$ still matches the monopoly solution and $d_a^{i/high*}$ becomes

$$d_a^{i/high*} = \frac{\left(p_c^{j*} - ms_c\right)\cdot\eta}{2}.$$

Using $d_a^{i/high*}$ as well as equations (20) and (22), the resulting consumer price is desribed by

$$p_c^{i/high*} = \frac{e\cdot\left(ms_c - p_c^{j*}\right)\cdot\eta}{2} + p_c^{j*}.$$

Therefore, operator i's deviation profit can be expressed as

[4] It can be shown that there is no equilibrium in the non-binding case as deviation to the monopoly solution would always be profitable, and the monopoly solution is not an equilibrium.

$$(23)\ \Pi_i^{high}\left(p_c^{j*}\right)=\frac{1}{4}\cdot\left(ms_c-p_c^{j*}\right)\cdot\left[\eta^2\cdot\left(ms_c-p_c^{j*}\right)+4\cdot p_c^{j*}\right].$$

The symmetric equilibrium is characterized by the results of Propositions 2 and 3. As stated before, Proposition 2 implies that both platform operators charge the monopoly advertising price in equilibrium, given any consumer price. The equilibrium consumer price can be obtained by the results of Proposition 3, because there is no incentive for deviation, if the deviation profit, given by (23), equals the platform's profit in the symmetry case. Therefore, in equilibrium

$$\Pi_i^{high}\left(p_c^*\right)=\Pi_i^s\left(p_c^*,p_a^*\right)$$

must hold. Given (18) and using equations (15) and (23), the equilibrium consumer price[5] is

$$(24)\ p_c^*=\frac{\eta\cdot\left[e\cdot\left(20+\eta^2\cdot(e\cdot\eta-8)\right)-12\cdot ms_a\right]\cdot ms_c}{\zeta},\text{ where}$$

$$\zeta=\eta\cdot\left[e\cdot\left(44+\eta\left(e\cdot\left(\eta^2-12\right)+8\cdot ms_a\right)\right)-12\cdot ms_a\right]-32.$$

The equilibrium is therefore characterized by (17) and (24), yielding

$$(25)\ d_a^*=d_a^{i*}=d_a^{j*}=\frac{4\cdot ms_c\cdot\eta\cdot(2-e\cdot\eta)}{\zeta}$$

$$(26)\ d_c^*=d_c^{i*}=d_c^{j*}=\frac{8\cdot \mathrm{ms_c}\cdot(e\cdot\eta-2)}{\zeta}$$

$$(27)\ \Pi^*=\Pi^{i*}=\Pi^{j*}=\frac{8\cdot ms_c^2\cdot\eta\cdot\left(e\cdot\left(\eta^2-6\right)+2\cdot ms_a\right)(e\cdot\eta-4)(e\cdot\eta-2)}{\zeta^2},$$

where an economically plausible symmetric equilibrium solution obviously requires

$$\Pi_i^s\left(p_c^*,p_a^*\right),d_c^s\left(p_c^*,p_a^*\right),d_a^s\left(p_c^*,p_a^*\right)\geq 0.$$

To obtain economically plausible results, we need to restrict the parameter values as follows: In case of ad-aversion ($e > 0$), economically plausible values result for

$$\left(ms_a,e\right)\in\left\{\left(ms_a>0,\alpha\leq e\leq ms_a\right)\right\},\text{ where}$$

$$\alpha=\mathrm{R}_2\text{ of }x^3+2\cdot\mathrm{ms_a}\cdot\left(1-\mathrm{x}^2\right)+\left(\mathrm{ms_a}^2-6\right)\cdot\mathrm{x}.$$[6]

[5] There is another symmetric equilibrium for $p_c^* = ms_c$, which generates a corner solution and will be ignored for the rest of the paper.

[6] R_l, $l = 1,...,n$, denotes the l-th real-valued polynomial root in ascending order of the corresponding polynom of degree n.

For ad-likers ($e < 0$) various intervals yield plausible solutions. Since we are eventually interested in a comparison of monopoly and duopoly, we will only present the parameters that are compatible with the monopoly model as well:[7]

$$(ms_a, e) \in \left\{ \left(0 < ms_a < 1, ms_a - 2 < e < \frac{ms_a - \sqrt{ms_a^{\ 2} + 8}}{2} \right) \right\}.$$

In case of ad-neutrality ($e = 0$), profit is strictly negative, so that there is no economically plausible solution.

4. Analysis of the Model

In the previous sections we developed a competitive bottleneck two-sided market model and determined the monopolist's optimum as well as the duopolists' equilibrium outcomes. In this section we are going to study these outcomes more deeply. We will specifically focus on the comparison of monopoly and duopoly in terms of prices, quantities, and welfare. Furthermore, we will focus rather on those cases that are counterintuitive or contrary to common economic knowledge from traditional one-sided markets, i.e. cases in which equilibrium duopoly prices equal or even exceed optimal monopoly prices.

We restrict parameter sets to those sets that yield plausible values for both models simultaneously. This rules out the case of ad-neutrality, because there are no plausible parameter sets in this case for the duopoly model. It is easy to see that the parameter restrictions necessary for the duopoly model are tighter than the ones for the monopoly model. Therefore, the analysis of this section is limited to parameter sets satisfying

$$(ms_a, e) \in \{(ms_a > 0, \alpha \le e \le ms_a)\}$$

in the case of ad-aversion, and

$$(ms_a, e) \in \left\{ \left(0 < ms_a < 1, ms_a - 2 < e < \frac{ms_a - \sqrt{ms_a^{\ 2} + 8}}{2} \right) \right\}$$

in the case of ad-liking.

The first question we are interested in is, if there exist economically plausible parameter triples (ms_a, ms_c, e) that equalize consumer prices in monopoly and duopoly. Remember that by Proposition 2, the equilibrium price for advertising is equal to the monopoly price given by (8). In other words, we are now searching for cases in which there is no observable price

[7] A full list of parameter ranges for this case is available upon request from the authors.

effect of competition. To do so, we equate (7) with (24) and solve for economically plausible triples (ms_a, ms_c, e) that satisfy the equation. Since ms_c already turned out to be a nonnegative scaling factor only, we will suppress it in the notation, that is, we will give tuples (ms_a, e) only.

In the case of ad-liking consumers, the only case in which consumer prices become equal is the corner solution $p_c = ms_c$. Therefore we will not further discuss the case of ad-liking, and continue with ad-averse consumers. In this case there are two types of valid tuples (ms_a, e) for which consumer price equality holds. First there are solutions that yield negative consumer prices. We will call these solutions "subsidization solutions", since the platform operator charges consumer prices below marginal cost in order to increase consumer demand, which in turn will attract more advertisers. Note that we assumed marginal costs to be zero. Therefore a negative price in our model can generally be interpreted as a price below marginal cost. The second type of solutions yields positive prices towards both market sides. (28) gives parameter sets that yield equal prices in monopoly and duopoly.

$$\text{(28)} \quad (ms_a, e) \in \left\{\left(2\sqrt{2}, \sqrt{2}\right), \left(\beta, \sqrt{2} < e < \gamma\right), \left(\delta, e \geq \gamma\right)\right\}, \text{ where}$$

$\beta = \mathrm{R}_1$ of $x^4\mathrm{e}^2 + 4\mathrm{x}^3\mathrm{e}\,(1 - \mathrm{e}^2) + 2\mathrm{x}^2(3\mathrm{e}^4 - 8\mathrm{e}^2 - 2) + 4\mathrm{x}\,(-\mathrm{e}^5 + 5\mathrm{e}^3 - 2\mathrm{e}) + \mathrm{e}^6 - 8\mathrm{e}^4 + 12\mathrm{e}^2 + 32$,

$\gamma = \mathrm{R}_4$ of $3\mathrm{x}^4 - 47\mathrm{x}^2 + 1$, and

$\delta = \mathrm{R}_3$ of $x^4\mathrm{e}^2 + 4\mathrm{x}^3\mathrm{e}\,(1 - \mathrm{e}^2) + 2\mathrm{x}^2(3\mathrm{e}^4 - 8\mathrm{e}^2 - 2) + 4\mathrm{x}\,(-\mathrm{e}^5 + 5\mathrm{e}^3 - 2\mathrm{e}) + \mathrm{e}^6 - 8\mathrm{e}^4 + 12\mathrm{e}^2 + 32$.

The first set $\left(2\sqrt{2}, \sqrt{2}\right)$ is a corner solution yielding

$$d_c^M = ms_c < d_c^* = \frac{4 \cdot ms_c}{3} \text{ and } p_c^M = p_c^* = -ms_c,$$

which implies zero profits in the duopoly case. The second set $\left(\beta, \sqrt{2} < e < \gamma\right)$ yields negative consumer prices, while the third set $\left(\delta, e \geq \gamma\right)$ yields positive prices.

Evaluated at any (ms_a, e) of (28), we see that

$$d_c^* > d_c^M,\ d_a^* < d_a^M < 2 \cdot d_a^*, \text{ and } 2 \cdot \Pi^* < \Pi^M.$$

Remember that d_a^* is advertising per duopolist, hence $2 \cdot d_a^*$ is total advertising on the duopoly market. (28) describes a situation, in which the mere fact that -all else equal- the market is served by two identical firms instead of one, causes an increase in total consumer demand. The reason for this is the two-sidedness of the market or more precisely, the effect of a decreasing amount of advertising on consumer utility. To illustrate the economics of this case, we do the following gedankenexperiment: Starting from some monopolistic optimum $d_c^M > 0$,

$d_a^M > 0$, we imagine that -all else equal- the monopolist is replaced by two identical, but independent platforms. In this case, total consumer demand d_c^M will be equally divided among the two platforms. As a consquence, advertising with one platform only reaches half of the consumers, which will reduce advertising demand per platform to some $d_a^n < d_a^M$. Since we assumed $e > 0$, the decrease in advertising exposure increases total consumer demand to some $d_c^n > d_c^M$. This increase in consumer demand in turn increases advertising demand. However, eventually the increase does not offset the decreasing effect, so that the total effect on advertising per platform is negative, from which follows that the total effect on consumer demand is positive.

The follow-up question to the results established above is, whether duopoly equilibrium consumer prices might even exceed optimal monopoly consumer prices. Again, we only need to consider the case of ad-aversion, since there is no parameter set that creates this effect if consumers are ad-likers. As before, there are subsidization solutions and solutions with positive prices. However, it is now also possible that the monopolist charges negative consumer prices, while the duopolists do not. In the case of subsidization, less negative prices in the duopoly case can be interpreted as a lower subsidization of consumers as compared to the monopoly case. (29) describes parameter sets given which the duopoly equilibrium consumer price exceeds the optimal monopoly consumer price.

$$(29)\ (ms_a, e) \in \left\{ \left(\beta < ms_a \leq \frac{e^2 - 1 + \sqrt{4 \cdot e^2 + 1}}{e}, \sqrt{2} < \mathrm{e} < \gamma \right), \left(\delta < ms_a \leq \frac{e^2 - 1 + \sqrt{4 \cdot e^2 + 1}}{e}, e \geq \gamma \right) \right\}$$

Evaluated at any (ms_a, e) of (29), we see that

$$2 \cdot \Pi^* < \Pi^M \text{ and } d_a^* < d_a^M ,$$

which is so far consistent with the results obtained from (28). However, we cannot draw general conclusions about the relationship of d_c^* and d_c^M, and of d_a^M and $2d_a^*$.

Refining (29) for those solutions that yield negative consumer prices, we obtain

$$(30)\ (ms_a, e) \in \left\{ \begin{array}{l} \left(\beta < ms_a \leq \dfrac{e^2 - 1 + \sqrt{4 \cdot e^2 + 1}}{e}, \sqrt{2} < \mathrm{e} \leq \kappa \right), \\ \left(\omega < ms_a \leq \dfrac{e^2 - 1 + \sqrt{4 \cdot e^2 + 1}}{e}, e > \kappa \right) \end{array} \right\}, \text{ where}$$

$\kappa = R_2$ of $x^6 - 10x^4 + 32x^2 - 36$, and

$\omega = R_1$ of $e^2x^3 + (8e - 3e^3)x^2 + (12 - 16e^2 + 3e^5)x - e^5 + 8e^3 - 20e$.

In case

$$\left(\beta < ms_a \le \frac{e^2 - 1 + \sqrt{4 \cdot e^2 + 1}}{e}, \sqrt{2} < \mathrm{e} \le \kappa \right),$$

$2 \cdot d_a^* > d_a^M$ and $d_c^* > d_c^M$ hold, while for

$$\left(\omega < ms_a \le \frac{e^2 - 1 + \sqrt{4 \cdot e^2 + 1}}{e}, e > \kappa \right)$$

the relationships of d_c^* and d_c^M and d_a^M and $2 \cdot d_a^*$ are still ambigous.

Parameter sets that yield a mixed case, in which a monopolist will subsidize while the duopolists will not, are described by (31).

$$(31)\ (ms_a, e) \in \left\{ \left(\frac{e + \sqrt{8 + e^2}}{2} < ms_a \le \delta, e > \kappa \right) \right\}$$

The results in case of (31) are analogue to the results of the second solution of (30): $d_a^* < d_a^M$ and $2 \cdot \Pi^* < \Pi^M$ hold, while the relationship of d_c^* and d_c^M as well as d_a^M and $2 \cdot d_a^*$ is ambigous.

Positive prices result, if

$$(32)\ (ms_a, e) \in \left\{ \left(\beta < ms_a < \frac{e + \sqrt{8 + e^2}}{2}, \kappa < e < \gamma \right), \left(\delta < ms_a < \frac{e + \sqrt{8 + e^2}}{2}, e > \beta \right) \right\}.$$

For both solutions of (32) it holds that $d_c^* > d_c^M$, $d_a^* < d_a^M < 2d_a^*$, and $2 \cdot \Pi^* < \Pi^M$.

Table 1 summarizes the results.

Case		$d_c^* \gtreqless d_c^M$	$d_a^* \gtreqless d_a^M$	$2d_a^* \gtreqless d_a^M$	$2 \cdot \Pi^* \gtreqless \Pi^M$
$p_c^* = p_c^M$	Subsidization	>	<	>	<
	Positive Prices	>	<	>	<
$p_c^* > p_c^M$	Subsidization 1	>	<	>	<
	Subsidization 2	$\gtreqless$	<	$\gtreqless$	<
	Mixed	$\gtreqless$	<	$\gtreqless$	<
	Positive Prices	>	<	>	<

Table 1: Relationship of quantities and profits, if consumer prices are equal or if the duopoly consumer price exceeds the monopoly consumer price.

Most notably, even though prices in the monopoly are not higher than in duopoly, monopoly profit still is strictly greater than total profit in the duopoly. This is in line with Chaudhri (1998), who compares monopoly and perfectly competitive newspaper markets, and finds that "(u)pon attaining monopoly control of a newspaper market, a proprietor, for reasonable parameter values […] opts to lower the price for her newspaper, which increases circulation, and hence, increased advertising revenue" (p.74).

Finally, we will study the welfare effects imposed by our model. Since we assumed zero costs of production, monopoly profit, resp. the sum of both provider's profits is equal to producer surplus. Traditional "consumer surplus" here is the sum of the surplus created on both market sides. In the monopoly case, the market side we labeled "consumers" realizes a benefit of

$$\int_0^{d_c^M} p_c^M\left(d_c, d_a^M\right) dd_c - d_c^M \cdot p_c^M = \frac{2 \cdot ms_c}{\left(\eta^2 - 4\right)^2},$$

where d_c^M is given by (5), d_a^M is given by (6), p_c^M is given by (7), and $p_c^M(\cdot)$ is the inverse of (2). In the duopoly case the consumer side realizes a surplus of

$$\int_0^{2d_c^i} p_c^d\left(d_c, d_a^*\right) dd_c - 2d_c^* \cdot p_c^* = \frac{128 \cdot (2 - e \cdot \eta)^2 \cdot {ms_c}^2}{\zeta^2},$$

where d_c^* is given by (26), d_a^* is given by (25), p_c^* is given by (24), and $p_c^d(\cdot)$ is the inverse of (13). Advertisers obtain a surplus of

$$\int_0^{d_a^M} p_a^M\left(d_a, d_c^M\right) dd_a - d_a^M \cdot p_a^M = \frac{\eta^2 \cdot ms_c}{4 \cdot \left(4 - \eta^2\right)},$$

where d_c^M is given by (5), d_a^M is given by (6), p_a^M is given by (8), and $p_a^M(\cdot)$ is the inverse of (4) in the monopoly case, and

$$2 \cdot \left(\int_0^{d_a^*} p_a^d\left(d_a, d_c^*\right) dd_a - d_a^* \cdot p_a^* \right) = \frac{2 \cdot \eta^2 \cdot (e \cdot \eta - 2) \cdot ms_c}{\zeta}$$

in the duopoly case, where d_c^* is given by (26), d_a^* is given by (25), $p_a^* = p_a^M$ is given by (8), and $p_a^d(\cdot)$ is the inverse of (12).

Welfare is given by the sum of producer, consumer, and advertiser surplus. Table 2 summarizes the relation of total welfare in monopoly optimum and duopoly equilibrium as well as the relations of the individual welfare components.

Case		Consumer Surplus	Advertiser Surplus	Producer Surplus	Total Welfare
$p_c^* = p_c^M$	Subsidization	$\gtreqless$	$\gtreqless$	$<$	$\gtreqless$
	Positive Prices	$>$	$>$	$<$	$>$
$p_c^* > p_c^M$	Subsidization 1	$>$	$>$	$<$	$\gtreqless$
	Subsidization 2	$\gtreqless$	$\gtreqless$	$<$	$\gtreqless$
	Mixed	$\gtreqless$	$\gtreqless$	$<$	$\gtreqless$
	Positive Prices	$>$	$>$	$<$	$>$
Left hand side = duopoly equilibrium; right hand side = monopoly optimum					

Table 2: Comparison of welfare effects for ad-averse consumers.

5. Conclusion and Implications

The results of the above analysis have implications on multiple fields of economic research. Keep in mind that we are studying a competitive bottleneck two-sided private goods market with perfect information, and that the analysis of our model is focussed on those parameter sets that are economically plausible *and* yield price effects contrary to economic intuition from one-sided markets, namely duopoly equilibrium prices being at least as high as monopoly prices. Furthermore it turned out that the cases we deem interesting require ad-averse consumers, that is a negative impact of demand from the multi-homing market side to the single-homing one.

As summarized in Table 1, there are cases in which total consumer demand in the duopoly equilibrium exceeds consumer demand in the monopoly optimum. On first sight, this result does not seem too surprising as it is a well-known relationship and a reason to foster competition. On second thought, the reason for higher equilibrium consumer demand in textbook oligopoly models is that oligopolistic competition yields lower consumer prices than in the monopoly case, and therefore demand increases. In our analysis, we explicitly study cases in which prices are equal in monopoly and duopoly. Hence, the two-sidedness of the market holds a demand enhancing effect, caused by the mere fact that total consumer demand is now equally splitt between two platforms instead of being served by one platform only.

The two-sidedness of the market also contributes an alternative explanation for a missing price effect of competition. On one-sided markets, there are generally two explanations for missing price effects: Either the monopolist does not or cannot make use of her monopoly power for some reason or the oligopolists collude implicitly or explicitly. On two-sided markets, there might just be no price effect of competition. As Table 1 suggests, there is a competitive effect that causes total profits in the duopoly equilibrium to be strictly lower than in the monopoly case, and total advertising demand to be lower in monopoly than in duopoly. We neither restrict the monopolist's optimization problem artificially nor do we hinder competition between the duopolists. Still and regardless of competition taking place, there is no observable price effect, given any of the parameter sets described by (28), and competition even increases prices given any of the parameter sets described by (29)-(32).

A price effect of competition, however, is the underlying assumption of empirical price-concentration studies. These studies presume that prices increase with the concentration of the market, and try to estimate the magnitude of this effect. Our results suggest that this relationship might be negative, given certain exogenous conditions as described by (29)-(32). Therefore, empirical analyses yielding a negative price-concentration effect, e.g. in the movie theater industry, do not necessarily suffer from methodological or technological mistakes. If, however, the exogenous conditions are in such a way as described by (28), then there is obviously no price effect of competition that could be measured. This implies that the absence of significant empirical results cannot be interpreted as lack of competition, unless it is verified that none of the parameter sets of (28) is present in the industry under consideration. In this light, it is also not sensible to follow Weyl (2006) and study the sum of the prices, which he calls the "price level", instead of the "price balance" that describes the relation of the prices of the two market sides.

Regarding the welfare effects of competition, we obtain ambigous results and need to distinguish our conclusions by the price level as in Table 2. In case of positive prices, i.e. in case of prices above marginal cost, total welfare is always higher in the duopoly equilibrium than in the monopoly optimum, even though consumer prices might be lower under monopoly. In case of subsidization, this is not necessarily true. Therefore, policy makers as well as regulators aiming at welfare maximization will have to obtain in-dept knowledge of the environment (i.e. the parameter set) they are facing before being able to act optimally. A brief glance at the prevailing price level or price balance will not suffice to make a sensible judgement. Unlike on common one-sided markets, fostering competition will not necessarily increase welfare. Similarly, merger control becomes more difficult. Under conditions of positive prices, mergers generally have a negative impact on total welfare. Under conditions of subsidized consumer prices, we cannot draw general conclusions. If, for some exogenous reason, a merger has to take place anyway, it will virtually always imply that one platform closes down (proof: see Appendix 2). This is in line with the regulator's objective of welfare maximization, because welfare increases, if the operator of the two merged platforms closes down one of them. It even holds for distributive objectives, i.e. consumer surplus, advertiser surplus, and producer surplus all increase, if the operator closes down one platform in case of a merger (see also Appendix 2).

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Appendix 1:

Proof: Proposition 2 implies that operator *i*'s deviation profit is at least as great as the profit in the corresponding symmetric situation:

Respecting that $p_c^{i/equal}$ is restricted by (16), the deviation profit departing from any symmetric situation (p_a^s, p_c^s) is given by

$$\Pi_i^{equal} = (e \cdot d_a^s - e \cdot d_a^{i/equal} + p_c^s) \cdot d_c^{i/equal} + \frac{ms_a + e}{2} \cdot d_a^{i/equal} \cdot d_c^{i/equal}.$$

Deviation from the symmetric situation will be take place, iff the profit from doing so is not lower than the profit in the symmetric situation, which is

$$\Pi_i^s\left(p_c^s, p_a^s\right) = d_c^s \cdot \left(p_c^s + p_a^s \cdot d_a^s\right).$$

Proposition 2 assumes a deviation strategy which yields the same consumer utility as the corresponding symmetrical situation. Therefore $d_c^{i/equal} = d_c^s$ is fixed.

Using (17), evaluating the inverse demand function obtained from (11) at $(d_a^s,\ d_c^s)$, and respecting that economic plausibility implies $d_c^s \geq 0$, it is easy to see that $\Pi_i^{equal} \geq \Pi_i^s$ becomes after some algebraic manipulation

$$d_c^{i/equal} \frac{(-\eta)^2}{4} \geq -d_a^s \cdot \eta - \frac{\left(d_a^s\right)^2}{d_c^{i/equal}}$$

Note that all expressions in this inequality are fixed, except for d_a^s on the right hand side. The expression on the right hand side has its maximum at

$$d_a^s = \frac{-\eta \cdot d_c^{i/equal}}{2}.$$

Evaluating the inequality at this maximum yields

$$d_c^{i/equal} \frac{(-\eta)^2}{4} \geq d_c^{i/equal} \frac{(-\eta)^2}{4},$$

which is always true. q.e.d.

Appendix 2:

Explicit collusion or merger on the duopoly market:

Assume, both platform operators are able and willing to cooperate in order to maximize joint profits. Given (10) and (11), the operators have two options: Either they equally divide consumer demand between their platforms or they close down one platform and create a monopoly. In the first case -for reasons to be seen soon, we label it "hypothetical collusion case"- the optimization problem is

$$\max_{d_c, d_a^i} \Pi_k = \sum_{i=1}^{2} d_a^i \cdot \frac{d_c}{2} \cdot p_a^i\left(d_a^i, \frac{d_c}{2}\right) + \frac{d_c}{2} \cdot p_c^i\left(d_a^i, \frac{d_c}{2}\right)$$

which yields a maximum profit of

$$\Pi_k = \frac{2 \cdot ms_c^2}{8 - \eta^2},$$

optimal quantities

$$d_c^k = \frac{4 \cdot ms_c}{\eta^2 - 8} \text{ and } d_a^{1,k} = d_a^{2,k} = d_a^k = \frac{\eta \cdot ms_c}{\eta^2 - 8},$$

and optimal prices

$$p_c^{1,k} = p_c^{2,k} = p_c^{i,k} = \frac{(\eta \cdot ms_a + 4) \cdot ms_c}{8 - \eta^2} \text{ and } p_a^{1,k} = p_a^{2,k} = p_a^{i,k} = p_a^M.$$

Given the nonnegativity constraints on ms_a and ms_c, and the parameter restrictions implied by the nonnegativity of Π_k and Π_M, the maximum hypothetical collusion profit never exceeds the optimal monopoly profit (9). Furthermore, there is only one corner solution, in which both profits become equal. Therefore, explicit collusion or merger always implies that the operators close down one platform to play the monopoly solution, except, if $(ms_a, e) = (e, e > 0)$, in which case the operators are indifferent between keeping both platforms open and closing down one.

Assume that for some exogenous reason it is not possible to close down one location. In case of a merger, this might be due to obligations of a regulating authority. To study the welfare effects in this case, we compute hypothetical consumer surplus as

$$\int_0^{d_c^k} p_c^k\left(d_c, d_a^k\right) dd_c - d_c^k \cdot p_c^k = \frac{8 \cdot {ms_c}^2}{\left(\eta^2 - 8\right)^2}, \text{ where}$$

$$p_c^1 = p_c^2 = p_c^k = ms_c - e \cdot d_a - d_c.$$

Hypothetical advertiser surplus is

$$2 \cdot \left(\int_0^{d_a^k} p_a^k \left(d_a, d_c^k \right) dd_a - d_a^k \cdot p_a^k \right) = \frac{\eta^2 \cdot ms_c}{2 \cdot \left(8 - \eta^2 \right)}, \text{ where}$$

$$p_a^1 = p_a^2 = p_a^k = ms_a - \frac{2 \cdot d_a}{d_c}.$$

Comparing consumer, advertiser, and producer surplus of the hypothetical collusion case and the monopoly optimum, we find that each of these welfare components is at least as great in the monopoly case as it is in the hypothetical collusion case. Therefore total welfare in the hypothetical collusion case also never exceeds total welfare in the monopoly case.

Comparing welfare outcomes of hypothetical collusion and duopoly equilibrium case, we need to distinguish the cases known from Section 4 and obtain the results presented in Table A1.

Case		Consumer Surplus	Advertiser Surplus	Producer Surplus	Total Welfare
$p_c^* = p_c^M$	Subsidization	$\gtreqless$	$\gtreqless$	$\leq$	$\gtreqless$
	Positive Prices	$>$	$>$	$<$	$>$
$p_c^* > p_c^M$	Subsidization 1	$>$	$>$	$<$	$>$
	Subsidization 2	$\gtreqless$	$\gtreqless$	$\leq$	$\gtreqless$
	Mixed	$\gtreqless$	$\gtreqless$	$\leq$	$\gtreqless$
	Positive Prices	$>$	$>$	$<$	$>$
Left hand side = duopoly equilibrium; right hand side = duopoly collusion					

Table A1: Welfare in the case of explicit collusion