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30 June 2010

Private and Public Incentive to Reduce Seasonality: a Simple Theoretical Model

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June 30, 2010

Abstract

This paper presents a theoretical model to investigate the incentive of private producer and policy-maker to reduce seasonality in a given market, where consumers derive different utilities from the consumption of the good in different seasons. The (seasonal) product differentiation is modeled along the lines of the contributions of Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). We take into consideration that investments are possible to reduce the degree of seasonality. We show that, for a wide set of parameter configuration, the policy maker finds it optimal to make more effort to reduce seasonality as compared to private producers. The theoretical conclusion is consistent with empirical and anecdotical evidence, especially in the field of tourism markets.

JEL classification: D29; L12; L83
Keywords: Seasonality; Tourism; Public Spending

1 Introduction

Seasonality is a major concern in several markets of very different sectors (tourism, transport, energy, agricultural and food items, arts and movies, till to financial products). A large body of literature, in each of the different fields, deals with causes and effects of seasonality. Even if some causes for seasonality are truly exogenous, there is no doubt that the seasonal pattern of markets can be affected largely by the institutional (or cultural) framework, and also by the choices of firms producing the goods. How strong is the incentive, for firms selling a goods, to reduce the demand seasonality, if possible, is an open question.

Pros and cons, indeed, are associated with seasonal pattern of quantity and price. For sure, seasonality is associated with private costs and benefits for sellers and for consumers; and social costs and benefits are also present in most cases, diverging from private costs and benefits (see different contributions in Baum and Lundtorp (2001), and specifically Butler (2001); or Soo Cheong (2004) and Cuccia and Rizzo (2010), for a short discussion concerning the tourism sector). Private costs include the volatility of revenues for producers; the higher prices of the item and its complements in the peak season for consumers. The social costs include possibly the congestion, the carrying capacity of productive system, among other other factors.

In industries with fixed capacity (let us think of the cases of tourism sector, e.g., beds in hotels, or seats in airplanes) it is a common-place that sellers have strong incentive to reduce demand seasonality, and a consistent incentive also holds for policy-makers for reducing seasonality and avoiding peaks with congestion or underutilization of capacity. In the field of tourism, for instance, a large set of interventions may be taken to reduce seasonality: institutional measures, ranging from school time-table to holiday design; or public actions at national level (organization of national or international living events) or events at local level (in

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the cultural field, in sport, and so on), till to private promotion (e.g., special offers from single hotels or travel agencies). Several times, private subjects complain about the lack of public initiatives aiming at reducing seasonality of demand. They claim that they are unable to do business because of the lack of adequate public initiatives attracting consumers. On the other hand, in several occasions, public initiatives do not find consistent answers by part of private firms. Some shocking cases can be reported. In mountain resorts, several hotels remain closed at the beginning of December and in April, even if sky stations are open. In minor islands or in specific seaside destinations, hotels and resort remain closed in May or September, even if connections are open and other public initiatives and interventions are operative.

In this paper we show that a conflict does arise between social and private incentive to invest for reducing seasonality, even we do not consider social costs emerging from externality effects. We consider the case in which consumers derive different utility levels from the consumption of a good in high or low season; the preference for consumption in high (low) season vary across consumers. The proposed model can be interpreted as an extension of the Gabszewicz and Thisse (1979) seminal model to the case of seasonal differentiation (see also Gabszewicz, 2009). We assume that costly investments are possible to reduce the demand seasonality.

We consider the alternative cases of private or public investments aimed at reducing seasonality. We find that only in some parameter regions private and public incentive to reduce seasonality coincide; in the other regions the policy-makers find it optimal to make more efforts to reduce seasonality as compared to private sellers. More specifically, it can happen that a policy maker (caring for the utility of consumers and profits of the firms) finds it optimal to have the market served over both seasons, while private suppliers find it optimal to serve the market only in high season; or, policy maker finds it optimal a larger amount of investment, leading to complete market coverage, whereas private firms make smaller efforts and leave the market partially uncovered, even if they operate over both seasons. Eventually, it can happen that both the social planner and the private firms find it optimal to serve the market in both seasons (though partially uncovered) but the optimal effort for reducing seasonality from a private perspective is smaller as compared to the social choice. The reason for the conflict between public and private incentive to reduce seasonality rests on the fact that policy-makers take into account also the utility of consumers, whereas firms are interested in their own profits only. No further considerations concerning (negative) externality of high season and congestion (that is, social costs of seasonality) are taken into account; thus, we believe that our conclusions are very strong and robust to more complicate (and realistic) hypothesis design, where additional reasons may exist to suggest a policy maker to reduce congestion in high season.

The structure of the paper is as follows. Section 2 introduces the basics of the model, and explains that it can be considered an extension of well-known models of product differentiation to the case of seasonal demand. Section 3 takes into account the possibility of investment for reducing seasonality, by part of private firms selling the goods. Section 4 takes the social welfare perspective, maintaining that the planner cares of producers and consumers only. Section 5 compares the private and social perspectives, and concludes. For the sake of simplicity we limit ourselves to the analysis of a monopolistic market. Extensions to oligopoly or other market forms are left to further analysis.

2 The model

Consider a monopolistic firm operating in a market characterized by seasonality, i.e. in which consumers get different utility levels depending on whether they consume in *high season* or *low season*.

Consumers are heterogeneous with respect to the evaluation of seasonal characteristic, and $\theta \in [0, \overline{\theta}]$ measures the differential in utilities they get consuming in high season *versus* low season. Each consumer can choose between buying one unit of the good (either in high or low season) or not buying at all.

We define consumer θ's utility function as:

$$U(\theta, u_i) = \begin{cases} U_0 + \theta u_h - p_h & \text{if buys in high season} \\ U_0 + \theta u_l - p_l & \text{if buys in low season} \\ 0 & \text{does not buy at all} \end{cases} \tag{1}$$

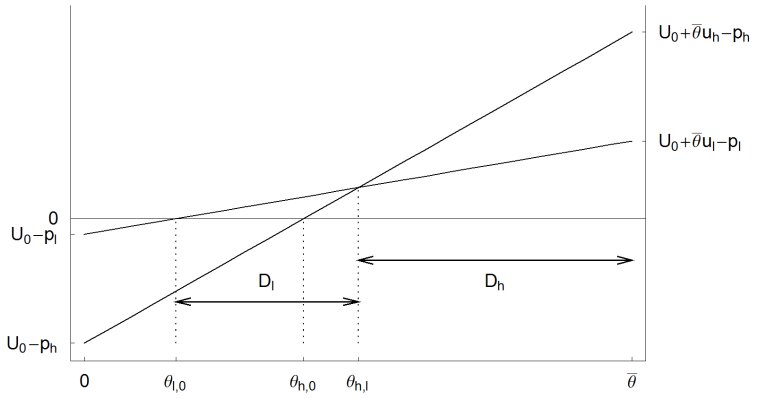

Figure 1: Utility in high and low season depending on θ (case (5)).

where U_0 is the utility derived from consuming the good, whatever the season, and p_h, p_l are the set prices for each season.

Solving for θ the equation $U_0 + \theta u_h - p_h = U_0 + \theta u_l - p_l$, it is possible to identify the consumer indifferent between h and l, that is:

$$\theta_{h,l} = \frac{p_h - p_l}{\Delta u} \tag{2}$$

where $\Delta u = u_h - u_l$.

In the same way, solving for θ the equation $U_0 + \theta u_l - p_l = 0$, we find the consumer indifferent between l and not consuming:

$$\theta_{l,0} = \frac{p_l - U_0}{u_l} \tag{3}$$

Solving for θ the equation $U_0 + \theta u_h - p_h = 0$, we find the consumer indifferent between h and not consuming:

$$\theta_{h,0} = \frac{p_h - U_0}{u_h} \tag{4}$$

It is easy to show that

$$\theta_{h,l} \geq \theta_{h,0} \Leftrightarrow \theta_{h,0} \geq \theta_{l,0}$$

while:

$$\theta_{h,l} \leq \theta_{h,0} \Leftrightarrow \theta_{h,0} \leq \theta_{l,0}$$

so that one of the following inequality must be true:

$$\theta_{h,l} \geq \theta_{h,0} \geq \theta_{l,0} \tag{5}$$

$$\theta_{h,l} < \theta_{h,0} < \theta_{l,0} \tag{6}$$

Figure 1, in which the abscissa is the set of consumers ordered by θ and the ordinate is the utility level in high and low season, graphically shows case (5), where consumer indifferent between h and l is at the *right* of the consumer indifferent between l and not consume. In this case the demand functions in high and low season (both positive) are:

$$D_h \equiv \frac{\overline{\theta} - \theta_{h,l}}{\overline{\theta}} = \frac{\overline{\theta}\Delta u - p_h + p_l}{\overline{\theta}\Delta u} \tag{7}$$

$$D_l \equiv \frac{\theta_{h,l} - \theta_{l,0}}{\overline{\theta}} = \frac{U_0\Delta u + u_l p_h - u_h p_l}{\overline{\theta} u_l \Delta u} \tag{8}$$

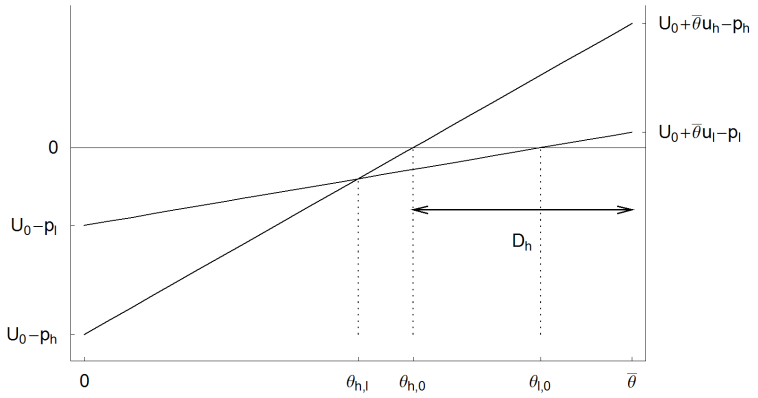

Figure 2: Utility in high and low season depending on θ (case (6)).

Figure 2, instead, represents case (6), where the consumer indifferent between h and l is at the *left* of the consumer indifferent between l and not consume. In this case the demand functions in high and low season are:

$$D_h \equiv \frac{\overline{\theta} - \theta_{h,0}}{\overline{\theta}} = \frac{U_0 + \overline{\theta} u_h - p_h}{\overline{\theta} u_h} \tag{9}$$

$$D_l = 0 \tag{10}$$

The profit function of the firm is:

$$\pi(p_h, p_l) = D_h(p_h, p_l)(p_h - c_m) + D_l(p_h, p_l)(p_l - c_m) \tag{11}$$

where c_m is the marginal cost of production.

Maximizing the profits wrt high and low season prices, we get the equilibrium prices:

$$p_h^* = \frac{c_m + U_0 + \overline{\theta} u_h}{2} \tag{12}$$

$$p_l^* = \frac{c_m + U_0 + \overline{\theta} u_l}{2} \tag{13}$$

Substituting the equilibrium prices in the equations (2),(3) and (4), we get the following lemmas:

Lemma 2.1. *If the marginal cost of production is low ($c_m < U_0$), inequalities* (5) *are true and the firm operates in both seasons. The equilibrium prices are given by* (12) *and* (13) *and the profit is:*

$$\pi^* = \frac{(U_0 - c_m)^2 + \overline{\theta} u_l (2U_0 + \overline{\theta} u_h - 2c_m)}{4\overline{\theta} u_l} \tag{14}$$

Lemma 2.2. *If the marginal cost of production is high ($c_m > U_0$), inequalities* (6) *are true and the firm operates only in high season. The high season equilibrium price is given by* (12) *and the profit is:*

$$\pi_0^* = \frac{(U_0 + \overline{\theta} u_h - c_m)^2}{4\overline{\theta} u_h} \tag{15}$$

Moreover if:

$$c_m < U_0 - \overline{\theta} u_l \tag{16}$$

then the market is covered ($\theta_{l,0} \leq 0$). If

$$c_m > U_0 + \overline{\theta} u_h \tag{17}$$

then the firm does not operate, not even in high season ($\theta_{h,0} \geq \overline{\theta}$).

(For a different modelling approach to seasonal pricing applied to the case of hotels, see Baum and Mudambi (1995)).

3 Deseasonalization effort

Consider now the case in which the firm, before setting the equilibrium prices, can choose to make an effort e in order to deseasonalize the demand. Effort e, by part of firm, is defined in a way such that consumer θ's utility function is:

$$U(\theta, u_i) = \begin{cases} U_0 + \theta u_h - p_h & \text{if in high season} \\ U_0 + \theta u_l - p_l + e & \text{if in low season} \\ 0 & \text{if not consume} \end{cases} \tag{18}$$

In figures 1 and 2 the effort e shifts up the low season utility, moving $\theta_{h,l}$ rightward and $\theta_{l,0}$ leftward.

In this case the equations (2) and (3) are substituted, respectively, by the following:

$$\theta_{h,l} = \frac{p_h - p_l + e}{u_h - u_l} \tag{19}$$

$$\theta_{l,0} = \frac{p_l - e - U_0}{u_l} \tag{20}$$

whereas $\theta_{h,0}$ does not change, since it is independent of e. As before, we can show that one between (5) and (6) must be true. We will show below that effort e entails a quadratic cost for the firm. Just to give an example, one can imagine that e represents the supply of additional service in the general case of a good with seasonal pattern, like the organization of entertainment events during the low season in the case of tourism markets.

In case (5), where the firm operates in both seasons, this implies a reduction of D_h to the good of D_l ($\theta_{h,l}$ moves *rightward*), and an higher market coverage, still to the good of D_l ($\theta_{l,0}$ moves *leftward*). Therefore, the demand functions become:

$$D_h = \frac{\overline{\theta}\Delta u - p_h + p_l - e}{\overline{\theta}\Delta u} \tag{21}$$

$$D_l = \frac{U_0 \Delta u + u_l p_h - u_h p_l + u_h e}{\overline{\theta} u_l \Delta u} \tag{22}$$

In case (6), where the firm operates only in high season, if the deseasonalization effort is sufficiently high, the firm starts operating in low season, otherwise the investment does not affect demand and prices, and the demand functions remain (9) and (10).

3.1 Equilibrium

The firm profit function is:

$$\pi(p_h, p_l) = D_h(p_h, p_l)(p_h - c_m) + D_l(p_h, p_l)(p_l - c_m) - ce^2 \tag{23}$$

where c captures the cost of deseasonalization.

Substituting the demand functions in the profit function and maximizing wrt the prices, we get the following equilibrium prices:

$$p_h^* = \frac{c_m + U_0 + \overline{\theta} u_h}{2} \tag{24}$$

$$p_l^* = \frac{c_m + U_0 + \overline{\theta} u_l + e}{2} \tag{25}$$

Lemmas 2.1 and 2.2 are still valid for $e = 0$. As e increases, we have some particular cases:

1. If $e \geq e^c \equiv \overline{\theta} u_l + c_m - U_0$ then the market is covered.
2. If $e \geq e^i \equiv \overline{\theta} \Delta u$ then the firm operates only in low season.

In what follow we assume that $e^c < e^i$ (i.e. $c_m < U_0 + \overline{\theta}(\Delta u - u_l)$), which means that investing in deseasonalization, *first* the firm covers the market and *then* the high season is erased.

This implies that the firm has incentive to invest in e only in order to increase D_l by an increase of the market coverage and not by a reduction of D_h, since this would imply a reduction of profits, because the high season price is always higher than the low season price ($e < e^i$); therefore the firm will never invest $e > e^c$.

3.2 Case 1: Low production cost

Proposition 3.1 (Low production cost (Lemma 2.1))**.** *If the marginal cost of production is low ($c_m < U_0$) then:*

(i) if $c > \frac{u_h}{4\overline{\theta} u_l \Delta u}$ then there are two cases:

$$c_m > \psi_\pi(c) \Rightarrow e^* = e^m \equiv \frac{(U_0 - c_m)\Delta u}{4\overline{\theta} c u_l \Delta u - u_h}$$

$$c_m < \psi_\pi(c) \Rightarrow e^* = e^c$$

where ψ_π is a decreasing convex function defined in appendix;

(ii) if $c < \frac{u_h}{4\overline{\theta} u_l \Delta u}$ then the firm finds optimal to invest in deseasonalization up to the complete coverage of the market $e^* = e^c$.

Proof. In appendix A.1 ■

By lemma 2.1, if marginal cost is low, the firm tends to operate in both seasons, but in general without completely covering the market.

If the investment cost is lower than a certain threshold, the profit function is convex and divergent in e. Hence the firm has incentive to invest up to the complete coverage of the market.

If, on the contrary, the investment cost is high then the profit function has a maximum in e^m: therefore the firm will invest in deseasonalization, but not necessarely completely covering the market. In particular, if $c_m < \psi_\pi(c)$ then $e^m > e^c$, and the firm will cover the market completely.

3.3 Case 2: High production cost

Proposition 3.2 (High production cost (Lemma 2.2))**.** *If the marginal cost of production is high ($c_m > U_0$) then:*

(i) if $c > \frac{u_h}{4\overline{\theta} u_l \Delta u}$ then the firm finds optimal not to invest in deseasonalization ($e^* = 0$);

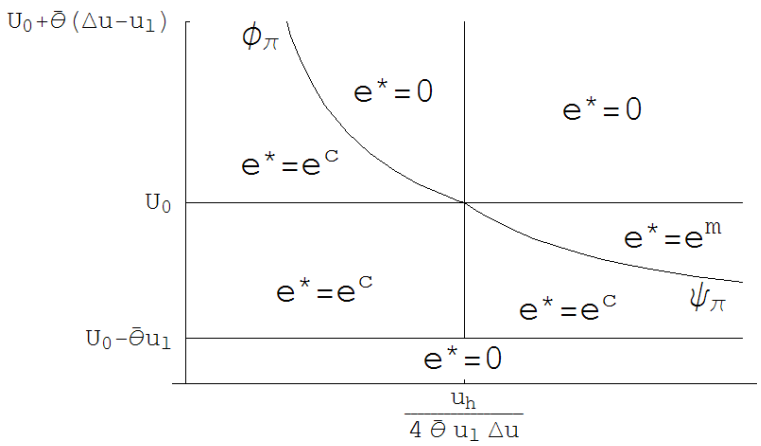

Figure 3: Deseasonalization effort e in the space (c, c_m)

(ii) if $c < \frac{u_h}{4\bar{\theta}u_l\Delta u}$ *then there are two cases:*

$$c_m > \phi_\pi(c) \Rightarrow e^* = 0$$
$$c_m < \phi_\pi(c) \Rightarrow e^* = e^c$$

where ϕ_π *is a convex decreasing function defined in appendix.*

Proof. In appendix A.2 ■

By lemma 2.2, if the production cost is high, the firm will tend to operate only in high season.

However, if the investment cost is lower than a certain threshold, the profit function is convex and divergent in e, with a first decreasing part. Therefore if the marginal cost is not excessively high with respect to the investment cost ($c_m < \phi_\pi(c)$), the firm may find convenient to invest in deseasonalization up to the complete coverage of the market.

In particular, if the firm can invest in deseasonalization at least a threshold level (namely, e^σ defined in appendix), it will reach profits at least equal to those obtainable without deseasonalization (π_0^*).

If, on the contrary, the deseasonalization cost is high, the firm will not be able to recover such cost through the low season activity, hence will continue to operate only in high season.

Figure 3 illustrates the optimal behaviour of the firm, depending on the levels of production cost c_m and deseasonalization cost c. Under the curve defined by functions ϕ_π and ψ_π, the firm will completely cover the market with deseasonalization investments (if the market was not already covered from the beginning); above such curve, instead, the market will remain partially uncovered and, if the production cost is high ($c_m > U_0$), the firm will continue to operate only in high season, without deseasonalization investment.

4 Welfare

Suppose now that in the first stage it is a policy-maker to choose how much to invest in deseasonalization, and that such policy-maker wants to maximize total welfare, defined as the sum of firm profits and consumer surplus minus the investment cost in deseasonalization. The welfare function is:

$$w(e) \equiv \frac{1}{\bar{\theta}} \left[\int_{\theta_{l,0}}^{\theta_{h,l}} (U_0 + \theta u_l + e) d\theta + \int_{\theta_{h,l}}^{\bar{\theta}} (U_0 + \theta u_h) d\theta \right] - c_m(D_h + D_l) - ce^2 \tag{26}$$

With an almost identical analysis to the previous one, we get what follows:

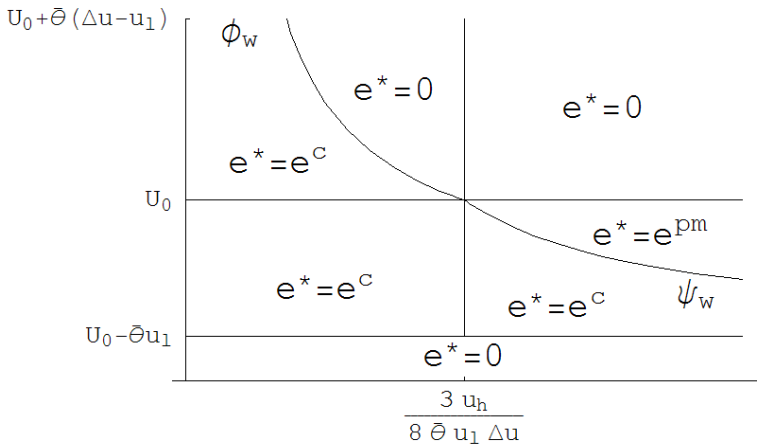

Figure 4: Deseasonalization effort e by the policy-maker in the space (c, c_m)

Proposition 4.1 (Low production cost (Lemma 2.1))**.** *If the marginal cost of production is low ($c_m < U_0$) then:*

(i) if $c > \frac{3u_h}{8\bar{\theta}u_l\Delta u}$ then there are two cases:

$$c_m > \psi_w(c) \Rightarrow e^* = e^{pm} \equiv \frac{3(U_0 - c_m)\Delta u}{8\bar{\theta}cu_l\Delta u - 3u_h}$$
$$c_m < \psi_w(c) \Rightarrow e^* = e^c$$

where ψ_w is a convex decreasing function defined in appendix;

(ii) if $c < \frac{3u_h}{8\bar{\theta}u_l\Delta u}$ then the policy-maker finds optimal to invest in deseasonalization up to the complete coverage of the market $e^* = e^c$.

Proof. In appendix A.3 ■

Proposition 4.2 (High production cost (Lemma 2.2))**.** *If the marginal cost of production is high ($c_m > U_0$) then:*

(i) if $c > \frac{3u_h}{8\bar{\theta}u_l\Delta u}$ then the policy-maker finds optimal not to invest in deseasonalization ($e^* = 0$);

(ii) if $c < \frac{3u_h}{8\bar{\theta}u_l\Delta u}$ then there are two cases:

$$c_m > \phi_w(c) \Rightarrow e^* = 0$$
$$c_m < \phi_w(c) \Rightarrow e^* = e^c$$

where ϕ_w is a convex decreasing function defined in appendix.

Proof. In appendix A.4 ■

Figure 4 (analogous to the 3) illustrates the optimal choices by the policy-maker, depending on the cost levels (c, c_m).

It can be shown that $\phi_\pi(c) > \phi_w(c)$ and $\psi_\pi(c) > \psi_w(c)$, hence in the space (c, c_m) the area in which the market is covered is larger in the case of a public investor; moreover it turns out that $e^{pm} > e^m$, hence the policy-maker always invests at least as much as the monopolistic firm, since he takes into account the increase in the consumer surplus.

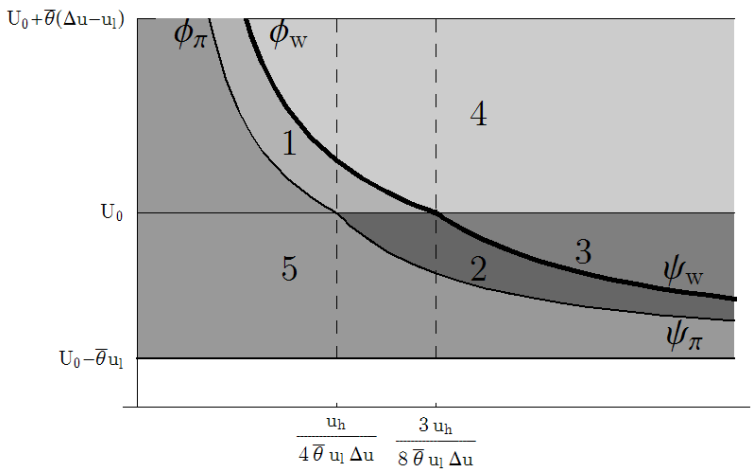

Figure 5: Public vs. private deseasonalization investment in the space (c, c_m)

Figure 5 puts together the optimal behaviour of the firm and the policy-maker.

In figure 5, in the area included between curves ϕ_w and ϕ_π, and above U_0 (area 1), the policy-maker would invest up to the complete coverage of the market, whereas the monopolistic firm would not invest in deseasonalization and would continue to operate only in high season.

In the area included between curves ψ_w and ψ_π, and below U_0 (area 2), the policy-maker would invest up to the complete coverage of the market, whereas the firm would invest in deseasonalization e^m, therefore covering the market just partially (and operating in both seasons).

In the area included between U_0 and ψ_w (area 3) the market would remain partially covered in both cases, however the policy-maker would invest more than the firm ($e^{pm} > e^m$).

In the area above ϕ_w and U_0 (area 4) no investment effort is judged optimal (either by private or public subjects) while in area 5 the socially optimal choice coincides with the private choice.

It is clear that the cases of areas 1, 2 and 3 in figure 5 are our points of interest, where a conflict emerges between private and social incentives for reducing seasonality: there are different parameter configurations where the public subject finds it optimal to make effort for reducing seasonality larger than the private actors. In some cases the public actor finds it optimal to have a complete coverage of the market over both seasons while the private suppliers prefer to serve only in the high season. In a second case both public and private subjects find optimal to serve in both seasons, but the public finds it optimal to cover the market while the private leaves the market partially covered. Eventually for some values of the parameters both the public planner and private actors find optimal to leave the market partially covered, but the public effort is larger than the private one.

Our conclusions have nothing to do with negative externalities due to congestion — in the case of tourism, upon local residents — which can represent a further reason to reduce seasonality, from a social welfare perspective. The consideration of this point would simply strengthen our conclusion that public incentive to mitigate seasonality is stronger than the private incentive.

5 Conclusions

In this paper we have proposed to use the Gabszewicz and Thisse (1979)-Shaked and Sutton (1982) theoretical framework to model market behaviours in the case of a good for which seasonality is relevant. The application to markets of tourism items, which we have provided in the paper, is straightforward but not unique.

Our content has been that a planner taking a social welfare perspective finds it optimal to reduce seasonality to a larger extent as compared to private firms supplying the item. In fact, the elaboration of the

present theoretical model has been suggested by the observation that, in the field of tourism, in some cases local authorities take actions to sustain demand in low seasons but private firms do not follow this actions; this observation suggests that the incentive of public sector to mitigate seasonality is higher than the incentive of private firms. An obvious reason could be that the congestion in high season generates negative externality to the local population. This point has not been considered in the model. We have simply shown that, apart from the negative externality upon residents, the social incentive to reduce seasonality is stronger than the private incentive.

The theoretical model is very simple and a more complicate – and more realistic – modelling is perhaps necessary to grasp all the relevant aspects of markets for seasonal items. However, we believe that our model, though very simple, is robust to further modifications, and can provide an explanation of the smaller private incentive to reduce seasonality as compared to the social welfare perspective.

Appendices

A Proofs

A.1 Proof of proposition 3.1

If the production cost is low, for $e = 0$ lemma 2.1 is valid, therefore the firm operates in both seasons.

If the market is covered (eq. (16)), the firm does not have incentive to deseasonalize and $e^* = 0$, otherwise the profit function in e is:

$$\pi^*(e) = \frac{u_h - 4\overline{\theta}u_l c\Delta u}{4\overline{\theta}u_l\Delta u}e^2 + \frac{(U_0 - c_m)(U_0 + 2e - c_m) + \overline{\theta}u_l(2U_0 + \overline{\theta}u_h - 2c_m)}{4\overline{\theta}u_l} \tag{27}$$

for which the derivatives are:

$$\frac{d\pi^*}{de} = \frac{u_h e + (U_0 - c_m)\Delta u}{2\overline{\theta}u_l\Delta u} - 2ce \tag{28}$$

$$\frac{d^2\pi^*}{de^2} = \frac{u_h - 4\overline{\theta}u_l c\Delta u}{2\overline{\theta}u_l\Delta u} \tag{29}$$

Setting the equation (28) equal to zero, we find an extremum in:

$$e^m = \frac{(c_m - U_0)\Delta u}{u_h - 4\overline{\theta}cu_l\Delta u} \tag{30}$$

The profit is concave (resp. convex) in e if the following inequality (resp. the opposite inequality) is valid:

$$c > \frac{u_h}{4\overline{\theta}u_l\Delta u} \tag{31}$$

If the (31) is false then $e^m < 0$, and it is a minimum. In this case the profit function diverges in e and the firm will completely cover the market, investing $e^* = e^c$. And this proves the second point of the proposition.

If the (31) is true then $e^m > 0$, and it is a maximum. The firm tends to invest e^m, however if $e^m \geq e^c$ it will not invest more than e^c, having already completely covered the market.

So we can define a (decreasing convex) function ψ_π which solves the equation $e^m = e^c$:

$$\psi_\pi(c) \equiv U_0 - \overline{\theta}u_l + \frac{\overline{\theta}\Delta u}{4c\overline{\theta}\Delta u - 1} \tag{32}$$

and it is such that:

$$c_m > \psi_\pi(c) \Rightarrow e^m < e^c; e^* = e^m$$
$$c_m < \psi_\pi(c) \Rightarrow e^m > e^c; e^* = e^c \quad \blacksquare$$

A.2 Proof of proposition 3.2

If the production cost is high, for $e = 0$ lemma 2.2 is true, therefore the firm operates in high season only and gets a profit π_0^*, defined by (15).

The firm can operate also in low season only if $\theta_{l,0} < \theta_{h,l}$, i.e. if $e > e^s \equiv \frac{\Delta u(c_m - U_0)}{u_h}$: under such threshold, the firm does not deseasonalize because it would not be able to take advantage of such investment. Over this threshold, the profit function is the (27), whose first and second derivatives were calculated above.

If the (31) is valid, then $e^m < 0$ and it is a maximum, therefore profits are decreasing in e and the firm does not invest in deseasonalization ($e^* = 0$). And this proves the first point of the proposition.

If the (31) is not valid, then $e^m > e^s > 0$, but it is a minimum. In this case, profits are decreasing up to e^m and increasing afterwards, hence the firm has to choose between either not investing in e, getting π_0^* operating in high season only, or investing in $e > e^m > e^s$, operating in both seasons and getting $\pi^*(e)$ which diverges in e.

Clearly the firm does invest in deseasonalization only if $\pi^*(e) \geq \pi_0^*$, i.e. if:

$$e > e^\sigma \equiv \frac{\Delta u(c_m - U_0)}{u_h - \sqrt{4c\bar{\theta}u_h u_l \Delta u}} > e^m \tag{33}$$

Therefore e^σ is the least necessary investment so that the firm chooses to operate in low season as well. Once e^σ is invested, profits are increasing in e.

If $e^\sigma \leq e^c$, the firm aims to completely cover the market, investing $e^* = e^c$; otherwise the complete coverage of the market implies profits lower than π_0^*, then $e^* = 0$.

So we can define a (decreasing convex) function ϕ_π which solves the equation $e^\sigma = e^c$:

$$\phi_\pi(c) \equiv \frac{\sqrt{4\bar{\theta}cu_h u_l \Delta u}(U_0 - \bar{\theta}u_l) - u_l(U_0 - \bar{\theta}u_h)}{\sqrt{4\bar{\theta}cu_h u_l \Delta u} - u_l} \tag{34}$$

and it is such that:

$$c_m > \phi_\pi(c) \Rightarrow e^\sigma > e^c; e^* = 0$$
$$c_m < \phi_\pi(c) \Rightarrow e^\sigma < e^c; e^* = e^c \quad \blacksquare$$

A.3 Proof of proposition 4.1

If the production cost is low, for $e = 0$ lemma 2.1 is valid, therefore the firm operates in both seasons.

If the market is covered (eq. (16)), the policy-maker does not have incentive to deseasonalize and $e^* = 0$, otherwise the welfare function in e is:

$$w^*(e) = \frac{3(U_0 - c_m)[U_0 - c_m + 2(e + \bar{\theta}u_l)] + 3\bar{\theta}^2 u_h u_l}{8\bar{\theta}u_l} + \frac{e^2(3u_h - 8\bar{\theta}u_l c\Delta u)}{8\bar{\theta}u_l \Delta u} \tag{35}$$

for which the derivatives are:

$$\frac{dw^*}{de} = \frac{3(U_0 - c_m)}{4\bar{\theta}u_l} + \frac{3u_h - 8\bar{\theta}u_l c\Delta u}{4\bar{\theta}u_l \Delta u} e \tag{36}$$

$$\frac{d^2 w^*}{de^2} = \frac{3u_h - 8\bar{\theta}u_l c\Delta u}{4\bar{\theta}u_l \Delta u} \tag{37}$$

Setting the equation (36) equal to zero, we find an extremum in:

$$e^{pm} = \frac{3(c_m - U_0)\Delta u}{3u_h - 8\bar{\theta}cu_l \Delta u} \tag{38}$$

The welfare function is concave (resp. convex) in e if the following inequality (resp. the opposite inequality) is true:

$$c > \frac{3u_h}{8\overline{\theta}u_l\Delta u} \tag{39}$$

If the (39) is false then $e^{pm} < 0$, and it is a minimum. In this case the welfare function diverges in e and the policy-maker will aim to the complete coverage of the market, investing $e^* = e^c$. And this proves the second point of the proposition.

If the (31) is true then $e^{pm} > 0$, and it is a maximum. The policy-maker tends to invest e^{pm}, however if $e^{pm} \geq e^c$ it will not invest more than e^c, having already completely covered the market.

So we can define a (decreasing convex) function ψ_w which solves the equation $e^{pm} = e^c$:

$$\psi_w(c) \equiv U_0 - \overline{\theta}u_l + \frac{\overline{\theta}\Delta u}{\frac{8}{3}c\overline{\theta}\Delta u - 1} \tag{40}$$

and it is such that:

$$c_m > \psi_w(c) \Rightarrow e^{pm} < e^c; e^* = e^{pm}$$
$$c_m < \psi_w(c) \Rightarrow e^{pm} > e^c; e^* = e^c \quad \blacksquare$$

A.4 Proof of proposition 4.2

If the production cost is high, for $e = 0$ lemma 2.2 is true, therefore the firm operates in high season only and gets a profit π_0^*, defined by (15). Moreover, in this case, we define the welfare as:

$$w_0^* \equiv \frac{1}{\overline{\theta}} \int_{\theta_{h,0}}^{\overline{\theta}} (U_0 + \theta u_h) d\theta - c_m D_h = \frac{3(U_0 + \overline{\theta}u_h - c_m)^2}{8\overline{\theta}u_h} \tag{41}$$

The firm can operate also in low season only if $\theta_{l,0} < \theta_{h,l}$, i.e. if $e > e^s \equiv \frac{\Delta u(c_m - U_0)}{u_h}$: under such threshold, the policy-maker does not deseasonalize because the firm would continue to operate in high season only. Over this threshold, the welfare function is the (35), whose first and second derivatives were calculated above.

If the (39) is valid, then $e^{pm} < 0$ and it is a maximum, therefore welfare is decreasing in e and the policy-maker does not invest in deseasonalization ($e^* = 0$). And this proves the first point of the proposition.

If the (39) is not valid, then $e^{pm} > e^s > 0$, but it is a minimum. In this case, welfare is decreasing up to e^{pm} and increasing afterwards, hence the policy-maker has to choose between either not investing in e, getting w_0^* with high season only, or investing in $e > e^{pm} > e^s$, with both seasons and getting $w^*(e)$ which diverges in e.

Clearly the policy-maker does invest in deseasonalization only if $w^*(e) \geq w_0^*$, i.e. if:

$$e > e^{p\sigma} \equiv \frac{\Delta u(c_m - U_0)}{u_h - \sqrt{\frac{8}{3}c\overline{\theta}u_h u_l \Delta u}} > e^{pm} \tag{42}$$

Therefore $e^{p\sigma}$ is the least necessary investment so that the policy-maker chooses to invest in deseasonalization, allowing the firm to operate in low season as well. Once $e^{p\sigma}$ is invested, welfare is increasing in e.

If $e^{p\sigma} \leq e^c$, the policy-maker aims to the complete coverage of the market, investing $e^* = e^c$; otherwise the complete coverage of the market implies welfare lower than w_0^*, then $e^* = 0$.

So we can define a (decreasing convex) function ϕ_w which solves the equation $e^{p\sigma} = e^c$:

$$\phi_w(c) \equiv \frac{\sqrt{\frac{8}{3}\overline{\theta}cu_h u_l \Delta u}(U_0 - \overline{\theta}u_l) - u_l(U_0 - \overline{\theta}u_h)}{\sqrt{\frac{8}{3}\overline{\theta}cu_h u_l \Delta u} - u_l} \tag{43}$$

and it is such that:

$$c_m > \phi_w(c) \Rightarrow e^{p\sigma} > e^c; e^* = 0$$
$$c_m < \phi_w(c) \Rightarrow e^{p\sigma} < e^c; e^* = e^c \quad \blacksquare$$

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