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RINGRAZIAMENTI

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This work is mainly intended for applied econometricians and students interested in development and application of estimation methods for structural econometric models. For the Klein-I model, detailed numerical tables of the parameters of the structural and restricted reduced form, of their covariance matrices and of the covariance matrices of the related disturbances, obtained by 8 different estimation methods, are displayed.

The authors will welcome any comment, correction and additional results for this model.

CONTENTS

1. INTRODUCTION

No econometric model has probably been as deeply analyzed as the Klein-I model; almost any new algorithm for the estimation of structural form or reduced form parameters is, generally, tested, or at least experimented, on Klein's model I. The model, in fact, has the advantage of being linear, sufficiently small and manageable, but at the same time has the structure of a complete econometric model, including current endogenous variables on the right hand side of the stochastic equations, lagged endogenous variables which make the model dynamic, overidentifying restrictions, etc.

However, it frequently happens that the same algorithms, applied to this model by different authors, give rather different numerical results; see, for example, the results in Goldberger, Nagar and Odeh (1961) and the related discussion in Bianchi, Calzolari and Corsi (1979) on some restricted reduced form parameters derived from 2SLS estimates, or the results in Brundy and Jorgenson (1974,p.695), Theil (1971,p.517) and Zellner and Thornber (1966) for the 3SLS estimates, or the results in Brundy and Jorgenson (1974,p.695), Chernoff and Divinsky (1953,pp.250,284), Chow (1968,pp.109,110) and Belsley (1980,p.218) for the FIML estimates of parameters and related standard errors.

Typing or printing errors are also rather frequent in the published numerical tables and may be sometimes misleading for the researcher who wants to use the contents of those tables as input data.

It also happens that accurate results do not display enough significant digits, leading sometimes to serious troubles; this is the case of the estimated asymptotic covariance matrix of the 2SLS structural coefficients displayed with great accuracy and without any printing error in Theil (1971,p.518), but whose significant digits are insufficient if the experimenter wants to use that matrix for Monte Carlo experiments that require the triangular decomposition of the matrix. Using the numerical values in Theil (1971,p.518), the matrix, in fact, seems to be not positive definite, but it is positive definite when more decimal digits are taken into account.

Clearly this paper will not supply any definitive answer to all the possible problems concerning estimation of this model. However, the authors hope that the published tables will give a concrete help to the interested researcher and not simply increase his doubts when he is testing his algorithms and programs for estimation of the structural or of the restricted reduced form parameters of econometric models.

For each of the 8 estimation methods which have been taken into account, this paper will display the 12 estimated structural coefficients, their estimated asymptotic covariance matrix (12 × 12) (except for OLS) and the (3 × 3) estimated covariance matrix of structural disturbances; as far as the restricted reduced form is concerned, for each estimation method the reduced form coefficients and their asymptotic standard errors (again, except for OLS) as well as the (6 × 6) covariance matrix of reduced form disturbances will be displayed.

For each of the estimation methods, some comments will be made on the numerical results available in the literature.

2. STRUCTURAL FORM OF THE KLEIN-I MODEL

The structural form equations of the Klein-I model are reported in the version displayed in Goldberger, Nagar and Odeh (1961,p.563). Formally, this version differs very slightly from the original one displayed in Klein (1950,pp.65,66) which contains (Y+T-W2)-1 instead of (P+W1+T)-1 in the third equation; the two terms, however, are perfectly equivalent, due to the forth and fifth equations. Several other representations are available (see, for example, Theil (1971,pp.432-434)). Sometimes, as in Bergström (1974,p.96), Goldberger (1964,p.303), Hendry (1971,pp.270,271), or Rothenberg (1973,p.104), a different number of equations (7 or 8) is presented. However, all these different representations can be reconducted, by means of suitable variables transformations, to the structure here displayed.

$$C_t = a_1 + a_2 P_t + a_3 P_{t-1} + a_4 (W1_t + W2_t) + u_{1t}$$
$$I_t = a_5 + a_6 P_t + a_7 P_{t-1} + a_8 K_{t-1} + u_{2t}$$
$$W1_t = a_9 + a_{10} (Y+T-W2)_t + a_{11} (P+W1+T)_{t-1} + a_{12} t + u_{3t}$$
$$Y_t + T_t = C_t + I_t + G_t$$
$$Y_t = P_t + W1_t + W2_t$$
$$K_t = K_{t-1} + I_t$$

C = Consumption

I = Net investment

W1 = Private wage bill

Y = National income

P = Profits

K = Stock of capital goods at the end of the year

W2 = Government wage bill

T = Business taxes

t = Time measured from 1931 as origin

G = Government expenditure

3. SAMPLE PERIOD DATA

The sample estimation period is from 1921 to 1941; the historical data are reported in the following table. They are taken by Klein (1950,p.135); the values for T have been obtained by difference of data in columns 4 (Y+T) and 9 (Y) of page 135. The value of K for 1941 has been obtained by K(1940)+I(1941) according to the relation $K_t=K_{t-1}+I_t$. Correct data are presented also in Theil (1971,p.456) and Maddala (1977,p.238); it should be noted that both present the series X (total production of private industry, given by Y+T-W2) instead of Y and that they indicate by G only the government non-wage expenditure, so that their G is equal to our G-W2.

Endogenous variables

Year	C	I	W1	Y	P	K
1920	39.8	2.7	28.8	43.7	12.7	182.8
1921	41.9	-0.2	25.5	40.6	12.4	182.6
1922	45.0	1.9	29.3	49.1	16.9	184.5
1923	49.2	5.2	34.1	55.4	18.4	189.7
1924	50.6	3.0	33.9	56.4	19.4	192.7
1925	52.6	5.1	35.4	58.7	20.1	197.8
1926	55.1	5.6	37.4	60.3	19.6	203.4
1927	56.2	4.2	37.9	61.3	19.8	207.6
1928	57.3	3.0	39.2	64.0	21.1	210.6
1929	57.8	5.1	41.3	67.0	21.7	215.7
1930	55.0	1.0	37.9	57.7	15.6	216.7
1931	50.9	-3.4	34.5	50.7	11.4	213.3
1932	45.6	-6.2	29.0	41.3	7.0	207.1
1933	46.5	-5.1	28.5	45.3	11.2	202.0
1934	48.7	-3.0	30.6	48.9	12.3	199.0
1935	51.3	-1.3	33.2	53.3	14.0	197.7
1936	57.7	2.1	36.8	61.8	17.6	199.8
1937	58.7	2.0	41.0	65.0	17.3	201.8
1938	57.5	-1.9	38.2	61.2	15.3	199.9
1939	61.6	1.3	41.6	68.4	19.0	201.2
1940	65.0	3.3	45.0	74.1	21.1	204.5
1941	69.7	4.9	53.3	85.3	23.5	209.4

Exogenous Variables

Year	W2	T	t	G
1920	2.2	3.4	-11.0	4.6
1921	2.7	7.7	-10.0	6.6
1922	2.9	3.9	-9.0	6.1
1923	2.9	4.7	-8.0	5.7
1924	3.1	3.8	-7.0	6.6
1925	3.2	5.5	-6.0	6.5
1926	3.3	7.0	-5.0	6.6
1927	3.6	6.7	-4.0	7.6
1928	3.7	4.2	-3.0	7.9
1929	4.0	4.0	-2.0	8.1
1930	4.2	7.7	-1.0	9.4
1931	4.8	7.5	0.0	10.7
1932	5.3	8.3	1.0	10.2
1933	5.6	5.4	2.0	9.3
1934	6.0	6.8	3.0	10.0
1935	6.1	7.2	4.0	10.5
1936	7.4	8.3	5.0	10.3
1937	6.7	6.7	6.0	11.0
1938	7.7	7.4	7.0	13.0
1939	7.8	8.9	8.0	14.4
1940	8.0	9.6	9.0	15.4
1941	8.5	11.6	10.0	22.3

4. ALTERNATIVE STRUCTURAL FORM ESTIMATES

The estimation methods considered in this paper and the respective abbreviations are the following:

- Ordinary Least Squares (OLS)
- Two Stage Least Squares (2SLS)
- Three Stage Least Squares (3SLS)
- Iterative Three Stage Least Squares (Iterative 3SLS)
- Limited Information Instrumental Variables Efficient (LIVE)
- Iterative Instrumental Variables (IIV)
- Full Information Instrumental Variables Efficient (FIVE)
- Full Information Maximum Likelihood (FIML)

5. ORDINARY LEAST SQUARES

The numerical results related to Ordinary Least Squares are presented just for completeness sake. Of course, the covariance matrix of coefficients is not reported; the standard errors of coefficients with correction for the degrees of freedom are displayed, in parentheses, under the related coefficients. These numerical values are not presented very often in the literature; however, our results are in agreement with those by Seaks (1974,p.2) and by Maddala (1977,p.242). In the latter, very minor differences, probably due to typing errors, have been found for the standard error of the constant term of the first equation and for the coefficient (and related standard error) of the constant term of the third equation.

1	P	P-1	W1+W2
16.2366	.192935	.089886	.796218
(1.303)	(.0912)	(.0906)	(.0399)

1	P	P-1	K-1
10.1258	.479635	.333039	-.111795
(5.466)	(.0971)	(.1009)	(.0267)

1	Y+T-W2	(P+W1+T)-1	t
1.49705	.439477	.146090	.130246
(1.270)	(.0324)	(.0374)	(.0319)

OLS covariance matrix of structural form disturbances
(no correction for degrees of freedom)

.851404		
.0494972	.824891	
-.380816	.121170	.476417

OLS reduced form coefficients

	C	I	W1	Y	P	K
1	61.6690	35.5429	44.2194	97.2119	52.9925	35.5429
P-1	.917706	.697107	.855763	1.61481	.759050	.697107
W2	.454409	-.200616	-.327941	.253793	-.418266	-.200616
K-1	-.187518	-.221853	-.179910	-.409372	-.229462	.778147
T	-1.32106	-1.14176	-1.08235	-3.46282	-2.38047	-1.14176
(W1+T)-1	.118433	-.052287	.175160	.066146	-.109013	-.052287
t	.105588	-.046616	.156162	.058972	-.097190	-.046616
G	1.67734	.984466	1.60928	3.66181	2.05253	.984466

OLS covariance matrix of reduced form disturbances

	C	I	W1	Y	P	K
C	7.85791					
I	5.37951	4.42433				
W1	5.38743	4.00361	4.28052			
Y	13.2374	9.80384	9.39104	23.0413		
P	7.84998	5.80023	5.11052	13.6502	8.53969	
K	5.37951	4.42433	4.00361	9.80384	5.80023	4.42433

6. TWO STAGE LEAST SQUARES

The numerical results of coefficient estimates are exactly equal to those published in Zellner and Theil (1962,p.77) and Zellner and Theil (1970,p.404). Other correct results are in Bergström (1974,p.225) (four coefficients are multiplied by a scaling factor 10), Brundy and Jorgenson (1974,pp.691,694), Goldberger (1964,p.365), Goldberger, Nagar and Odeh (1961,p.564), Kloek and Mennes (1960,p.59), Rothenberg (1973,p.110), Rothenberg and Leenders (1964,p.75), Theil (1971,p.517) and Zellner and Thornber (1966,p.728, columns 2 and 4). Results slightly different are reported in Maddala (1977,p.242).

1	P	P-1	W1+W2
16.5548	.017302	.216235	.810182
1	P	P-1	K-1
20.2782	.150222	.615944	-.157788
1	Y+T-W2	(P+W1+T)-1	t
1.50030	.438859	.146673	.130396

The covariance matrix of structural disturbances for this estimation method is practically equal to the matrix displayed in Rothenberg (1973,p.120); only a minor difference can be found in the first element of the last row. Slightly larger differences can be noticed with respect to the matrix displayed in Goldberger, Nagar and Odeh (1961,p.571).

2SLS covariance matrix of
structural form disturbances

1.04405		
.437854	1.38327	
-.385294	.192535	.476444

The covariance matrix of structural coefficients, hereunder displayed, has been computed using the formula in Theil (1971,p.500). The numerical results are in agreement with those presented in Theil (1971,p.518), except a scaling factor 10 or 100 regarding variances and/or covariances of the constant terms of the various equations. However, as already pointed out in the introduction, the few significant digits displayed in Theil make the matrix not positive definite, contrarily to our results.

The numerical values presented in Goldberger, Nagar and Odeh (1961,p.565) contain several rounding and sign errors. As a result of these errors, some minor inaccuracy can be found also in the standard errors of coefficients displayed in Goldberger (1964,p.365).

The asymptotic standard errors of reduced form coefficients have been computed, also for the other estimation methods considered in this paper, by means of the method by Goldberger, Nagar and Odeh (1961), but numerically differ from those there displayed. For detailed discussion, see Bianchi, Calzolari and Corsi (1979a and 1979b).

The covariance matrix of reduced form disturbances is in agreement, at least as far as the first 3-4 decimal digits are concerned, with that in Goldberger, Nagar and Odeh (1961,p.571) and Rothenberg (1973,p.123).

2SLS estimated asymptotic covariance matrix of the structural coefficients

(all the numbers are multiplied by 10^4)

	Consumption Equation				Investment Equation				Private Wage Bill Equation			
	1	P	P-1	W1+W2	1	P	P-1	K-1	1	Y+T-W2	(P+W1+T)-1	t
1	17444.9	-153.438	-47.5203	-327.329	21123.7	-451.881	173.718	-80.4383	-6127.36	92.1117	7.10438	27.7191
P		139.357	-95.7104	-15.2599	579.364	40.3753	-31.1103	-3.74998	-5.22286	-18.1495	18.8879	6.60346
P-1			115.064	-5.30847	204.000	-46.4228	51.3948	-1.30451	328.398	11.7994	-17.8843	4.76236
W1+W2				16.2004	-820.657	12.7806	-11.8103	3.98111	15.7719	.511380	-.801645	-5.23720
1					568923.	-9248.47	7775.96	-2690.31	3689.38	134.362	-201.205	-320.617
P						300.084	-257.716	41.8978	-17.0442	5.94335	-5.86173	4.76616
P-1							264.991	-38.7169	-155.477	-3.31248	6.11211	-6.69910
K-1								13.0510	-3.80888	-.900280	.998127	1.74478
1									13174.1	-66.8422	-154.053	156.083
Y+T-W2										12.6964	-11.9972	-3.09153
(P+W1+T)-1											15.0826	.510228
t												8.4920

2SLS reduced form coefficients and asymptotic standard errors

	C	I	W1	Y	P	K
1	42.8261	25.8412	31.6355	68.6673	37.0317	25.8412
	(8.235)	(6.473)	(6.940)	(14.04)	(7.974)	(6.473)
P-1	.947303	.735788	.885312	1.68309	.797778	.735788
	(.1155)	(.0806)	(.0911)	(.1863)	(.1063)	(.0806)
W2	.684221	-.029069	-.151340	.655152	-.193508	-.029069
	(.0785)	(.0390)	(.0463)	(.1017)	(.0584)	(.0390)
K-1	-.104706	-.181952	-.125803	-.286658	-.160856	.818048
	(.0379)	(.0321)	(.0319)	(.0666)	(.0377)	(.0321)
T	-.128469	-.175877	-.133565	-1.30435	-1.17078	-.175877
	(.2834)	(.2390)	(.2110)	(.4832)	(.2727)	(.2390)
(W1+T)-1	.178844	-.007598	.221826	.171246	-.050580	-.007598
	(.0422)	(.0103)	(.0506)	(.0443)	(.0203)	(.0103)
t	.158997	-.006755	.197209	.152242	-.044967	-.006755
	(.0315)	(.0095)	(.0394)	(.0320)	(.0194)	(.0095)
G	.663587	.153143	.797288	1.81673	1.01944	.153143
	(.2367)	(.2087)	(.1977)	(.4206)	(.2426)	(.2087)

2SLS covariance matrix of reduced form disturbances

	C	I	W1	Y	P	K
C	3.92210					
I	2.40420	2.00291				
W1	2.84409	2.07243	2.72454			
Y	6.32630	4.40711	4.91653	10.7334		
P	3.48221	2.33467	2.19198	5.81688	3.62490	
K	2.40420	2.00291	2.07243	4.40711	2.33467	2.00291

7. THREE STAGE LEAST SQUARES

The numerical estimates of coefficients for this method are (except the number of significant decimal digits) exactly equal to those published in Zellner and Theil (1970,pp.403,404) and in Zellner and Thornber (1966,p.729, third and fourth column of Table 2). Other correct results are in Hausman (1974,p.649) and Theil (1971,p.517). Unfortunately, the results published in Brundy and Jorgenson (1974,p.695), Rothenberg and Leenders (1964,p.75) and Zellner and Theil (1962,pp.76,77) are in some way wrong. The errors in the Zellner and Theil(1962) were recognized in Zellner and Theil (1970,p.397, fn.12a). Correct results are mentioned in Belsley (1979), who makes reference to detailed results presented in an earlier draft of his paper (1977,p.22).

1	P	P-1	W1+W2
16.4408	.124891	.163145	.790081
1	P	P-1	K-1
28.1779	-.013079	.755724	-.194848
1	Y+T-W2	(P+W1+T)-1	t
1.79723	.400492	.181291	.149674

3SLS covariance matrix of
structural form disturbances

.891760		
.411317	2.09305	
-.393615	.403045	.520026

The following table displays the asymptotic covariance matrix of structural coefficients. The numerical values of this table are in substantial agreement with those presented in Theil (1971,p.518) and in Zellner and Theil (1970,p.403), except again the scaling factor in the variances and covariances of the constant terms. As already mentioned for the values of the estimated coefficients, the results in Zellner and Theil (1962,p.76) were wrong, as recognized in Zellner and Theil (1970,p.397, fn.12a). With respect to the correct numbers above mentioned, the only differences are some minor rounding errors for some values and the values 169.5 (instead of 196.5) for the covariance between the constant terms of the first and second equation and -44.8 (instead of -43.8) for the covariance between the constant term of the second equation and the term (P+W1+T)-1 of the third. Zellner and Thornber (1966,p.729) give the correct standard errors in the third and fourth column of the table there displayed. Also in Hausman (1974,p.649) the standard errors are presented; there is a minor typing error in the standard error of the coefficient of time in the last equation (it is wrongly set equal to the corresponding value of the preceeding term).

3SLS estimated asymptotic covariance matrix of the structural coefficients

(all the numbers are multiplied by 10^4)

	Consumption Equation				Investment Equation				Private Wage Bill Equation			
	1	P	P-1	W1+W2	1	P	P-1	K-1	1	Y+T-W2	(P+W1+T)-1	t
1	17018.5	-169.812	-55.7862	-307.119	19645.2	-430.048	153.133	-73.2226	-6153.08	99.4907	-.094590	33.9275
P		116.919	-80.6243	-11.6847	-156.536	60.9356	-49.4273	-.315535	8.49344	-13.5445	13.8818	6.39459
P-1			100.879	-5.65169	546.903	-59.0258	63.3691	-2.93111	285.767	8.82945	-14.0731	4.42875
W1+W2				14.3929	-620.742	8.85792	-8.58289	3.05086	27.6368	-.369105	-.094324	-5.17011
1					461554.	-7322.50	6045.01	-2175.66	2765.46	379.188	-438.844	-330.119
P						262.104	-223.468	32.6940	-16.7675	.323827	-.046229	4.92808
P-1							233.886	-30.4281	-147.862	1.76514	.721784	-6.62560
K-1								10.5825	.154070	-2.06271	2.13374	1.77253
1									12451.4	-79.0641	-128.931	140.623
Y+T-W2										10.1210	-9.11903	-2.82723
(P+W1+T)-1											11.6683	.503103
t												7.80382

3SLS reduced form coefficients and asymptotic standard errors

	C	I	W1	Y	P	K
1	46.7274	27.6185	31.5722	74.3459	42.7737	27.6185
	(8.800)	(5.519)	(6.246)	(13.60)	(8.167)	(5.519)
P-1	.945928	.744849	.859334	1.69078	.831444	.744849
	(.1112)	(.0649)	(.0763)	(.1655)	(.1007)	(.0649)
W2	.656855	.002670	-.136358	.659525	-.204118	.002670
	(.0759)	(.0326)	(.0383)	(.0918)	(.0561)	(.0325)
K-1	-.123661	-.192370	-.126568	-.316031	-.189463	.807630
	(.0411)	(.0271)	(.0290)	(.0646)	(.0387)	(.0271)
T	-.195853	.014501	-.072630	-1.18135	-1.10872	.014501
	(.2562)	(.1767)	(.1598)	(.4012)	(.2416)	(.1767)
(W1+T)-1	.199619	.000811	.262462	.200430	-.062032	.000811
	(.0360)	(.0099)	(.0417)	(.0393)	(.0208)	(.0099)
t	.163991	.000667	.215618	.164658	-.050960	.000666
	(.0267)	(.0081)	(.0357)	(.0288)	(.0195)	(.0081)
G	.634655	-.012718	.649573	1.62194	.972364	-.012718
	(.2140)	(.1549)	(.1471)	(.3477)	(.2162)	(.1549)

3SLS covariance matrix of reduced form disturbances

	C	I	W1	Y	P	K
C	3.85421					
I	2.39570	2.03356				
W1	2.68518	2.17912	2.70338			
Y	6.24990	4.42926	4.86430	10.6792		
P	3.56473	2.25014	2.16092	5.81486	3.65394	
K	2.39570	2.03356	2.17912	4.42926	2.25014	2.03356

8. ITERATIVE THREE STAGE LEAST SQUARES

The authors do not know any published results obtained by this method. The method consists in iterating three stage least squares until convergence is reached up to six significant digits.

1	P	P-1	W1+W2
16.5590	.164510	.176564	.765801

1	P	P-1	K-1
42.8963	-.356532	1.01130	-.260200

1	Y+T-W2	(P+W1+T)-1	t
2.62477	.374779	.193651	.167926

Iterative 3SLS covariance matrix of

structural form disturbances

.914909		
.641739	4.55536	
-.434985	.734498	.605649

Iterative 3SLS estimated asymptotic covariance matrix of the structural coefficients

(all the numbers are multiplied by 10^4)

	Consumption Equation				Investment Equation				Private Wage Bill Equation			
	1	P	P-1	W1+W2	1	P	P-1	K-1	1	Y+T-W2	(P+W1+T)-1	t
1	14991.6	-145.995	-47.7547	-272.606	26843.9	-601.150	197.629	-97.8628	-6974.54	111.936	.772880	44.0983
P		92.5403	-64.0659	-8.86908	-1118.49	130.513	-107.917	3.39825	14.9395	-10.5602	10.6798	7.14102
P-1			81.1803	-4.81102	1090.98	-113.318	119.768	-5.67757	255.720	6.61675	-11.2632	4.03619
W1+W2				12.0825	-615.045	6.08574	-8.10488	3.21694	56.1063	-1.01077	.079286	-5.56427
1					1122300.	-16573.5	13321.9	-5278.73	7687.25	1049.96	-1214.01	-899.828
P						676.817	-570.203	72.2184	-80.5849	.680921	.684491	12.6025
P-1							618.889	-68.9591	-494.151	6.72052	1.56134	-18.3467
K-1								25.8770	10.5535	-5.84311	5.86985	4.92488
1									14293.6	-113.199	-124.287	148.972
Y+T-W2										9.67380	-8.06719	-3.36854
(P+W1+T)-1											10.4988	.919756
t												8.36891

Iterative 3SLS reduced form coefficients and asymptotic standard errors

	C	I	W1	Y	P	K
1	47.1385	27.2501	30.5040	74.3885	43.8845	27.2501
	(9.165)	(6.864)	(6.896)	(15.43)	(9.116)	(6.864)
P-1	.935721	.712856	.811503	1.64858	.837074	.712856
	(.1111)	(.0834)	(.0853)	(.1841)	(.1085)	(.0834)
W2	.656198	.062668	-.105363	.718866	-.175771	.062668
	(.0647)	(.0334)	(.0314)	(.0784)	(.0490)	(.0334)
K-1	-.121772	-.190575	-.117061	-.312346	-.195285	.809425
	(.0431)	(.0340)	(.0325)	(.7406)	(.4387)	(.0340)
T	-.074645	.305150	.086388	-.769495	-.855884	.305150
	(.1988)	(.1528)	(.1251)	(.3308)	(.2061)	(.1528)
(W1+T)-1	.203245	.019410	.277098	.222656	-.054442	.019410
	(.0325)	(.0108)	(.0391)	(.0376)	(.0176)	(.0108)
t	.176246	.016832	.240287	.193078	-.047210	.016832
	(.0237)	(.0094)	(.0360)	(.0284)	(.0180)	(.0094)
G	.467993	-.267585	.449888	1.20041	.750520	-.267585
	(.1664)	(.1338)	(.1157)	(.2880)	(.1823)	(.1338)

Iterative 3SLS covariance matrix of reduced form disturbances

	C	I	W1	Y	P	K
C	3.89940					
I	2.53259	2.39487				
W1	2.75142	2.56177	2.99265			
Y	6.43199	4.92746	5.31319	11.3595		
P	3.68057	2.36569	2.32054	6.04627	3.72573	
K	2.53259	2.39487	2.56177	4.92746	2.36569	2.39487

9. LIMITED INFORMATION INSTRUMENTAL VARIABLES EFFICIENT

The method used to compute the numerical values displayed in this section is described in Brundy and Jorgenson (1971) (LIVE method). It requires initial consistent but possibly inefficient estimates. These have been obtained, as suggested in Brundy and Jorgenson (1974,p.680), starting from ordinary least squares, computing fitted values of endogenous variables and using these as instruments for a first round estimator. Finally, fitted values of endogenous variables are again computed and used for the second round estimator. The numerical values obtained for the coefficient estimates are exactly equal to those presented in Brundy and Jorgenson (1974,p.694, last column).

1	P	P-1	W1+W2
16.8014	-.115631	.312132	.820507
1	P	P-1	K-1
21.6005	.107318	.652790	-.163778
1	Y+T-W2	(P+W1+T)-1	t
1.60111	.419711	.164768	.135058

LIVE covariance matrix of

structural form disturbances

1.44650		
.726599	1.53808	
-.339636	.273550	.486842

To obtain the numerical values displayed in the following table, the formulae in Brundy and Jorgenson (1971,p.215) have been used.

LIVE estimated asymptotic covariance matrix of the structural coefficients

(all the numbers are multiplied by 10^4)

	Consumption Equation				Investment Equation				Private Wage Bill Equation			
	1	P	P-1	W1+W2	1	P	P-1	K-1	1	Y+T-W2	(P+W1+T)-1	t
1	26248.3	-340.942	-14.3300	-470.044	28369.8	-703.511	209.865	-97.5226	-7429.83	115.355	4.78457	27.5467
P		254.026	-176.559	-25.5136	674.227	95.4291	-74.7891	-5.29344	-5.00866	-20.3692	21.1832	8.14311
P-1			197.227	-5.62531	178.485	-92.9807	99.2907	-1.16711	389.845	14.7676	-22.0183	5.10824
W1+W2				23.9412	-1019.98	14.8101	-13.8050	4.96720	21.8360	-.316950	-.048303	-5.99651
1					508267.	-7946.57	6540.95	-2396.27	2308.32	82.3157	-124.282	-119.651
P						319.809	-270.587	34.7940	-16.6612	2.66976	-2.47780	1.64775
P-1							276.743	-32.4327	-67.7421	-1.55710	2.78098	-2.61384
K-1								11.6696	-4.35009	-.508290	.601468	.671462
1									13520.5	-42.5745	-184.992	154.749
Y+T-W2										12.7105	-12.4304	-2.65201
(P+W1+T)-1											16.0647	.078004
t												8.62800

LIVE reduced form coefficients and asymptotic standard errors

	C	I	W1	Y	P	K
1	34.9797	25.1748	26.8486	60.1545	33.3059	25.1748
	(7.872)	(6.525)	(6.280)	(13.48)	(7.990)	(6.525)
P-1	.928815	.738969	.864755	1.66778	.803029	.738969
	(.1129)	(.0796)	(.0839)	(.1815)	(.1085)	(.0796)
W2	.745150	-.016925	-.114067	.728225	-.157708	-.016925
	(.0810)	(.0314)	(.0416)	(.0961)	(.0565)	(.0314)
K-1	-.068759	-.179221	-.104080	-.247980	-.143900	.820779
	(.0358)	(.0323)	(.0284)	(.0635)	(.0376)	(.0323)
T	.119121	-.106534	.005283	-.987413	-.992696	-.106534
	(.2595)	(.1998)	(.1746)	(.4158)	(.2412)	(.1998)
(W1+T)-1	.211579	-.004806	.251553	.206773	-.044780	-.004806
	(.0464)	(.0090)	(.0517)	(.0479)	(.0193)	(.0090)
t	.173428	-.003939	.206194	.169489	-.036705	-.003939
	(.0342)	(.0074)	(.0411)	.(0348)	(.0174)	(.0074)
G	.419831	.094294	.635494	1.51412	.878629	.094294
	(.2123)	(.1772)	(.1610)	(.3610)	(.2162)	(.1772)

LIVE covariance matrix of reduced form disturbances

	C	I	W1	Y	P	K
C	3.91261					
I	2.39371	1.98945				
W1	2.90461	2.09278	2.79873			
Y	6.30632	4.38316	4.99739	10.6895		
P	3.40172	2.29038	2.19866	5.69209	3.49344	
K	2.39371	1.98945	2.09278	4.38316	2.29038	1.98945

10. ITERATIVE INSTRUMENTAL VARIABLES

LIVE has been iterated until convergence has been reached up to six significant digits. Estimated coefficients are practically equal to those published in Bergström (1974,p.225), apart from a scaling factor 10 in four coefficients and the inverted order of two digits in the constant term of the second equation.

1	P	P-1	W1+W2
16.7858	-.117503	.312609	.821456
1	P	P-1	K-1
21.6064	.107126	.652955	-.163805
1	Y+T-W2	(P+W1+T)-1	t
1.59635	.420614	.163914	.134838

IIV covariance matrix of
structural form disturbances

1.45233		
.729292	1.53882	
-.341983	.270235	.485912

To obtain the numerical values displayed in the following table, the formulae in Brundy and Jorgenson (1971,p.215) have been used.

IIV estimated asymptotic covariance matrix of the structural coefficients

(all the numbers are multiplied by 10^4)

	Consumption Equation				Investment Equation				Private Wage Bill Equation			
	1	P	P-1	W1+W2	1	P	P-1	K-1	1	Y+T-W2	(P+W1+T)-1	t
1	24435.0	-283.152	-21.1759	-448.737	28361.8	-641.567	188.490	-101.074	-5436.94	81.8593	6.17151	23.7613
P		207.290	-145.902	-19.9790	574.258	81.8576	-64.3999	-4.50009	1.76302	-15.4270	15.9477	5.55266
P-1			170.572	-7.41996	255.446	-81.5228	88.9461	-1.67128	278.426	11.0145	-16.2096	3.98064
W1+W2				21.8824	-1010.03	14.3195	-13.4361	4.92881	16.5079	-.040150	-.243105	-4.40530
1					524036.	-7934.69	6572.70	-2478.46	6501.36	232.136	-350.330	-344.993
P						302.391	-257.556	35.1377	-48.3576	8.02050	-7.47306	4.42747
P-1							267.942	-32.9700	-191.543	-4.75305	8.22613	-7.25428
K-1								12.0945	-12.0659	-1.44527	1.70498	1.94024
1									13244.7	-47.7221	-174.996	152.676
Y+T-W2										13.6065	-13.2695	-2.87612
(P+W1+T)-1											16.7615	.345859
t												8.50121

IIV reduced form coefficients and asymptotic standard errors

	C	I	W1	Y	P	K
1	34.9688	25.1679	26.8907	60.1367	33.2460	25.1679
	(7.639)	(6.453)	(6.594)	(13.30)	(7.489)	(6.453)
P-1	.929367	.738943	.865629	1.66831	.802682	.738943
	(.1136)	(.0799)	(.0892)	(.1832)	(.1046)	(.0799)
W2	.746460	-.016778	-.113699	.729682	-.156618	-.016778
	(.0756)	(.0302)	(.0400)	(.0902)	(.0523)	(.0302)
K-1	-.068805	-.179198	-.104313	-.248003	-.143689	.820802
	(.0344)	(.0321)	(.0295)	(.0624)	(.0354)	(.0321)
T	.121862	-.106151	.006608	-.984289	-.990897	-.106151
	(.2366)	(.1933)	(.1638)	(.3894)	(.2256)	(.1933)
(W1+T)-1	.211181	-.004747	.250743	.206434	-.044309	-.004747
	(.0472)	(.0085)	(.0531)	(.0489)	(.0179)	(.0085)
t	.173720	-.003905	.206265	.169816	-.036449	-.003905
	(.0351)	(.0072)	(.0411)	(.0350)	(.0162)	(.0072)
G	.420042	.093971	.636815	1.51401	.877198	.093971
	(.2021)	(.1714)	(.1628)	(.3472)	(.2019)	(.1714)

IIV covariance matrix of reduced form disturbances

	C	I	W1	Y	P	K
C	3.91787					
I	2.39476	1.98953				
W1	2.90909	2.09351	2.80179			
Y	6.31263	4.38429	5.00260	10.6969		
P	3.40354	2.29077	2.20082	5.69431	3.49349	
K	2.39476	1.98953	2.09351	4.38429	2.29077	1.98953

11. FULL INFORMATION INSTRUMENTAL VARIABLES EFFICIENT

The method used to compute the numerical values displayed in this section is described in Brundy and Jorgenson (1971) (FIVE method). It requires initial consistent but possibly inefficient estimates. These have been obtained, as suggested in Brundy and Jorgenson (1974,p.680), starting from ordinary least squares, computing fitted values of endogenous variables and using these as instruments for a first round estimator. Finally, fitted values of endogenous variables are again computed and used for the second round estimator. The results obtained do not coincide with those published in Brundy and Jorgenson (1974,p.695, col.5, IIV method).

1	P	P-1	W1+W2
16.4570	.091267	.198055	.789598

1	P	P-1	K-1
24.7860	.029683	.717039	-.178374

1	Y+T-W2	(P+W1+T)-1	t
1.93827	.383522	.196435	.157667

FIVE covariance matrix of
structural form disturbances

.937915		
.442160	1.86804	
-.379889	.452247	.565780

The formulae in Brundy and Jorgenson (1971,p.215) have been used to compute the values in the following table.

FIVE estimated asymptotic covariance matrix of the structural coefficients

(all the numbers are multiplied by 10^4)

	Consumption Equation				Investment Equation				Private Wage Bill Equation			
	1	P	P-1	W1+W2	1	P	P-1	K-1	1	Y+T-W2	(P+W1+T)-1	t
1	24447.2	-420.089	-36.7052	-385.563	22896.0	-625.812	135.899	-70.7251	-7310.93	128.703	-11.0903	35.0015
P		200.330	-144.179	-14.5240	-460.351	119.988	-96.8379	.097394	9.28678	-14.1087	14.4525	6.81209
P-1			169.209	-7.20913	726.845	-106.989	112.305	-3.78495	344.231	11.1502	-17.4850	5.12053
W1+W2				18.0550	-642.516	8.46724	-8.18160	3.15960	31.1566	-1.75978	1.28533	-5.63911
1					364431.	-5715.72	4470.71	-1697.71	4017.96	437.958	-522.113	-192.857
P						283.957	-237.389	23.9760	-47.2683	-3.40482	4.34162	2.86028
P-1							245.925	-22.3867	-34.9999	4.01509	-3.55492	-3.58453
K-1								8.27624	-12.9731	-2.22550	2.52873	1.01372
1									12737.3	-53.0237	-160.663	134.862
Y+T-W2										9.79320	-9.22861	-1.91790
(P+W1+T)-1											12.3290	-.339371
t												7.70685

FIVE reduced form coefficients and asymptotic standard errors

	C	I	W1	Y	P	K
1	42.3451	25.9787	28.1420	68.3238	40.1819	25.9787
	(10.27)	(6.285)	(6.631)	(15.79)	(9.821)	(6.285)
P-1	.938770	.741964	.841033	1.68073	.839700	.741964
	(.1294)	(.0720)	(.0793)	(.1912)	(.1216)	(.0720)
W2	.668248	-.006184	-.129606	.662064	-.208330	-.006184
	(.0888)	(.0365)	(.0435)	(.1099)	(.0686)	(.0365)
K-1	-.102878	-.183617	-.109877	-.286494	-.176618	.816383
	(.0479)	(.0307)	(.0305)	(.0747)	(.0464)	(.0307)
T	-.161025	-.033238	-.074504	-1.19426	-1.11976	-.033238
	(.3164)	(.1966)	(.1839)	(.4824)	(.2987)	(.1966)
(W1+T)-1	.212931	-.001970	.277343	.210961	-.066383	-.001970
	(.0406)	(.0117)	(.0427)	(.0448)	(.0250)	(.0117)
t	.170907	-.001582	.222607	.169326	-.053281	-.001582
	(.0289)	(.0094)	(.0359)	(.0326)	(.0224)	(.0094)
G	.576754	.029391	.615992	1.60614	.990153	.029391
	(.2679)	(.1740)	(.1637)	(.4192)	(.2680)	(.1740)

FIVE covariance matrix of reduced form disturbances

	C	I	W1	Y	P	K
C	3.68908					
I	2.36746	1.99796				
W1	2.59795	2.12294	2.65395			
Y	6.05654	4.36542	4.72089	10.4220		
P	3.45859	2.24249	2.06694	5.70108	3.63414	
K	2.36746	1.99796	2.12294	4.36542	2.24249	1.99796

12. FULL INFORMATION MAXIMUM LIKELIHOOD

The results presented in this section have been obtained and checked using the three following algorithms:

a) The program described in Chapman and Fair (1972).
b) The maximization of the concentrated log-likelihood function without restrictions on the covariance matrix of the structural disturbances.
c) The iteration of the full information instrumental variables method described in section 11.

The estimated coefficients are equal (except the number of significant decimal digits or the use of scaling factors) to those in Bergström (1974,p.225), Chernoff and Divinsky (1953,pp.250,284, method F.I.N.D.), Chow (1968,pp.109,110) and Hausman (1974,p.649). Correct results are mentioned also in Belsley (1979) and displayed in detail in Belsley (1977). Results which do not seem correct are in Brundy and Jorgenson (1974,p.695). In Hendry (1969, appendix 1, p.2), correct values for all the estimated coefficients, except that of the constant term of the third equation, are reported. This happens because the computation has been performed using 1920 as base year for the variable time, even if this choice does not appear explicitly in Hendry's paper (where Golberger (1964) is cited as source of data). The numerical values in Hendry (1971,p.264, table 2) should not be considered, being based on a sample period 1922 to 1941. However, as a matter of comparison, the authors have repeated also these computations, obtaining the same results as Hendry, except again the constant term of the last equation, for the reasons just above mentioned.

Due to the absence of restrictions on the covariance matrix of the structural disturbances, the results in this section are different from and should not be confused with the results presented in Chernoff and Divinsky (1953,pp.250,299, method F.I.O.), Klein (1950,p.68), Seaks (1974,p.2) and Theil and Boot (1962,pp.136,137), which are, sometimes, referred to as FIML estimates.

1	P	P-1	W1+W2
18.3433	-.232389	.385673	.801844
1	P	P-1	K-1
27.2639	-.801006	1.05185	-.148099
1	Y+T-W2	(P+W1+T)-1	t
5.79429	.234118	.284677	.234835

The following covariance matrix of structural form disturbances coincide with the S matrix displayed in Hendry (1969, appendix 1, p.2).

FIML covariance matrix of
structural form disturbances

2.10437		
3.87955	12.7728	
.481826	3.85778	1.80119

The asymptotic covariance matrix of the structural coefficients has been computed either from the Hessian matrix of the concentrated log-likelihood function (methods (a) and (b) above mentioned), or by the same formula used for the FIVE method. As the numerical results are quite different, both of them are displayed in the following pages.

The numerical values of the following table, obtained from the Hessian matrix of the concentrated log-likelihood function, coincide, at least in the first digits, with those in Chernoff and Divinsky (1953,p.288) where, however, the variances and covariances related to the constant terms of the three stochastic equations are not displayed.

FIML estimated asymptotic covariance matrix of the structural coefficients

(all the numbers are multiplied by 10^4)

	Consumption Equation				Investment Equation				Private Wage Bill Equation			
	1	P	P-1	W1+W2	1	P	P-1	K-1	1	Y+T-W2	(P+W1+T)-1	t
1	213918.	-24891.3	11138.9	605.044	159879.	-34865.6	12887.6	1096.36	77440.4	-2783.88	1551.77	1437.19
P		3370.40	-1642.24	-123.977	-13960.5	4575.21	-1942.41	-157.149	-9690.97	367.872	-213.887	-160.037
P-1			910.336	40.7793	5687.69	-2241.20	1143.58	67.0328	3536.19	-150.609	95.0052	68.6069
W1+W2				19.7966	-370.654	-137.645	28.7672	11.0947	688.626	-23.2217	12.1754	3.43264
1					909064.	-39340.7	14826.4	-2400.55	84275.0	624.682	-2068.69	-119.437
P						7057.67	-3029.19	-150.928	-14988.7	494.187	-253.351	-224.594
P-1							1800.59	34.1716	1611.09	-119.718	96.2101	31.0067
K-1								21.8967	719.941	-34.9695	23.8029	16.9838
1									105010.	-2471.54	763.660	1482.55
Y+T-W2										90.2657	-50.8670	-44.2438
(P+W1+T)-1											39.5143	20.2567
t												31.9523

FIML reduced form coefficients and asymptotic standard errors

	C	I	W1	Y	P	K
1	24.6776	10.3912	14.0045	35.0689	21.0643	10.3912
	(8.799)	(6.976)	(5.044)	(15.41)	(10.81)	(6.976)
P-1	.692003	.530129	.570800	1.22213	.651332	.530129
	(.1846)	(.1456)	(.1422)	(.3231)	(.1876)	(.1456)
W2	.800640	.075801	-.028927	.876441	-.094632	.075801
	(.0947)	(.0307)	(.0348)	(.1069)	(.0733)	(.0307)
K-1	-.000900	-.091447	-.021620	-.092346	-.070727	.908553
	(.0480)	(.0414)	(.0282)	(.0876)	(.0600)	(.0414)
T	.238667	.405701	.150858	-.355631	-.506490	.405701
	(.2629)	(.1582)	(.0795)	(.4016)	(.3448)	(.1582)
(W1+T)-1	.297597	.028175	.360946	.325771	-.035175	.028175
	(.0617)	(.0121)	(.0542)	(.0676)	(.0263)	(.0121)
t	.245493	.023242	.297751	.268735	-.029016	.023242
	(.0499)	(.0113)	(.0504)	(.0576)	(.0223)	(.0113)
G	.006076	-.382530	.145983	.623545	.477562	-.382530
	(.3225)	(.1567)	(.1565)	(.4617)	(.3131)	(.1567)

As regards the following covariance matrix of structural coefficients, computed from Iterative FIVE, comparable results for the standard errors can be found in Hendry (1969, appendix 1, p.2, FIML method with 21 observations); once again, the standard error of the constant term of the last equation is different, due to the base shift for the variable time. Correct numerical results for the standard errors are displayed also in Hausman (1974,p.649), with the exception of the standard errors for the constant terms of the various equations.

FIML estimated asymptotic covariance matrix of the structural coefficients

computed from Iterative FIVE (all the numbers are multiplied by 10^4)

	Consumption Equation				Investment Equation				Private Wage Bill Equation			
	1	P	P-1	W1+W2	1	P	P-1	K-1	1	Y+T-W2	(P+W1+T)-1	t
1	61753.9	-5842.57	3025.48	-279.981	90931.7	-8778.32	4241.50	-51.2432	280.698	28.4010	-30.3006	167.875
P		973.166	-622.990	-9.46118	-8091.47	1394.78	-898.293	-3.77272	-189.216	9.45562	-6.53025	1.37123
P-1			472.442	-5.77796	3628.03	-899.733	690.225	1.32483	-262.317	-1.57533	6.15542	1.69030
W1+W2				12.8832	-285.169	-1.10834	-8.97194	2.24851	179.369	-3.91295	.959408	-5.27268
1					630071.	-21833.7	9252.47	-2028.61	37904.9	1276.77	-1944.40	-485.911
P						2414.95	-1542.45	31.4395	-684.497	12.2665	-.900100	5.96340
P-1							1242.28	-17.6738	-1432.04	5.23728	19.2720	-20.0034
K-1								8.91301	-5.26299	-7.82923	8.19968	3.55502
1									32559.6	-407.447	-124.716	307.232
Y+T-W2										23.8320	-17.6567	-9.29463
(P+W1+T)-1											20.4382	4.32826
t												11.9027

FIML reduced form coefficients and asymptotic standard errors
computed from Iterative FIVE

	C	I	W1	Y	P	K
1	24.6776	10.3912	14.0045	35.0689	21.0643	10.3912
	(5.397)	(4.203)	(3.472)	(9.105)	(6.187)	(4.203)
P-1	.692003	.530129	.570800	1.22213	.651332	.530129
	(.0960)	(.0786)	(.0685)	(.1677)	(.1081)	(.0786)
W2	.800640	.075801	-.028927	.876441	-.094632	.075801
	(.0497)	(.0236)	(.0153)	(.0534)	(.0397)	(.0236)
K-1	-.000900	-.091447	-.021620	-.092346	-.070727	.908553
	(.0231)	(.0193)	(.0119)	(.0392)	(.0284)	(.0193)
T	.238667	.405701	.150858	-.355631	-.506490	.405701
	(.1620)	(.1019)	(.0685)	(.2488)	(.1905)	(.1019)
(W1+T)-1	.297597	.028175	.360946	.325771	-.035175	.028175
	(.0397)	(.0093)	(.0431)	(.0431)	(.0154)	(.0093)
t	.245493	.023242	.297751	.268735	-.029016	.023242
	(.0272)	(.0085)	(.0363)	(.0326)	(.0137)	(.0085)
G	.006076	-.382530	.145983	.623545	.477562	-.382530
	(.1558)	(.0961)	(.0687)	(.2384)	(.1790)	(.0961)

The covariance matrix of reduced form disturbances displayed in the following table coincides (at least for the initial 3x3 block, which is the dimension to which Hendry (1969) reconducts the computation) with the $\hat{\Omega}$ matrix displayed in Hendry (1969, appendix 2, $\hat{\Omega}$ matrix for FIML with 21 observations).

FIML covariance matrix of reduced form disturbances

	C	I	W1	Y	P	K
C	5.20882					
I	4.17322	3.79775				
W1	4.58763	4.24216	4.98448			
Y	9.38204	7.97098	8.82980	17.3530		
P	4.79441	3.72881	3.84532	8.52322	4.67791	
K	4.17322	3.79775	4.24216	7.97098	3.72881	3.79775

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