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Testing the New Keynesian Phillips curve through Vector Autoregressive models: Results from the Euro area*

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Abstract

This paper addresses the issue of testing the 'hybrid' New Keynesian Phillips Curve (NKPC) through Vector Autoregressive (VAR) systems and likelihood methods, giving special emphasis to the case where the variables are non-stationary. The idea is to use a VAR for both the inflation rate and the explanatory variable(s) to approximate the dynamics of the system and derive testable restrictions. Attention is focused on the 'inexact' formulation of the NKPC. Empirical results over the period 1971-1998 show that the NKPC is far from providing a 'good first approximation' of inflation dynamics in the Euro area.

Keywords: Inflation dynamics, Forecast model, New Keynesian Phillips Curve, Forward-looking behaviour, VAR expectations.

J.E.L. Classification: C32, C52, E31, E32.

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1 Introduction

The Phillips curve plays a central role in our understanding of business cycles and the management of monetary policy. Several of the New Keynesian models of inflation dynamics, including the models of staggered contracts of Taylor (1979) and Calvo (1983), and the quadratic price adjustment cost model of Rothemberg (1982), have a common formulation which is similar to the expectations-augmented Phillips curve of Friedman and Phelps (Roberts, 1995). The empirical literature on the so-called New Keynesian Phillips curve (NKPC) has expanded rapidly without consensus as to the role of forward-looking components in inflation dynamics.

The recent success of the NKPC can be attributed to Galí and Gertler (1999) and Galí et al. (2001), where the so-called 'hybrid' version of the Phillips curve is found to provide a 'good first approximation' for inflation in the US and the Euro area. On the other hand, the use of the NKPC as a consensus model of inflation dynamics seems to disregard the idea that there exists many sources of price growth, see e.g. Hendry (2001). Furthermore, aside from the subtle issue of empirically disentangling between forward and backward-looking behaviour, the process of data aggregation can blur the actual single-agent behavioural relationships connecting prices and other macroeconomic variables at the country level.

This paper contributes to the empirical literature by addressing the econometric investigation of the NKPC through Vector Autoregressive (VAR) systems, giving special emphasis to the case where the variables are non-stationary. VARs are used extensively to proxy agents' expectations and to derive a set of (testable) cross-equation restrictions with the theoretical model, which can be used to estimate and test the NKPC, see e.g. Fuhrer and Moore (1995), Fuhrer (1997), Sbordone (2005), Rudd and Whelan (2006) and Kurmann (2006). However, when the roots of the VAR are close to the unit circle, the application of standard asymptotic inference may result in considerable size distortion and power losses, given the relatively small sample lengths which typically characterize macroeconomic analysis, see e.g. Johansen (2006). We show that in these circumstances the econometric investigation of the NKPC can be carried out by treating (aggregate) variables as realization of integrated of order one (I(1)) processes. Indeed, although theory at the individual (firm) level is based on stationary variables, non-stationarity may result from the aggregation of

sectoral and regional/national Phillips curves.

Our method is inspired by Sargent's (1979) VAR-based analysis of Euler equations, and generalizes to some extent the likelihood-based estimation and testing strategy set out in Johansen and Swensen (1999) and Fanelli (2002) for forward-looking models with I(1) variables. The idea is to nest the NKPC within a dynamic system (the VAR) serving as agents' forecast model. The VAR, including inflation and its explanatory variable(s), can be reparameterized in Vector Equilibrium Correction (VEqC) form when time-series are non stationary.

We focus on the 'inexact' version of the NKPC, namely on a formulation of the forward-looking model of inflation dynamics which incorporates an exogenous disturbance term modelled as a martingale difference sequence (MDS) which captures (unexplained) transitory deviations from the theory. Aside from studies based on 'miniature' DSGE models (e.g. Lindé, 2005), Bårdsen et al. (2004) and Kurmann (2006) provide existing examples where the 'inexact' NKPC is dealt with. We extend the analysis to the case where the agents' forecast model is a non-stationary, possibly cointegrated, VAR.

The proposed method is applied to quarterly inflation dynamics in the Euro area over the period 1971-1998. In line with the conclusions of Bårdsen et al. (2004), based on the encompassing principle, our results suggest that the hybrid formulation of the NKPC suffers from 'missing dynamics', in the sense we explain in the paper.

The paper is organized as follows. Section 2 introduces the hybrid NKPC and Section 3 addresses the empirical issue of non-stationarity. Section 4 sketches the VAR-based investigation of the 'inexact' NKPC. Section 5 summarizes empirical results for the Euro area over the period 1971-1998, and Section 6 contains some concluding remarks. Technical details are outlined in the Appendix.

2 The New Keynesian Phillips curve

The hybrid formulation of the NKPC reads as a Linear Rational Expectations (LRE) model where the inflation rate depends on the expected future value of inflation rate, lagged inflation and a single or a set of driving variables. Following Gali et al. (1999) and Galì et al. (2001), the 'final' structural form

of the NKPC can be formulated as

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda' x_t + u_t \tag{1}$$

where π_t is the inflation rate at time t, x_t the vector of explanatory variable(s), $E_t \pi_{t+1}$ is the expected value at time t of the inflation rate prevailing at time $t+1$, u_t a disturbance term which we discuss in Section 4 and γ_f, γ_b and λ are structural parameters, with λ a scalar or vector, depending on the dimensions of x_t. Expectations are conditional upon the information set available at time t, i.e. $E_t \pi_{t+1} = E(\pi_{t+1} \mid \mathcal{F}_t)$.

In most recurrent specifications, x_t is a single driving variable (λ is thus a scalar) capturing demand pressure, and proxied by the output gap, the unemployment rate, or a measure of firms' real marginal costs. In small-open economy versions of the NKPC, x_t is a vector incorporating unit labour costs and the price of imported intermediate goods, see e.g. Petursson (1998) and Batini et al. (2005).

Equation (1) can be derived through several routes within the New Keynesian paradigm, see e.g. Roberts (1995). Galì et al. (2001) refer to the rational expectations staggered-contracting model of Calvo (1983). In general, $\gamma_f \geq 0$, $\gamma_b \geq 0$, $\lambda > 0$ and $\gamma_b + \gamma_f \leq 1$; in the Calvo model γ_f, γ_b and λ depend on other 'deep' structural parameters related to firms' discount factor, the fraction of backward-looking firms, and the average time over which prices are kept fixed, see Galì and Gertler (1999).[1]

The NKPC can be also regarded as the aggregate supply equation of 'miniature' dynamic stochastic general equilibrium (DSGE) policy models, derived under the hypothesis of intertemporal micro-optimizing households and firms. Typically these models include a forward-looking IS curve, the NKPC and an interest rate rule; Henry and Pagan (2004) provide an overview. The present paper focuses on the econometric analysis of model (1) by assuming that the process generating x_t is in reduced form.[2]

[1] From the policy point of view, the NKPC implies that a fully credible disinflation implies a positive sacrifice ratio which increases with the fraction of backward-looking firms. On the other hand, if $\gamma_b = 0$, the purely forward-looking NKPC entails that a fully credible disinflation has no output costs. The inclusion of lagged inflation terms in the base 'pure forward-looking' version of the model ($\gamma_b = 0$) can be also motivated by referring to models with two (or more) period overlapping wage contracts as in Fuhrer and Moore (1995).

[2] The literature on LRE models shows that the dynamic specification of the x_t variable(s) is crucial for the identification of the structural parameters, even when these are thought to

3 Addressing the empirical analysis

The estimation of the NKPC (1) is usually carried out by treating inflation and its driving variable(s) as the realization of stationary processes; Petursson (1998), Bårdsen et al. (2004) and Boug et al. (2006) represent remarkable exceptions. Using US quarterly data, Fuhrer and Moore (1995), Section 2, recognize the empirical relevance of unit roots when dealing with inflation data, but do not appeal to I(1) techniques in the estimation of their forward-looking model of inflation dynamics. This limited attention to non-stationarity has its roots in the underlying theory, which is intrinsically built on mean-reverting variables, and in the observation that DSGE models are obtained as linearized approximations of nonlinear models around some steady state.

However, while theory is formulated at the single-agent level, estimation is usually based on aggregate data. Aggregation may have both theoretical and empirical consequences. For instance, Hughes Hallet (2000) shows that the aggregation of sectoral, regional/national Phillips curves may yield an inflation-unemployment trade-off which is not vertical in the long run, despite the 'individual' curves being vertical. On the other hand, the time-series literature shows that the aggregation of simple, possibly dependent, dynamic micro-relationships may result in aggregate series which possibly display long-memory and unit root behaviour, e.g. Granger (1980). In line with these considerations, O'Reilly and Whelan (2005), for example, find that the persistence of Euro area inflation is very close to one and stable over time.

Whether inflation is best described as an highly persistent stationary process or as a unit root process, has a number of economic and empirical implications which are not addressed in the present paper. A detailed discussion may be found in Culver and Papell (1997). Similarly, although the output gap is conceptually a stationary variable, there is no guarantee that methods based, for example, on the Hodrick-Prescott (HP) filter, or on regressions of output on deterministic terms, actually deliver stationary time-series. Computing, for instance, the log of labour income share 'in deviation from the steady state' by removing some constant from the corresponding time-series, does not guarantee that the resulting variable is stationary. Moreover, test statistics based on standard asymptotic theory and the typical sample lengths of macroeconomic

be exogenously given (Pesaran, 1987, Ch. 6). Bårdsen et al. (2004) and Mavroeidis (2005) show that the empirical analysis of the NKPC (1) can be hardly carried out by ignoring the process-generating explanatory variables.

analysis may suffer considerable size distortion and power losses when the roots of the characteristic equation are close to the unit circle. Johansen (2006) shows that if in DSGE models one insists that a root very close to unity is a stationary root, then many more observations than those usually available for conducting inferences on steady state values are needed. Hence, fixing the number of unit roots of the system when there is a strong suspicion that its variables might be approximated by stochastic trends, may in principle relieve some small sample inferential issues.

4 Testing the NKPC

As with many other economic theories, the NKPC specifies a relationship involving future expectations (forecasts) of a set of variables. This relationship implies a set of restrictions which may be tested, along the lines of Sargent (1979), against some general unrestricted dynamic model for $Y_t = (\pi_t, x_t')'$ such as a VAR serving as agents' forecast system.

In deriving VAR restrictions and testing the model, however, a relevant issue is whether the NKPC (1) is specified in 'exact' form ($u_t = 0$), or as an 'inexact' LRE model ($u_t \neq 0$). Abstracting from contributions based on 'small scale' DSGE models, empirical investigations of the NKPC through 'full-information' methods are usually based on cross-equation restrictions derived with respect to the 'exact' model, see, among others, Sbordone (2005) and Ruud and Whelan (2006). A part from the myriad of possible economic interpretations that one can attach to a non zero u_t term in (1), the 'inexact' specification of the NKPC is more flexible and appealing since, if for example u_t obeys a MDS with respect to the information set $\mathcal{F}_t$, that is $E(u_t \mid \mathcal{F}_{t-1}) = 0$, the model embodies a quantity capturing temporary (unexplained) deviations from theoretical predictions. For this reason, a NKPC with $u_t = 0$ results in tighter, although algebraically less involved, VAR constraints. Bårdsen et al. (2004) and Kurmann (2006) take an explicit stand on the 'inexact' NKPC. However, whereas the former recognize that Euro area inflation dynamics resembles the behaviour of a unit root process, the latter treats variables as stationary time-series.[3] In this section we generalize the VAR-based analysis of the NKPC to

[3] Unlike previous likelihood-based findings on the US economy, Kurmann (2006) shows that results coincide by and large with Gali and Gertler's (1999) GMM estimates, confirming that conditional on marginal cost being (correctly) measured by labour income share, forward-

the non-stationary framework.

To approximate the agents' expectation-generating system, we consider the $p \times 1$ vector $Y_t = (\pi_t,\ x_t')'$, $p = (q+1)$, where x_t can be a scalar ($q = 1$) or a vector ($q \geq 2$) of explanatory variables, and the VAR(k) representation

$$Y_t = A_1 Y_{t-1} + \ldots + A_k Y_{t-k} + \mu_0 + \mu_d D_t + \varepsilon_t \tag{2}$$

where $A_1, \ldots, A_k$ are $p \times p$ matrices of parameters, k is the lag length, Y_{-p}, ..., Y_{-1}, Y_0, are given, μ_0 is a $p \times 1$ constant, D_t is a $d \times 1$ vector containing deterministic terms (linear trend, seasonal dummies, intervention dummies and so on) and μ_d is the corresponding $p \times d$ matrix of parameters. Moreover, $\varepsilon_t \sim N(0, \Omega)$ is a $p \times 1$ MDS with respect to the sigma-field $I_t = \sigma\{Y_t,\ Y_{t-1},\ ...,\ Y_1\} \subseteq \mathcal{F}_t$, and it is assumed that the parameters $(A_1, \ldots, A_k,\ \mu_0,\ \mu_d,\ \Omega)$ are time invariant and that the roots of the characteristic equation associated with the VAR

$$\det(A(z)) = \det(I_p - A_1 z - A_2 z^2 - ... - A_k z^k) = 0 \tag{3}$$

are such that $\mid z \mid > 1$ or $z = 1$.

The VAR(k) (2) can be written in Vector Equilibrium Correction (VEqC) form

$$\Delta Y_t = \Pi Y_{t-1} + \Phi_1 \Delta Y_{t-1} + \ldots + \Phi_{k-1} \Delta Y_{t-k+1} + \mu_0 + \mu_d D_t + \varepsilon_t \tag{4}$$

where $\Pi = -(I_p - \sum_{i=1}^{k} A_i)$ is the long run impact matrix, and $\Phi_j = -\sum_{i=j+1}^{k} A_i$, $j = 1, ..., k-1$. When there are exactly $p - r$ unit roots in the system, $rank(\Pi) = r$, $0 < r < p$, in (4), and $\Pi = \alpha\beta'$, with α and β two $p \times r$ full rank matrices, see Johansen (1996).

Using simple algebra, the NKPC (1) can be expressed in error-correction form

$$\Delta \pi_t = \psi E_t \Delta \pi_{t+1} + \omega z_t + u_t^* \tag{5}$$

where, provided that $\gamma_b + \gamma_f < 1$, $z_t = (\pi_t - \xi'\ x_t)$, $\xi = \frac{\lambda}{1-\gamma_f-\gamma_b}$, $\psi = \frac{\gamma_f}{\gamma_b}$, $\omega = \left(\frac{\gamma_f+\gamma_b-1}{\gamma_b}\right)$ and $u_t^* = \gamma_b u_t$. In the parameterization (5) z_t reads as the driving variable of the acceleration rate. Interestingly, if π_t and x_t are generated by I(1) processes, it turns out that z_t must be stationary for (5) to be a balanced model.[4] Apparently (5) involves only two parameters, ψ and ω, which in turn

looking behaviour is an important feature of price setting.

[4] Observe that $\gamma_f + \gamma_b = 1$ is at odds with a NKPC model where π_t and x_t are cointegrated. It can be easily proved, however, that $\gamma_f + \gamma_b = 1$ is consistent with the presence of unit roots in the system.

depend on γ_f and γ_b; however, from the definitions above it turns out that the third structural parameter, λ, is embedded in the definition of z_t. Hence, given γ_f and γ_b and an estimate of ξ, $\widehat{\xi}$, λ is determined by $\lambda = (1 - \gamma_f - \gamma_b)\widehat{\xi}$.

By conditioning both sides of (5) with respect to I_{t-1}, use of the law of iterated expectations and exploiting the MDS property of u_t (u_t^*) yields the relation

$$E(\Delta\pi_t \mid I_{t-1}) = \psi E(\Delta\pi_{t+1} \mid I_{t-1}) + \omega E(z_t \mid I_{t-1}) \tag{6}$$

which can be used to derive cross-equation restrictions once expectations are replaced by the corresponding VEqC-based forecasts. Using the companion form representation of the system, and incorporating the restriction $z_t = (\pi_t - \xi' x_t) = \beta' Y_t$ (implying that the cointegration rank of the system is $r = 1$), it is possible to retrieve a set of nonlinear restrictions between the VEqC and the NKPC. In the Appendix we outline a simple method for deriving the cross-equation restrictions between (4) and (5). The procedure is based on a particular representation of the VEqC (4) subject to $\Pi = \alpha\beta'$: we show that for given cointegration rank r and cointegration matrix β, the VEqC (4) can be represented as a stable VAR(k) of the form

$$W_t = B_1 W_{t-1} + ... + B_k W_{t-k} + \mu^0 + \mu_d^0 D_t + \varepsilon_t^0 \tag{7}$$

where the $p \times 1$ vector W_t is defined as

$$W_t = \begin{pmatrix} \beta' Y_t \\ v' \Delta Y_t \end{pmatrix} \equiv \begin{pmatrix} W_{1t} \\ W_{2t} \end{pmatrix} \begin{matrix} r \times 1 \\ (p-r) \times 1 \end{matrix}, \tag{8}$$

v is a $p \times (p-r)$ matrix such that $\det(v'\beta_\perp) \neq 0$, $\beta_\perp$ is the orthogonal complement of β (Johansen, 1996), and B_i, $i = 1, ..., k$, μ^0, μ_d^0 and ε_t^0 are defined (and constrained) suitably. The attractive feature of the representation (7)-(8) is that for $r = 1$ and $\beta = (1, -\xi')'$,[5] and for a suitable choice of v, the conditional expectations entering (6) can be computed through standard methods and therefore a set of cross-equation restrictions can be retrieved along the lines of Campbell and Shiller (1987). Indeed, by using the system (7)-(8) to compute the forecasts $E(\Delta\pi_t \mid I_{t-1})$, $E(\Delta\pi_{t+1} \mid I_{t-1})$ and $E(z_t \mid I_{t-1})$, and substituting these forecasts into (6), yields the following set of cross-equation restrictions

$$g_\pi' B(I_{pk} - \psi B) - \omega g_z' B = 0_{pk}' \tag{9}$$

[5] As shown in Section 5, $z_t = \beta' Y_t$ may also include a constant when μ_0 in (4) is restricted to lie in the cointegration space (Johansen, 1996).

where B is the companion matrix of (7), and g_π and g_z are two (known) selection vectors (see Appendix for details). Using the definitions of ψ and ω, the cross-equation restrictions (9) can be also written as

$$g'_\pi \gamma_b B(I_{pk} - \gamma_f B) - g'_z(\gamma_f + \gamma_b - 1)B = 0'_{pk}. \tag{10}$$

It can be shown that for suitable values of k, the VAR(k) (7) is locally identifiable under the cross-equation restrictions (10), with a number of overidentifying constraints depending on p and r, see Appendix. Hence, once β is fixed at its super-consistent estimate, the system (7)-(8) can be estimated both unrestrictedly and subject to the constraints (10), and likelihood ratio (LR) tests for the NKPC can be computed.

5 Results from the Euro area

Using Euro area data, Bårdsen et al. (2004) have investigated the 'inexact' version of the NKPC. These authors conclude, using encompassing techniques, that the forward-looking model of inflation dynamics is almost indistinguishable from standard dynamic mark-up equations. Bårdsen et al. (2004) also recognize that Euro area inflation resembles the dynamics of a unit root process over the sample they analyze, but they do not implement VAR-based techniques for the NKPC. This section fills the gap by applying the method discussed in Section 4 and in the Appendix.

We consider quarterly data for the Euro area covering the period 1971:1-1998:2. Fagan et al. (2001) provide a detailed analysis and definition of variables.[6] The empirical analysis is based on two bivariate VARs of the form $Y_t = (\pi_t, x_{1t})'$, with x_{1t} proxied by the wage share (ws_t) and the output gap ($\widetilde{y}_t$) respectively.[7] Each VAR is estimated over the period mentioned ($T = 110$

[6] The inflation rate is calculated as in Galì et al. (2001), i.e. as the growth rate on a quarterly basis of the log of the implicit GDP deflator. The wage share is calculated as in Bårdsen et al. (2004) except for a scale factor. The output gap is defined as the deviations in real GDP from potential output, measured in terms of a constant returns to scale Cobb-Douglas production function and neutral technical progress (Fagan et al., 2001); this variable starts at 1971:4. We consider the data release up to 1998 in order to compare results with Galì et al. (2001) and Bårdsen et al. (2004).

[7] We have also considered two trivariate systems of the form $Y_t = (\pi_t, x_{1t}, x_{2t})'$, with x_{1t} defined as before, and $x_{2t} \equiv i_t$ representing a short term nominal interest rate. The role of interest rates in forming inflation expectations is discussed in Fuhrer and Moore (1995). Results obtained through trivariate VARs as well as LR tests for cointegration rank are not reported in

observations) with the deterministic part given by a constant and a dummy, taking value 1 at the fourth quarter of 1974 in correspondence of the oil shock, and zero elsewhere.[8] The VAR lag length is selected by combining standard information criteria (AIC, SC, HQ) with residual-based diagnostic tests; in all cases a VAR(5) seems to describe the dynamics of the system sufficiently well.

Preliminary results are summarized in Table 1; the table reports the highest eigenvalues of the estimated VAR companion matrix, and the estimated long run relationships (when cointegration is detected).[9] It is worth noting that the highest roots of the estimated VARs' companion matrices are almost indistinguishable from unity, suggesting that treating variables as stationary might, in this case have dramatic effect, on both size and power of the test of cross-equation restrictions. Surprisingly, a cointegrating relation is also found between π_t and $\widetilde{y}_t$. This means that from the statistical point of view the chosen measure of the output gap, based on a production function, is perceived to be I(1) over the sample. From the economic point of view, the result can be motivated by referring to Hughes Hallet (2000) who shows that a non-vertical Phillips curve may follow from the aggregation of the underlying (national, regional and sectoral) curves, especially in view of the structural differences and mismatch between supply and demand which characterizes the labour markets of European countries.

The empirical analysis of the 'inexact' hybrid NKPC (1), or more precisely, of its error-correcting formulation (5), is summarized in Table 2. Here we consider two VARs of the form (7)-(8), i.e. $W_t = (z_t, \Delta\pi_t)'$ with $z_t = \widehat{\beta}' Y_t = (\pi_t - \widehat{\xi} x_t)$ defined as in the upper panel (wage share model, $x_t \equiv ws_t$) and in the lower panel (output gap model, $x_t \equiv \widetilde{y}_t$) of Table 1, respectively. As detailed in Section 4 and in the Appendix, the empirical assessment of the model is based on the evaluation of the cross-equation restrictions which (5) imposes on the VAR for W_t. The LR statistics in the last column of Table 2 compare the log-likelihood of the unrestricted system with the log-likelihood of the system

this paper due to space constraints but can be found in the working paper version of the article at http://www.rimini.unibo.it/fanelli/fanelli_WP_nkpc.pdf. Observe that, except where explicitly indicated, results obtained through trivariate VARs do not change substantially with respect to those obtained with bivariate systems.

[8] Computations have been performed using PcGive 10.0.

[9] In the upper panel of Table 1 a cointegrating relation between π_t and ws_t is detected only after the short-term interest rate is included in the system. Note, however, that π_t and ws_t prove to be cointegrated when the bivariate system $Y_t = (\pi_t, ws_t)'$ is estimated with a lower number of lags.

subject to the cross-equation restrictions (10).[10] Overall, Table 2 reveals that the 'inexact' NKPC is sharply rejected over the period 1971:1-1998:2, however, relatively high values of the forward-looking parameter, γ_f, and relatively low values of the backward-looking parameter, γ_b, tend to be favoured in terms of likelihood.

6 Concluding remarks

In this paper we address the issue of testing the hybrid NKPC under VAR expectations, giving special emphasis to the case where variables are treated as realizations of non-stationary, possibly cointegrated, processes. The paper derives the cross-equation restrictions between the agents' forecast system and the 'inexact' version of the NKPC. The estimation and testing procedure can be implemented with any existing econometric software.

The empirical investigation of the NKPC on Euro area data for the period 1971-1998 suggests two considerations. First, the persistence of variables over the selected period appears to be consistent with that of unit root cointegrated processes. This evidence is surprisingly overlooked, with few exceptions, in the literature on the NKPC where the issue of non-stationarity is usually dismissed as empirically irrelevant. The present paper shows that the assessment of the NKPC is more involved and more controversial when the 'highly persistent' stationary world is replaced by the unit root alternative. Secondly, the restrictions that the NKPC imposes on the VARs describing data dynamics are sharply rejected, irrespective of whether firms' real marginal costs are proxied by the wage share or the output gap.

These results do not necessary imply that forward-looking behaviour is unimportant in modelling Euro area inflation. Additional lags (or leads) in (1) might better capture inflation persistence. More complex dynamic specifications of the NKPC can be motivated by relying on sluggish intertemporal costs of adjustment (Rotemberg, 1982), Taylor-type contracting (Fuhrer, 1997), sticky information models (Mankiw and Reis, 2002), or even on empirical grounds (Bårdsen et al., 2004). Alternatively, further driving variables

[10] The grid for γ_f and γ_b (ψ and ω) and λ has been constructed by considering the range [0.1, 0.95], incremental value of 0.01, and the restrictions: $\gamma_f + \gamma_b < 1$, $0.03 \leq (1-\gamma_f-\gamma_b)\widehat{\xi} \leq 0.30$, where the latter constraint is motivated by the necessity of considering, given the estimates of $\widehat{\xi}$, values of the structural parameter $\lambda = (1-\gamma_f-\gamma_b)\widehat{\xi}$ which are compatible with the Calvo set-up and previous evidence.

might be needed. For instance, using data from the eighties onwards, Gerlach and Svensson (2003) show that also the real money gap (the difference between the real money stock and the long run equilibrium real money stock) also plays a role in forecasting European inflation. Quantifying the empirical relevance of all these issues is the topic of ongoing research.

Appendix

In this Appendix we establish the link between the VEqC (4) and the VAR (7)-(8) and derive the restrictions implied by the 'inexact' NKPC (5).

Paruolo (2003), Theorem 2, shows that given the I(1) cointegrated VEqC (4), the W_t vector

$$W_t = \begin{pmatrix} \beta' Y_t \\ v' \Delta Y_t \end{pmatrix} \equiv \begin{pmatrix} W_{1t} \\ W_{2t} \end{pmatrix} \quad \begin{matrix} r \times 1 \\ (p-r) \times 1 \end{matrix} \tag{11}$$

admits the following VAR(k) representation:

$$B(L) W_t = \mu^0 + \mu_d^0 D_t + \varepsilon_t^0 \tag{12}$$

where μ^0 and μ_d^0 are function of μ_0, μ_d and $(\beta,v)'$, $\varepsilon_t^0 = (\beta,v)'\varepsilon_t$, $B(L) = I_p - \sum_{i=1}^k B_i L^i$ is a characteristic polynomial with B_i, $i = 1, ..., k-1$ $p \times p$ matrices of parameters, and with the roots of the characteristic equation, $\det[B(L)] = 0$, lying outside the unit circle. Furthermore, by partitioning parameters conformably with (11), the B_k matrix in (12) is restricted as

$$B_k = B_k^* \equiv \begin{bmatrix} B_{w1,k} \vdots O \end{bmatrix} \quad \begin{matrix} p \times r & \quad p \times (p-r) \end{matrix} \tag{13}$$

where we have reported dimensions of sub-matrices alongside blocks. Due to the super-consistency result, one can replace the cointegration parameters β ($\beta_\perp$) in (11)-(12) by the estimates $\widehat{\beta}$ ($\widehat{\beta}_\perp$) obtained through cointegration methods, and treat $\widehat{\beta}$ ($\widehat{\beta}_\perp$) as the 'true' parameter value, see e.g. Johansen (1996). Clearly, when $r = 0$ (I(1) not cointegrated variables) the 'natural' choice in (11) is $v = I_p$, and the system (12) corresponds to a DVAR($k-1$) for $W_t \equiv W_{2t} = \Delta Y_t$; conversely, when $r = p$ (I(0) variables) and given $\beta' = I_p$, the system (12) collapses to a VAR(k) for $W_t \equiv W_{1t} = Y_t$. If the NKPC with I(1) variables is supported by the data, one expects the cointegration rank to be $r = 1$, and $W_{1t} = z_t = \beta' Y_t = (\pi_t - \xi' x_t)$ in (12). However, also $r > 1$ is in principle consistent with the NKPC.[11]

[11] Of course, this may happen when x_t in (1) (or in (5)) is a vector. When $r > 1$ it is necessary to identify the 'additional' cointegrating relation(s).

The companion form representation of (11)-(12) is given by

$$\widetilde{W}_t = B\ \widetilde{W}_{t-1} + \widetilde{\varepsilon}_t^0 \tag{14}$$

where $\widetilde{W}_t = (W_t', ..., W_{t-k+1}')'$, $\widetilde{\varepsilon}_t^0 = (\mu^{0\prime} + D_t'\mu_d^{0\prime} + \varepsilon_t^{0\prime}, 0', ..., 0')'$ and the $pk \times pk$ companion matrix B is defined accordingly, with B_k subject to (13). VAR (VEqC) forecasts can be therefore computed, abstracting from deterministic terms,[12] using $E(\widetilde{W}_{t+j} \mid I_t) = B^j\widetilde{W}_t$. Let g_π and g_z be two selection vectors such that $g_\pi'\widetilde{W}_t = \Delta\pi_t$ and $g_z'\widetilde{W}_t = z_t$, where z_t corresponds to $\beta' Y_t \equiv W_{1t}$ if $r = 1$, or is an element of W_{1t} if $r > 1$. Using these definitions it turns out that $E(\Delta\pi_t \mid I_{t-1}) = g_\pi' B\widetilde{W}_{t-1}$, $E(\Delta\pi_{t+1} \mid I_{t-1}) = g_\pi' B^2\widetilde{W}_{t-1}$, and $E(z_t \mid I_{t-1}) = g_z' B\widetilde{W}_{t-1}$, so that the relation (6) of Section 4 can be written as

$$g_\pi' B\widetilde{W}_{t-1} = \psi g_\pi' B^2\widetilde{W}_{t-1} + \omega g_z' B\widetilde{W}_{t-1}.$$

Since the expression above must hold a.s. for every $\widetilde{W}_{t-1}$, it must be the case that

$$g_\pi' B(I - \psi B) - \omega g_z' B = 0_{pk}' \tag{15}$$

as in (9).

To see how things work in practice, suppose first, without loss of generality, that x_t in (1) is a scalar ($q = 1$, hence $p = 2$) and that π_t and x_t are cointegrated with cointegrating vector $\beta = (1, -\xi)'$. This means that the cointegration rank in the VEqC is equal to $r = 1$, and that $W_{1t} = \beta' Y_t = (\pi_t - \xi\ x_t) = z_t \sim$I(0). Assume further that the cointegrating vector is fixed at its super-consistent estimate $\beta = \widehat{\beta} = (1, -\widehat{\xi})'$ and that k in (4) is equal to 2. Given $v = (1, 0)'$, $W_{2t} = v'\Delta Y_t = \Delta\pi_t$ ($\det(v'\beta_\perp) \neq 0$), hence $W_t = (z_t, \Delta\pi_t)'$ and the VAR (12)-(13) specializes in

$$\left(I_2 - \begin{bmatrix} b_{1,11} & b_{1,12} \\ b_{1,21} & b_{1,22} \end{bmatrix} L + \begin{bmatrix} b_{2,11} & 0 \\ b_{2,21} & 0 \end{bmatrix} L^2\right) \begin{pmatrix} z_t \\ \Delta\pi_t \end{pmatrix} = \begin{pmatrix} \mu_z^0 \\ \mu_\pi^0 \end{pmatrix} + \begin{pmatrix} \varepsilon_{zt}^0 \\ \varepsilon_{\pi t}^0 \end{pmatrix} \tag{16}$$

where L is the lag operator ($L^j Y_t = Y_{t-j}$), and $b_{i,jh}$ is the jh element of B_i, $i = 1, 2$. Observe that $b_{2,12} = 0$, $b_{2,22} = 0$ by construction because of (13). Therefore the total number of free parameters of the unrestricted system is

[12] For the sake of simplicity we ignore the role of deterministic components in the derivation of cross-equation restrictions. In general, however, it is possible to account for deterministic terms to the extent that these components are also included in the forward-looking model; see e.g. Fanelli (2002) for an example in a related context.

$p^2k - p(p-r)$. Using simple algebra and provided that $b_{1,21} \neq -(\omega/\psi)$, the cross-equation restrictions (15) can be written in explicit form as

$$b_{1,11} = \frac{b_{1,21}(1-\psi b_{1,22}) - \psi b_{2,21}}{\omega + \psi b_{1,21}} \tag{17}$$

$$b_{1,12} = \frac{b_{1,22}(1-\psi b_{1,22})}{\omega + \psi b_{1,21}} \tag{18}$$

$$b_{2,11} = \frac{b_{2,21}(1-\psi b_{1,22})}{\omega + \psi b_{1,21}}. \tag{19}$$

Observe that the equations in (17)-(19) represent the unique mapping relating the parameters of the z_t-equation of the VAR (16) to the structural parameters (ψ, ω), and the remaining VAR coefficients. The number of free parameters of the restricted system is $(p-r)[pk-(p-r)]+2$, where 2 is the dimension of $(\psi, \omega)'$. Hence, the number of overidentifying restrictions under the cross-equation restrictions is $f = p^2k - (p-r)(pk+r) - 2$, where the VAR lag length must satisfy $k \geq 1 + (3-r^2)/pr$ to guarantee that $f \geq 1$.

To compute LR tests of the NKPC, the VAR (16) must be estimated by ML under the restrictions (17)-(19) and unrestrictedly (i.e. only under the zero constraints characterizing B_2). The unrestricted estimation is standard. The estimation under (17)-(19) requires numerical optimization methods. Kurmann (2006) recommends the simulated annealing algorithm. Nevertheless, since the range of values that γ_f and γ_b (hence ψ and ω) can take is bounded by construction (see Section 5), the maximization of the likelihood of the system under the restrictions (17)-(19) can be achieved by combining grid search techniques for ψ and ω (γ_f, γ_b) with quasi-Newton methods. Provided that the LR test for overidentifying restrictions does not reject the model, ML estimates of ψ and ω (γ_f and γ_b) can be recovered from the constrained VAR estimation. An indirect ML estimate of λ can be retrieved from the estimated cointegration relation (recall that $\beta' Y_t = (\pi_t - \xi\ x_t) = z_t$) using $\lambda = (1-\gamma_f-\gamma_b)\widehat{\xi}$. The procedure works similarly if the VAR includes three or more variables and $k > 2$.

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Tables

VAR roots and estimated cointegrating relations
wage share model: VAR(5), $Y_t = (\pi_t \ , \ ws_t)' \ , \quad x_t \equiv ws_t$
highest roots: 0.9664±0.0431
Estimated cointegrating relation[a]
$\widehat{\beta}' Y_t = \pi_t - \underset{(0.05)}{0.79} ws_t - \underset{(0.12)}{2.00}$
output gap model: VAR(5), $Y_t = (\pi_t \ , \ \widetilde{y}_t)' \ , \quad x_t \equiv \widetilde{y}_t$
highest root: 0.9863
Estimated cointegrating relation
$\widehat{\beta}' Y_t = \pi_t - \underset{(0.013)}{0.09} \widetilde{y}_t - \underset{(0.013)}{0.09}$

Table 1: Estimated highest eigenvalues of VAR companion matrix and cointegrating relations. NOTES: a= see footnote 8; standard errors in parentheses.

Testsof the "inexact" NKPC			
wage share model : $z_t = \pi_t - 0.79ws_t - 2.00 \quad (\widehat{\xi} = 0.79)$			
VAR(5)	Unr. log-lik [a]	Restr. log-lik [b,c]	LR test
$W_t = \begin{pmatrix} z_t \\ \Delta\pi_t \end{pmatrix}$	880.02	870.21 ($\widehat{\gamma}_f$=0.71 , $\widehat{\gamma}_b$=0.19 , $\widehat{\lambda}$=0.079)	$\chi^2(7)$= 19.62 [0.0065]
output gap model: $z_t = \pi_t - 0.09\widetilde{y}_t - 0.09 \quad (\widehat{\xi} = 0.09)$			
VAR(5)	Unr. log-lik [a]	Restr. log-lik [b,c]	LR test
$W_t = \begin{pmatrix} z_t \\ \Delta\pi_t \end{pmatrix}$	627.26	605.18 ($\widehat{\gamma}_f$=0.80 , $\widehat{\gamma}_b$=0.07 , $\widehat{\lambda}$=0.03)	$\chi^2(7)$ = 44.16 [0.000]

Table 2: LR tests of the "inexact" NKPC in the Euro area, see Section 4 and Appendix. NOTES: a= value of the log-likelihood of the VAR (7)-(8) (k=5 lags); b= value of the log-likelihood of the VAR (7)-(8) (k=5 lags) subject to the cross-equation restrictions (10); c = the vector $(\gamma_f, \gamma_b, \lambda)$ is estimated through grid search as detailed in footnote 10; p-values in squared brackets.