Fiscal (and external) sustainability

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Fiscal (and External) Sustainability

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Abstract. Nothing new here—just a concise yet detailed presentation of the simple but inexorable algebra of sustainability.

> "The progress of the enormous debts which at present oppress, and will in the long-run probably ruin, all the great nations of Europe has been pretty uniform. Nations, like private men, have generally begun to borrow upon what may be called personal credit, without assigning or mortgaging any particular fund for the payment of the debt; and when this resource has failed them, they have gone on to borrow upon assignments or mortgages of particular funds." —Adam Smith (1776), *An Inquiry into the Nature and Causes of the Wealth of Nations*, Book V, Ch. 3, 'Of Public Debts.'

UNSUSTAINABLE DEBT PATHS may eventually lead to sharp adjustments if not to crises—*i.e.*, to generalised failure of economic agents to meet their obligations. Hence, sustainability is a most desirable quality. The emphasis of this merely expository note shall be on the *ex-post* algebra of sustainability. While the *ex-post* algebra of sustainability is rather straightforward, *ex-ante* analysis is not: the key variables involved are either endogenous, uncertain, or both; expectations playing a prominent role driving equilibria and, therefore, driving expectations themselves. Nonetheless, the simple *ex-post* algebra of sustainability provides a useful framework to identify vulnerabilities and risks—after all, a most fertile source for insight is hindsight.

1. Simple Algebra of Fiscal Sustainability

Indicators of fiscal sustainability are constructed to shed light on the following question:[1]

> "Can the current course of fiscal policy be sustained, without exploding debt? Or

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[1] O.J. Blanchard (1990), "Suggestions for a New Set of Fiscal Indicators," OECD Dept. of Economics and Statistics Working Paper no. 79. Available from http://www.oecd.org and reprinted in H.A.A. Verbon and F.A.A.M. van Winden (editors), *The Political Economy of Government Debt*, New York: North-Holland.

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will the government have to sharply increase taxes, decrease spending, have recourse to monetisation, or even repudation?"

Let D_t denote the stock of government debt at the end of year t, let i_t be the (average) nominal interest rate, B_t the primary (*i.e.*, non-interest) government balance ($B_t > 0$ means that the government runs a surplus), and let M_t denote the end-of-period stock of high-powered money. The **government budget constraint** implies that:

$$D_t = (1 + i_t)D_{t-1} - B_t - \Delta M_t \tag{1}$$

$$= D_{t-1} - \underbrace{(B_t - i_t D_{t-1})}_{\text{Overall Balance}} - \underbrace{\Delta M_t}_{\text{Seignorage}} \tag{2}$$

Equation (1) always holds *ex-post*. It simply states that the government meets its debt obligations, and that any gap, $B_t < 0$, must either be financed by new debt, or monetised, or a mix of the two. In contrast, a surplus, $B_t > 0$, can be used to reduce the stock of existing debt or the stock of money—*e.g.*, see the flow of funds displayed in Table 1.

Note that any gross **amortization** due during t will require a rollover unless a government surplus—*i.e.*, $(B + \Delta M) > 0$—allows for a net amortization effectively reducing the debt stock. For example, assume that a portion α of D_{-1} comes due and that $(B + \Delta M) = 0$, then we'd have $D = [1 + \alpha i + (1 - \alpha)i_{-1}]D_{-1}$, which simplifies to $D = (1 + i)D_{-1}$ when $i = i_{-1}$.[2]

It is useful to **normalise** the quantities in equation (1) by some measure of the government's ability to service and repay its debt—*e.g.*, government revenues, GDP, exports in the case of external debt, etc. The most common choice used for normalising government debt is GDP. Dividing equation (1) by nominal GDP, $P_t Y_t$, results in:

$$\frac{D_t}{P_t Y_t} = \frac{(1 + i_t)D_{t-1}}{P_t Y_t} - \frac{B_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t} \tag{3}$$

$$= \frac{(1 + i_t)}{(1 + g_t)(1 + \pi_t)} \left(\frac{D_{t-1}}{P_{t-1} Y_{t-1}} \right) - \frac{B_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t} \tag{4}$$

where g_t is the real growth rate and π_t is the inflation rate (measured as the rate of change of the GDP deflator, P).

Use lowercase symbols to denote ratios to GDP and Δm_t for seignorage, to rewrite (4), and obtain the **law of motion of the government debt-to-GDP ratio**:

$$d_t = \frac{(1 + i_t)}{(1 + g_t)(1 + \pi_t)} d_{t-1} - (b_t + \Delta m_t) \tag{5}$$

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[2] The trick in equation (1) is to define i_{t-1} implictly as an effective rate. See also W. Easterly and S. Fischer (1990), "The Economics of the Government Budget Constraint," *World Bank Research Observer*, 5:2, 127–42.

$$= \underbrace{\left(\frac{1+r_t}{1+g_t}\right)}_{\varphi_t} d_{t-1} - (b_t + \Delta m_t) \qquad (6)$$

$$= \varphi_t d_{t-1} - (b_t + \Delta m_t) \qquad (7)$$

where r_t is the real interest rate,[3] and φ_t is a *discount factor* defined as $\varphi_t = (1+r_t) \div (1+g_t)$. Equation (7) is also our fundamental fiscal-sustainability *identity*. It will always hold, as it is simply derived from accounting identities, with no behavioral assumptions intervening.[4] For notational simplicity, from now on we shall drop the seignorage—*i.e.*, whenever be encounter b from now on it is understood that any seignorage, Δm, must be added to it.

Solving (7) recursively (Appendix 5.2) we obtain:

$$d_t = d_0 \prod_{i=1}^{t} \varphi_i - \sum_{i=1}^{t} b_i \prod_{j=i+1}^{t} \varphi_j \qquad (8)$$

If we wish to use this equation in a forward-looking analysis—say, looking ahead over a period of 5 years or longer—we can assume constant balances and discount factors, which can then be interpreted as *averages*, to simplify (8). In this context, by dropping the time subscripts in φ and b, we obtain:

$$d_t = d_0 \varphi^t - b \sum_{i=0}^{t-1} \varphi^i \qquad (9)$$

Equation (9) can be used for different exercises. Assume, *e.g.*, that there is a **target of debt ratio**, $\bar{d}$, to be achieved by time T. Then the required average primary surplus (plus seignorage) can be obtained by simply solving for b. A polar case to consider is when the real interest rate equals the growth rate, $\varphi = 1$, then solving equation (9) becomes: $b = (d_0 - \bar{d})/T$ so that the primary balance plus seignorage must fill one-T^{th} of the gap each year. For a general $\varphi > 1$, the annual effort must now be larger than that:

$$b = \frac{d_0 \varphi^T - \bar{d}}{\sum_{i=0}^{T-1} \varphi^i} > \frac{d_0 - \bar{d}}{T} \qquad (10)$$

Of course, (10) characterises only one of many possible paths that lead to the desired debt ratio in T years: the path with *identical* surplus as a percent to GDP during the adjustment period—the *smoother* path in this sense.

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[3] The real interest rate is given by $r = (i - \pi) \div (1 + \pi)$, which is safely approximated by $r \approx i - \pi$, when both i and π are not large (*e.g.*, smaller than 10%).

[4] Note, however, that, *ex ante*, φ_t is an endogenous factor, jointly determined with b_t and Δm_t. However, since no behavioural modelling is attempted here, we shall simply take φ_t as given.

Explosive Debt-Dynamics. In forward-looking analysis, the most common assumption is to have a positive interest-growth differential, $r_t > g_t$. If the interest-growth differential is positive, then the debt ratio will blow up unless the last term in equation (7), b_t, is large enough to compensate. Refer to equation (9). Assume now that $\varphi > 1$; in order to stay in a non-explosive path, we need that eventually (sooner rather than later): $b > 0$—*i.e.*, that the government runs a large enough **primary surplus**.

1.1. Stabilising Debt-to-GDP Ratios

Substract d_{t-1} on both sides of equation (6) to obtain an expression for the *change* in the debt ratio:

$$\Delta d_t = \left(\frac{r_t - g_t}{1 + g_t} \right) \cdot d_{t-1} - b_t \tag{11}$$

Note that if *real* growth, g, is zero, then the change in the debt ratio is driven by the *operational* balance—*i.e.*, the overall government balance less the inflation component of interest payments, πd.[5]

If we want to stabilise the debt-to-GDP ratio, make zero the left-hand side of equation (11) and solve for b to obtain the **debt-stabilising balance**:

$$b_t^* = \left(\frac{r_t - g_t}{1 + g_t} \right) \cdot d_{t-1} \tag{12}$$

which is the required government primary surplus (including seignorage). A larger surplus, $b_t > b_t^*$, will bring the debt ratio down.

The larger is the (real) interest-growth differential, $(r_t - g_t)$, the larger the required debt-stabilising surplus. Note that if the interest-growth differential is zero, $r_t = g_t$, then it follows from (11) that $\Delta d_t = -b_t$. In this context, a balanced primary keeps the debt-to-GDP ratio constant.

Note that while the left-hand side of equation (12) displays the *primary* balance, what finally matters for the debt dynamics is the *overall* balance. **The overall balance is what ultimately determines the change in the debt stock**—*i.e.*, from equation (2), we have that

$$\Delta D_t = - \underbrace{(B_t - i_t D_{t-1})}_{\text{Overall Balance}} \tag{13}$$

When we are dealing with ratios to GDP, then there is an additional a growth 'dividend' term driving the debt dynamics. Let's start out with a version of equation

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[5] See Vito Tanzi, M.I. Blejer and M.O. Tejeiro (1993), "Effects of Inflation on Measurement of Fiscal Deficits: Conventional *vs* Operational Measures," in M.I. Blejer and A. Cheastey (editors), *How to Measure the Fiscal Deficit*, Washington: IMF.

(11) that uses nominal rates of interest, i_t, and nominal growth, $\gamma_t = g_t + \pi_t + g_t\pi_t$, given by:

$$\Delta d_t = \frac{i_t - \gamma_t}{1 + \gamma_t}d_{t-1} - b_t \tag{14}$$

$$= \frac{i_t - \gamma_t}{1 + \gamma_t}d_{t-1} - (b_t - i_t d_{t-1}) - i_t d_{t-1} \tag{15}$$

$$= -\underbrace{(b_t - i_t d_{t-1})}_{\text{overall balance}} - \underbrace{\frac{1 + i_t}{1 + \gamma_t}\gamma_t d_{t-1}}_{\text{growth dividend}} \tag{16}$$

It becomes apparent that when *nominal* growth, γ, is zero, the change in the debt ratio is simply driven by the government overall balance—*cf.* equation (11) expressed for real rates of interest and growth.

1.2. Cashflow, liquidity and rollover risk

While equation (13) determines the evolution of the stock of debt, it is also important to pay attention to the gross debt issuance, which depends not only on the government balance but also on the amortizations, A_t.

$$D_t = \underbrace{(D_{t-1} - A_t)}_{\text{Old Debt}} + \underbrace{[A_t - (B_t - i_t D_{t-1})]}_{\text{New Debt Issued}} \tag{17}$$

Thus, the gross issuance of debt depends on the debt service, $(A + iD_{-1})$, and the primary balance, B. Or, equivalently, on the overall balance, $(B - iD_{-1})$, and the amortizations, A.

1.3. Sustainable Fiscal Policy

At any given time s, the **net present value** of the stream of all the future payments, both of principal and interest, $\mathbf{p} = \{p_s, p_{s+1}, p_{s+2}, \ldots\}$, discounted by the nominal rate $i = r + \pi + i\pi$, is given by:[6]

$$\mathcal{D}(\mathbf{p}, r, \pi) = \sum_{t=s}^{\infty} \frac{p_t}{[(1 + \pi)(1 + r)]^{t-s}} \tag{18}$$

Since we used before D_s to denote the market value of debt, in equilibrium we must have $\mathcal{D}(\mathbf{p}, r, \pi) = D_s$. Equation (18) shows, for example, that a raise in the country's sovereign risk that increases r will unequivocally decrease $\mathcal{D}(\mathbf{p}, r, \pi)$—*i.e.*, the market value of the government's debt declines. The other side of the coin is that now a given stream of future payments raises less funds for the government.

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[6] At s there is a set of prevailing interest rates for government debt issues at different maturities. Here, however, for the sake of simplicity we shall use a single (average) rate, r, constant over s.

We can similarly compute the net present value of a stream of primary surpluses, $\mathcal{B}(\mathbf{b}, r, \pi)$. **Fiscal policy is sustainable if the government's solvency condition is satisfied.** The solvency condition requires the equalisation of the net present value of primary surpluses and debt obligations:[7]

$$\mathcal{D}(\mathbf{p}, r, \pi) = \mathcal{B}(\mathbf{b}, r, \pi) \tag{19}$$

As we shall discuss below, the upper bound for the time index in the primary-balances net-present-value sum is a crucial assumption. Making $t \to \infty$ in (19) allows for the public debt to be simply serviced with no need to be entirely paid out.

As Robert Eisner states: "... the notion that the debt incurred by current generations has somehow to be paid off *eventually* by future generations is a confusing, or confused, effort to use a mathematical principle of solvency where it does not apply."[8] In a growing economy there is no reason why the public debt ever has to be paid out entirely. At the heart of the government's solvency condition is only the requirement of a non-explosive debt ratio, all or part of the existing debt could always be rolled over. As we shall see below, simply stabilising the debt ratio does satisfy the government's solvency condition (19).

Assume that primary balances evolve over time in line with nominal GDP growth, $B_t = (1 + g)(1 + \pi)B$, so that the balance as a percent-to-GDP remains constant, $b_t = B/Y_s = b$. The net present value of this stream of primary balances from $t = s + 1$ on, expressed as a percentage of GDP, is given by

$$\frac{B}{Y_s} \sum_{t=s+1}^{\infty} \left[\frac{(1+\pi)(1+g)}{(1+\pi)(1+r)} \right]^{t-s} = \frac{1+g}{1+r} b \sum_{t=s}^{\infty} \left(\frac{1+g}{1+r} \right)^{t-s} = \frac{1+g}{r-g} b \tag{20}$$

The solvency condition requires equating this net present value to the current debt ratio, d_s, which gives: $d_s = b(1+g)/(r-g)$. This equation can be solved for b:

$$\tilde{b} = \left(\frac{r-g}{1+g} \right) \cdot d_s \tag{21}$$

Equation (21) is identical to equation (12), which determines the required balance to mantain a stable the debt-to-GDP ratio, b^*. The constant primary surplus required for solvency never brings the debt ratio to zero, it simply stabilises it.[9]

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[7] In a money-dominant regime, the solvency condition is satisfied by adjustments in the sequence of fiscal balances, $\mathbf{b}$. On the contrary, in a fiscal-dominant regime, where the sequence $\mathbf{b}$ is assumed to be given, it is the inflation rate, π that adjusts to satisfy the solvency condition. Note, however, that when (18) is taken as an equilibrium condition, rational expectations will rule out *ad hoc* revaluations of the debt stock, which can only follow after genuine shocks.

[8] R. Eisner (1994), *The Misunderstood Economy: What Counts and How to Count It*, Boston MA: Harvard University Press, [Page 126].

[9] Of course, alternatively, we could have a constant primary, b^* over T years to bring the debt ratio to a target $\bar{d} \geq 0$—using equation (10) to determine b^*—and then switch to $\bar{b}$—determined by equation (12)—from $T + 1$ onwards to stabilise it. If we make $\bar{d} = 0$ the public debt is then paid off by time T.

The 'missundertood' solvency requirement truncates the sum in (20) at a terminal T—requiring that the public debt be completely paid off by then.

2. Government Debt in an Open Economy

So far we haven't been too specific about what exactly is the interest rate, i. We think of i as an *effective interest rate on government debt*. When a substantial portion of debt is denominated in foreign currency, we must take into account the debt composition and exchange rate movements.

Let's return to equation (4), we must worry about two things now. One is the effective interest on government debt, which will be a weighted average of domestic and foreign rates, and exchange-rate movements. Consequently, the nominal exchange rate will play a crucial role here. We must also acknowledge that the inflation rate (*i.e.*, the change in the GDP deflator) will be determined by domestic inflation for the non-tradable sector and by world inflation and exchange-rate movements for the tradable sector.

2.1. Foreign-denominated Debt

Let's use an f-superscript to denote foreign-denominated variables and h-superscript to denote (home) domestically-denominated variables. Let e be the exchange rate in domestic currency per unit of foreign currency. We have that:

$$D = D^h + eD^f \tag{22}$$

Let α^f be the portion of foreign-denominated government debt, $\alpha^f = e_{t-1}D^f_{t-1}/D_{t-1}$, and $\alpha^h = 1 - \alpha^f$ the portion of foreign-denominated debt (both computed at the end of the previous period). The debt-dynamics equation (1) now becomes (Appendix 5.3):

$$D_t = [1 + \underbrace{\alpha^h i_t^h + \alpha^f(i_t^f + \varepsilon_t + i_t^f \varepsilon_t)}_{i_t}]D_{t-1} - (B_t + \Delta M_t) \tag{23}$$

where $\varepsilon_t = \Delta e_t/e_{t-1}$ is the rate of depreciation of the local currency (*i.e.*, $\varepsilon_t > 0$ means depreciation). The *effective nominal interest rate* is given now by a weighted average of domestic rates and a term involving foreign rates and exchange-rate movements:[10]

$$i_t = \alpha^h i_t^h + \alpha^f(i_t^f + \varepsilon_t + i_t^f \varepsilon_t) \tag{24}$$

which makes readily apparent how movements in the exchange rate may affect the debt burden. (The third term within the parentheses will generally be rather small and sometimes it is dropped to simplify expressions.) Define a weighted average of

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[10]   For simplicity, we have only one foreign country here, i^f itself will be generally substituted by a weighted average of foreign rates, i^{f_j}'s, where the j index runs over foreign creditor countries.

domestic and foreign interest rates as $\hat{\imath}_t = \alpha^h i_t^h + \alpha^f i_t^f$, then we can rewrite (24) as:

$$i_t = \hat{\imath}_t + \varepsilon_t \alpha^f (1 + i_t^f). \tag{24$'$}$$

Thus, the effective interest rate has two components: (1) a weighted average of domestic and foreign interest rates, $\hat{\imath}$; and (2) the exchange-rate induced valuation gains or losses in the foreign-debt obligations. This second element may grow to be substantial during an exchange-rate crisis.

2.2. Composition of GDP: Tradable and Non-Tradable Sectors

Consider equation (3) where the government's current budget constraint (1) was divided by nominal GDP. Now movements in the exchange rate will produce price changes in the tradable sector. Let's use the superscripts f and h to denote the tradable and nontradable sectors, respectively. Then nominal GDP is expressed as:

$$PY = P^h Y^h + e P^f Y^f \tag{25}$$

As before, define $\beta^h = P_{t-1}^h Y_{t-1}^h / P_{t-1} Y_{t-1}$ as the share of the non-tradable sector, and $\beta^f = 1 - \beta^h$, as the share of the tradable sector, then:

$$P_t Y_t = (1 + g_t)[1 + \underbrace{\beta^h \pi_t^h + \beta^f (\pi_t^f + \varepsilon_t + \varepsilon_t \pi_t^f)}_{\pi_t}] P_{t-1} Y_{t-1} \tag{26}$$

(see Appendix 5.4), so that now:

$$\pi_t = \beta^h \pi_t^h + \beta^f (\pi_t^f + \varepsilon_t + \varepsilon_t \pi_t^f) \tag{27}$$
$$= \hat{\pi}_t + \varepsilon_t \beta^f (1 + \pi_t^f) \tag{27$'$}$$

where $\hat{\pi}_t = \beta^h \pi_t^h + \beta^f \pi_t^f$ is a weighted average of domestic and foreign inflation rates.[11] Compare equation (27$'$) with equation (24$'$). In an open economy, the inflation rate has two components: (1) a weighted average of domestic and foreign inflation rates, $\hat{\pi}$; and (2) the exchange-rate induced valuation gains or losses in the tradable-sector output.

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[11] For long-term analysis, we must also acknowledge that tradable and non-tradable sectors may grow at different rates, $g_t = \beta_t^h g_t^h + \beta_t^f g_t^f$, altering the next period β's.

2.3. Putting it all Together

We can still use our fundamental sustainability equation (5)—as well as all the subsequent equations in the previous section—but now we must keep in mind that i_t and π_t are defined by equations (24) and (27). Consequently, exchange-rate movements, $\varepsilon \neq 0$, play a prominent role in debt dynamics. We have:

$$d_t = \frac{1 + r_t}{(1 + g_t)(1 + \pi_t)} d_{t-1} - b_t \tag{5}$$

$$i_t = \hat{\imath}_t + \varepsilon_t \alpha^f (1 + i_t^f) \tag{24'}$$

$$\pi_t = \hat{\pi}_t + \varepsilon_t \beta^f (1 + \pi_t^f) \tag{27'}$$

These equations can be combined to obtain:

$$d_t = \frac{1 + \hat{\imath}_t + \varepsilon_t \alpha^f (1 + i_t^f)}{(1 + g_t)[1 + \hat{\pi}_t + \varepsilon_t \beta^f (1 + \pi_t^f)]} d_{t-1} - b_t \tag{28}$$

Substracting d_{t-1} from both sides of equation (28) yields:[12]

$$\Delta d_t = \left\{ \frac{1 + \hat{\imath}_t + \varepsilon_t \alpha^f (1 + i_t^f)}{(1 + g_t)[1 + \hat{\pi}_t + \varepsilon_t \beta^f (1 + \pi_t^f)]} - 1 \right\} \cdot d_{t-1} - b_t \tag{29}$$

In this context, with foreign and domestic inflation, one could define a real interest rate, ρ as:

$$1 + \rho_t = \frac{1 + \hat{\imath}_t + \varepsilon_t \alpha^f (1 + i_t^f)}{1 + \hat{\pi}_t + \varepsilon_t \beta^f (1 + \pi_t^f)}$$

which implies

$$\rho_t = \frac{(\hat{\imath}_t - \hat{\pi}_t) + \varepsilon_t [\alpha^f (1 + i_t^f) - \beta^f (1 + \pi_t^f)]}{1 + \hat{\pi}_t + \varepsilon_t \beta^f (1 + \pi_t^f)}$$

and then rewrite equation (29) as:

$$\Delta d_t = \left(\frac{\rho_t - g_t}{1 + g_t} \right) \cdot d_{t-1} - b_t \tag{30}$$

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[12] The Excel spreadsheet developed by the IMF's Policy Development Review department for assesing fiscal sustainability makes $\beta^f \approx 0$ and further assumes that $i^f \approx \hat{\imath}$ to write (29) as:

$$\Delta d_t \approx \left\{ \frac{1 + \hat{\imath}_t + \varepsilon_t \alpha^f (1 + \hat{\imath}_t)}{(1 + g_t)(1 + \pi_t)} - 1 \right\} d_{t-1} - b_t$$

See "Assessing Sustainability," Paper prepared by the IMF Policy Development Review Department, May 2002, SM/02/166. Available from http://www.imf.org.

which is analogous to (11).

Finally, note that all the variables—*i.e.*, nominal interest, inflation and growth rates—involved in the discount factors $\varphi_t = (1 + i_t) \div [(1 + g_t)(1 + \pi_t)]$, are endogenously determined, and uncertain *ex-ante*. Obviously, interest and growth rates are interconnected. In addition, market expectations may push up a country's risk premium, increasing i_t to levels that turn any given situation unsustainable. A vicious cycle may start, and self-fullfilling expectations may lead an economy to a bad equilibrium possibly involving a sharp adjustment or, worse, a crisis.

2.4. More on Exchange-Rate Movements

Exchange-rate movements have three immediate effects: *(i)* they affect the domestic cost of servicing interest of foreign-denominated debt, *(ii)* they affect the value in domestic currency of the foreign-denominated debt, and *(iii)* they affect the value of GDP in domestic currency through changes in prices in the tradable sector.

Since the debt ratio involves foreign-denominated variables both in the numerator and denominator,

$$d = \frac{D^h + eD^f}{P^h Y^h + eP^f Y^f} \tag{31}$$

movements in the exchange rate, e, will immediately affect d.[13]

If $(D^h/eD^f) = (P^h Y^h/eP^f Y^f)$ so that the composition of debt and output are perfectly matched (*i.e.*, $\alpha^f = \beta^f$), then movements in the exchange rate, e, will not affect sustainability directly via changes in d.[14]

When $\alpha^f > \beta^f$, an exchange depreciation may have severe effects on sustainability. In an extreme case, if the tradable sector is negligible ($\beta^f \to 0$), exchange rate depreciation will have maximum effects on d.

3. External-Debt Sustainability

While fiscal-sustainability analysis looks at the 'Government' column in the Flow of Funds table, external sustainability focuses on the 'Rest of the World' column (Table 1). Similarly to the government accumulating public debt because its revenues fall short of its expenditures, a country accumulates external debt when it has a current account deficit: the country spends more than its national income (Appendix 5.1).

The algebra of external-debt sustainability is formally quite similar to the analysis of fiscal sustainability. There is, however, a difference in presentation; in this

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[13] G. Calvo, A. Izquierdo, and E. Talvi (2003), "Sudden Stops, the Real Exchange Rate and Fiscal Sustainability: Argentina's Lessons," NBER Working Paper No. w9828. Available from http://papers.nber.org/papers/W9828

[14] Note that for $(a + \Delta a) \div (b + \Delta b) = a \div b$ we must have that $\Delta a \div \Delta b = a \div b$.

Table 1. Schematic Flow of Funds

| | | Domestic Economy $(1) = (2) + (3) + (4)$ | | | Rest of the World |
| | | Govern-ment | Private Sector | Banking System | |
	(1)	(2)	(3)	(4)	(5)
Gross National Domestic Income	Y	Y_g	Y_p		
$\oplus$ Final Consumption	$-C$	$-C_g$	$-C_p$		
$\oplus$ Gross Investment	$-I$	$-I_g$	$-I_p$		
$\oplus$ Exports of Goods & Nonfactor Serv.					$-X$
$\ominus$ Imports of Goods & Nonfactor Serv.					M
$\oplus$ Net Factor Income					$-Y_f$
$\oplus$ Net Transfers					$-Y_r$
0. Non-Financial Balances	$(S-I)$	$(S_g - I_g)$	$(S_p - I_p)$	0	CAB
1. Domestic Financing					
1.1. Monetary Financing					
1.1.1. Money		ΔM	$-\Delta M2$	$\Delta M2 - \Delta M$	
1.1.2. Domestic Credit		ΔNDC_g	ΔNDC_p	$-\Delta NDC$	
1.2. Non-Monetary Financing					
1.2.1. Non-Bank Financing		NB	$-NB$		
2. Foreign Financing					
2.1. Monetary Financing					
2.1.1. Change in Net Foreign Assets	$-\Delta NFA$			$-\Delta NFA$	ΔNFA
2.2. Non-Monetary Financing					
2.2.1. Direct Investment	FDI		FDI		$-FDI$
2.2.2. Net Foreign Borrowing	NFB	NFB_g	NFB_p		$-NFB$
Vertical Check $(0 - (1+2)) =$	0	0	0	0	0

context, it is customary to express all variables in U.S. dollars (USD).[15]

Let D_t denote the stock of all (public and private) external debt expressed in USD. The analogue to the law of motion of the debt stock (1) is now the country's budget constraint in relation to the rest of the world:

$$D_t = (1 + i_t^f)D_{t-1} - CA_t \qquad (32)$$

where CA_t is the non-interest current account balance. Output shall be expressed in USD too:

$$PY/e = P^hY^h/e + P^fY^f \qquad (25')$$

and we define the tradable and non-tradable sector shares, β^f and β^h, as before.

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[15] Not coincidentaly, the U.S. is one of the few countries whose flow of funds table shows a significant nonzero entry on holdings of domestic currency by foreign residents—*i.e.*, row 1.1.1 in the last column.

The equation analogue to (11) and (29) now becomes:

$$\Delta d_t = \left\{ \frac{1 + \hat{\imath}_t + \xi_t \alpha^h (1 + i_t^h)}{(1 + g_t)[1 + \hat{\pi}_t + \xi_t \beta^h (1 + \pi_t^h)]} - 1 \right\} \cdot d_{t-1} - ca_t \qquad (33)$$

where ξ is the rate of change of the inverse of the exchange rate $(1/e)$, and $\alpha^h = (1 - \alpha^f)$ is the portion of domestic debt in total debt. This equation makes apparent that a key variable for external sustainability is the current-account balance, ca. It plays the same role that the government primary balance played for fiscal sustainability.

4. Other Issues

While this note deals with *ex-post* identities, sustainability analysis is inherently forward-looking, and *endogeneity* plays a central role. In particular, some of the things to keep in mind: *(i)* the interest rate is not independently determined from the stock of debt; *(ii)* the growth rate depends, in turn, on the interest rate; and *(iii)* foreign and domestic interest rates will not be determined independently.

When doing cross-country comparisons of debt-to-GDP ratios, we must keep in mind that both revenue-to-GDP ratios and the revenue composition also vary substantially. Therefore, we must also take into account tax-to-GDP ratios (and possibly other measures) for assesing sustainability. The same applies to external sustainability, the more open the economy and the larger the tradable sector, the easier to adjust rapidly.

The stock of present debt ignores *contingent government liabilities*. Private debt may suddenly become a public liability. In the event of a systemic banking crisis, for example, recapitalization of the banking system can greatly increase government debt. Some authors have emphasized that an economic definition for 'debt' ought to be used instead of the legal definition that is usually employed.

Some authors have wisely emphasised the need to also take a long-term perspective and focus on the government's intertemporal budget constraint, (19): "Today's long-run becomes tomorrow's short-run."[16] Nonetheless, from a pragmatic point of view, the danger of shifting the focus to infinite net-present-value sums is that the short-run can easily become unimportant and, hence, ignored. Since, in turn, the long-run is decidedly determined by all past short-run's, this approach, if narrowly understood, could unintentionally lead to a series of imprudent short-run's wishfully compensated by perpetually shifted-away corrections.

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[16] W.H. Buiter (1995), "Measuring Fiscal Sustainability," mimeo, Cambridge University. Available from /www.nber.org/~wbuiter/sustain.pdf.

5. Appendix

5.1. Basic Macro Indentities and the Twin Deficits

The most basic macro identity equates aggregate supply to aggregate demand:

$$\underbrace{Q + M}_{\text{Aggregate Supply}} = R + \overbrace{\underbrace{(C + I + X)}_{\text{Aggregate Demand}}}^{\text{Final Demand}}, \tag{34}$$

where Q denotes total output, R denotes intermediate consumption, M are imports, X denotes exports, C is aggregate consumption and I denotes aggregate investment. The gross domestic product, GDP, is defined as:

$$GDP = (Q - R) = C + I + \underbrace{(X - M)}_{\text{Trade Balance}} \tag{35}$$

If we add to GDP net factor foreign income Y_f and net transfers from abroad Y_r we obtain domestic income, Y,

$$Y = GDP + Y_f + Y_r \tag{36}$$
$$= C + I + \underbrace{(X - M) + Y_f + Y_r}_{CAB}, \tag{37}$$

where CAB is the current-account balance. Rearranging, we have that:

$$CAB = Y - (C - I) = (S - I), \tag{38}$$

that is, the CAB is determined by the aggregate saving-investment balance. When domestic demand exceeds domestic income, the CAB must be negative. Decomposing the national saving-investment balance into the private-sector balance, $(S_p - I_p)$ and the government-sector balance, $(S_g - I_g) = (T - G)$, where T denotes government revenues and G government expenditures, we obtain:

$$CAB = (S_p - I_p) + (T - G). \tag{39}$$

Equation (39) makes apparent the impact of the public-sector deficit on the current account (*i.e.*, the so-called *twin deficits*). Rearranging terms in (39) we obtain $-(T - G) = (S_p - I_p) - CAB$, or:

$$\underbrace{(G - T)}_{\text{Budget Deficit}} = \underbrace{(S_p - I_p)}_{\text{Private Sector Balance}} + \underbrace{CAD}_{\text{CA Deficit}} \tag{40}$$

5.2. Derivation of Equation (8)

Starting at time 0, applying (6) recursively we obtain:

$$d_1 = \varphi_1 d_0 - b_1 \tag{41}$$

$$d_2 = \varphi_2 d_1 - b_2 \tag{42}$$

$$= \varphi_2[\varphi_1 d_0 - b_1] - b_2 \tag{43}$$

$$= \varphi_2\varphi_1 d_0 - \varphi_2 b_1 - b_2 \tag{44}$$

$$d_3 = \varphi_3 d_2 - b_3 \tag{45}$$

$$= \varphi_3[\varphi_2\varphi_1 d_0 - \varphi_2 b_1 - b_2] - b_3 \tag{46}$$

$$= \varphi_3\varphi_2\varphi_1 d_0 - \varphi_3\varphi_2 b_1 - \varphi_3 b_2 - b_3 \tag{47}$$

$$\vdots$$

$$d_t = d_0 \prod_{j=1}^{t} \varphi_j - \sum_{j=1}^{t} b_j \prod_{i=j+1}^{t} \varphi_i \tag{48}$$

Where it is understood that a sum over an empty index set is equal to zero, and a product over an empty index set is equal to 1.

5.3. Derivation of Equation (23)

$$D_t = (1 + i_t^h)D_{t-1}^h + (1 + i_t^f)e_t D_{t-1}^f - (B_t + \Delta M_t) \tag{49}$$

$$= (1 + i_t^h)D_{t-1}^h + (1 + \varepsilon_t)(1 + i_t^f)e_{t-1}D_{t-1}^f - (B_t + \Delta M_t) \tag{50}$$

$$= [\alpha^h(1 + i_t^h) + \alpha^f(1 + \varepsilon_t)(1 + i_t^f)]D_{t-1} - (B_t + \Delta M_t) \tag{51}$$

$$= [1 + \underbrace{\alpha^h i_t^h + \alpha^f(i_t^f + \varepsilon_t + i_t^f \varepsilon_t)}_{i_t}]D_{t-1} - (B_t + \Delta M_t) \tag{23}$$

5.4. Derivation of Equation (26)

$$P_t Y_t = (1 + g_t)[(1 + \pi_t^h)P_{t-1}^h Y_{t-1}^h + (1 + \varepsilon_t)(1 + \pi_t^f)e_{t-1}P_{t-1}^f Y_{t-1}^f] \tag{52}$$

$$= (1 + g_t)[(1 + \pi_t^h)\beta^h + (1 + \varepsilon_t)(1 + \pi_t^f)\beta^f]P_{t-1}Y_{t-1} \tag{53}$$

$$= (1 + g_t)[(\beta^h + \beta^f) + \beta^h\pi_t^h + \beta^f(\pi_t^f + \varepsilon_t + \varepsilon_t\pi_t^f)]P_{t-1}Y_{t-1} \tag{54}$$

$$= (1 + g_t)[1 + \underbrace{\beta^h\pi_t^h + \beta^f(\pi_t^f + \varepsilon_t + \varepsilon_t\pi_t^f)}_{\pi_t}]P_{t-1}Y_{t-1} \tag{26}$$

5.5. Pesky Details on the Dynamics of Deficit and Debt

In order to compute debt paths, we must link flows and stocks in discrete time—
e.g., in monthly or annual intervals. While the overall balance will include payments
on new debt, the change in debt, in turns, will depend on the size of the overall
balance:

$$OB_t = B_t - [i_{t-1}D_{t-1} + i_t \Delta D_t] \tag{55}$$

$$\Delta D_t = OB_t \tag{56}$$

which implies:

$$\Delta D_t = OB_t = (1 + i_t)[B_t - i_{t-1}D_{t-1}]. \tag{57}$$

Equation (57) assumes, however, that all new debt is issued at the beginning of
the period t. It basically says that the government must finance the gap between
revenues and expenditures plus the interest payments on the gap itself.

In the absence of detailed cashflow projections, a better assumption for long
periods (*e.g.*, a quarter or longer), is that the debt is issued uniformly along the
duration of the period. In this case, on average, the stock of the net increase in
debt is given by $\Delta D_t/2$, so that equation (57) becomes:

$$\Delta D_t = \left(1 + \frac{i_t}{2}\right)[B_t - i_{t-1}D_{t-1}]. \tag{58}$$

Finally, there is the issue that some of the old debt will be amortized duing t
and any new debt issued to roll over those obligations may carry a different interest
rate, i_t. Assume that a portion α_t of the existing debt is amortized in t,[17] then
equation (58) becomes:

$$\Delta D_t = \left(1 + \frac{i_t}{2}\right)\{B_t - [(1 - \alpha_t)i_{t-1} + \alpha_t i_t]D_{t-1}\}. \tag{59}$$

For the sake of notational simplicity, all these details were ignored throughout
the main body of the handout. Nonetheless, they may be rather important for
empirical analyses, especially when the sovereign spreads change and the interest
on newly-issued debt differs significantly from the interest on old debt.

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[17] Note that this portion is a weighted average of durations and amounts—*i.e.*, as before, 10
percent of the debt amortized in January or December have distinct consequences if i_{t-1} and i_t
are sufficiently different.