Banks as Better Monitors and Firms' Financing Choices in Dynamic General Equilibrium

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Abstract

This paper builds a dynamic general equilibrium model that emphasizes banks' comparative advantage in monitoring financial distress in order to explain firms' choice between bank loans and market debt. Banks can deal with financial distress more cheaply than bond holders, but this requires a higher initial expenditure proportional to the loan size. In contrast, bond issues may involve a small fixed cost. Entrepreneurs' choice of bank or bond financing depends on their net worth. The steady state of the model can explain why smaller firms tend to use more bank financing and why bank financing is more prevalent in Europe than in the US. We find that a higher fixed cost of issuing market debt is a key factor in replicating the higher use of bank financing relative to market debt in Europe. Finally, we find that for plausible calibrations one can predict aggregate quantities just as well using a model with only one type of loan with costs of financial distress that are an average of the costs for bank loans and market debt.

JEL classification: E4, G3
Key words: financial frictions, costly state verification, financial distress

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1 Introduction

Debt financing is the most prevalent form of external financing in most developed countries(see for example Damodaran[10] and Gorton and Winton[19]). One of the most important characteristics of debt is whether it is issued by a bank(or a similar institution such as a finance company) or whether it is market debt. There is a vast theoretical litterature discussing the difference between these two types on debt(Diamond[11] and Rajan[29] are seminal contributions), emphasizing the trade off between the better loan monitoring or information gathering abilities of banks and the extra costs attached to borrowing from a bank. Because the level of financial frictions attached to the two types of loans is different, the composition of financing between them may matter for the overall level of financial frictions affecting firms and for macroeconomic outcomes. To complicate matters, the choice between bank and market debt almost certainly depends on aggregate macroeconomic conditions.

Most dynamic general equilibrium models with financial frictions ignore the distinction between bank loans and market debt[6][2]. Models that examine the effect of frictions between banks and their depositors assume that banks are responsible for all lending in the economy[24][8]. This assumption leads to a potential overestimate of the impact of bank lending on the transmission of shocks, since it eliminates the possibility of using other types of financing. Market debt accounts for 57.5% of total non financial sector debt in the US and for 12% of non financial sector debt in the Euro area[14]. Clearly assuming 100% bank or bond lending may be misleading in analyzing financial frictions in the US. For the Euro area such an assumption may seem like a good approximation, except that it is still possible that the low average proportion of bond financing hides important variation across the business cycle which may be relevant for the propagation of various shocks.

Investigation of the effect of the financing choice between bank and market debt on macroeconomic outcomes has been mostly based on reduced form models. In studies with aggregate level VAR's such as Kashyap, Stein and Wilcox[23] or Oliner and Rudebusch[26] it is difficult to distinguish between the hypothesis that financial frictions in general affect the transmission of economic shocks from the hypothesis that that the source of financing matters. Micro level data as in Cantillo and Wright[5] can provide stronger evidence on the importance of macroeconomic conditions for the choice be-

tween bank and market debt, but without a general equilibrium framework it is impossible to go in the other direction and judge the impact of financing choice on macroeconomic outcomes.

In this paper we develop a dynamic general equilibrium model of the firm's choice between bank and bond market financing based on the idea that banks are better loan monitors than markets in financial distress situations. Firms facing financial frictions can use either bank financing or bond financing. Lending to firms is subject to a costly state verification problem as in [6]. Banks are better monitors than debt markets, but their superior lending technology requires them to spend more resources per dollar of loans before the firm produces. Market debt has higher monitoring costs, and it may require a fixed under-writing cost. This paper studies the implications of the model for the steady state of such an economy.

The use of a dynamic general equilibrium framework allows us to make a more quantitative assessment of the magnitude of frictions required to generate realistic financing choice patterns. It also allows investigation of financing choice dynamics in reaction to structural changes in financing costs as well as in reaction to business cycles.

Our modeling of banks as better monitors in the costly state verification framework is based on empirical evidence suggesting that banks' key advantage is in dealing with financial distress situations. Bank loans are easier to renegotiate, and banks have a better understanding of the businesses they are dealing with than bondholders, for example by forcing borrowers to maintain a transactions account at the bank[25]. As a result banks are more capable of dealing with problems such as risk shifting in default, and they are less likely to engage in inefficient liquidation of firms[18][5][3].

In contrast to most papers that model costly state verification in dynamic general equilibrium, such as Carlstrom and Fuerst, [6], and Bernanke Gertler and Gilchrist[2], we assume that firms' production technology exhibits decreasing returns to scale. As a result, firms' financing choice depend on their net worth. Our analysis shows that differences in net worth may be important in accounting for the choices between bank and bond financing observed in the data. In particular, as in the data, higher net worth reduces the likelihood of choosing bank financing(see Cantillo and Wright (2000)[5] and Sufi (2005)[33] for evidence on this point). This occurs in our model despite the fact that all firms are equally productive ex-ante, before signing the financial contract. One could imagine that the link between net worth and financing choice is natural in the presence of a fixed cost of issuing bonds. However,

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the benchmark model produces a strong negative correlation between bank financing and net worth even without the fixed cost. Intuitively, higher net worth reduces the probability of financial distress, which makes the bank's comparative advantage in handling financial distress less valuable. The direct link between net worth and financing choice in the model is in contrast with most previous theoretical work that has explained why less productive firms may prefer bank loans, with the link between firm size and bank financing explained through a positive correlation between firm size and productivity(see for example [29]and [11]).

The benchmark model , where the only cost of bond financing is the cost of auditing distressed firms, cannot explain the high relative use of bank financing in Europe without unrealistically low costs of bank financing. [1] Therefore, we extend the model by assuming that issuing a bond in Europe also requires a small fixed cost. The extra cost is motivated by evidence that until recently bond financing was more expensive in Europe than in the US[32]. The fixed cost assumption is motivated by evidence of large economies of scale in market debt issue costs, which are not present for bank debt[10][7]. We find that a small fixed cost of bond issue (around 0.22% of the average value of issued bonds) can explain most of the discrepancy between the relative amount of bank financing in Europe and the US. To the degree that bond markets in Europe have become more competitive[32], the ratio of bond to bank financing in Europe may converge to that in the US.

Finally, we examine the importance of explicitly modeling the choice between bank and market debt for aggregate output and consumption. We find that for reasonable calibrations, the steady state aggregates of the model are virtually identical to those of a model with only one type of financial intermediary with monitoring costs that are an average of those of the bank and bond contracts. This suggests that at least for the analysis of the steady state, a researcher interested only in aggregates may choose to ignore the choice between bank and market financing.

Several papers have studied the choice between bank and market debt in partial equilibrium (prominent examples include Rajan[29], Diamond[11], Holmstrom and Tirole[21]and Bolton and Freixas[3]). The general message of most of these papers is that more profitable(for a fixed loan size) or higher quality firms tend to prefer market debt, while lower quality firms will pre-

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[1]Europe refers to the Euro Area in this paper.

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fer bank loans. Holmstrom and Tirole[21] distinguish between unmonitored lending(market bonds) and monitored lending(bank lending), where monitoring increases entrepreneur effort and the success probability of projects. Their model generates a negative link between bank finacncing and net worth, just like our model. Meh and Moran(2007)[24] incorporate the Holmstrom and Tirole model into a fully specified dynamic general equilibrium model with entrepreneur and bank capital dynamics.The model in this paper examines a mechanism for the link betwee net worth and financing choice which complements the moral hazard based mechanism in Holmstrom and Tirole[21], while being more focused on the role of banks in managing financial distress emphsized by the empirical evidence. Furthermore, Holmstrom and Tirole only explore the extreme assumptions of a fixed project size and variable project size with linear returns. Assuming linear returns as in Meh and Moran's paper eliminates any possible link between the size of firms and financing choice, while assuming only a fixed project size is usually unrealistic.

Perhaps the closest papers to this one are Cantillo and Wright[5] and De Fiore and Uhlig[14]. Cantillo and Wright's model generates a negative link between net worth and financing and bank financing due to a higher default rate for smaller firms and banks providing cheaper reorganisation in default. They use a partial equilibrium framework with a fixed project size. As a result their framework ignores the possibility that larger firms may prefer bank financing if they are allowed to undertake a larger project and (as in their model and in essentially all applications of the costly state verification framework) monitoring costs are increasing in the size of the project. This effect is eliminated by assumption in a fixed project size model. This paper explores the intermediate case of variable project size with nonlinear returns and provides conditions that generalize some of the insights from the fixed project size model to the more general setup.

De Fiore and Uhlig[14] model the firm's choice between bank and bond financing in a costly state verification framework and integrate this choice into a standard RBC framework. They model banks as offering better ex ante screening of projects at a cost, generating a tradeoff in which lower productivity entrepreneurs prefer bank financing. The realism of their focus on superior ex ante screening by banks is unclear. In fact, it may be more realistic to model bond market lenders as better screeners of projects due to the screening activities of bond rating agencies and investment banks[17]. Furthermore, the prediction of their model that more profitable firms prefer

market debt is empirically controversial(see Sufi (2005)[33] and Cantillo and Wright[5]). Furthermore, their model does not capture the key stylized fact that the use of bank financing relative to market debt decreases with the size of firms. [2]

In the rest of the paper we proceed as follows: section 2 describes the model and provides a sufficient condition for the model to match the empirical link between firm size and the choice between bank and market debt. Section 3 discusses the results of numerical simulations of the model's steady state. Section 4 concludes.

2   The Model

The model features overlapping generations of risk neutral entrepreneurs, a representative risk averse worker and a continuum of pefectly competitive financial intermediaries. Entrepreneurs produce all the output in the economy using capital and labour. They accumulate net worth using capital. When their accumulated net worth is insufficient to fully fund their desired output level, they require loans from financial intermediaries. Due to information frictions, loans require using one of two types of financial intermediaries:banks and bond mutual funds. The differences between these intermediaries and the choice between them will be described in greater detail below.

2.1   Entrepreneurs

The heart of the model is the entrepreneur's intratemporal financing and production decision. Therefore we start by describing the financial contracting environment in any given period.

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[2]A recent paper by Champonnois[7] estimates a structural model of bank versus bond financing that reproduces the empirical link between the use of bank loans and firm size, assuming that larger firms have systematically higher productivity levels than smaller firms. The two period nature of the model and the assumption that entrepreneurs do not have any net worth(no equity) makes integration of the model into standard dsge's difficult, and may miss important dynamics of financing choice. Also, it is not clear that larger firms are always more productive.

2.1.1 Entrepreneur Production and Financing Decisions for a Fixed Level of Net Worth

Entrepreneur j produces final output using the production function

$y_{jt} = z_t \omega_{jt} k_{jt}^\alpha l_{jt}^\gamma, \ , \ 0 < \theta \equiv \alpha + \gamma < 1,$

The firm specific productivity shock $\omega_{jt} \in [0, \infty)$ is i.i.d across both entrepreneurs and time, has a CDF $\Phi(\omega)$, a PDF $\phi(\omega)$ and $E\omega_{jt} = 1$. ω_{jt} is unknown when signing the financial contract. We define $\bar{y}_{jt} \equiv Ey_{jt} = z_t k_{jt}^\alpha l_{jt}^\gamma$. The realisation of ω_{jt} is the private information of the entrepreneur, but can be observed by a type i intermediary at a cost of $\mu_i \bar{y}_{jt}$. z_t is an aggregate productivity shock with a mean of 1.

Entrepreneurs rent capital at a rate r_t and buy labour from households at a wage rate w_t. Production requires spending $x_{jt} = r_t k_{jt} + w_t l_{jt}$ before output is obtained. The entrepreneur has n_{jt} in internal funds available, of which he devotes $\bar{n}_{jt} \le n_{jt}$ to the project. If the desired x_{jt} exceeds the entrepreneur's internal funds dedicated to the project, the entrepreneur will require external financing from a lender. Alternatively, we can think of the entrepreneur as being able to post a collateral of n_{jt} before output is realized. Any loan below n_{jt} does not involve any financing frictions or other costs. But any part of the loan above n_{jt} will be subject to information frictions.

There is a continuum of fully diversified financial intermediaries of two types: banks and mutual funds, indexed by $i \in \{b, m\}$. Both intermediaries collect funds from households, and use them to make loans to entrepreneurs. Competition for borrowers ensures that the financial intermediaries make zero profits. Because financial intermediaries can fully diversify the idiosyncratic risk of the entrepreneurs, households are risk neutral with respect to intermediaries' loan portfolios. Because the loans are intratemporal and risk free, the required gross rate of return is 1.

The difference between the two types of intermediaries is that banks are better informed about borrowers than mutual funds. This superior information makes banks better monitors in case of financial distress. In particular, banks can observe ω_{jt} at a lower cost than mutual funds.

Banks can learn ω_{jt} at a cost of $\mu_b \bar{y}_{jt}$. Mutual funds can learn ω_{jt} at a cost of $\mu_m \bar{y}_{jt}$, where $\mu_m > \mu_b$. With a more general interpretation of audit costs as financial distress costs one can imagine for example that auditing prevents entrepreneur from taking on risky projects that may benefit him but reduces the expected value of the assets obtained by lender, and banks are better at controlling this risk-shifting. In order to offer lower cost monitoring, banks

must spend $\tau(x_{jt} - \bar{n}_{jt})$, where $\tau > 0$. These could be interpreted directly as costs of gathering more information on lenders. They could also be seen as a reduced form for costs related to frictions between banks and depositors in a model where banks cannot accumulate any capital. [3] At the same time, bond mutual funds may require a fixed cost C_m in order to issue a bond.

To simplify notation we can abstract from time and entrepreneur subscripts. It is convenient to first pick the optimal amounts of capital and labour given a total expenditure $x = rk + wl$ and characterize the solution to the contract in terms of x and the bankruptcy threshhold $\bar{\omega}$. For a given expenditure level the optimal expected output is $\bar{y} = Mx^\theta$, where $M = z\frac{\alpha^\alpha \gamma^\gamma}{\theta^\theta r^\alpha w^\gamma}$. Once we have solved the financial contract, factor demands are given by $k = \frac{\alpha}{\theta r}x$ and $l = \frac{\gamma}{\theta w}x$. We start by solving for the entrepreneur's production level when he is restricted to self financing. The expenditure level without access to external financing solves

$\pi_a^e = \max_x Mx^\theta + (n - x)$

subject to

$x \leq n.$

$x \geq 0.$

Depending on the availability of internal funds, the entrepreneur either picks the first-best interior solution $x_a = \hat{x} \equiv (\theta M)^{1/(1-\theta)}$, or he sets $x_a = n$. In fact since $\hat{x}$ is independent of n, any entrepreneur with n above a certain threshhold value will prefer self-financing.

We now turn to the contract conditional on the entrepreneur requiring external financing and having chosen a type i intermediary. Let R_i be the required gross rate of return to the lender($R_b = 1 + \tau, R_m = 1$). Because the entrepreneur has access to a storage technology, his opportunity cost of funds is 1. The optimal contract specifies a state contingent repayment schedule and the set of audited states in order to maximize entrepreneur profits subject to incentive compatibility constraints for the entrepreneur and the lender's break-even constraint. The distribution of ω and the auditing costs satisfy the conditions in Gale and Hellwig [16] for the optimal contract to be a debt contract with a threshhold $\bar{\omega}$ such that the repayment is $b(\omega) = y$ for $\omega \leq \bar{\omega}$ and $b(\omega) = \bar{\omega}\bar{y}$ if $\omega > \bar{\omega}$.

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[3]For example, we can imagine 1 period bankers that can run away with and consume a proportion τ of the loan. In this case the loan contract must ensure that the expected repayments by entrepreneurs net of the audit costs and the deposit repayments exceed $\tau(x_{jt} - \bar{n}_{jt})$, which is exactly the bank's break-even constraint in the financial contract.

We define the entrepreneur's and the lenders' expected shares of $\bar{y}$ as

$$f(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega d\Phi - \bar{\omega}[1 - \Phi(\bar{\omega})] \text{ for the entrepreneur.}$$

$$m(\bar{\omega}) = \int_{0}^{\bar{\omega}} \omega d\Phi + \bar{\omega}[1 - \Phi(\bar{\omega})] - \mu\Phi(\bar{\omega}) \text{ for the lender.}$$

Like other papers(e.g Bernanke, Gertler and Gilchrist[2] and Covas and Den Haan[9]), we assume that the hazard rate of ω is increasing in ω at the optimal $\bar{\omega}$,

Assumption 1:

$$\frac{d}{d\bar{\omega}} \left(\frac{\phi(\bar{\omega})}{1 - \Phi(\bar{\omega})} \right) > 0, \ \phi(\bar{\omega}) > 0 \text{ when } \bar{\omega} > 0.$$

This assumption is satisfied by commonly used distributions such as the lognormal or uniform distributions. Define $C_i \geq 0$ to be the fixed cost of issuing debt for type i intermediary, where $C_b = 0$. Define $1(x - \bar{n} > 0)$ as the indicator function for $x - \bar{n} > 0$.

The optimal contract with type $i \in \{b, m\}$ financial intermediary solves

$$\pi_i^e = \max_{x, \bar{\omega}, \bar{n}} f(\bar{\omega}_i) M x_i^\theta + (n - \bar{n}_i)$$

subject to

$$m(\bar{\omega}_i)\bar{y}_i \geq R_i(x_i - \bar{n}_i) + C_i 1(x - \bar{n} > 0) \quad (1)$$
$$\bar{n}_i \leq n. \quad (2)$$
$$\bar{n}_i \geq 0 \quad (3)$$
$$x_i - \bar{n} \geq 0 \quad (4)$$

If $\phi(0) = 0$,$R = 1$ and $C_m = 0$, then $x - \bar{n} > 0$ whenever $n < \hat{x}$, the first best level of output. Otherwise, when $C_m = 0$ we can guarantee that $x - \bar{n} > 0$ on an interval of $n's$ $(0, \bar{n})$, where $\bar{n}$ tends to $\hat{x}$ as μ and τ go to 0 (See the appendix for the proof). When $C_m > 0$, $x - \bar{n} > 0$ as long as C_m is not too large. In this section, we assume $x - \bar{n} > 0$. [4] The numerical

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[4]Alternatively one can think of all the proofs in this section as applying only to those firms that are not rationed.

algorithm in the next sections allows for the possibility of rationing. Let $\lambda_i, \xi_i,$ ψ_i be the lagrange multipliers for constraints (1-3) respectively. Besides the complementary slackness conditions, the first order conditions are

$$x \quad : \quad \theta M x_i^{\theta-1}[f(\bar{\omega}_i) + \lambda_i m(\bar{\omega}_i)] = \lambda_i R_i \tag{5}$$

$$\bar{\omega} \quad : \quad f'(\bar{\omega}_i) + \lambda_i m'(\bar{\omega}_i) = 0 \tag{6}$$

$$\bar{n} \quad : \quad \psi_i + \lambda_i R_i = \xi_i + 1. \tag{7}$$

Finally, the entrepreneur picks $E y^e = \max[\pi_a^e, \pi_m^e, \pi_b^e]$.

The following lemma collects some straightforward results that simplify the solution of the model and help us characterize the financing choice of entrepreneurs:

Lemma 1 *a)   At the optimum, $m'(\bar{\omega}) > 0$ and the bank's break-even constraint is binding.b) Under the assumption that $\Phi'(\bar{\omega}) = \phi(\bar{\omega}) > 0$ the entrepreneur always chooses Maximal Equity Participation(MEP): $\bar{n} = n.$* [5] *c)external financing is never optimal if the entrepreneur's wealth constraint*

does not bind.   d)Under assumption 1,$\lambda'(\omega) > 0.e)\frac{d\bar{\omega}}{dn} < 0$ and $\frac{d\bar{\omega}}{dC_m} > 0.$ *f)$\frac{d\bar{\omega}}{d\tau} < 0$ as long as C_m is not too large.*

Proof. see the appendix. ∎

Due to the i.i.d nature of the productivity shock ω_{jt}, the only ex-ante heterogeneity among entrepreneurs is due to different levels of net worth n_{jt}. There are several effects that determine the desirability of bank or bond financing for a given level of net worth. The model emphasizes the role of banks in reducing costs of financial distress. Since the default rate is decreasing in net worth $(\frac{d\bar{\omega}}{dn} < 0)$ for a fixed financing type, it seems natural then that holding $\bar{y}$ constant, smaller firms that are more likely to default for a given financing type will gravitate towards banks. At the same time a

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[5]The   strict optimality of MEP contracts relies on our limited liability assumption concerning $n - \bar{n}$. Gale and Hellwig[16]assume $n - \bar{n}$ can be used as collateral. In that case the optimal level of equity participation is indeterminate, and the MEP contract only weakly dominates any other contract.

higher net worth increases the project's expected output $\bar{y}$, which increases expected default costs $\mu\Phi(\bar{\omega})\bar{y}$ for a given μ and $\bar{\omega}$. For a fixed project size x, the extra cost per dollar of bank loan $\tau(x-n)$ penalizes small firms with higher $x-n$. Things are less clear cut when firms can adjust the size of their project. Higher net worth reduces the desired leverage ratio $\frac{x-n}{n}$. [6] Decomposing the ex-ante cost of bank financing $\tau(x-n)$ as $\tau\left(\frac{x-n}{n}\right)n$, we see that it may actually decline in n, though this does not have to be the case. We cannot theoretically rule out that the last two effects overwhelms the effect of a lower $\bar{\omega}$ and makes bank financing more attractive for larger firms for general values of n and θ but, we will find conditions for larger firms to prefer bond financing conditions on certain range of $n's$ and $\theta's$. The numerical analysis in the next section will examine the plausibility of those conditions.

Before proceeding with the analysis for the decreasing returns to scale, we can verify that under the standard assumption in for example Carlstrom and Fuerst[6] or Bernanke, Gertler and Gilchrist[2], of constant returns to scale and no fixed cost of market finance the choice of financing type does not depend on net worth. Therefore, all firms in our setup would choose the same financial intermediary if $\theta = 1$. The only possibility for a link between net worth and financial intermediation choice in the constant returns to scale case arises when there is a fixed cost of market financig $C_m > 0$ in an environment in which market financing would dominate when $C_m = 0$ or when some firms would be rationed at $x - n = 0$ with market financing:

Proposition 2 $a)$Suppose $\theta = 1$ and for each i $M > \frac{R_i}{1-\mu\phi(0)}$. If $C_m = 0$,then the optimal choice of financial intermediary is independent of n. Therefore, all firms choose the same financial intermediary type. All firms prefer bank financing when $\tau < \bar{\tau}$ and all firms prefer bond financing when $\tau > \bar{\tau}$, for some $\bar{\tau} > 0$. b) If $C_m > 0$, any firm rationed at $x - \bar{n}$ by the bond contract will pick bank financing. Among non-rationed firms, If the optimal form of intermediation is bank financing when $C_m = 0$ then this is also the optimal choice for all firms when $C_m > 0$. If the optimal financial intermediation

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[6]For the leverage ratio, note that combining the foc for x and the break even constraint we have $\theta(1 - n/x + \frac{C}{Rx})\left(\frac{f+\lambda m}{\lambda m}\right) = 1$. The second term in brackets is decreasing in $\bar{\omega}$. So x/n must be increasing in $\bar{\omega}$ if $C = 0$. The comparative statics with respect to n and M now follow from the relation between $\bar{\omega}$ and those parameters. With $C > 0$ its is possible for x/n to decrease in $\bar{\omega}$, but by continuity $\frac{d(x-n)/n}{dn} < 0$ should still hold for a small enough C.

is market financing when $C_m = 0$, then in the economy with $C_m > 0$ there exists a threshold $\hat{n}$ such that all non rationed firms with $n < \hat{n}$ prefer bank financing while all non rationed firms with $n \geq \hat{n}$ prefer market financing.

Proof. a) Consider first the case when $C_m = 0$. Our assumption that $M > \frac{R_i}{1-\mu\phi(0)}$ ensures that $x - n > 0$ for all firms(see the appendix for the proof when $\theta < 1$. The proof for $\theta = 1$ is similar). Using the break-even constraint of the financial intermediary to solve for x, the entrepreneur's expected profit with type i intermediary is $\pi_i^e = f(\bar{\omega}_i) M \frac{nR_i}{R_i - m(\bar{\omega})M}$. $\pi_b^e - \pi_m^e = Mn \left(\frac{(1+\tau)f(\bar{\omega}_b)}{(1+\tau)-m(\bar{\omega}_b)M} - \frac{f(\bar{\omega}_m)}{1-m(\bar{\omega}_m)M} \right) \equiv Mn\Delta > 0$ iff $\Delta > 0$. From the first order conditions, $\frac{d\omega}{dn} = 0$ when $\theta = 1$. Since n does not directly affect $f(\bar{\omega})$ or $m(\bar{\omega})$ Δ is independent of n, making the sign of $\pi_b^e - \pi_m^e$ independent of n. Since $\frac{d\pi^e}{d\mu} < 0$, $\pi_b^e - \pi_m^e > 0$ when $\tau = 0$. $\frac{d\pi^e}{d\tau} < 0$, and therefore $\pi_b^e - \pi_m^e$ is decreasing in τ. Finally, $\lim_{\tau \to \infty} \pi_b^e - \pi_m^e < 0$. By the continuity of $\pi_b^e - \pi_m^e$ in τ, there exists a unique $\bar{\tau}$ such that $\pi_b^e - \pi_m^e > 0$ whenever $\tau < \bar{\tau}$ and $\pi_b^e - \pi_m^e < 0$ for $\tau > \bar{\tau}$. b) Next, consider the case when $C_m > 0$. Any firm that would be rationed by the bond contract will obviously pick the bank contract, since by our assumption on M the optimal bank contract dominates choosing $x = n$. Now consider firms that are not rationed by the bond contract. The relative profit of bank financing versus bond financing is $\pi_b^e - \pi_m^e = M \left[n\Delta + \frac{f(\bar{\omega}_m)C_m}{1-m(\bar{\omega}_m)M} \right]$. If $\Delta > 0$ (all firms prefer bank financing when $C_m = 0$), then bank financing is preferred iff $n > -\frac{1}{\Delta}\frac{f(\bar{\omega}_m)C_m}{1-m(\bar{\omega})M}$. Since the last expression is negative and $n \geq 0$, this constraint never binds and all firms pick bank financing in this case. If $\Delta = 0$, then clearly all firms prefer bank financing. If $\Delta < 0$ (all firms prefer bond financing when $C_m = 0$), then a firm prefers bank financing iff $n < -\frac{1}{\Delta}\frac{f(\bar{\omega}_m)C_m}{1-m(\bar{\omega}_m)M} \equiv \hat{n}$. All other firms prefer bond financing. Note that $\bar{\omega}$ remains independent of n if $x - n > 0$ regardless of C_m. From the same first order conditions, $\bar{\omega}$ is also independent of C_m. As a result $\hat{n}$ can be computed as the product of C_m and a term that depends only on M. ∎

The empirical calibrations in the next section suggests that the extra intermediation fee of the bank τ is quite low, casting doubt on the ability of the constant returns to scale model to generate a negative relation between net worth and financing choice, except through the rationing of small firms by the bond contract due to the fixed cost. [7]

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[7] We did a few quick tests of the constant returns to scale model in a partial equilibrium

We would like to establish some conditions guaranteeing that the model with $\theta < 1$ reproduces the pattern observed in the data where smaller firms prefer bank financing and larger firms prefer market financing. In particular if the derivative of the relative bank versus bond contract profit with respect to wealth $\frac{d(\pi_b^e - \pi_m^e)}{dn} < 0$, and firms choose both bank and bond financing then it must be the case that for some $\bar{n}$ firms with $n < \bar{n}$ prefer bank financing while firms with $n \geq \bar{n}$ prefer bond financing.

The following proposition gives a sufficient condition guaranteeing the existence of an interval of values of $\tau \in (0, \tau^*)$ and an interval of $n \in (0, n^*)$ values for which bond financing becomes more attractive as n increases. The key requirement is that bank financing lowers the shadow cost of external finance λ:

Proposition 3 *Suppose that $\frac{d\lambda_m}{d\mu_m} > 0$ at $n = 0$. Then there exists a neighbourhood $N_\varepsilon \subset R_+^2$ of $n = 0 = \tau$ such that $\frac{d(\pi_b^e - \pi_m^e)}{dn} < 0$ whenever $(\tau, n) \in N_\varepsilon$. Therefore there exist a τ^* and a n^* such that whenever $\tau < \tau^*$ and $n < n^*$ we have $\frac{d(\pi_b^e - \pi_m^e)}{dn} < 0$.*

Proof. *By the envelope theorem $\frac{d(\pi_b^e - \pi_m^e)}{dn} = (1+\tau)\lambda(\bar{\omega}_b(\tau, n)) - \lambda(\bar{\omega}_m(0, n)) \equiv S(\tau, n) - \lambda_m(0, n)$. Since $\mu_b < \mu_m$, $\frac{d\lambda_m}{d\mu_m} > 0$ at $n = 0$ $\frac{d\bar{\omega}}{dC} > 0$ and $\lambda_i'(\bar{\omega}) > 0$, we have $S(0,0) - \lambda_m(0,0) = \lambda_b(0,0) - \lambda_m(0,0) < 0$. By the maximum theorem, for either $i = b$ or m, $\bar{\omega}_i(\tau, n)$ is continuous on R_+^2 at $\tau = n = 0$. Therefore $S(\tau, n) - \lambda_m(\tau, n)$ is continuous in (τ, n) at $(0,0)$. Together with $S(0,0) - \lambda_m(0,0) < 0$ this implies the existence of the required N_ε neighbourhood. The existence of τ^* and n^* is immediate by taking any rectangle contained in N_ε.* ∎

$\frac{d\lambda}{d\mu} = \frac{\partial\lambda}{\partial\mu} + \lambda'(\bar{\omega})\frac{d\bar{\omega}}{d\mu}$. Since $\frac{\partial\lambda}{\partial\mu} > 0$ and $\lambda'(\bar{\omega})$, $\frac{d\bar{\omega}}{d\mu} \geq 0$ is a sufficient condition for $\frac{d\lambda}{d\mu} > 0$. $\frac{d\bar{\omega}}{d\mu} \geq 0$ holds for the standard costly state verification model with a fixed project size. It may still hold with a variable project size as long as x does not decline too much when μ increases. [8] Intuitively, increasing

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setting, with the wage and interest rate fixed by the steady state of an open economy where there are no financial frictions in the rest of the world. Bank financing was optimal in those tests even for $\tau = 0.08$ which is much higher than the evidence presented in Erosa(2001)[13].

[8] x must be decreasing in μ : Let $A \equiv \theta M x_i^{\theta-1} \frac{f(\bar{\omega}_i) + \lambda_i m(\bar{\omega}_i)}{\lambda_i}$. From the first order condition for x, $\frac{dA(x(\mu), \bar{\omega}(\mu), \mu)}{d\mu} = \frac{\partial A}{\partial x}\frac{dx}{d\mu} + \frac{\partial A}{\partial\bar{\omega}}\frac{d\bar{\omega}}{d\mu} + \frac{\partial A}{\partial\mu} = 0$. $\frac{\partial A}{\partial x} < 0$ and $\frac{\partial A}{\partial\bar{\omega}} < 0$. Since

μ raises the required repayments for a given expenditure x. If x does not react too strongly , this requires an increase in the coupon rate and hence an increase in the default rate. If we had constant returns to scale ($\theta = 1$), the entrepreneur would react to a higher μ by lowering x so much that $\bar{\omega}$ would decline. Intuition suggests that with sufficiently decreasing returns to scale the reaction of x is small enough that $\bar{\omega}$ may actually increase as in the model with an exogeneously fixed project size. At least, the decrease in $\bar{\omega}$ in response to a higher μ would be small enough to allow the positive direct effect on λ of a higher μ to dominate. More formally, we have:

Lemma 4 *Suppose that at $n = \tau = 0$, $\lim\limits_{\theta \to 0}\phi(\bar{\omega}) > 0$, $\lim\limits_{\theta \to 0}\bar{\omega} < \infty$, and $\lim\limits_{\theta \to 0}m'(\bar{\omega}) > 0$. Then $\lim\limits_{\theta \to 0}\frac{d\omega_m}{d\mu} \geq 0$, and there exists a $\bar{\theta} > 0$ such that $\frac{d\lambda}{d\mu} > 0$ at $\tau = n = 0$ whenever $\theta < \bar{\theta}$.*

Proof. See the appendix. ∎

The requirement that $\lim\limits_{\theta \to 0}\phi(\bar{\omega}) > 0$ may be problematic. Certainly this condition holds for the uniform distribution. For the lognormal distribution the condition is always satisfied if the standard deviation σ of $\ln\bar{\omega}$ is high enough, but for typical calibrations(as well for the calibration in section 3), $\lim\limits_{\theta \to 0}\phi(\bar{\omega}) > 0$ requires the presence of a fixed cost $C_m > 0$. The numerical calibration in section 3 shows that the model with decreasing returns to scale can generate a realistic negative relation between net worth and bank financing even without the fixed cost. This is in contrast to the constant returns to scale model which requires a fixed cost in order to have a non degenerate financing choice. Also, the net worth-financing choice link emerges in the numerical analysis for $\theta = 0.9$, suggesting that the degree of decreasing returns to scale required by the theoretical results above is plausible.

If we have the stronger condition that $\lim\limits_{\theta \to 0}\frac{d\omega_m}{d\mu} > 0$, we can also derive an interesting result about the effect of a general improvement in the auditing technology for both banks and bond funds. Let $s \equiv \mu_m/\mu_b > 1$. We are interested in the consequences of a reduction in the cost of auditing μ_b for a fixed ratio of market to bank debt auditing efficiency s. If a reduction in μ_b leads more firms to prefer bond financing, then according to the

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$\frac{\partial\lambda(\bar{\omega},\mu)}{\partial\mu} > 0$ and $\frac{\partial m(\bar{\omega},\mu)}{\partial\mu} < 0$, $\frac{\partial A}{\partial\mu} < 0$. This implies that $\frac{\partial A}{\partial x}\frac{dx}{d\mu} + \frac{\partial A}{\partial\bar{\omega}}\frac{d\bar{\omega}}{d\mu} > 0$. Let $\Delta\mu > 0$. Since $\frac{d\pi^e}{d\mu} < 0$, $\Delta x \geq 0$ implies that $\Delta\bar{\omega} > 0$. But then $\frac{\partial A}{\partial x}\frac{dx}{d\mu} + \frac{\partial A}{\partial\bar{\omega}}\frac{d\bar{\omega}}{d\mu} < 0$, which is impossible. Therefore, $\frac{dx}{d\mu} < 0$.

model a general improvement in the cost of dealing with financial frictions causes a switch towards more market debt. Intuitively this should be the case: if auditing technology in general improves the importance of banks as better auditors should diminish. Thus the model also provides a potential explanation for the shift towards more market debt financing in the last 30 years(Samolyk,2004)[31]. [9]We can show that this is what happens if we have sufficiently strong decreasing returns to scale(low θ):

Proposition 5 *Suppose $\frac{d\bar{\omega}_m}{d\mu} \geq 0$ and $\lim\limits_{\theta \to 0}\bar{\omega}_m > 0$. Then for a fixed $s > 1$, there exists a $\theta_* > 0$ such that$\frac{d(\pi_b^e - \pi_m^e)}{d\mu_b} > 0$ for $\theta \in (0, \theta_*)$.*

Proof. By the envelope theorem, ,$\frac{d(\pi_b^e - \pi_m^e)}{d\mu_b} = s\lambda(\omega_m, \mu_m)\Phi(\bar{\omega}_m)Mx_m^\theta - \lambda(\omega_b, \mu_b)\Phi(\bar{\omega}_b)Mx_b^\theta$. Since $\frac{d\bar{\omega}_m}{d\mu} \geq 0$, and $\frac{d\bar{\omega}}{d\tau} < 0$ (from lemma 1f) $\omega_m > \omega_b$. This, together with $\frac{\partial\lambda(\bar{\omega},\mu)}{\partial\mu} > 0$ and $\lambda'(\bar{\omega}) > 0$ implies that $\lambda(\omega_m, \mu_m)\Phi(\bar{\omega}_m) > \lambda(\omega_b, \mu_b)\Phi(\bar{\omega}_b)$ for any $\theta > 0$. Since $\lim\limits_{\theta \to 0}\bar{\omega}_m > 0$ and $s > 1$, this implies that $\lim_{\theta \to 0} s\lambda(\omega_m, \mu_m)\Phi(\bar{\omega}_m) > \lim_{\theta \to 0} \lambda(\omega_b, \mu_b)\Phi(\bar{\omega}_b)$. But then since $\lim_{\theta \to 0} Mx^\theta = M$, we have that $\lim_{\theta \to 0} \frac{d(\pi_b^e - \pi_m^e)}{d\mu_b} > 0$. Therefore, there exists a $\theta_* > 0$ such that $\frac{d(\pi_b^e - \pi_m^e)}{d\mu_b} > 0$ when $\theta < \theta_*$. ■

The sufficient condition in proposition 5 holds when $C_m > 0$ in a neighbourhood of $n = 0$, as long as $\bar{\omega}$ and $\frac{d\bar{\omega}}{d\mu}$ are continuous in θ at 0 :

Lemma 6 *Suppose that $\bar{\omega}$, $\frac{d\bar{\omega}_m}{d\mu}$ is right continous in θ at $\theta = n = 0$ and $C_m > 0$. Then $\lim\limits_{\theta \to 0}\bar{\omega}_m > 0$ and there exists a $\theta^* > 0$ such that $\frac{d\bar{\omega}_m}{d\mu} > 0$ at $n = 0$ for any $\theta < \theta^*$. Furthermore, these properties continue to hold for $n \in (0, n^{**})$, where n^{**} is a positive number.*

Proof. See the appendix. ■

2.1.2 The Dynamic Behaviour of Entrepreneurs

[9]Samolyk [31] finds that the proportion of short term nonfinancial business lending done by commercial banks in the US has declined from around 75% to about 50% between 1974 and 2004. This figure may overestimate the decline of bank-like lending to the degree that many finance company loans(that have increased significantly during this period) may be very similar to bank loans.

We model entrepreneur savings in a similar way to Bernanke,Gertler and Gilchrist[2]. There are overlapping generations of two-period lived entrepreneurs. There is a measure 1 of old entrepreneurs in each period that can operate a project and then exit the economy. Each period, they are replaced by a measure 1 of young entrepreneurs. Young entrepreneurs cannot produce or work, and they are born without any endowment. Entrepreneurs are risk neutral and only care about consumption when old. An old entrepreneur begins the period with a stock of capital k_{jt}^e. The entrepreneur can rent out his capital to obtain $n_{jt} = k_{jt}^e(1 + r_t - \delta)$, where δ is the depreciation rate of capital. Then, the entrepreneur makes the financing decision in order to maximise expected income Ey_{jt}^e. Based on net worth n_{jt} and the aggregate state of the economy, the entrepreneur decides whether to produce with a bank loan, produce with a mutual fund loan or rely only on n_{jt} for funding. Next, if the entrepreneur has decided to contract with a financial intermediary the contract is signed for a loan of $r_t k_{jt} + w_t l_{jt} - n_{jt}$, and the entrepreneur rents capital and hires labour. If the entrepreneur prefers autarky he uses part of his net worth to finance his production, and he stores the remaining funds $n_{jt} - \bar{n}_{jt}$ till the end of the period.

Finally, the idiosyncratic shocks ω_{jt} are realised, entrepreneurs produce, pay for capital and labour and deliver the loan repayment $b(\omega_{jt})$. This leaves entrepreneurs with income y_{jt}^e. At this point old entrepreneurs get to consume all their income with probability π_e. In this case they consume $c_{jt} = y_{jt}^e$, leaving the young without any capital in the beginning of the next period. With a probability $1 - \pi_e$, the young get all the income as a bequest from the old entrepreneurs. In this case the young entrepreneur saves $k_{j,t+1}^e = y_{jt}^e$. [10]In the case of constant returns to scale this structure gives exactly the same saving function as the original Bernanke et al. [2] model with infinitely lived entrepreneurs with a constant death probability of π_e. With constant returns to scale, the risk neutrality of entrepreneurs would make it optimal for them to maximise expected profits from production inside each period, despite the infinite horizon. Therefore, we can use the previously derived financial contract to describe entrepreneur production decisions. The decreasing returns to scale assumption complicates matters. If the default rate were independent of net worth, then the value of the firm would still

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[10]This outcome can be derived from a negotiation between old and young entrepreneurs over $y_{j,t}^e$. The old make a take it or leave it offer to the young with probability π_e, while the young make a take it or leave it offer to the old with probability $1 - \pi_e$.

be increasing in expected current profits, and the static contract would still be optimal. Since the default rate is decreasing in net worth maximising expected current profits is no longer optimal. In particular the entrepreneur may prefer a lower project size relative to the static case in order to lower the default rate and reduce the chances of entering the next period with zero net worth. In combination with the discrete financing choice this makes the optimisation problem with decreasing returns to scale considerably more challenging, particularly if the goal is to eventually study financing choices in general equilibrium with aggregate shocks. The overlapping generations assumption sidesteps this issue. [11]

2.2 Workers

There is a measure 1 of risk averse workers. The representative worker chooses sequences of consumption and saving to maximise $E_0 \sum_{t=0}^{\infty} \beta^t \ln c_{h,t}$ subject to the sequence of budget constraints

$$c_{h,t} + k_{h,t+1} = k_{h,t}(1 + r_t - \delta) + w_t \tag{8}$$

where r_t and w_t are the rental rate and the real wage rate. Workers rent out capital and work. They can then use their income to lend to financial intermediaries $L_t \leq r_t k_{ht} + w_t l_{ht}$. Both banks and bond mutual funds are completely diversified with respect to entrepreneurs' idiosyncratic risk.This, in addition to their intratemporal nature, makes the gross rate of return on loans 1. At the end of the period all payments are made and workers consume and save. From the workers' optimisation problem, we get the standard Euler equation:

$$\frac{1}{c_{ht}} = \beta E_t \frac{1}{c_{h,t+1}} \left(1 + r_{t+1} - \delta\right) \tag{9}$$

In the steady state, this equation pins down the interest rate at $r = \frac{1}{\beta} - 1 + \delta$.

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[11]Another assumption that would preserve the optimality of static expected profit maximisation would be allowing the entrepreneur to diversify away the risk of default by holding a continuum of projects, and generating differences among entrepreneurs through ex-ante idiosyncratic shocks(occuring before the financial contracting decision is made).

2.3 The Competitive Equilibrium

We are now in a position to define the steady state competitive equilibrium for this economy. The state of each entrepreneur is described by $S_{jt} = \left(\omega_{jt}, k_{jt}^e, 1_d^e\right)$, where 1_d^e indicates entrepreneur death, with a joint probability measure $F_t(S)$. Let $C_t = c_{h,t} + \int c_{jt}dF$, $n_t = \int n_{jt}dF$, $x_t = \int x_{jt}dF$, $l_t = \int l_{jt}dF$, and $i_t = K_{t+1} - (1 - \delta)K_t$ where $K_t = k_{h,t} + \int k_{jt}^e dF$. Finally define 1_b, 1_m, 1_a and 1_s as the indicator functions respectively for bank financing, market debt financing, autarky and entrepreneur default.

A steady state competitive equilibrium consists of capital rental rate and wage rates (r, w) entrepreneur and worker consumption and saving policies $\left\{c^e(S_{jt}), k_{j,+1}^e(S_{jt}), c_h(k_t^h), k_{h,+1}(k_{h,t})\right\}$ and decisions $\left\{1_m(k_{jt}^e), 1_b(k_{jt}^e), 1_a(k_{jt}^e), 1_s(k_{jt}^e, \omega_{jt})\right\}$ such that

1. The capital market clears: $k_h + \int k_{jt}^e dF = \int k_{jt}dF$.
2. The labour market clears: $l_t = 1$.
2. The output market clears: $y = C + i + \int 1_s 1_b \mu_b \bar{y}_{jt} dF + \int 1_s 1_m \mu_m \bar{y}_{jt} dF + \tau \int 1_b(x_{jt} - n_{jt})dF + C_m \int 1_m dF$.
3. The loan market clears: $rk + wl - n = x - n$.
4. Financial contracts are optimal, entrepreneurs maximise their expected income Ey_{jt}^e and pick $k_{j,t+1}^e$ optimally.
5. Households pick consumption and saving optimally.
7. F is an invariant distribution: given the conditional probability function $Q(S, A)$ and given any event A,
$F(A) = \int_A Q(S, S')F(dS')$.

3   Results

3.1   Calibration

The model period is one quarter. Following Carlstrom and Fuerst[6], we set $\beta = 0.99$, $\delta = 0.02$. We set aggregate productivity to $z = 1$. Following the discussion in Restuccia and Rogerson[30] and Jaimovich and Rebelo(08) we set $\theta = 0.9$, implying a profit share of around 10%. A θ of 0.9 is in the upper bound of empirical estimates. If anything, choosing a relatively high value of θ should make it more difficult for the model to generate a link between firm size and financing choice. We set the share of capital to one third, giving $\alpha = 0.3$. The entrepreneurs' death rate is 3%, based on Bernanke,Gertler and Gilchrist[2].

The calibration of μ_m and μ_b does not have any precedents in the litterature, requiring us to make some extra assumptions. Let $s = \frac{\mu_m}{\mu_b}$. We define the average audit cost,

$$\mu \equiv \mu_b \frac{\int 1_s 1_b \bar{y}_{jt} dF}{\int 1_s \bar{y}_{jt} dF} + \mu_m \frac{\int 1_s 1_m \bar{y}_{jt} dF}{\int 1_s \bar{y}_{jt} dF} = \mu_b \hat{p} + \mu_m (1 - \hat{p}), \text{ where } \hat{p} \equiv \frac{\int 1_s 1_b \bar{y}_{jt} dF}{\int 1_s \bar{y}_{jt} dF}.$$

To simplify the calibration we approximate $\hat{p}$ by $p \equiv \frac{\int 1_b (x_{jt} - n_{jt}) dF}{\int (1 - 1_a)(x_{jt} - n_{jt}) dF}$ and set $\mu = p\mu_b + (1-p)\mu_m = \mu_b[p + (1-p)s]$. [12] Given μ and p, we could solve for μ_b and μ_m if we knew s. We approximate p by the average ratio of bank finance to total debt finance over 1997-2003 in the US and in the Euro area, as reported in De Fiore and Uhlig[14]. This gives us $p = 0.425$ for the US and $p = 0.88$ for the Euro area. As there are several estimates of μ in the litterature, we take an intermediate estimate of $\mu = 0.15$ from Carlstrom and Fuerst[6]. This leaves s. In line with our focus on banks' lower cost of dealing with financial distress, we use evidence from Gilson,Kose and Lang[18] on the probability of private restructuring as opposed to formal bankruptcy and on the relative costs of these procedures to determine s. Let π_i be the probability of private restructuring for a debt of type i. Let $\bar{m}$ be the proportional cost of private restructuring and $\hat{m}$ be the proportional cost of formal bankruptcy, where due to lack of evidence we assume these costs are the same for both bank and non-bank debt. Since our model does not distinguish between these two forms of financial distress, we assume that $\mu_i = \pi_m \bar{m} + (1 - \pi_m)\hat{m}$. This implies

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[12] The approximation is exact if we have constant returns to scale.

that $s = \frac{\mu_m}{\mu_b} = \frac{\pi_m \bar{m} + (1-\pi_m)\hat{m}}{\pi_b \bar{m} + (1-\pi_b)\hat{m}} = \frac{\pi_m h + (1-\pi_m)}{\pi_b h + (1-\pi_b)}$, where $h = \bar{m}/\hat{m}$. Gilson,Kose and Lang[18] estimate $\pi_b = 0.9$ and $\pi_m = 0.375$. They also report that the average successful private restructuring takes 15.4 months, as opposed to 28.5 months for the average unsuccesful restructuring and formal bankruptcy. Assuming that the costs of these procedures is proportional to their duration, we get an estimate of $h = 15.4/28.5 = 0.54$. With these numbers we find $s = 1.412$, $\mu_b = 0.121$ and $\mu_m = 0.171$ for the US. For the Euro area, we assume the same s and the same μ, giving $\mu_b = 0.143$ and $\mu_m = 0.202$.[13]

In the US calibration we assume that there are no fixed costs of issuing bonds($C_m = 0$). In this case, we pick the other parameters to roughly match the quarterly default rate for the US of 0.974% reported in Carlstrom and Fuerst[6], the ratio of bank financing to market debt financing from De Fiore and Uhlig[14] and the costs of bank intermediation per dollar of loans in developed economies from Erosa[13]. [14] For the European calibration, we pick C_m to match the difference between bond issue costs in the US and Europe from Santos and Tsatsaronis[32].

We assume that ω follows a log-normal distribution where, $\ln \omega$ has a standard deviation σ and mean $-\sigma^2/2$.

We set $\sigma = 0.185$. We try several values of τ ranging from 0.005% to 2%. The most successful calibration has $\tau = 0.25\%$. In combination with the costs of auditing financially distressed firms, this value leads to a reasonable estimate of total bank intermediation costs.

3.2 General Equilibrium Results

For the US, the model provides a rough match to default rates, the ratio of bank to market debt and the costs of financial intermediation in the US with$\tau = 0.25\%$ (table 1). The 2.8% (10.74% at an annual rate) default rate obtained in our simulations is high relative to the 1% default rate used as a target. However, the 1% estimate is biased downwards due to underrepresentation of small unincorporated firms in the sample[15]. Such firms are

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[13]The idea that costs of financial distress are higher in Europe is supported by Djankov et al's analysis of bankruptcy costs around the world[12]

[14]In the interpretation of the model where n_{jt} is partly used as collateral for loans that are not subject to frictions instead of just being directly used for self financing, we assume that lending not subject to frictions is allocated between banks and markets in the same proportion as lending subject to frictions.

included in our model, and they are potentially important in explaining the prevalence of bank financing. The higher default rate in the simulations may be quite compatible with the behaviour of those firms, particularly once one realises that default in our model does not just represent formal bankruptcy or liquidation but any failure to fully repay the promised coupon $\bar{\omega}\bar{y}$. The model's annual capital to output ratio of approximately 2.5 is not too far from the average ratio for the US of 3 reported in[4], even though this ratio was not targetted in the calibration.

The model generates a negative correlation between firm size and bank financing. In particular, the correlation between the bank financing indicator (1 if $i = b$, 0 otherwise) and n (our measure of the value of equity) is significantly negative. As discussed in the theoretical analysis of the financial contract, the negative link between n and the default threshhold $\bar{\omega}$ is probably the key mechanism producing this effect.

The smallest firms in our model use bank loans. Next, intermediate size firms use market debt. Finally the largest firms are financially unconstrained. Note that this cross sectional pattern also holds for the time series evolution of a typical firm. So we can also interpret the results as a model of the life-cycle of firm financing choices. As the firm becomes older it evolves from bank debt to market debt to a regime where financing choices do not matter very much.

Comparing the strength of this effect in the model with actual data is difficult due to the limited availability of data on the division of debt between bank and market sources. Nevertheless, this qualification we compare the model's predicted correlation between net worth and market debt issues with the one indicator available in Compustat data- the existence of a bond rating. Cantillo and Wright(2000) [5] were able to obtain more precise data on the decomposition of debt between banks and markets for a subset of Compustat firms. For that subsample, they find a nearly perfect correlation between the existence of a debt rating and the existence of outstanding market debt in a given year. On the assumption that this strong correlation continues to hold in the general Compustat sample, as well as using the fact that firms in our model do not issue bank and market debt simultaneously and that debt in our model lasts for only one period, we can associate the existence of a bond rating in a given firm-year with the issuance of market debt. Therefore, we compare the correlation between using market debt and net worth in the model(calibrated at an annual frequency to match Compustat's ratings information) to the correlation between having a bond rating and net worth

in Compustat between 1997 and 2006. From this perspective, our model is rejected: the model's correlation between net worth and issuance of market debt is around 0.77 while in Compustat the correlation is only around 0.25. There are several possible explanations for this failure. One possibility is that Compustat contains very large firms that are financially unconstrained in our model. To check this, we reexamined the correlation in the Compustat data excluding from the analysis the top 31% of firms by net worth that would be financially unconstrained according to the model. The correlation in the restricted Compustat sample is around 0.21, again, far from the model's prediction. [15]

The model without fixed bond issue costs cannot match the ratio of bank to market debt in Europe without assuming an extremely low bank administration cost parameter τ(table 2). Even with $\tau = 0.005\%$, we can only get a ratio of bank financing to market financing of 2.43 in comparison to a ratio of 7.33 in the data. The problem is that lowering τ in Europe even further leads to an implausibly low estimate of the cost of bank intermediation per dollar of loan in Europe relative to the cost in the US. It is certainly possible that bank monitoring is cheaper in Europe. For example, we know that banks in many European countries can acquire equity stakes in firms that they lend to more easily than in the US. This may lower the cost of monitoring loans(showing up in reduced form in the model either as a lower μ_b/μ_m or as a lower τ for a given μ_b/μ_s). [16] An alternative interpretation of this result is that market financing is relatively more expensive in Europe. Santos and Tsatsaronis(2003)[32] find that until 2001 average bond underwriting fees in Europe exceeded American average underwriting fees by approximately $0.05\% - 0.8\%$. [17] The introduction of the Euro led to greater competition among investment banks in the Euro area, lowering bond underwriting fees.

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[15]There are several measurement issues that could also explain this discrepancy. The assumption of perfect correlation between having a bond rating and having outstanding market debt may be a bad approximation in our sample. Also, Compustat data is biased biased towards larger firms that can issue equity.

[16]In contrast American banks could not hold equity in firms that they lend to until 1999, with the repeal of the Glass-Steagall act. Even with the repeal of the Glass-Steagall act, European banks still have more freedom to own equity in borrowing firms(Barthet al 2000). Santos(97) argues that in practice European banks' equity holdings are small, though one cannot exclude the possibility that even small equity holdings translate into significant reductions in bank monitoring costs.

[17]Their sample covers only international bond issues, which includes almost all European corporate bond issues, but excludes many American bonds issued only domestically. One

We can model the difference in underwriting fees by allowing for positive bond issuance costs in Europe. To capture this we specify a fixed cost of issuing market debt $C_m > 0$ in Europe in addition to the expected costs of financial distress. We assume that $\tau = 0.25\%$ in Europe as in our preferred specification for the US. Like Santos and Tsatsaronis, we use the loan size weighted average of issue costs per dollar of lending as our measure of average issue costs.

The addition of a fixed bond issue cost leads to a large increase in the relative desirability of bank financing (table 3). An average issue cost of $0.17\%(C_m = 0.25\%)$ almost triples the relative proportion of bank loans to market loans in Europe. With an average issue cost of $0.22\%(C_m = 0.35\%)$ we get a ratio of bank to bond financing of 5.54 in Europe. The cost of bank intermediation per dollar of loans in Europe is still estimated to be significantly lower than in the US (1.78% versus 2.72% in the US), but the difference is much more plausible than the one obtained trying to match the relative amount of bank financing in Europe without bond issue costs. This estimate is significantly lower than the estimates reported by Erosa[13] for European countries. However, his estimates are for the year 1985. It is quite possible that due to technological progress in the financial sector, costs of bank intermediation have declined significantly since 1985.

Finding a lower cost of bank intermediation in Europe despite having the same loan administration cost τ, a higher audit cost parameter μ and the same average default rate as in the US may seem counterintuitive at first. The explanation lies in the audit cost function used. Recall that the audit cost is $\mu M x^\theta$, which is concave in x. The average x financed by the bank in Europe is 1.91. The average x financed by the bank in the US is 1.34. This difference occurs due to the larger number of high n firms using bank financing in Europe, reflected in a lower magnitude of the negative correlation between n and bank financing choice in Europe. The cost of bank intermediation per dollar of loans is $\tau + \frac{\mu M \int 1_b x_j^\theta dF}{\int 1_b (x_j - n_j) dF}$. The amount of bank loans in Europe is more than double the amount in the US(the denominator). At the same time, while the total amount of expenditure by bank financed firms is larger, due to the diminishing marginal cost of auditing and the higher average expenditure financed by a bank loan in Europe, the total auditing costs in Europe are smaller relative to the amount of loans. As a result, we get a lower cost of

should also bear in mind that the market finance in our model is closer to commercial paper, while Santos and Tsatsaronis cover longer term bonds.

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bank intermediation per dollar of loans in Europe.

The model has more limited success in matching some other stylized facts about the distribution of firms. The model predicts a negative relation between the debt to equity ratio $\frac{x-n}{n}$ and n, with a correlation of around -0.3 for the US, and -0.53 in Europe. This prediction is at odds with the empirical evidence for Compustat data reported in Frank and Goyal(2005)[34]. In fact the negative correlation between the leverage ratio(leverage is $\frac{x}{n}$, that is the debt to equity ratio minus 1) and net worth is so strong that it also leads to a negative correlation between loan sizes and n. This prediction is again probably unrealistic. There are several possible reactions to this problem. First, the evidence for a positive link between leverage and size is not conclusive. Arellano,Bai and Zhang(2007) [1] examine data from the UK and find a negative correlation between leverage and size for the whole sample. They only find a positive leverage and size correlation for the largest firms in the UK. To the degree that the compustat data oversamples the largest American firms, Frank and Goyal's conclusion on the correlation between size and leverage is consistent with Arellano et al. [18] Second, these correlations emerge in the model when restricting the analysis to firms that require external financing. For higher net worth firms, the Modigliani-Miller theorem applies in our model, and the financing choice is indeterminate. Those firms could for example prefer higher leverage due to unmodeled tax trade-offs between debt and equity. Therefore, our model is not necessarily at odds with a positive size and leverage relation for the largest firms. Third, the negative link between $x - n$ and n does not necessarily mean that the model predicts a negative correlation between n and overall lending. Recall the alternative interpretation of the model according to which $x - n$ is the part of lending subject to frictions, with the total amount of lending relative to self-financing being indeterminate. This interpretation is consistent with an economy where firms with higher net worth borrow more, but the amount of their borrowing subject to information frictions is lower than for low net worth firms. Finally, like virtually all implementations of the CSV model, we have assumed that the idiosyncratic shocks ω are i.i.d. Suppose instead that ω follows an AR(1) process. In this case M is increasing in $E(\omega_t|\omega_{t-1})$. A high sequence of ω's would raise M. Since $\frac{d(x-n)/n}{dM} > 0$, the firm's leverage ratio would also increase for a given n. At the same time,

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[18]Cooley and Quadrini(2001) also report that the leverage ratio is negatively correlated with firm equity when the sample is not restricted to corporations.

the firm's net worth n should increase due to the higher recent profitability. In this case we may see a rise in n accompanied by a rise in $\frac{x-n}{n}$, despite the negative direct relation between these two variable for a fixed M. [19] Finally, we have followed the standard costly state verification framework in restricting entrepreneurs to using only internal equity. Allowing firms in the model to issue external equity subject to the typical quadratic issuance costs (see for example Henessy and Whited[20]) would increase the cost of equity disproportionately for the largest firms and encourage them to increase the proportion of debt financing.

Only around 40% of firms in our calibrations actually borrow. The result that most firms in our simulations are not financially constrained is unrealistic. For example Henessy and Whited (2006) [20] estimate a more quantitative model of investment with financial frictions and find the presence of moderate financing frictions even for large US corporations, though as in our model they find that financial frictions are considerably less important for larger firms. One result of the high proportion of self-financing firms in the model is the extremely low aggregate debt to equity ratio of around 0.07 predicted in our simulations. The actual aggregate debt to equity ratio for 1997-2003 was 0.41 in the US and 0.61 in Europe [14].The debt to equity ratio of borrowing firms is actually around 1.36 for our US calibrations, but these firms only hold around 20% of assets and even less of the aggregate equity. As a result, the total amount of debt to equity for all firms is low.

The key factor explaining the large number of financially unconstrained firms in our simulations is the aggressive saving behaviour of the entrepreneurs. This allows many firms to accumulate enough net worth to make financial frictions irrelevant. The largest firms in the model do not require any external financing. The simplest way to improve the model's performance in this respect is to increase the bargaining power of old entrepreneurs π_e. This would directly reduce the entrepreneur dynasties' ability to accumulate large net worth levels. For example, increasing π^e to 10% while keeping all other parameters at their level in the preferred US calibration increases the aggregate debt to equity ratio to 0.36, at the cost of an unrealistically high default rate of 6.25%. A more realistic but more challenging approach

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[19]One potential question is how would financing choice be affected by a model with persistent productivity shocks. Computing the derivative of $\pi_b^e - \pi_m^e$ at $n = 0$ and taking the limit as $\theta \to 0$, it can be shown that if n and θ are not too large and there is a fixed cost of issuing bonds, then bank financing becomes less profitable relative to bond financing as M increases.

would be to model risk averse/consumption smoothing entrepreneurs as in Zha(2001)[35].

To summarize, the main factor explaining higher bank financing in Europe in our model is the higher cost of issuing market financing in Europe. The higher audit costs for a given loan size (higher μ) do not seem to play a large role in explaining European firms' stronger preference for bank financing. To the degree that bond underwriting costs have declined with the introduction of the Euro, our model predicts increased reliance on market financing in Europe. One should note though, that while the introduction of the Euro has lowered international bond issue costs, it is not clear that it had a similar effect on the costs of issuing commercial paper (which is a better description of the short term market debt in the model). If differences in the cost of issuing commercial papers persist, bank financing should remain more popular in Europe in the future.

Finally, we compare the model with bank versus market debt choice to the standard Carlstrom and Fuerst[6] model with only one intermediary type(table 4). This is a special case of the benchmark model with $\mu_b = \mu_m = 0.15$ and $\tau = 0$. The other parameter values are the same. We compare this model to our preferred calibration for the US with two types of financial intermediaries. The differences in the prediction of the two models for aggregate output and consumption are practically indistinguishable. Output and total consumption are higher by about 0.11% in the model with bank and market debt. Worker consumption is slightly higher than in the Carlstrom and Fuerst model, while entrepreneur consumption is slightly lower. In the context of this model, allowing for two types of financing does not seem to matter very much for predicting aggregate quantities. An economist could have done just as well in predicting aggregate output and consumption in this economy by assuming a single type of intermediary with audit costs being an average of bank and bond audit costs. This provides some evidence that, at least for some purposes, the usual practice of modeling a single type of financial intermediary may provide a good approximation as long as the lending technology of that financial intermediary is appropriately calibrated.

4 Conclusion

We have examined the ability of a simple extension of the standard costly state verification model of financing, based on the idea of banks as better monitors, to account for firms' choice between bank and market debt. The model successfuly captures the tendency of larger firms to use more market debt. The intuition for this result is that financial distress is a smaller problem for these firms, and this makes the advantages offered by banks in dealing with default less valuable. In order to capture the higher use of bank financing in Europe relative to the US we depart from the benchmark model by assuming higher fixed costs of issuing market debt in Europe. The fixed cost model can generate a realistic bank to market financing ratio in Europe.

There are several extensions of this paper that should be explored in future research. First, we have only examined the steady state behaviour of the model. Once we allow for aggregate shocks, we can study how financing choices change over the business cycle. Many authors(see for example Oliner and Rudebusch , and Cantillo and Wright[26][5]) have argued that the impact of different financing types on business cycle dynamics depends on the degree to which firms switch from one mode of financing to another in reaction to aggregate shocks. Beyond the usual productivity and demand shocks, an interesting type of aggregate shock that may be worthwhile to investigate within this model is a shock to the cost of monitoring μ_b. At the end of section 2 we discussed the possibility of explaining the switch to more market based financing in the US as a decline in μ_b in recent decades. Such an explanation seems plausible. At the same time, it is sometimes argued that the switch away from standard bank based loans in recent years has been excessive and may have even contributed to the current financial problems in the US. One way to formalize such a story in this model is to model a news shock to μ_b, where agents in the economy overestimate the future declines in μ_b.

Another interesting extension is to endogenize part of the ex-ante cost of bank financing τ, by introducing bank capital effects. Reduced form empirical research has found significant effects of the strength of the bank's balance-sheet on interest rates and lending[27][28][22]. An extension of this model with bank capital would allow investigation of these effects in a more structural framework.This could be done in our model by limiting the ability of the bank to diversify part of the firms' risk across its loan portfolio. For example, we could introduce an industry level shock in an environment where the bank must specialize in a specific industry. In this case, the rate

of return on bank deposits is no longer certain. Assuming that the realized return on the bank's loan portfolio is freely observable only to the banker, the depositors will have to audit the bank in case of low returns, just as the bank has to audit the entrepreneur when output is low. The financial friction between banks and depositors makes bank capital(the banker's net worth) valuable in this environment, just as the entrepreneur's net worth reduces frictions in the current model.

Finally, while we focus on bank and market financing in developed economies, in principle the framework that we present can be used to study any situation in which the financing options available to firms differ by the degree of monitoring. For example, we could also study the introduction of various intermediate forms of financing such as market debt backed by bank repayment guarantees to investors. Or we could examine the choice of farmers and entrepreneurs in a developing country between local lenders with better information on borrowers and outside financial intermediaries (for example foreign banks) that with a more efficient lending infrastructure but less access to information on local borrowers.

Table 1,
Benchmark model, US

τ	Pr(default)	bankcostratio	btomf#firms	btomfloans	Debt/Equity	Pr(extfin)
0.01	0.026	0.045	0.248	0.21	0.069	0.403
0.005	0.029	0.04	0.372	0.353	0.0697	0.401
0.0025	0.029	0.027	0.619	0.671	0.07	0.399

τ	corr(i=a,n)	corr(i=b,n)	Y	K
0.01	0.592	-0.543	2.631	26.314
0.005	0.591	-0.639	2.633	26.336
0.0025	0.589	-0.754	2.634	26.347

Bankcostratio is $\tau + \frac{\mu M \int 1_b x_j^{\theta} dF}{\int 1_b (x_j - n_j) dF}$. btomf#firms is the ratio of the number of firms choosing bank to those choosing market debt.

btomfloans is the ratio of bank loans to market loans. Debt/Equity is the aggregate debt to equity ratio.

Table 2,
Europe, benchmark calibration

τ	Pr(default)	bankcostratio	btomf#firms	btomfloans	Debt/Equity	Pr(extfin)
0.01	0.0263	0.052	0.247	0.188	0.067	0.404
0.005	0.0288	0.0481	0.386	0.337	0.068	0.404
0.0025	0.0288	0.03	0.643	0.656	0.068	0.403
0.0005	0.0288	0.021	1.221	1.624	0.069	0.403

τ	corr(i=a,n)	corr(i=b,n)	Y	K
0.01	0.592	-0.539	2.629	26.286
0.005	0.593	-0.644	2.63	26.301
0.0025	0.592	-0.759	2.631	26.312
0.0005	0.592	-0.871	2.632	26.327

Table 3
Fixed bond issue cost in Europe ($\tau = 0.0025$)

C_m	$E(\frac{1_m C_m}{x-n})$	Pr(default)	bankcostratio	btomf#firms	btomfloans	Debt/Equity
0.0025	0.174%	0.0288	0.022	2.602	1.855	0.068
0.003	0.199%	0.0288	0.019	4.466	3.138	0.068
0.0035	0.224%	0.0288	0.0178	7.81	5.544	0.068

C_m	Pr(extfin)	corr(i=a,n)	corr(i=b,n)	Y	K
0.0025	0.396	0.586	-0.196	2.6307	26.302
0.003	0.396	0.587	-0.122	2.6309	26.303
0.0035	0.396	0.586	-0.069	2.6306	26.299

Table 4,
Model with only 1 financial intermediary type.
US calibration uses $\tau=0.25\%$, $\mu_b=0.121$, $\mu_m=0.171$
Single financial intermediary model uses $\mu=0.15, \tau=0$

	Y	K	C_h	C_e
1FI	2.631	26.315	1.772	0.325
2FIs,US	2.634	26.347	1.775	0.326
1FI/2FIs	1.001	1.001	1.002	0.998

Appendix A:
Sufficient conditions for $x - \bar{n} > 0$ (no rationing):

Consider first the case when $C_m = 0$. Define the relative profit between external financing and autarky as a function of x as $\Delta(x) \equiv \pi^e(x) - \pi_a^e(n)$. We will show that $\Delta'(n) > 0$ when $n < \left(\frac{\theta M[1-\mu\phi(0)]}{R}\right)^{\frac{1}{1-\theta}}$. Therefore, by continuity and the mean value theorem, for small enough $\varepsilon = x - n > 0$,$\Delta(n+\varepsilon) - \Delta(n) = \Delta(n+\varepsilon) > 0$. Taking on some positive loan must lead to higher profits than those available with just $x = n$. To establish this result, we go through the following steps:

1) $m'(\bar{\omega}) > 0$. Otherwise, since $f'(\bar{\omega}) < 0$, for a given x one could always raise $\bar{\omega}$ and raise π^e while satisfying the lender's break-even constraint

2) $m(\bar{\omega})\bar{y} = R(x - \bar{n})$. Otherwise, by 1) and $f'(\bar{\omega}) < 0$ we could reduce $\bar{\omega}$ and tighten the lender's break even constraint without violating it, while increasing π^e for any given x.

3) $x = 0$ is never optimal. If $x = \bar{n} = 0$, then $\lim_{x \to 0}(1-\mu)\theta M x^{\theta-1} - R = \infty$. This implies that we can always guarantee a positive loan surplus by picking

a small $x' > 0 = \bar{n}$. Continuity allows us to find a finite $\bar{\omega} > 0$ such that $m(\bar{\omega})Mx'^\theta \geq x'$ and $\pi^e = f(\bar{\omega})Mx'^\theta > 0$. So $x = \bar{n} = 0$ can never be optimal.

4) $x - \bar{n} = 0$ if and only if $\bar{\omega} = 0$. The only if comes from realizing that $f(\omega) = 1 - m(\bar{\omega}) - \mu\Phi(\bar{\omega})$, and that 2) and 3) require that $m(\bar{\omega}) = 0$. This can be achieved without any monitoring by setting $\bar{\omega} = 0$. For any given x, picking any other $\bar{\omega} > 0$ such that $m(\bar{\omega}) = 0$ (if it exists) would generate positive expected monitoring costs. Therefore $\bar{\omega} = 0$ is optimal. For the if, setting $\bar{\omega} = 0$ when $x - \bar{n} > 0$ would mean that the borrower could make expected repayments arbitrarily small while evading all auditing by always reporting that $\omega = \varepsilon > 0$ whenver $\omega \geq \varepsilon$ for some small ε such that $\varepsilon Mx^\theta < R(x - \bar{n})$. But this violates the lender's break-even constraint.

Since $f(0) = 1$, $\pi^e(n) = \pi_a^e$ and $\Delta(n) = 0$.

5) $\bar{n} = n$. Therefore $\Delta(n) = 0$. See the proof of lemma 1b) to see that this is optimal when $x - \bar{n} > 0$. When $x - \bar{n} = 0$, $\pi^e = M\bar{n}^\theta + n - \bar{n} = \pi_a^e$.Since $x_a = n$ whenever $n \leq \hat{x}$, it is also optimal for the financial intermediary to set $\bar{n} = n$.

6)Define $\Delta(x) \equiv \pi^e - \pi_a^e = f(\bar{\omega}(x))Mx^\theta - Mn^\theta$, where $\bar{\omega}(x)$ is implicitly defined by the break even constraint $m(\bar{\omega})Mx^\theta = R(x - \bar{n})$ (the function$\bar{\omega}(x)$ is well defined since $m'(\bar{\omega}) > 0$). Using the break-even constraint to find $\frac{d\bar{\omega}}{dx}$, $\Delta'(x) = f'(\bar{\omega})\frac{d\bar{\omega}}{dx}Mx^\theta + \theta f(\bar{\omega})Mx^{\theta-1} = -[1 - \Phi(\bar{\omega})]\frac{R - \theta m(\bar{\omega})Mx^{\theta-1}}{m'(\bar{\omega})} + \theta f(\bar{\omega})Mx^{\theta-1}$ .Evaluating this at n, $\Delta'(n) = \theta Mn^{\theta-1} - \frac{R}{1-\mu\phi(0)} > 0$ iff $n < \left(\frac{\theta M[1-\mu\phi(0)]}{R}\right)^{\frac{1}{1-\theta}}$. Note that if $\phi(0) = 0$ and $R = 1$ then the right hand side of this inequality is the first best level of output with no financial frictions $\hat{x}$. In this case, the condition holds for any $n < \hat{x}$ by the concavity of $\theta Mx^{\theta-1}$ and the first order condition defining $\hat{x}$.

Therefore, for any $n < \left(\frac{\theta M[1-\mu\phi(0)]}{R}\right)^{\frac{1}{1-\theta}}$ by the mean value theorem there exists an $\varepsilon > 0$ such that $\Delta(n+\varepsilon) - \Delta(n) = \Delta(n+\varepsilon) > 0$. This implies that $x - \bar{n} = x - n = 0$ cannot be optimal when there are no fixed costs.

Furthermore, note that $\left(\frac{\theta M[1-\mu\phi(0)]}{R}\right)^{\frac{1}{1-\theta}}$ is decreasing in both μ and τ. As a consequence for any value of n for which $x - n > 0$ with a given τ and μ, $x - n > 0$ still holds for lower values of τ and μ.

When $C_m > 0$, $\Delta'(n) > 0$ continues to hold if C_m is low enough. By the mean value theorem, we can then once again find $x = n + \varepsilon > n$ such that $\Delta(n + \varepsilon) - \Delta(n) > 0$.

Proof of lemma 1:

1a) First note that $f'(\bar{\omega}) = -[1 - \Phi(\bar{\omega})] < 0$ for any finite $\bar{\omega}$. From the f.o.c for $\bar{\omega}$, we see that this requires that $\lambda > 0$ and $m'(\bar{\omega}) > 0$.

1b) If $\bar{n} = 0$, we have $\psi + \lambda_l R = 1$. Since $R \geq 1$, $\lambda > 1$ is sufficient for $\psi = 0$. If $0 < \bar{n} < n$, the f.o.c for $\bar{n}$ implies that $\xi > 0$ is equivalent to $\lambda_l R - 1 > 0$. Since $R \geq 1$, a sufficient condition for this is again that $\lambda > 1$. By (1a) and the f.o.c for $\bar{\omega}$, $\lambda = -\frac{f'(\bar{\omega})}{m'(\bar{\omega})}$. Note that $f(\bar{\omega}) + m(\bar{\omega}) = 1 - \mu\Phi(\bar{\omega})$, implying that $f'(\bar{\omega}) + m'(\bar{\omega}) = -\mu\phi(\bar{\omega}) < 0$. Since $m'(\bar{\omega}) > 0$, this is equivalent to $\lambda = -\frac{f'(\bar{\omega})}{m'(\bar{\omega})} > 1$.

1c) Using the MEP result and the lender rationality constraint, for any financial intermediary $\pi^e = f(\bar{\omega}^*)Mx^{*\theta} = [1 - \mu\Phi(\bar{\omega}^*)]Mx^{*\theta} - R(x^* - n) - C$, where starred variables indicate optimal choices. By the MEP, when the entrepreneur uses external financing $x \geq n$. If the first best level with no financial frictions $x_{nofrictions} \leq n$, then $Mx^{\theta} - (x_{nofrictions} - n) \geq Mx^{*\theta} - (x^* - n) \geq [1 - \mu\Phi(\bar{\omega}^*)]Mx^{*\theta} - R(x^* - n) - C$.

i.e, whenever external financing may be optimal($x \geq n$), internal financing of $x_a = x_{nofrictions}$ is preferred.

1d)$\lambda'(\bar{\omega}) = \frac{m''(\bar{\omega})f'(\bar{\omega}) - f''(\bar{\omega})m'(\bar{\omega})}{m'(\bar{\omega})^2} = \frac{\mu\left[\phi'(\bar{\omega})(1-\Phi(\bar{\omega})) + \phi(\bar{\omega})^2\right]}{m'(\bar{\omega})^2}$. The denominator is clearly positive. The numerator is positive whenever $\frac{d}{d\bar{\omega}}\left(\frac{\phi(\bar{\omega})}{1-\Phi(\bar{\omega})}\right) = \frac{\phi'(\bar{\omega})(1-\Phi(\bar{\omega})) + \phi(\bar{\omega})^2}{[1-\Phi(\bar{\omega})]^2} > 0$, which is assumption 1. Therefore, $\lambda'(\bar{\omega}) > 0$.

e) The proof technique is from Covas and Den Haan[9]. From the first order conditions, $\theta M x_i^{\theta-1}\frac{f(\bar{\omega}_i) + \lambda_i m(\bar{\omega}_i)}{\lambda_i} = R_i$. Suppose C_m falls. Under assumption 1, $\Delta\bar{\omega} \geq 0 \implies \Delta\frac{f(\bar{\omega}_i) + \lambda_i m(\bar{\omega}_i)}{\lambda_i} \leq 0 \implies \Delta\theta M x_i^{\theta-1} \geq 0 \implies \Delta x \leq 0$. $\Delta\bar{\omega} \geq 0$ and $\Delta x \leq 0$ imply that $\Delta\pi^e \leq 0$, but this contradicts $\frac{d\pi^e}{dC} < 0$. Therefore, $\frac{d\bar{\omega}}{dC} > 0$. The proof that $\frac{d\bar{\omega}}{dn} < 0$ follows the same argument, except that now $\frac{d\bar{\omega}}{dn} > 0$ leads to a contradiction.

f) We will start by showing that $\frac{dx}{d\tau} < 0$. This will imply that $\frac{d\bar{\omega}}{d\tau} < 0$. From the first order condition $\theta M x_i^{\theta-1}\frac{f(\bar{\omega}_i) + \lambda_i m(\bar{\omega}_i)}{\lambda_i} = 1 + \tau$, $\Delta\tau > 0$ and $\Delta x \geq 0$ imply that $\Delta\frac{f(\bar{\omega}_i) + \lambda_i m(\bar{\omega}_i)}{\lambda_i} > 0$ and $\Delta\bar{\omega} < 0$. But this contradicts $\frac{d\pi^e}{d\tau} < 0$. Therefore $\frac{dx}{d\tau} < 0$. When $C_m = 0$, substituting the financial intermediary's break-even constraint into the first order condition for x, we get $\theta(1-\frac{n}{x})(\frac{f}{\lambda m} + 1) = 1$. Since $\frac{dx}{d\tau} < 0$, $\Delta\tau > 0$ implies that $\Delta(1 - \frac{n}{x}) < 0$ and $\Delta(\frac{f}{\lambda m} + 1) > 0$. Since $\frac{f}{\lambda m}$ is decreasing in ω, $\Delta\bar{\omega} < 0$ when τ increases. By continuity, the result continues to hold if C_m is close to 0.

Proof of lemma 4:

$\frac{d\lambda}{d\mu} = \frac{\partial\lambda}{\partial\mu} + \lambda'(\bar{\omega})\frac{d\bar{\omega}}{d\mu}$. $\frac{\partial\lambda}{\partial\mu} = \phi(\bar{\omega})\frac{1-\Phi(\bar{\omega})}{m'(\bar{\omega})^2} > 0$. By our assumption that $\lim_{\theta\to 0}\bar{\omega} < \infty$ and $\lim_{\theta\to 0}\phi(\bar{\omega}) > 0$, the limit of the numerator as $\theta \to 0$ is positive. $\lim_{\theta\to 0}m'(\bar{\omega}) > 0$ implies that the limit of the denominator is also positive as $\theta \to 0$ (note that since $m'(\bar{\omega}) > 0$ for any $\theta > 0$, $\lim_{\theta\to 0}m'(\bar{\omega}) \geq 0$. The only assumption is that the inequality is strict.). Therefore $\lim_{\theta\to 0}\frac{\partial\lambda}{\partial\mu} > 0$. Since $\lambda'(\bar{\omega}) = \frac{\mu\left[\phi'(\bar{\omega})(1-\Phi(\bar{\omega}))+\phi(\bar{\omega})^2\right]}{m'(\bar{\omega})^2} > 0$ whenever $\theta > 0$, $\lim_{\theta\to 0}\lambda'(\bar{\omega}) \geq 0$. Moreover, it is clear from examining the expression for $\lambda'(\bar{\omega})$ that its limit is bounded. Totally differentiating the first order conditions with respect to μ, we get $\frac{d\bar{\omega}}{d\mu} =$

$$-\frac{\frac{\phi(\bar{\omega})\lambda(\bar{\omega})}{m'(\bar{\omega})} - \frac{\lambda(\bar{\omega})^2}{f(\bar{\omega})+\lambda(\bar{\omega})m(\bar{\omega})}\left(\frac{\phi(\bar{\omega})m(\bar{\omega})}{m'(\bar{\omega})}-\Phi(\bar{\omega})\right)}{\lambda'(\bar{\omega})\frac{f(\bar{\omega})}{f(\bar{\omega})+\lambda(\bar{\omega})m(\bar{\omega})} + \frac{1-\theta}{\theta}\frac{\lambda(\bar{\omega})^2}{f(\bar{\omega})}m'(\bar{\omega})} - \frac{-\frac{\lambda(\bar{\omega})^2}{f(\bar{\omega})}\Phi(\bar{\omega})}{\frac{\theta}{1-\theta}\lambda'(\bar{\omega})\frac{f(\bar{\omega})}{f(\bar{\omega})+\lambda(\bar{\omega})m(\bar{\omega})} + \frac{\lambda(\bar{\omega})^2}{f(\bar{\omega})}m'(\bar{\omega})} \equiv -A(\theta) -$$

$B(\theta)$. $\lim_{\theta\to 0}A(\theta) = 0$ and $\lim_{\theta\to 0}B(\theta) \leq 0$. Therefore $\lim_{\theta\to 0}\frac{d\bar{\omega}}{d\mu} \geq 0$. Together with our results for the limits of $\frac{\partial\lambda}{\partial\mu}$ and $\lambda'(\bar{\omega})$, this implies that $\lim_{\theta\to 0}\frac{d\lambda}{d\mu} > 0$. But then there must be a $\bar{\theta} > 0$ such that $\frac{d\lambda}{d\mu} > 0$ still holds when $\theta < \bar{\theta}$.

Proof of lemma 6:

Suppose $\theta = n = 0$, and consider the (suboptimal) pseudo-bond contract where the fixed cost is paid even when $x = n = 0$. The lender's break even constraint is now $m(\bar{\omega})\bar{y} \geq C_m$. Note that now $\bar{y}$ is fixed and therefore the optimal $x = 0$. The optimal contract with external financing contract must have $m'(\bar{\omega}) > 0$ and $m(\bar{\omega}_m)\bar{y} = C_m$, otherwise we could increase $\bar{\omega}_m$ and improve the borrower's expected profits without violating the lender's break-even constraint. Since $m(0) = 0$ and $C_m > 0$, $\bar{\omega}_m > 0$. The right continuity of $\bar{\omega}_m$ at $\theta = 0$ then implies that $\lim_{\theta\to 0}\bar{\omega}_m > 0$. Since $\frac{\partial m(\bar{\omega}_m,\mu)}{\partial\mu} < 0$, $m'(\bar{\omega}_m) > 0$ and $\bar{y}$ is fixed, an increase in μ implies an increase in $\bar{\omega}_m$: $\frac{d\bar{\omega}_m}{d\mu} > 0$ when $\theta = 0$. The right continuity in θ of $\frac{d\bar{\omega}_m}{d\mu}$ at $\theta = n = 0$ imply the existence of a $\theta^* > 0$ such that $\frac{d\bar{\omega}_m}{d\mu} > 0$ whenever $\theta < \theta^*$. Since for any $n > 0$ and $\theta > 0$ $\frac{d\bar{\omega}_m}{d\mu}$ is a continuous function of n, we can find a $n^{**} > 0$ such that $\frac{d\bar{\omega}_m}{d\mu} > 0$ and $\lim_{\theta\to 0}\bar{\omega}_m > 0$ continue to hold whenever $\theta < \theta^*$ and $n < n^{**}$.

Model Solution:

Partial Equilibrium:

We can solve the financial contract recursively by first solving a nonlinear equation for $\bar{\omega}$ and then finding $x(\bar{\omega})$. We use the first order conditions for

x and $\bar{\omega}$ to find $x(\bar{\omega}) = \left[\frac{\theta M(f(\bar{\omega})+\lambda_l m(\bar{\omega}))}{\lambda_l R}\right]^{1/(1-\theta)}$. We then replace x with this equation in the lender's break even constraint and solve for $\bar{\omega}$. We solve for $\bar{\omega}$ using Brent's algorithm, which is a refinement of the standard bisection algorithm.

As for the uniqueness of the the solution found by this method we have the following result:

Proposition 7 $\lambda'(\bar{\omega}) > 0$ and $m'(\bar{\omega}) \geq 0$ imply a unique solution(if it exists).

Proof. We're looking for $F(\bar{\omega}) = m(\bar{\omega})Mx^\theta - Rx + Rn - C = 0$. It is

sufficient to prove the strict monotonicity of $F(\bar{\omega})$. $F'(\bar{\omega}) = Mx^\theta m'(\bar{\omega}) + x'(\bar{\omega})[\theta Mm(\bar{\omega})x^{\theta-1} - R]$. The first term is positive or zero if $m'(\bar{\omega}) \geq 0$. As for the second term, after substituting our expression for $x(\bar{\omega})$ and simplifying $\theta Mm(\bar{\omega})x^{\theta-1} - R = R(\frac{\lambda m(\bar{\omega})}{f(\bar{\omega})+\lambda m(\bar{\omega})} - 1) < 0$. $x'(\bar{\omega}) = -\frac{R\theta M}{1-\theta}x^\theta \frac{\lambda_l'(\bar{\omega})f(\bar{\omega})}{(\lambda R)^2} < 0$ if $\lambda'(\bar{\omega}) > 0$. Therefore under our assumptions the second term is also positive and $F'(\bar{\omega}) > 0$. ∎

We know from lemma 1 that $\lambda'(\bar{\omega}) > 0$ iff the hazard rate $h(\omega)$ satisfies $h'(\bar{\omega}) > 0$. For the lognormal distribution there exists a ω' such that the lognormal distribution's hazard rate is increasing for $\omega < \omega'$ and is decreasing for $\omega > \omega'$. Therefore we can restrict our search for a solution to $\bar{\omega} < \omega'$.The other condition is that $m'(\bar{\omega}) \geq 0$. This must be true at an optimum, but need not hold for all $\bar{\omega}$. However, once again restricting the search for a solution to the region where $h'(\bar{\omega}) > 0$ helps. $m'(\bar{\omega}) = (1 - \Phi)(1 - \mu h) \geq 0$ iff $1 - \mu h \geq 0$. Since $h'(\bar{\omega}) > 0$ there exists a unique ω'' such that for any $\bar{\omega} < \omega''$, $1 - \mu h(\bar{\omega}) \geq 0$ and $1 - \mu h(\bar{\omega}) < 0$ for any $\bar{\omega} > \omega''$. Therefore there is a unique region $[0, \min(\omega'', \omega')]$ on which the conditions of the proposition are satisfied. This implies that if our candidate solution satisfies $\lambda'(\bar{\omega}) > 0$ and $m'(\bar{\omega}) \geq 0$, then we have found the optimal $\bar{\omega}$.

General Equilibrium computation:
Our goal is to find fixed steady state values for capital stocks, prices, aggregate consumption and an invariant distribution of entrepreneur net worth levels. Since the workers' euler equation fixes $r = 1/\beta - 1 + \delta$, solving the GE model amounts to finding the labour market clearing wage w.

As long as for at least some positive measure of entrepreneurs x_j is lower in the presence of auditing costs relative to the frictionless economy(implying that labour demand is lower with financial frictions), we can bound the equilibrium $w_{frictions} \leq w_{nofrictions}$. In fact, define $\bar{l}(w)$ to be the labour demand in the economy where there are no financial frictions. We have $l(w) \leq \bar{l}(w)$[20]. Then, $l(w_{nofrictions}) - 1 \leq \bar{l}(w_{nofrictions}) - 1 = 0$.Furthermore, if $l(w) - 1$ is continuous, there exists a small enough $\varepsilon > 0$ such that $l(\varepsilon) > 1$. We can then guarantee that using $[\varepsilon, w_{nofrictions}]$ as the starting interval the bisection algorithm will converge to $w_{frictions}$. Absent switches between external financing types, firm and hence aggregate labour demands are continuous. The possibility of switching financing types makes showing continuity difficult. Even if the true $l(w)$ function is continuous, we must approximate it with a finite number N of entrepreneurs, and this approximation may be discontinuous. In practice finding a market clearing wage was never a problem. In all of the model parametrisations tried the bisection algorithm did converge to a solution according to the stopping criterion $\left| z^l \right| = \left| \frac{1}{N} \sum_{t=1}^{N} l_{jt} - 1 \right| < 10^{-4}$. As for uniqueness, we know that for each entrepreneur labour supply is still downward sloping in w in the economy with financial frictions, as long as the change in w does not make him switch between bank and bond financing[21]. If there were no such switches, aggregate labour demand would be clearly downward sloping in w, ensuring a unique equilibrium $w_{frictions}$. If the entrepreneur switches between external financing types, it is still clear that an increase in the wage must reduce profits, but this may occur through an increase in both x and $\bar{\omega}$. In practice, partial equilibrium analysis for our calibrations suggested that entrepreneur labour demand is still declining in the wage despite the possibility of switching financial contracts.

We start the algorithm with the initial interval $[w_{low}, w_{nofrictions}]$ for a small $w_{low} > 0$. We iterate on the following:

1. Given the interval $[w_l^i, w_h^i]$ and $w^i = \frac{w_l^i + w_h^i}{2}$ approximate the invariant distribution of $\{n_j, k_{jt}\}$. To do this we simulate a long time series of obser-

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[20]In our model$x_{j frictions} \leq x_{jno\ frictions}$ whenever $x_{jno\ frictions} \geq n$, since $f(\bar{\omega})Mx_j^\theta = (1 - \mu \Pr(s_j < \bar{\omega}_j))M_j x_j^\theta - R(x_j - n_j)$ is concave in x, holding $\bar{\omega}$ constant. In this case, by proposition 5 in Gale and Hellwig[16] and the concavity of the revenue function$x_{j frictions} \leq x_{jno\ fricitons}.l_{j frictions} \leq l_{jno\ fricitons}$ from the experssion for labour demand as a function of total expenditure.

[21]To see this it is sufficient to note that the labour demand function decreases in w as long as x decreases in w. But raising w decreases M, which lowers x(see Covas and Den Haan[?] for the argument that $\partial x / \partial M > 0$).

vations for a single entrepreneur starting from an initial net worth of n_0 and discard a subset of the observations to reduce the impact of the initial n_0. This leaves a sample of N observations. Appealing to a law of large numbers we then take the sampe averages of c_{jt}^e, k_{jt}^e n_{jt}, l_{jt} and k_{jt} to approximate aggregate entrepreneur consumption capital supply, net worth, labour demand and capital demand.

2. Use the capital,output and consumption markets clearing conditions and the estimates of aggregate$y$, c^e and k^e from the previous step to solve for c_h, c, k_h.

3. If the labour excess demand function satisfies $\left| z^l \right| = \left| \frac{1}{N} \sum_{t=1}^{N} l_{jt} - 1 \right| < \varepsilon_l$ stop. Otherwise, update the bisection interval and proceed to the next iteration.

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