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# Changing Threat Perceptions and the Efficient Provisioning of International Security\*

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## Abstract

This paper extends Gupta (2010), which proposes a mechanism for the structuring of international institutions for the efficient provision of global security. In that paper, the level of threat by a rogue nation was assumed as being exogenously determined. This paper uses a similar framework to the one seen in Gupta (2010) to analyze the robustness of the results seen in that paper, in the case where the threat is endogenized. Additionally, this paper investigates how the evolution of public opinion in the respective countries facing the rogue nation's threat, impacts the efficient provisioning of global security.

## 1 Introduction

This paper extends Gupta (2010), which proposes a mechanism for the structuring of international institutions for the multilateral provision of global security by an alliance of nations, acting in concert against the threat of a rogue nation. In Gupta (2010), the level of threat by the rogue nation was assumed as being exogenously determined. This paper uses a similar framework to analyze the robustness of the results seen in that paper, in the case where the threat is endogenized in the model (due to the presence of a rogue nation which acts strategically vis-à-vis the alliance, hence making the threat level variable in response to action by the alliance).

As in Gupta (2010), security effort by an alliance member is assumed to be non-rival and non-excludable, so the benefits of the effort jointly accrue to every other member. This effort has both positive and negative effects, as security measures prevent attacks by the rogue nation, but also involves loss of personal rights, commercial benefits, etc. I characterize the equilibrium of a non-cooperative game of joint effort provision among the nations in case of an exogenously specified level of threat. In this equilibrium a single nation unilaterally provisions effort for the whole alliance. For specified conditions, this level of effort may be either greater or lesser than the efficient level.

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In Gupta (2010), I suggested an institutional structure for the alliance which would lead to the achievement of the efficient level of world security through multilateral contribution. This structure required that voting on the issue of security provision for the alliance be restricted to a subset of member nations, as well as the requirement of unanimity among these nations to adopt a resolution. Interestingly, the set of voters included nations that have the lowest preference of security effort provision and the one that has the strongest preference, but excluded those having preferences close to the latter. The current paper demonstrates that the results of Gupta (2010) are qualitatively robust in the case of endogenous threat levels, i.e. a comparable institutional structure would lead to the achievement of efficiency even in this case.

An additional contribution of the paper is to characterize the dependency of the multilateral response to rogue nations on the evolution of national public opinion against threat of such nations, within the respective member nations of the global alliance. Results show that the maintenance of the alliance is dependent on whether the public in the ally nations harden or soften their proclivity for proactive action against the rogue nation, upon the escalation of conflict. Interestingly, it is seen that often the divergence of public opinion among the respective nations (for example, public opinion in nation A hardening in favor of action against the rogue nation, while that in nation B moving against action) actually helps in the multilateral provisioning of global security in certain cases, while it hampers such efforts in other situations. Other combinations of evolving public opinion in the respective member nations of the alliance, and the implications for the maintenance of the alliance against the rogue nation, are also examined.

As with Gupta (2010) the contribution of this paper to the economic theory of alliances<sup>1</sup> can best be seen in the post Cold-War context in which the tastes of traditional allies regarding global security issues have diverged to an extent not seen previously. In fact, this divergence is so marked that there seems to be disagreement among these nations whether after a certain level security effort is intrinsically ‘good’ or ‘bad’, as seen by the French and German reactions to the Second Gulf War. In contrast, during most of the Cold War, US defence provisions in Europe were mostly supported by all NATO members. This forces us to rethink our earlier conclusions on how best to structure alliances like the NATO or the United Nations. In this process we add to the literature on the economics of alliances that began with Olson and Zeckhauser’s seminal contributions (1966 and 1967), and continued with later contributions by Murdoch and Sandler (1982), McGuire (1990), Bruce (1990), and McGuire and Groth (1985). In the context of the current paper, Weber and Wiesmeth’s (1991) analysis of a supranational institutional structure for NATO, that leads to quasi-egalitarian cost-sharing among the members, is of special interest,<sup>2</sup> as is Niou and Gan’s (2005) analysis

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<sup>1</sup>For a comprehensive review of this literature see Hartley & Sandler (1995a & 1995b). In addition to the alliance literature, there is some literature on terrorism that is interesting, including Lee (1988), Lee and Sandler (1989), Sandler and Lapan (1988), and Sandler, Tschirhart, and Cauley (1983).

<sup>2</sup>For cooperative solutions to such problems, see Moulin (1995). For discussions of institutional arrangements within a nation for the allocation of defense budgets, see Murdoch, Sandler, and Hansen (1991) and Jones (1992).

of Olson's (1965) propositions of suboptimality and exploitation (in the context of collective action in the face of endogenized external threat). As with Gupta (2010), the current paper adds to the literature by assuming that security effort may not only have positive externalities, but also have negative externalities beyond a point. The institutional structure suggested by the paper takes into consideration this important assumption, which is driven by realities currently observed in the international arena. Section 2, below, develops a model of global security provision. Section 3 discusses the implications of the findings of the model. I conclude in section 4.

## 2 The Model

### 2.1 Environment

In this section we extend the model seen in Gupta (2010) to make the level of threat by the rogue nation endogenous. In reality, rogue states react to the actions of their adversaries, so we need to take that into account. This analysis will reveal if the findings of the model with an exogenous level of threat undergo any modifications for the level of threat being endogenous. As in Gupta (2010), there is a finite number of countries (governments)  $i = 1, 2, \dots, I$  forming an alliance, to fight against a level of global threat  $t \in [0, \infty)$ . The utility of government  $i$  is given by:

$$\begin{aligned} U^i(m^i, e; t) &= m^i + S^i(e; t) - N^i(e) \\ &= m^i + \lambda^i(t)S(e) - \varpi^i N(e) \\ &= m^i + \lambda^i(t)e - \varpi^i e^2 \end{aligned}$$

Here  $m^i$  is a private good (money) consumed by  $i$ ,  $e = \sum_{i=1}^I e^i$  is the amount of joint effort expended by the alliance against the rogue nation,  $e^i$  is  $i$ 's contribution to the joint effort. Effort is assumed to be proactive, non-rival, and non-excludable - its results jointly accrue to every member. Note that this effort might include military action, trade embargoes, and other kinds of punitive action. Let  $\lambda^i(\bar{t}) \in (0, \infty)$  be different for each nation (we explain below what  $\lambda$  is), or  $\lambda^i(\cdot) \neq \lambda^j(\cdot), \forall i, j$ . Let  $\varpi^i = 1$ . For the present I assume that  $\lambda$  is increasing in the level of threat  $t$ , hence  $\lambda_t^i > 0$  (this assumption will be modified in later sections, to explore certain other plausible situations that might occur). The value of  $\lambda(\cdot)$  is greatest for country  $I$  for any  $t$ , so  $\lambda^I(\cdot) > \lambda^i(\cdot), \forall i \neq I$ . The other  $(I - 1)$  alliance members are ranked according to the value of their  $\lambda$ s, such that for all  $t$ ,  $\lambda^{I-1}(\cdot) > \lambda^{I-2}(\cdot) > \dots > \lambda^1(\cdot)$ . The marginal benefit of effort for  $i$  is  $[\lambda^i(t) - 2e]$ , which implies  $\frac{\partial^2 U}{\partial e \partial t} > 0$ . The marginal benefit is more for higher  $\lambda^i(t)$ .  $\lambda^i$  may be thought of as an index of public support for security effort in a nation.<sup>3</sup> Briefly, the governments' utility is dependent on the amount of

<sup>3</sup>Some readers might like a nation's effort to have some effect on public support for the government, as the public might

private good consumed and the security effort expended by the alliance. However, such effort does not only have the positive effect of increasing security  $S(e) = e$  by eliminating the threat,<sup>4</sup> but also has a negative effect  $N(e)$  on utility in case the effort put in by the alliance infringes on human rights, trade contacts, etc. Both these elements are captured in the government's utility function by the term  $S^i(e; t) - N^i(e) = [\lambda^i(t)e - e^2]$ , where  $S^i(e; t) = \lambda^i(t)S(\cdot) = \lambda^i(t)e$  and  $N^i(e) = \varpi^i N(e) = \varpi^i e^2 = e^2$  (assuming  $\varpi^i$  is 1 for all nations). For a given level of  $t$ , an increase in joint effort  $e$  leads to greater utility by providing security, but also has a disutility that is captured by the part  $e^2$ . For a more detailed discussion and justification of the government's utility function see Gupta (2010).

There is a rogue nation  $L$  which makes the decision to make effort  $t \in [0, \infty)$ , which gives the level of threat against the alliance of countries seen above.<sup>5</sup> The utility of the rogue nation is given by:

$$U^L(m^L, t; e) = m^L + \alpha^L(e)t - t^2$$

where  $m^L \in [0, \infty)$  is a private good (money) consumed by  $L$ , and  $0 < \alpha^L(e) < \infty$  is a preference index of the rogue government, and is a measure of the support it has for its activities from within its constituency. Let  $\alpha_e < 0$ .

The rogue government have a positive benefit from undertaking effort, as well as disutility from that effort. The positive benefit would come from causing harm to what they consider enemy nations. However, a rogue state may suffer various effects of trade embargoes, and there might be restrictions on its citizens in travelling to other parts of the world or having business contacts, and so on. All this increases if effort by the security alliance,  $e$ , goes up. This reflected by the fact that  $\alpha_e < 0$ , which leads to the disutility of the rogue's effort being more for more  $e$ . The marginal utility of the rogue's effort (activity) decreases with an increase in the given level of  $e$ , i.e.  $\frac{\partial^2 V^L}{\partial t \partial e} < 0$ . The budget constraint of the rogue country is given by:  $m^L + C(t^L) \leq M^L$ ; where  $\infty > M^L > 0$  is the initial endowment of the private good of  $L$  and  $C(t^L) = vt^L, v > 0$ , is the cost of threat activity level  $t^L$ .

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have to ultimately pay for effort, perhaps by way of higher taxes. I make the simplifying assumption that even if such an effect is present, it is purely decomposable from the support for security effort that arises from the public's perception of threat. If that is the case, then if for the sake of simplicity I assume that some part of public support for the government suffers from an increase in effort in a linear fashion, then it can be made part of effort costs, and will not affect the results of this paper qualitatively. In other words, if the effort based support for the government suffers for increasing effort, then a cost function written as  $\delta(e) = \delta e$  can be subsumed into the overall cost structure of effort provision in this model. Hence, in what follows I choose to ignore this detail.

<sup>4</sup>I assume there is a simple linear technology converting effort to a level of security (by destroying the threat). The process how effort eliminates the threat is not modeled.

<sup>5</sup>For the detail oriented, I could have modeled a linear technology that would have mapped effort by the rogue nation one-to-one onto a level of threat. However, I choose to neglect this technical detail and interchangeably use the concepts of effort by the rogue nation and the level of threat presented by it. This shortcut does not affect the results of my model.

## 2.2 The basic game

I consider that all countries play a simultaneous move game with respect to its alliance members and with the rogue enemy nation. The rogue nation also moves simultaneously with respect to the actions taken by the alliance members. In the overall game there are  $(I + 1)$  players, with the alliance members choosing effort  $e^i$ , and the rogue nation choosing threat level (effort)  $t^L$ . The payoff for an alliance member is  $V^i$  and that of the rogue nation is  $V^L$ , such that  $V^i(\cdot) = M^i + \sum_{i=1}^I e^i[\lambda^i(t) - \sum_{i=1}^I e^i] - ce^i$ , for  $i = 1, 2, \dots, I$ , and  $V^L(\cdot) = M^L + \alpha^L(e)t^L - (t^L)^2 - vt^L$ , for  $l$ . We have  $e = \sum_{i=1}^I e^i$ .

We can solve for the Nash equilibrium of the overall game by solving for the Nash equilibrium level of effort of each country in a game within its alliance, taking the joint effort level of the enemy as given. This will give us the joint effort level of the alliance, as well as the choice of threat by the rogue nation, as a reaction function of the other. Using these reaction functions, we can arrive at the equilibrium level of threat and security effort. In the Nash equilibrium of the intra-bloc game for the alliance, effort is provided solely by country  $I$  and is given by:<sup>6</sup>

$$e^N = e^I = \frac{1}{2}[\lambda^I(t) - c]$$

The threat by the rogue nation is given by:

$$t^L = \frac{1}{2}[\alpha^L(e) - v]$$

Now, we must solve for the equilibrium of the game between the alliance and the rogue nation, which effectively reduces to a game between the two countries  $I$  and  $L$ , having effort choices  $e^I$  and  $t^L$ , and payoff functions  $V^I$  and  $V^L$ . Here  $V^I(\cdot) = M^I + e^I[\lambda^I(t^L) - e^I] - ce^I$  and  $V^L(\cdot) = M^L + t^L[\alpha^L(e^I) - t^L] - vt^L$  and  $e^I = e^N$  and  $t^L = t^N$ . So, the Nash equilibrium effort outcome for this game is described by the pair  $(e^N, t^N)$  given by the simultaneous solution of the equations:

$$e^N = \frac{1}{2}[\lambda^I(t^N) - c]$$

and

$$t^N = \frac{1}{2}[\alpha^L(e^N) - v]$$

for  $t^N = t^L$  and  $e = e^N = e^I$ . Let us call  $e^N$  the unilateral effort level for the alliance.

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<sup>6</sup>See Gupta (2010) for a detailed formulation and solution of the effort provisioning game within the alliance. I am using this result, derived at length in that paper, here.

**Remark 1** The slope of the reaction function of the alliance is  $\frac{\partial e^N}{\partial t^N} > 0$ , and that of the rogue nation is  $\frac{\partial t^N}{\partial e^N} < 0$ .

### 2.3 Efficiency with endogenous threat

I now solve for the efficient level of joint effort of the alliance:

$$\begin{aligned} \text{Maximize}_{\{e, \sum m^i\}} \sum_{i=1}^I V^i(\cdot) &= \sum_i m^i + \sum_i e[\lambda^i(t) - e], \\ \text{s.t. } \sum_i m^i + ce &= \sum_i M^i; \sum_i m^i \in [0, \infty); e \in [0, \infty) \end{aligned}$$

or

$$\text{Maximize}_{\{e\}} \sum M^i + e[\sum \lambda^i(t) - Ie] - ce; e \in [0, \infty)$$

Solution to the FOC of the above problem gives us the efficient solution:

$$e^E = \frac{1}{2I} [\sum_{i=1}^I \lambda^i(t) - c]$$

If the alliance provisions the efficient level of effort  $e^E$  and not the unilateral level  $e^N$ , the rogue country  $L$  will make effort

(A)

$$t^L = t^E = \frac{1}{2} [\alpha^L(e^E) - v]$$

We notice that this level of effort is different than that seen in the last section, since it is a best-response to the efficient effort level by the alliance, and not the unilateral level  $e^N$ . Let us call this  $t^E$ , though we must be careful to remember that it is the Nash outcome for the game between the alliance and the rogue nation and not an "efficient" threat level, in any sense.

Thus, putting  $t = t^E$  in the equation for  $e^E$  we get:

(B)

$$e^E = \frac{1}{2I} [\sum_{i=1}^I \lambda^i(t^E) - c]$$

The equilibrium at the inter-bloc level is given by the pair  $(e^E, t^E)$  got by simultaneous solution of equations (A) and (B). The efficient level of effort for the alliance may be more or less than the unilateral level, as seen in the following result.

**Lemma 1** The efficient level of joint effort  $e^E$  is lesser (greater) than the unilateral outcome  $e^N$  for  $\lambda^I(t^N) \geq \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I}$ .

**Proof.**  $e^E \leq e^N$  for  $\frac{1}{2I}[\sum_i \lambda^i(t^E) - c] \leq \frac{1}{2}[\lambda^I(t^N) - c]$ .

Rearranging the terms of the latter inequality, we arrive at the above result. ■

Comparing the above result to its counterpart for endogenous threat, seen in Gupta (2010), we notice that here the index  $\lambda$  shifts for a change in effort level, since the level of threat is sensitive to the effort level of the alliances. Thus, whether the efficient effort level is more or less than the unilateral level depends not just on the ex ante values of the  $\lambda$ s, but their ex post values, if the alliance were to shift to the efficient level of security from the unilateral level. In other words, the efficiency level is dependent on the magnitudes of shift of these indices, for a change in the level of threat that would occur from a shift in the security level.

**Assumption:** I assume that ex post nation  $I$  still has the highest  $\lambda$ .

The next result relates the level of threat observed in our model, to the level of security that is provisioned by the alliance. It seems fairly intuitive that if the level of effort at the efficient outcome is less than the unilateral outcome for the alliance, then the equilibrium amount of threat in the former situation will be more compared to the latter. The opposite should hold for the situation where the level of efficient security effort is more than the unilateral level. Conversely, if the level of threat is higher for the alliance making the unilateral effort level, compared to the level of threat if they play the efficient outcome, then it must be true that the effort in the unilateral outcome is lower than that in the efficient outcome. These results are proved below.

**Lemma 2** *If security effort at the unilateral outcome is greater (lesser) than that at the efficient outcome, then the threat level is lesser (greater) at those respective outcomes, and vice versa, i.e.  $e^N \geq e^E \iff t^N \leq t^E$ .*

**Proof.** See appendix 1. ■

We note that since  $t^E > t^N$  implies  $e^E < e^N$ , it must be true for  $t^E > t^N$  that the condition  $\lambda^I(t^N) > \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I}$  must hold (using the above result and lemma 1). In other words, if ex-post efficiency requires a drop in security levels, then the average of the public opinion indices (even after the increase in the threat level), must be still lesser than the ex-ante public opinion index in country  $I$ .

## 2.4 Payees

In Gupta (2010), I outlined an institutional structure which would enable the alliance to move to the efficient level of security provision, from the unilateral level. For that purpose, I constructed a game of security effort provision among the members of the alliance, with an additional "neutral player" participating in the game along with the original members of the alliance. This game is described in detail in Gupta (2010). The institutional structure implied by this game (which leads to the efficient provisioning of global security), and the details of the real life implementation of the institution are the main contributions of the mentioned



paper. For the benefit of the reader, a short description of a slightly modified version of this “institutional game” is provided in appendix 2. As mentioned in the introduction, one of the purposes of this paper is to test the robustness of this proposed institution in the presence of endogenous threat (threat was exogenous in Gupta(2010)), as well as derive insights into the conditions that must prevail to preserve the effectiveness of this institution in this modified environment. In order to achieve these goals, the first step is to find out in our current model which of the alliance partners would be willing to pay to move from an allocation with effort vector with joint effort provision at  $e^N$ , to one at which the joint effort is  $e^E$ , and how much.

For a country to be willing to pay a positive amount  $z^i$  for this movement, it has to be true that its utility from  $e^E$  must be greater than from  $e^N$ , even after it pays  $z^i$ . For the unilateral outcome, no country pays anything. In our current environment, the change in the threat level in response to the alliance’s action, becomes important. The individual rationality condition for  $i$  being willing to pay  $z^i > 0$  to achieve the efficient effort outcome over the unilateral outcome is  $[V^i(m^E, e^E; t^E) | z^i > 0] \geq [V^i(M^i, e^N; t^N) | z^i = 0]$ . The notation is as seen in earlier sections, and the superscripts for the security effort, threat levels and the private goods are self explanatory. We now find out that for a certain country willing to pay for the change, what is the maximum amount that it is willing to pay.

**Lemma 3** *If the utility of a country rises for a change in the effort level, the maximum amount it might be willing to pay for the change, given that it makes no effort contribution in the efficient allocation, is  $e^E \lambda^i(t^E) - e^N \lambda^i(t^N) + (e^N)^2 - (e^E)^2$ .*

**Proof.** See appendix 1. ■

**Remark 2** *Note that  $z^i > 0 \implies \lambda^i(t^E) - \frac{e^N}{e^E} \lambda^i(t^N) + \frac{(e^N)^2}{e^E} - e^E > 0$ , as we have assumed that effort levels are non-negative.*

We will now group the countries according to their willingness to contribute to a fund for transfers that need to be given to move to the efficient outcome.

Case (I).  $e^E > e^N$  : We will group the countries according to their willingness to contribute for a change in outcome. In order to do this, we categorize the nations according to their shift in  $\lambda$  between the unilateral outcome and the efficient outcome.

For nation  $i$  we have

$$z^i = \lambda^i(t^E) - \frac{e^N}{e^E} \lambda^i(t^N) + \kappa, \text{ where } \kappa = \frac{(e^N)^2}{e^E} - e^E$$

Since  $t^E < t^N$  for  $e^E > e^N$  (vide lemma 2), and  $\lambda^i(t^E) < \lambda^i(t^N)$  since  $\lambda_t > 0$ , it follows that  $z^i > 0$  iff  $\lambda^i(t^E) > \frac{e^N}{e^E} \lambda^i(t^N) - \kappa$  (note that  $\kappa = \frac{(e^N + e^E)(e^N - e^E)}{e^E}$  is negative, and that  $\frac{e^N}{e^E} < 1$ ). So, the willingness to contribute to the fund depend on the change in  $\lambda(\cdot)$  and the values of  $e^E$  and  $e^N$ .

One of the main differences with the results of Gupta (2010) is the crucial importance of how the public support index evolves between the unilateral and efficient states (rather than the ex ante level) in determining the contribution to the transfers' fund. In fact, given that  $\frac{e^N}{e^E} < 1$ , the ex post level of public support  $\lambda^i(t^E)$  should be sufficiently close to the ex ante support  $\lambda^i(t^N)$ , for a nation to be willing to contribute a positive amount to the transfers' fund, in the case where the ex post efficient level of security is higher than the ex ante unilateral level. This is not surprising, when one realizes that greater security would reduce threat levels, hence reducing the public's appetite for security (even though, on one hand, greater security related benefits accrue to the nation). Hence a positive contribution towards enhanced security levels can be supported only if the appetite for security remains high enough, even with a reduction in the threat level. In order to make our task simpler, we can construct  $\theta^i = \lambda^i(t^E) - \frac{e^N}{e^E} \lambda^i(t^N)$  and rank countries according to the value of their  $\theta$ s, such that  $\theta^1 < \theta^2 < \dots < \theta^j < \theta^{j+1} < \dots < \theta^{I-1}$ . We note that this ranking of a country in this case is different from its ranking according to the value of  $\lambda(\cdot)$ . The value of the  $\theta$ s obviously depend on the change in  $\lambda(\cdot)$  and the values of  $e^E$  and  $e^N$ .<sup>7</sup>

**Remark 3** *It follows that if there are  $j$  nations in the alliance with  $\theta^i = \theta^1, \dots, \theta^j$  such that  $\theta^i > \kappa$ , they would be willing to pay a positive amount for the movement to the efficient outcome.*

Case (II).  $e^E < e^N$  :

For nation  $i$  let

$$z^i = \lambda^i(t^E) - \frac{e^N}{e^E} \lambda^i(t^N) + \kappa, \text{ where } \kappa = \frac{(e^N)^2}{e^E} - e^E$$

Since  $t^E > t^N$  for  $e^E < e^N$ , we have  $\lambda^i(t^N) < \lambda^i(t^E)$ . Also, in this case  $\kappa = \frac{(e^N + e^E)(e^N - e^E)}{e^E}$  is positive, and that  $\frac{e^N}{e^E} > 1$ .

Now, let

$$\mu = \kappa - \frac{e^N}{e^E} \lambda^i(t^N)$$

Note that for  $\mu > 0$ ,  $z^i > 0$ . In this case,  $\lambda^i(t^N) < e^N - \frac{(e^E)^2}{e^N}$  (solving for  $\mu > 0$ ). This could happen for a

low enough  $\lambda^i(t^N)$ , or a low enough  $\frac{e^N}{e^E}$  (i.e. the ex post and ex ante effort levels are close), or both.

However, for  $\mu < 0$  (which entails a high enough  $\lambda^i(t^N)$ , i.e.  $\lambda^i(t^N) > e^N - \frac{(e^E)^2}{e^N}$ ), for  $z^i > 0$  we need either

a high enough  $\lambda^i(t^E)$  (i.e.  $\lambda^i(t^E) > \mu$ ), or a low enough  $\frac{e^N}{e^E}$  (i.e. the ex post and ex ante effort levels are

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<sup>7</sup>This easily seen. Consider a country having a shift in  $\lambda$  from 9 to 1, and another having a higher initial value of  $\lambda$ , 11, which shifts to 10. The ranking of the  $\lambda$ s for the countries is preserved, but the change for the first country is higher. However, in an alternate scenario, if the first country has a shift from 5 to 1, and the second a shift from 11 to 6, then the change for the second country is higher.

close),<sup>8</sup> or both.

So to make our task simpler, let us construct  $\phi^i = \lambda^i(t^E) - \frac{e^N}{e^E} \lambda^i(t^N)$ , and rank countries according to the value of their  $\phi$ s, such that  $\phi^1 < \phi^2 < \dots < \phi^J < \phi^{J+1} < \dots < \phi^{I-1}$ . Of these, let  $J$  nations have  $\phi^i > 0$ . Loosely speaking, for a large enough difference between the unilateral and efficient security levels, these would typically be nations whose ex post levels of the public support index are sufficiently greater than the ex ante levels. In other words, lesser security (due to the movement to  $e^E$ ) raises the threat, making the public in these nations sufficiently raise their appetite for security. In reality one would expect the public support for security in these nations low to begin with, causing them to advocate for a reduction in the security level. The reduced security would raise threat levels, but would cause a decline in the negative effects of security efforts (as seen earlier), and also enhance the public's appetite for security. So, for the governments of these nations, the movement to the efficient level would cause an ex post increase in utility levels.

**Remark 4** *It follows that these  $J$  nations (with  $\phi^i = \phi^{I-J}, \dots, \phi^{I-1} > 0$ ) may be willing to pay a positive amount for the movement to the efficient outcome.*

However, as we will see in the next section, I will not be able to include all these nations that are potentially willing to pay for a movement to the efficient outcome among my set of "payee nations" in the institutional scheme designed by me for achieving the efficient level of joint security by the alliance of nations facing the rogue nation's threat.

## 2.5 Recipients

In this section, I will analyze who will need to be paid for the alliance to move to the efficient level of security. In what follows, I assume that country  $I$ , which had the largest ex ante public support index among the countries in the alliance still has the largest public support index ex post. In other words, the  $\lambda$  ranking of country  $I$  is preserved, even if the alliance moves from the unilateral to the efficient security level. From appendix 2, the reader will observe that I have the neutral player propose that the alliance shift to the efficient level with the members nations provisioning the effort profile  $(e^i)_{i=1}^I = (0, 0, \dots, 0, e^E)$ . In other words, in both the unilateral, as well as the efficient scenarios, country  $I$  is the only nation undertaking security effort. In what follows, I will analyze in spirit of the institution suggested in Gupta (2010), which countries need to be paid (and how much), to realize this proposed effort profile.

For country  $I$ , a movement to the efficient level will not entail a loss in utility, if it is given a transfer  $\tau$ , seen below. As the unilateral effort level was chosen by country  $I$  in the benchmark model, even when the efficient effort was available, this transfer level should be positive.

$$\begin{aligned} \tau^I &= V^I(m^N, e^N; t^N) - V^I(m^E, e^E; t^E) \\ &= \{M^I + e^N[\lambda^I(t^N) - e^N] - ce^N\} - \{M^I + e^E[\lambda^I(t^E) - e^E] - ce^E\} \\ &= e^N \lambda^I(t^N) - e^E \lambda^I(t^E) - (e^N - e^E)[(e^N + e^E) + c] \end{aligned}$$

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<sup>8</sup>This is easily seen. For  $e^E \rightarrow e^N, \frac{e^N}{e^E} \rightarrow 1$ , reducing  $z^i = \lambda^i(t^E) - \lambda^i(t^N) + e^N - e^E$  which is positive in this particular case.

Let us now move on to the other nations that are adversely affected by a movement from the unilateral to the efficient level of joint effort. The main purpose of this exercise is to determine which countries have to be given a transfer (which I would like to be the minimally required amount, rather than one which would be Pareto improving for all member nations of the alliance) to maintain the effort profile  $(e^i)_{i=1}^I = (0, 0, \dots, 0, e^E)$ .

1. Case (I):  $e^E > e^N$

First, note that the ex post level of effort provision for each country, given joint effort from other allies at  $e^E$  and threat level  $t^E$ , is  $\hat{e}^i = \frac{1}{2}[\lambda^i(t^E) - 2e^E - c]$ . This is the level of effort that a country would supply, if it had to fight the rogue nation with joint effort fixed at  $e^E$  from the side of all its other allies. Now, for  $e^E > e^N$ , no country would want to deviate from zero effort. This is easily seen, as  $\hat{e}^i < e^E$  (since  $\hat{e}^i < e^N$ , as  $t^E < t^N$ ). Hence, if no other country other than  $I$  was supplying effort in the unilateral (Nash) outcome, in the efficient outcome no one else would have an incentive to make effort. This means that for effort  $e^E$  by  $I$ , the best response of other countries would be to make no effort. This means that to achieve a profile  $(e^i)_{i=1}^I = (0, 0, \dots, 0, e^E)$  with  $e^E > e^N$ , no other country other than  $I$  needs to be compensated by the payee nations (discussed in the last section).

2. Case (II):  $e^E < e^N$

Now, let  $e^{i*} = \frac{1}{2}[\lambda^i(t^E) - c]$  be the private effort level of a nation, i.e. the effort level it would provision if it had to fight a threat level  $t^E$  alone, without the help of any allies. For joint effort  $e^E < e^N$  (supplied by  $I$ ), countries having private provision  $e^{i*}$  levels greater than  $e^E$  would have an incentive to deviate from zero effort (and make up the difference between  $e^E$  and  $e^{i*}$ , gaining utility in the process),<sup>9</sup> and hence make it difficult to sustain the effort profile  $(e^i)_{i=1}^I = (0, 0, \dots, 0, e^E)$ . It is easily verified that these are countries for which  $\lambda^i(t^E) > \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I}$ . However, this can be prevented by having a transfer scheme in which they would be compensated up to their utility level for their private provision level (conditional on making no effort). This level of transfer is given by  $\tau^i = V^i(m^{i'}, e^{i*} - e^E, 0 | e^{i*}) - V^i(m^E, 0, \tau^i | e^E)$

$$= e^{i*}[\lambda^i(t^E) - e^{i*}] - c(e^{i*} - e^E) - e^E[\lambda^i(t^E) - e^E]$$

$$= (e^{i*} - e^E)[\lambda^i(t^E) - (e^{i*} + e^E + c)]^{10}$$

The set of countries for which these transfers are needed contains not only nations which suffer a loss in utility due to a movement from the unilateral to the efficient effort level, but may also contain some countries which gain from the movement. The reason they get compensated is because their ex post private provision level is more than the efficient level. Hence they must be compensated to maintain zero effort levels, if the effort profile  $(0, 0, \dots, 0, e^E)$  has to be maintained.

### 2.5.1 The subgame perfect equilibrium of the institutional game

In this section I will outline the main result of this paper. The proposition below describes the subgame perfect equilibrium of the institutional game described in appendix 2. This proposition is relevant for the case  $e^E < e^N$ , and a similar result can easily be derived for the case  $e^E > e^N$ , which I leave to the interested reader, for the sake of brevity.

<sup>9</sup>Note that for  $e^I = e^E$  and  $e^j = 0$ , for  $j \neq i, I$ ,  $e^i = \frac{1}{2}[\lambda^i(t^E) - c - 2e^E] = \frac{1}{2}[\lambda^i(t^E) - c] - e^E = e^{i*} - e^E$ , where  $e^{i*}$  is the private provision level of  $i$ .

<sup>10</sup>Notice that this compensation amount is one which puts a recipient country at its utility level for the joint effort provision of the alliance being at its ex post private provision level, but it having to bear the cost of provision only for the amount of this effort which is above the efficient level. Simple algebra shows this transfer amount to be positive, substituting for  $e^{i*}$ ,  $e^E$ , and using the facts that  $\lambda^i(t^E) > \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I}$  as  $e^{i*} > e^E$

In what follows,  $S$  is the set of all nations in the alliance, the set  $P$  consists of a set of payees among the nations in the alliance: This set contains nations with  $\phi^i > 0$  (see section 2.4 above) and  $\lambda^i(t^E) > \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I}$ . The set  $R$  consists of all other nations in the alliance (for the sake of simplicity I make the minor assumption that there are no nations with the same ex ante and ex post utilities). I have also assumed that if a country gets the same payoff from making zero effort and a positive effort, then it makes no effort. As detailed in appendix 2, other than these players, there is a "neutral" player in the "institutional game" who acts as a proposer and facilitator. I assume that the ex-post efficiency condition  $\sum_{i \in P} z^i \geq \sum_{i \in R} \tau^i + \tau^I$  holds.

**Proposition 1** *The profile ( $\{Agree, e^i = 0 \text{ for } NP\}_{i \in P}, \{e^i = 0 \text{ for } P \ \& \ NP\}_{i \in R \setminus I \text{ and } \phi^i < 0}, \{Agree, e^i = 0 \text{ for } NP\}_{i \in R \setminus I \text{ and } \phi^i > 0}, \{Agree, e^I = e^N \text{ for } NP\}$ ) is a subgame perfect equilibrium of the institutional game, for the elements proposal by the neutral player being such that:*

- (i).  $z^i$  for all  $i \in P$  such that  $0 \leq z^i \leq e^E \lambda^i(t^E) - e^N \lambda^i(t^N) + (e^N)^2 - (e^E)^2$ , and  $\sum_{i \in P} z^i = \sum_{i \in R} \tau^i + \tau^I$ .
- (ii). The neutral player proposing to compensate player  $I$  an amount  $\tau^I = e^N \lambda^I(t^N) - e^E \lambda^I(t^E) - (e^N - e^E)[(e^N + e^E) + c]$  for  $e^I = e^E$ , and 0 otherwise.
- (iii). Proposing to compensate players  $i \in R \setminus I$  an amount  $\tau^i = (e^{i*} - e^E)[\lambda^i(t^E) - (e^{i*} + e^E + c)]$ , for choosing  $e^i = 0$  in the effort choice subgame with transfers, and 0 otherwise.
- (iv). The proposal requiring unanimity support of nation  $I$ , nations  $i \in P$ , and nations  $i \in R \setminus I$  and  $\phi^i > 0$  who are the only nations invited to vote on the proposal.

*This subgame perfect outcome of the institutional game has all invited voters agreeing to pass the neutral player's proposal in the second round and all alliance members  $i \in S$  making effort choices in the third round such that the effort outcome is  $(e^i)_{i=1}^I = (0, 0, \dots, e^E)$ . Hence, the joint effort of the alliance is at the efficient level.*

**Proof.** See appendix 1. ■

This proposition gives the central result of this paper. This result suggests a particular institutional structure for the alliance (closely mirroring the one suggested in Gupta (2010)) that would help it reach its efficient effort level. For such an institutional structure, unilateral action by a single nation would be tempered towards the efficient outcome by multilateral participation by other alliance members. Comparison of the results obtained in the two papers is discussed in section 3. Note that in the above mechanism, nation  $I$  is getting "cheated" a little bit, compared to the other recipients, because they are getting compensated up to the utility of their ex post "private effort" levels, while nation  $I$  is getting compensated only up to the utility of its ex ante private effort level (or in other words the unilateral effort level). However, it can do nothing about it, because if it does not agree to the proposal, the status quo remains, hence as the unilateral provider in the benchmark case, it can do no better.

## 2.6 Declining public support

In the previous section I have assumed that  $\lambda$  is increasing in the level of threat  $t$ , hence  $\lambda_t^i > 0$ . However, there are some real world occurrences where for some nations the reverse is true, i.e. an increase in the

threat level actually weakens public support for their contributions security efforts of the alliance. This may particularly be true if these countries are initially low targets for the threats, but then become targets - making the public feel a that a lower profile in security activities would again make them low priority targets.<sup>11</sup> In order to model this phenomenon, I now assume that for the countries belonging to set  $P$  in the earlier section,  $\lambda_t^i < 0$  (so public support begins at a certain level, but as threat levels increase, they go down). For all other nations  $\lambda_t^i$  is still positive.

1. Case (I):  $e^E > e^N$

Since  $t^E < t^N$  for  $e^E > e^N$  (vide lemma 2), for the nations in set  $P$ ,  $\lambda^i(t^E) > \lambda^i(t^N)$ . This means that in this case  $e^E = \frac{1}{2I}[\sum_{i=1}^I \lambda^i(t^E) - c]$  will be higher than the efficient effort level when all nations had  $\lambda_t^i > 0$ . Thus the transfer  $\tau^I$  needed to make nation  $I$  supply the higher efficient level of effort will be higher:

$$\frac{\partial \tau^I}{\partial e^E} = -\lambda^I(t^E) - e^E \frac{\partial \lambda^I(t^E)}{\partial t^E} \frac{\partial t^E}{\partial e^E} + 2e^E + c = -\lambda^I(t^E) - e^E \frac{\partial \lambda^I(t^E)}{\partial t^E} \frac{\partial t^E}{\partial e^E} + \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I} > 0$$

(substituting for  $e^E$  and using the facts that  $\lambda^I(t^E) < \lambda^I(t^N) < \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I}$ ,  $\frac{\partial \lambda^I(t^E)}{\partial t^E} > 0$ , and  $\frac{\partial t^E}{\partial e^E} < 0$ )

Recall that there are no other recipients of transfers (except for  $I$ ) in the case where  $e^E > e^N$ .

Now, for nation  $i \in P$  we have

$$z^i = \lambda^i(t^E) - \frac{e^N}{e^E} \lambda^i(t^N) + \kappa, \text{ where } \kappa = \frac{(e^N)^2}{e^E} - e^E$$

Since for  $e^E > e^N$  we now have  $\lambda^i(t^E) > \lambda^i(t^N)$ , rather than  $\lambda^i(t^E) < \lambda^i(t^N)$  as before, it follows that the value of  $z^i$  is higher in this case (or the maximum amount country  $i$  is willing to pay for a movement to the efficient level has gone up compared to before). So even though a higher transfer amount has to be supported by the payee countries, given that reduction of the threat actually increases the public support for the conflict, it might be easier to support the (higher) efficient amount of joint effort in this case.

Case (II).  $e^E < e^N$  :

Since  $t^E > t^N$  for  $e^E < e^N$ , for the nations in set  $P$ ,  $\lambda^i(t^E) < \lambda^i(t^N)$ . This means that in this case  $e^E = \frac{1}{2I}[\sum_{i=1}^I \lambda^i(t^E) - c]$  will be lower than the efficient effort level when all nations had  $\lambda_t^i > 0$ . We see below the transfer  $\tau^I$  needed to make nation  $I$  supply a lower efficient level of effort might be higher or lower:

<sup>11</sup>The experience in Spain after the Madrid train bombings of 2004, with resultant decline in public support for the war in Iraq (which was low enough to start with), is a case to the point.

$$\begin{aligned}
\frac{\partial \tau^I}{\partial e^E} &= -\lambda^I(t^E) - e^E \frac{\partial \lambda^I(t^E)}{\partial t^E} \frac{\partial t^E}{\partial e^E} + 2e^E + c = -\lambda^I(t^E) - e^E \frac{\partial \lambda^I(t^E)}{\partial t^E} \frac{\partial t^E}{\partial e^E} + \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I} \leq 0 \\
\frac{\partial \tau^I}{\partial e^E} &< 0 \text{ if } \left| e^E \frac{\partial \lambda^I(t^E)}{\partial t^E} \frac{\partial t^E}{\partial e^E} \right| < \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I} - \lambda^I(t^E) \\
\text{and } \frac{\partial \tau^I}{\partial e^E} &> 0 \text{ if } \left| e^E \frac{\partial \lambda^I(t^E)}{\partial t^E} \frac{\partial t^E}{\partial e^E} \right| > \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I} - \lambda^I(t^E)
\end{aligned}$$

(substituting for  $e^E$  and using the facts that  $\lambda^I(t^E) > \lambda^I(t^N) > \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I}$ ,  $\frac{\partial \lambda^I(t^E)}{\partial t^E} > 0$ , and  $\frac{\partial t^E}{\partial e^E} < 0$ )

We see in one of the cases above that a lower transfer might be required to get to the lower efficient effort level in the case where a combination of factors occur:  $e^E$  is high enough, the gain in public support due to the increase in the threat level is sufficiently high, and the absolute level of public support in the efficient state is also suitably high. Thus, the perverse effect of decreasing security, leading to a greater threat, drives up public support, and also saves on costs (as effort provision falls), hence requiring less compensation in form of transfers (as this increasing public support and cost savings increases the utility of government  $I$  to a sufficient level). In the other, more intuitive case, a higher transfer is needed to compensate nation  $I$ , for it to move to a lower (efficient) level of effort. As seen below, a similar result does not obtain for other recipients since they are compensated for their ex post private provision levels (which increases for a lower efficiency level) - this is not the case for  $I$ , which is compensated up to its ex ante private provision level.

Recall that there are other recipients of transfers (other than  $I$ ) in the case where  $e^E < e^N$ . For these other recipients of transfers, a reduction in the efficient effort level will also impact the transfer levels. In fact, the transfer  $\tau^i$  needed to make nation  $i$  make no effort on its own will be unambiguously higher (for a lower efficient level of joint effort):

$$\begin{aligned}
\frac{\partial \tau^i}{\partial e^E} &= (e^{i*} - e^E) \left[ \frac{\partial \lambda^i(t^E)}{\partial t^E} \frac{\partial t^E}{\partial e^E} - 1 \right] - [\lambda^i(t^E) - (e^{i*} + e^E + c)] \\
&= (e^{i*} - e^E) \left[ \frac{\partial \lambda^i(t^E)}{\partial t^E} \frac{\partial t^E}{\partial e^E} - 1 \right] - \frac{1}{2} \left[ \lambda^i(t^E) - \left( \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I} \right) \right] < 0
\end{aligned}$$

(substituting for  $e^{i*}$ ,  $e^E$ , and using the facts that  $\lambda^i(t^E) > \frac{\sum_{i=1}^I \lambda^i(t^E)}{I} + \frac{c(I-1)}{I}$  as  $e^{i*} > e^E$ ,  $\frac{\partial \lambda^i(t^E)}{\partial t^E} > 0$ , and  $\frac{\partial t^E}{\partial e^E} < 0$ )

Now, for nation  $i \in P$  we have

$$z^i = \lambda^i(t^E) - \frac{e^N}{e^E} \lambda^i(t^N) + \kappa, \text{ where } \kappa = \frac{(e^N)^2}{e^E} - e^E$$

Since for  $e^E < e^N$  we now have  $\lambda^i(t^E) < \lambda^i(t^N)$ , rather than  $\lambda^i(t^E) > \lambda^i(t^N)$  as before, it follows that the value of  $z^i$  is lower in this case (or the maximum amount country  $i$  is willing to pay for a movement to the efficient level has gone down compared to before). The escalation of the threat decreases the public support for the conflict. The above analysis mostly suggests it might be harder to support the (lower) efficient amount of joint effort in this case, when public opinions in the allied countries diverge.

### 3 Discussion

In this section I will discuss some of the implications of the above results, and compare them those found in Gupta (2010). First, the institutional structure suggested for the realization of the efficient level of global security in this paper is more "inclusive" than the one suggested in the earlier paper. This structure gives all nations in the alliance, except for those that enjoy hugely positive free-riding benefits, a chance to participate in the decision of whether or not to accept the neutral player's proposal. Not only that, since all voting nations have de facto "veto rights", all these nations have a strong say in the decision-making process. From a democratic perspective, this certainly seems desirable.

The most important contribution of this paper, the one which becomes possible to make due to the endogeneity of the threat level, is to study how the evolution of public opinion in the member nations in the alliance might influence the movement to efficiency. In the case where the efficient security level is more than the unilateral level, one of the main differences with the results of Gupta (2010) is the crucial importance of how the public support index evolves in the payee states, between the unilateral and efficient states, in determining the contribution to the transfers' fund. In fact, the ex post level of public support should be sufficiently close to the ex ante support for a nation to be willing to contribute a positive amount to the transfers' fund. This occurs because greater security would reduce threat levels, hence reducing the public's appetite for security (even though, on one hand, greater security related benefits accrue to the nation). Hence a positive contribution towards enhanced security levels can be supported only if the appetite for security remains high enough, even with a reduction in the threat level.

For the case where the efficient level is lower than the unilateral level, for a large enough difference between the unilateral and efficient security levels, the payee nations would typically need to have ex post levels of the public support which are sufficiently greater than the ex ante levels. In other words, lesser security (due to the movement to the efficient security level) raises the threat, and that in turn should cause the public in these nations sufficiently raise their appetite for security. In reality one would expect the public support for security in these nations low to begin with, causing them to advocate for a reduction in the security level. The reduced security would raise threat levels, but would cause a decline in the negative effects of security



efforts, and also enhance the public's appetite for security. So, for the governments of these nations, the movement to the efficient level would cause an ex post increase in utility levels.

This paper also analyzes the implications for the scenario where the public opinion supporting the conflict actually drops in a subset of member nations of the alliance, when the level of threat increases. It turns out that the transfer needed to shift to the efficient level of security, when the efficient level is higher than the unilateral level, is more than in the earlier scenario (when the public support in all nations increase for a higher threat). However, as the maximum amount the payee countries are willing to pay for a movement to the efficient level goes up compared to before (given that reduction of the threat actually increases the public support for the conflict) it might be easier to support the (higher) efficient amount of joint effort in this case. On the other hand, if the efficient security level is less than the unilateral level, the maximum amount payee countries are willing to pay for a movement to the efficient level has goes down compared to before, as the escalation of the threat decreases the public support for the conflict. This suggests that it might be harder to support the (lower) efficient amount of joint effort in this case. Interestingly, in this case a scenario is possible where a lower transfer might be required to get the unilateral provider to the lower efficient effort level: for that to happen, a combination of factors must occur - the efficient level of security is high enough, the gain in public support due to the increase in the threat level is sufficiently high, and the absolute level of public support in the efficient state is also suitably high. Thus, the perverse effect of decreasing security, leading to a greater threat, simultaneously drives up public support and saves on cost, hence requiring less compensation in form of transfers (as this increasing public support increases the utility of government of the country supplying the security effort to a sufficient level, to compensate for the actual loss in security itself). Of course, as seen in the earlier section, the case where a higher level of transfers is needed, is possible as well under certain circumstances.

## 4 Conclusion

Often rogue nations react to the actions of an alliance of nations (for example, the NATO) fighting them. We extended our model (seen in Gupta 2010) in this paper, by taking that into account. In order to that, we have considered a rogue nation that strategically makes threatening effort. The rogue nation and the alliance interact, and the efforts of both are endogenously determined. The findings obtained when threat is exogenous (as seen in Gupta (2010)) do not qualitatively change when it is endogenized, as regards the existence of an institutional structure that would facilitate the alliance's movement from a unilateral to an efficient level of global security. However, there are interesting implications of how endogeneity of the threat level makes the evolution of public support for the conflict (in the allied nations combating the threat) very

important. The effect of evolving public support in the achievement of an institutional structure, that would help the alliance reach an efficient level of global security, is analyzed in detail in this paper. Further work would involve extending this analysis, when there is uncertainty regarding the evolution of such support.

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## Appendices

### Appendix 1: Proofs

#### Proof of Lemma 2

*Proof.* If  $e^N \geq e^E$ , then  $\alpha(e^N) \leq \alpha(e^E)$  (since  $\alpha_e < 0$ )

Thus, it follows from the solutions for  $t^N$  and  $t^E$  that  $t^N \leq t^E$ .

Now, for  $t^N > t^E$ , let  $e^N > e^E$ .

If  $e^N > e^E$ , then  $\alpha(e^N) < \alpha(e^E)$ , since  $\alpha_e < 0$ .

But this violates the original premise that  $t^N > t^E$  (as per the solutions for  $t^N$  and  $t^E$ , this would follow from  $\alpha(e^N) < \alpha(e^E)$ ).

Thus, it must be true that  $e^N < e^E$ .

So, for  $t^N > t^E$ , it must be that  $e^N < e^E$ . Similarly, for  $t^N < t^E$ , it must be that  $e^N > e^E$ .

#### Proof of Lemma 3

*Proof.* From the individual rationality condition, a country  $i$  might be willing to pay a positive amount, i.e.  $z^i > 0$ , only when  $[V^i(m^E, e^E; t^E) | z^i > 0] \geq [V^i(M^i, e^N; t^N) | z^i = 0]$

Or,  $[V^i(m^E, e^E; t^E) | z^i > 0] - [V^i(M^i, e^N; t^N) | z^i = 0] \geq 0$

We note that the utility of  $i \neq I$  for the unilateral outcome is  $M^i + e^N[\lambda^i(t^N) - e^N]$ ,

And its utility in the efficient outcome is  $M^i + e^E[\lambda^i(t^E) - e^E] - z^i$

So,  $[V^i(m^E, e^E; t^E) | z^i > 0] - [V^i(M^i, e^N; t^N) | z^i = 0] \geq 0$

$\implies \{M^i + e^E[\lambda^i(t^E) - e^E] - z^i\} - \{M^i + e^N[\lambda^i(t^N) - e^N]\} \geq 0$

$\implies z^i \leq e^E\lambda^i(t^E) - e^N\lambda^i(t^N) + (e^N)^2 - (e^E)^2$

Thus, the maximum amount  $i$  would be willing to pay for the change is  $e^E\lambda^i(t^E) - e^N\lambda^i(t^N) + (e^N)^2 - (e^E)^2$

#### Proof of Proposition 1

*Proof.* Part A: The effort choice for  $i \in R \setminus I$  in the third stage of the institutional game is zero

Note that for countries  $i \in R \setminus I$  the best response function in the status quo game is:  $e^i = \frac{1}{2}[\lambda^i(t^N) - c - 2\sum_j e^j]$ , for  $\lambda^i > c + 2\sum_j e^j, j \neq i$  and 0 otherwise.

In the institutional game with transfers, given  $\tau^i = (e^{i*} - e^E)[\lambda^i(t^E) - (e^{i*} + e^E + c)]$  for  $e^i = 0$  and 0 otherwise, the best response effort level is zero, for  $e^E$  by  $I$  and no effort by others.

This is because from the best response function  $e^i = \frac{1}{2}[\lambda^i(t^E) - c - 2\sum_j e^j]$ , for  $\lambda^i > c + 2\sum_j e^j, j \neq i$  and 0 otherwise, its best response to  $e^E$  by  $I$  and no effort by others, is to make effort  $e^{i*} - e^E$ . The payoff from making this effort is  $V^i(m^{i'}, e^{i*} - e^E, 0 | e^{i*})$  if  $e^I = e^E$ . But payoff from making no effort is  $V^i(m^E, 0, \tau^i | e^E)$ . Since by construction  $V^i(m^E, 0, \tau^i | e^E) = V^i(m^{i'}, e^{i*} - e^E, 0 | e^{i*})$ ,  $i$ 's effort level zero is payoff equivalent to making effort.<sup>12</sup> Note that if a country gets the same payoff from making zero effort and a positive effort, then it makes no effort (by assumption).

Further, for  $e^i = \theta \neq 0$ ,  $\tau^i = 0$  and  $e^I = e^E - \theta$ , for no effort by the other players. Using the best response functions, it is easy to check that these strategies are indeed best responses to each other.

<sup>12</sup>In this proof  $V^i(m^i, e^i, \tau^i(e^i) | e)$  denotes  $i$ 's utility from its consumption of the private good  $m^i$ , the amount of effort  $e^i$  that it puts in, and the transfer  $\tau^i(e^i)$  it gets (dependent on its effort), given the joint effort level  $e$ . Note that an effort level  $e^i$  is weakly preferred by  $i$  to an alternate level  $e^{i'}$ , if  $V^i(m^i, e^i, \tau^i(e^i) | e) \geq V^i(m^{i'}e^{i'}, \tau^{i'}(e^{i'}) | e')$  for given effort levels of all  $j \neq i$ .

Hence, i's payoff from deviation is  $M^i + e^E[\lambda^i(t^E) - e^E] - c\theta < M^i + e^E[\lambda^i(t^E) - e^E] + \tau^i$ .

Thus,  $e^i = 0$  maximizes (weakly) i's payoff, given  $e^I = e^E - \theta$  and  $e^j = 0, j \neq i, I$ .

Part B: In the second stage of the institutional game, voters unanimously "agree" to the neutral player's first stage proposal

By construction  $V^i(m^E, 0, \tau^i | e^E) + \tau^i = V^i(m^{i^*}, e^{i^*} - e^E, \tau^i = 0 | e^{i^*})$  for  $i \in R \setminus I$  and  $\phi^i > 0$ .

By construction  $V^I(m^E, e^i = e^E; t^E) + \tau^I = V^I(m^N, e^N; t^N) - V^I(m^E, e^E; t^E)$  for  $I$ .

As  $0 \leq z^i \leq e^E \lambda^i(t^E) - e^N \lambda^i(t^N) + (e^N)^2 - (e^E)^2$  for all  $i \in P$ ,  $V^i(m^E, e^i = 0, \tau^i | e^E) \geq V^i(M^i, e^i = 0, \tau^i = 0 | e^N)$  for these players.

Hence agreeing to the neutral players proposal at least (weakly) dominates not agreeing for all voters. Hence the proposal is unanimously adopted.

## Appendix 2: The Institutional Game

This game is described in detail in Gupta (2010). In brief, let the set of all the  $I$  members of the alliance be called  $S$ . The transfers will be paid by a set of payees  $P \subset S$  to a set of recipients  $R \subset S$ . In what follows, I will outline a game of perfect and complete information in which all the members belonging to the sets  $P$  and  $R$  participate, along with a neutral player (think of the neutral player as an entity within a supranational agency, like the Office of Security within the NATO). There will be certain rules of interaction among the players. From these rules it is possible to identify an institutional structure for the alliance that would lead to the efficient outcome. I call the game described below the 'institutional game'. All players in this game are rational and have complete information. This game exists only if  $P$  is non-empty. It is assumed that the ex-post efficiency condition outlined in the paper holds. The "institutional game" is as follows:

There are four stages in this game. In the first stage, the neutral player makes a proposal to the other players. The proposal is a collection of elements  $[P, R, (T^i)_{i=1}^I, (e^i)_{i=1}^I]$ , where  $P$  is a set of payees,  $R$  is a set of recipients,  $(T^i)_{i=1}^I$  is a vector of transfers paid by payees and received by recipients, and  $(e^i)_{i=1}^I$  is a particular effort vector. For what follows, let the effort vector proposed by the neutral player be  $(e^i)_{i=1}^I = (0, 0, \dots, 0, e^E)$ .  $P$  and  $R$  are such that  $P \cup R = S$ , and  $P \cap R = \phi$ .

In the second stage, the players in the set  $P$ , player  $I$ , and players in set  $R$  with  $\phi^i > 0$  simultaneously vote either *Agree* or *Not Agree* to the proposal. As mentioned, for the payees the proposal contains a total amount that they need to pay and a rule to divide the payment among them. For  $I$ , the proposal commits to pay an amount of transfer  $\tau$  to him, dependent on it making the efficient effort level. For the proposal to be adopted, it must be adopted *unanimously* by all players in the set  $P$  and player  $I$ . Otherwise the proposal fails, and no transfers are made. Once a player votes for the proposal, it is committed to adhering to it. It is not possible (by membership rules of the alliance) for any member of  $P$  to make a private transfer to any other player, other than through the neutral player. If the proposal succeeds, the neutral player

takes the amounts given in vector  $(T^i)_{i=1}^I$  and holds them. If it does not, no payments are made, that is  $(T^i)_{i=1}^I = (0^i)_{i=1}^I$ .

In the third stage, the alliance members  $i \in (R \setminus I \text{ and } \phi^i < 0)$  play a simultaneous-move non-cooperative game of effort choice for adoption of the proposal. For non-adoption of the proposal, there is the status quo effort choice game with all players in  $S$ . If the proposal was adopted in the second stage, there is an effort choice game where transfer amounts are committed by the neutral player to recipients according to a scheme outlined in the proposal (which is discussed in detail later). In brief, the neutral player commits to pay players  $i \in R$  a transfer sum  $z^i \in \mathfrak{R}^-$  from the transfer amounts handed over to it by the payees, if the effort chosen by them is zero. If, however, they make positive effort then they do not receive this transfer. For the proposal being adopted and the set  $(R \setminus I \text{ and } \phi^i < 0)$  being empty, there is no third stage, the fourth stage described below follows the second. In this paper I assume the more general case, so the set  $(R \setminus I \text{ and } \phi^i < 0)$  is assumed to be non-empty.

The fourth stage is the payments stage (for the game with transfers). Payments are made to all recipients upon observation of effort or money given back to payees, in full or in part (dependent on the effort choices of the players in set  $R$ ).

Lastly, it is assumed that the neutral player does not retain any money itself (thus, the amount paid by the payees equals the amount received by the recipients) and conforms to all the rules of the game described above.