A Critical Note on Marx's Theory of Profits

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ABSTRACT

This paper shows that Marx's theory of profits is based, implicitly, on the existence of a vertically integrated sector that (i) can produce the exact amount of commodities received as wages; (ii) includes all the processes of production actually used in the economy considered; and (iii) constitutes a quasi-one-commodity system. Nevertheless, the said sector does not always exist, whilst when it exists, positive surplus labour is a necessary and sufficient condition for positive profits in this sector, pure and simple. Consequently, Marx's theory of profits cannot be sustained.

INTRODUCTION

As is well known, Marx tried to show that the 'exploitation of workers', as estimated by surplus labour (unpaid labour/surplus value), is the sole source of the *actual* profits.

The purpose of this note is to make clear the conditions under which positive surplus labour is *both* necessary and sufficient for the existence of positive profits in the economy as *a whole*, i.e., to examine the robustness of the 'exploitation theory of profits'. It is shown that these conditions are very special and therefore Marx's theory of profits cannot be sustained.

The paper is organized as follows. In the next section we consider a simple model for a multi-sector economy.[1] The following section allows for reducible systems, heterogeneous labour and pure joint products. The final section concludes.

THE BASIC ARGUMENT

Consider a world with single-product industries, only basic commodities (in the sense of Sraffa (1960), pp.7-8)), full capacity utilization, constant returns, circulating capital, and homogeneous labour. For simplicity, assume that input coefficients are fixed, all wages are consumed and the input of labour in the households equals zero.

The price side of our simple economy is described by the system

$$\pi = \pi A + w\mathbf{a} + \mathbf{k} = \pi(\mathbf{A} + \mathbf{da}) + \mathbf{k} = \pi\mathbf{B}(\mathbf{I} + \mathbf{R}) \tag{1}$$

where π ($> \mathbf{0}$) denotes a (1 x n) vector of market prices, $\mathbf{A}$ the semi-positive, irreducible and profitable matrix of material input coefficients, w the money wage rate, paid at the beginning of the common production period, $\mathbf{a}$ the positive vector of direct labour inputs, $\mathbf{k}$ ($= [k_j]$) the vector of profits per unit activity level, $\mathbf{d}$ ($\geq \mathbf{0}$) a given column vector representing the real wage rate, $\mathbf{B}$ ($= \mathbf{A} + \mathbf{da}$) the augmented matrix of inputs, $\mathbf{I}$ the (n x n) identity matrix, and $\mathbf{R}$ the diagonal matrix of the sectoral rates of profit.[2]

The quantity side is described by the system

$$\mathbf{U} = (\mathbf{I} - \mathbf{B})\mathbf{X} = \mathbf{Y} - \mathbf{d}L = \mathbf{BGX} + \mathbf{C} \tag{2}$$

where $\mathbf{U}$ denotes the 'surplus product', $\mathbf{X}$ ($= [X_j] > 0$) the intensity vector, $\mathbf{Y}$ the net output vector, L ($= \mathbf{aX}$) the level of total employment, $\mathbf{G}$ the diagonal matrix of the sectoral rates of growth ($\mathbf{BGX}$ symbolizes the net investment vector), and $\mathbf{C}$ ($\geq \mathbf{0}$) the vector of consumption out of profits.

Equations (1) and (2) imply that

$$(\mathbf{I} - \mathbf{B})\mathbf{X^j} = \mathbf{BGX^j} + \mathbf{C^j} + \mathbf{E^j} \tag{3}$$

$$k_j X_j = \pi\mathbf{BGX^j} + \pi\mathbf{C^j} + \pi\mathbf{E^j} \tag{4}$$

where $\mathbf{X^j}$, $\mathbf{C^j}$, $\mathbf{E^j}$ are, respectively, gross output vector, vector of consumption out of profits and net 'export' vector of the jth process.[3]

If $\mathbf{v}$ ($= \mathbf{a}(\mathbf{I} - \mathbf{A})^{-1}$) is the vector of 'labour values', i.e., the vector of the quantities of labour 'crystallized' in the different commodities or, equivalently, the vector of the quantities of labour required, directly and indirectly, to produce one (net) unit of each commodity, and $\mathbf{x}$ is the so-called 'necessary intensity vector', i.e., a vector which satisfies $\mathbf{x} = \mathbf{Ax} + \mathbf{d}L$, then

$$S = L - \mathbf{ax} = (1 - \mathbf{vd})L \tag{5}$$

$$S_u = L - \mathbf{vd}L \tag{5a}$$

$$S_v = \mathbf{vU} = (1 - \mathbf{vd})L \tag{5b}$$

$$e = S/\mathbf{ax} = (1/\mathbf{vd}) - 1 \tag{6}$$

are, respectively, total 'surplus labour', 'unpaid labour', 'surplus value' and 'rate of surplus labour' (*or* 'rate of exploitation'; see Fujimori (1982, pp. 4-13) for a detailed exposition). Consequently, $S_v = S_u = S$.

Finally, from (1) and (5) we obtain

$$\mathbf{k}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{d}L = \boldsymbol{\pi}\mathbf{d}S \tag{7}$$

which provides an explicit relationship between $\mathbf{k}$ and S.

From the well-known Perron-Frobenius theorems (for semi-positive matrices) it is easily found that the following hold:

(i) Positive surplus labour is a necessary (but *not* a sufficient) condition for the existence of positive profit in *every* process (this is the so-called 'Fundamental Marxian Theorem'; Okishio (1955, pp. 75-8, 1993, Essays 3 and 6) and Morishima and Seton (1961, pp. 207-9)), i.e.,

$$S >(\leq) 0 \Leftrightarrow \{\exists(\nexists) \boldsymbol{\pi}: \mathbf{k} >(\geq) \mathbf{0}\} \tag{8}$$

$$S > 0 \nRightarrow \mathbf{k} > \mathbf{0} \tag{8a}$$

When $\boldsymbol{\pi}$ is proportional to $\mathbf{v}$, i.e., $\boldsymbol{\pi} = \beta\mathbf{v}$, we get

$$\mathbf{k} = \beta\mathbf{a}(S/L) \tag{8b}$$

Hence, $S > 0$ is *both* necessary and sufficient for $\mathbf{k} > \mathbf{0}$ (see also Marx (1967, pp. 190-4)). As is well known, however, the proportionality of prices to values is not associated with the capitalist economy.

(ii) Positive surplus labour assures that at least one element of $\mathbf{k}$ (and of $\mathbf{U}$) is positive. Zero surplus labour does not necessarily imply that $\mathbf{k}$ ($\mathbf{U}$) equals zero unless $\boldsymbol{\pi}$ ($\mathbf{X}$) is the left (right) Perron-Frobenius eigenvector of $\mathbf{B}$.

(iii) The co-existence of positive (non-positive) surplus labour with non-positive (positive) *total* profits, $\boldsymbol{\pi}\mathbf{U}$, is entirely possible and it implies that at least one process is reproduced on a lower scale,[4] i.e.,

$$\{S >(\leq) 0, \boldsymbol{\pi}\mathbf{U} = \mathbf{k}\mathbf{X} \leq(>) \mathbf{0}\} \Rightarrow (\mathbf{U}, \mathbf{k}, \mathbf{G}) \ngeq \mathbf{0} \tag{9}$$

(iv) A non-profitable process may be reproduced on a higher scale, i.e.,

$$\{k_j X_j \leq 0, \mathbf{B}\mathbf{G}\mathbf{X}^\mathbf{j} \geq \mathbf{0}\} \Rightarrow \boldsymbol{\pi}\mathbf{E}^\mathbf{j} < 0 \tag{10}$$

Consider now the vertically integrated sector producing the total real wages (WS hereafter; see also Sraffa (1960, Appendix A), Pasinetti (1973) and in particular Garegnani (1984, pp. 311-20)). If $\mathbf{X_w}$, $\mathbf{Y_w}$, $\mathbf{U_w}$ are, respectively, intensity vector, net output vector ($= \mathbf{d}L$) and surplus product of the WS, then, first, $\mathbf{X_w}$ equals the 'necessary intensity vector', second,

$$\mathbf{U_w} = \mathbf{d}S = \mathbf{Y_w}(S/L) \tag{11}$$

i.e., the WS is a *quasi-one-commodity* economy in the sense that $\mathbf{U_w}$ is proportional to $\mathbf{Y_w}$, third,

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$$\pi\mathbf{U_w}(\mathbf{aX_w}/\pi\mathbf{Y_w}) = \mathbf{vU_w} \tag{11a}$$

and, fourth,

$$r_w = e/(c_w + 1) = e/[(\pi\mathbf{HY_w}/w\mathbf{vY_w}) + 1] \tag{11b}$$

where r_w, c_w are, respectively, average rate of profit and price 'composition of capital' in the WS, and $\mathbf{H}\ (= \mathbf{A(I - A)}^{-1})$ is the vertically integrated technical coefficients matrix, the jth column of which shows the vector of commodities required, directly and indirectly, to produce one (net) unit of commodity j.[5] From (11) it follows that $\mathbf{U_w}$ is (semi-)positive if and only if S is positive. Hence positive surplus labour is necessary and sufficient for positive profits in the WS, *pure and simple* (see also (11b)).

When a *uniform* rate of profit, r, is postulated, system (1) becomes

$$\mathbf{p} = \mathbf{pB}(1 + r) \tag{12}$$

where $\mathbf{p}$ represents a vector of 'production prices'. Basically there are two equivalent, but rather *different*, ways to determine r (and $\mathbf{p}$):

(1) Since a non-positive vector of commodity prices is economically insignificant, it follows that

$$(1 + r)^{-1} = \lambda[\mathbf{B}] \tag{13}$$

where $\lambda[\mathbf{.}]$ is the Perron-Frobenius eigenvalue of a matrix. Equations (12) and (13) determine a unique, positive solution for $(\mathbf{p}, r)$, provided only that $\lambda[\mathbf{B}] < 1$. Consequently, the sign of profits is independent of the pattern of output, $\mathbf{X}$, and therefore positive profits may co-exist with diminishing reproduction.

It should be emphasized that this determination, in terms of the augmented matrix of inputs, makes no reference to any labour values.[6]

(2) From (11b) and (12) it follows that

$$r = r_w = \mathbf{pU_w}/\mathbf{pBX_w} = \mathbf{pd}S/\mathbf{p(H + dv)}\mathbf{d}L \tag{14}$$

or

$$\mathbf{v(I - H}r)^{-1}\mathbf{d}(1 + r) = 1 \tag{14a}$$

Equation (14a) determines a unique, positive profit rate, provided only that $S > 0$. Thus we may state the following proposition: r and $\mathbf{p}$ are *all* positive if and only if S is positive.[7]

It should be emphasized that surplus labour enters into the picture *because* the profit rate is determined in terms of the WS.

The above analysis has shown that, when commodity prices deviate from the production prices, positive surplus labour (unpaid labour/surplus value) is neither necessary nor sufficient for positive total profits. Positive surplus labour is necessary and sufficient for positive profits in the WS. But this is no basis for saying that 'surplus value (profit) is the sole source (is the transformed form) of profit (of surplus value)'. Nevertheless, when commodity prices are given as the production prices, the sectoral rates of profit equal the profit rate in the WS. Therefore positive surplus labour *converts* to a necessary and sufficient condition for the existence of positive prices yielding a positive profit rate.

SOME EXTENSIONS

In this section we shall extend the argument to the following cases: (i) reducibility; (ii) heterogeneous labour; and (iii) pure joint production. It is shown that the conversion of positive surplus labour into a necessary and sufficient condition for the existence of a positive profit rate is *not* always possible.

Reducibility

In order to simplify the analysis, consider an economy in which (i) there are $n + 1$ commodities; (ii) system (12) is associated with the first n commodities; (iii) the $n + 1$st commodity is a self-reproducing non-basic (in the sense of Sraffa (1960, Appendix B)); and (iv) surplus labour is positive or, equivalently, $\lambda[\mathbf{B}] < 1$. Then the $n + 1$st price is determined by the equation

$$p_{n+1} = (\mathbf{p}\mathbf{F} + p_{n+1}f)(1 + r) \tag{12a}$$

where $\mathbf{F}$ is a column vector and $f\ (< 1)$ is a scalar.

It is easily shown that, with reducibility, the two ways to determine the profit rate (considered above) are not always equivalent and that the latter cannot be regarded as acceptable from an economic point of view. The profit rate must be determined by

$$(1 + r)^{-1} = \max \{\lambda[\mathbf{B}], f\} \tag{15}$$

Thus the system has a unique, positive solution for the profit rate and prices if and only if $f < \lambda[\mathbf{B}]$.[8]

We conclude that $S > 0$ is necessary but not sufficient for $r > 0$. This is the consequence of the fact that, when $f \geq \lambda[\mathbf{B}]$, the non-basic process cannot achieve the profit rate in the WS.

Finally, it may be noted that, with differential rates of profit, zero surplus labour is compatible with a semi-positive vector of profits (compare to (8)), i.e., with zero (positive) profit in every process of the WS (in the non-basic process). Given that there is no connection between surplus labour and the profits in those processes which play no role, direct or indirect, in the production of the wage bundle, this does not come as a surprise.

Heterogeneous Labour

Assume that there are m types of heterogeneous labour. Let $\mathbf{D}$ be the ($n \times m$) matrix of real wage rates and let $\mathbf{L}$ be the ($m \times 1$) vector of employment levels. If the augmented matrix of inputs is irreducible, then we can safely write

$$r = r_w = \mathbf{pDS}/\mathbf{p(H + DV)D}L, \quad \mathbf{p} > \mathbf{0} \qquad (16)$$

where $\mathbf{V}$ denotes the ($m \times n$) matrix of labour values and $\mathbf{S}$ the ($m \times 1$) vector of surplus labours, whilst $\mathbf{DS}$ equals the surplus product of the WS.

It is quite clear that the WS ceases to be a quasi-one-commodity economy. Thus $\mathbf{S} \geq \mathbf{0}$ is sufficient but not necessary for $\mathbf{DS} \geq \mathbf{0}$ and therefore $\mathbf{S} \geq \mathbf{0}$ does not convert to a necessary condition for $r > 0$.[9]

Pure Joint Production

Consider an economy, which produces n commodities by n linear processes of joint production, i.e., a 'square', profitable and productive system of joint production defined by the triplet $[\mathbf{J}, \mathbf{A}, \mathbf{a}]$, where $\mathbf{J}$ is the ($n \times n$) output matrix. Moreover, in order to be in a position to compare this system directly with irreducible single-product systems, presume that we can safely write[10]

$$r = r_w = \mathbf{p}\mathbf{d}S^*/\mathbf{p}(\mathbf{H}^* + \mathbf{d}\mathbf{v}^*)\mathbf{d}L, \quad \mathbf{p} > \mathbf{0} \qquad (17)$$

where $\mathbf{H}^* (= \mathbf{A}(\mathbf{J} - \mathbf{A})^{-1})$ denotes the vertically integrated technical coefficients matrix, $\mathbf{v}^*$ $(= \mathbf{a}(\mathbf{J} - \mathbf{A})^{-1})$ the vector of *additive* values and S^* the total surplus labour (unpaid labour/surplus value) calculated in terms of the additive values, whilst $\mathbf{d}S^*$ equals the surplus product of the WS.[11]

Undoubtedly, positive surplus labour is necessary and sufficient for the (semi-) positiveness of $\mathbf{d}S^*$. However, it is neither necessary nor sufficient for the positiveness of the denominator in (17) and thus $S^* >(<) 0$ is compatible with negative (positive) profits. *By contrast*, when the WS can produce the exact amount of commodities received as wages, i.e., when $\mathbf{X_w} \geq 0$, positive surplus labour converts to a necessary and sufficient condition for $r > 0$, whether or not $\mathbf{v}^*$ contains non-positive elements.[12]

Ceteris paribus, now consider a 'rectangular' system in which m processes operate.[13] The intensity vector of the WS can be defined by

$$(\mathbf{J} - \mathbf{A})\mathbf{X_w} = \mathbf{d}L \tag{18}$$

where $\mathbf{X_w}$ is now an (m x 1) vector. When (18) is consistent, the general solution is

$$\mathbf{X_w} = (\mathbf{K}^-)\mathbf{d}L + [\mathbf{I_m} - (\mathbf{K}^-)\mathbf{K}]\mathbf{y} \tag{19}$$

where $\mathbf{K} = \mathbf{J} - \mathbf{A}$, $\mathbf{K}^-$ is a '(1)-inverse' of $\mathbf{K}$ (i.e., $\mathbf{K}(\mathbf{K}^-)\mathbf{K} = \mathbf{K}$), $\mathbf{I_m}$ is the (m x m) identity matrix and $\mathbf{y}$ is an *arbitrary* (m x 1) vector (see, e.g., Barnett (1990, pp. 260-73)). In that case the WS continues to be a quasi-one-commodity system, i.e.,

$$\mathbf{U_w} = \mathbf{d}(L - \mathbf{a}\mathbf{X_w}) = \mathbf{d}S \tag{20}$$

Consequently, when there is a semi-positive solution for $\mathbf{X_w}$, positive *surplus labour* is a necessary and sufficient condition for $\mathbf{U_w} \geq \mathbf{0}$ and hence for $r (= r_w) > 0$. However, when there is a non-positive solution for $\mathbf{X_w}$, profit and surplus labour can be of opposite sign. Finally, when (18) is inconsistent, we can write

$$\mathbf{h}L = \mathbf{K}\mathbf{X_w} - \mathbf{d}L \tag{21}$$

where $\mathbf{h}$ denotes the *residual*. In that case the WS ceases to be a quasi-one-commodity economy, i.e.,

$$\mathbf{U_w} = \mathbf{d}S + \mathbf{h}L \tag{22}$$

and therefore r and S can be of opposite sign.[14]

Until now we have considered the relationship(s) between profits and surplus labour. Nothing has been said about the determination of the labour values. One possible way forward would be to stipulate that they are given by (Fujimori (1982, p. 48))

$$\mathbf{v}\mathbf{K} = \mathbf{a} \tag{23}$$

which constitutes a 'dual' system of (18). When (23) is consistent, $\mathbf{v}$ represents a vector of additive values. In that case, first, surplus value, S_v, equals unpaid labour, S_u, and second, (21) implies that surplus value is given by

$$S_v (= S_u) = L - \mathbf{a}(\mathbf{K}^-)\mathbf{d}L = S + \mathbf{v}\mathbf{h}L \tag{24}$$

from which we note that, when (18) is consistent (i.e., $\mathbf{h} = \mathbf{0}$), S_u necessarily equals S. Thus it becomes clear that the traditional relationship between profits and 'exploitation' cannot be established unless the following special conditions hold: (i) the net output of the WS contains only the total real wages; and (ii) the value system is consistent. The fact that the former condition is necessary and sufficient (is necessary) for the validity of the said relationship in terms of surplus labour (in terms of unpaid labour/surplus value) indicates that Marx's theory of profits is based, *implicitly*, on the concept of the WS.

As is well known, Morishima (1974) has proposed an alternative approach in which negative value magnitudes are impossible, *by definition* (see also Morishima and Catephores (1978, ch. 2) and Fujimori (1982, ch. 4)). Within the framework of Morishima's approach, and in terms of our above notation and assumptions, the 'necessary intensity vector' is defined as an optimal solution to the following linear programming problem:

$$\text{Min } \{\mathbf{ax}|\ \mathbf{Jx} \geq \mathbf{Ax} + \mathbf{d}L,\ \mathbf{x} \geq \mathbf{0}\} \tag{25}$$

Denoting the solution to this problem by $\mathbf{x}^o$, the necessary labour is (re-) defined by $\mathbf{ax}^o$ and the surplus labour by $S^o = L - \mathbf{ax}^o$. Thus we get

$$(\mathbf{J} - \mathbf{B})\mathbf{x}^o \geq \mathbf{d}S^o \tag{26}$$

Pre-multiplying (26) by a (semi-) positive vector of market prices gives

$$\mathbf{kx}^o = \boldsymbol{\pi}\mathbf{BRx}^o \geq \boldsymbol{\pi}\mathbf{d}S^o \tag{27}$$

where $\mathbf{k}$ is now the ($1 \times m$) vector of profits per unit activity level, and $\mathbf{R}$ is now the ($m \times m$) diagonal matrix of the rates of profit. Thus we may state that $S^o > 0$ is sufficient for $\mathbf{kx}^o > 0$, provided that $\boldsymbol{\pi}\mathbf{d} > 0$. However, this statement is no more than a logical truth: (25) *defines* a WS (which we may call 'the Morishima WS') producing a net output, which is greater than or equal to the total real wages, and therefore $S^o > 0$ is sufficient (but not necessary) for the semi-positiveness of the surplus product, $(\mathbf{J} - \mathbf{B})\mathbf{x}^o$, of this WS.

Furthermore, the vector of labour values is defined as an optimal solution to the 'dual' problem of (25), i.e.,

$$\text{Max } \{\mathbf{vd}L|\ \mathbf{vJ} \leq \mathbf{vA} + \mathbf{a},\ \mathbf{v} \geq \mathbf{0}\} \tag{28}$$

The duality theorem of linear programming guarantees that $\mathbf{v^o}dL = \mathbf{ax^o}$, where $\mathbf{v^o}$ is the solution to (28), and hence unpaid labour (*à la* Morishima) equals S^o.[15] From (25), (28) and the quantity system of our economy, it follows that

$$\mathbf{v^o U} = \mathbf{v^o}(\mathbf{BGX} + \mathbf{C}) \leq S^o \qquad (29)$$

where $\mathbf{v^o U}$ represents the surplus value (*à la* Morishima) and $\mathbf{G}$ is now the ($m \times m$) diagonal matrix of the rates of growth.[16] Thus we may state that S^o is positive if the surplus product (or the net investment vector to be precise) is positive. However, this statement is no more than a logical truth in the sense that (28) *defines* a price vector, $\mathbf{v^o}$, which implies that the vector of profits per unit activity level is less than or equal to $\mathbf{a}(S^o/L)$ (compare to (8b)).

Finally, when a uniform rate of profit and a uniform rate of growth, g, are postulated, (27) and (29) imply

$$g\mathbf{v^o BX} + \mathbf{v^o C} \leq S^o \leq r(\mathbf{pBx^o/pd}) \qquad (30)$$

So we can conclude that $S^o > 0$ is not necessary for $r > 0$ unless at least one of the following two special conditions holds:[17] (i) $\mathbf{Kx^o} = \mathbf{d}L$. In that case (26) holds with equality or, equivalently, (18) has a semi-positive solution. In other words, the Morishima WS coincides with the 'actual' WS and thus $r > 0$ if and only if $S^o (= S) > 0$; or[18] (ii) $g = \gamma r$, where $0 < \gamma \leq 1$, i.e., a fraction of profits is saved. In that case $S^o > 0$ implies $r > 0$, by definition; $r > 0$ implies $g > 0$, by assumption; and $g > 0$ implies $S^o > 0$, by definition. Thus $S^o > 0$, $r > 0$ and $g > 0$ are all equivalent (this is the so-called 'Strong General Fundamental Marxian theorem'; Morishima (1980, 1989, pp. 88-92)).

Consequently, like Marx's, Morishima's approach is unable to identify the (newly defined) 'exploitation of workers' as the sole source of the *actual* profits.

CONCLUDING REMARKS

It has been shown that positive surplus labour, as defined by Marx, is both necessary and sufficient for the existence of positive profits if and only if, *first*, the economy is irreducible, *second*, there is a vertically integrated sector that (i) can produce the exact amount of commodities received as wages; and (ii) constitutes a quasi-one-commodity system, and *third*, commodity prices assure a uniform rate of profit in all sectors of the economy. Given that these conditions are very special, it follows that Marx's theory of

profits cannot be sustained. It has also been shown that Morishima's approach does not constitute an 'exploitation theory of profits' (in the sense of Marx).

Consequently, it must be said either that the 'exploitation of workers' is not the sole source of the actual profits or that surplus labour (unpaid labour/surplus value) provides no adequate measure of the 'exploitation of workers' in capitalist economies.

Notes

1. This section is partially borrowed from Mariolis (1998, 2000).
2. Matrices (and vectors) are denoted by boldface letters. If all elements of a matrix (or vector) $\mathbf{A}$ are greater than those of $\mathbf{B}$, we write $\mathbf{A} > \mathbf{B}$; if they are greater or equal, we write $\mathbf{A} \geq \mathbf{B}$; we write $\mathbf{A} \geq \mathbf{B}$, if $\mathbf{A} \geq \mathbf{B}$ and $\mathbf{A} \neq \mathbf{B}$.
3. When $\boldsymbol{\pi}\mathbf{E}^{\mathbf{j}}$ is positive (negative), the jth process is a lender (borrower).
4. *Ceteris paribus*, it cannot be excluded that an economy with excess capacity may reproduce itself on a higher scale. Changes in the elements of $\mathbf{B}$ may lead to the same result.
5. It goes without saying that (11a) expresses the validity of a modified (with respect to the net output) 'Marxian double equality'.
6. It goes without saying that (13) 'is both the most explicit form of determination of r and the best form for generalizing to more complex cases' (Steedman (1991, p. 208)).
7. Steedman (1977, p. 58) stresses: 'The very fact that [this proposition] 'runs both way' (r is positive *if and only if S* is positive) means at once that it does not constitute a theory of why r is positive. Any theory of *why profits are positive* will at the same time, be a theory of why surplus value is positive.' See also Steedman (1991, pp. 207-9).
8. When $f \geq \lambda[\mathbf{B}]$, we obtain $\mathbf{p} = \mathbf{0}$ and $p_{n+1} > 0$. This means that a uniform rate of profit is impossible.
9. The reader may consider the 2×2 numerical examples provided by Morishima (1978), where $\mathbf{U}$ and $\mathbf{S}$ contain one negative element, whilst $\mathbf{DS}$ equals zero (*ibid.*, p. 306) or it is positive (*ibid.*, p. 307), and by Krause (1981, p. 65), where $\mathbf{U}$, $\mathbf{S}$ and $\mathbf{DS}$ contain one negative element, whilst r is positive.
10. This set of presumptions (squareness, positiveness of commodity prices, non-singularity of $(\mathbf{J} - \mathbf{A})$, and $r = r_w$) is not innocent. See Kurz and Salvadori (1995, ch. 8) and Bidard (1997) for the particular features of joint production systems.
11. In other words, labour values are defined as the labour-commanded prices corresponding to zero profits. It is quite clear that $\mathbf{v}^*$ and/or $\mathbf{H}^*$ can contain one or more negative elements. However, this does not mean that the labour requirement, or the requirement for some capital stock, is negative for a commodity; it means only that the net output of the system cannot contain only that commodity (see Sraffa (1960, ch. 9) and Steedman (1975, pp. 118-20, 1977, pp. 165-9)). For Morishima's (1974) alternative definition of values, see below.
12. It is easily checked that the notorious 2×2 numerical examples provided by Steedman (1975) imply a negative denominator in (17) and $S^* < 0$, $r > 0$ (*ibid.*, p. 115) or $S^* > 0$, $r < 0$ (*ibid.*, p.117, n. 4; note that wages are paid *ex post* and thus the said denominator equals $\mathbf{pH}^*\mathbf{d}L$). The former case also appears in the attractive 3×3 numerical example provided by Hosoda (1993, pp. 37-9). Finally, it must be remarked that in Steedman's examples there is a case in which the said denominator equals zero, $S^* = 0$, $\mathbf{p} > \mathbf{0}$ and $r > 0$ (*ibid.*, pp.

117-8). Consequently, r_w is an indeterminate, in terms of the production prices, and thus (17) does not hold.

13. See Fujimori (1982, ch. 3) for a detailed algebraic analysis.

14. It may be noted that there is a difference between the case in which (18) has a non-positive solution and the case in which (18) is inconsistent. In the latter case $S = 0$ can be compatible with $r\ (=r_w) \neq 0$.

15. It is well known that Marx's labour values satisfy *additivity* and *actuality*. Morishima's values are neither additive nor actual ($\mathbf{v^o}$ is not necessarily related to the processes actually used in the economy considered). See Steedman (1976, 1977, ch. 13) for a thorough discussion.

16. According to Morishima and Catephores (1978, pp. 38-45), the ratio $S^o/\mathbf{ax^o}$ (i.e., the re-defined rate of surplus labour), which is equal to the re-defined rate of unpaid labour, $(L - \mathbf{v^o d}L)/\mathbf{v^o d}L$, is a complete measure of 'exploitation', whilst the ratio $\mathbf{v^o U}/\mathbf{v^o d}L$ (i.e., the re-defined rate of surplus value), which is not uniquely determined, is an imperfect one. Petri (1987, p. 68) notes: 'The rate of exploitation is then re-defined as $S^o/\mathbf{v^o d}L$ [using our notation]: a notion, it would seem, only interesting for purposes of comparison of reality with possible utopias ('how much less workers could afford to work if the social goal were the minimization of their working time, given their consumption').'

17. It should be noted that there is no inconsistency between this result and the so-called 'Generalized Fundamental Marxian Theorem' (Morishima (1974)), since the latter states that the *warranted* rate of profit (which cannot exceed r) and the *capacity* rate of growth (which is at least as high as g) are both positive if and only if $S^o > 0$.

18. It is easily checked that the numerical examples provided by Roemer (1980, pp. 520-1) and Petri (1980), with $m = 1$ and $n = 2$, imply that (18) is inconsistent, $\mathbf{U} = \mathbf{h}L \geq \mathbf{0}$, $g = 0$, $S^o = 0$ and $r > 0$.

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