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SUMMARY

This paper used Monte Carlo simulations to analyze the small sample properties of cross-sectionally augmented panel unit root test (CIPS test). We considered situations involving two types of time-series heteroskedasticity (unconditional and ARCH) in the unobserved common factor and idiosyncratic error term. We found that the CIPS test could be extremely robust versus the two types of heteroskedasticity in the unobserved common factor. However, we found under-size distortion in the case of unconditional heteroskedasticity in the idiosyncratic error term, and conversely, over-size distortion in the case of ARCH. Furthermore, we observed a tendency for its over-size distortion to moderate with low volatility persistence in the ARCH process and exaggerate with high volatility persistence.

Keywords: panel unit root test, CIPS test, heteroskedasticity, cross-section dependence

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1. INTRODUCTION

In recent years, research on the panel unit root test methodology has focused on how to consider correlation between cross sections. Representative examples of initial work are the pooled t-test (LLC test) proposed by Levin *et al.* (2002), the averaged t-test (IPS test) developed by Im *et al.* (2003), and the combination test developed by Maddala and Wu (1999), all of which are referred to as first-generation panel unit root tests. Under the strong assumption of cross-section independence, these methodologies proved more powerful than the unit root test applied to univariate time-series data. The data, however, often do not satisfy the assumption of cross-section independence; cross-country (or regional) data used in testing, for example, the purchasing power parity hypothesis, income convergence hypothesis, and current account stability are indicative. In addition, prior research has clearly shown that application of the panel unit root test, despite the existence of cross-section dependence in the data, results in serious size distortions (O'Connell, 1998; Strauss and Yigit, 2003; Pesaran, 2007). Many researchers have proposed "second-generation panel unit root tests," which consider correlation between cross sections, to overcome this problem.[1]

One such second-generation panel unit root test is the cross-sectionally augmented IPS (CIPS) test proposed by Pesaran (2007). Pesaran introduced an unobserved single common factor to the regression equation used for testing in order to explicitly consider correlation between data cross sections. Other researchers, including Bai and Ng (2004), Moon and Perron (2004), and Phillips and Sul (2003), have also proposed panel unit root tests applying a common factor model, but Pesaran's methodology is exceptional

[1] For more details, refer to Choi (2006) and Breitung and Pesaran (2008). These papers are comprehensive surveys of nonstationary panel analysis, including panel unit root tests.

for its simplicity and clarity. Whereas other approaches require the use of principal component analysis to estimate factor loadings for common factors, Pesaran's methodology instead introduces an appropriate proxy for the common factor and uses OLS estimation, making it easier to apply.

For this paper, we tested the small sample properties of Pesaran's CIPS test. More specifically, we used Monte Carlo simulation to test the degree of distortion in the CIPS test size when there is time-series unconditional and conditional heteroskedasticity in the unobserved common factor and idiosyncratic error term. Tests of the small sample performance of the CIPS test have already been performed by De Silva *et al.* (2009), Cerasa (2008), and Pesaran (2007) as well, but none of these tests considered cases in which there is time-series heteroskedasticity. Many papers have analyzed the impacts of heteroskedasticity on the Dickey-Fuller test and other tests for a unit root in a univariate series,[2] but there have been few, if any, efforts to examine the impacts of heteroskedasticity on panel unit root tests, let alone the CIPS test in particular. Since there are many economic variables with time-variant distributions, like equity prices and exchange rates, it is only natural to presume the existence of time-series heteroskedasticity. It is extremely important, therefore, that its impact on the panel unit root test be quantitatively evaluated. The rest of this paper is organized as follows: Section 2 explains the panel unit root test proposed by Pesaran (2007); Section 3 presents the design and results of our Monte Carlo simulations; and Section 4 concludes the paper.

[2] For example, refer to Kim and Schmidt (1993), Hamori and Tokihisa (1997), and Sjölander (2008).

2. PANEL UNIT ROOT TEST OF PESARAN (2007)

Let us consider y_{it} generated by a heterogeneous panel autoregressive process as follows:

$$y_{it} = (1-\phi_i)\mu_i + \phi_i y_{i,t-1} + u_{it}, \quad i = 1,2,\ldots,N; t = 1,2,\ldots T, \tag{1}$$

where i and t denote a cross section unit and time, μ_i and ϕ_i are parameters, and u_{it} is an error term that has a single common factor structure:

$$u_{it} = \gamma_i f_t + \varepsilon_{it}, \tag{2}$$

or, in the vector notation,

$$\mathbf{u}_t = \boldsymbol{\gamma} f_t + \boldsymbol{\varepsilon}_t, \tag{3}$$

in which $\mathbf{u}_t = (u_{1t}, u_{2t}, \ldots, u_{Nt})'$, f_t is an unobserved common factor, $\boldsymbol{\varepsilon}_t$ is an $N \times 1$ vector of idiosyncratic shocks ε_{it}, and $\boldsymbol{\gamma}$ is an $N \times 1$ vector of parameters (factor loadings), which are assumed to be $N^{-1}\sum_{i=1}^{N}\gamma_i \neq 0$. It is assumed that ε_{it} is independently distributed across both i and t with $\mathrm{E}(\varepsilon_{it}) = 0$ and $\mathrm{E}({\varepsilon^2}_{it}) = \sigma_i^2$; f_t is serially uncorrelated with $\mathrm{E}(f_t) = 0$ and $\mathrm{E}(f_t^2) = 1$; and γ_i, f_t, and ε_{it} are independent of each other. Under these assumptions, the covariance matrix of $\mathbf{u}_t$ is given by

$\mathrm{E}(\mathbf{u}_t \mathbf{u}_t') = \mathbf{\Gamma} + \mathbf{\Omega}$ where

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_1^2 & \gamma_1\gamma_2 & \cdots & \gamma_1\gamma_N \\ \gamma_2\gamma_1 & \gamma_2^2 & \cdots & \gamma_2\gamma_N \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_N\gamma_1 & \gamma_N\gamma_2 & \cdots & \gamma_N^2 \end{bmatrix}, \quad \mathbf{\Omega} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_N^2 \end{bmatrix}, \tag{4}$$

and hence, it is clear that $\mathbf{u}_t$ is contemporaneously correlated due to the existence of off-diagonal elements in $\mathbf{\Gamma}$.

Equation (1) can be rewritten as

$$\Delta y_{it} = \alpha_i + \beta_i y_{i,t-1} + \gamma_i f_t + \varepsilon_{it}, \tag{5}$$

where $\Delta y_{it} = y_{it} - y_{i,t-1}$, $\alpha_i = (1-\phi_i)\mu_i$, and $\beta_i = \phi_i - 1$. The null hypothesis of interest is

$$\mathrm{H}_0 : \beta_i = 0, \quad \textit{for all } i, \tag{6}$$

and the alternative is

$$\mathrm{H}_1 : \beta_i < 0, i = 1, 2, \ldots, N_1; \quad \beta_i = 0, i = N_1 + 1, N_1 + 2, \ldots, N. \tag{7}$$

According to Pesaran (2006), the cross-section mean of Δy_{it} and $y_{i,t-1}$ can be used as the proxies for the unobserved common factor f_t. Pesaran (2007) exploits the results of his study to derive the test statistics for the hypothesis, and proposes the cross-sectionally augmented Dickey-Fuller (CADF) regression model:

$$\Delta y_{it} = a_i + b_i\, y_{i,t-1} + c_i\, \overline{y}_{t-1} + d_i\, \Delta\overline{y}_t + e_{it}\ , \qquad (8)$$

where $\overline{y}_{t-1} = N^{-1}\sum_{i=1}^{N} y_{i,t-1}$ and $\Delta\overline{y}_t = N^{-1}\sum_{i=1}^{N}\Delta y_{it}$. The *t*-ratio of the OLS estimate of b_i in Equation (8), defined by $t_i(N,T)$, is referred to as a CADF statistic for i, and the average of its *t*-ratio

$$CIPS(N,T) = N^{-1}\sum_{i=1}^{N} t_i(N,T) \qquad (9)$$

yields the panel unit root test statistic. $CIPS(N,T)$ is a cross-sectionally augmented version of the test statistic proposed by Im *et al*. (2003) and is referred to as a CIPS statistic.

While the deterministic term of Equation (8) is the intercept only, it can be easily extended to the model including the linear time trend:

$$\Delta y_{it} = a_i + \delta_i t + b_i\, y_{i,t-1} + c_i\, \overline{y}_{t-1} + d_i\, \Delta\overline{y}_t + e_{it}\ . \qquad (10)$$

Although the distributions of both CADF and CIPS statistics are non-standard, the critical values in the instances of both the intercept only and the linear time trend are tabulated by Pesaran (2007).[3]

[3] It is possible to extend the CADF and CIPS statistics to the case in which both ε_{it} and f_t are serially correlated. For details, refer to Pesaran (2007, pp. 279-282).

3. MONTE CARLO SIMULATION

In the panel unit root test proposed by Pesaran (2007), the variance of both the common factor f_t and the idiosyncratic term ε_{it} is assumed to be time invariant. We consider the case in the presence of time-series conditional and unconditional heteroskedasticity in f_t and ε_{it}, and investigate the extent to which such heteroskedasticity influences the size of CIPS statistics using Monte Carlo simulations. The following subsections explain the data generating process (DGP) for our investigation and show the simulation results.

3.1. Design of Monte Carlo Simulation

Under the null hypothesis of unit root, Equation (5) is rewritten as

$$y_{it} = y_{i,t-1} + \gamma_i f_t + \varepsilon_{it} . \tag{11}$$

Based on the above equation, the DGP for our investigation is as follows. First, we consider the DGP where the variance of ε_{it} is time invariant, but f_t has the following two types of time-series heteroskedasticity:

$$\text{DGP}_f 1 \qquad \begin{aligned} & f_t \sim \begin{cases} N(0,1) & \text{if } t = 1,2,\ldots T/2 \\ N(0,10) & \text{if } t = (T/2)+1,\ldots,T \end{cases} \\ & \varepsilon_{it} \sim N(0,\sigma_i^2), \quad \sigma_i^2 \sim U[0.5,1.5] , \end{aligned} \tag{12}$$

and

$$\mathrm{DGP}_f\,2 \qquad \begin{aligned} & f_t = \sqrt{h_t}\,\eta_t, \quad \eta_t \sim N(0,1) \\ & h_t = \alpha_0 + \alpha_1 f_{t-1}^2 \\ & \varepsilon_{it} \sim N(0,\sigma_i^2), \quad \sigma_i^2 \sim U[0.5, 1.5], \end{aligned} \tag{13}$$

where $N(\,)$ and $U(\,)$ denote Normal and Uniform distributions, respectively. $\mathrm{DGP}_f\,1$ indicates that the variance of f_t changes in the middle point of the sample period, implying that f_t has unconditional heteroskedasticity. $\mathrm{DGP}_f\,2$ is the case that f_t follows the autoregressive conditional heteroskedasticity (ARCH) with order one: ARCH(1). The parameters α_0 and α_1 will be specified later.

Next, we consider the reverse situation where the variance of f_t is time invariant, but ε_{it} has time-series heteroskedasticity, specifying as follows:

$$\mathrm{DGP}_\varepsilon\,1 \qquad \begin{aligned} & \varepsilon_{it} \sim \begin{cases} N(0,\sigma_{i1}^2) & \text{if } t = 1,2,\ldots T/2 \\ N(0,\sigma_{i2}^2) & \text{if } t = (T/2)+1,\ldots,T \end{cases} \\ & \sigma_{i1}^2 \sim U[0.3, 0.5], \quad \sigma_{i2}^2 \sim U[3, 5] \\ & f_t \sim N(0,1), \end{aligned} \tag{14}$$

and

$$\mathrm{DGP}_\varepsilon\,2 \qquad \varepsilon_{it} = \sqrt{h_{it}}\,\eta_{it}, \quad \eta_{it} \sim N(0,1) \tag{15}$$

$$h_{it} = \alpha_{0i} + \alpha_{1i}\varepsilon_{i,t-1}^2$$

$$f_t \sim N(0,1)\,.$$

$\mathrm{DGP}_\varepsilon 1$ and $\mathrm{DGP}_\varepsilon 2$ represent unconditional heteroskedasticity in ε_{it} and ARCH(1) process in ε_{it}, respectively.

The remaining parameters we do not specify are α_0, α_1, α_{0i}, and α_{1i} in the ARCH(1) models, and γ_i in Equation (11). As for the parameters in the ARCH models, we adopt the results of the study for single time series unit root tests. According to those studies, the Dickey-Fuller test tends to show the over-size distortion when the error term has conditional heteroskedasticity. Although the degree of the distortion is not so serious in finite sample size, it has a tendency to increase as the volatility persistence parameters (α_1 and α_{i1}) enlarge and come close to 1, and the intercepts (α_0 and α_{i0}) come close to 0 (Kim and Schmidt, 1993; Sjölander, 2008).[4] On the basis of these observations, we consider two cases of low and high volatility persistence,

$$\begin{aligned} &\alpha_0 = 0.5, && \alpha_1 = \begin{cases} 0.1 & \text{Low persistence} \\ 0.9 & \text{High persistence} \end{cases} \\ &\alpha_{0i} \sim U[0.3, 0.5], && \alpha_{1i} \sim \begin{cases} U[0.05, 0.25] & \text{Low persistence} \\ U[0.75, 0.95] & \text{High persistence} \end{cases}, \end{aligned} \tag{16}$$

and examine whether, as in the case of the Dickey-Fuller test, the size distortion of CIPS test is more serious in high persistence case than in low persistence case.

Finally, we specify the parameter γ_i, which represents the degree of cross-section

4 Note that Kim and Schmidt (1993) and Sjölander (2008) exploit GARCH(1) model.

dependence. Following Pesaran (2007), we consider the low and high cross-section dependence as follows:

$$\gamma_i \sim \begin{cases} U[0, 0.2] & \text{Low cross-section dependence} \\ U[-1, 3] & \text{High cross-section dependence.} \end{cases} \tag{17}$$

Combining these parameter settings in Equations (16) and (17) and the DGPs in Equations (12) through (15), 12 different DGPs are obtained for our Monte Carlo simulations, which are summarized in Table I. On the basis of the DGPs, we compute the empirical size (one-sided lower probability) of CIPS test at the critical values of the 5% nominal level, which are proposed by Pesaran (2007), and investigate the extent of the difference between the empirical size and the nominal size. The computation procedure for the empirical size is as follows: [5]

(i) Generate y_{it} for $i = 1, 2, \ldots, N$ and $t = -50, 49, \ldots, 1, 2, \ldots, T$ from $y_{i,-50} = 0$, under the DGPs in Table I. The initial values of f_t in Equation (13) and ε_{it} in Equation (15) are $f_{-50} = 0$ and $\varepsilon_{i,-50} = 0$ for all i, respectively.

(ii) Calculate CADF statistic for each i, using y_{it} for $t = 1, 2, \ldots, T$ generated in (i) and the CADF regression models with the intercept only (Equation (8)) and with the linear time trend (Equation (10)).

(iii) Calculate CIPS statistic for both the intercept case and the linear time trend case, using the CADF statistics obtained in (ii).

Replicate (i) to (iii) 50,000 times.

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[5] All computations are implemented using *Ox* version 4.10 (Doornik, 2006).

(iv) Calculate the empirical size at the critical values of the 5% nominal level, using 50,000 CIPS statistics.

For the values of N and T, we choose $N = \{10, 20, 30, 50, 100\}$ and $T = \{20, 30, 50, 100, 200\}$.

3.2. Simulation Results

Tables II through IV present the simulation results in the case where f_t has heteroskedasticity. Table II shows the case of the unconditional heteroskedasticity, and Tables III and IV indicate the case of ARCH(1) with low and high volatility persistence, respectively. As these tables suggest, irrespective of the difference in type of heteroskedasticity and in the parameter settings, the computed values of the empirical size are almost the same as the 5% nominal size. This finding implies that Pesaran's (2007) CIPS test is substantially robust for the presence of conditional and unconditional heteroskedasticity in f_t.

Tables V through VII report the results in the case where ε_{it} has heteroskedasticity. In contrast to the data where f_t has heteroskedasticity, size distortions are observed. As Table V shows, CIPS test suffers from the problem of under-size distortions when the unconditional heteroskedasticity exists in ε_{it}. The degree of the distortion tends to be large in the case of the linear time trend model and the high cross-section dependence, and it has a tendency to increase as N enlarges. According to Tables VI and VII, the presence of ARCH(1) in ε_{it} leads to the problem of over-size distortions. Interestingly, the direction of the size distortion is opposite to that of the unconditional heteroskedasticity. Since the values of the empirical size in the low persistence case (Table VI) are around 0.06, the degree of the distortions is not too large. However, in the high persistence case (Table VII), the minimum value of the empirical size is 0.114, and the maximum value is 0.273. Hence, it is clear that, in finite samples, the size distortions of CIPS test due to the presence of ARCH(1) in ε_{it} are more serious in the high persistence case than in the low persistence case. This property of CIPS test is similar to that observed with the Dickey-Fuller test (Kim and Schmidt, 1993; Sjölander, 2008). Table VII indicates that, as N enlarges, the degree of the distortion increases in the range of $T = 20$ to $T = 50 \text{ or } 100$, and that its degree slightly decreases in more than $T = 50 \text{ or } 100$ at arbitrary sample size N.

4. CONCLUSION

This paper used Monte Carlo simulations to analyze the small sample properties of the panel unit root test (CIPS test) proposed by Pesaran (2007). While there are several previous examples of work examining the small sample properties of the CIPS test, this paper is unique for its analysis of the impacts of time-series heteroskedasticity on CIPS test size. Numerous papers have analyzed the impacts of heteroskedasticity on the Dickey-Fuller test and other tests for a unit root in a univariate series, but there has been very little, if any, work done to perform the same analysis on panel unit root tests, including the CIPS test. Analyzing the impact of heteroskedasticity on panel unit root tests is extremely important, given the large number of economic variables with time-variant distributions, like equity prices and exchange rates.

For this paper, we considered two types of heteroskedasticity, unconditional heteroskedasticity and ARCH, as characteristics of the unobserved common factor f_t and the idiosyncratic error term ε_{it}, and we analyzed the impacts on CIPS test size. For ARCH, we examined cases of high and low volatility persistence separately. Our results showed that when there is heteroskedasticity only in f_t, there was almost no CIPS test size distortion, regardless of heteroskedasticity types (unconditional or conditional) and degree of volatility persistence. The CIPS test, therefore, could be extremely robust versus heteroskedasticity in the unobserved common factor.

In contrast, when there is heteroskedasticity only in ε_{it}, our analysis found distortion in the CIPS test size. Importantly, we found under-size distortion in the case of unconditional heteroskedasticity and, conversely, over-size distortion in the case of ARCH. Furthermore, we observed a tendency for over-size distortion to moderate with low volatility persistence in the ARCH process and exaggerate with high volatility persistence. It follows, then, that the problem of under-rejection of the null hypothesis emerges when there is unconditional heteroskedasticity in ε_{it}—for example, in the form of a distribution shift due to a structural change—and serious over-rejection emerges when ε_{it} is characterized by an ARCH process with high volatility persistence.

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Table I. Data generating process

Heteroskedasticity in f_t	Results
1. $\mathrm{DGP}_f 1$ with low CD.	Table II
2. $\mathrm{DGP}_f 1$ with high CD.	
3. $\mathrm{DGP}_f 2$ with low CD and low VP.	Table III
4. $\mathrm{DGP}_f 2$ with high CD and low VP.	
5. $\mathrm{DGP}_f 2$ with low CD and high VP.	Table IV
6. $\mathrm{DGP}_f 2$ with high CD and high VP.	
Heteroskedasticity in ε_{it}	**Results**
7. $\mathrm{DGP}_\varepsilon 1$ with low CD.	Table V
8. $\mathrm{DGP}_\varepsilon 1$ with high CD.	
9. $\mathrm{DGP}_\varepsilon 2$ with low CD and low VP.	Table VI
10. $\mathrm{DGP}_\varepsilon 2$ with high CD and low VP.	
11. $\mathrm{DGP}_\varepsilon 2$ with low CD and high VP.	Table VII
12. $\mathrm{DGP}_\varepsilon 2$ with high CD and high VP.	

Note: The $\mathrm{DGP}_f 1$, $\mathrm{DGP}_f 2$, $\mathrm{DGP}_\varepsilon 1$, and $\mathrm{DGP}_\varepsilon 2$ are defined in Equations (12) through (15). CD and VP designate the cross-section dependence (CD) and the volatility persistence (VP).

Table II. Empirical size of CIPS tests with unconditional heteroskedasticity in f_t

	Low cross-section dependence					High cross-section dependence				
	T = 20	T = 30	T = 50	T = 100	T = 200	T = 20	T = 30	T = 50	T = 100	T = 200
Intercept case										
N = 10	0.051	0.050	0.051	0.049	0.051	0.055	0.055	0.055	0.056	0.056
N = 20	0.052	0.050	0.046	0.049	0.048	0.050	0.049	0.049	0.050	0.049
N = 30	0.048	0.049	0.046	0.047	0.050	0.048	0.048	0.047	0.046	0.047
N = 50	0.048	0.047	0.050	0.047	0.048	0.050	0.050	0.051	0.049	0.050
N = 100	0.047	0.044	0.044	0.044	0.041	0.045	0.044	0.041	0.040	0.037
Linear time trend case										
N = 10	0.053	0.050	0.052	0.050	0.050	0.059	0.058	0.058	0.059	0.057
N = 20	0.053	0.048	0.051	0.051	0.049	0.056	0.053	0.056	0.056	0.054
N = 30	0.048	0.053	0.050	0.050	0.050	0.054	0.053	0.055	0.054	0.051
N = 50	0.053	0.049	0.051	0.052	0.049	0.061	0.060	0.059	0.057	0.061
N = 100	0.053	0.052	0.054	0.049	0.051	0.057	0.055	0.055	0.052	0.053

Note: The empirical size (one-sided lower probability) of CIPS test is computed at the critical values of the 0.05 (5%) nominal size, which are proposed by Pesaran (2007).

Table III. Empirical size of CIPS tests with ARCH(1) in f_t: low volatility persistence

	Low cross-section dependence					High cross-section dependence				
	T = 20	T = 30	T = 50	T = 100	T = 200	T = 20	T = 30	T = 50	T = 100	T = 200
Intercept case										
N = 10	0.048	0.049	0.053	0.051	0.050	0.055	0.053	0.054	0.056	0.056
N = 20	0.051	0.049	0.048	0.052	0.048	0.053	0.052	0.051	0.052	0.052
N = 30	0.050	0.050	0.049	0.048	0.052	0.050	0.050	0.050	0.050	0.051
N = 50	0.048	0.049	0.051	0.051	0.054	0.053	0.054	0.054	0.056	0.055
N = 100	0.050	0.051	0.049	0.051	0.049	0.052	0.051	0.048	0.050	0.049
Linear time trend case										
N = 10	0.052	0.051	0.050	0.051	0.050	0.056	0.053	0.055	0.056	0.056
N = 20	0.051	0.049	0.050	0.053	0.048	0.057	0.051	0.056	0.055	0.053
N = 30	0.048	0.050	0.048	0.050	0.050	0.050	0.051	0.050	0.050	0.052
N = 50	0.050	0.047	0.050	0.050	0.049	0.056	0.057	0.057	0.058	0.057
N = 100	0.052	0.052	0.050	0.049	0.051	0.054	0.051	0.052	0.050	0.049

Note: The empirical size (one-sided lower probability) of CIPS test is computed at the critical values of the 0.05 (5%) nominal size, which are proposed by Pesaran (2007).

Table IV. Empirical size of CIPS tests with ARCH(1) in f_t : high volatility persistence

	Low cross-section dependence					High cross-section dependence				
	T = 20	T = 30	T = 50	T = 100	T = 200	T = 20	T = 30	T = 50	T = 100	T = 200
Intercept case										
N = 10	0.052	0.050	0.052	0.050	0.050	0.055	0.056	0.057	0.057	0.057
N = 20	0.050	0.049	0.048	0.049	0.048	0.054	0.053	0.051	0.053	0.052
N = 30	0.050	0.049	0.050	0.047	0.052	0.050	0.050	0.048	0.046	0.050
N = 50	0.049	0.049	0.050	0.050	0.050	0.055	0.054	0.055	0.055	0.057
N = 100	0.052	0.047	0.047	0.049	0.045	0.051	0.047	0.045	0.047	0.044
Linear time trend case										
N = 10	0.052	0.047	0.049	0.052	0.051	0.058	0.056	0.057	0.059	0.057
N = 20	0.054	0.048	0.051	0.052	0.048	0.056	0.051	0.053	0.055	0.054
N = 30	0.049	0.052	0.049	0.048	0.049	0.051	0.052	0.051	0.049	0.049
N = 50	0.050	0.049	0.049	0.049	0.051	0.059	0.057	0.056	0.055	0.058
N = 100	0.053	0.049	0.049	0.050	0.049	0.055	0.052	0.048	0.046	0.046

Note: The empirical size (one-sided lower probability) of CIPS test is computed at the critical values of the 0.05 (5%) nominal size, which are proposed by Pesaran (2007).

Table V. Empirical size of CIPS tests with unconditional heteroskedasticity in ε_{it}

	Low cross-section dependence					High cross-section dependence				
	T = 20	T = 30	T = 50	T = 100	T = 200	T = 20	T = 30	T = 50	T = 100	T = 200
<u>Intercept case</u>										
N = 10	0.056	0.054	0.054	0.053	0.053	0.036	0.035	0.036	0.036	0.035
N = 20	0.061	0.056	0.055	0.052	0.050	0.029	0.029	0.028	0.026	0.027
N = 30	0.060	0.053	0.052	0.047	0.049	0.027	0.026	0.025	0.024	0.026
N = 50	0.057	0.052	0.049	0.048	0.048	0.027	0.028	0.028	0.027	0.028
N = 100	0.053	0.046	0.044	0.042	0.041	0.026	0.025	0.024	0.024	0.022
<u>Linear time trend case</u>										
N = 10	0.046	0.045	0.045	0.045	0.043	0.033	0.032	0.034	0.034	0.034
N = 20	0.040	0.035	0.034	0.034	0.033	0.022	0.020	0.022	0.022	0.021
N = 30	0.032	0.030	0.028	0.027	0.027	0.017	0.017	0.015	0.016	0.015
N = 50	0.026	0.020	0.020	0.019	0.019	0.016	0.013	0.013	0.013	0.014
N = 100	0.017	0.012	0.012	0.010	0.010	0.008	0.007	0.005	0.006	0.005

Note: The empirical size (one-sided lower probability) of CIPS test is computed at the critical values of the 0.05 (5%) nominal size, which are proposed by Pesaran (2007).

Table VI. Empirical size of CIPS tests with ARCH(1) in ε_{it} : low volatility persistence

	Low cross-section dependence					High cross-section dependence				
	T = 20	T = 30	T = 50	T = 100	T = 200	T = 20	T = 30	T = 50	T = 100	T = 200
Intercept case										
N = 10	0.059	0.060	0.060	0.056	0.052	0.068	0.063	0.064	0.061	0.058
N = 20	0.065	0.060	0.058	0.055	0.052	0.065	0.062	0.059	0.059	0.055
N = 30	0.064	0.062	0.055	0.052	0.054	0.064	0.063	0.058	0.056	0.055
N = 50	0.062	0.061	0.058	0.055	0.055	0.065	0.066	0.066	0.063	0.061
N = 100	0.066	0.063	0.060	0.057	0.051	0.066	0.063	0.059	0.055	0.053
Linear time trend case										
N = 10	0.062	0.061	0.060	0.057	0.054	0.068	0.067	0.065	0.063	0.061
N = 20	0.064	0.063	0.062	0.060	0.052	0.068	0.064	0.067	0.064	0.055
N = 30	0.063	0.064	0.061	0.057	0.054	0.063	0.066	0.062	0.058	0.056
N = 50	0.063	0.060	0.061	0.056	0.054	0.070	0.068	0.068	0.065	0.061
N = 100	0.064	0.065	0.065	0.058	0.055	0.063	0.065	0.064	0.059	0.056

Note: The empirical size (one-sided lower probability) of CIPS test is computed at the critical values of the 0.05 (5%) nominal size, which are proposed by Pesaran (2007).

Table VII. Empirical size of CIPS tests with ARCH(1) in ε_{it} : high volatility persistence

	Low cross-section dependence					High cross-section dependence				
	T = 20	T = 30	T = 50	T = 100	T = 200	T = 20	T = 30	T = 50	T = 100	T = 200
Intercept case										
N = 10	0.124	0.126	0.132	0.122	0.114	0.132	0.132	0.138	0.132	0.127
N = 20	0.145	0.151	0.150	0.146	0.135	0.159	0.159	0.158	0.159	0.148
N = 30	0.153	0.159	0.158	0.151	0.147	0.162	0.166	0.162	0.159	0.151
N = 50	0.166	0.171	0.175	0.172	0.158	0.174	0.177	0.183	0.178	0.164
N = 100	0.186	0.185	0.183	0.180	0.163	0.182	0.183	0.185	0.181	0.162
Linear time trend case										
N = 10	0.123	0.135	0.143	0.147	0.141	0.135	0.140	0.152	0.157	0.155
N = 20	0.150	0.164	0.184	0.192	0.183	0.164	0.172	0.193	0.203	0.197
N = 30	0.152	0.184	0.201	0.207	0.203	0.161	0.189	0.203	0.212	0.212
N = 50	0.171	0.195	0.222	0.238	0.232	0.185	0.204	0.228	0.242	0.238
N = 100	0.184	0.222	0.252	0.273	0.259	0.184	0.220	0.250	0.265	0.261

Note: The empirical size (one-sided lower probability) of CIPS test is computed at the critical values of the 0.05 (5%) nominal size, which are proposed by Pesaran (2007).