International parity relations between Poland and Germany: a cointegrated VAR approach

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A Cointegrated VAR Approach[*]

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Abstract

This paper analyses empirically the purchasing power parity, the uncovered interest parity and the real interest parity (Fisher parity) between Poland and Germany. The international parity relations are investigated jointly within the cointegrated VAR framework. Our analysis fails to find evidence that the parities, or any linear combinations of them, hold for our data set. We identify two long-run equilibrium relations: one imposing a long-run homogeneity restriction on the domestic (i.e. Polish) and foreign (i.e. German) inflation and the domestic interest rate and one that brings together the domestic real interest rate and the foreign inflation. Another interesting result is the weak exogeneity of the deviation of the real exchange rate from the PPP and the strong exogeneity of the German interest rate.

Keywords: cointegrated VAR, PPP, UIP, Fisher parity, Poland

JEL: E32, E43, F31

1   Introduction

With its accession to the European Union (EU) on 1 May 2004, Poland took on a commitment to join the Economic and Monetary Union (EMU) upon fulfilling the convergence criteria set in the Maastricht treaty. The timing of the EMU accession depends to a large extent on the country's economic policy decisions, which affect the level and stability of prices, long-term interest rates, the fiscal position and the nominal exchange rate. However, the adoption of the euro is inevitable, as none of the ten countries that became EU members in 2004 has the formal right, as that exercised by Denmark and the United Kingdom, to opt out from EMU arrangements.

For a candidate country with a flexible nominal exchange rate regime, as it is the case with Poland currently, joining the euro implies giving up monetary policy independence. The question arises, then, whether the economy is "ripe" for the common monetary policy. This problem has usually been analysed from the point of view of the optimum currency area theory (see Mundell 1961; McKinnon 1963; Kenen 1969), which weighs the benefits of the accession to a monetary union (increased microeconomic efficiency) against its costs (potentially more painful adjustment to asymmetric shocks). A number of empirical studies in this area concentrate on the symmetry of shocks and shock transmission mechanisms in a given country and its potential partners in the monetary union (see De Grauwe 2003 for a survey).

This paper asks a similar question – to what extent Poland has already achieved a sufficient degree of convergence with the current euro area members – but applies a different perspective, namely that of international parity relations: the purchasing power parity, the uncovered interest parity and the real interest parity. The basic logic behind this approach is that the three parities between two economies hold if goods and asset markets of these economies are perfectly integrated, i.e. when goods and capital are perfectly mobile. If this is the case, the economies in question can form a common currency area without fearing serious turbulence in case of asymmetric shocks; indeed, the probability of such shocks is very low under such conditions.

There is vast empirical literature on the parity conditions that we are analysing but usually each of them is treated separately, whereas in our paper they are modelled jointly within the cointegrated vector autoregressive (VAR) framework. This joint modelling approach is originally due to Juselius and MacDonald (2004a), who scrutinised the parity relations between Germany and the US. Essentially, the analysis in this paper is an application of their approach to the Polish data – to the best of our knowledge, the first such one.

Thus, we analyse empirically to what extent Poland's macroeconomic aggregates of interest are interrelated with those of the current EMU countries. The most important empirical questions are the following: do the international parity relations postulated by economic theory

hold for Poland relative to the euro area? What are the common stochastic trends driving inflation, interest rates and the real exchange rate against the EMU? Do the developments in Poland significantly affect those in the common currency area, or can the latter be treated as exogenously given when analysing the Polish economy? The EMU is represented by Germany, the largest of its members and a neighbour of Poland, which makes it the most natural reference economy for studying Poland's international trade and payments relations with the euro area.[1]

The remainder of this paper is structured as follows. The next section presents the three international parity relations, briefly reviews the relevant literature, and derives hypotheses that can be tested within the cointegrated VAR framework. Section 3 visually inspects the data used in the VAR model, which is presented in Section 4. Section 5 reports the outcome of the cointegration analysis. Section 6 summarises the main findings and concludes.

2   International parity conditions[2]

The purchasing power parity (PPP) is one of the most extensively studied relationships in the international economics. In its strong form it can be written as follows:

$$ppp_t = p_t + p_t^* - s_t \qquad (1)$$

where ppp_t is the deviation from PPP (alternatively, the real exchange rate multiplied by -1), p_t and p_t^* are, respectively, domestic and the foreign price levels, and s_t is the spot exchange rate (in price notation, i.e. the price of foreign currency in units of domestic currency). All lowercase variables in this paper, except for the bond yields or interest rates, are in logs so that their first differences can be interpreted as the rates of change in the underlying variable. Empirically, the PPP condition is verified if ppp_t is a stationary process.

The second important relationship is the uncovered interest parity (UIP):

$$E_t\left(\Delta s_{t+m}\right) = i_t^m - i_t^{m*} \qquad (2)$$

where E_t denotes the expected value given the information set available at time t, Δ is the difference operator and i_t^m and i_t^{m*} are, respectively, the domestic and the foreign bond yields with maturity m.[3] Thus, the UIP postulates that the expected rate of denomination of the domestic currency should be equal to the home vs. foreign interest spread (the terms "interest rates", "bond

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[1] Admittedly, interest rates and exchange rates have been heavily influenced by financial flows, where the German mark was not always the dominant currency. Nevertheless, of all EMU countries Germany seems to be the best *single* reference country due to its economic size and geographic proximity to Poland. For similar reasons, Germany was treated as a natural anchor country in virtually every article written in the 1990s on the optimality (or simply viability) of the future monetary union in EC/EU countries (see, e.g. Bayoumi, Eichengreen 1992a; 1992b and the vast literature that was pioneered by these papers).

[2] The beginning of this section draws heavily on Juselius, MacDonald (2004a), Section 2.

[3] Note that UIP may apply to short or to long bond yields; see Juselius, MacDonald (2004a) for a discussion.

yields", and "Treasury bill rates" are used interchangeably in this paper). Assuming rational expectations, we have:

$$\Delta s_{t+m} = E_t\left(\Delta s_{t+m}\right) + \varepsilon_{t+m} \tag{3}$$

where ε_t is a white noise error term. Combining (2) and (3) leads to:

$$\Delta s_{t+m} - \left(i_t^m - i_t^{m*}\right) = \varepsilon_{t+m} \tag{4}$$

Under the assumption of rational expectations, testing the UIP amounts to testing whether ε_t in (4) is stationary. The third parity relation that we are interested in is the real interest parity (RIP):

$$r_t^m = r_t^{m*} \tag{5}$$

or rather its testable version:

$$r_t^m - r_t^{m*} = v_t \tag{6}$$

where r_t^m and r_t^{m*} are the domestic and the foreign real bond yields with maturity m, respectively. If the RIP holds, then v_t in (6), which is the empirically observed real interest differential between home and foreign country, should be a stationary process. Now, a useful relation is the Fisher decomposition stating that the nominal bond yield is the sum of the real yield and the expected inflation rate over a given period (t to $t + m$):

$$i_t^m = r_t^m + E_t\left(\Delta p_{t+m}\right) \tag{7}$$

Using the Fisher decomposition, equation (6) can be rewritten in the following way:

$$i_t^m - i_t^{m*} = E_t\left(\Delta p_{t+m} - \Delta p_{t+m}^*\right) + v_t \tag{8}$$

Again assuming rational expectations, we have:

$$\left(i_t^m - i_t^{m*}\right) - \left(\Delta p_{t+m} - \Delta p_{t+m}^*\right) = v_t \tag{9}$$

i.e. the RIP holds empirically if the difference between the interest spread and the inflation differential is stationary.

The economic rationale behind the three parities is given by arbitrage on goods and asset markets. Specifically, if goods are perfectly mobile across countries, then arbitrage ensures that their prices – after accounting for expected changes in the value of the various currencies – are ultimately equalised, which is reflected in the PPP condition. Further, if capital is perfectly mobile across countries, then arbitrage ensures that yields on assets of these countries – again after accounting for expected changes in the value of their respective currencies – are also equalised, which is reflected in the UIP. It can be shown that the PPP and the UIP, taken together, imply the RIP (see Lambelet, Mihailov 2005); in other words, arbitrage on goods and asset markets ultimately leads to an equalisation of real returns on assets. An implication that the three parities hold is, thus, that the goods and asset markets of two economies are to a large extent integrated. This, in turn, means that these economies can share a currency and a common monetary policy

without fearing serious turbulence when large asymmetric shocks occur. Indeed, the probability of such shocks is very low, because economies whose markets are integrated also share a common business cycle and usually have similar output structures (see Mongelli 2005).

The three parities have been analysed very extensively using various methods; theoretical and empirical studies in this field are discussed at length in the meta-studies of MacDonald (1998) and Sarno, Taylor (2002).[4] The general upshot of this literature is that the parities, taken alone, seldom hold empirically in typical data samples. Only for very long time series, spanning a century or so, or for panel data of large dimensions can the parities be empirically verified.

As mentioned in Section 1, the empirical methodology in this paper follows the approach put forward by Juselius and MacDonald (2004a), who scrutinise the international parity relations (the three discussed above and the term structure of interest rates) between Germany and the USA. The analysis strongly rejects the stationarity of single parities, but by allowing the latter to be interrelated it recovers their stationarity. The authors also argue that the apparent non-stationarity of the simple parities is due to very slow adjustment to sustainable exchange rates. The approach of Juselius and MacDonald is based on earlier work by Juselius (1990; 1992; 1995), Johansen and Juselius (1992), and MacDonald and Marsh (1997), and it was also applied to Japan vs. the USA by Juselius and MacDonald (2004b). Another important exception to the rule that empirical research in this area concentrates on only one of the parities is a recent paper by Lambelet and Mihailov (2005), who also model the three parities jointly using various single equation and system equation estimation methods. The authors refer to the parities as the *triple-parity law*, stressing that they are closely interrelated. Robust evidence is found that the parities hold "in the long run, on average, and ex post".

The joint modelling of the various parities within the cointegrated VAR framework can help understand the forces driving the entire system of variables of interest. We believe that the VAR methodology itself is superior to structural simultaneous equation models, because all relevant variables entering the parities are jointly determined so that none of them can from the outset be treated as exogenously given, and because the direction of causality is uncertain. The cointegration approach, moreover, allows one to determine not only the short-run dynamics of the system, as in the case of (structural) VAR models, but also the long-run equilibrium relations between the variables. Specifically, our aim is to find cointegration relations that reflect the three parity relations. If the simple parities do not hold, i.e. if the linear combinations of variables that define the parities are non-stationary, we can still test whether stationary linear combinations of these non-stationary relations exist.

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[4] For recent empirical analyses of the parities for the case of emerging economies and in particular of the Central and East European countries see e.g. Bekő, Boršič (2007); Sideris (2006); Giannellis, Papadopoulos (2006); Singh, Banerjee (2006).

Before we proceed to the empirical analysis, an important caveat is in order. The above equations define the three parities in their strong form, which does not allow for persistent departures of the real exchange rate, the nominal interest spread and the real interest rate from the levels implied by the respective parity condition. The weaker form of these equations, in contrast, allows for permanent (or at least persistent) departures from these levels. Such departures can result from institutional or structural characteristics of economies in question. Empirical tests of the parities in their weaker form consist in testing whether the equations (4) and (6) each include a non-zero constant term or a deterministic time trend, with the term ε_t in equation (4) and v_t in equation (6) being white noise (zero mean) error terms. Similar remarks apply to equation (1), where the term ppp_t need not be stationary but can also be trend-stationary. This is the strategy that we follow in our empirical analysis. After all, it only seems natural that the RIP between Poland, a former centrally planned transition economy, and Germany, a stable market economy, cannot hold in its strong form throughout any reasonable sample period, which must cover years of catching up and thus of falling real interest differential. The same applies to the remaining two conditions.

3   A visual inspection of the parities

Before analysing the international parities presented in Section 2 within the cointegrated VAR framework, we first inspect them graphically. An ocular analysis of various linear combinations of the relevant time series can suggest a first tentative answer to the question whether the parities hold empirically. The underlying time series in Figures 1 to 4 are defined in Section 5 and their levels and differences are depicted in Figure A.1 in the Appendix.

From the cross plot of the nominal exchange rate and the price differential between Poland and Germany (see the upper panel of Figure 1) it is difficult to tell whether and to what extent the former has mirrored the latter. The reason for this is that the prices seem to be integrated of order 2, $I(2)$; this was confirmed by formal tests which will be discussed in Section 5. The middle panel of Figure 1 depicts the deviation from PPP (the real exchange rate multiplied by -1)[5] and the inflation differential. If the PPP held, then the real exchange rate and the price differential would move together and the deviation from PPP would be stationary. As can be seen from the figure, there is hardly any evidence of PPP holding.

However, the picture might be blurred by the fact that the sample period has been the time of intensive transition from a centrally planned to a market economy and high productivity growth in Poland relative to Germany. As a consequence, both the real exchange rate and the price

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[5] The deviation from PPP in Figure 1 and the rate of depreciation in Figure 2 were scaled by the factor 10 to ease interpretation of the cross plots.

differential have exhibited pronounced trends: the former a positive[6], the latter a negative one, which might make it difficult to tell whether the exchange rate is at least trend-stationary or not. The bottom panel of Figure 1, which depicts the detrended series, shows that the deviation from PPP is not even trend-stationary.

Figure 1: The behaviour of prices and exchange rates
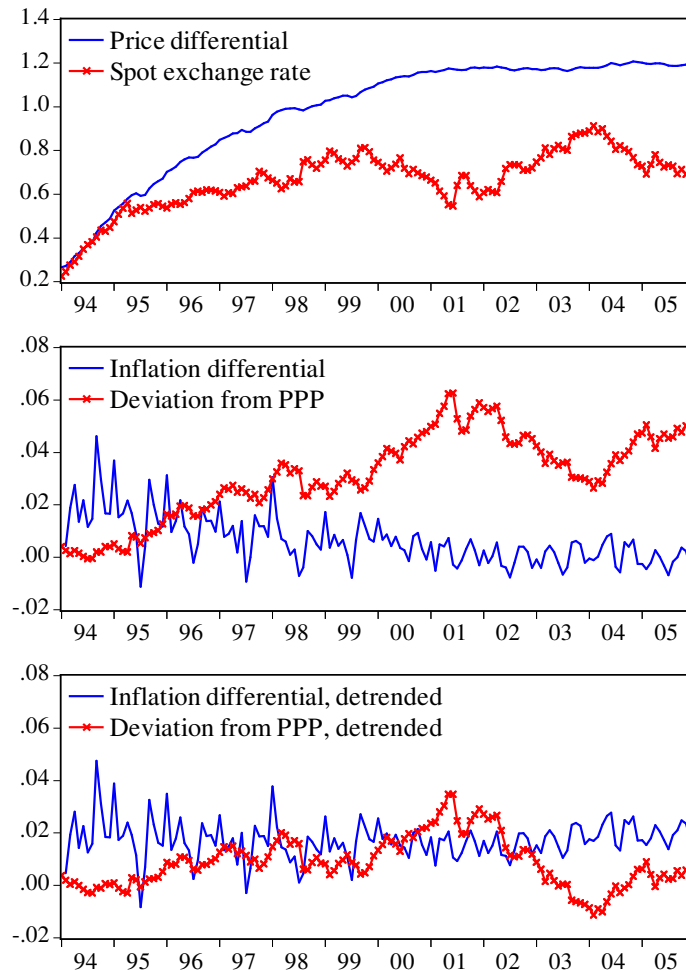
Further we look at the depreciation rate and the home vs. foreign interest differential (Figure 2). If the UIP held, the two series would move together and the difference between the two would be stationary (see equation (4)). The upper panel of the figure is again difficult to interpret because the interest rate spread is trending (which is again a by-product of the economic transition), whereas the depreciation rate is not. The bottom panel shows the detrended series[7], which reveal a similar picture: there is hardly any evidence supporting the UIP.

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[6] Note that ppp_t is the real exchange rate multiplied by -1 so that a positive trend in ppp_t means a real appreciation trend, although a rise in s_t means nominal *depreciation* of the home currency.
[7] The series were detrended by means of an OLS regression on a constant and a linear time trend. Each detrended series was computed as the difference between the original series and the trend term times its estimated coefficient.

Figure 2: Depreciation rate and home vs. foreign interest rate spread
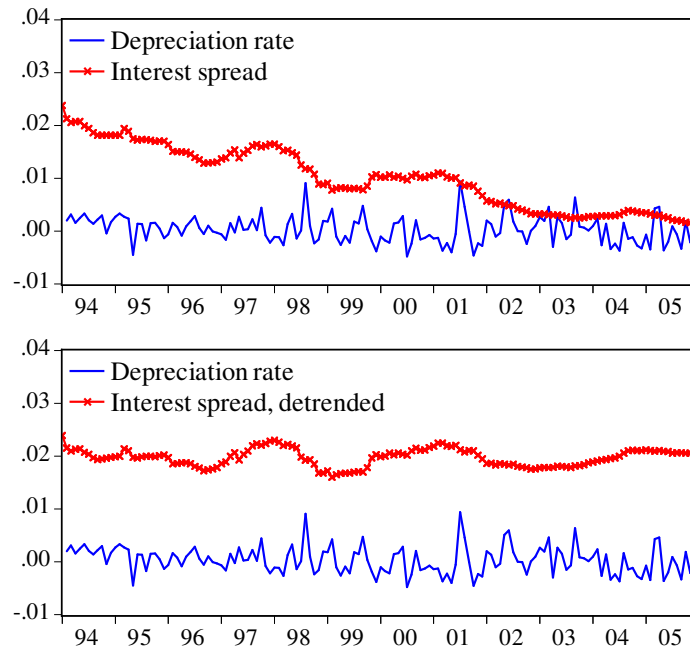
Source: IMF International Financial Statistics, National Bank of Poland, own calculations

The third condition to look at is the RIP, postulating that the deviation between the real interest rates in the two countries should be stationary. Figure 3, especially the bottom panel depicting the series smoothed by taking 12-month moving averages, shows that this is probably not the case, as the deviations between the two series are rather persistent. Recall that using the Fisher decomposition, the RIP condition could also be written in the form (9), i.e. as a relation between the nominal interest rates and the inflation differential, which are graphed in Figure 4. Here, the impression is that the difference between the two series is $I(0)$.

To summarise, the impression from the graphical analysis is that the three parities presented in Section 2 do not hold. Obviously, a visual inspection is only an informal way of investigating whether the given relations are stationary. The results of formal tests will be discussed in Section 5; before that, the next section will present the cointegrated VAR model.

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Figure 3: Real interest rates
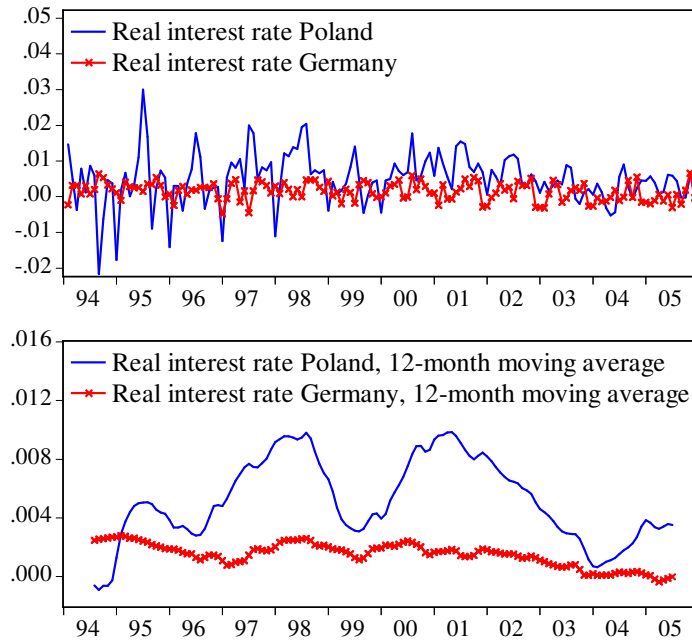
Source: IMF International Financial Statistics, National Bank of Poland, own calculations

Figure 4: Home vs. foreign interest rate spread and inflation differential
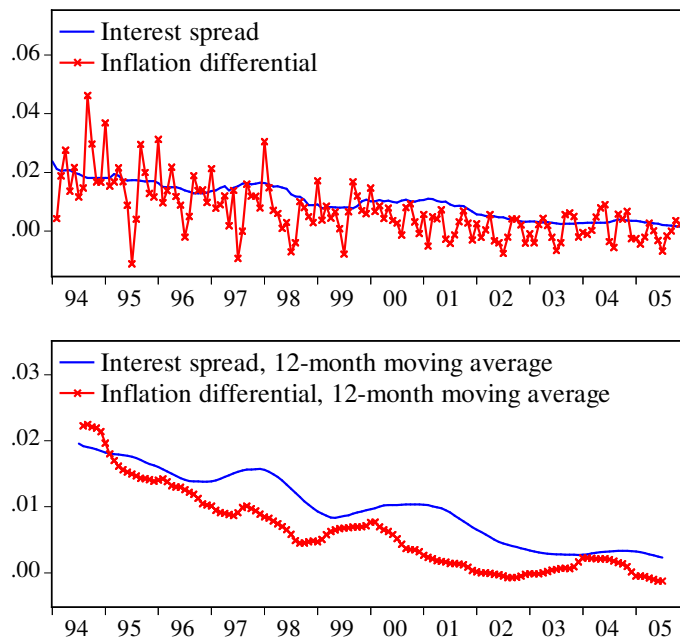
Source: IMF International Financial Statistics, National Bank of Poland, own calculations

4   The cointegrated VAR model[8]

The j-dimensional cointegrated VAR(k) model in the vector equilibrium correction (VEC) form is given by the following equation:

$$\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{k-1} \Delta x_{t-k+1} + \Phi D_t + \varepsilon_t \tag{10}$$

where x_t is a $j \times 1$ vector of endogenous variables, D_t is a $b \times 1$ vector of deterministic components (such as a constant, a linear time trend, seasonal or intervention dummies, or strictly exogenous variables), ε_t is a $j \times 1$ vector of i.i.d. Gaussian error terms, and Π, Γ_i $(i = 1,...,k-1)$, and Φ are coefficient matrices of appropriate dimension. Based on the assumption that all variables in (10) are at most $I(1)$, the cointegration hypothesis can be formulated as a reduced rank restriction on the matrix Π:

$$\Pi = \alpha \beta' \tag{11}$$

where α and β are $j \times r$ coefficient matrices with full column rank and $r \leq j$, which implies that the rank of Π is also r. As the variables in x_t are $I(1)$, their first differences on the left hand side of (10) are stationary; therefore, all terms on the right hand side of the equation must also be stationary. Thus, the matrix Π translates the non-stationary vector x_{t-1}, into a stationary one, Πx_{t-1}. More precisely, it is the expression $\beta' x_{t-1}$ that defines the stationary linear combinations (cointegration relations) of the $I(1)$ vector x_{t-1}, whereas the matrix α describes how the variables in the system adjust to the equilibrium error from the previous period, $\beta' x_{t-1}$. The rank r of the matrix Π gives the number of cointegration relations (steady states, long-run equilibrium relations) between the j variables of the VAR system, whereas $j - r$ gives the number of common stochastic trends that drive their behaviour. The former can be interpreted as the pulling forces and the latter as the pushing forces of the system; each time a variable is pushed away from the steady state, it is pulled back to it. The analysis in the next section aims at finding cointegration relations between the variables of interest that can be given a meaningful economic interpretation, and at identifying the common stochastic trends.

The vector of variables that are relevant for our analysis is defined as follows:

$$x_t = \begin{bmatrix} p_t & p_t^* & i_t & i_t^* & s_t \end{bmatrix}' \tag{12}$$

where p_t = the Polish ("home country") consumer price index,

   p_t^* = the German ("foreign country") consumer price index,

   i_t = the Polish Treasury bill rate,

   i_t^* = the German Treasury bill rate,

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[8] The cointegrated VAR analysis is discussed in depth in Juselius (2006).

s_t = the spot exchange rate (defined as PLN/DM).

The data are monthly, not seasonally adjusted, and cover the period 1994:1 to 2006:1. All series except for the exchange rate are taken from the IMF International Financial Statistics whereas the exchange rate is the end-of-month rate as announced by the National Bank of Poland[9]. From January 1999 onwards, the PLN/DM exchange rate is represented by the PLN/EUR rate, divided by the irrevocable DM/EUR conversion rate. The Treasury bill rates are not the usual annualised rates but monthly rates so they are directly comparable to the monthly changes in the remaining variables. Our choice of the proxy for long-term interest rates was not straightforward. Ideally, we should have used long (e.g. ten-year) government bond yields. However, first emissions of longer-term government bonds in Poland took place in 1999 so the time series are rather short.[10] As the Treasury bill rate is the only interest rate that has been available throughout the whole sample period, we could only use this rate as a proxy for long bond yields. The data in levels and in differences are depicted in Figure A.1 in the Appendix.

Both the graphical analysis of the time series in the previous section and the formal tests to be discussed in the next section suggest that the price variables are $I(2)$, whereas our model is based on the $I(1)$ assumption. Therefore, we transformed the data so that the resulting series are at most $I(1)$, while at the same time preserving information about the long-run trends driving the prices.[11] The transformed vector of variables whose joint behaviour is to be explained within the cointegrated VAR framework now becomes:

$$x_t = \begin{bmatrix} \Delta p_t & \Delta p_t^* & i_t & i_t^* & ppp_t \end{bmatrix}' \sim I(1) \tag{13}$$

where ppp_t was defined in Section 2. Note that the VEC model is defined for differenced data, which means that the price variables in the vector Δx_t are differenced twice:

$$\Delta x_t = \begin{bmatrix} \Delta^2 p_t & \Delta^2 p_t^* & \Delta i_t & \Delta i_t^* & \Delta ppp_t \end{bmatrix}' \sim I(0) \tag{14}$$

The point of departure for our analysis is the following stylised scenario. In a neoclassical world we would expect prices of goods, capital and foreign exchange to be driven by no more than two different stochastic trends. These could be defined e.g. as cumulated supply and demand shocks, or as cumulated domestic and foreign shocks. Alternatively, one trend could be associated with shocks to the current account and the other with capital account shocks. Therefore, we would expect the rank of the matrix Π to be equal to 3. However, in a world with nominal rigidities, barriers to trade with goods and to capital and labour movements across countries, asymmetric information, risk aversion etc., there might be more than two common stochastic trends driving

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[9] When average monthly exchange rates are used instead of end-of-month rates, the qualitative results of the analysis are identical and the quantitative results are very similar.

[10] This problem is typical of Central and East European former centrally planned economies.

[11] See the discussion in Juselius & MacDonald (2004a).

our variables. In a similar data set for Germany and the US, Juselius and MacDonald (2004a) identify a third common trend associated with the special role of the US dollar in the international monetary system, which manifests itself in agents' willingness to hold dollars irrespective of the developments in the US economy. The presence of a similar trend, which the authors term a "safe haven" or portfolio balance effect, in the Polish-German data seems plausible because of the traditionally important role of the German mark as a medium of exchange and, especially, as a store of value in the formerly centrally planned economies of Central and Eastern Europe. In that case the rank of Π would be equal to 2.

To summarise, we expect to find two or three cointegration relations, and, correspondingly, three or two common stochastic trends driving the system. More specifically, if the simple parities discussed in Section 2 describe the variation in our data correctly, then they can be modelled individually because the relations defining them are stationary by themselves. From the graphs in Section 3, we reckon that the parities do not hold for our data set. Therefore, we aim at finding out whether there exist stationary linear combinations of the simple parities. In other words, we seek to find parameter values for a, b and c such that:

$$a\left(i_t - i_t^*\right) - b\left(\Delta p_t - \Delta p_t^*\right) - c\ ppp_t \tag{15}$$

or, alternatively:

$$a\left(i_t - \Delta p_t\right) - b\left(i_t^* - \Delta p_t^*\right) - c\ ppp_t \tag{16}$$

define stationary equilibrium relations which pull the system variables whenever they are pushed away from equilibrium. Note that the simple parity relations are special cases of the above equations as they result from setting two of the parameters a, b, c to zero and normalising the remaining parameter. We expect the steady-state relations found in our data to be special cases of equations (15) and (16), or perhaps the equations themselves.

5   The empirical analysis[12]

A. Specification and estimation of the unrestricted VAR model

As a first step of our analysis, we specified and estimated the unrestricted VAR model presented in Section 4. By setting the maximum lag length to two, we were able to obtain a parsimonious model with well-behaved residuals. We based our choice of the lag length primarily on residual analysis, although we also checked the information criteria and performed lag reduction tests.[13]

In terms of deterministic components, the model was specified so as to include an unrestricted constant, which means that the data in levels show trending behaviour but the

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differenced data have no trend. This is exactly what the graphs of levels and differences of our time series show (see Figure A.1). Originally we included a trend term restricted to appear in the cointegration space in order to account for the possibility that the trends in data do not cancel out in the cointegration relations. Long-run variable exclusion tests showed, however, that the trend term could be excluded from the cointegration space without loss of information.

Apart from the constant, centred seasonal dummies and other dummies were included. Specifically, we used innovational dummies to account for large interventions as well as a shift dummy restricted to lie in the cointegration space, $C_{1995:05}$. The latter picks up a level shift in the equilibrium relation involving the Polish bond rate, which we believe to have taken place in May 1995. The shift, whose occurrence is suggested by our data, can be put down to important structural changes in the monetary regime in Poland. Specifically, on 16th May 1995 there was a changeover to a crawling bands exchange rate regime with a ±7% fluctuation band. Moreover, starting from 1st June 1995 the Polish zloty became convertible in accordance with Article VIII of the Articles of Agreement of the International Monetary Fund (IMF 1945). The unrestricted estimate of the long-run matrix Π, with significant coefficients typed in bold face, is given in Table A.1.a in the Appendix.[14]

B. Determination of the cointegration rank

The second step of the analysis consisted in the determination of the cointegration rank, r, i.e. the number of steady-state relations between the variables of the system. As the choice of the cointegration rank is crucial for all subsequent analysis, we used all information that was available from the data before deciding upon the "correct" rank.[15] The only formal test that we applied was the trace test, or the Johansen test[16], whose results for the model described above are reported in Table A.2.a in the Appendix.[17] The largest two eigenvalues are significantly different from zero at every standard significance level; the significance of the third-largest eigenvalue is borderline. The trace test thus points to $r = 2$, but at this point we cannot exclude the possibility that the third cointegration relation is also stationary. The reason is the fact that the trace test has low power to reject the unit root hypothesis when the true root is lower that but near one, i.e. when it is in the "near unit root region". The low power problem is aggravated by our relatively small sample size.

Therefore, we need to use other sources of information concerning r. As a first sensitivity check, we recalculated the trace test for a different model specification, namely one that includes

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[14] Prior to estimation, additive outliers (measurement errors) were removed from the time series of the German price level; the figures in the previous section and in the Appendix depict the corrected data.

[15] All the tests and procedures used here are discussed at length in Juselius (2006, ch. 6).

[16] See Johansen (1996).

[17] We simulated the asymptotic distribution of the trace test statistics using the automatic CATS procedure with 1,000 random walks and 10,000 replications.

no dummy variables except for seasonal dummies. The results of this test are reported in Table A.2.b in the Appendix. The test this time very clearly points to $r = 2$: with a p-value of over 0.8, the significance of the smallest three eigenvalues cannot be rejected.

Secondly, we looked at graphs depicting the individual cointegration relations of the unrestricted model (see Figure A.2.a in the Appendix) to assess whether they look stationary. The first two cointegration relations behave like stationary processes, the opposite holds for the last two. The third relation is of special interest because if it looked stationary, then we would consider $r = 3$ in spite of the above-reported results of the trace tests. As can be seen from the figure, this is hardly the case. The two cointegration relations of the model where the cointegration rank was restricted to 2 (see Figure A.2.b in the Appendix) seem again to be very stationary, which again points to $r = 2$.

Thirdly, we computed the roots of the companion matrix for different values of r (see Table A.3 in the Appendix). Note that choosing a given r automatically leads to $j - r$ unit roots, which does not necessarily mean that there are $j - r$ stochastic common trends in the data. Looking at the largest eigenvalues for different choices of r reveals that for $r \geq 3$, the third-largest eigenvalue is near unity, whereas the fourth and the fifth are distinctly far from the unit circle. This leads us to the tentative conclusion[18] that the trace test has picked up the "correct" cointegration rank.

A further source of information on the cointegration rank is the unrestricted estimate of the matrix α and more specifically, the significance of its parameters. As can be seen from Table A.4 in the Appendix, which gives the unrestricted estimates of α given different values of r, the coefficients in the first two columns have generally high t-ratios, but the third column contains only one coefficient that is borderline significant.[19] This can again be interpreted as evidence that the third cointegrating relation might be stationary, although rather borderline so.

Furthermore, we used the recursively calculated trace test statistics (see Figure A.3 in the Appendix) to draw conclusions on the cointegration rank. The upper two lines, depicting the trace test statistics for the two "most stationary" cointegration relations, exhibit pronounced linear growth, whereas the other three remain roughly constant as more and more observations are added to the base period. This result again suggests that $r = 2$.

Finally, one can draw on economic theory to hypothesise about the number of cointegrating relationships in our model. As argued in Section 4, we expected the variables in our system to be driven by two or three stochastic common trends, and therefore the cointegration

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[18] The conclusion is only tentative because we do not know the distribution of the eigenvalues, which makes it impossible to test which values are significantly different from unity.

[19] Note that the exact distribution of these coefficients is unknown. If the corresponding equilibrium relations are stationary, the t-statistics are distributed as Student's t and in the non-stationary case as Dickey-Fuller's τ.

rank to be equal to three or two, which is consistent with the results discussed above. Thus, based on all sources of information we conclude that the rank of the matrix Π and thus the number of steady-state relationships between our variables of interest is equal to two. The estimate of Π based on this reduced rank is given in Table A.1.b.

C. Specification tests

Prior to the actual cointegration analysis we performed various specification tests of the estimated VAR model to check the assumption of the error terms being independently normally distributed. The results of these tests, both for the full rank and the restricted rank VAR model, are reported in Table A.5 in the Appendix. An important point to note is that valid statistical inference is sensitive to violation of certain assumptions, such as autocorrelated or skewed residuals and parameter inconstancy, and quite robust to violation of others, such as residual heteroskedasticity or excess kurtosis.

The most important assumptions regarding the residuals are therefore those of no autocorrelation and zero skewness. As can be seen from the table, none of the tests rejects the former hypothesis for the whole system. As for the latter, normality is strongly rejected for the whole system and for equations explaining the Polish inflation rate and both bond rates. This result is, however, primarily due to the fact that the kurtosis of the respective empirical distributions is too large to be associated with normal distribution, whereas the skewness seems to be less of a problem. Table A.5 shows that the residuals from the equation explaining the Polish interest rate exhibit ARCH effects, whereas no such effects are detected in any the other equation or the system as a whole. All in all, we conclude that the assumption of independent multivariate normal distribution of the residuals is by and large confirmed by the data.

Furthermore, Table A.5 reports goodness-of-fit measures for the whole model (trace correlation) and for individual equations (determination coefficient, R^2). The trace correlation is fairly large and the same holds for R^2 for the equations explaining the inflation rates and the Polish bond rate. The low values of R^2 for the remaining two equations can be explained by the weak exogeneity of the German bond rate and the deviation from PPP (see Section 5.D).

The third assumption that is crucial for valid statistical inference based on a VAR model is that the sample period defines a reasonably constant parameter regime. To check this, we performed various recursive tests of parameter constancy for the reduced rank model ($r = 2$): the recursively calculated test for constancy of the log-likelihood function, the recursively calculated trace test statistics, eigenvalues and transformed eigenvalues, the max test of constant beta, and the 1-step prediction test.[20] Virtually all tests, whose results are not reported here to save space[21],

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[20] All tests are extensively discussed in Juselius (2006).

show that the model's parameters have been constant throughout the sample period. This is especially true with regard to the concentrated model, i.e. one where the short-run dynamics, $\Gamma_1 \Delta x_{t-1}$, and deterministic components, ΦD_t, have been concentrated out.

The results presented in this section and the previous one suggest that our VAR model satisfies the $I(1)$ assumptions, which postulate that (i) the rank of the matrix Π is equal to r, (ii) the companion matrix has exactly $j - r$ unit roots, corresponding to the stochastic trends that drive the system variables, (iii) the residuals are independent, (iv) the sample size is large (our relatively small sample size is accounted for by the Bartlett correction of various test statistics) and (v) the parameters of the VAR model are stable throughout the sample. These conditions are the prerequisite for the Granger representation theorem to hold, i.e. for the VAR model to have a moving average representation (see equation (17) in the next section).

D. Testing restrictions on long-run parameters

The next step is to test restrictions on parameters of the long-run structure, i.e. of the matrices α and β. The point of departure for all tests discussed below are the estimates of α and β subject to rank restriction $r = 2$. The parameters of the former matrix are termed adjustment coefficients because they describe how the variables of the system adjust when they are pushed away from the steady state. An important test is that of a zero row in α, which is tantamount to weak exogeneity of the variable corresponding to that row. The hypothesis of long-run weak exogeneity, or no levels feedback, of a variable x_{it} for the long-run parameters β means that the variable x_{it} has influenced the long-run stochastic path of the other variables in the system but has itself not been influenced by them. This can be seen from the moving average (MA) representation, which in its simplest form (without short-run dynamics and deterministic components) is given by:

$$x_t = \tilde{\beta}_\perp \alpha'_\perp \sum_{s=1}^{t} \varepsilon_s + C^*(L)\varepsilon_t + A \tag{17}$$

where $\tilde{\beta}_\perp \equiv \beta_\perp (\alpha'_\perp \beta_\perp)^{-1}$, $\alpha_\perp$ and $\beta_\perp$ are the respective orthogonal complements to α and β[22], $C^*(L)$ is a lag polynomial and A depends on initial values. The term $\alpha'_\perp \sum \varepsilon_s$ defines the common stochastic trends driving the system and $\tilde{\beta}_\perp$ their loadings, describing how the common trends are transmitted to the system variables. The hypothesis of a zero row in α corresponds to a unit vector in its complement, $\alpha_\perp$. Thus, if the hypothesis of weak exogeneity of a given variable is accepted, the cumulated shocks to that variable alone define one of the common trends driving

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[21] With the exception of the recursively calculated trace test statistics discussed in the last section, see Figure A.3 in the Appendix.

[22] I.e. $j \times (j - r)$ matrices of full column rank such that $\mathrm{rank}(\alpha, \alpha_\perp) = \mathrm{rank}(\beta, \beta_\perp) = j$, $\alpha'\alpha_\perp = 0$ and $\beta'\beta_\perp = 0$.

the system. As there are $j - r$ common trends, the number of weakly exogenous variables cannot exceed $j - r$, i.e. three in our case.

The tests results (see Table A.6 in the Appendix) show that the German bond rate and the deviation from PPP are both weakly exogenous when tested individually. Moreover, the hypothesis of the two variables being jointly weakly exogenous is also accepted. We can conclude that the cumulated shocks to each of these variables define two of the three common trends pushing the system. As will be shown in Section 5.E, the German bond rate is also strongly exogenous, which means that this variable itself, and not just the cumulated shocks to it, represent a common trend. The third common trend is a linear combination of cumulated shocks to the Polish and the German inflation rates and to the Polish bond rate (see also equation (21) in Section 5.E). Accepting the hypothesis of no long-run levels feedback for the two variables in question means that our VAR model does not explain the stochastic path of the deviation from PPP, which would be a problem if modelling this path was the goal of our analysis. From that follows that we could reduce the dimension of our system to three and only include the German bond rate and the deviation from PPP as weakly exogenous variables in the cointegration space.

A second test involving the adjustment coefficient matrix is that of a unit vector in α, meaning that the variable corresponding to this vector is exclusively adjusting (i.e. shocks to that variable have only temporary effects on the other variables of the system). This can again be seen from (17): as a unit vector in the matrix α corresponds to a zero row in $\alpha_\perp$, shocks to the given variable do not enter the term $\alpha'_\perp \sum \varepsilon_s$, i.e. do not influence the level of x_t in the long run. We performed the test for each of the endogenous variables in our system (see Table A.7 in the Appendix for results) and found no evidence of a unit vector in α at the 5 percent significance level. Thus, we conclude that none of the variables in the system is exclusively adjusting.

When testing restrictions on the parameters of β, the aim is to find out which of the model variables and which linear combinations of them are stationary. This leads to the identification of the "final" set of cointegration relations that are, ideally, economically meaningful equilibrium relations. As a first step, we performed tests of the long-run exclusion of variables from all cointegrating relations, i.e. tests of zero row restrictions on β. The results, reported in Table A.8 in the Appendix, show that only the German bond rate (which is also weakly exogenous to the system) can be excluded from the long-run equilibrium relations. Interestingly enough, the shift dummy, $C_{1995:05}$, cannot be excluded from the cointegration space. We will draw on these results when formulating our final cointegration relations.

In a second step, we tested the stationarity of a variety of linear combinations of the system variables, starting from the variables themselves (see Table A.9 in the Appendix). We first

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tested for stationarity of each single variable (hypotheses H_1 to H_5), coming to the conclusion that only the German inflation rate is by itself $I(0)$. However, the p-value associated with that latter test is so low that we do not, in fact, believe that Δp_t^* is stationary.[23] Then we tested a number of relations involving the inflation differential $\Delta p_t - \Delta p_t^*$ (H_6 to H_8), the interest spread $i_t - i_t^*$ (H_9 to H_{13}), the domestic and the foreign real interest rates, $i_t - \Delta p_t$ and $i_t^* - \Delta p_t^*$ (H_{14} to H_{18}). We do not report the results of all performed tests but rather present the outcome for the given simple relation and all its stationary combinations with other variables that we have found. For each of the hypotheses we also tested whether the relations are stationary when the shift dummy is included in the relationship but we only report the outcome when it was changed by the inclusion of the dummy.

The general outcome of this exercise is that none of the simple parity conditions is satisfied by the data. If PPP held, then the real exchange rate should be stationary or at least cointegrated with the inflation differential. However, the two variables can only be made stationary if the German bond rate or both bond rates are added to the linear combination (see H_7 and H_8). If UIP held, then the interest spread should be stationary or at least cointegrated with the nominal depreciation rate. We were not able to test the latter hypothesis directly within our VAR framework because the nominal rate is not one of the system variables.[24] However, the stationarity of the interest spread is decisively rejected (H_6). If RIP held, then the real bond rates would be $I(0)$ or at least cointegrated with each other, and the interest spread would be cointegrated with the inflation differential (we have already shown that these both simple relations are non-stationary). These hypotheses are also rejected, though (H_{14}, H_{18}, H_{19}, and H_{20}, respectively). A linear combination of the interest spread and the inflation spread can only be made stationary by augmenting it with both the real exchange rate and the shift dummy (H_{22}); in case of the real bond rates stationarity cannot be achieved even in this way (H_{21}).

Recall from Section 4 that we expected our cointegration relations to be special cases of equations (15) and (16), or these equations themselves. Relation (16) turned out to be non-stationary even when augmented by a shift dummy (H_{21}); therefore, there is no equilibrium relation between real interest rates in both countries and the real exchange rate. As for relation (15), describing a linear combination of the interest spread, the price differential and the real

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[23] If the German inflation rate is stationary, it cannot be cointegrated with any non-stationary single variable or linear combination of variables in the system so there was, theoretically, no point in testing e.g. the hypotheses of the inflation differential or the German real bond rate being stationary. However, the fact that one cannot reject a hypothesis does not necessarily mean that the latter is true: the probability of accepting a false hypothesis is never zero (unless one adopts the strategy of never accepting the null). We thus decided to test such combinations that, from the purely theoretical point of view, could not be stationary if the German inflation rate *really* was $I(0)$.

[24] We tested the hypothesis of the nominal exchange rate being $I(1)$, i.e. of its first difference being $I(0)$, using a different specification of the VAR model where the vector of variables included the price differential, both interest rates, the spot rate and the domestic inflation rate, and could not reject this hypothesis.

exchange rate, it is stationary when the level shift is accounted for (H_{22}). This equation thus became our primary candidate for a cointegration relation. However, when testing the restrictions imparted in relation (15) jointly with those incorporating any other stationary combination of the system variables, we found that the restrictions were only borderline accepted. Moreover, previous tests showed that the German bond rate can be excluded from the cointegration space altogether. These results made us look for other stationary combinations which could be thought of as "irreducible cointegration relations" and, ideally, should have a plausible economic interpretation as long-run steady-states.[25]

One candidate for an irreducible cointegration relation is the linear combination defined by H_{15}, $\left(i_t - \Delta p_t\right) - a\Delta p_t^* - bC_{1995:05}$, which relates the domestic real interest rate to the foreign inflation. A relation that can be given economic interpretation, on the other hand, is the one defined by H_{23}, $\Delta p_t - a\Delta p_t^* - \left(1-a\right)i_t - b\, ppp_t - cC_{1995:05}$, which imposes a long-run homogeneity restriction (sum of the coefficients equal to zero) on the domestic and foreign inflation and the domestic interest rate. Its interpretation is as follows: the domestic inflation is partly imported and partly the result of inflation expectations, reflected in the domestic bond rate; it is also affected by the real exchange rate. These two linear combinations of the system variables are the ones that we eventually adopted as our cointegration relations.

E. Identification of the long-run and the short-run structure

In the previous section we established two stationary relations linear combinations of the system variables that are our potential cointegration relations. The restricted rank VAR model was then estimated subject to restrictions defining the two relations as well as two zero row restrictions on the matrix α (recall from the previous section that i_t^* and ppp_t are individually and jointly weakly exogenous). The result is given in Table A.10 and the corresponding restricted estimate of the matrix Π in Table A.1.c (both tables are in the Appendix). The restrictions on α and β have hardly changed the estimate when compared with previous results. Our cointegration relations are defined as follows:

$$CR1_t = \Delta p_t - 0.543\Delta p_t^* - 0.457\, i_t + 0.022\, ppp_t + 0.003C_{1995:05} \tag{18}$$

$$CR2_t = \left(i_t - \Delta p_t\right) - 14.881\Delta p_t^* - 0.014C_{1995:05,t} \tag{19}$$

As can be easily seen, the first relation is just identified and the second is over-identified. The system as a whole is therefore formally (generically) over-identified[26] and the restrictions are

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[25] An "irreducible cointegration relation" is a stationary linear combination of non-stationary variables that becomes non-stationary once any of them is dropped from the relation; see Davidson (1998). A theoretically meaningful equilibrium relation can be a linear combination of two or more irreducible cointegration relations.

[26] See Juselius (2006) for an intuitional exposition of generic identification and Johansen (1995) for technical details.

testable. The restrictions were accepted with a fairly large p-value based on a Likelihood Ratio (LR) test. Moreover, the cointegration relations are also empirically identified, i.e. the coefficients which have not been set to zero when formulating the restrictions are in fact significantly different from zero in the estimated system. As for economic identification, i.e. interpretability of the results, we already discussed this issue at the end of the previous section.

From the economic point of view, not only the cointegration relations but also the adjustment coefficients are of special interest. Based on the results in Table A.10, we have:

$$
\begin{bmatrix} \Delta^2 p_t \\ \Delta^2 p_t^* \\ \Delta i_t \\ \Delta i_t^* \\ \Delta ppp_t \end{bmatrix} = \begin{bmatrix} -0.922 & 0 \\ 0 & 0.067 \\ 0.018 & -0.004 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} CR1_{t-1} \\ CR2_{t-1} \end{bmatrix} + \ldots \tag{20}
$$

The zero coefficient values in the last two rows of α are the result of the imposed restrictions; however, the unrestricted coefficients were insignificantly different from zero anyway. This means that both weakly exogenous variables, the German bond rate and the real exchange rate, do not equilibrium-adjust, i.e. their change in the present period is unaffected by the departure from equilibrium in the previous period. The coefficients α_{12} and α_{21} are insignificant so we set them to zero in the above equation. The reactions of the "truly endogenous" system variables to the departure from steady-states are plausible in the sense that the respective α coefficients are significant, have the signs consistent with error-correcting behaviour (i.e. there is no overshooting in the system)[27], and are of magnitude which by and large "makes sense". The Polish inflation rate adjusts to the first cointegration relation, $CR1$, which is the equilibrium relation for this variable. If the departure from $CR1$ in a given month is positive, then Δp_t would fall in the following month, correcting approximately 92% of the equilibrium error, which amounts to very fast adjustment. The German inflation rate exhibits equilibrium-correcting behaviour with respect to $CR2$ and the Polish bond rate with respect to both relations, although the adjustment is much slower than that of Δp_t. Apart from the surprisingly high speed of adjustment of the Polish inflation rate, the estimated system seems to be economically plausible.

The over-identified long-run structure described above was the point of departure for the identification of the short-run structure: when testing restrictions on short-run parameters, we kept the parameters $\hat{\beta}^c$ fixed at their previously estimated values.[28] The VAR model discussed so far

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[27] The i-th cointegrating relation is significantly equilibrium-correcting if the parameters in the i-th column of the matrix α are significantly different from zero and have the signs consistent with equilibrium-correcting behaviour, i.e. the signs opposite to those of the corresponding coefficients in the matrix β.

[28] The statistical motivation for this is the superconsistency of the estimator $\hat{\beta}$ (or $\hat{\beta}^c$).

is heavily overparametrised; especially the short-run matrix Γ_1 and the deterministic components matrix Φ contains many insignificant coefficients. Our goal is now to achieve a parsimonious parametrisation of the short-run reduced-form VAR model. Based on parameter significance and the results of the LR test of over-identifying restrictions, we were able to impose a total of 56 restrictions on the short-run structure.

The results are reported in Table A.11 in the Appendix; the columns represent the equations of the system. The unlagged "endogenous" variables[29] have only been included in their own equations and the corresponding unit matrix of coefficients is not reported to save space. As can be seen from the table, most of the coefficients of the matrix Γ_1 could be set to zero without significantly changing the value of the likelihood function; only in the equation of the Polish interest rate and the deviation from PPP are the lagged differences of (some) system variables significant.

A particularly striking result is that of all coefficients in the German interest rate equation equal to zero. Combined with the results of the analysis in Section 5.D, where i_t^* was found to be weakly exogenous (individually and jointly with the real exchange rate), this means that the German bond rate is strongly exogenous to the system and that the corresponding equation could be excluded from the model with no loss of information. As already mentioned in Section 5.D, another conclusion is that one of the stochastic trends to the system is i_t^* itself, not just shocks to it.

As for the adjustment coefficients, the results are similar to those described above, with the difference that the German inflation rate now adjusts to both cointegration relations and the speed of adjustment of the Polish inflation rate is somewhat lower. All in all, our restricted reduced-form VAR does not entail any results that are inconsistent with economic theory or with the outcome of our previous analysis. Moreover, the residuals are essentially uncorrelated, as can be seen from the bottom panel of Table A.11: only the correlation coefficient between the residuals of the first and the fifth equation is significantly different from zero. Thus, our reduced-form model can be interpreted as a structural VAR model.

Based on the estimated over-identified system (20), the MA representation is as follows:

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[29] The term "endogenous" is in quotation marks because it stands for the variables that stand on the left-hand side of the system (including the weakly exogenous ones, like the real exchange rate and the German bond rate in our model), not necessarily those that are actually explained by the system.

$$
\begin{bmatrix} \Delta p_t \\ \Delta p_t^* \\ i_t \\ i_t^* \\ ppp_t \end{bmatrix} = \begin{bmatrix} 0.484 & 0 & -0.026 \\ 0.062 & 0 & 0.002 \\ 1.408 & 0.978 & 0 \\ 0 & 1.034 & 0 \\ 0 & 0 & 1.208 \end{bmatrix} \begin{bmatrix} 0.024 \sum_{s=1}^{t} \varepsilon_{\Delta p,s} + 0.059 \sum_{s=1}^{t} \varepsilon_{\Delta p^*,s} + \sum_{s=1}^{t} \varepsilon_{i,s} \\ \sum_{s=1}^{t} \varepsilon_{i^*,s} \\ \sum_{s=1}^{t} \varepsilon_{ppp,s} \end{bmatrix} + \ ... \tag{21}
$$

The estimates of $\alpha_\perp$, defining the common trends, and $\tilde{\beta}_\perp$, defining their loadings, are given in Table A.12 in the Appendix; for simplicity we set insignificant coefficients to zero in the above equation. Bearing in mind the result of strong exogeneity of the German bond rate, we have:

$$
\sum_{s=1}^{t} \varepsilon_{i^*,s} = i_t^* \tag{22}
$$

i.e. the German bond rate itself, and not just shocks to it, constitutes the second common trend, which drives both bond rates in the long run. The third common trend, driving prices in both countries and the real exchange rate, is the cumulated sum to that latter variable. The first trend is a linear combination of cumulated shocks to the three endogenous variables, and it determines the levels of these three variables in the long run. We have not tried to find the structural MA representation or to give the shocks labels, i.e. to interpret them as "structural" shocks; this is a task for our future research. However, we note that the second trend, the German bond rate, can be interpreted as a "safe haven" or portfolio balance effect (see Juselius, MacDonald 2004a), which is related to the important role of the German mark – or rather, the (future) EMU for which Germany is a proxy – for the Polish economy.

6 Summary and conclusions

In this paper we tried to identify a set of economically meaningful long-run equilibrium relations that would reflect the international parity conditions: the purchasing power parity, the uncovered interest parity and the real interest parity. As these simple parities seldom hold empirically, the general idea was to model them jointly in order to uncover the dynamic structure underlying the stochastic behaviour of prices, interest rates and the real exchange rate in Poland versus the EMU, represented by Germany. The empirical analysis, based on a cointegrated VAR model, not only showed that the simple parities are inconsistent with our data set but it also failed to identify cointegration relations that would be linear combinations of all three parities.

Therefore, the question arises why the parities that are so well-established in the economic theory could not be pinned down when analysing the Polish-German data set, even when we analysed them jointly and allowed for time trends and level shifts in the data. We see the rationale for this in the fact that our sample was rather short, and covered the period of Poland's transition from a centrally planned to a market economy. Therefore, the parities which are supposed to hold

in the long run could not (yet) be identified within our model. One has probably to wait several years before these long-run relations can actually be reflected in the data.

What the analysis did establish, though, is a VAR model with reasonably stable parameters and remarkably well-behaved residuals, which let us draw interesting conclusions about the stochastic behaviour of the variables of interest. We identified two meaningful long-run equilibrium relations that the system was adjusting to: one describing the domestic (i.e. Polish) inflation rate as being partly imported (from Germany), partly the result of inflation expectations, and partly affected by the real exchange rate, and the other bringing together the domestic real interest rate and the foreign inflation. The three variables of the system that can be considered endogenous – the Polish inflation and interest rate as well as the German inflation rate – exhibit equilibrium-adjusting behaviour, i.e. they are pulled back to the steady-state once they have been pushed away from it. The two remaining variables – the real exchange rate and the German interest rate – are weakly exogenous to the system, i.e. they affect the stochastic behaviour of the endogenous variables but are not affected by them.

The system is pushed by three stochastic common trends: one defined as cumulated shocks to the real exchange rate, one defined as the cumulated shocks to the German bond rate (and the bond rate itself, as it turned out to be strongly exogenous), and one being a linear combination of shocks to the endogenous variables. The second of these common trends can be interpreted as the "safe haven" effect, reflecting the important international role of the German mark in formerly communist economies of Central and Eastern Europe. We did not try to label the other two common trends driving the system or to identify structural shocks hitting it; we leave this task for our future research.

Referring to the question asked in the introduction to this paper – whether Poland is "ripe" for the common monetary policy – the answer is not a clear-cut "no", despite the empirical failure of the parities. As the Polish-German inflation rates, interest rates, and the real exchange rate have followed a pattern that is consistent with long-run equilibrium-correcting behaviour, and because the estimated system shows such remarkable degree of stability, it can be argued that Poland has shown a tendency to converge to Germany both in nominal and in real terms. Therefore, we believe that it is rather sooner than later that Poland will be able to join the euro without fearing major turbulences.

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Appendix

Tables

Table A.1: Estimate of the matrix Π [a]

<table>
<tr><td colspan="7">a The unrestricted model</td></tr>
<tr><td>$\Delta^2 p_t$</td><td>-0.922 (-10.42)</td><td>0.383 (1.54)</td><td>0.337 (2.32)</td><td>0.666 (0.71)</td><td>-0.024 (-5.71)</td><td>-0.002 (-1.15)</td></tr>
<tr><td>$\Delta^2 p_t^*$</td><td>0.010 (0.24)</td><td>-1.063 (-8.93)</td><td>0.013 (0.19)</td><td>-0.138 (-0.31)</td><td>0.002 (0.82)</td><td>-0.001 (-1.43)</td></tr>
<tr><td>Δi_t</td><td>0.027 (3.17)</td><td>0.059 (2.46)</td><td>-0.050 (-3.61)</td><td>0.202 (2.26)</td><td>-0.001 (-1.84)</td><td>0.001 (3.33)</td></tr>
<tr><td>Δi_t^*</td><td>0.005 (1.26)</td><td>0.006 (0.56)</td><td>0.008 (1.41)</td><td>-0.095 (-2.50)</td><td>0.000 (1.69)</td><td>-0.000 (-0.62)</td></tr>
<tr><td>Δppp_t</td><td>0.160 (0.28)</td><td>2.457 (1.54)</td><td>-0.407 (-0.44)</td><td>6.399 (1.07)</td><td>-0.026 (-0.98)</td><td>0.025 (2.42)</td></tr>
<tr><td colspan="7">Log-Likelihood = 4698.703 Trace correlation = 0.539</td></tr>
<tr><td colspan="7">b The model with rank restriction: $r = 2$</td></tr>
<tr><td>$\Delta^2 p_t$</td><td>-0.934 (-10.65)</td><td>0.419 (1.70)</td><td>0.371 (8.55)</td><td>0.438 (3.31)</td><td>-0.022 (-10.13)</td><td>-0.003 (-5.35)</td></tr>
<tr><td>$\Delta^2 p_t^*$</td><td>0.003 (0.08)</td><td>-1.031 (-8.74)</td><td>0.090 (4.31)</td><td>-0.546 (-8.60)</td><td>0.004 (3.97)</td><td>-0.002 (-6.93)</td></tr>
<tr><td>Δi_t</td><td>0.025 (2.86)</td><td>0.070 (2.90)</td><td>-0.017 (-3.97)</td><td>0.031 (2.41)</td><td>0.000 (1.22)</td><td>0.000 (4.11)</td></tr>
<tr><td>Δi_t^*</td><td>0.005 (1.26)</td><td>0.005 (0.46)</td><td>-0.002 (-1.34)</td><td>0.001 (0.26)</td><td>0.000 (0.90)</td><td>0.000 (1.16)</td></tr>
<tr><td>Δppp_t</td><td>0.008 (0.01)</td><td>2.785 (1.75)</td><td>-0.249 (-0.89)</td><td>1.470 (1.72)</td><td>-0.011 (-0.77)</td><td>0.005 (1.40)</td></tr>
<tr><td colspan="7">Log-Likelihood = 4686.647 Trace correlation = 0.518</td></tr>
<tr><td colspan="7">c The model with rank restriction ($r = 2$) and restricted long-run parameters [b]</td></tr>
<tr><td>$\Delta^2 p_t$</td><td>-0.938 (-10.72)</td><td>0.266 (1.07)</td><td>0.437 (10.40)</td><td>0 (.NA)</td><td>-0.021 (-10.62)</td><td>-0.003 (-9.22)</td></tr>
<tr><td>$\Delta^2 p_t^*$</td><td>0.009 (0.22)</td><td>-1.038 (-8.72)</td><td>0.032 (1.60)</td><td>0 (.NA)</td><td>0.002 (1.83)</td><td>-0.001 (-3.96)</td></tr>
<tr><td>Δi_t</td><td>0.022 (2.56)</td><td>0.055 (2.24)</td><td>-0.013 (-3.00)</td><td>0 (.NA)</td><td>0.000 (2.07)</td><td>0.000 (3.39)</td></tr>
<tr><td>Δi_t^*</td><td>0 (.NA)</td><td>0 (.NA)</td><td>0 (.NA)</td><td>0 (.NA)</td><td>0 (.NA)</td><td>0 (.NA)</td></tr>
<tr><td>Δppp_t</td><td>0 (.NA)</td><td>0 (.NA)</td><td>0 (.NA)</td><td>0 (.NA)</td><td>0 (.NA)</td><td>0 (.NA)</td></tr>
<tr><td colspan="7">Log-Likelihood = 4682.817 Trace correlation = 0.515</td></tr>
<tr><td colspan="7">[a] t-statistics in brackets [b] Two last rows in α equal to 0; restrictions on β : see equations (18)-(19) in the text</td></tr>
</table>

Table A.2: Trace test of cointegration rank

<table>
<tr><td colspan="8">a For the full model</td></tr>
<tr><td>$j - r$</td><td>r</td><td>Eigenvalue</td><td>Trace test statistics</td><td>Trace test statistics*</td><td>95% critical value</td><td>p-value</td><td>p-value *</td></tr>
<tr><td>5</td><td>0</td><td>0.496</td><td>194.3</td><td>183.7</td><td>63.8</td><td>0.000</td><td>0.000</td></tr>
<tr><td>4</td><td>1</td><td>0.401</td><td>97.0</td><td>92.3</td><td>42.8</td><td>0.000</td><td>0.000</td></tr>
<tr><td>3</td><td>2</td><td>0.107</td><td>24.1</td><td>23.1</td><td>26.4</td><td>0.090</td><td>0.115</td></tr>
<tr><td>2</td><td>3</td><td>0.031</td><td>8.0</td><td>7.5</td><td>13.5</td><td>0.281</td><td>0.325</td></tr>
<tr><td>1</td><td>4</td><td>0.024</td><td>3.5</td><td>2.1</td><td>3.9</td><td>0.065</td><td>0.148</td></tr>
<tr><td colspan="8">b For the model without deterministic components (except for seasonal dummies)</td></tr>
<tr><td>$j - r$</td><td>r</td><td>Eigenvalue</td><td>Trace test statistics</td><td>Trace test statistics*</td><td>95% critical value</td><td>p-value</td><td>p-value *</td></tr>
<tr><td>5</td><td>0</td><td>0.425</td><td>152.4</td><td>144.0</td><td>69.6</td><td>0.000</td><td>0.000</td></tr>
<tr><td>4</td><td>1</td><td>0.340</td><td>73.9</td><td>70.3</td><td>47.7</td><td>0.000</td><td>0.000</td></tr>
<tr><td>3</td><td>2</td><td>0.056</td><td>14.9</td><td>14.2</td><td>29.8</td><td>0.792</td><td>0.829</td></tr>
<tr><td>2</td><td>3</td><td>0.034</td><td>6.7</td><td>6.1</td><td>15.4</td><td>0.615</td><td>0.685</td></tr>
<tr><td>1</td><td>4</td><td>0.013</td><td>1.8</td><td>1.4</td><td>3.8</td><td>0.176</td><td>0.233</td></tr>
<tr><td colspan="8">* = trace test statistics and p-values based on the Bartlett small-sample correction</td></tr>
</table>

Table A.3: Roots of the companion matrix for different ranks of the matrix Π

Modulus of:	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}
$r = 0$	1.00	1.00	1.00	1.00	1.00	0.43	0.38	0.19	0.13	0.13
$r = 1$	1.00	1.00	1.00	1.00	0.41	0.41	0.41	0.38	0.14	0.14
$r = 2$	1.00	1.00	1.00	0.44	0.44	0.41	0.36	0.36	0.09	0.09
$r = 3$	1.00	1.00	0.84	0.44	0.44	0.43	0.36	0.36	0.09	0.09
$r = 4$	1.00	0.98	0.85	0.44	0.44	0.43	0.36	0.36	0.12	0.12
$r = 5$	0.98	0.98	0.85	0.43	0.43	0.42	0.36	0.36	0.11	0.11

Table A.4: Unrestricted estimate of the matrix α [a]

	α_1	α_2	α_3	α_4	α_5
$\Delta^2 p_t$	**1.027**	**-0.608**	0.490	-0.027	-0.235
	(10.42)	(-2.71)	(0.59)	(-0.13)	(-0.66)
$\Delta^2 p_t^*$	**-0.157**	**-0.875**	0.531	-0.063	-0.059
	(-3.34)	(-8.16)	(1.34)	(-0.61)	(-0.35)
Δi_t	-0.015	**0.085**	**0.204**	-0.029	-0.004
	(-1.59)	(3.96)	(2.58)	(-1.39)	(-0.13)
Δi_t^*	-0.004	0.009	-0.069	-0.012	-0.016
	(-1.01)	(0.96)	(-2.05)	(-1.36)	(-1.09)
Δppp_t	0.407	2.378	6.801	1.488	-3.359
	(0.65)	(1.66)	(1.29)	(1.08)	(-1.48)

[a] t-statistics in brackets

Table A.5: Specification tests

	Full rank model		Restricted rank model: $r = 2$	
	$\chi^2(v)$	p-value	$\chi^2(v)$	p-value
Tests for autocorrelation:				
Ljung-Box	807.9 (825)	0.658	808.9 (840)	0.774
LM(1)	29.5 (25)	0.243	33.4 (25)	0.120
LM(2)	21.9 (25)	0.642	23.3 (25)	0.558
Test for normality	22.4 (10)	0.013	22.8 (10)	0.012
Tests for ARCH:				
LM(1)	241.6 (225)	0.213	231.4 (225)	0.217
LM(2)	465.6 (450)	0.296	473.2 (450)	0.370
Trace correlation	0.539		0.518	

Univariate residual analysis	Full rank model		Restricted rank model: $r = 2$	
Equation	Skewness / kurtosis	R^2	Skewness / kurtosis	R^2
$\Delta^2 p_t$	0.23 3.90	0.829	0.25 3.92	0.828
$\Delta^2 p_t^*$	0.10 3.07	0.717	0.12 3.01	0.713
Δi_t	0.10 4.08	0.632	0.16 4.07	0.610
Δi_t^*	0.07 3.95	0.196	-0.05 3.83	0.155
Δppp_t	-0.24 2.87	0.388	-0.35 3.08	0.367
Equation	ARCH(2) [a]	Normality [a]	ARCH(2) [a]	Normality [a]
$\Delta^2 p_t$	4.6 (0.100)	6.5 (0.038)	4.3 (0.117)	6.7 (0.036)
$\Delta^2 p_t^*$	0.1 (0.969)	0.6 (0.746)	0.2 (0.895)	0.5 (0.769)
Δi_t	10.5 (0.005)	8.7 (0.013)	9.7 (0.008)	8.4 (0.015)
Δi_t^*	0.8 (0.677)	7.3 (0.027)	0.7 (0.696)	6.0 (0.050)
Δppp_t	4.6 (0.099)	1.6 (0.457)	3.8 (0.147)	3.0 (0.221)

[a] p-values in brackets

Table A.6: Tests of weak exogeneity (zero row in α) [a]

Δp_t	Δp_t^*	i_t	i_t^*	ppp_t	i_t^* and ppp_t jointly
79.2 (0.000)	51.3 (0.000)	14.2 (0.001)	1.7 (0.426)	2.9 (0.240)	1.7 (0.426)

[a] LR test, $\chi^2(2)$, p-values in brackets

Table A.7: Tests of unit vector in α [a]

Δp_t	Δp_t^*	i_t	i_t^*	ppp_t
6.8 (0.077)	11.0 (0.012)	50.6 (0.000)	63.4 (0.000)	65.7 (0.000)

[a] LR test, $\chi^2(3)$, p-values in brackets

Table A.8: Tests of long-run exclusion [a]

Δp_t	Δp_t^*	i_t	i_t^*	ppp_t	$C_{1995:05}$
77.7 (0.000)	60.0 (0.000)	7.2 (0.028)	1.5 (0.463)	26.9 (0.000)	9.0 (0.011)
59.9* (0.000)	46.3* (0.000)	5.5* (0.063)	1.2* (0.552)	20.7* (0.000)	7.0* (0.031)

[a] LR test, $\chi^2(2)$, p-values in brackets; * = Bartlett-corrected values

Table A.9: Tests of stationarity of single relations

	Δp_t	Δp_t^*	i_t	i_t^*	ppp_t	$C_{1995:05}$	$\chi^2(v)$	p-value
Tests of stationarity of single variables								
H_1	1	0	0	0	0	0	67.9 (5)	0.000
H_2	0	1	0	0	0	0	8.0 (5)	**0.155**
H_3	0	0	1	0	0	0	70.5 (5)	0.000
H_4	0	0	0	1	0	0	68.3 (5)	0.000
H_5	0	0	0	0	1	0	70.3 (5)	0.000
Tests of inflation spread relations								
H_6	1	-1	0	0	0	0	70.6 (5)	0.000
H_7	1	-1	0	-2.93	0.03	0	4.21 (3)	**0.240**
H_8	1	-1	-0.24	-1.65	0.03	0	1.6 (2)	**0.461**
Tests of interest spread relations								
H_9	0	0	1	-1	0	0	70.8 (5)	0.000
H_{10}	0	-893.3	1	-1	0	-0.83	4.8 (3)	**0.185**
H_{11}	-1.38	-9.8	1	-1	0	-0.02	0.6 (2)	**0.725**
H_{12}	-1.86	0	1	-1	-0.04	-0.01	0.5 (2)	**0.783**
H_{13}	0	-43.74	1	-1	0.11	-0.06	1.2 (2)	**0.538**
Tests of real interest rate relations								
H_{14}	-1	0	1	0	0	0	41.9 (5)	0.000
H_{15}	-1	-13.7	1	0	0	-0.02	3.4 (3)	**0.341**
H_{16}	-1	-5.3	1	-3.18	0	-0.01	0.1 (2)	**0.956**
H_{17}	-1	-26.91	1	0	0.04	-0.04	1.255 (2)	**0.534**
H_{18}	0	-1	0	1	0	0	25.4 (5)	0.000
Tests of other relevant relations								
H_{19}	-1	166.35	1	-166.35	0	0	25.4 (4)	0.000
H_{20}	1	-1	-1.07	1.07	0	0	43.7 (4)	0.000
H_{21}	-1	4.14	1	-4.14	-0.02	-0.00	10.1 (2)	0.007
H_{22}	1	-1	-0.51	0.51	0.02	0.00	1.3 (2)	**0.515**
H_{23}	1	-0.54	-0.46	0	0.02	0.00	0.2 (2)	**0.891**

Table A.10: Estimates of the long-run matrices for the restricted model [a]

	α_1	α_2		Δp_t	Δp_t^*	i_t	i_t^*	ppp_t	$C_{1995:05}$
$\Delta^2 p_t$	**-0.922** (-10.62)	0.016 (0.98)	β_1'	1 (.NA)	**-0.543** (-8.58)	**-0.457** (-7.22)	0 (.NA)	**0.022** (8.00)	**0.003** (2.71)
$\Delta^2 p_t^*$	0.076 (1.83)	**0.067** (8.62)	β_2'	-1 (.NA)	**14.881** (-9.11)	-1 (.NA)	0 (.NA)	0 (.NA)	**-0.014** (-2.12)
Δi_t	**0.018** (2.07)	**-0.004** (-2.71)							
Δi_t^*	0 (0.00)	0 (0.00)							
Δppp_t	0 (0.00)	0 (0.00)							

Test of restricted model:	$\chi^2(7) = 7.7$ p-value = 0.364	Log-Likelihood = 4682.817

[a] Two last rows in α equal to 0; restrictions on β: see equations (18)-(19) in the text; t-statistics in brackets

Table A.11: A parsimonious parameterisation of the short-run reduced-form VAR model [a]

	$\Delta^2 p_t$	$\Delta^2 p_t^*$	Δi_t	Δi_t^*	Δppp_t
$\Delta^2 p_{t-1}$			-0.0232 (-3.34)		
$\Delta^2 p_{t-1}^*$					
Δi_{t-1}			0.3004 (4.89)		
Δi_{t-1}^*			0.6825 (3.25)		
Δppp_{t-1}					0.1915 (2.57)
$CR1_{t-1}$ [b]	-0.7543 (-10.9)	0.0930 (2.83)	0.0144 (1.79)		
$CR2_{t-1}$		0.0673 (10.6)	-0.0028 (-2.12)		

LR test of over-identifying restrictions: $\chi^2(56) = 69.2$ p-value = 0.110

Residual correlations [c] (residual standard deviations on the diagonal):

$\varepsilon_{\Delta^2 p_t}$	0.0038				
$\varepsilon_{\Delta^2 p_t^*}$	0.0750	0.0018			
$\varepsilon_{\Delta i_t}$	-0.0449	0.0984	0.0004		
$\varepsilon_{\Delta i_t^*}$	0.0694	0.0978	0.1664	0.0002	
$\varepsilon_{\Delta ppp_t}$	0.2887	-0.0727	0.0105	0.0029	0.0236

[a] Unlagged "endogenous" variables only appear in their own equations; seasonal and other dummies are not reported; t-statistics in brackets
[b] $CRi = i$-th cointegration, $i=1,2$ (see equations (18)-(19) in the text)
[c] significant correlations: ± 0.1667 or larger

Table A.12: MA representation of the restricted model [a]

	$\tilde{\beta}_{\perp 1}$	$\tilde{\beta}_{\perp 2}$	$\tilde{\beta}_{\perp 3}$		$\varepsilon_{\Delta^2 p_t}$	$\varepsilon_{\Delta^2 p_t^*}$	$\varepsilon_{\Delta i_t}$	$\varepsilon_{\Delta i_t^*}$	$\varepsilon_{\Delta ppp_t}$
Δp_t	**0.484** (3.20)	0.528 (1.46)	**-0.026** (-10.35)	$\alpha_{\perp 1}'$	**0.024** (2.42)	**0.059** (2.31)	1 (NA)	0 (NA)	0 (NA)
Δp_t^*	**0.062** (5.11)	0.030 (1.05)	**0.002** (9.07)	$\alpha_{\perp 2}'$	0 (NA)	0 (NA)	0 (NA)	1 (NA)	0 (NA)
i_t	**1.408** (9.89)	**0.978** (2.89)	0.001 (0.49)	$\alpha_{\perp 3}'$	0 (.NA)	0 (.NA)	0 (.NA)	0 (.NA)	1 (.NA)
i_t^*	0.033 (0.87)	**1.034** (11.35)	0.001 (1.67)						
ppp_t	8.571 (1.24)	-2.883 (-0.18)	**1.208** (10.67)						

[a] Two last rows in α equal to 0; restrictions on β: see equations (18)-(19) in the text; t-statistics in brackets
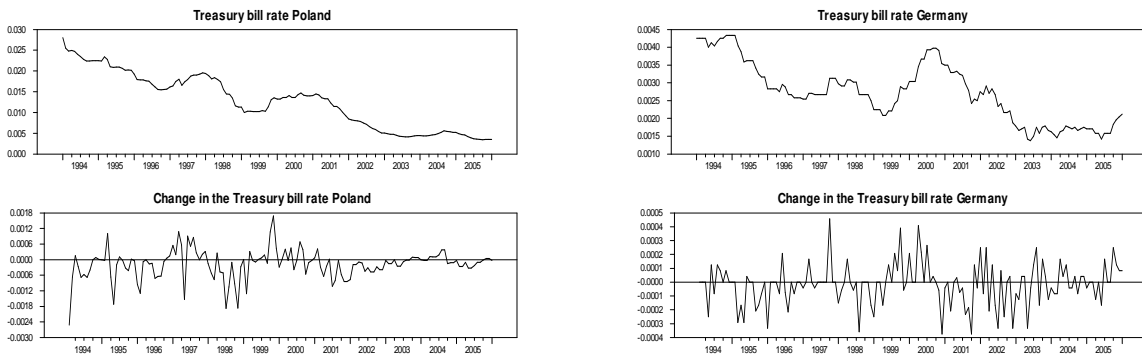(NA = not available)

Figures

Figure A.1: Data in levels and differences

a Prices and inflation
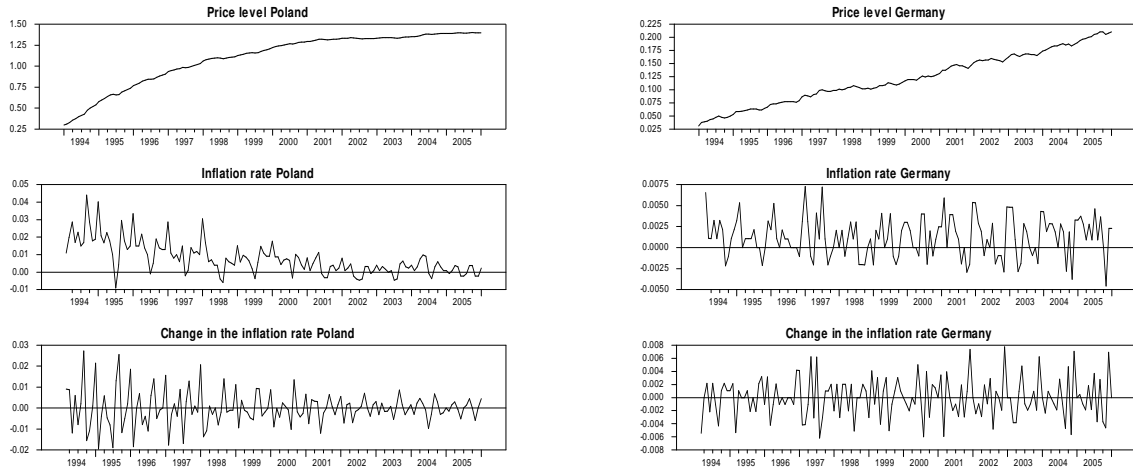
b Treasury bill rates
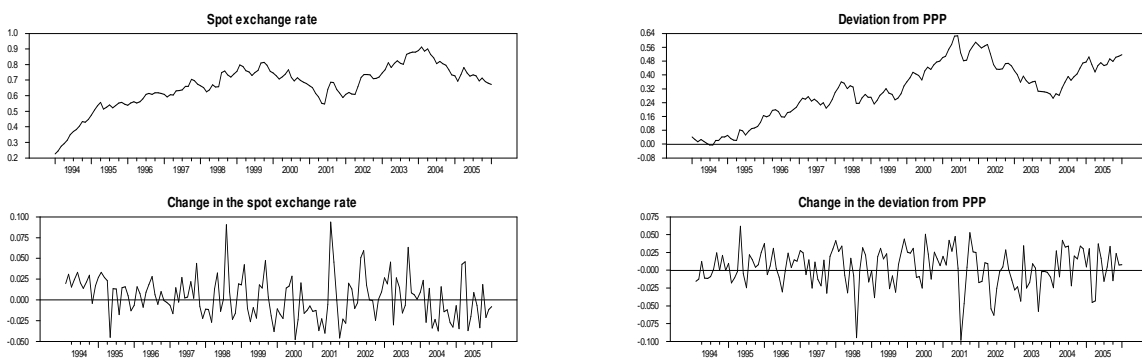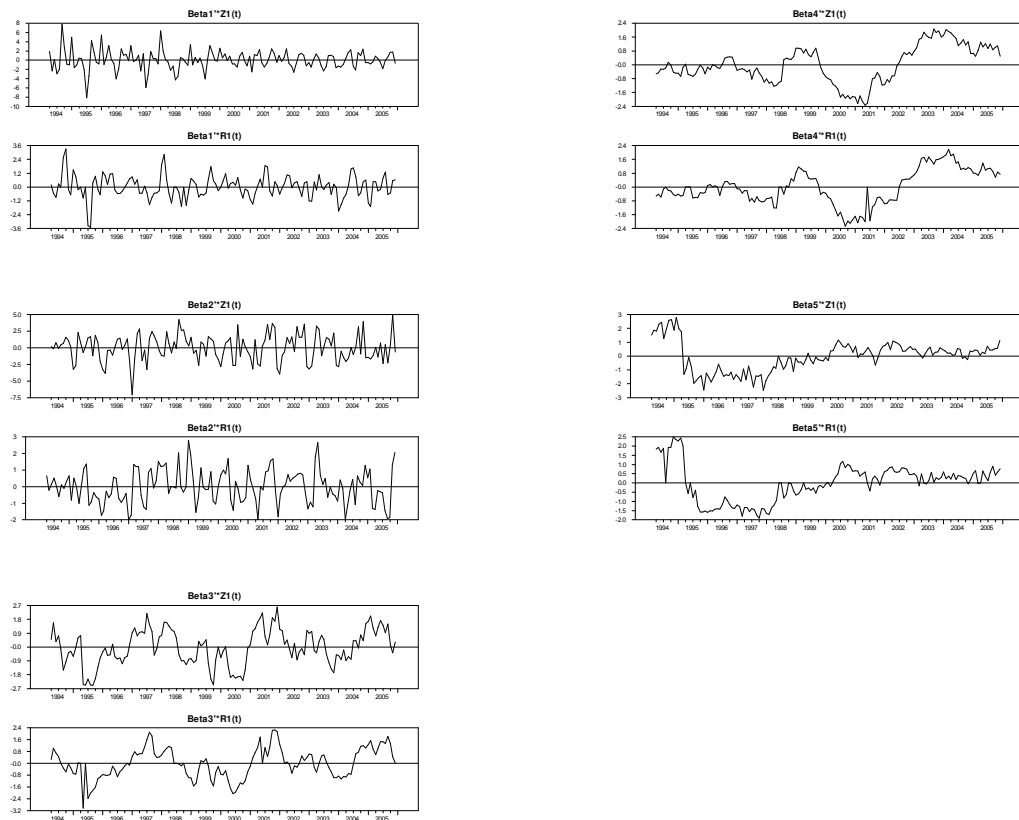

c Exchange rate and the deviation from PPP
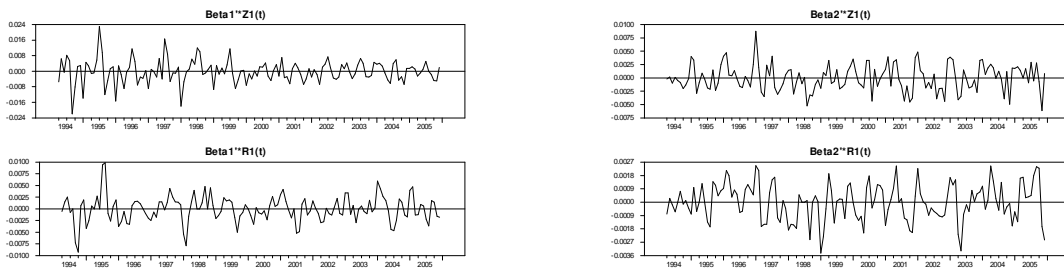
Figure A.2: Cointegration relations [a]

a Unrestricted model ($r = 5$)


b Restricted model ($r = 2$)

[a] The upper panel of each graph depicts the given cointegration relation based on the full model and the lower panel the same cointegration relation based on the concentrated model (without the short-run dynamics and the deterministic components). The order of the cointegration relations is that of decreasing stationarity.

Figure A.3: Recursively calculated trace test statistics [a]

a Forward recursive test
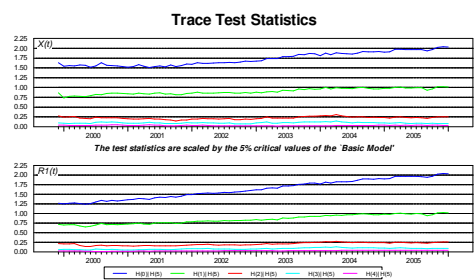
b Backward recursive test
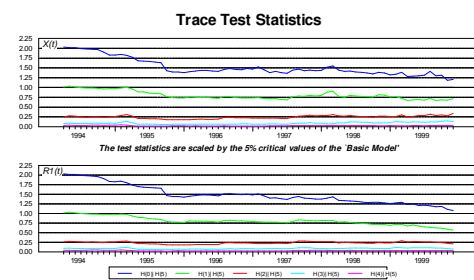
[a] The base period for the forward test is 1994:04 to 1999:12 and for the backward test 1999:12 to 2006:01