Asset Bubbles, Endogenous Growth, and Financial Frictions

Hirano, Tomohiro and Yanagawa, Noriyuki

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Tomohiro Hirano[†]and Noriyuki Yanagawa[‡]

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Abstract

This paper analyzes the effects of bubbles in an infinitely-lived agent model of endogenous growth with financial frictions and heterogeneous agents. We provide a complete characterization on the relationship between financial frictions and the existence of bubbles. Our model predicts that if the degree of pledgeability is sufficiently high or sufficiently low, bubbles can not exist. They can only arise at an intermediate degree. This suggests that improving the financial market condition might enhance the possibility of bubbles. We also examine whether bubbles are growth-enhancing or growth-impairing in the long run. We show that when the degree of pledgeability is relatively low, bubbles boost long-run growth. On the other hand, when it is relatively high, bubbles lower long-run growth. Moreover, we examine the effects of the burst of bubbles, and show that the effects much depend on the degree of the pldgeability, i.e., the quality of financial system.

Key words: Asset Bubbles, Endogenous Growth, and Financial Frictions

[†]Tomohiro Hirano, Financial Research and Training Center, Financial Services Agency, The Japanese Government Email: tomohih@gmail.com

[‡]Noriyuki Yanagawa, Graduate School of Economics, The University of Tokyo, Email: yanagawa@e.u-tokyo.ac.jp

1 Introduction

Many countries have experienced large movements in asset prices called asset bubbles. The boom and bust of asset bubbles is associated with significant fluctuations in real economic activity. A notable example is the recent global economic up and down before and after the financial crisis. For this reason, it has been of great concern for many economists or policy makers to understand why bubbles emerge and how they affect economies. For example, it is recognized that emerging market economy often experienced bubble-like dynamics. As explored by Caballero and Krishnamurthy (2006), the financial imperfection or less developed financial market is a key element of the existence of bubbles in emerging market economy. However, if the financial imperfection is the reason of bubbles, why do less developed countries such as African countries not experience the bubbly economy? In order to examine this point, we address the following questions: How is the emergence of bubbles related to financial conditions? Are bubbles more likely to occur in financially developed economies or less-developed ones? How do bubbles affect investment and long run growth, in other words, do they crowd investment in and enhance growth or do they crowd it out and impair growth? How does the effects of burst of bubbles on the growth rate become different according to financial conditions?

To answer these questions, we develop an infinitely lived agent model of endogenous growth with two important characteristics. One is the presence of heterogeneous investment opportunities. In our model, there are two types of investments. Some of the entrepreneurs have high productive investment (H-projects) and the others have low productive ones (L-projects). A key assumption is that the productivity of each entrepreneur's investment changes over time. That is, the entrepreneurs who have L-projects (H-projects) in the current period may have H-projects (L-projects) in the future with some probabilities. This assumption allows for a situation that the entrepreneurs purchase bubbles when they have L-projects, because the rate of return of them is greater than that of L-projects, and use them for H-projects when they meet the projects.

The other characteristics is the presence of financial frictions. We assume that the entrepreneurs can pledge only a fraction of the return from the investment to creditors. This fraction reflects the degree of financial market imperfections. Higher (lower) fraction implies that the degree of financial imperfection is low (high). Recently, this financial imperfection has been

focused in many theoretical and empirical papers. Because of this limited pledgeability, the entrepreneurs are credit constrained when they have H-projects, and their investment depends on the amount of their net worth. Moreover, bubbles affect the net worth level crucially.

Under these two key frameworks, the theoretical challenges of this paper are twofold. First, we identify the relation between the existence of bubbles and the degree of financial imperfection. Second, we characterize the macroeconomic consequences of bubbles according to the degree of imperfection.

Concerning the existence of bubbles in infinite horizon economies, it is commonly thought that bubbles can not arise in deterministic sequential market economies with a finite number of infinitely lived agents (Tirole, 1982). In Tirole model, the financial market is assumed to be perfect, that is, agents are allowed to borrow and lend freely. Tirole has shown that in such an environment, no equilibrium with bubbles exists. This result holds true in our model too. That is, when the pledgeability is equal to one, which means financial market is perfect, bubbles can not arise. However, if the pledgeability is less than one and the financial market is imperfect, our model shows that even in infinite horizon economies, bubbles can occur. A complete characterization of the steady state with bubbles is provided. Our model shows that bubbles can not exist if the pledgeability is sufficiently low or sufficiently high. This suggests that bubbles can not occur in financially underdeveloped or well-developed economies. They can only occur in financially intermediate-developed ones (intermediate pledgeability level). This result suggests that improving the financial market condition might enhance the emergence of bubbles if the initial condition of the financial market is underdeveloped.[1] Of course, the possibility of bubbles in infinite horizon economies with borrowing constraint has been recognized even in the previous papers, such as Scheinkman and Weiss (1986), Kocherlakota (1992), Santos and Woodford (1997), and Hellwig and Lorenzoni (2009). All of these studies are based on an endowment economy, however. Our paper's contribution is that we consider a heterogenous investments model and give a full characterization on the relation between the existence of bubbles and financial frictions in the production economy.

Our model also has macroeconomic implications of bubbles. The conven-

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[1] In this sense, ours is related to Matsuyama (2007, 2008), in which Matsuyama shows that a better credit market might be more prone to financing what he calls bad investments, which do not have positive spillover effects on future generations.

tional wisdom (Samuelson, 1958; Tirole, 1985) suggests that bubbles crowd investment out and lower output. In the traditional view, the financial market is perfect, and all the saving in the economy flow to investment. In such a situation, once bubbles appear in the economy, they crowd savings away from investment. Saint-Paul (1992), Grossman and Yanagawa (1993), and King and Ferguson (1993) extend Samuelson-Tirole model to economies with endogenous growth, and have shown that bubbles reduce investment and retard long run economic growth.[23]

More recently, however, some researchers such as Woodford (1990), Caballero and Krishnamurthy (2006), Kiyotaki and Moore (2008), Kocherlakota (2009), and Martin and Ventura (2010) develop a model with financial frictions, and show that bubbles crowd investment in and increase output. In these studies, because of the presence of financial market imperfections, enough resources can not be transfered to those who have investment from those who do not. As a result, underinvestment occurs. Bubbles help to transfer resources between them.

A natural question is why do we observe such contradicting views? Where does such a discrepancy in opinion come from? The second contribution of our paper is to unify these two conflicting views within one theoretical framework. The key point is the limited pledgeability. In our model, when bubbles emerge, the interest rate rises. This produces two competing effects. One is a crowd-out effect. That is, the entrepreneurs who have H-projects are forced to cut back on their investment because the borrowing constraint becomes tight. This reduces their net worth and crowd investment out. On the other hand, the rise in the interest rate produces a crowd-in effect too. That is, in our model, the entrepreneurs who have L-projects purchase bubbles, and when they meet an opportunity to invest in H-projects in the future, they sell bubbles. Since the rate of return of bubbles is high together with the rise in the interest rate, by selling bubbles, their net worth increases, which crowds investment in. If the latter dominates the former, bubbles enhance growth. Otherwise, they lower it. We show that the balance between these

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[2] This crowd-out effect of bubbles has been criticized, because it seems inconsistent with historical evidence that investment and economic growth rate tend to surge when bubbles pop up, and then stagnate when they burst.

[3] Olivier (2000) shows that the conclusions in Saint-Paul (1992), Grossman and Yanagawa (1993), and King and Ferguson (1993) crucially depend on the type of asset that is being speculated on. Bubbles on equity markets can be growth-enhancing while bubbles on unproductive assets are growth-impairing.

two conflicting forces changes according to the degree of financial imperfection. In other words, there is a threshold value of the degree of pledgeability below which bubbles are growth-enhancing and above which they are growth-impairing.

The crowd-in effect caused by the rise in the interest rate in our model is in sharp contrast with a standard investment theory. In the standard view, when the interest rate goes up, firms always want to invest less. However, in our model with imperfect financial markets, the rise in the interest rate generates not only the crowd-out effect as in the traditional view, but also the crowd-in effect by increasing net worth. In this respect, our paper shares with Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) the idea that an improvement in borrower net worth, which is caused by positive productivity shocks in their paper, produces positive balance sheet effects, thereby increasing investment. In our paper, the rise in the interest rate caused by bubbles generates positive balance sheet effects.

Moreover, in our model, bubbles may improve the efficiency in production. Without bubbles and when the pledgeability is relatively low, the financial markets can not transfer all the saving of the entrepreneurs who have L-projects to the ones who have H-projects. As a result, the entrepreneurs with L-projects cannot use their savings efficiently and end up with investing in their own projects with low returns. However, the emergence of bubbles eliminates the production of those L-projects, transferring more resources to the entrepreneurs with productive investment. This also implies that, when the pledgeability is relatively low, bubbles burst produces productive inefficiency.

Our paper is related to a number of research on bubbles. In the traditional literature (Samuelson; 1958, Tirole; 1985), bubbles can only occur in an equilibria where overinvestment occurs. In our study, however, overinvestment does not occur, but underinvestment occurs because of limited pledgeability, which implies that some of the saving in the economy flow to L-projects. Bubbles can arise in such equilibria as well as in equilibria where all the savings are allocated to H-projects.

The role of bubbles in supporting investment when there is limited pledgeability has been addressed for example in Woodford (1990), Kocherlakota (2009), and Martin and Ventura (2010). In their study, none of the returns from investment can be pledgeable, and thus nobody can borrow and lend.[4]

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[4]In Kocherlakota (2009), agents can borrow against bubbles in land prices. However,

In our study, however, the entrepreneurs are allowed to borrow as long as debts are secured by collateral. In this sense, our study examines a general case of their arguments.

Caballero and Krishnamurthy (2006) develop a theory of stochastic bubbles in emerging markets using an overlapping generations model. In their model, even if bubbles emerge, the loan rate does not rise, and thus the crowd-out effect does not operate. However, in our model, the loan rate increases, which produces the crowd-out effect.

Farhi and Tirole (2010) are closely related to ours in the point that bubbles affect investment through a rise in the interest rate. In their model, the effects of the rise in the interest rate on investment crucially depends on what they call outside liquidity. Positive outside liquidity means that the entrepreneurs are net receivers of interest, in which case bubbles increase the interest income and their net worth, thereby crowding investment in, while negative means that they need to repay interest on net, in which case the opposite happens. In our model, however, the effects of the rise in the interest rate on investment depends on the pledgeability, and we can characterize the effects of bubbles by the degree of the pledgeability.

Kiyotaki and Moore (2008) are also related to ours. In their theory, since fiat money (bubbles) has more advantage in lubricating exchange, people hold money, even though the rate of return of it is low. They emphasize the role of money (bubbles) as a medium of exchange. In our model, however, we focus on the role of bubbles as a store of value. The entrepreneurs buy bubbles, because the rate of return of them is high.

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without such bubbles, nobody can borrow and lend.

2   The Model

Consider a discrete-time economy with one homogenous good and a continuum of entrepreneurs. A typical entrepreneur has the following expected utility,

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \log c_t^i \right], \tag{1}$$

where i is the index for each entrepreneur, and c_t^i is the consumption of him at date t. $\beta \in (0, 1)$ is the subjective discount factor, and $E_0[x]$ is the expected value of x conditional on information at date 0.

At each date, each entrepreneur meets high productive investment projects (thereafter H-projects) with probability p, and low productive investments (L-projects) with probability $1 - p$.[5] The investment technologies are as follows,

$$y_{t+1}^i = \alpha_t^i z_t^i, \tag{2}$$

where $z_t^i (\geq 0)$ is the investment level at date t and y_{t+1}^i is the output at date $t + 1$. α_t^i is the marginal productivity of investment at date t. $\alpha_t^i = \alpha^H$ if the entrepreneur has H-projects, and $\alpha_t^i = \alpha^L$ if he has L-projects. We assume $\alpha^H > \alpha^L$.[6] The probability p is exogenous, and independent across entrepreneurs and over time. At the beginning of each date t, the entrepreneur knows his own type at date t, whether he has H-projects or L-projects. Assuming that the initial population measure of each type of the entrepreneur is one at date 0, the population measure of each type after date 1 is $2p$ and $2 - 2p$, respectively. We call the entrepreneurs with H-projects (L-projects) "H-entrepreneurs" ("L-entrepreneurs").

In this economy, we assume that because of frictions in a financial market,

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[5]A similar setting is used in in Kaas (2009), Kiyotaki and Moore (2008), and Kocherlakota (2009). In Woodford (1990), the entrepreneurs have investment opportunities in alternating periods.

[6]We can also consider the model where capital goods is produced by the investment technology. For example, let $k_{t+1}^i = \alpha_t^i z_t^i$ be the investment technology, where k is capital goods. Capital fully depreciates in one period. Consumption goods is produced by the following aggregate production function: $Y_t = K_t^\sigma N_t^{1-\sigma} \bar{k}_t^{1-\sigma}$, where K and N are the aggregate capital and labor input, and $\bar{k}$ is per-labor capital of the economy, capturing the externality in order to generate endogenous growth. In this type of the model, we can obtain the same results as this paper.

the entrepreneur can pledge at most a fraction θ of the future return from his investment to creditors.[7] In such a situation, in order for debt contracts to be credible, debts repayment cannot exceed the pledgeable value. That is, the borrowing constraint becomes

$$r_t b_t^i \leq \theta \alpha_t^i z_t^i, \tag{3}$$

where r_t and b_t^i are the gross interest rate, and the amount of borrowing at date t, respectively. The parameter $\theta \in [0,1]$, which is assumed to be exogenous, can be naturally taken to be the degree of imperfection of the financial market.

The entrepreneur's flow of funds constraint is given by

$$c_t^i + z_t^i = y_t^i - r_{t-1} b_{t-1}^i + b_t^i. \tag{4}$$

The left hand side of (4) is expenditure on consumption and investment. The right hand side is financing which comes from the return from investment in the previous period minus debts repayment, and borrowing. We define the net worth of the entrepreneur as $e_t^i \equiv y_t^i - r_{t-1} b_{t-1}^i$.

2.1 Equilibrium

Let us denote the aggregate consumption of H-and L-entrepreneurs at date t as $\sum_{i \in H_t} c_t^i \equiv C_t^H$ and $\sum_{i \in L_t} c_t^i \equiv C_t^L$, where H_t and L_t mean a family of H- and L-entrepreneurs at date t. Similarly, let $\sum_{i \in H_t} z_t^i \equiv Z_t^H$, $\sum_{i \in L_t} z_t^i \equiv Z_t^L$, $\sum_{i \in H_t} b_t^i \equiv B_t^H$, and $\sum_{i \in L_t} b_t^i \equiv B_t^L$ be aggregate investment, and borrowing of each type. Then, the market clearing condition for goods, and the market clearing condition for credit are

$$C_t^H + C_t^L + Z_t^H + Z_t^L = Y_t, \tag{5}$$

$$B_t^H + B_t^L = 0, \tag{6}$$

where $\sum_{i \in (H_t \cup L_t)} y_t^i \equiv Y_t$ is the aggregate output at date t.

The competitive equilibrium is defined as a set of prices $\{r_t\}_{t=0}^{\infty}$ and quantities $\{c_t^i, b_t^i, z_t^i, y_{t+1}^i, C_t^H, C_t^L, B_t^H, B_t^L, Z_t^H, Z_t^L, Y_t\}_{t=0}^{\infty}$, such that (i) the market clearing conditions, (5) and (6), are satisfied, and (ii) each entrepreneur

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[7]See Hart and Moore (1994) and Tirole (2006) for the foundations of this setting.

chooses consumption, borrowing, investment, and output to maximize his expected utility (1) under the constraints (2), (3), (4), and the following transversality condition.

$$\lim_{t\to\infty}\beta^t\frac{1}{c_t^i}(z_t^i - b_t^i) = 0 \tag{7}$$

Since the utility function is log-linear, each entrepreneur consumes a fraction $1 - \beta$ of the net worth every period, that is, $c_t^i = (1 - \beta)(y_t^i - r_{t-1}b_{t-1}^i).$[8] Thus, the entrepreneur's maximization problem is replaced with the following one,

$$\underset{z_t^i, b_t^i}{Max} \quad \alpha_t^i z_t^i - r_t b_t^i, \quad subject \; to \; z_t^i = \beta e_t^i + b_t^i, and \; r_t b_t^i \leq \theta \alpha_t^i z_t^i.$$

Given e_t^i, and r_t, the entrepreneur chooses z_t^i and b_t^i to maximize the net worth of the next period $\alpha_t^i z_t^i - r_t b_t^i$ subject to the flow of funds, and the borrowing constraints. If and only if $\alpha_t^i > r_t$, the borrowing constraint, (3), becomes binding because the rate of return on investment is strictly greater than the interest rate.

2.2 The Case with $\theta = 1$: Perfect Financial Market

First, let us consider the case with a perfect financial market, that is $\theta = 1$. In this case, if $\alpha^H > r_t$, H-entrepreneur must be willing to borrow an unlimited amount. On the other hand, if $\alpha^L < \alpha^H < r_t$, nobody would borrow. Thus, the equilibrium interest rate must be

$$r_t = \alpha^H.$$

Since each entrepreneur saves a fraction β of the net worth every period, the aggregate saving in the economy is βY_t, which flows to finance H-projects.

Thus, the law of motion of the aggregate output becomes

$$Y_{t+1} = \alpha^H \beta Y_t, \tag{8}$$

and we get the growth rate of the aggregate output as follows,

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[8]See, for example, chapter 1.7 of Sargent (1988).

$$g_t \equiv \frac{Y_{t+1}}{Y_t} = \alpha^H \beta. \tag{9}$$

We observe that the growth rate is independent of wealth distribution. Since we have assumed simple linear production functions, the interest rate is equal to the marginal productivity of H-projects and the steady state growth rate is positive. Moreover, the interest rate is strictly greater than the growth rate of the economy and the trasversarity condition is satisfied, as explored in the traditional literature.

2.3 The Case with $\theta < 1$: Imperfect Financial Market

Next, we examine the case with an imperfect financial market, that is $\theta < 1$. Even if $\theta < 1$, all of the total saving is used for H-projects and $r_t = \alpha^H$ as long as θ is sufficiently high. Hence, in this section, we focus on the case where the interest rate is strictly lower than α^H and the borrowing constraint is binding for H-enterpreneurs,

$$\alpha^L \le r_t < \alpha^H.$$

In equilibrium, the interest rate must be at least as high as α^L, since nobody lends to the projects if $r_t < \alpha^L$. We will explore later that which range of θ satisfies the above condition about r_t.

Since each entrepreneur consumes a fraction $1 - \beta$ of the net worth, we get the following relation from (4),

$$z_t^i - b_t^i = \beta(y_t^i - r_{t-1}b_{t-1}^i).$$

On the other hand, as long as $r_t < \alpha^H$, the borrowing constraint, (3), is binding and b_t^i satisfies the following relation,

$$b_t^i = \frac{\theta \alpha^H}{r_t} z_t^i.$$

From those two relations, we can derive the following investment function for the H-entrepreneurs at date t,

$$z_t^i = \frac{\beta(y_t^i - r_{t-1}b_{t-1}^i)}{1 - \dfrac{\theta\alpha^H}{r_t}}. \tag{10}$$

This is a popular investment function under financial constraint problems.[9] We see that the investment equals the leverage, $1/\left[1 - (\theta\alpha^H/r_t)\right]$, times savings, $\beta(y_t^i - r_{t-1}b_{t-1}^i)$. The leverage increases with θ and is greater than one in equilibrium. This implies that when θ is larger, H-entrepreneurs can finance more investment, z_t^i.

By aggregating (10), we get

$$Z_t^H = \frac{\beta E_t^H}{1 - \dfrac{\theta\alpha^H}{r_t}}, \tag{11}$$

where $\sum_{i \in H_t} e_t^i \equiv E_t^H$ is the aggregate net worth of H-entrepreneurs at date t. The movement of the aggregate net worth of H-entrepreneurs evolves according to the following equation.

$$E_t^H = p(\alpha^H Z_{t-1}^H - r_{t-1}B_{t-1}^H) + p(\alpha^L Z_{t-1}^L - r_{t-1}B_{t-1}^L) = pY_t. \tag{12}$$

The first term of (12) represents the aggregate net worth of the entrepreneurs who continue to have H-projects from the previous period (we call H-H entrepreneurs). The second term represents the aggregate net worth of the entrepreneurs who switch from the state with L-projects to the state with H-projects (we call L-H entrepreneurs). Since every entrepreneur has the same chance to meet H-projects at each period, the aggregate net worth of H-entrepreneurs at date t is a fraction p of the aggregate output at date t. Hence, (11) can be rewritten as

$$Z_t^H = \frac{\beta p Y_t}{1 - \dfrac{\theta\alpha^H}{r_t}}. \tag{13}$$

For L-entrepreneurs, if $r_t = \alpha^L$, lending and borrowing to invest are indifferent. Thus, how much they invest in their own projects is indeterminate at

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[9]See, for example, Bernanke et al. (1999), Holmstrom and Tirole (1997), and Kiyotaki and Moore (1997).

an individual level. However, their aggregate investment's level is determined by the goods market clearing condition,

$$Z_t^H + Z_t^L = \beta Y_t. \tag{14}$$

This implies that the aggregate investment of L-entrepreneurs equals the aggregate saving minus the aggregate investment of H-entrepreneurs. On the other hand, if $r_t > \alpha^L$, Z_t^L must be zero. Thus, the following conditions must be satisfied.

$$Z_t^L(r_t - \alpha^L) = 0, \ Z_t^L \geq 0, \ r_t - \alpha^L \geq 0. \tag{15}$$

The aggregate output is

$$Y_{t+1} = \alpha^H Z_t^H + \alpha^L Z_t^L,$$

and can be rewritten as

$$Y_{t+1} = \beta \alpha^H Y_t - (\alpha^H - \alpha^L)Z_t^L.$$

Hence, the growth rate of Y_t becomes as follows,

$$g_t \equiv \frac{Y_{t+1}}{Y_t} = \beta \alpha^H - (\alpha^H - \alpha^L)\beta l_t, \tag{16}$$

where $l_t \equiv Z_t^L/\beta Y_t$, the ratio of the low productive investment to the total investment. The interpretation of this relation is simple. As long as the amount of L-projects, l_t, is zero, the total saving is allocated to the H-projects, and the growth rate of this economy becomes $\beta \alpha^H$, which is just same as that under the perfect financial market. If $l_t > 0$, however, the difference of productivity between H-projects and L-projects, $\alpha^H - \alpha^L$, decreases the growth rate and g_t becomes $\beta \alpha^H - (\alpha^H - \alpha^L)\beta l_t$.

Next, we examine the equilibrium level of l_t and how the equilibrium l_t is affected by the degree of financial imperfection, θ. Since $l_t \equiv Z_t^L/\beta Y_t = (\beta Y_t - Z_t^H)/\beta Y_t$, we can rewrite l_t and g_t as follows,

$$l_t = 1 - \frac{p}{1 - \dfrac{\theta \alpha^H}{r_t}} = \frac{r_t(1-p) - \theta \alpha^H}{r_t - \theta \alpha^H} = l(r_t, \theta), \tag{17}$$

$$g_t \equiv \frac{Y_{t+1}}{Y_t} = \alpha^H \beta - (\alpha^H - r_t)\beta l(r_t, \theta). \tag{18}$$

From (15), the following relations must be satisfied,

$$(r_t - \alpha^L)\frac{r_t(1-p) - \theta\alpha^H}{r_t - \theta\alpha^H} = 0, \ l_t \geq 0, \ r_t - \alpha^L \geq 0. \tag{19}$$

Those imply that r_t must be α^L or $\theta\alpha^H/(1-p)$. If $\theta\alpha^H/(1-p) \leq \alpha^L$, $r_t = \alpha^L$ since r_t cannot be lower than α^L. On the other hand, if $\theta\alpha^H/(1-p) > \alpha^L$, $r_t = \theta\alpha^H/(1-p)$ since l_t cannot be negative. Hence, we get the following relation.

$$r_t = r(\theta) = \begin{cases} \alpha^L, & \text{if } 0 \leq \theta < (1-p)\dfrac{\alpha^L}{\alpha^H}, \\[2ex] \dfrac{\theta\alpha^H}{1-p}, & \text{if } (1-p)\dfrac{\alpha^L}{\alpha^H} \leq \theta < 1-p, \end{cases}$$

and

$$l_t = l(r(\theta), \theta) = \begin{cases} \dfrac{\alpha^L(1-p) - \theta\alpha^H}{\alpha^L - \theta\alpha^H}, & \text{if } 0 \leq \theta < (1-p)\dfrac{\alpha^L}{\alpha^H}, \\[2ex] 0 & \text{if } (1-p)\dfrac{\alpha^L}{\alpha^H} \leq \theta < 1-p. \end{cases}$$

From those results, we get the following equilibrium growth rate.

$$g_t = g(\theta) = \begin{cases} \beta\alpha^H - (\alpha^H - \alpha^L)\beta\dfrac{\alpha^L(1-p) - \theta\alpha^H}{\alpha^L - \theta\alpha^H}, & \text{if } 0 \leq \theta < (1-p)\dfrac{\alpha^L}{\alpha^H}, \\[2ex] \beta\alpha^H, & \text{if } (1-p)\dfrac{\alpha^L}{\alpha^H} \leq \theta < 1-p, \end{cases} \tag{20}$$

(20) implies that the growth rate of the economy, g_t, is an increasing function of θ, because more savings flow to H-projects from L-projects through the relaxation of the borrowing constraint, which improves the aggregate total factor productivity and enhance growth.[10]Moreover, from the above

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[10]The recent macroeconomic literature emphasizes the role of TFP in accounting for

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relation, we can find that if $\theta \geq 1 - p$, $r_t = \alpha^H$ and $g_t = \beta\alpha^H$.

In summary, we get the following proposition.

Proposition 1 *When $\theta < 1$ and bubbles do not exist, the equilibrium interest rate, r_t, and the equilibrium growth rate, g_t, are the following increasing functions of θ, respectively.*

$$r_t = r(\theta) = \begin{cases} \alpha^L, & \text{if } 0 \leq \theta < (1-p)\dfrac{\alpha^L}{\alpha^H}, \\[3mm] \dfrac{\theta\alpha^H}{1-p}, & \text{if } (1-p)\dfrac{\alpha^L}{\alpha^H} \leq \theta < 1 - p, \\[3mm] \alpha^H, & \text{if } 1 - p \leq \theta. \end{cases}$$

$$g_t = g(\theta) = \begin{cases} \beta\alpha^H - (\alpha^H - \alpha^L)\beta\dfrac{\alpha^L(1-p) - \theta\alpha^H}{\alpha^L - \theta\alpha^H}, & \text{if } 0 \leq \theta < (1-p)\dfrac{\alpha^L}{\alpha^H}, \\[3mm] \beta\alpha^H, & \text{if } (1-p)\dfrac{\alpha^L}{\alpha^H} \leq \theta < 1 - p, \\[3mm] \beta\alpha^H, & \text{if } 1 - p \leq \theta. \end{cases}$$

If $0 \leq \theta < (1-p)\alpha^L/\alpha^H$, the degree of pledgeability is so small, i.e., financial frictions are so severe. Then, it is difficult for the financial system to transfer all the savings of L-entrepreneurs to H-projects. As a result, L-entrepreneurs hold idle savings and end up with investing such idle savings in their own investment with low returns. The severer financial frictions are, the more L-projects are financed in the economy. However, as financial frictions improve, more savings flow to H-projects. This improvement in saving allocations increases the aggregate total factor productivity, which leads to higher growth. On the other hand, the interest rate is suppressed at a low level of α^L because of the severity of financial frictions.

If $(1-p)\alpha^L/\alpha^H \leq \theta < 1 - p$, L-entrepreneurs can lend all of their savings to H-entrepreneurs, even though the financial market is still imperfect. As a result, only H-entrepreneurs invest. Hence, the economy's growth rate is

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business cycles or growth. In our model, the aggregate TFP is endogenously determined depending on saving allocations between H-projects and L-projects, which in turn depends on θ in the steady state.

$\beta \alpha^H$. In this region, together with an improvement in financial frictions, the interest rate rises due to the tightness in the financial market. Note that the borrowing constraint is still binding for H-entrepreneurs in this region, because the interest rate is strictly lower than the rate of return on H-projects.

If $1 - p \le \theta$, and the degree of pledgeability is large enough, the interest rate is equal to the rate of return on H-projects. The interest rate becomes equal to the rate of return on H-projects, and the borrowing constraint no longer binds for H-entrepreneurs. The financial system can allocate all the saving in the economy to H-projects, and resource allocation is efficient, even though θ is strictly less than one.[11] In this region, the characteristics of the economy is the same as the one with the perfect financial market.

Figure 1 depicts this situation. In horizontal axis, we take θ, and in vertical axis, we take g and r. It is shown that the relation between g and θ is nonlinear. As we will show below, this nonlinearity plays a crucial role in creating regions where bubbles can arise (bubble regions), or regions where they can not arise (non-bubble regions).

3   Existence of Asset Bubbles

Now we describe the economy with asset bubbles (we call "bubble economy"). Let x_t^i be the level of bubble assets purchased by type i entrepreneur at date t, and let P_t be the per unit price of them at date t in terms of consumption goods. In the bubble economy, each entrepreneur faces the following three constraints; flow of funds condition, the borrowing constraint, and the short sale constraint:

$$c_t^{*i} + z_t^{*i} + P_t x_t^i = y_t^{*i} - r_{t-1}^* b_{t-1}^{*i} + b_t^{*i} + P_t x_{t-1}^i, \tag{21}$$

$$r_t^* b_t^{*i} \le \theta \alpha_t^i z_t^{*i} + \theta P_{t+1} x_t^i, \tag{22}$$

$$x_t^i \ge 0, \tag{23}$$

where * represents the case of bubble economy. Both sides of (21) include bubbles. $P_t x_{t-1}^i$ in the right hand side is the sales of the bubble assets, and

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[11] In $\theta \in (0, 1-p)$, H-entrepreneurs earn $\alpha^H (1 - \theta) / \left(1 - \theta \alpha^H / r_t\right)$, which is strictly greater than r_t that L-entrepreneurs earn. Thus, income distribution is different between the entrepreneurs. However, in $\theta \in [1-p, 1]$, both entrepreneurs earn the same rate of return, which is α^H. Hence, there is no difference in income distribution.

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$P_t x_t^i$ in the left hand side is the new purchase of them. We define the net worth of the entrepreneur in the bubble economy as $e_t^{*i} \equiv y_t^{*i} - r_{t-1}^* b_{t-1}^{*i} + P_t x_{t-1}^i$. (22) is the borrowing constraint under the bubble economy. For simplicity, we assume here that the degree of pledgeability of the return from the bubbles is equal to that of the return from H or L investments, θ. Even if we assume that the pledgeable fraction of bubbles' return, $\theta^x < 1$, is different from that of the investment's return, θ, our results which will be explained below are not affected.[12]

We should add a few remarks about the short sale constraint (23). As Kocherlakota (1992) has shown, the short sale constraint plays an important role for the existence of bubbles in deterministic economies with a finite number of infinitely lived agents. Without the constraint, bubbles always represent an arbitrage opportunity for an infinitely-lived agent; he can gain by permanently reducing his holdings of the asset. However, it is well-known that in such economies, equilibria can only exist if agents are constrained not to engage in Ponzi schemes. Kocherlakota (1992) has demonstrated that the short sale constraint is one of no-Ponzi-game conditions and hence, it can support bubbles by eliminating the agent's ability to permanently reduce his holdings of the asset.[13]

In order that bubble assets are held in equilibrium, the rate of return of bubbles has to be equal to the interest rate:

$$r_t^* = \frac{P_{t+1}}{P_t}. \tag{24}$$

Each entrepreneur chooses the levels of consumption, investment, output, borrowing, and bubble assets $\left\{ c_t^{*i}, z_t^{*i}, y_{t+1}^{*i}, b_t^{*i}, x_t^i \right\}$, to maximize the expected utility (1) subject to (21), (22), and (23), given the interest rate and the current and future price of bubbles, r_t^*, P_t, and P_{t+1}. Moreover, on the optimal feasible plan, the following transversality condition must be satisfied.[14]

$$\liminf_{t \to \infty} \beta^t \frac{1}{c_t^{*i}} P_t x_t^i = 0 \tag{25}$$

---

[12] As will be explained below, the crucial point for our results is that the previous return from bubbles increases the net worth for a borrower.

[13] See Kocherlakota (1992) for details.

[14] See Kocherlakota (1992) for the transversality condition in economies with the short sale constraint.

As in the previous case, since the entrepreneur consumes a fraction $1 - \beta$ of the net worth, the maximization problem for the entrepreneur can be replaced with the following one:

$$\underset{z_t^i, b_t^i, x_t^i}{Max} \quad \alpha_t^i z_t^{*i} - r_t^* b_t^{*i} + P_{t+1} x_t^i$$

$$subject \ to \quad z_t^{*i} + P_t x_t^i = \beta e_t^{*i} + b_t^{*i}, \ (22), \ (23), (25).$$

As in the previous section, we focus on the case where the equilibrium interest rate is strictly lower than the productivity of H-projects, that is, L-entrepreneurs purchase bubbles and the borrowing constraint of H-entrepreneurs is binding. The investment function of the entrepreneurs who have H-projects at date t is

$$z_t^* = \frac{\beta e_t^{*i}}{1 - \dfrac{\theta \alpha^H}{r_t^*}}.$$

where $e_t^{*i} = -r_{t-1}^* b_{t-1}^{*i} + P_t x_{t-1}^i$ for L-H entrepreneurs, and $e_t^{*i} = y_t^{*i} - r_{t-1}^* b_{t-1}^{*i}$ for H-H entrepreneurs. For L-H entrepreneurs, since they purchased bubbles in the previous period, they are able to sell bubbles at the time they meet H-projects. As a result, their net worth becomes higher (compared to the bubbleless case) and boosts their investments, that is, the "balance seat effect" works.[15] Moreover, the expansion level of the investment is more than the direct increase of net worth because of the leverage effect. For H-H entrepreneurs, they are not able to take advantage of this merit, because they didn't buy bubbles in the previous period.

Next, we describe the aggregate economy. Since both H-and L-entrepreneurs consume a fraction $1 - \beta$ of their net worth, the goods market clearing condition can be written as

$$Z_t^{*H} + P_t X = \beta(Y_t^* + P_t X), \tag{26}$$

where $Y_t^* + P_t X$ and $\beta(Y_t^* + P_t X)$ are the aggregate wealth (total asset) and the aggregate saving in the bubble economy. X is the aggregate quantity of bubbles, which is exogenously fixed. We see that some of the aggregate saving flow to bubble assets as well as H-projects, which can be the source for raising

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[15]In Kiyotaki and Moore (1997), the rise in the land price increases the entrepreneurs' net worth, which results in producing balance sheet effects, thereby increasing investment. In this paper, bubbles play a similar role as the land in Kiyotaki and Moore's paper.

the interest rate in the financial market. The aggregate demand for bubbles, $P_t X$, is equal to the aggregate saving minus the aggregate investment of H-entrepreneurs, $\beta(Y_t^* + P_t X) - Z_t^{*H}$.

The aggregate investment function becomes,

$$Z_t^{*H} = \frac{\beta E_t^{*H}}{1 - \frac{\theta \alpha^H}{r_t^*}},$$

where $E_t^{*H} = p(Y_t^* - r_{t-1}^* B_{t-1}^{*H}) + p(P_t X - r_{t-1}^* B_{t-1}^{*L}) = p(Y_t^* + P_t X)$. The first term is the aggregate net worth of H-H entrepreneurs at date t and the second term represents the one of L-H entrepreneurs at date t. Hence, it can be written as

$$Z_t^{*H} = \frac{\beta p \left(Y_t^* + P_t X\right)}{1 - \frac{\theta \alpha^H}{r_t^*}}. \tag{27}$$

The aggregate wealth under the bubble economy can be written as,

$$Y_{t+1}^* + P_{t+1} X = \alpha^H Z_t^{*H} + r_t^* P_t X. \tag{28}$$

In order to characterize the economic growth rate, let us define as follows,

$$k_t = \frac{P_t X}{\beta(Y_t^* + P_t X)},$$

$$g_t^k = \frac{Y_{t+1}^* + P_{t+1} X}{Y_t^* + P_t X},$$

where k_t is the relative size of bubbles and g_t^k is the growth rate of the aggregate wealth, $Y_t^* + P_t X$. From (27) and these definitions, (28) can be written as

$$g_t^k = \alpha^H \frac{\beta p}{1 - \frac{\theta \alpha^H}{r_t^*}} + r_t^* \beta k_t. \tag{29}$$

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Moreover, from (26), we get

$$\frac{\beta p}{1 - \frac{\theta \alpha^H}{r_t^*}} + \beta k_t = \beta. \tag{30}$$

From (29) (30), the growth rate of the aggregate wealth, g_t^k, becomes

$$g_t^k = \alpha^H \beta (1 - k_t) + r_t^* \beta k_t = \alpha^H \beta - (\alpha^H - r_t^*)\beta k_t.$$

Furthermore,

$$k_t = 1 - \frac{p}{1 - \frac{\theta \alpha^H}{r_t^*}} = \frac{r_t^*(1 - p) - \theta \alpha^H}{r_t^* - \theta \alpha^H} = l(r_t^*, \theta). \tag{31}$$

This means that given r_t and θ, the relative size of bubbles, k_t, is just equal to the relative size of L-projects, $l_t = l(r_t, \theta)$ under the bubbleless economy. Hence, we can derive the following growth rate function,

$$g_t^k(r_t^*, \theta) = \beta \alpha^H - (\alpha^H - r_t^*)\beta l(r_t^*, \theta).$$

This means the growth rate function in the bubble economy is just same as the bubbleless economy. However, the growth rate becomes different since the equilibrium interest rate is different.

Next we examine the determination process of r_t^*. From the definition of k_t

$$\frac{k_{t+1}}{k_t} = \frac{r_t}{g_t^k} = \frac{r_t}{\alpha^H \beta - (\alpha^H - r_t)\beta k_t}. \tag{32}$$

In order to satisfy the transversality condition (25),

$$\frac{k_{t+1}}{k_t} = \frac{r_t^*}{g_t^k} \leq 1 \tag{33}$$

should be satisfied. Moreover, if $k_{t+1}/k_t < 1$, the economy converges to the asymptotically bubbleless economy. Hence, in this paper, we focus on the case where $k_{t+1}/k_t = 1$, that is, the share of the bubble assets is constant

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under the steady state.[16] Hence, at each period, the following condition must be satisfied,

$$r^*(\theta) = \alpha^H \beta - (\alpha^H - r^*(\theta))\beta l(r^*(\theta), \theta). \tag{34}$$

From (31) and $k_{t+1}/k_t = 1$, we get the equilibrium interest rate and the relative size of bubbles as follows,

$$
\begin{aligned}
r_t^* &= r^*(\theta) = \alpha^H \frac{(1-\beta)\theta + p\beta}{1 - \beta + p\beta}, \\
k_t &= l(r^*(\theta), \theta) = \frac{\beta(1-p) - \theta}{\beta(1-\theta)}.
\end{aligned}
$$

Furthermore, since the growth rate of the total output, $g_t^* \equiv Y_{t+1}^*/Y_t^*$ is just equal to g_t^k from $k_{t+1}/k_t = 1$, we get

$$g_t^* = g_t^k = r^*(\theta) = \beta\alpha^H - (\alpha^H - r^*(\theta))\beta l(r^*(\theta), \theta) = \alpha^H \frac{(1-\beta)\theta + p\beta}{1 - \beta + p\beta}. \tag{35}$$

Obviously, the equilibrium growth rate is an increasing function of θ. An increase of θ decreases the relative size of bubbles, k_t, and raises the growth rate.

3.1 Existence condition of bubbles

In this subsection, we examine the existence condition of bubbles. For the existence of bubbles, the following two conditions must be satisfied. First, the equilibrium interest rate must not be lower than α^L at each period. Second, k_t must be non-negative. In other words. the following two conditions must be satisfied.

$$
\begin{aligned}
\alpha^L &\leq r^*(\theta) = \alpha^H \frac{(1-\beta)\theta + p\beta}{1 - \beta + p\beta} \leq \alpha^H, \\
k_t &= l(r^*(\theta), \theta) = \frac{\beta(1-p) - \theta}{\beta(1-\theta)} > 0.
\end{aligned}
$$

From these conditions, we get the following proposition.

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[16]In our model, the steady state equilibrium with bubbles is unstable, while the one without bubbles is stable. Thus, the bubble economy is vulnerable to shocks.

Proposition 2 *Bubbles can exist as long as θ salsifies the following condition,*

$$\underline{\theta} \equiv Max\left[\frac{\alpha^L - \beta\{\alpha^L + (\alpha^H - \alpha^L)p\}}{\alpha^H(1-\beta)}, 0\right] \le \theta < \overline{\theta} \equiv \beta(1-p).$$

Proof. If $\beta\alpha^H < \alpha^L$, the growth rate of this economy cannot be equal to the interest rate and bubbles cannot exist. This situation is the case where $\frac{\alpha^L - \beta\{\alpha^L + (\alpha^H - \alpha^L)p\}}{\alpha^H(1-\beta)} > \beta(1-p)$ and θ which satisfies the above condition does not exist. Thus, we focus on the cases where $\beta\alpha^H \ge \alpha^L$. When $\frac{\alpha^L - \beta\{\alpha^L + (\alpha^H - \alpha^L)p\}}{\alpha^H(1-\beta)} > 0$, $r^*(\underline{\theta}) = \alpha^L$ and $l(r^*(\underline{\theta}),\underline{\theta}) > 0$. Since $r^*(\theta)$ is a strictly increasing function of θ, $r^*(\theta) < \alpha^L$ and bubbles cannot exist if $\theta < \underline{\theta} = \frac{\alpha^L - \beta\{\alpha^L + (\alpha^H - \alpha^L)p\}}{\alpha^H(1-\beta)}$. On the other hand, $r^*(\theta) \ge \alpha^L$ if $\theta \ge \underline{\theta} = \frac{\alpha^L - \beta\{\alpha^L + (\alpha^H - \alpha^L)p\}}{\alpha^H(1-\beta)}$. When $\frac{\alpha^L - \beta\{\alpha^L + (\alpha^H - \alpha^L)p\}}{\alpha^H(1-\beta)} \le 0$, $\underline{\theta} = 0$ and $r^*(0) \ge \alpha^L$. But θ cannot be negative. Hence, we do not have to consider the case of $\theta < \underline{\theta}$ and $r^*(0) \ge \alpha^L$ as long as $\theta \ge \underline{\theta} = 0$. However, $l(r^*(\theta),\theta)$ is a decreasing function of θ and $l(r^*(\theta),\theta)$ becomes zero when $\theta = \beta(1-p) = \overline{\theta}$. Therefore, bubbles can exist as long as $\underline{\theta} \le \theta < \overline{\theta}$. ∎

From this proposition, we can understand that bubbles tend to exist when the degree of financial imperfection, θ, is in the middle range. In other words, improving the financial market condition might enhance the existence of bubbles when the initial condition of θ is low.[17] This result is quite contrast to the results in the previous literature such as Farhi and Tirole (2010), in which bubbles are more likely to emerge when the finacial market is more imperfect (when the pledgeability is more limited).

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[17]The papers such as Kaminsky and Reinhart (1999) and Allen and Gale (1999)) point out that financial liberalization causes bubbles. If we understand this from our model, one interpretation goes as follows. For instance, before financial liberalization, the economy is in non-bubble regions. After the liberalization, θ increases, and the borrowing constraint is relaxed, so that the economy enters bubble regions. Like this, we might be able to think of the increase in θ as a measure of financial liberalization.

Figure 2 is a typical case representing the relation between θ and bubble regions.[18] It is shown that if the degree of financial frictions is sufficiently large or small, bubbles can not exist. This suggests that in financially under-developed or well-developed economies, bubbles can not emerge. They can only arise in financially intermediate-developed ones.[19] An intuitive reason of this result is as follows. If θ is low, H-entrepreneurs cannot borrow sufficiently and the growth rate must be low even with bubbles. On the other hand, the interest rate cannot be lower than α^L since there is an opportunity to invest in L-projects even if θ is low. Hence, under very low θ level, the interest rate becomes higher than the growth rate and bubbles cannot exist. Since we assume heterogenous investment opportunities, the interest rate has the lowest bound and we can get the result which is different from the previous literature[20].

Moreover, we can characterize the existence condition by using the structure of the bubbleless economy. The existence condition of bubble is that the growth rate is not lower than the interest rate under the bubbleless economy. This condition is consistent with the existence condition in the previous literature such as Tirole(1985).

Proposition 3 *The necessary condition for the existence of bubble is that the equilibrium growth rate is not lower that the equilibrium interest rate*

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[18] Even though the growth rate is strictly greater than the interest rate, bubbles can not arise in the economy unless people expect that they are able to pass bubbles on to other people. This expectation is the sufficient condition for the existence of bubbles. Here, we assume that the condition is satisfied when bubbles appear.

[19] Readers may wonder why the phenomenon which looks like bubbles occurs repeatedly in the real world where the development of the financial system keeps increasing over time, even though our model suggests that bubbles do not appear in high θ regions. We propose one interpretation from our model. In the paper, we assume a common θ on both high and low investment. However, we can put different θ on those projects. In such a case, the important factor for the existence of bubbles is θ^H, which is placed on high profitable investment, not on low profitable investment. Taking this into account, think about the situation where the existing projects with α^L disappear, and new investment opportunities with higher profitability than the existing α^H appear into the economy. In such a situation, the θ which is placed on those new projects matters for the existence of bubbles. If the θ is low, the economy will get into bubble regions again even if it is in non-bubble regions with high θ before. In the real world, this process might repeat itself.

[20] Martin and Ventura (2010) assume two types of investment opportunities but bubbles may be able to exist under the economy without credit market. The crucial difference is they allow bubble creations at each period.

under the bubbleless economy.

Proof. If $\beta\alpha^H < \alpha^L$, bubbles cannot exist as explained in the proposition 1, and the growth rate under the bubbleless economy is lower than the interest rate under the bubbleless economy for any θ. Next we check the case where $\beta\alpha^H \geq \alpha^L$. From the definition of $\underline{\theta}$, the following relation is satisfied.

$$\beta\alpha^H - (\alpha^H - \alpha^L)\beta l^*(\alpha^L, \underline{\theta}) = \alpha^L.$$

This relation means that at $\underline{\theta}$, the growth rate under the bubbleless economy (the left hand side) is equal to the interest rate under the bubbleless economy (the right hand side). When θ is a little higher than $\underline{\theta}$ but smaller than $(1-p)\alpha^L/\alpha^H$, the growth rate under the bubbleless economy becomes higher than α^L but the interest rate is still α^L. Thus, the growth rate is higher than the interest rate under the bubbleless economy. If θ becomes higher than $(1 - p)\alpha^L/\alpha^H$, the interest rate becomes $\dfrac{\theta\alpha^H}{1 - p}$ which is higher than α^L and the growth rate becomes $\beta\alpha^H$. Hence, the growth rate becomes lower than the interest rate when θ becomes higher than $\bar{\theta} \equiv \beta(1-p)$. In summary, the growth rate is higher than the interest rate under the bubbleless economy when $\underline{\theta} \leq \theta \leq \bar{\theta}$, and $\underline{\theta} \leq \theta \leq \bar{\theta}$ is exactly the necessary condition of existence of bubbles. ∎

4   Asset Bubbles and Economic Growth

In this section, we examine how bubbles affect the economic growth rate. We will show here that the effect of bubbles on the growth rate is dependent upon the financial market condition, θ, even if the existence condition of bubbles is satisfied. We can derive that there is a threshold level of θ, $\theta^* = \dfrac{\alpha^L}{\alpha^H}\beta(1-p)$.

Proposition 4 *Let us define $\theta^* \equiv \dfrac{\alpha^L}{\alpha^H}\beta(1-p)$. If $\underline{\theta} \leq \theta \leq \theta^*$, the growth rate under the bubble economy is higher than that under the bubbleless economy at each period. If $\theta^* < \theta < \bar{\theta}$, the growth rate under the bubble economy is lower than that under the bubbleless economy at each period.*

Proof. From (18) and (35), $g_t = g(\theta) = \beta\alpha^H - (\alpha^H - \alpha^L)\beta\frac{\alpha^L(1-p)-\theta\alpha^H}{\alpha^L - \theta\alpha^H}$, and $g_t^* = r^*(\theta) = \alpha^H\frac{(1-\beta)\theta+p\beta}{1-\beta+p\beta}$. We derive θ which satisfies $g_t = g_t^*$, that is

$$\beta\alpha^H - (\alpha^H - \alpha^L)\beta\frac{\alpha^L(1-p)-\theta\alpha^H}{\alpha^L - \theta\alpha^H} = \alpha^H\frac{(1-\beta)\theta+p\beta}{1-\beta+p\beta}.$$

This relation can be rewritten as

$$\theta^2 - \frac{-p\beta\alpha^H + \alpha^L(1-\beta) + \beta\alpha^L(1-\beta+p\beta)}{\alpha^H(1-\beta)}\theta$$
$$+\frac{-p\beta\alpha^L + \left[\alpha^L + (\alpha^H - \alpha^L)p\right]\beta\frac{\alpha^L}{\alpha^H}(1-\beta+p\beta)}{\alpha^H(1-\beta)} = 0, \tag{36}$$

and we already know that $\theta = \underline{\theta} = \dfrac{\alpha^L(1-\beta+p\beta)-p\beta\alpha^H}{\alpha^H(1-\beta)}$ satisfies this (36). Thus,

$$(\theta - \underline{\theta})(\theta - \theta^*) = 0$$

is equal to the above quadratic function (36). In other words,

$$\underline{\theta} + \theta^* = \frac{-p\beta\alpha^H + \alpha^L(1-\beta) + \beta\alpha^L(1-\beta+p\beta)}{\alpha^H(1-\beta)}$$

$$\underline{\theta}\theta^* = \frac{-p\beta\alpha^L + \left[\alpha^L + (\alpha^H - \alpha^L)p\right]\beta\frac{\alpha^L}{\alpha^H}(1-\beta+p\beta)}{\alpha^H(1-\beta)}.$$

By solving these equations, we can derive that

$$\theta^* = \frac{\alpha^L}{\alpha^H}\beta(1-p).$$

Furthermore, from the quadratic function (36), we can derive that $g_t < g_t^*$ if $\theta < \theta^*$, and $g_t > g_t^*$ if $\theta > \theta^*$. ∎

Proposition 5 implies that in the economies within the bubble regions and with relatively low θ, bubbles enhance growth while in the economies with relatively high θ, they impede it. Here we explain an intuitive reason of this

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result. From (20) and (35), we get that

$$g_t = \beta\alpha^H - (\alpha^H - \alpha^L)\beta l(\alpha^L, \theta)$$

$$g_t^* = \beta\alpha^H - (\alpha^H - r^*(\theta))\beta l(r^*(\theta), \theta)$$

The difference between the two growth rates mainly comes from the difference in the interest rate. When bubbles appear in the economy, the interest rate rises, which produces two competing effects. One is a crowd-out effect. That is, H-entrepreneurs are forced to cut back on their investment because the borrowing constraint becomes tight. This reduces the growth rate of the aggregate net worth of H-entrepreneurs, which in turn crowds investment out. The other is a crowd-in effect. Due to the hike in the interest rate, the interest income for L-entrepreneurs, which is the returns from purchasing bubbles, rises. This increases the growth rate of the aggregate net worth of H-entrepreneurs, which in turn crowds investment in. More precisely, the difference of the growth rates can be written as follows.

$$g_t^* - g_t = (r^*(\theta) - \alpha^L)\beta l(\alpha^L, \theta) - (\alpha^H - r^*(\theta))\beta(l(r^*(\theta), \theta) - l(\alpha^L, \theta)). \quad (37)$$

In this formulation, the first term of the right hand side represents the crowd-in effect and the second term of the right hand side represents the crowd-out effect. Since $r^*(\theta) \geq \alpha^L$, the first term is (weakly) positive. This term captures the effect that an increase of the interest rate raises the income of the entrepreneurs who invested in the bubbles and enhances the economic growth rate. More precisely, if there is no bubble, L-entrepreneurs have to invest in L-projects, $l(\alpha^L, \theta)$, as long as the borrowing constraint of the H-entrepreneurs is binding. If they have a chance to invest in bubble assets instead of L-projects, they can earn $r^*(\theta)l(\alpha^L, \theta)$ instead of $\alpha^L l(\alpha^L, \theta)$. This increased earning contributes to enhance the H-investment at the time they become H-entrepreneurs in the future. Thus, this income effect increases the growth rate by the increase of H-investment.

The second term represents the crowd-out effect. As you can see from (17), $l(r^*, \theta)$ is an increasing function of r^*. A rise of the interest rate tightens the borrowing constraint of the H-entrepreneuers and increases the investments towards the L-projects or bubbles. Hence, $l(r^*(\theta), \theta) - l(\alpha^L, \theta)$ is positive. Under the bubble economy, if θ is low, $\beta l(\alpha^L, \theta)$ is high and the crowd-in effect is high. On the other hand, if θ is high, $\beta l(\alpha^L, \theta)$ becomes low and the crowd-in effect is dominated by the crowd-out effect. Thus, in

the economies with relatively low θ, the crowd-in effect dominates the crowd-out effect, but, in the economies with relatively high θ, the crowd-out effect dominates the crowd-in effect[21].

Here we will add a few remarks on the effect of bubbles on the aggregate productivity. In the bubble economy, L-entrepreneurs stop investing and only H-entrepreneurs invest, i.e., bubbles improve efficiency in production by eliminating low productive investment. Thus, the total factor productivity increases together with the emergence of bubbles. It moves procyclically with economic growth if θ is relatively low. This implies that bubble burst produces productive inefficiency.

4.1 A Change in Investment Opportunities and the Existence Condition of Bubbles

In this subsection, we examine how changes of investment opportunities affect the existence condition of bubbles. It is sometimes claimed that the shortage of investment opportunities causes of bubbles. Here we check this intuition. In this model, the probability that an agent faces H-projects, p, represents the degree of investment opportunities. Hence, let us imagine that p declines, which implies that it becomes more difficult to find the investment with high profitability. From proposition 2, we can derive that

$$d\underline{\theta}/dp < 0, \ d\bar{\theta}/dp < 0.$$

Intuition is as follows. In financially low-developed regions, because of the decline in p, the growth rate of the economy decreases while the interest rate is unchanged at α^L. As a result, the existence condition for bubbles becomes tighter. On the other hand, in financially high-developed regions, due to the decline in p, the interest rate falls while the growth rate remain unchanged at $\beta\alpha^H$. As a result, the condition gets loosened. We can do the same thought experiment on changes in α^H/α^L. We summarize the above results in the following proposition.

Proposition 5 *Both $\underline{\theta}$ and $\bar{\theta}$ are decreasing functions of p. This means that*

---

[21]In our model, the presence of L-projects plays a crucial role to show that bubbles crowd investment in and enhance growth. Without it, in the bubbleless economy, the interest rate adjusts such that all the savings in the economy flow to H-projects. In such a situation, once bubble assets appear into the economy, they crowd savings away from H-projects, thereby lowering the growth.

a decrease of p tends to depress bubbles when θ is low and tends to generate bubbles when θ is high.

5   Effects of the burst of bubbles

In this section, we examine the effects of burst of bubbles. In this perfect foresight model, an unexpected shock may generate the burst of bubbles. Let us suppose that there is an unexpected shock at $t = s$ which decreases the productivity from α^H to $\alpha^S < \alpha^H$. First, we examine the case where this shock is permanent (or at least this shock is expected to be permanent at $t = s$.). As we have shown in the previous section, $\beta\alpha^H \geq \alpha^L$ is a necessary condition for the existence of bubbles. Hence, if α^S is strictly smaller than α^L/β, bubbles must burst for any θ. Even if $\alpha^S \geq \alpha^L/\beta$, bubbles may burst if θ is relatively low. Since $\underline{\theta}$ is a decreasing function of α^H, bubbles must collapse in the countries whose pledgeability is lower than $\underline{\theta}(\alpha^S)$. This result shows that even if the shock is common, the effect of the shock is different from country to country and in particular, the effect on the stock price in a country is crucially affected by the financial market condition of this country.

Next we examine how the growth rate in each country is affected by the unexpected shock at $t = s$. After the collapse of bubbles, the growth rate is determined by the mechanism explained in Section 2, Hence, the growth rate after the shock becomes,

$$
g_t = g(\theta) = \begin{cases} \beta\alpha^S - (\alpha^S - \alpha^L)\beta\dfrac{\alpha^L(1-p) - \theta\alpha^S}{\alpha^L - \theta\alpha^S} & \text{if } 0 \leq \theta < (1-p)\dfrac{\alpha^L}{\alpha^S}, \\[3ex] \beta\alpha^S, & \text{if } (1-p)\dfrac{\alpha^L}{\alpha^S} \leq \theta \leq 1. \end{cases}
$$

Since the productivity is lower than before, the growth rate becomes lower than that of the bubble periods. Although the growth rate under the bubble economy is not lower than α_L, the growth rates must be lower than α_L after the burst when $\beta\alpha^S < \alpha_L$. Furthermore, the variance of growth rates among countries becomes higher after the burst. The reason is as follows. The countries whose θ is $\underline{\theta} \leq \theta \leq \theta^*$ experienced relatively high growth rate by the existence of bubbles. This means that after the burst of bubbles, those

countries experience the decrease in the growth rate from the two reasons, the decrease in productivity and the burst of bubbles. Hence, those countries experience relatively very low growth rate after the burst of bubbles. On the other hand, the countries whose θ is $\theta^* < \theta < \bar{\theta}$ suffer the decrease in the growth rate by the decrease in productivity but this effect must be offset by the positive effect of the burst on the growth rate, since the growth rates of those countries were decreased by the existence of bubbles. In summary, the low (high) θ countries experience relatively lower (higher) growth rates, thus the variance of the growth rate becomes higher even though the average growth rate must be lower than before the burst. This result may be consistent with an empirical observation. Figure 3 shows the growth rates of Asian countries before and after the financial crisis. The figure shows that the variance of the growth rates becomes higher after the crisis. Of course, actual growth rates must be affected by many factors, our result is not inconsistent with this interesting observation.

Next we examine the case where the unexpected shock is temporary and it is expected so after the shock. In this case, bubbles might exist even after the shock since all agents can expect that this shock is temporary. In order to sustain the bubble path after the shock, however, the price of bubbles, P_s, must drop according to the shock. The reason is as follows. Let us suppose the shock is temporary and the productivity recovers to α^H after $t = s + 1$. Under the shock, from $t = s + 1$, the growth rate of each country can recover to $g_t^*(\theta)$ but Y_t must be lower since Y_{s+1} is decreased by the shock. Hence, in order to sustain the bubble path, the price of bubbles must be adjusted with the decrease of Y_t and must decrease at $t = s$. This result suggests that the decrease of asset prices does not directly mean the burst of bubbles. It might be an adjustment process of bubbles. Even after the drop of asset prices, bubbles can exist even under the perfect foresight economy as long as there is an unexpected shock.

It is not necessary, however, that people continue to choose the bubble path even after the unexpected shock. People may choose the bubbleless path after the shock. Hence, bubbles may burst if agents revise their expectation by the shock and expect that the value of the bubble is zero even if the productivity shock is temporary and α^H recovers to the original level at $t = s+1$, Next, we examine how the bubble bursts affect the economic growth rates in this case. Since the bubbles have bursted at $t = s$, the growth rate follows (20) from $t = s+1$. This implies that the difference in the growth rate between before and after the burst of bubbles can be characterized by the

difference in the growth rate between the bubble economy and the bubbleless economy. Hence, we get that If $\underline{\theta} \leq \theta \leq \theta^*$, the growth rate becomes lower after the burst of bubbles but if $\theta^* < \theta < \overline{\theta}$, the growth rate becomes higher (except $t = s$) after the burst of bubbles. This result suggests that the effect of bubble bursts is not uniform. It is crucially affected by the financial condition of each country. If the imperfection of financial market is relatively high, the burst of bubbles decreases the growth rate of the country but the burst may enhance the long run growth rate if the financial market condition is relatively good. This point is shown in Figure 4[22]. In other words, the burst of bubbles explores the "true" economic condition of each country. This result also means that the variance of growth rates among countries becomes higher and once again this result is consistent with the observation in Figure 3.

6   Conclusion

In this paper, we assumed the imperfection of the financial market and examined the effects of bubbles under the imperfect financial market. We explored that the existence condition of bubbles is related to the financial market condition and the middle range of pledgeability allows the existence of bubbles. This suggests that improving the financial market condition might enhance the possibility of bubbles if the initial condition of the financial market is underdeveloped. Moreover, the effects of bubbles on the economic growth rates are also related to the financial market condition. If the pledgeability is relatively low, bubbles increase the growth rate but bubbles decrease the growth rate if the pledgeability is relatively high. This result has an important implication for the effects of bubble bursts. The burst of bubbles decreases the growth rate when the financial market condition is not so good, but the burst may enhance the growth rate when the financial market condition is relatively good.

Our model can be extended to several directions. One would be to consider stochastic bubbles. In the case of stochastic bubbles, the entrepreneurs' portfolio decision is more complicated than in the deterministic bubbles. Risk

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[22]If $\underline{\theta} \leq \theta \leq \theta^*$, the growth rate at $t = s$ might be higher than $t = s-1$ if the temporary shock is not so large since the growth rate is enhanced by the burst of bubbles even at $t = s$.

averse L-entrepreneurs want to hedge themselves by investing in their own L-projects as well as buy bubbles and lend to H-entrepreneurs. If θ is relatively low, the interest rate may stick to α^L and may not rise together with the emergence of bubbles, and hence only growth-enhancing effects of bubbles are generated. Another direction would be to endogenous θ. In this model, we assume that θ is exogenously given and constant over time. However, in the real world, it may be natural to think that θ keeps increasing over time. It would be worthwhile to endogenous θ by, for example, developing a theory of secondary markets such as Broner et al. (2010), and how this might change bubble regions. Finally, we have not analyzed welfare implications of bubbles, policy-oriented issues such as government's intervention after bubble bursts or how regulations in the financial markets will affect the emergence of bubbles. These would be also promising works.

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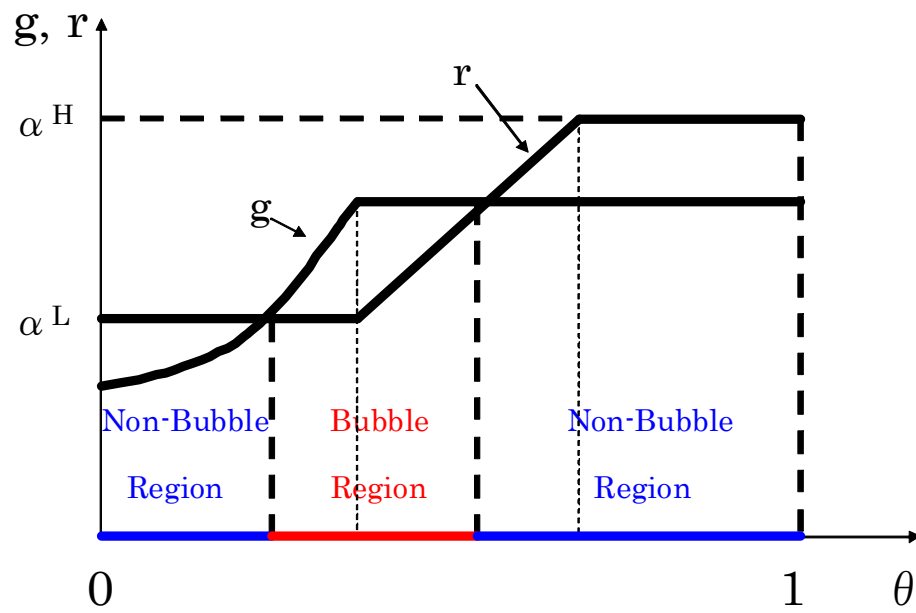[31] Woodford, M., (1990), "Public Debt as Private Liquidity," *American Economic Review*, 80, pp382-388.
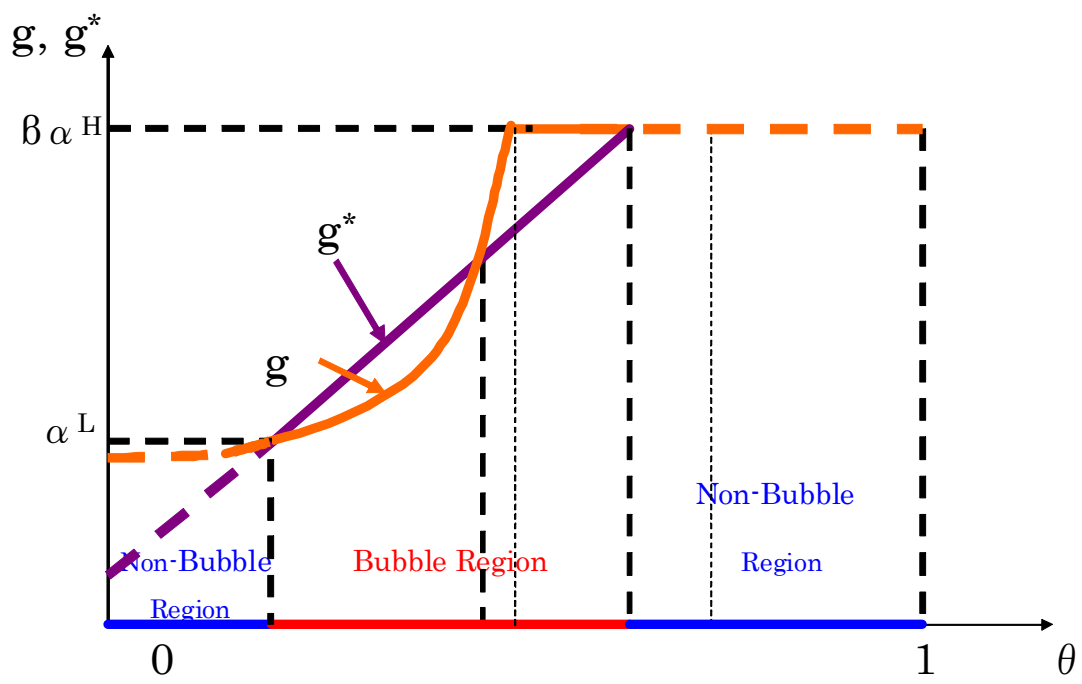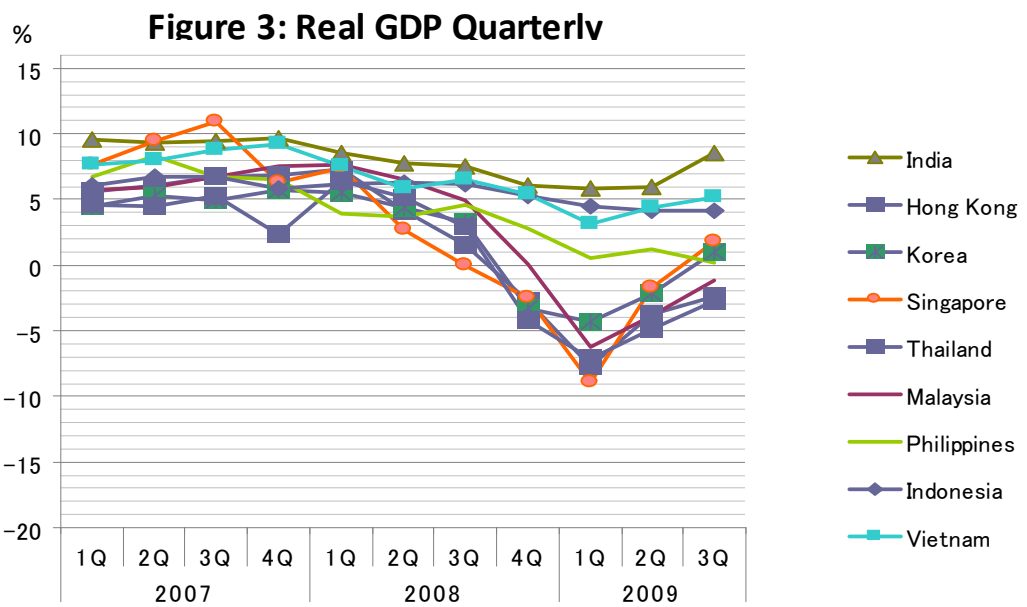
Figure 1: Bubble region and θ

Figure 2: Bubbles and Economic Growth

Figure 3: Real GDP Quarterly

Thomson Reuter, Datastream
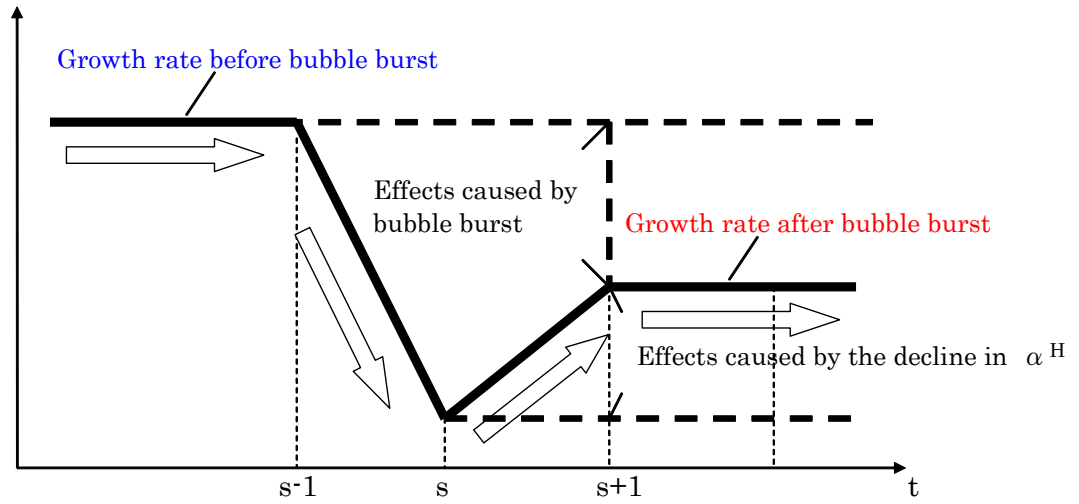* year-over-year basis

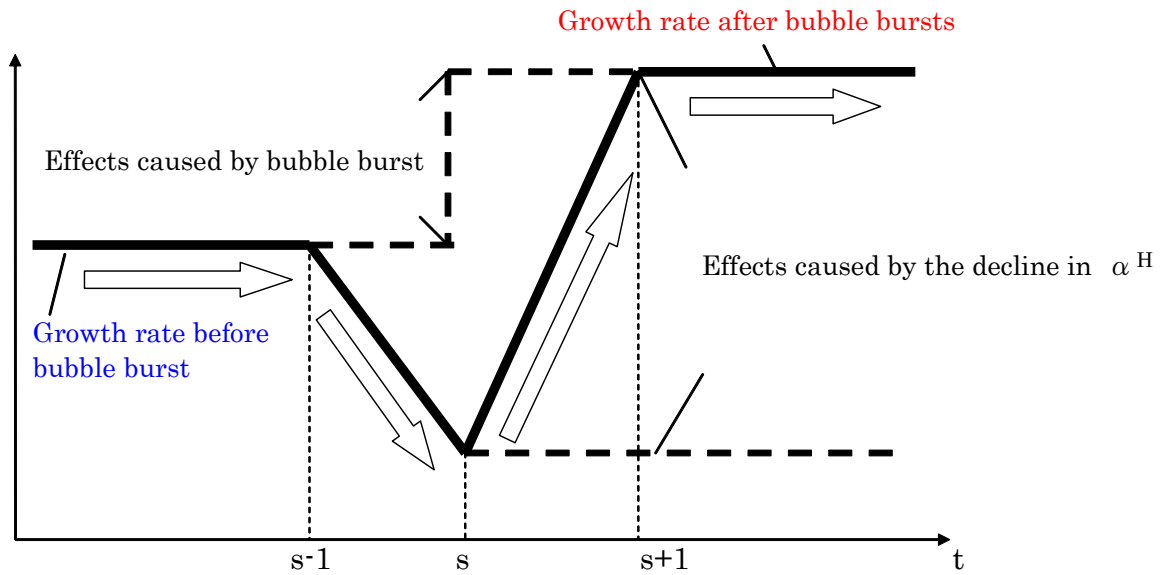Figure 4-1: The effect of bubbles' bursting in relatively low θ

Figure 4-2: The effects of bubbles' bursting in relatively high θ