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Abstract: Some analytic simulation techniques for the analysis of the reduced form and of the dynamic properties of econometric models are described. Comparisons are made with analytical methods available for linear models.

Keywords: Econometric models; structural form; reduced form; analytic simulation; stochastic simulation; impact multipliers; dynamic multipliers; forecast errors; asymptotic standard errors.

1. Introduction

The evaluation of an econometric model as a simultaneous equations system and the analysis of its dynamic properties are crucial steps in the model building process, particularly when the model is used for forecasting and for simulating alternative economic policies.
In contrast with linear econometric models, where analytic methods are always applicable, in nonlinear models one must generally resort to simulation techniques.

Purpose of this paper is to briefly describe some analytic simulation techniques (combination of numerical simulation and analytical methods, according to the definition by Howrey and Klein [10] and by Klein [12]) for the analysis of the reduced form and of the dynamic properties of the models. The proposed techniques, which integrate the program for stochastic simulation described in [2], extend, to nonlinear models, methods that are available in the literature for linear econometric models only. Also for linear models, however, these methods can be sometimes preferred, both for their greater simplicity in the input of data and for their computational performances. In particular, these techniques allow a fast and reliable computation of the following:

- Standard errors of the reduced form equations.
- Reduced form coefficients (impact multipliers, in particular) and

covariance matrix of their asymptotic distribution.

- Dynamic (interim) multipliers and related asymptotic covariance matrices.
- Asymptotic variances of the forecast errors.

An example of the application of the above mentioned techniques to a "test" model will be presented in section 2; in this section, dealing with the standard errors of the reduced form equations, the analytic simulation methodology will also be briefly described. The extension of the methodology to the computation of the covariance matrices of impact and dynamic multipliers and of forecast errors will be shortly discussed in sections 3, 4 and 5, together with a short comment on the computational performances; finally, in section 6 the main features of the package implemented by the authors at the IBM Scientific Center of Pisa will be presented.

2. Standard Errors of the Reduced Form Equations

Let

(2.1) $Ay_t + Bz_t = u_t \qquad t=1,2,\ldots,T$

be a linear econometric model in its structural form, where y_t, z_t and u_t are, respectively, the vectors of the endogenous and predetermined variables and of the structural stochastic disturbances at time t, A and B are matrices of structural coefficients (A nonsingular square matrix). Furthermore, let u_t be distributed as

(2.2) $u_t \sim N(0,\Sigma); \qquad \mathrm{cov}(u_t, u_s) = \delta_{ts}\Sigma \qquad$ all t,s

δ_{ts} being the Kronecker delta; in other words the vectors u_t are supposed to be independent and identically distributed, with a multivariate normal distribution, zero mean and covariance matrix constant over time.

The estimated structural model is

(2.3) $\hat{A}y_t + \hat{B}z_t = \hat{u}_t$

where $\hat{u}_t$ are the estimated residuals and

(2.4) $\hat{\Sigma} = 1/T(\sum_{t=1}^{T} \hat{u}_t \hat{u}_t')$

is an estimate of the covariance matrix of the structural disturbances (or, that is the same, of the structural form equations).

The restricted reduced form (i.e. the reduced form derived from the structural, which takes into account all the restrictions on coefficients) is:

(2.5) $y_t = -A^{-1}Bz_t + v_t$

where

(2.6) $v_t = A^{-1}u_t$

is the vector of the reduced form disturbances at time t. It is clearly

(2.7) $v_t \sim N(0, A^{-1}\Sigma A'^{-1})$

so that an estimate of the reduced form covariance matrix (Ω) is immediately available as

(2.8) $\hat{\Omega} = \hat{A}^{-1}\hat{\Sigma}\hat{A}'^{-1}$

provided the estimated $\hat{A}$ is nonsingular.

If the model is nonlinear, equation (2.8) cannot be applied. In fact, in the nonlinear case, for the structural econometric model

(2.9) $f(y_t, z_t, a) = u_t$

(where a is a vector including all the structural coefficients, because a clear distinction between coefficients of y_t and coefficients of z_t, i.e. between the elements of A and B, is no more possible), an explicit analytic expression of the reduced form

(2.10) $y_t = g(z_t, a, u_t)$

is, in general, unknown. Nevertheless, the covariance matrix of the reduced form equations can be computed by an analytic simulation procedure. This procedure is based on a nonexplicit linearization of the model in the neighbourhood of the solution point corresponding to the period t under examination. It is clear from equations (2.5) and (2.6) that the elements of the matrix A^{-1} (such that $A^{-1}u_t = v_t$ are the reduced form disturbances) are the partial derivatives of the endogenous variables with respect to the structural disturbances at time t (elements of the vector u_t). These derivatives can be computed via numerical simulation, stored in a matrix $\hat{D}_t$ and the reduced form covariance matrix ($\hat{\Omega}$) at time t can be computed as:

(2.11) $\hat{\Omega}_t = \hat{D}_t \hat{\Sigma} \hat{D}_t'$

where $\hat{\Sigma}$ is estimated as in (2.4), given the additive hypothesis on the structural disturbances in (2.9). It must be pointed out that, while in case of linear models, being $D_t = A^{-1}$, D_t will be constant, for nonlinear models D_t (and consequently Ω_t) will be time-varying, so that, in equation (2.11), the subscript t has been introduced. The computation of the $\hat{D}_t$ matrix (partial derivatives of the endogenous variables, with respect to the structural disturbances, in the solution point at time t) required by the analytic simulation method, can be performed using finite increments on the structural disturbances. More exactly, first a deterministic control solution is computed, at time t, with all the $\hat{u}_t$ set to zero. Then a value ϵ is assigned to the disturbance of the first equation, all the other being still zero, and the model is solved again. The procedure is then repeated for all the structural stochastic equations and the differences between the disturbed solutions and the control solution, divided by the values adopted for ϵ, supply the numerical values of the partial derivatives. The way in which $\hat{D}_t$ is computed (by numerical simulation) and the use of equation (2.11) define the analytic simulation procedure as a combination of numerical simulation and analytical methods. The proposed procedure, when applied to linear models, is an alternative to the use of equation (2.8). The advantages in using analytic simulation techniques are related to the fact that, for medium or large-size models, the econometricians generally use the standard Gauss-Seidel iterative algorithm for the solution, expressing the model as a set of equations, each equation being normalized with respect to a different endogenous variable. In such a case it is much easier to compute the $\hat{D}_t$ ($=\hat{A}^{-1}$) matrix by simulation, rather than to invert $\hat{A}$; in fact, for the inversion of the $\hat{A}$ matrix, the model should be expressed according to (2.1), which involves the practical problem of the correct correspondence between variables and coefficients.

For nonlinear models, an alternative way of estimating the reduced form covariance matrix ($\hat{\Omega}_t$) is based on the stochastic simulation of the model [4]. Without entering into the details of this methodology, it is important to remark that, by means of stochastic simulation, the accuracy of the estimates (asymptotically exact), increases with the number of replications; the analytic simulation procedure, on the contrary, is not exact, but involves (via linearization) a systematic approximation.

In order to check the size of this approximation, for some variables of the nonlinear Klein-Goldberger model (revised version by Klein [11], estimated by 2SLS with 4 principal components), the standard errors of the reduced form equations have been computed by means of the analytic simulation procedure and by means of the stochastic simulation approach after 50, 500, 5000 and 50000 replications. The results are displayed in table 1.

From table 1 one could get the strong impression that the stochastic simulation results converge to those of analytic simulation as the number of replications goes to infinity. This is of course impossible, due to the nonlinearity of the model, but clearly gives an idea of the great accuracy of the analytic simulation method, which requires one control solution and as many disturbed solutions as the number of stochastic equations (16, for this model).

In regard to the computational performances of the analytic simulation procedure in estimating the $\hat{\Omega}$ matrix, it must be remarked that, in case of linear models, the computation time required by the procedure is comparable with that required by the use of the (analytic) formula (2.8). In case of nonlinear models, the comparison

Table 1

Klein-Goldberger Model

Reduced Form Standard Errors at 1965

Cd = consumption of durables; X = gross national product; W = wages and salaries and supplements to wages and salaries; Pc = corporate profits including inventory valuation adjustment; p = implicit GNP deflator.

Variab. Name	Computed Value	Standard Errors: Stochastic Simulation, Number of Replications 50	500	5000	50000	Analytic Simulation
Cd	55.33	2.78	2.48	2.44	2.42	2.42
X	530.1	9.05	8.44	8.52	8.54	8.53
W	310.8	5.24	4.77	4.73	4.77	4.78
Pc	41.97	6.44	6.21	6.16	6.11	6.11
p	1.225	.040	.035	.036	.036	.036

between analytic and stochastic simulation is, in general, largely in favour of the analytic simulation procedure; in fact, this procedure supplies results whose accuracy is similar to that of the results obtained, in a much more expensive way, via stochastic simulation after a very large number of replications. For example, the results displayed in table 1 require, on a computer IBM/370 model 168, less than one second of CPU time for analytic simulation, 6 seconds for 500 replications of stochastic simulation and about 10 minutes for 50000 replications.

3. Impact Multipliers and Asymptotic Standard Errors

Following Dhrymes [6] and Goldberger [8], the reduced form coefficients are defined as the partial derivatives of the conditional expectation of each current endogenous variable with respect to each predetermined variable, with all other z_t's held constant; properly speaking, the impact multipliers are the subset of the reduced form coefficients corresponding to the current exogenous variables, but in this section the two terms will be indifferently used.

For linear models, the matrix of the reduced form coefficients (Π)

can be directly estimated by the matrix product $-A^{-1}B$, as follows from equation (2.5). For nonlinear models, as pointed out in the previous section, the reduced form is unknown; disregarding the effects of nonlinearities on the conditional expectation of the endogenous variables (effects which are always very small, according to the experience of the authors), the computation at time t, of the multiplier of the j-th exogenous variable with respect to the i-th endogenous variable ($\hat{\pi}_{ijt}$), can be performed, as in [12], using simulation techniques, by the ratio:

(3.1) $\hat{\pi}_{ijt} = (\hat{y}^d_{it} - \hat{y}^c_{it}) / (z^d_{jt} - z^c_{jt})$

where $\hat{y}^c_{it}$ is the deterministic control solution corresponding to the control value z^c_{jt} and $\hat{y}^d_{it}$ is the disturbed solution corresponding to the value $z^d_{jt} = z^c_{jt} + \epsilon$, all the other predetermined variables being equal to their control values.

Attempts to derive the small-sample distribution of the impact multipliers have been performed using Monte Carlo methods (see, for example, [15]). These methods, however, have a major drawback in the possible non-existence of finite moments in the small-sample distribution of the structural and reduced form coefficients, even for linear models when estimated with simultaneous consistent methods [13], [14]. Truncation must be, therefore, performed on the distribution of the pseudo-random disturbances to be used in the Monte Carlo experiment [17,p.1004], thus involving some arbitrariness.

As suggested by Theil [18,p.377], resort to asymptotic distribution could be sometimes preferable; under quite general assumptions, it can be shown that the asymptotic covariance matrix of the reduced form coefficients is given by

(3.2) $\Psi_t = J_t \Delta J_t'$

where J_t is the matrix of the second order derivatives, properly arranged, of (2.10) with respect to z_j and a, $(\partial^2 g_i / \partial z_j \partial a_k)$, computed in the point $(z_{jt}, a, u_t = 0)$, and Δ is the asymptotic covariance matrix of all the structural coefficients.

The problem of computing the asymptotic covariance matrix of impact multipliers ($\hat{\Psi}$) was dealt with in 1961 by Goldberger, Nagar and Odeh [9] for linear models (in which case $\hat{\Psi}$ is not time varying): they have proposed the explicit formula

(3.3) $\hat{\Psi} = (\hat{A}^{-1} \otimes [\hat{\Pi}' \ I]) \hat{\Delta} (\hat{A}^{-1} \otimes [\hat{\Pi}' \ I])'$

where $\otimes$ denotes Kronecker product; the above formula, in order to be applicable to nonlinear models, would require an explicit linearization of the model, thus making extremely laborious the process also for small models. Even in case of linear models the procedure is quite laborious; this is probably one of the reasons of the quite different and contradictory results displayed in the literature, for the same test model [3], [5], [7], [9], [16].

Analytic simulation overcomes most of the difficulties, allowing a fast and reliable computation, even for moderately complex models and with no difference between linear and nonlinear models; $(\partial^2 g_i / \partial z_j \partial a_k)$, in fact, can be simply computed in the point $(z_{jt}, \hat{a}, u_t = 0)$, using finite increments, as

(3.4) $\Delta(\Delta g_i / \Delta \hat{a}_k) / \Delta z_j$

thus requiring, for the complete computation of the $\hat{J}_t$ matrix, approximately as many solutions of the model at time t as the product of the number of exogenous variables with the number of estimated structural coefficients.

4. Dynamic Multipliers and Asymptotic Standard Errors

The dynamic properties of an econometric model are related to the presence of lagged endogenous variables in z_t (vector of the predetermined variables): in this case equation (2.1) could be more properly rewritten as:

(4.1) $Ay_t + Bx_t + Cy_{t-1} = u_t$

where B is the matrix of the structural coefficients of the exogenous variables x_t and C is the matrix of the structural coefficients of the lagged endogenous variables y_{t-1}.

In analogy with the impact multiplier, the k-lag dynamic (delay as in [8], or interim as in [16]) multiplier, could be defined as the partial derivative of the conditional expectation of an endogenous variable at time t with respect to an exogenous variable at time t-k.

The k-lag interim multipliers in linear models could be computed in the following way:

(4.2) $\hat{\Pi}_k = (-\hat{A}^{-1}\hat{C})^k (-\hat{A}^{-1}\hat{B})$;

for nonlinear models, the computation of the elements of $\hat{\Pi}_k$ is always performed using simulation techniques, by the simple ratio (3.1), where instead of z^d_{jt} and z^c_{jt}, the values of x_j at time t-k are used.

Analytic simulation can be profitably used for estimating the covariance matrix of the asymptotic distribution of dynamic multipliers. In fact, even if Schmidt [16] gave an analytic solution to the problem, the proposed method (revised by Brissimis and Gill [5], [7]) is applicable to linear models only, and has the practical drawbacks of requiring the use of large sparse matrices whose non-zero elements are hard to be filled automatically and of requiring a large computation time. For example, for the Klein-I model, the computation of the standard errors of interim multipliers up to lag 15 using the method proposed by Gill and Brissimis [7] requires about 6 minutes of CPU time on a computer IBM/370 model 168; an ad-hoc program which uses the analytic simulation approach performs the same computation in less than one second.

The analytic simulation procedure used for this purpose, is quite similar to the one discussed in the previous section; the only difference is that the partial numerical derivatives must be computed with respect to the values of x_j at time t-k.

5. Asymptotic Covariance Matrix of the Forecast Errors

Under the assumption of independence among structural disturbances in different periods, the analysis of the forecast error, or briefly of the forecast, can be performed decomposing the error into two independent components: a first component that depends only on errors on the estimated coefficients and a second component that depends only on the vector of the structural disturbances.

Following the mentioned decomposition, for linear models and in one-step simulation, Goldberger, Nagar and Odeh [9] have proposed, for the estimation of the covariance matrix of forecasts $\hat{\Phi}$, the following formula:

(5.1) $$\hat{\Phi} = F\hat{\Psi}F' + \hat{\Omega}$$

where F is a matrix containing, in a convenient order, the values of the predetermined variables in the forecast period, $\hat{\Psi}$ is the asymptotic covariance matrix of the reduced form coefficients defined in (3.3) and $\hat{\Omega}$ is the covariance matrix of the reduced form disturbances defined in (2.8).

The previous formula could be applied also to nonlinear models; in fact, using analytic simulation techniques, eqs. (3.2,3.4) and (2.11) will provide, respectively, an estimation of the matrices $\hat{\Psi}$ and $\hat{\Omega}$ to be used in (5.1). It is, however, much simpler and more precise, as suggested in [1], to compute directly the asymptotic covariance matrix of the component due to the errors on the structural coefficients (i.e. $F\hat{\Psi}F'$), using an analytic simulation approach, without the intermediate step of the computation of $\hat{\Psi}$, covariance matrix of the reduced form coefficients. This method is simply based on the numerical computation of first order derivatives of the endogenous variables with respect to the structural coefficients. In this way it is not necessary to compute the second order derivatives involved by the matrix $\hat{\Psi}$, thus ensuring a greater numerical accuracy and a considerable shortening of execution time (for example, 0.05 seconds, instead of 10 seconds of CPU time, for the linear Klein-I model).

6. Main Features of the Package

The package is written in FORTRAN IV and ASSEMBLER languages; its basic structure is similar to that described in [2].

For each model, two data sets must be prepared: one is a FORTRAN subroutine containing the model's equations in suitable form for the Gauss-Seidel solution algorithm; the other is a data set containing the time series of the endogenous and exogenous variables, the estimated coefficients, residuals and asymptotic covariance matrix of the structural coefficients.

The additional analytic simulation techniques have been introduced into the program in form of four separate subroutines (less than 1000 FORTRAN statements on the whole) which perform, respectively, the computations described in sections 2, 3, 4 and 5. 512K of main storage are generally sufficient for the simulation of medium-size models (for example, the already mentioned Klein-Goldberger model).

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