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Klein-I model: revised computations to
complete "A note on the numerical
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Econometrica, 47 (1979)**

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ON THE RESTRICTED REDUCED FORM OF THE KLEIN-I MODEL

Revised Computations to Complete
"A Note on the Numerical Results by Goldberger,
Nagar and Odeh", *Econometrica*, 47 (1979)

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The numerical example which completes the paper by Goldberger, Nagar and Odeh [3], on the estimated asymptotic covariance matrix of the reduced form coefficients for the Klein-I model estimated by Two Stage Least Squares (2SLS), has led to some misinterpretations of the properties of the model, (see [5], [6] and again [3]). In this paper, revised computations are presented, completing, with numerical tables, the note recently published by the authors on *Econometrica* [2].

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CONTENTS

1. INTRODUCTION	p. 7
2. ONE-PERIOD FORECASTS	p. 9
3. SIMULTANEOUS CONFIDENCE INTERVALS	p. 11
4. ASYMPTOTIC STANDARD ERRORS OF INTERIM MULTIPLIERS	p. 12
APPENDIX 1	p. 13
APPENDIX 2	p. 16
REFERENCES	p. 17

1. INTRODUCTION

The importance of the covariance matrix (Ω) of the asymptotic distribution of restricted reduced form coefficients proposed by Goldberger, Nagar and Odeh [3] has been underlined in several applications that require the covariance matrix Ω as a starting point.

Some of these applications present also numerical results referred to the 2SLS estimate of the Klein-I model; they are:

- a) the estimated asymptotic covariance matrix of the one-period forecasts [3];
- b) the simultaneous confidence intervals of the forecast errors [5];
- c) the estimated asymptotic standard errors of the interim multipliers [6].

Unfortunately, the $\hat{\Omega}$ matrix computed by Goldberger, Nagar and Odeh for the Klein-I model, displayed in [3, pp.567-570], contains several errors. These errors are sufficiently slight and perhaps acceptable (as relative errors) on the main diagonal, but are larger in many off-diagonal elements and this causes misleading results when $\hat{\Omega}$ is used for further computations.

Purpose of this paper is to present revised numerical computations of the $\hat{\Omega}$ matrix and of the other statistics which have been mentioned above.

The $\hat{\Omega}$ matrix, used for the computations of this paper, has been computed by the authors in the three following different ways which have led to the same results up to 5 significant decimal digits:

- 1) with a computer program developed by the authors directly applying the formulae by Goldberger, Nagar and Odeh;
- 2) using Havenner's program [4];
- 3) via simulation, by means of special features inserted into the program described in [1].

This matrix is displayed in Appendix 1 with the same format as in [3].

The 2SLS asymptotic covariance matrix of the structural coefficients, $\hat{\Sigma}$, used to derive $\hat{\Omega}$, has also been recomputed by the authors; it is slightly different from $\hat{\Sigma}$ in [3,p.565], but it is practically equal to that published by Theil [7,p.518]. $\hat{\Sigma}$ is displayed in Appendix 2 with 6 significant decimal digits.

2. ONE-PERIOD FORECASTS

Re-computing properly the $F_*\hat{\Omega}F_*'$ matrix (for this notation, refer to [3]) the following results are obtained:

$$F_*\hat{\Omega}F_*' = \begin{bmatrix} 2.4187 & 1.0513 & 1.6663 & 1.8037 & 3.4701 & 1.0513 \\ & 0.8412 & 0.8485 & 1.0440 & 1.8925 & 0.8412 \\ & & 1.6364 & 0.8784 & 2.5148 & 0.8485 \\ & & & 1.9693 & 2.8477 & 1.0440 \\ & & & & 5.3626 & 1.8925 \\ & & & & & 0.8412 \end{bmatrix}$$

The elements of this matrix are much smaller than those published in [3,p.571, (7.5)], which are the following:

$$\begin{bmatrix} 53.6532 & 30.9937 & 44.9810 & 39.6659 & 84.6469 & 30.9937 \\ & 31.3230 & 27.8137 & 34.5031 & 62.3167 & 31.3230 \\ & & 47.2123 & 25.5824 & 72.7947 & 27.8137 \\ & & & 48.5866 & 74.1690 & 34.5031 \\ & & & & 146.9636 & 62.3167 \\ & & & & & 31.3230 \end{bmatrix}$$

The $\hat{\Delta}$ and $\hat{\nu}$ matrices displayed in [3,p.571, (7.3,7.4)] are practically exact; they have been recomputed with more significant digits and are hereunder displayed for completeness sake.

$$\hat{\Delta} = \begin{bmatrix} 1.04405 & .437854 & -.385294 & 0. & 0. & 0. \\ & 1.38327 & .192535 & 0. & 0. & 0. \\ & & .476444 & 0. & 0. & 0. \\ & & & 0. & 0. & 0. \\ & & & & 0. & 0. \\ & & & & & 0. \end{bmatrix}$$

$$\hat{\nu} = (\hat{r}^{-1})' \hat{\Delta} (\hat{r}^{-1}) = \begin{bmatrix} 3.9221 & 2.4042 & 2.8441 & 3.4822 & 6.3263 & 2.4042 \\ & 2.0029 & 2.0724 & 2.3347 & 4.4071 & 2.0029 \\ & & 2.7245 & 2.1920 & 4.9165 & 2.0724 \\ & & & 3.6249 & 5.8169 & 2.3347 \\ & & & & 10.7334 & 4.4071 \\ & & & & & 2.0029 \end{bmatrix}$$

The matrix $F_*\hat{\Omega}F_*' + \hat{\nu}$ [3,p.572] should be therefore modified as follows:

$$F_* \hat{\Omega} F_*' + \hat{v} = \begin{bmatrix} 6.3409 & 3.4555 & 4.5104 & 5.2859 & 9.7964 & 3.4555 \\ & 2.8442 & 2.9210 & 3.3787 & 6.2996 & 2.8442 \\ & & 4.3610 & 3.0704 & 7.4314 & 2.9210 \\ & & & 5.5942 & 8.6646 & 3.3787 \\ & & & & 16.0960 & 6.2996 \\ & & & & & 2.8442 \end{bmatrix}$$

As a consequence, the vector s_* of the estimated standard errors of forecast of the individual endogenous variables (C, I, W1, P, Y, K) for 1948 should be as follows:

$$s_* = \begin{bmatrix} 2.5 \\ 1.7 \\ 2.1 \\ 2.4 \\ 4.0 \\ 1.7 \end{bmatrix} \quad \text{instead of} \quad \begin{bmatrix} 7.6 \\ 5.8 \\ 7.1 \\ 7.2 \\ 12.6 \\ 5.8 \end{bmatrix} \quad \text{as in [3,p.572, (7.7)].}$$

The statement in [3,p.572] "each of the forecast errors for 1948 was less than its estimated standard error" does not hold; in fact, the forecast error for the variables C, I, and K [3,p.571, (7.2)] is larger than its estimated standard error.

3. SIMULTANEOUS CONFIDENCE INTERVALS

The simultaneous confidence intervals for the forecast errors at 1948, derived by Hymans [5, pp.28,29], should be correspondingly modified as follows:

$$\begin{aligned}
 |e_1| &\leq \sqrt{(15.23)(6.3409)} = 9.83 \\
 |e_2| &\leq \sqrt{(15.23)(2.8442)} = 6.58 \\
 |e_3| &\leq \sqrt{(15.23)(4.3610)} = 8.15 \\
 |e_1+e_2| &\leq \sqrt{(15.23)(16.0960)} = 15.66 \\
 |e_1+e_2-e_3| &\leq \sqrt{(15.23)(5.5942)} = 9.23 \\
 |e_2| &\leq \sqrt{(15.23)(2.8442)} = 6.58
 \end{aligned}$$

so that the joint intervals for the endogenous variables are:

$$\begin{aligned}
 68.40 &\leq C_{48} \leq 88.06 \\
 2.72 &\leq I_{48} \leq 15.88 \\
 51.71 &\leq W_{1,48} \leq 68.01 \\
 80.08 &\leq Y_{48} \leq 111.40 \\
 17.95 &\leq R_{48} \leq 36.41 \\
 200.43 &\leq K_{48} \leq 213.58
 \end{aligned}$$

much narrower than those in [5, p.29].

4. ASYMPTOTIC STANDARD ERRORS OF INTERIM MULTIPLIERS

Schmidt's estimate of the asymptotic standard errors of the interim multipliers [6, p. 163], based on the numerical results by Goldberger, Nagar and Odeh, should be correspondingly modified as follows:

Interim Multipliers of Government Expenditure (G)
and Taxes (T) on National Income (Y),
for Klein-I Model

Lag	G	T	Lag	G	T
0	1.8167 (0.421)	-1.3044 (0.483)	8	-0.6752 (0.373)	0.8698 (0.463)
1	1.8085 (0.393)	-1.7717 (0.475)	9	-0.4575 (0.341)	0.6098 (0.424)
2	1.1919 (0.334)	-1.4489 (0.408)	10	-0.2218 (0.299)	0.3173 (0.372)
3	0.4548 (0.334)	-0.6487 (0.404)	11	-0.0144 (0.263)	0.0525 (0.327)
4	-0.1779 (0.355)	0.1311 (0.429)	12	0.1364 (0.245)	-0.1460 (0.304)
5	-0.6072 (0.371)	0.6939 (0.453)	13	0.2205 (0.241)	-0.2625 (0.299)
6	-0.8102 (0.382)	0.9833 (0.472)	14	0.2420 (0.236)	-0.2999 (0.295)
7	-0.8144 (0.385)	1.0208 (0.478)	15	0.2151 (0.222)	-0.2742 (0.277)

The conclusion by Schmidt in [6, p. 164] that "the estimated interim multipliers are not significantly different from zero (for reasonable significance levels) for lags of more than one period" does not hold; there are several other cases in which they are significantly different from zero (still for reasonable significance levels).

$\hat{\Omega}(1, 2)$

	1	P-1	W2	K-1	T	(W1+T)-1	t	G
1	43.6998	-0.0806	-0.1237	-0.2070	-0.6867	-0.0310	-0.0292	0.5874
P-1	-0.0517	0.0074	0.0002	-0.0004	0.0018	0.0001	0.0000	-0.0017
W2	-0.2085	-0.0002	0.0013	0.0010	0.0060	0.0004	0.0004	-0.0049
K-1	-0.1974	-0.0002	0.0006	0.0010	0.0033	0.0001	0.0001	-0.0028
T	-0.7963	0.0013	0.0082	0.0036	0.0480	0.0022	0.0019	-0.0419
(W1+T)-1	-0.0855	0.0002	0.0004	0.0004	0.0019	0.0000	0.0001	-0.0015
t	-0.0584	-0.0003	0.0001	0.0003	0.0003	0.0000	-0.0000	-0.0003
G	0.6942	-0.0012	-0.0073	-0.0032	-0.0444	-0.0019	-0.0017	0.0387

 $\hat{\Omega}(1, 3)$

	1	P-1	W2	K-1	T	(W1+T)-1	t	G
1	51.1316	-0.0832	-0.2461	-0.2305	-1.0777	-0.1042	-0.0454	0.9604
P-1	-0.0117	0.0095	-0.0005	-0.0005	0.0017	-0.0015	0.0004	0.0002
W2	-0.2651	-0.0005	0.0033	0.0011	0.0090	0.0007	-0.0006	-0.0063
K-1	-0.2226	-0.0002	0.0010	0.0011	0.0051	0.0003	0.0004	-0.0046
T	-0.9350	0.0034	0.0091	0.0045	0.0555	-0.0008	0.0009	-0.0414
(W1+T)-1	-0.1906	-0.0008	0.0012	0.0007	0.0017	0.0020	-0.0005	-0.0036
t	-0.0459	-0.0003	0.0002	0.0004	0.0012	-0.0004	0.0012	-0.0011
G	0.9559	-0.0008	-0.0075	-0.0046	-0.0473	-0.0018	-0.0016	0.0419

 $\hat{\Omega}(1, 4)$

	1	P-1	W2	K-1	T	(W1+T)-1	t	G
1	60.3758	-0.1366	-0.2928	-0.2766	-1.3587	-0.0674	-0.0707	1.1126
P-1	-0.1792	0.0113	0.0000	-0.0000	0.0028	0.0003	-0.0001	-0.0034
W2	-0.3586	-0.0005	0.0042	0.0015	0.0116	0.0011	0.0013	-0.0083
K-1	-0.2750	-0.0001	0.0012	0.0013	0.0065	0.0003	0.0003	-0.0054
T	-1.6111	0.0007	0.0137	0.0074	0.0728	0.0043	0.0035	-0.0637
(W1+T)-1	-0.0356	-0.0003	0.0007	0.0002	0.0015	-0.0002	0.0003	-0.0003
t	-0.0995	0.0003	0.0002	0.0004	0.0015	0.0002	-0.0002	-0.0013
G	1.2239	-0.0019	-0.0095	-0.0058	-0.0603	-0.0025	-0.0022	0.0528

 $\hat{\Omega}(1, 5)$

	1	P-1	W2	K-1	T	(W1+T)-1	t	G
1	111.5074	-0.2199	-0.5389	-0.5071	-2.4364	-0.1717	-0.1161	2.0730
P-1	-0.1910	0.0208	-0.0005	-0.0005	0.0046	-0.0012	0.0003	-0.0032
W2	-0.6237	-0.0009	0.0075	0.0026	0.0206	0.0018	0.0007	-0.0147
K-1	-0.4976	-0.0003	0.0022	0.0024	0.0115	0.0006	0.0006	-0.0100
T	-2.5460	0.0040	0.0229	0.0119	0.1283	0.0035	0.0043	-0.1051
(W1+T)-1	-0.2262	-0.0011	0.0018	0.0009	0.0032	0.0018	-0.0002	-0.0040
t	-0.1454	-0.0001	0.0004	0.0008	0.0027	-0.0003	0.0010	-0.0024
G	2.1798	-0.0027	-0.0170	-0.0104	-0.1076	-0.0043	-0.0038	0.0947

 $\hat{\Omega}(2, 3)$

	1	P-1	W2	K-1	T	(W1+T)-1	t	G
1	39.7120	-0.0281	-0.1307	-0.1837	-0.5215	-0.0726	-0.0427	0.4949
P-1	-0.0626	0.0064	0.0001	-0.0003	0.0018	0.0003	-0.0003	-0.0015
W2	-0.0868	0.0003	0.0013	0.0004	0.0077	0.0004	-0.0000	-0.0068
K-1	-0.1904	-0.0004	0.0006	0.0009	0.0023	0.0003	0.0002	-0.0022
T	-0.4940	0.0017	0.0068	0.0022	0.0463	0.0023	0.0003	-0.0422
(W1+T)-1	-0.0175	0.0001	0.0003	0.0001	0.0020	0.0000	0.0000	-0.0017
t	-0.0208	0.0001	0.0003	0.0001	0.0018	0.0001	-0.0001	-0.0016
G	0.4165	-0.0017	-0.0058	-0.0019	-0.0404	-0.0018	-0.0003	0.0366

$\hat{\Omega}(2,4)$

	1	P-1	W2	K-1	T	(W1+T)-1	t	G
1	45.8862	-0.0690	-0.1451	-0.2165	-0.6473	-0.0292	-0.0309	0.5166
P-1	-0.0634	0.0075	0.0002	-0.0003	0.0024	-0.0000	0.0001	-0.0023
W2	-0.1043	0.0004	0.0016	0.0005	0.0098	0.0004	0.0004	-0.0085
K-1	-0.2193	-0.0003	0.0006	0.0011	0.0028	0.0001	0.0001	-0.0022
T	-0.5652	0.0030	0.0084	0.0025	0.0589	0.0021	0.0022	-0.0521
(W1+T)-1	-0.0297	0.0001	0.0004	0.0001	0.0026	0.0001	0.0001	-0.0023
t	-0.0235	0.0001	0.0004	0.0001	0.0023	0.0001	0.0001	-0.0020
G	0.4882	-0.0026	-0.0072	-0.0022	-0.0514	-0.0018	-0.0019	0.0456

$\hat{\Omega}(2,5)$

	1	P-1	W2	K-1	T	(W1+T)-1	t	G
1	85.5982	-0.0970	-0.2758	-0.4002	-1.1688	-0.1018	-0.0736	1.0115
P-1	-0.1259	0.0139	0.0003	-0.0005	0.0042	0.0003	-0.0002	-0.0038
W2	-0.1910	0.0007	0.0029	0.0008	0.0174	0.0008	0.0004	-0.0153
K-1	-0.4097	-0.0007	0.0012	0.0020	0.0050	0.0005	0.0004	-0.0044
T	-1.0592	0.0047	0.0152	0.0047	0.1052	0.0043	0.0025	-0.0942
(W1+T)-1	-0.0473	0.0002	0.0008	0.0002	0.0046	0.0001	0.0001	-0.0040
t	-0.0444	0.0002	0.0007	0.0002	0.0041	0.0002	0.0001	-0.0036
G	0.9046	-0.0043	-0.0130	-0.0040	-0.0917	-0.0036	-0.0022	0.0822

$\hat{\Omega}(3,4)$

	1	P-1	W2	K-1	T	(W1+T)-1	t	G
1	42.6845	-0.0916	-0.1612	-0.2017	-0.7699	-0.0078	-0.0419	0.5793
P-1	-0.1286	0.0076	0.0004	0.0000	0.0033	0.0003	0.0001	-0.0038
W2	-0.1862	0.0001	0.0024	0.0008	0.0088	0.0005	0.0007	-0.0067
K-1	-0.2028	-0.0001	0.0007	0.0010	0.0036	0.0001	0.0001	-0.0028
T	-0.9401	0.0017	0.0096	0.0043	0.0573	0.0027	0.0025	-0.0502
(W1+T)-1	0.0235	-0.0002	0.0001	-0.0000	0.0004	-0.0005	0.0002	0.0005
t	-0.0634	0.0005	-0.0004	0.0003	0.0006	0.0000	-0.0005	-0.0008
G	0.6621	-0.0027	-0.0065	-0.0031	-0.0461	-0.0011	-0.0016	0.0395

$\hat{\Omega}(3,5)$

	1	P-1	W2	K-1	T	(W1+T)-1	t	G
1	90.8436	-0.0743	-0.3519	-0.4131	-1.4290	-0.2081	-0.0667	1.3724
P-1	-0.1113	0.0159	-0.0002	-0.0006	0.0051	-0.0007	-0.0003	-0.0025
W2	-0.3768	-0.0004	0.0045	0.0016	0.0160	0.0015	0.0005	-0.0133
K-1	-0.4142	-0.0007	0.0015	0.0020	0.0067	0.0008	0.0004	-0.0065
T	-1.5992	0.0036	0.0167	0.0074	0.1018	0.0038	0.0030	-0.0876
(W1+T)-1	-0.1768	-0.0012	0.0010	0.0007	0.0015	0.0021	-0.0003	-0.0037
t	-0.0881	0.0001	-0.0006	0.0006	0.0012	-0.0005	0.0011	-0.0019
G	1.4553	-0.0013	-0.0132	-0.0068	-0.0836	-0.0053	-0.0027	0.0786

$\hat{\Omega}(4,5)$

	1	P-1	W2	K-1	T	(W1+T)-1	t	G
1	106.2620	-0.2426	-0.4629	-0.4942	-2.1762	-0.0653	-0.1231	1.7121
P-1	-0.2056	0.0189	-0.0000	-0.0004	0.0036	-0.0002	0.0004	-0.0045
W2	-0.4379	0.0002	0.0058	0.0019	0.0221	0.0011	0.0006	-0.0166
K-1	-0.4931	-0.0003	0.0019	0.0024	0.0099	0.0003	0.0005	-0.0079
T	-2.0060	0.0052	0.0213	0.0092	0.1317	0.0041	0.0039	-0.1117
(W1+T)-1	-0.0966	0.0003	0.0016	0.0004	0.0064	-0.0001	0.0003	-0.0043
t	-0.1016	0.0000	0.0017	0.0004	0.0057	0.0004	-0.0001	-0.0041
G	1.6292	-0.0056	-0.0168	-0.0076	-0.1157	-0.0026	-0.0033	0.0984

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