Capital Taxation with Entrepreneurial Risk

Panousi, Vasia

Federal Reserve Board

2009

Capital Taxation with Entrepreneurial Risk

Vasia Panousi

Federal Reserve Board[*]

November 6, 2009

Abstract

This paper studies the effects of capital taxation in a dynamic heterogeneous-agent economy with uninsurable entrepreneurial risk. Although it allows for rich general-equilibrium effects and a stationary distribution of wealth, the model is highly tractable. This permits a clear analysis, not only of the steady state, but also of the entire transitional dynamics following any change in tax policies. Unlike either the complete-markets paradigm or Bewley-type models where idiosyncratic risk impacts only labor income, here it is shown that capital taxation may actually stimulate capital accumulation. This possibility emerges because of the general-equilibrium effects of the insurance aspect of capital taxation. In particular, for the preferred calibrated version of the model, when the tax on capital is 25%, output per work-hour is 2.2% higher than it would have been had the tax rate been zero. Turning to the welfare effects of a reform in capital taxation, it is examined how these effects depend on whether one focuses on the steady state or also takes into account transitional dynamics, as well as how they vary in the cross-section of the population (rich versus poor, entrepreneurs versus non-entrepreneurs).

1 Introduction

This paper studies the macroeconomic and welfare effects of capital-income taxation in an environment where agents face uninsurable idiosyncratic investment risk. Such risk is empirically important for entrepreneurs and wealthy agents, who, even though they represent a small fraction of the population, yet they hold most of an economy's wealth. In this context, capital taxation raises an interesting tradeoff between the distortion of investment versus the provision of insurance against idiosyncratic capital-income risk. On the one hand, capital taxation comes at a cost, since it distorts agents' saving decisions. On the other hand, it has benefits, since it provides agents with partial insurance against idiosyncratic investment risk. This suggests that a positive tax on capital income could be welfare-improving, even if it reduced capital accumulation.

Most surprisingly though, it is shown that a positive tax on capital income may actually stimulate capital accumulation. Indeed, the steady-state levels of the capital stock, output and employment may all be maximized at a positive value of the capital-income tax. This possibility emerges because of the general-equilibrium effects of the insurance aspect of capital taxation. This result stands in stark contrast to the effect of capital taxation both under complete-markets models, and under incomplete-markets models with uninsurable labor-income risk alone. In these models, capital-income taxation, irrespective of whether it is welfare-improving or not, necessarily discourages capital accumulation.

Model. This paper represents a first attempt to study the effects of capital-income taxation in a general-equilibrium incomplete-markets economy, where agents are exposed to uninsurable idiosyncratic investment risk. The framework builds on Angeletos (2007), who develops a variant of the neoclassical growth model that allows for idiosyncratic investment risk, and studies the effects of such risk on macroeconomic aggregates. Agents own privately held businesses that operate under constant returns to scale. Agents are not exposed to labor-income risk, and they can freely borrow and lend in a riskless bond, but they cannot diversify the idiosyncratic risk in their private business investments. Abstracting from labor-income risk, borrowing constraints, and other market frictions, isolates the impact of the idiosyncratic investment risk and preserves the tractability of the general-equilibrium dynamics. The present model extends Angeletos's model in the following ways. First, a government is introduced, imposing proportional taxes on capital and labor income, along with a non-contingent lump-sum tax or transfer. Second, agents have finite lives, which ensures the existence of a stationary wealth distribution. Third, there is stochastic, though exogenous, transition in and out of entrepreneurship, which helps capture the observed heterogeneity between entrepreneurs and non-entrepreneurs without the complexity of endogenizing occupational choice. Fourth, labor supply is endogenous. Clearly the first element is essential for the novel contribution of the paper. The other three improve the quantitative performance of the model and demonstrate the robustness of the main result.

Preview of results. The main result of the paper is that an increase in capital-income taxation

may actually stimulate capital accumulation. The intuition behind this result comes from recognizing that the overall effect of the capital-income tax on capital accumulation can be decomposed in two parts. The first part captures the response of capital to the tax in a setting with endogenous saving but exogenously fixed interest rate. This is isomorphic to examining the effects of the capital tax in a "small open economy". In this context, it is shown that an increase in the capital-income tax unambiguously decreases the steady-state capital stock. The second part, which is the core result of this paper, captures the importance of the general-equilibrium adjustment of the interest rate for wealth and capital accumulation. Here, an increase in the tax reduces the effective variance of the risk agents are exposed to. This reduces the demand for precautionary saving, and therefore increases the interest rate, which in turn increases steady-state wealth. With decreasing absolute risk aversion, wealthier agents are willing to undertake more risk, and hence they will increase their investment in capital. In other words, the general-equilibrium effect of the interest rate adjustment is a force that tends to increase investment and the steady-state capital stock.

For plausible parameterizations of the closed economy, the general equilibrium effect dominates for low levels of the capital-income tax, so that steady-state capital at first increases with the tax. In particular, for the preferred calibrated version of the model, the steady-state capital stock is maximized when the tax on capital is 40%. So, for example, when the tax on capital is 25%, output per work-hour is 2.2% higher than what it would have been had the tax rate been zero. The result that the steady-state capital stock is inversely U-shaped with respect to the capital-income tax is robust for a wide range of empirically plausible parameter values. Furthermore, the tax that maximizes steady-state capital is increasing in risk aversion and/or the volatility of idiosyncratic risk. This finding reinforces the insurance interpretation of the tax system.

Subsequently, the paper examines the aggregate and welfare effects of eliminating the capital-income tax. This is because an extensive discussion has been conducted within the context of the complete-markets neoclassical growth model about the welfare benefits of setting the capital-income tax to zero. In light of the main result here, revisiting this discussion is worthwhile. In particular, the aggregate and welfare effects are presented from two different perspectives. On the one hand, one might be interested in examining the welfare of the current generation immediately after the policy reform, taking into account the entire transitional dynamics of the economy towards the new steady state with the zero tax. On the other hand, one might be interested in examining the welfare of the generations that will be alive in the distant future, i.e. at the new steady state with the zero tax.

First, consider the macroeconomic effects of eliminating the capital-income tax. When markets are complete, investment increases in the short run, and it is also higher at the new long-run steady state with the zero tax, compared to the old steady state with the positive tax. By contrast, in the present model of incomplete markets, investment falls in the short run, as well as in the long run.

Second, consider the welfare effects of eliminating the capital-income tax. These vary across the

different types of agents, the different levels of wealth, and the current versus the future generations. In the current generation, poor agents, whether entrepreneurs or non-entrepreneurs, prefer the zero tax. This is because most of their wealth comes from wage income, and, with capital fixed, the present value of wages increases due to a fall in the interest rate. Rich agents, on the other hand, prefer a positive tax, since they benefit more from insurance provision.

In the long run, all types of agents, and at all levels of wealth, prefer a positive tax on capital income. However, the cost of switching to a zero-tax regime is much higher for poorer than for wealthier agents of all types. This is because, in the long run, the elimination of the tax decreases the steady-state capital stock, thereby decreasing the present value of wages. Therefore poorer agents will suffer the most, since human wealth constitutes a big part of their total wealth.

Literature review. This paper focuses on entrepreneurial risk, because such risk is in fact empirically relevant, even in a financially developed country like the United States. For example, Moskowitz and Vissing-Jørgensen (2002) find that 75% of all private equity is owned by agents for whom such investment constitutes at least half of their total net worth. Furthermore, 85% of private equity is held by owners who are actively involved in the management of their own firm.[1] Given this evidence about the US, one expects that entrepreneurial risk must be even more prevalent in less developed economies, where a large part of production takes place in small unincorporated businesses and where risk-sharing arrangements are much more limited.

Furthermore, idiosyncratic investment risk need not be interpreted as affecting solely the owners of privately held businesses. In recent work, Panousi and Papanikolaou (2008) find a significant and robust negative relationship between idiosyncratic risk and the investment of publicly traded firms in the US. In addition, they show that this relationship is stronger in firms where the insider mangers hold a larger fraction of the firm's shares, and they provide evidence for a possible explanation that has to do with managerial risk aversion. Combined with the work of Moskowitz and Vissing-Jørgensen (2002), this demonstrates that a large fraction of total investment in the US, whether by publicly traded or privately held businesses, is sensitive to idiosyncratic risk, and therefore strengthens the empirical applicability of the present model setup.

This paper relates to the strand of the macroeconomic literature discussing optimal taxation and the effects of taxation. However, most of this literature has focused on labor income risk. Chamley (1986) and Judd (1985) first established the result of zero optimal capital taxation in the long run when markets are complete. Atkeson, Chari and Kehoe (1999) generalized this result to most of the short run for an interesting class of preferences, and to the case of finitely lived agents. Aiyagari (1995) extended the complete-markets framework to include uninsurable labor income risk and borrowing constraints. In this context, when only a limited set of policy instruments are available, it becomes optimal to tax capital in the long run: a positive capital tax increases welfare,

---

[1]Further evidence for these observations is also provided by Quadrini (00), Carroll (02), Gentry and Hubbard (00), and Cagetti and DeNardi (06).

but it unambiguously lowers the level of the capital stock.[2]

A related but different normative exercise is conducted by Davila et al. (2007). They examine constrained efficiency, in the spirit of Geanakoplos-Polemarchakis, within a version of Aiyagari's model. This exercise does not allow for risk-sharing through taxes or any other instruments, and instead considers an efficiency concept where the planner directly dictates to the agents how much to invest and to trade. Angeletos and Werning (2006) examine a similar constrained efficiency problem in a two-period version of a model with idiosyncratic investment risk. Albanesi (2006) considers optimal taxation in a two-period model of entrepreneurial activity, in a constrained efficiency setting, and following the Mirrlees optimal policy tradition. The benefit of her approach is that the source of incomplete risk-sharing is endogenously specified as the result of a private information (moral hazard) problem, and that there are no ad hoc restrictions placed on the tax instruments. However, her model does not allow for dynamics, for long-run considerations, or for general-equilibrium effects like those studied in the present paper. In general, the extensive theoretical work on taxation originating from the Mirrlees and the new dynamic public finance tradition focuses on labor-income risk, as does the literature that examines the optimal progressivity of the tax code.[3]

The growing literature on the effects of borrowing constraints on entrepreneurial choices has examined policy questions, and especially the implications of replacing a progressive with a proportional income-tax schedule, in an Aiyagari-type environment, i.e. with decreasing returns to scale at the individual level, borrowing constraints, and undiversifiable labor income risk. Some examples in this area include Li (2002), Domeij and Heathcote (2003), Meh (2005), Cagetti and DeNardi (2007), and Kitao (2007). Benabou (2002) develops a tractable dynamic general-equilibrium model of human capital accumulation with endogenous effort and missing credit and insurance markets. Within this framework he examines the long-run tradeoffs of progressive taxation and education finance. Finally, Erosa and Koreshkova (2007) examine the long-run effects of switching from progressive to proportional income taxation in a quantitative dynastic model of human capital.

This paper also relates to the branch of the public finance literature that considers the effects of capital taxation on portfolio allocation and risk-taking. Domar and Musgrave (1944) first proposed the idea that proportional income taxation may increase risk-taking, by having the government

---

[2]Alvarez et al. (1992), Erosa and Gervais (2002), and Garriga (2003), show that in life-cycle models the optimal capital-income tax is in general different from zero, at least if the tax code cannot explicitly be conditioned on the age of the household. Consea et al. (2008) quantitatively characterize the optimal capital- and labor- income tax in an overlapping-generations model with idiosyncratic uninsurable labor-income shocks and permanent productivity differences across households, and find for an optimal capital-income tax of 36%. Uhlig and Yanagawa (1995) show that, under mild conditions, higher capital-income taxes lead to faster growth in an overlapping-generations economy with endogenous growth. It should be noted, however, that the results of the present paper do not depend on a life-cycle or overlapping-generations setup. Instead, they arise in the context of the standard neoclassical framework of infinitely-lived agents.

[3]Some examples here include Golosov et al. (2003), Albanesi and Sleet (2005), Consea and Krueger (2006), Werning (2007), and Reiter (2004).

bear part of the risk facing the agents.[4] This idea was formalized by Stiglitz (1969), within a two-period single-agent model, where asset returns and the level of saving are exogenously given, but where the agent optimally chooses the allocation of his fixed amount of saving between a risky and a riskless asset. Ahsan (1974) extended Stiglitz by endogenizing the intertemporal consumption-saving decision in a two-period model. He showed that the partial-equilibrium effect of capital-income taxation on risk-taking is in general ambiguous. By contrast, in the "small open economy" version of the present model, which differs from Ahsan's in that the horizon is infinite and the return to capital is endogenous, it is shown that the steady-state capital stock is decreasing in the capital-income tax. This finding highlights that the results here are driven by, novel to the literature, general-equilibrium effects.

As already mentioned, the present model builds on Angeletos (2007), who abstracted from policy questions and considered instead the effect of investment risk on macroeconomic aggregates. The contribution of the present paper is to study the effects of capital-income taxation on aggregates and welfare. Angeletos and Panousi (2009), in a framework like the one in Angeletos (2007), examine the effects of government spending on macroeconomic aggregates, but for the case where government spending is financed exclusively through lump-sum taxation.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes individual behavior and the aggregate equilibrium dynamics. Section 4 characterizes the steady state in terms of aggregates and distributions. Section 5 presents and discusses the main theoretical result. Section 6 presents the calibration methodology and the parameter choices, along with the implications of the preferred calibrated model for aggregates and distributions. Section 7 quantifies the steady-state effects of capital taxation, as well as the short-run and long-run effects of eliminating the capital-income tax. Section 8 examines the robustness of the results to the availability of a safe asset in positive net supply. Section 9 concludes. All proofs are delegated to the appendix.

2   The Model

Time is continuous and indexed by $t \in [0, \infty)$. There is a continuum of agents distributed uniformly over $[0, 1]$. At each point in time an agent can be either an entrepreneur, denoted by E, or a laborer, denoted by L. The probabilities of switching between these two types are exogenous. In particular, the probability that an agent will switch from being an entrepreneur to being a laborer is $p_{EL} \, dt$, and the probability that he will switch from being a laborer to being an entrepreneur is $p_{LE} \, dt$. The measure of entrepreneurs in the economy at time t is denoted by χ_t.

In what follows, and for expositional simplicity, labor is taken to be exogenous. All of the proofs, which are delegated to the appendix, and all of the calibrations, will consider the general

---

[4]Sandmo (1977) extended this idea to the case of multiple risky assets.

case of endogenous labor, where preferences are homothetic between consumption and leisure, i.e. they are of the King-Plosser-Rebelo (1988) specification.

2.1 Preferences

All agents are endowed with one unit of time. Preferences are Epstein-Zin over consumption, c, and they are defined as the limit, for $\Delta t \to 0$, of [5]:

$$U_t = \left\{ (1 - e^{-\beta \Delta t}) \, c_t^{1-1/\theta} \; + \; e^{-\beta \Delta t} \, (E_t \, [\, U_{t+\Delta t}^{1-\gamma} \,])^{\frac{1-1/\theta}{1-\gamma}} \right\}^{\frac{1}{1-1/\theta}} , \qquad (1)$$

where $\beta > 0$ is the discount rate, $\gamma > 0$ is the coefficient of relative risk aversion, and $\theta > 0$ is the elasticity of intertemporal substitution. For $\theta = 1/\gamma$, this reduces to the case of standard expected utility, $U_t = E_t \int_t^\infty e^{-\beta s} U(c_s) \, ds$, where $U(c_t) = \frac{c_t^{1-1/\theta}}{1-1/\theta}$.

2.2 Entrepreneurs

When an agent is an entrepreneur, he owns and runs a firm operating a constant-returns-to-scale neoclassical production function $F(k, l)$, where k is capital input and l is labor input. An entrepreneur can only invest in his own firm's capital, although he supplies and employs labor in the competitive labor market. Capital investment in his firm is subject to uninsurable risk. The idiosyncratic shocks are i.i.d., hence there is no aggregate uncertainty. An entrepreneur can also save in a riskless bond.

The financial wealth of an entrepreneur i, denoted by x_t^i, is the sum of his holdings in private capital, k_t^i, and the riskless bond, b_t^i:

$$x_t^i = k_t^i + b_t^i . \qquad (2)$$

The evolution of x_t^i is given by:

$$dx_t^i = (1 - \tau_t^K) \, d\pi_t^i + [\, (1 - \tau_t^K) \, R_t \, b_t^i + (1 - \tau_t^L) \, \omega_t + T_t - c_t^i \,] \, dt , \qquad (3)$$

where $d\pi_t^i$ are firm profits (capital income), R_t is the interest rate on the riskless bond, τ_t^K is the proportional capital-income tax, ω_t is the wage rate in the aggregate economy, τ_t^L is the proportional labor-income tax, T_t are non-contingent lump-sum transfers received from the government, and c_t^i is consumption. Finally, a no-Ponzi game condition is imposed.

Firm profits are given by:

$$d\pi_t^i = [\, F(k_t^i, l_t^i) - \omega_t \, l_t^i - \delta \, k_t^i \,] \, dt \; + \; \sigma \, k_t^i \, dz_t^i , \qquad (4)$$

where $F(k, l) = k^\alpha l^{1-\alpha}$ with $\alpha \in (0, 1)$, and δ is the mean depreciation rate in the aggregate

---

[5]Lemma 1 in the appendix gives the formal description of preferences.

economy. Idiosyncratic risk is introduced through dz_t^i, a standard Wiener process that is i.i.d. across agents and across time[6]. The scalar σ measures the amount of undiversified idiosyncratic risk, and is an index of market incompleteness, with higher σ corresponding to a lower degree of risk-sharing, and $\sigma = 0$ corresponding to complete markets.

2.3 Laborers

When an agent is a laborer, he cannot invest in capital, and he can only hold the riskless bond. He also supplies labor in the competitive labor market. Financial wealth for a laborer i is therefore:

$$x_t^i = b_t^i \,, \tag{5}$$

and its evolution is given by:

$$dx_t^i = [\, (1 - \tau_t^K)\, R_t\, b_t^i + (1 - \tau_t^L)\, \omega_t + T_t - c_t^i \,]\, dt \,. \tag{6}$$

2.4 Government

At each point in time the government taxes capital and bond income at the rate τ_t^K, and labor income at the rate τ_t^L. Part of the tax proceeds is used by the government for own consumption at the deterministic rate G_t. Government spending does not affect the utility from private consumption or the production technology. The remaining tax proceeds are then distributed back to the households in the form of non-contingent lump-sum transfers, T_t. The government budget constraint is therefore:

$$0 = [\, \tau_t^L\, F_{L_t}(\int_i k_t^i, 1) + \tau_t^K\, (\, F_{K_t}(\int_i k_t^i, 1) - \delta\,) \int_i k_t^i - G_t\ - T_t \,]\, dt \,, \tag{7}$$

where $F_{K_t}(\int_i k_t^i, 1)$ is the marginal product of capital in the aggregate economy, $F_{L_t}(\int_i k_t^i, 1)$ is the marginal product of labor, and $\int_i l_t^i = 1$.

2.5 Finite lives and annuities

All households face a constant probability of death, with Poisson arrival rate $v\, dt$ at every instant in time.[7] There is no intergenerational altruism linking a household to its descendants, and utility

---

[6]Idiosyncratic risk is modeled here as coming from uninsurable i.i.d. depreciation shocks. However these shocks could also be modeled as or interpreted as i.i.d. productivity shocks.

[7]The (small) positive probability of death is introduced in order to guarantee the existence of a stationary wealth distribution. In general, with finite lives and no altruism, Ricardian equivalence might fail, since some of the tax burden associated with the current issue of a bond is borne by agents who are not alive when the bond is issued. Here, for $v = 0$, Ricardian equivalence holds, because all agents can freely borrow in the riskless bond. The theoretical steady-state results for the aggregates are derived for $v = 0$, and they carry through for v small but positive. However, it might still be the case that the dynamic effects of time-varying policy changes possibly depend on the validity of Ricardian equivalence. Nonetheless, for the purposes of this paper, the government budget constraint will be written

is zero after death. The discount rate in preferences can then be reinterpreted as $\beta = \tilde{\beta} + v$, where $\tilde{\beta}$ is the psychological or subjective discount rate and v is the probability of death[8].

In order to isolate the effects of capital-income risk, it is assumed that there exist annuity firms permitting all agents to get insurance against mortality risk, by freely borrowing the entire net present value of their future labor income. As a result, all agents have (safe) human wealth, denoted by h_t, and defined as the present discounted value of their net-of-taxes labor endowment plus government transfers:[9]

$$ h_t = \int_t^{\infty} e^{-\int_t^s \left((1-\tau_j^K)R_j + v \right) dj} \left((1-\tau_s^L)\omega_s + T_s \right) ds . \tag{8} $$

Then, the total effective wealth, w_t^i, for an agent is defined as the sum of his financial and human wealth, i.e. $w_t^i \equiv x_t^i + h_t$. Hence, effective wealth for an entrepreneur is given by:

$$ w_t^i = k_t^i + b_t^i + h_t , \tag{9} $$

and effective wealth for a laborer is given by:[10]

$$ w_t^i = b_t^i + h_t . \tag{10} $$

3   Equilibrium

This section characterizes individual behavior and the general equilibrium in the economy. The analysis will be in closed-form, since, as will be shown, the wealth distribution is not a relevant state variable for the characterization of aggregate equilibrium dynamics.

---

as in (7) for v positive but small.

[8]Since utility is zero after death, this is a valid interpretation that does not violate the axioms of expected utility.

[9]Let $\dot{h}_t = (R_t + v)h_t - \omega_t$, and $b_t = -h_t$. Then, $\dot{b}_t = R_t b_t - vh_t + \omega_t$. These equations are consistent with each other and with market clearing, and they have two alternative but isomorphic interpretations. First, in the beginning of time, every agent borrows from annuity firms an amount equal to his entire human wealth. From then on, he repays this debt every period by giving up his wage plus interest to the annuity firms, and this only stops when he dies. Second, the annuity firms borrow from the agent his entire human wealth, and every period from then on they repay the agent by giving him wage plus interest, until the agent dies. Either of these interpretations is consistent with the analysis here.

[10]It is assumed that capital is fully fungible upon exit from entrepreneurship. The assumption of exogenous transition probabilities is maintained here for tractability, in order to ensure a closed-form solution. This assumption could have the interpretation that, at some random point in time, the agent is given the chance to operate a high-return, high-risk technology, while at some other random point in time the option to save in this alternative technology is taken away (for example, the agent has an idea which depreciates at some exogenous rate).

3.1   Individual Behavior

Entrepreneurs choose employment after their capital stock has been installed and their idiosyncratic shock has been observed. Hence, since their production function, F, exhibits constant returns to scale, optimal firm employment and optimal profits are linear in own capital:

$$l_t^i = l(\omega_t)\, k_t^i \qquad \text{and} \qquad d\pi_t^i = r(\omega_t)\, k_t^i\, dt + \sigma\, k_t^i\, dz_t^i, \tag{11}$$

where $l(\omega_t) \equiv \arg\max_l [\, F(1,l) - \omega_t\, l\,]$ and $r(\omega_t) \equiv \max_l [\, F(1,l) - \omega_t\, l\,] - \delta$. Here, $r_t \equiv r(\omega_t)$ is an entrepreneur's expectation of the return to his capital prior to the realization of his idiosyncratic shock, as well as the mean of the realized returns in the cross-section of firms. The key result here is that entrepreneurs face risky, but linear, returns to their investment.

  The evolution of effective wealth for an entrepreneur is described by:

$$dw_t^i = [\, (1 - \tau_t^K)\, r_t\, k_t^i + (1 - \tau_t^K)\, R_t\, (b_t^i + h_t) - c_t^i\,]\, dt + \sigma\,(1 - \tau_t^K)\, k_t^i\, dz_t^i. \tag{12}$$

The first term captures the expected rate of growth of effective wealth, and it shows that wealth grows when the total return to saving for an entrepreneur exceeds consumption expenditures. The second term captures the impact of idiosyncratic risk. The evolution of effective wealth for a laborer is described by:

$$dw_t^i = [\, (1 - \tau_t^K)\, R_t\, (b_t^i + h_t) - c_t^i\,]\, dt. \tag{13}$$

  Let the fraction of effective wealth an agent saves in the risky asset be:

$$\phi_t^i \equiv \frac{k_t^i}{w_t^i}. \tag{14}$$

Let an agent's marginal propensity to consume out of effective wealth be:

$$m_t^i \equiv \frac{c_t^i}{w_t^i}. \tag{15}$$

  Let $\mu_t = (1 - \tau_t^K) r_t - (1 - \tau_t^K) R_t$ denote the risk premium. Since investment in capital is risky, it has to be the case that $r_t > R_t$, otherwise no one would invest in capital. In other words, agents require a positive risk premium as compensation for undertaking capital investment. Let $\rho_t \equiv \phi_t\,(1 - \tau_t^K)\, r_t + (1 - \phi_t)\,(1 - \tau_t^K)\, R_t$ denote the net-of-tax mean return to saving for an entrepreneur, and let $\hat{\rho}_t \equiv \rho_t - 1/2\,\gamma\, \phi_t^2\, \sigma^2\,(1 - \tau_t^K)^2$ denote the net-of-tax risk-adjusted return to saving for an entrepreneur. The net-of-tax return to saving for a laborer is simply $(1 - \tau_t^K)\, R_t$. Then, since $R_t < r_t$, it has to be that $(1 - \tau_t^K)\, R_t < \hat{\rho}_t < \rho_t < (1 - \tau_t^K)\, r_t$.

  Because of the linearity in assets of the budget constraints (12) and (13), and the homotheticity of the preferences, the optimal individual policy rules will be linear in total effective wealth, for

9

given prices and government policies. Hence, for given prices and policies, an agent's consumption-saving problem reduces to a tractable homothetic problem as in Samuelson's and Merton's classic portfolio analysis. Optimal individual behavior is then characterized by the following proposition.

Proposition 1. *Let $\{\omega_t, R_t, r_t\}_{t \in [0,\infty)}$ and $\{\tau_t^K, \tau_t^L, T_t, G_t\}_{t \in [0,\infty)}$ be equilibrium price and policy sequences. If an agent i is an entrepreneur, his optimal consumption, investment, portfolio, and bond holding choices, respectively, are given by:*

$$c_t^i = m_t^E\, w_t^i, \quad k_t^i = \phi_t\, w_t^i, \quad \phi_t = \frac{(1 - \tau_t^K)\, r_t - (1 - \tau_t^K)\, R_t}{\gamma\, \sigma^2\, (1 - \tau_t^K)^2}, \quad b_t^i = (1 - \phi_t)\, w_t^i - h_t\,. \tag{16}$$

If an agent i is a laborer, his optimal consumption, investment, and bond holding choices, respectively, are given by:

$$c_t^i = m_t^L\, w_t^i, \quad k_t^i = 0, \quad b_t^i = w_t^i - h_t\,. \tag{17}$$

The marginal propensities to consume satisfy the following system of ordinary differential equations:

$$\frac{\dot{m}_t^E}{m_t^E} = m_t^E - \theta\beta + (\theta - 1)\, \hat{\rho}_t + \frac{\theta - 1}{1 - \gamma}\, p_{EL}\, \left[\, \left(\frac{m^L}{m^E}\right)^{\frac{1-\gamma}{1-\theta}} - 1\, \right] \tag{18}$$

$$\frac{\dot{m}_t^L}{m_t^L} = m_t^L - \theta\beta + (\theta - 1)(1 - \tau_t^K)\, R_t + \frac{\theta - 1}{1 - \gamma}\, p_{LE}\, \left[\, \left(\frac{m^E}{m^L}\right)^{\frac{1-\gamma}{1-\theta}} - 1\, \right]. \tag{19}$$

From (16) and (17) it is clear that optimal consumption is a linear function of total effective wealth, where the marginal propensity to consume depends only on the type of the agent, and not on the level of wealth. In other words, all entrepreneurs share a common marginal propensity to consume, m_t^E, and all laborers share a common marginal propensity to consume, m_t^L. The fraction ϕ_t of wealth invested in the risky asset by an agent who happens to be an entrepreneur is increasing in the risk premium, decreasing in risk aversion, and decreasing in the effective variance of risk, $\sigma(1 - \tau_t^K)$. Because of homotheticity and linearity, ϕ_t is the same across all entrepreneurs, and independent of the level of wealth. The policy for optimal bond holdings follows from (9) or (10), and (14). The system of (18) and (19) is a system of two Euler equations. It shows that the marginal propensities to consume, conditional on being an entrepreneur or a laborer, depend on two factors. First, on the process of the corresponding net-of-tax anticipated (risk-adjusted) returns to saving, in accordance with whether the elasticity of intertemporal substitution, θ, is higher or lower than 1. Second, on the probability that the agent might switch between being an entrepreneur and being a laborer.

3.2 General equilibrium

The initial position of the economy is given by the distribution of (k_0^i, b_0^i) across households. An equilibrium is a deterministic sequence of prices $\{\omega_t, R_t, r_t\}_{t \in [0,\infty)}$, a deterministic sequence of poli-

cies $\{\tau_t^K, \tau_t^L, T_t, G_t\}_{t \in [0,\infty)}$, a deterministic macroeconomic path $\{C_t, K_t, Y_t, L_t, W_t, W_t^E, W_t^L\}_{t \in [0,\infty)}$, and a collection of individual contingent plans $(\{c_t^i, l_t^i, k_t^i, b_t^i, w_t^i\}_{t \in [0,\infty)})$ for $i \in [0,1]$, such that the following conditions hold: (i) given the sequences of prices and policies, the plans are optimal for the households; (ii) the labor market clears, $\int_i l_t^i = 1$, in all t; (iii) the bond market clears, $\int_t b_t^i = 0$, in all t; (iv) the government budget constraint (7) is satisfied in all t; and (v) the aggregates are consistent with individual behavior, $C_t = \int_i c_t^i$, $L_t = \int_i l_t^i = 1$, $K_t = \int_i k_t^i$, $Y_t = \int_i F(k_t^i, l_t^i) = F(\int_i k_t^i, 1)$, $W_t = \int_i w_t^i$, $W_t^E = \int_{i,\,E} w_t^i$, and $W_t^L = \int_{i,\,L} w_t^i$, in all t.

Because individual consumption and investment are linear in individual wealth, aggregates at any point in time do not depend on the extend of wealth inequality at that time. Therefore here, in contrast to other incomplete-markets models, it is not the case that the entire wealth distribution is a relevant state variable for aggregate dynamics. In fact, for the determination of aggregate dynamics, it suffices to keep track of the mean of aggregate wealth, and of the allocation of total wealth between the two groups of agents. To that end, let the fraction of total effective wealth held by entrepreneurs in the economy be:

$$\lambda_t \equiv \frac{W_t^E}{W_t} . \tag{20}$$

The aggregate equilibrium dynamics can then be described by the following recursive system.

Proposition 2. *In equilibrium, the aggregate dynamics satisfy:*

$$\dot{W}_t/W_t = \lambda_t(\rho_t - m_t^E) + (1 - \lambda_t)((1 - \tau_t^K)R_t - m_t^L) \tag{21}$$

$$\dot{\lambda}_t/\lambda_t = (1 - \lambda_t)\phi_t\mu_t + (1 - \lambda_t)(m_t^L - m_t^E) + p_{LE}(\frac{1}{\lambda_t} - 1) - p_{EL} \tag{22}$$

$$\dot{H}_t = ((1 - \tau_t^K)R_t + v)H_t - (1 - \tau_t^L)\omega_t - (\tau_t^L\omega_t + \tau_t^K(F_{K_t} - \delta)K_t - G_t) \tag{23}$$

$$K_t = \frac{\phi_t \lambda_t}{1 - \phi_t \lambda_t} H_t , \tag{24}$$

along with (18) *and* (19).

Equation (21) shows that the evolution of total effective wealth is a weighted average of two terms. The first term is positive when the mean net-of-tax return to saving for entrepreneurs exceeds their marginal propensity to consume, and is weighted by the fraction of total wealth the entrepreneurs hold in the economy. The second term is positive when the net-of-tax return to saving for laborers exceeds their marginal propensity to consume, and is weighted by the fraction of total wealth the laborers hold in the economy. Equation (22) shows the endogenous evolution of the relative distribution of wealth between the two groups of agents. The evolution of λ depends on three factors. First, on the differential excess return the entrepreneurs face on their saving, which is given by $\phi_t\mu_t$, where ϕ_t is the fraction of wealth invested in the risky asset, and μ_t is the risk premium. Second, on the difference in the level of saving between entrepreneurs and laborers, as captured by the difference in the marginal propensities to consume, $m_t^L - m_t^E$. Third, on the

adjustment made for the transition probabilities. Note here that the evolution of consumption can be recovered by aggregating across individual optimal policies, so that $C_t^E = m_t^E W_t^E$ and $C_t^L = m_t^L W_t^L$, and using (18), (19), (21), and (22). Equation (23) shows the evolution of total human wealth, using the government budget constraint $T_t = \tau_t^L \omega_t + \tau_t^K (F_{K_t} - \delta) K_t - G_t$, where F_{K_t} is the marginal product of capital in the aggregate production function $F(K, 1)$, and where $\omega_t = F_{L_t}(K_t, 1)$ from market clearing. Since $\dot{W} = \dot{K} + \dot{H}$, the resource constraint of the economy is also satisfied. Equation (24) is the bond market clearing condition. It comes from aggregating across individual capital and bond choices as given in (16) and (17), adding up, using $B_t^E + B_t^L = 0$, and using (20). From (24) it follows that, for given prices and human wealth, a decrease in λ decreases K. A fall in λ indicates that the entrepreneurs on average now borrow more from the laborers, hence their wealth will on average be lower. With decreasing absolute risk aversion, this will negatively affect their willingness to take risk, and therefore investment and the capital stock will fall for given prices.

3.3 Steady state: characterization of aggregates

A steady state is a competitive equilibrium as defined in section 3.2, where prices, policies, and aggregates are time-invariant. For expositional purposes, and to illustrate that the results about the effects of capital-income taxation on the aggregates are not due to the presence of two types of agents or to the probability of death, section 3.3 (as well as section 4 later on) will consider the case with $\lambda = 1$ and $v = 0$[11]. However, section 3.4 will characterize the invariant distributions for the general case.

The steady state is the fixed point of the dynamic system in Proposition 2. Let government spending, G, be parameterized as a fraction g of tax revenue. The following proposition characterizes the steady state.

Proposition 3. *(i) The steady state always exists and is unique. (ii) In steady state, the capital stock, K, and the interest rate, R, are the solution to:*

$$F_K(K) - \delta = R + \sqrt{\frac{2\,\theta\,\gamma\,\sigma^2}{\theta+1}\left(\beta - (1 - \tau^K)\,R\right)} \tag{25}$$

$$K = \frac{\phi(K, R)}{1 - \phi(K, R)} \frac{(1 - \tau^L)\,\omega(K) + (1 - g)\left(\tau^L\,\omega(K) + \tau^K\,(F_K(K) - \delta)\,K\right)}{(1 - \tau^K)\,R}, \tag{26}$$

where $F_K(K)$ is the marginal product of capital and $\omega(K)$ is the wage rate in the aggregate economy.

From (18) or (19) and (21) in steady state, and using the fact that $\phi\mu = (F_K - \delta - R)^2/\gamma\sigma^2$, we get equation (25). This condition gives the combinations of K and R that are consistent with

---

[11]The more general case is left for the appendix.

wealth and consumption stationarity. Using (24) and (23) in steady state yields equation (26). This condition gives the combinations of K and R that are consistent with stationarity of human wealth and bond market clearing.

At this point it is useful to briefly compare the steady state to its complete-markets counterpart. From (25) note that the difference from complete markets, in which case it would be $F_K(K) - \delta = R$, is the presence of the square-root term, which captures the risk premium, i.e. here $\mu(R) = \sqrt{2\,\theta\,\gamma\,\sigma^2\,(\,\beta - (1-\tau^K)\,R\,)\,/\,(\theta+1)} \neq 0$. In other words, agents here require a (private) risk premium in order to invest in capital. In addition, combining (18) or (19) with (21), and using the fact that $C = mW$, we get $\dot{C}/C = \theta\,(\hat{\rho}_t - \beta) + \frac{1}{2}\,\gamma\,\phi_t^2\,\sigma_t^2\,(1-\tau_t^K)^2$, from which, in steady state, we conclude that:

$$\hat{\rho} \;=\; \beta \;-\; \frac{1}{2}\,\frac{\gamma}{\theta}\,\phi^2\,\sigma^2\,(1-\tau^K)^2\;. \tag{27}$$

In other words, the risk-adjusted return to saving must be just low enough to offset the precautionary saving motive,[12] which is present here because agents face risk in their consumption stream. Since $(1-\tau^K)R < \hat{\rho}$, it follows that $(1-\tau^K)R < \beta$, i.e. the net interest rate is lower than it would have been under complete markets. This result is also true in Aiyagari (1994) and in other Bewley-type models, with the difference that in Bewley models it is labor-income that introduces the risk in the consumption stream. Furthermore here, because $F_K - \delta > R$, it could be the case either that $F_K - \delta > \beta$ or $F_K - \delta < \beta$. Hence, capital can be either lower or higher than under complete markets.[13] This is in contrast to the effects of labor-income risk on steady-state capital, and it is due to the fact that idiosyncratic investment risk introduces a wedge (the risk premium) between the return to the risky asset and the return to the riskless asset.

3.4   Steady state: characterization of invariant distributions

At each point in time, agents die and are replaced by newborn agents, and the assumption is that the newborn agents are endowed with the wealth of the exiting agents.[14] This force generates mean reversion and guarantees the existence of an invariant wealth distribution. Let $\xi_t^i \equiv w_t^i/W_t$ denote the distance between individual and aggregate effective wealth. Let Φ_L and Φ_E be the conditional invariant distributions for laborers and entrepreneurs respectively. The following proposition characterizes the invariant distributions.

---

[12]If the risk-adjusted return were higher than this critical level, consumption (and wealth) would increase over time without bound, which would be a contradiction of steady state. Conversely, if the risk-adjusted return were lower than this level, consumption (and wealth) would shrink to zero, which would once again be a contradiction of steady state.

[13]Angeletos (2007) gives a condition that determines whether steady-state capital is higher or lower than under complete markets, and quantifies the effects of idiosyncratic capital-income risk on steady-state aggregates.

[14]Hence, from a law of large numbers, each agent starts life with the sum of human wealth plus the mean wealth in the economy.

Proposition 4. *The conditional invariant distributions Φ_L and Φ_E are characterized by the following second order linear differential system:*

$$0 = \kappa_1 \, \xi \, \frac{\partial \Phi_L}{\partial \xi} \; + \; \kappa_2 \, \Phi_L \; + \; p_{EL} \, \Phi_E,$$

$$0 = \kappa_3 \, \xi^2 \, \frac{\partial^2 \Phi_E}{\partial \xi^2} \; + \; \kappa_4 \, \xi \, \frac{\partial \Phi_E}{\partial \xi} \; + \; \kappa_5 \, \Phi_E \; + \; p_{LE} \, \Phi_L \, ,$$

where $\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5$ are constants determined by steady-state aggregates.

The point to note here is that the tractability of the model allows for a very detailed characterization of the invariant distributions. This is particularly useful for the case of entrepreneurs, since it is reasonable to expect that the distribution of wealth over entrepreneurs will be, to a large extent, determined by the realization of entrepreneurial returns.[15]

4 Steady-State Effects of Capital Taxation

This section presents the core of the contribution of this paper, which is the study of the steady-state effects of capital-income taxation. Again, for illustration purposes, the assumption is that $\lambda = 1$ and $v = 0$. The main result here is that an increase in the capital-income tax may actually increase investment and the steady-state capital stock. This possibility arises because of the general-equilibrium effects of the insurance aspect of capital taxation, which operate mainly through the endogenous adjustment of the interest rate. In order to illustrate this, the analysis will proceed by making the distinction between the case where the interest rate is fixed, and the case where the interest rate is allowed to adjust endogenously.

Note then that equation (25) expresses capital, K, as a function of the tax, τ^K, and the interest rate, R. If the interest rate were fixed,[16] then the steady-state capital stock would be $K^o(\tau^K, R)$, as given by (25), and where both τ^K and R are exogenous. Next, by plugging $K^o(\tau^K, R)$ from (25) into (26), we can solve for the closed-economy steady-state interest rate, as a function of the capital-income tax. Let $R^c(\tau^K)$ denote the closed-economy solution for the interest rate. It follows then, that the closed-economy steady-state capital stock will be given by $K^c(\tau^K) = K^o(\tau^K, \, R^c(\tau^K))$.

Hence, the impact of the capital-income tax on the closed economy steady-state capital stock can be decomposed in two parts. The first part describes how steady-state capital changes with the tax when the interest rate is kept constant or exogenously fixed. The second part describes the general-equilibrium adjustment of the interest rate in the closed economy, and the subsequent

---

[15]Whereas the tractability of the aggregates follows from Angeletos (2007), the result about the tractability of the invariant distributions is novel to the present paper.

[16]This would be the case, for example, in a (small) open-economy version of the present model. This would be an economy with the same preferences, technologies, and risks, but which is open to an international market for the riskless bond, thus facing an exogenously fixed interest rate.

effects of this adjustment on capital accumulation. Thus, the total effect of the capital-income tax on the closed-economy steady-state capital stock can be decomposed as follows:

$$\frac{dK^c}{d\tau^K} = \frac{\partial K^o}{\partial \tau^K} + \frac{\partial K^o}{\partial R}\frac{dR^c}{d\tau^K},$$ (28)

where the first term is the effect when the interest rate is fixed, and the second term is the effect when the interest rate is allowed to adjust, i.e. it is the closed-economy or general-equilibrium effect.

Let's first turn to the fixed-interest rate effect. The following corollary characterizes the effect of capital-income taxation on capital accumulation when the interest rate is held constant.

Corollary 1. *When the interest rate is exogenously fixed, an increase in the capital-income tax unambiguously reduces the steady-state capital stock, i.e. $\partial K^o/\partial \tau^K < 0$.*

This result follows immediately from (25), for a given R. Hence, when the interest rate is kept constant, capital falls with the tax, despite a direct insurance aspect of the tax that is still present, namely that the tax reduces the variance of net returns, $\sigma(1-\tau^K)$. Clearly then, for a given interest rate, this channel is not strong enough to outweigh the distortionary effect of capital taxation on investment. This result stands in contrast to the findings of Ahsan (1974). Ahsan considers the simultaneous determination of the size and the composition of the optimal portfolio, in a two-period model with exogenous returns. He shows that the effect of an increase in capital-income taxation on risk-taking and capital is in general ambiguous.[17] The result here indicates that, once Ahsan's setting is extended to incorporate endogenous capital return and infinite horizon, the ambiguity disappears and capital taxation always leads to a fall in the steady-state capital stock. It is then clear that, in addition to the direct insurance role of the tax, the endogenous adjustment of the interest rate is also required for the effect of capital taxation on capital to become ambiguous once again.

Let's now turn to the general-equilibrium effect, which captures the fact that in the closed economy the interest rate endogenously adjusts to clear the bond market, according to equation (26). This effect further consists of two parts.

First, an increase in the capital-income tax reduces the effective volatility of risk for entrepreneurs, $\sigma(1-\tau^K)$, and this is the direct insurance effect mentioned above. As a result, the interest rate, which is below the discount rate in steady state, increases, essentially because of a reduction in the demand for precautionary saving, i.e. $dR^c/d\tau^K > 0$.[18] In fact, the increase in

---

[17]Ahsan's result is, in turn, a generalization of Stiglitz (1969), who examines the effects of proportional capital-income taxation in a two-period model, taking not only returns, but also the level of saving as exogenously given.

[18]This intuitive result has not been proven in the context of the infinite horizon model, although a proof is available for the two period version of the closed economy, for small τ^K. There, it can be shown in closed-form that steady-state capital is inversely U-shaped with respect to the capital-income tax. Nonetheless, simulations show that in the infinite-horizon closed-economy model the net interest rate is always increasing in the tax, as section 6.1 will demonstrate.

the interest rate is so high, that the net interest rate, $R(1 - \tau^K)$, ends up increasing, despite the increase in the capital-income tax.

Second, this increase in the (net) interest rate will generate two opposing effects on saving and wealth accumulation, as can be seen from (25). On the one hand, an increase in the interest rate increases the opportunity cost of capital, and thus it tends to lower the steady-state capital stock. On the other hand, an increase in the interest rate tends to increase the return to saving, and hence the steady-state wealth of entrepreneurs. With decreasing absolute risk aversion, this increases entrepreneurs' willingness to take risk, and hence it is a force that tends to increase the steady-state capital stock. This second effect is due to the fact that here investment is sensitive to wealth, a mechanism which is absent when markets are complete. In other words, agents require a (private) risk premium in order to invest in capital, but this premium is lower at higher levels of wealth.[19] Therefore, the overall effect of an increase in R on K is ambiguous, as is summarized in the following corollary.

Corollary 2. *When the interest rate is taken to be exogenous, $\partial K^o / \partial R \Leftrightarrow \theta > \phi/(1 - \phi)$.*

The proof for this corollary also follows from equation (25), and is left for the appendix. The intuition behind this result is a bit convoluted, so it is worth examining step-by-step. Combining equations (18) or (19) and (21) in steady state, we get:

$$\rho + (\theta - 1)\,\hat{\rho} = \theta\beta, \tag{29}$$

where ρ is the mean return to saving, and $\hat{\rho}$ is the risk adjusted return, both evaluated at the steady-state K and for given R. Of course, this condition is equivalent to (25), but it is more useful for developing intuition.

Note first that an increase in K necessarily reduces $\rho + (\theta - 1)\hat{\rho}$. This is because an increase in K reduces $f'(K)$, and, for given ϕ, this reduces ρ and $\hat{\rho}$ equally, thus also reducing $\rho + (\theta - 1)\hat{\rho}$. Of course, the optimal ϕ must also fall, but this only reinforces the negative effect on ρ (since the portfolio is shifted towards the low-return bond), while it does not affect $\hat{\rho}$ (because of the envelope theorem and the fact that ϕ maximizes $\hat{\rho}$).

Note next that an increase in R has an ambiguous effect on $\rho + (\theta - 1)\hat{\rho}$. This is because, for given ϕ, both ρ and $\hat{\rho}$ increase with R, but now the decrease in ϕ works in the opposite direction, contributing to lower ρ. Intuitively, though, this effect should be small if ϕ was small to begin with. Moreover, the impact of $\hat{\rho}$ is likely to dominate if θ is high enough. Therefore, $\rho + (\theta - 1)\hat{\rho}$ is expected to increase with R if and only if either ϕ is low or θ is high.

---

[19]To see this wealth effect more clearly, note that we can use (23) and (25) to write steady-state human wealth as $H(R) = H(K(R))$. Then, by bond market clearing, steady-state aggregate wealth is $W(R) = K(R) + H(R)$. The appendix[20] shows that $W'(R) > 0 \Leftrightarrow \mu'(R) < \bar{\mu} < 0$. But from (25) it is easy to show that $\mu''(R) < 0$. Hence, $W'(R) > 0 \Leftrightarrow \mu'(R) < \bar{\mu} \Leftrightarrow R > \underline{R}$. In other words, when the interest rate is above a certain threshold, then an increase in the interest rate increases aggregate steady-state wealth.

Combining these two observations leads to the conclusion that steady-state K increases with R if and only if $\theta > \phi/(1-\phi)$. As shown in section 3.3, this condition is more likely to be satisfied when R is sufficiently high, since the steady-state risk premium is a decreasing function of R. Intuitively, when R is close to β, a marginal increase in R has such a strong positive effect on steady-state wealth, that the consequent reduction in the risk premium more than offsets the increase in the opportunity cost of investment, ensuring that K increases with R.

The following proposition now summarizes the discussion above and the main result of this section.

Proposition 5. *If $\theta > \phi/(1-\phi)$ and $dR^c/d\tau^K$ sufficiently high, then $dK^c/d\tau^K > 0$, i.e. the closed-economy steady-state capital stock is increasing in the capital-income tax.*

In order to assess the empirical relevance of the relationship $\theta > \phi/(1-\phi)$, one can use a simple back-of-the-envelope calculation that does not require any reference to the degree of market incompleteness, σ. In particular, take labor income to be 65% of GDP, and take the safe rate to be 2%. Then, steady-state H is about 33 times GDP, or 11 times K, if the steady-state capital-output ratio is taken to be 3. Hence, $\phi/(1-\phi) = K/H = 0.1$, which, as section 5 will discuss, is lower than most of the empirical estimates of θ that use micro data for the United States. Hence, in all likelihood, the condition $\theta > \phi/(1-\phi)$ is satisfied in the data. At the same time, a high positive value for $dR^c/d\tau^K$ is intuitive, considering the (insurance) effect of the tax on the demand for precautionary saving.

It is therefore possible that, although $\partial K^o/\partial\tau^K < 0$, it could still be that $dK^c/d\tau^K > 0$ over some region, since $\partial K^o/\partial R \cdot dR^c/d\tau^K > 0$ is very likely positive. This means that the general-equilibrium effect of insurance on the adjustment of the interest rate, and the subsequent effect of this adjustment on wealth accumulation, is crucial for overthrowing the negative effect of the capital-income tax on capital when the interest rate is fixed. The next sections will demonstrate how, for empirically plausible parameter values, this general-equilibrium effect will produce the counter-intuitive result that increases in the capital-income tax will at first increase steady-state capital, even with the fixed interest-rate effect working in the opposite direction.

5   Calibration and Steady-State Implications

For the quantitative part of the paper, the benchmark model analyzed so far is extended to include endogenous labor. Preferences are assumed homothetic between consumption, c and leisure, n, according to the King-Plosser-Rebelo (1988) specification, and they are defined as the limit, for $\Delta t \to 0$, of:

$$U_t = \{ \, (1 - e^{-\beta\Delta t})\,(c_t^{1-\psi}n_t^{\psi})^{1-1/\theta} + e^{-\beta\Delta t}\,(\,E_t[U_{t+\Delta t}^{1-\gamma}]\,)^{\frac{1-1/\theta}{1-\gamma}} \, \}^{\frac{1}{1-1/\theta}} \,. \tag{30}$$

The appendix presents all proofs for the general case of endogenous labor. Note also that the calibration will treat the case with two types of agents, i.e. there will be both entrepreneurs and laborers in the model. This section will present the benchmark calibration, and will examine its implication for the steady-state aggregates and wealth distributions. The next section will then focus on the effects of capital-income taxation for aggregates and welfare.

5.1 Simulations

The dynamic system described in Proposition 2, and generalized to the case of endogenous labor, is highly tractable compared to other incomplete-markets models, where the entire wealth distribution is a relevant state variable for aggregate equilibrium dynamics. The steady state of the system is found by setting the dynamics of all equations in Proposition 2 to zero. The algorithm first solves for the steady-state aggregates, which are deterministic and characterized by Proposition 3. Subsequently, for any historically given (K_0, χ_0, X_0^E), where χ_0 is the initial measure of entrepreneurs in the economy, and X_0^E is the historically given financial wealth of the entrepreneur group, and using as boundary conditions the steady state values of (H, m^E, m^L), it integrates backward until the path of $(K_t, \lambda_t, H_t, m_t^E, m_t^L)$ is close enough to its steady-state value.

The method of finite differences is used on the general version of the system in Proposition 4. The first and second derivatives of the invariant distributions are replaced by their discrete time approximations. The only conditions imposed are that the probability density functions integrate to one, and that they do not explode to the right. The emerging functions Φ_L and Φ_E are well-behaved and stable.

Subsequently, Monte-Carlo simulations are performed. The processes of dying, of type-switching, and of the idiosyncratic capital-income shocks, are simulated using random number generators for series of 200,000 households and 100,000 years. The wealth distributions generated converge to those produced by the finite-differences method, and their variances are stable as time increases. Finally, using these distributions, welfare calculations are performed.

5.2 Parameter choice

The economy is parameterized by $(\alpha, \beta, \gamma, \delta, \theta, \sigma, \psi, v, p_{EL}, p_{LE}, \tau^K, \tau^L, G)$. Table 1 presents the parameter choices for the preferred benchmark model calibration.

Parameters	Values
Preferences	
β	0.024
γ	8
θ	1
ψ	0.75
Technology	
α	0.40
δ	0.06
Probabilities	
v	0.0067
p_{EL}	0.18
p_{LE}	0.025
Government	
τ^K	0.25
τ^L	0.35
G/GDP	0.20
Risk	
σ	0.15

Table 1. Benchmark Calibration Values.

The parameter values chosen refer to annual data from the United States. The discount rate is $\beta = 0.024$. The preference parameter is $\psi = 0.75$, which is standard in the macro literature[21]. The income share of capital is $\alpha = 0.40$. The depreciation rate is $\delta = 0.06$. The probability of death is chosen to be $v = 1/150$, a compromise between having an empirically relevant probability of death and allowing for some altruism across generations. The probability of exiting entrepreneurship is $p_{EL} = 0.18$. The probability of entering entrepreneurship is $p_{LE} = 0.025$. These two values were estimated from the PSID and SCF data, and subsequently used for calibrations, by Quadrini (2000). In Quadrini's model, as well as here, they imply a fraction of entrepreneurs in the total population of 12%,[22] which is in line with the data, as Quadrini and Cagetti and DeNardi (2006) document.

The elasticity of intertemporal substitution is chosen to be $\theta = 1$. The empirical estimates of the EIS vary a lot. Using aggregate British data and correcting for aggregation bias, Attanasio and Weber (1993) estimate θ to be about 0.7. Although the exact estimates from micro data vary across studies and specifications, in most cases they are around 1, especially for agents at the top layers

---

[21]For example, King, Plosser, and Rebelo (1988), and Christiano and Eichenbaum (1992).
[22]The proof can be found in Lemma 3 of the appendix.

of wealth and asset holdings. For example, using data from the Consumer Expenditure Survey (CEX) and an Epstein-Zin specification, as in the present paper, Vissing-Jørgensen and Attanasio (2003) report baseline estimates between 1 and 1.4 for stockholders.

The proportional tax on capital income is $\tau^K = 0.25$. The Congressional Budget Office Background Paper (December 2006) reports that the average marginal rate at which corporate profits are taxed is 35%, whereas the average marginal rate at which non-corporate business income is taxed is around $26\% - 27\%$. The CRS Report for Congress (October 2003) details the capital income tax revisions and effective tax rates due to provisions granted through bonus depreciations of 30% or 50%. If these provisions are taken into account, the average marginal capital income tax is between $20\% - 25\%$ for non-corporate businesses and between $25\% - 30\%$ for corporate businesses. The value of $\tau^K = 0.25$ is chosen to be in the middle of these estimates.[23] The proportional tax on labor income is $\tau^L = 0.35$. The Congressional Budget Office Background Paper (December 2006) reports that the median effective marginal tax rate on labor income is 32%, inclusive of federal, state and payroll taxes.[24] Incorporating the distortionary effect of social security taxes would further increase this number, hence the choice made here. The level of government spending, G, is chosen so that the steady-state government-spending-to-GDP ratio is 20%.

The coefficient of relative risk aversion is chosen to be $\gamma = 8$. The empirical estimation of γ is a complicated task, because, as Vissing-Jørgensen and Attanasio (2003) detail, it requires making additional assumptions about the covariance of consumption growth with stock returns, the share of stocks in the financial wealth portfolio, the properties of the expected returns to human capital, and the share of human capital in overall wealth. Using the Consumer Expenditure Survey (CEX), Vissing-Jørgensen and Attanasio find estimates of risk aversion for stockholders in the range of $5 - 10$, but with a broader sample and under different assumptions these estimates go up to $20 - 30$. They also compare their results to Campbell (1996), who estimates γ in the range of $17 - 25$, using data on monthly and annual returns, and assuming that the entire financial portfolio is held in stocks. Alan and Browning (2008), use the PSID data to structurally estimate the joint distribution of discount factors and relative risk aversion coefficients. They find that the lower educated households are less risk averse than the more educated households, and that the medians of the two relative risk aversion distributions are 6.2 and 8.4 respectively. Guiso and Paiella (2005), using data from the 1995 Bank of Italy Survey on Household Income and Wealth, estimate direct measures of risk aversion based on the maximum price a consumer is willing to pay to buy a risky asset. They find that the median relative risk aversion is 6, if consumers have a one-year horizon, and it is 16, if they have a lifetime horizon. Dohmen et el. (2005) present evidence on the distribution of risk attitudes in the population, using survey questions and a representative sample of 22,000 individuals living in Germany. The behavioral relevance of their survey is tested by conducting a

---

[23]Altig et al. (2001) report a proportional capital income tax of 20% at the federal level, but they also subject capital income to a 3.7% state tax.

[24]This number is also reported by Jokisch and Kotlikoff (2006).

complementary field experiment, based on a representative sample, and the conclusion is that the survey measure is a good predictor of actual risk-taking behavior. They find that the bulk of the mass in the γ-distribution is located between $1 - 10$. There is, however, a non-negligible mass of estimates in the range of higher values, up to 20. Barsky et al. (1997) measure risk aversion based on survey responses by participants in the Health and Retirement Study to hypothetical situations. They find that most individuals fall in the category that has mean relative risk aversion of 15.8. Cohen and Einav (2005) use a data set of 100,000 individuals' deductible choices in auto insurance contracts, to estimate the distribution of risk preferences. They find that the 82nd percentile in the distribution of the coefficient of relative risk aversion is about $13 - 15$.[25]

The volatility coefficient is chosen to be $\sigma = 0.15$. The empirical estimation of the standard deviation of idiosyncratic entrepreneurial returns is a very difficult task, and has not as yet received much attention in the literature. So far, the most thorough attempt to measure idiosyncratic risk is by Moskowitz and Vissing-Jørgensen (2002) and is at the cross-sectional level. Using the Survey of Consumer Finances (SCF) data, they document poor diversification and extreme concentration of entrepreneurial investment, significant heterogeneity in individual investment choices, and high risk at the individual level due to high bankruptcy rates. However, because of the problems arising when imputing labor income, and because of the lack of sufficient time dimension in the data, they cannot provide an accurate estimate of the volatility of entrepreneurial returns for unincorporated businesses. In the end, they conjecture that the volatility of returns for private firms cannot be lower than the corresponding volatility of publicly traded firms, which the find to be about 0.5 per annum. This number is also used in calibrations by Bitler et al. (2005), and by Roussanov (2009). On the other hand, Benhabib and Zhu (2008) use a value of 0.26.[26] The point to note here is that, because of the lack of an accurate measure, the finance literature attempting to quantify entrepreneurial risk has used plausible but arbitrary magnitudes for the private rate-of-return volatility.

An indirect estimate for the private-sector volatility could be motivated by the work of Davis et al. (2006), who use the Longitudinal Business Database (LBD), containing annual observations on employment and payroll for all establishments and firms in the private sector, to estimate the volatility of employment growth rates. They find that, in 2001, the ratio of private to public volatility was in the range $1.43 - 1.75$. Given that the average annual standard deviation for public firms over $1990 - 1997$ was 0.11,[27] and that there is, at least in the context of the present model, a close relationship between volatility of profits and volatility of labor demand, the choice of $\sigma = 0.15$ could also be justified from this perspective.

---

[25] Attanasio et al. (2002) also provide evidence of considerable heterogeneity in the point estimates of the relative risk aversion coefficient, using data from the UK Family Expenditure Survey over 1978-1995. Estimates of relative risk aversion in the range of 10 have also been reported by Pálsson (1999), who uses Swedish cross-sectional data from tax returns in 1985.

[26] This choice gives a volatility of the return to the risky asset three times as large as the (exogenous) mean return to the risky asset.

[27] As reported in Campbell et al. (2001).

As additional support for the choice of $\sigma = 0.15$, it is noted that this magnitude generates an annual variance for steady-state consumption growth in the range indicated by the micro data, once consumer heterogeneity is taken into account.[28]
[29]

Parameters γ and σ are especially important for the calibrated model, for two reasons. First, they directly influence λ, the fraction of wealth held by entrepreneurs in the economy. And then, for example, as mentioned in section 3.2, when λ falls, i.e. when the entrepreneurs borrow more from the laborers, then, for given prices, K falls as well.[30] Given this importance of λ, the calibrated model's implications about λ are a good criterion of model performance. As will be shown in section 5.3, the choices $\gamma = 8$ and $\sigma = 0.15$, which seem empirically relevant given the discussion above, produce, without an attempt to match it, a value for λ that is reasonably close to the values documented in the data. Second, parameters γ and σ relate to the interpretation of the capital-income tax as providing insurance. For this reason, comparative statics will also be performed, in section 6, to show how the tax that maximizes the steady-state capital stock varies with risk aversion and the volatility of risk. The main result, that steady-state capital is inversely U-shaped with respect to the capital-income tax, is preserved qualitatively for $\sigma \in (0, 1)$ and for $\gamma \in (2, 20]$.

5.3 Implications for steady-state aggregates and distributions

This section undertakes the examination of the quantitative performance of the model in terms of aggregates and wealth distributions, for three reasons. First, to show how wealth inequality is influenced by the random-walk component introduced in wealth by the idiosyncratic investment risk. Second, to demonstrate how wealth inequality depends on the excess returns to entrepreneurship, which is an important question in its own right, but also in view of the impact of agent heterogeneity on capital accumulation. Third, to provide some additional confidence in the main quantitative results presented in the next section (section 6) about the effects of capital-income taxation on capital accumulation, by showing that the model performs well in matching aspects of the US aggregate and welfare data.

Table 2 presents the implications of the model for steady-state aggregates, and compares them to the data from the US economy. The model's capital-output ratio is 2.8. Investment is 17% of GDP. The safe rate is 2.5%. The steady-state fraction of entrepreneurs, χ^{ss}, is 12%, and it matches the data by choice of the transition probabilities, as explained in section 5.2. Entrepreneurs hold

---

[28]For example, Aït-Sahalia et al. (2001), and Malloy et al. (2006).

[29]It is to be noted though, that their analysis focuses on the differences in the *cross-sectional* volatility facing private entrepreneurs. But what really matters in the present model is the investment volatility an entrepreneur faces *over time*, since the time dimension is the one relevant for capital accumulation. For this time dimension, Panousi and Papanikolaou (2008) find that the mean annual idiosyncratic investment volatility is approximately 0.4 for publicly traded firms in the US.

[30]This result also carries over to the steady-state analysis. For example, when $\lambda \neq 1$, then, for $\theta = 1$, equation (25) takes the form $F_K(K) - \delta = R + \sqrt{\lambda(R)^{-1} \gamma \sigma^2 (\beta - (1 - \tau^K) R)}$, where $\lambda = (\beta - (1 - \tau^K) R + p_{LE})/(\beta - (1 - \tau^K) R + p_{LE} + p_{EL})$. From this it is clear that, for given R, an increase in λ leads to a fall in steady-state K.

30% of total wealth in the economy, where the equivalent of λ in the data is the ratio X^E/X[31]. The share of total wealth held by entrepreneurs in the data ranges between $35\% - 55\%$. The model-generated value for λ is an indication that the model performs reasonably well, especially given the low value of σ used in the calibration, and also since the rest of the aggregates could have been matched by a standard neoclassical growth model. The fraction of entrepreneurs in the top 10% of the population is 18% in the model, whereas in the data this number ranges between $32\% - 54\%$.[32]

	K/Y	I/Y	G/Y	R	χ^{ss}	X^E/X	χ^{ss} in top 10%
US Data	2.7	17%	20%	2%	$10-19\%$	$35-55\%$	$32-54\%$
Model	2.8	17%	20%	2.5%	12%	30%	18%

Table 2. Steady-State Aggregates.

Next, Table 3 examines the wealth distribution generated by the model. The first two rows present the percentiles for wealth computed by Quadrini (2000), using the PSID and SCF samples for 1994 and 1992, respectively. The last row is the conditional wealth distribution of the benchmark calibrated model.[33]

	Top Percentiles				
	30%	20%	10%	5%	1%
SCF	87.6	79.5	66.1	53.5	29.5
PSID	85.9	75.9	59.1	44.8	22.6
Model	76.25	63.68	44.44	29.80	10.53

Table 3. Distribution of Wealth in the US and in the Model.

Aiyagari's (1994) benchmark calibration predictions for the wealth holdings of the top 5% and the top 1% of the population are 13.1% and 3.2%, respectively. Hence, the present model demonstrates how the random-walk component introduced in wealth by entrepreneurial risk helps generate a fatter right tail in the wealth distribution.[34]

---

[31]This is because, in the data, wealth is defined as total net worth, i.e. it is financial wealth, X, as defined in the present model, plus housing.

[32]The data on entrepreneurs and wealth concentration is as reported in Cagetti and De Nardi (2006).

[33]Compared to the data, the model's unconditional wealth distribution has a larger fraction of agents at negative levels of wealth, most likely because of the absence of borrowing constraints.

[34]A tractable extension that could improve the model's prediction about wealth concentration at the top would be to introduce a third state, in which an agent gets to be an entrepreneur operating a very high return or very low risk production function. Then, the transition probabilities between the three states can be freely chosen to match desired moments of the wealth distribution. In particular, making the good entrepreneurial state the least persistent and the most likely to transition to the state of being a laborer would increase the precautionary saving, and therefore the wealth concentration, of the very rich agents.

Next, Figure 1 plots the Lorenz curves for the model's aggregate wealth and consumption distributions. The model produces results in the right direction, in that the distribution of wealth over the population is much more unequal than the distribution of consumption. The model's Gini coefficient for wealth, conditional on wealth being positive, is 0.62. The model's Gini coefficient for consumption is 0.15.[35] In the data, the Gini coefficient for total net worth is 0.8, and the Gini coefficient for consumption is 0.32.

(a) Wealth   (b) Consumption
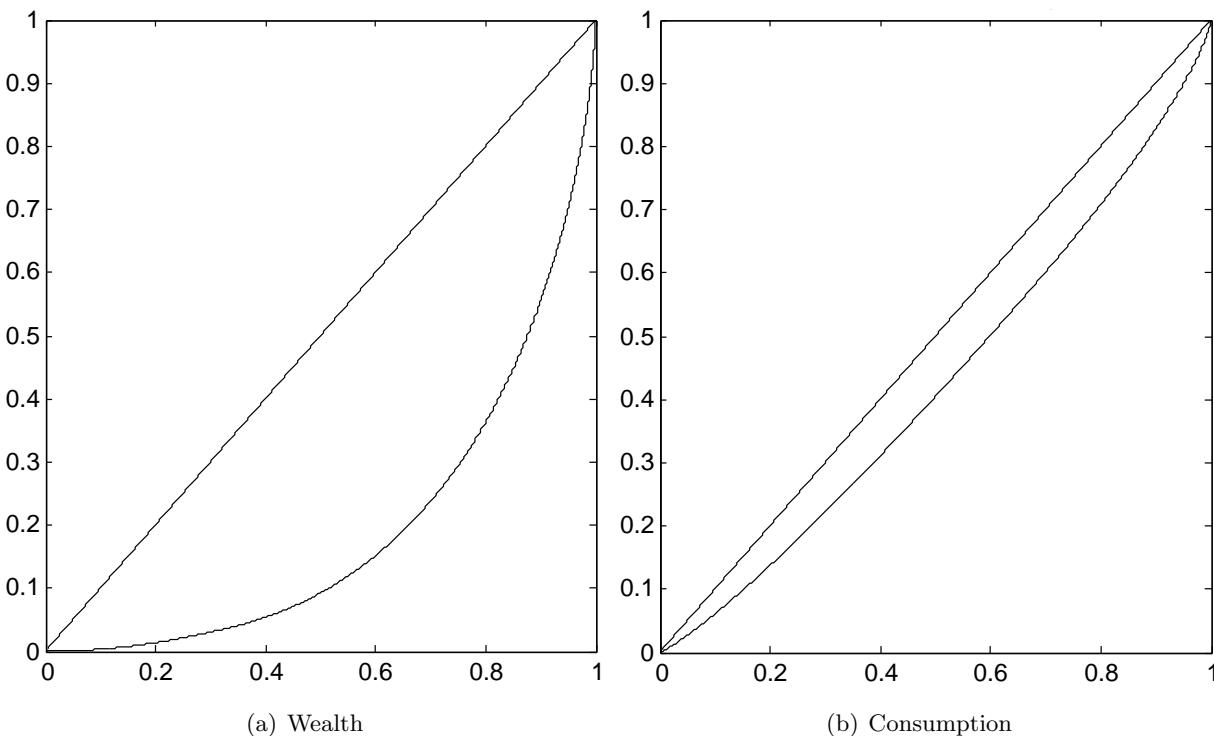
Figure 1: *Lorenz Curves for Wealth and Consumption*

Finally, Figure 2 presents the model's conditional wealth distributions over entrepreneurs and laborers. On the horizontal axis is wealth normalized by mean annual income in the economy. On the vertical axis are frequencies. The solid line represents entrepreneurs, and the dashed line laborers. Consistent with the data, the distribution of wealth for the population of entrepreneurs displays a fatter tail than the one for laborers. This is due to the random-walk component that the uninsurable investment risk introduces into entrepreneurial wealth. Furthermore, the entrepreneurial wealth distribution is shifted to the right, and it has lower frequencies at lower levels of wealth. This is due to the higher mean return of the total entrepreneurial portfolio. Finally, the distributions of wealth for both groups have significant mass of people with wealth higher than fifty times mean

---

[35]The differences in the Gini coefficients are due to the presence of human wealth: since poorer agents have higher human-to-financial-wealth ratios, they can sustain relatively high consumption. This would not be the case in the presence of borrowing constraints.

income. In the model, the laborers at the right tail of the wealth distribution are former successful entrepreneurs.
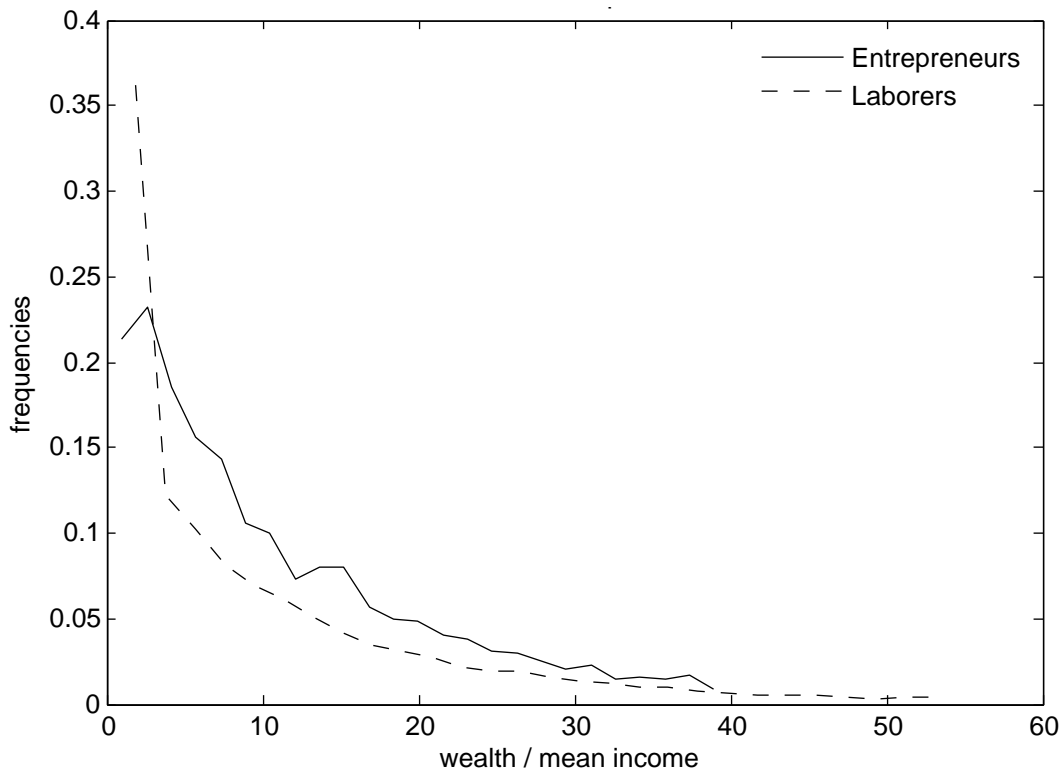

Figure 2: *Wealth Distribution for Entrepreneurs and Laborers*

6 Macroeconomic Effects of Capital-Income Taxation

This section now proceeds to study the macroeconomic effects of capital-income taxation under the benchmark calibration, where the relevant parameters are those of Table 1. Section 6.1 examines the steady-state effects of capital-income taxation. Section 6.2 examines the implications for macroeconomic aggregates of eliminating the capital-income tax.

6.1 Steady state

This section quantifies the main theoretical result of the paper, which is that an increase in the capital-income tax increases the steady-state capital stock, when the tax is low enough. As already explained, this result is due to the general-equilibrium effect of the insurance aspect of the capital-income tax, and it operates mainly through the endogenous adjustment of the interest rate.
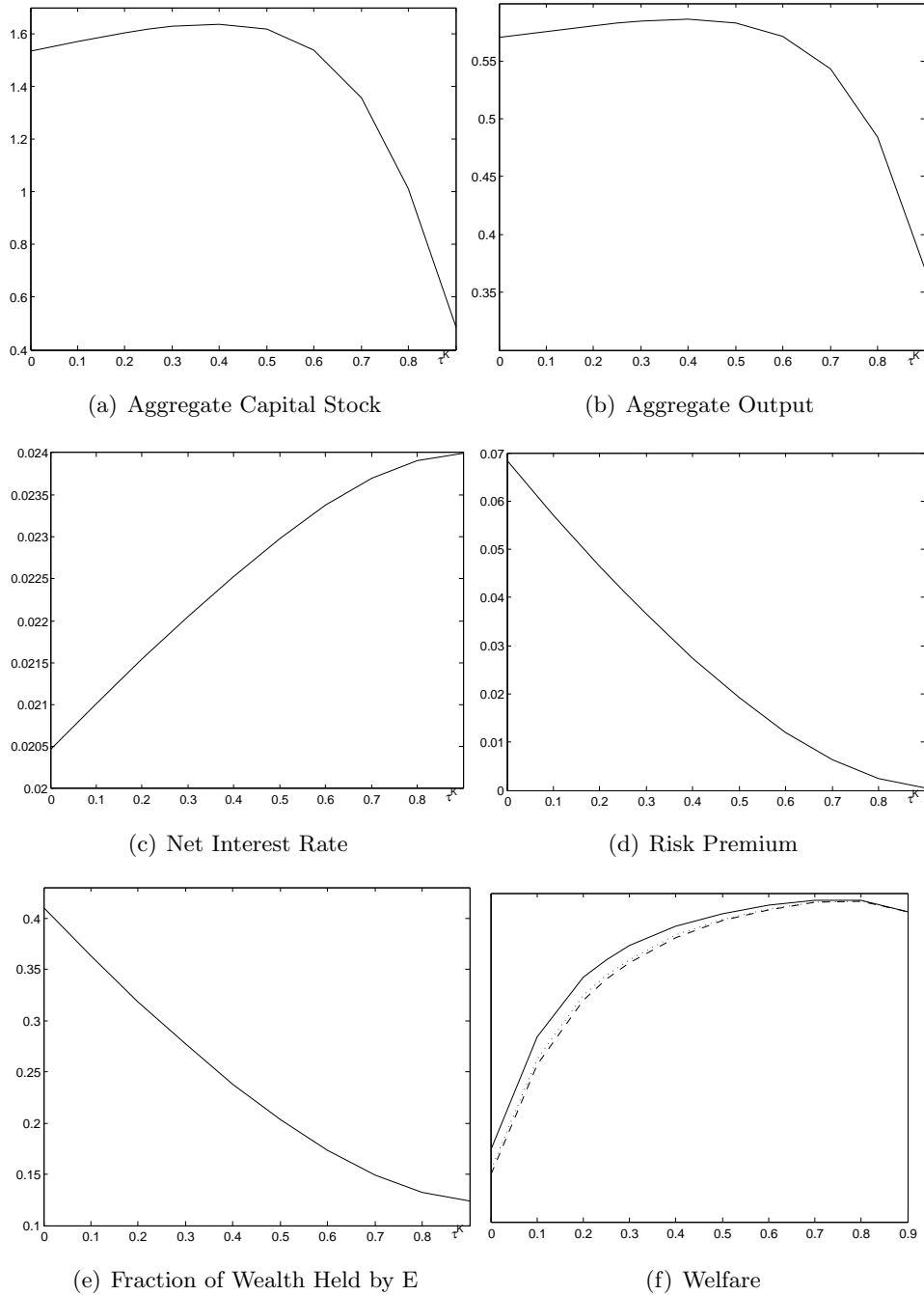
(a) Aggregate Capital Stock   (b) Aggregate Output

(c) Net Interest Rate   (d) Risk Premium

(e) Fraction of Wealth Held by E   (f) Welfare

Figure 3: *Steady State and Capital-Income Taxation*

Figure 3 shows the behavior of the steady-state aggregates with respect to the capital-income tax. Capital (panel (a)) and output (panel (b)) are inversely-U shaped with respect to the capital-income tax, and they reach a maximum when $\tau^K = 0.4$. The same is true for employment, the capital-labor (capital per work-hour) ratio, and output per work-hour. At $\tau^K = 0.4$, steady-state capital per work-hour is 6.75% higher than when $\tau_K = 0$, and output per work-hour is 2.65%

higher.

Figure 3(c) shows that the net interest rate increases with the tax, and that it tends to the discount rate, $\beta = 0.024$, as $\tau^K \rightarrow 1$. This is the demonstration of the precautionary saving motive mentioned in section 4: when the capital-income tax increases, the effective volatility of risk facing an entrepreneur decreases, which reduces the demand for precautionary saving, and therefore increases the interest rate. Figure 3(d) reinforces this interpretation of the capital-income tax as providing insurance: when the tax increases, the precautionary saving motive becomes weaker, and therefore entrepreneurs are satisfied with a lower risk premium. Figure 3(e) shows that the fraction of wealth held by entrepreneurs in the economy is decreasing in the capital-income tax. This results from the combination of the weaker precautionary saving motive, and the fall in the risk premium.

At this point, it is useful to compare the effects of capital-income taxation in the present model to those under complete markets, where there is no scope for insurance (assuming that agents are homogeneous), as well as to those in the open economy version of the model, where only the direct insurance aspect of the tax is present. As already mentioned, here, at $\tau^K = 0.4$, steady-state capital per work-hour and output per work-hour are 6.75% and 2.65%, respectively, higher than when $\tau_K = 0$. By contrast, under complete markets,[36] at $\tau^K = 0.4$, steady-state capital per work-hour and output per work-hour are 25% and 11% lower than when $\tau^K = 0$. And finally, in the open-economy version of the model the aggregates fall all the way with the tax, but less so than under complete markets. In particular, at $\tau^K = 0.4$, steady-state capital per work-hour and output per work-hour are 14% and 6% lower than when $\tau^K = 0$.

Finally, in order to reinforce the insurance interpretation of the tax system, Figure 4 presents robustness checks with respect to volatility, σ, and risk aversion, γ. On the vertical axis is the tax that maximizes the steady-state capital stock. When either the volatility of risk increases or risk aversion increases, the tax that maximizes the steady-state capital stock increases. These comparative statics also indicate that the main result of the paper is robust to the wide range of empirically plausible values of $\sigma \in (0, 1)$ and of $\gamma \in (2, 20]$. In particular, for the low value of $\sigma = 0.15$, the capital-income tax that maximizes the steady-state capital stock is positive for all $\gamma > 2$, and it is actually zero when $\gamma = 2$.

---

[36]The complete-markets calibration uses the relevant parameter values from the benchmark Table 1.

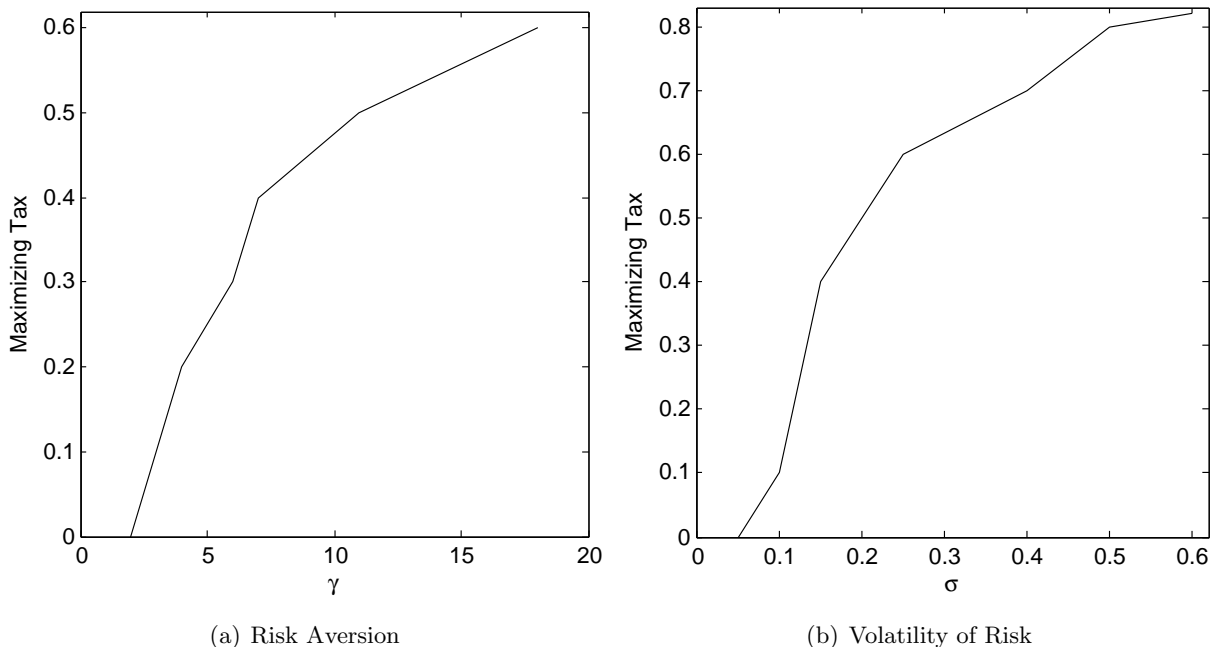(a) Risk Aversion   (b) Volatility of Risk

Figure 4: *Robustness Checks*

6.2   Aggregate dynamics of eliminating the capital-income tax

This section presents the immediate and long-run responses of the aggregate variables to the policy reform that eliminates the capital-income tax, taking into account the entire transitional path of the economy towards the new steady state with the zero tax. In the standard representative-agent complete-markets neoclassical model, the optimal capital-income tax is zero in the long run, as well as in most of the short run for an interesting class of preferences. These findings have initiated an extensive debate as to the possible benefits of eliminating the tax on capital income. By contrast, the main result of the present paper is that an increase in the capital-income tax may actually increase the steady-state capital stock. In light of this result, it is worthwhile to revisit the discussion on the implications of setting the capital-income tax to zero.

The present model can in fact examine the dynamic implications of policy reforms, because it is very tractable, compared to other incomplete-markets models, where the entire wealth distribution is a relevant state variable. Here, only the mean of the wealth distribution is relevant for aggregate dynamics, which constitutes a significant gain in tractability, and allows for the entire dynamic response of the economy, after a policy change, to be considered. This is important, since it has by now been recognized that the immediate effects of a policy change may well be very different from its long-run effects.

Here, the economy starts from the steady state described by the benchmark calibration parameters in Table 1, where the capital-income tax is $\tau^K = 0.25$. Subsequently, the tax is set to

zero, ceteris paribus. Table 4 shows the response of the aggregates on the impact of the policy reform (denoted by Current), as well as at the new steady state (denoted by Long Run), under both complete markets and the present model of incomplete markets. The effects on the interest rate, R, the risk premium, μ, and the investment-output ratio, I/Y, are in percentage units. The rest of the numbers denote percentage changes.

	Current		Long Run	
	Incomplete	Complete	Incomplete	Complete
L	-2.98	8.94	-0.0477	1.43
Y	-1.80	5.27	-2.12	7.77
C	2.48	-5.52	-2.06	5.87
Y/L	1.22	-3.37	-2.08	6.25
I/Y	-3.16	6.44	-0.52	2.48
R_{net}	-1.21	1.29	-0.13	0

Table 4. Dynamics of Eliminating the Capital-Income Tax.

Under complete markets, a permanent (unanticipated) tax cut leads to an immediate negative jump in consumption and an immediate positive jump in investment. Capital slowly increases and converges to a higher steady-state value, while consumption is initially lower and increases over time. In other words, the long-run increase in investment requires an initial period of lower consumption, which in turn allows for an immediate increase in investment as well. By contrast, under incomplete markets, the exact opposite is the case. In light of the main mechanism of the paper, investment decreases in the long run. This allows for an immediate increase in consumption, and therefore necessitates a fall in current investment. In particular, the investment-output ratio falls by more than 3 percentage units. These effects are also illustrated in Figure 5, which plots the impulse responses of the variables when the capital-income tax is eliminated.
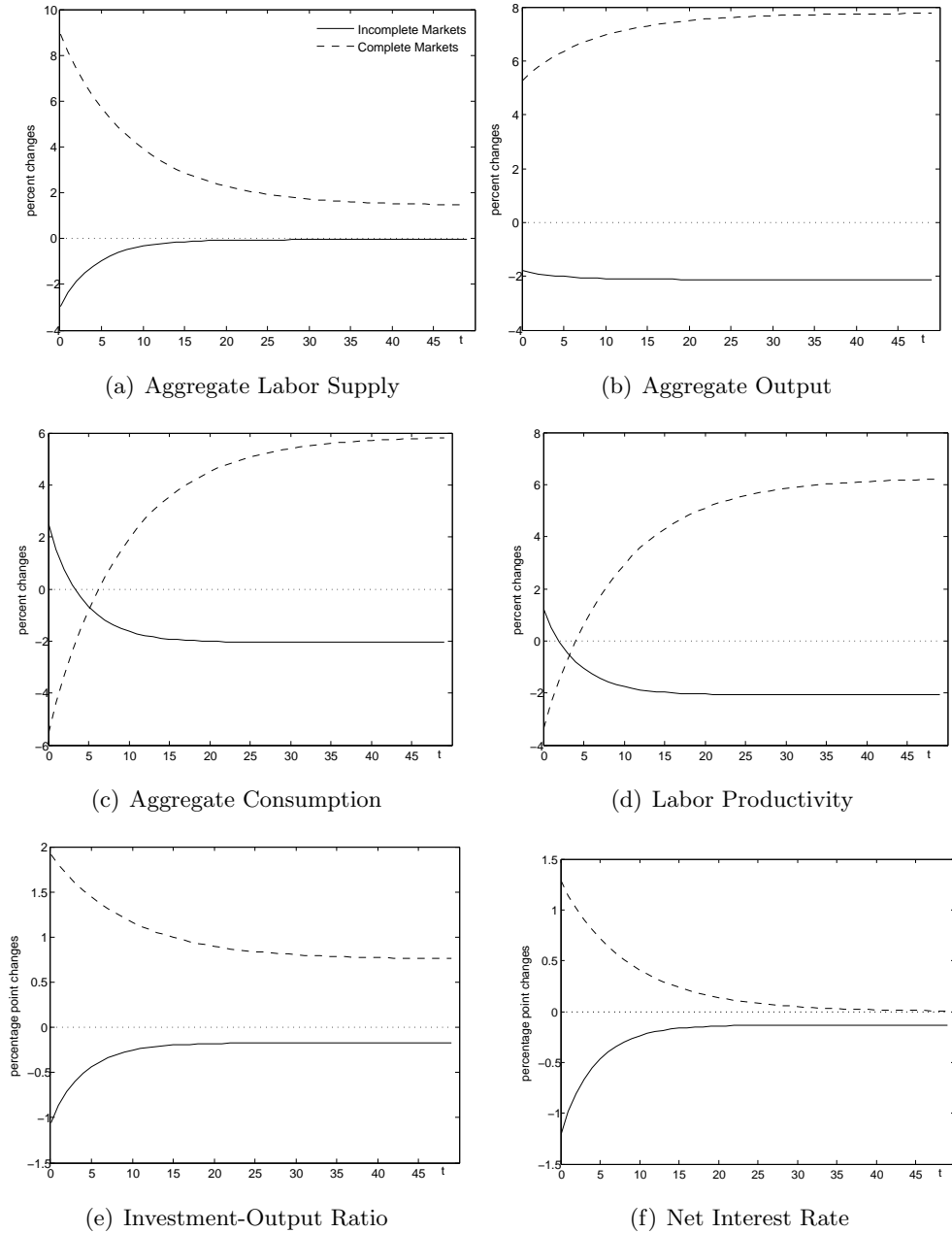
Figure 5: *Dynamics of Incomplete vs. Complete Markets: Eliminating the Capital-Income Tax*

7 Welfare Effects of Capital-Income Taxation

This section studies the welfare effects of capital-income taxation under the benchmark calibration, where again the relevant parameters are those of Table 1. Note that, under complete markets, steady-state welfare is decreasing in the level of the capital-income tax. However, this result will be upset here due the insurance aspect (both direct and indirect) of capital-income taxation. Section

7.1 examines the steady-state welfare effects of the capital-income tax. Section 7.2 examines the dynamic welfare effects of eliminating the capital-income tax. These welfare effects will be examined from two perspectives. On the one hand, one might be interested in examining the welfare of the current generation immediately after the policy reform, taking into account the entire transitional dynamics of the economy towards the new steady state with the zero tax. On the other hand, one might be interested in examining the welfare of the generations that will be alive in the distant future, i.e. at the new steady state in the long run. In the latter case, for example, an interpretation of aggregate welfare is that it is the ex ante utility of the agents who are going to be assigned to the stationary wealth distribution.[37]

7.1 Steady state

In steady state, as shown in Figure 3(f), aggregate welfare is maximized at $\tau^K = 0.7$, whether for entrepreneurs (solid line), laborers (dashed line), or the economy as a whole (dotted line).[38] This is because of the combined direct insurance effect of the tax, through the reduction in $\sigma(1 - \tau^K)$, and the effect of the tax on aggregates. It is worth noting that in the open-economy version of the model steady-state welfare is maximized when $\tau^K = 0.4$, which reflects solely the direct insurance aspect of the tax.

7.2 Welfare dynamics of eliminating the capital-income tax

The welfare implications of eliminating the capital-income tax are represented here in terms of a compensating differential for each level of wealth and each type of agent, whether entrepreneur or laborer. In particular, starting from the old regime with $\tau^K = 0.25$, the question is what fraction of his financial wealth would an agent be willing to give up in order to avoid the impact of the new regime initiated by the policy change, either immediately or in the long run.

Figure 6 presents the welfare implications of abolishing the capital-income tax for entrepreneurs (solid line) and laborers (dashed line). Panel (a) shows the welfare implications for the current generation, taking into account the entire transitional dynamics of the economy towards the new steady-state, and panel (b) shows the welfare implications for the generations alive at the new steady state. Financial wealth normalized by annual mean income is on the horizontal axis, and the compensating differentials are on the vertical axis. A negative number on the vertical axis indicates an agent who benefits from the reform: the agent would have to be paid to be indifferent

---

[37]The welfare effects are more likely to be sensitive to the assumption that the insurance possibilities are exogenously fixed. For example, in a richer model, there might be important welfare implications coming from the interaction between the insurance effects of capital taxation and the insurance endogenously provided by private markets. Still, the welfare implications of capital taxation are worth exploring here, especially since they will reveal how general equilibrium mechanisms in particular can affect different types of agents differently.

[38]Naturally, entrepreneur welfare is higher than laborer welfare for all tax levels. In addition, entrepreneur welfare is higher than laborer welfare for all levels of wealth, since entrepreneurs are unconstrained in their investment choices.

between the old regime and the regime initiated by the impact of the policy change, hence the agent prefers the new regime with the zero capital-income tax.
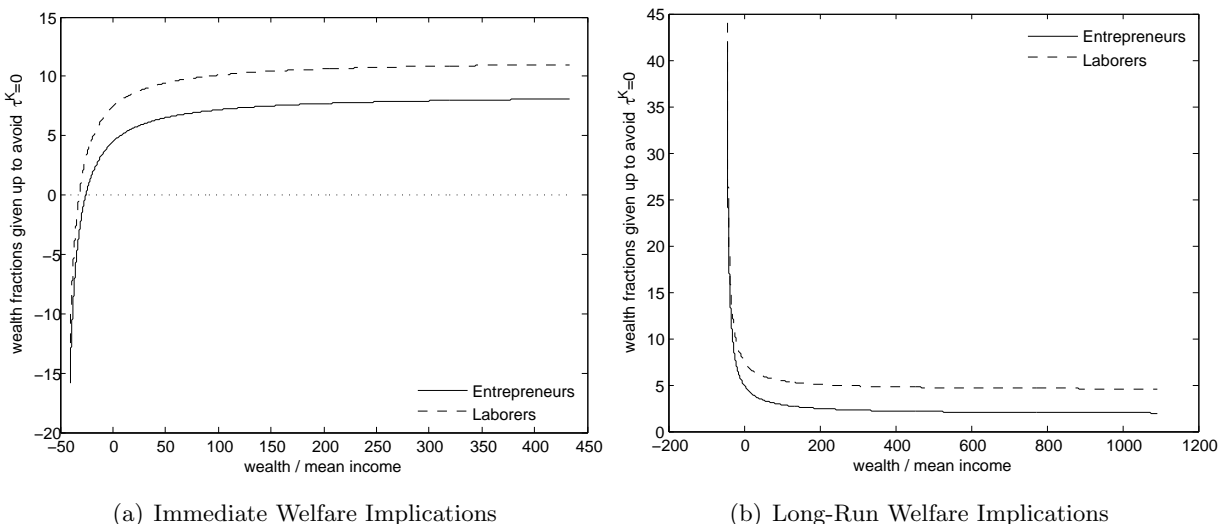

(a) Immediate Welfare Implications   (b) Long-Run Welfare Implications

Figure 6: *Welfare Implications of Eliminating the Capital-Income Tax*

Figure 6(a) shows that current-generation poor agents, whether entrepreneurs or laborers, prefer the zero capital-income tax regime. As wealth increases, both entrepreneurs and laborers prefer the positive capital-income tax regime. Finally, the mean cost of eliminating the tax is higher for the middle-class agents than for the very rich. Figure 6(b) shows that, in the long run, both types of agents and at all wealth levels prefer the steady state with the positive tax, the rich less so than the poor, and the entrepreneurs less so than the laborers. Table 5 presents these welfare benefits (if negative) and costs (if positive) of $\tau^K = 0$, where V_E^{SR} denotes the implications for entrepreneurs of the current generation, V_L^{LR} denotes the implications for laborers of the future generations, etc. Here b_1 and b_5 denote the bottom 1% and the bottom 5% of the distributions, respectively; t_{10}, t_5, and t_1 denote the top 10%, 5%, and 1%, respectively; and p_i, $i = 1, 2, 3, 4, 5$ denotes quintiles.[39]

	b_1	b_5	b_{10}	p_1	p_2	p_3	p_4	p_5	t_{10}	t_5	t_1
V_E^{SR}	−1.6	−2.1	0.7	9.4	21	22.6	23.3	23.7	11.9	5.9	1.2
V_L^{SR}	−0.8	0.1	3.6	12.7	20.7	21.8	22.3	22.5	12.3	5.6	1.1
V_E^{LR}	0.8	3.8	7.6	15.2	15.6	16.2	17.6	35.4	24.9	17.9	7.9
V_L^{LR}	0.9	4.4	8.8	17.6	17.8	18.1	18.8	27.6	17.3	11.4	4.4

Table 5. Welfare Implications of Eliminating the Capital-Income Tax.

---

[39]Under complete markets, and starting from the steady state with $\tau_K = 0.25$, the average long-run welfare gain (in terms of consumption equivalent) of eliminating the capital-income tax is 1.7%, whereas the average short-run welfare gain is 0.6%.

These cross-sectional differences can be explained by referring to Figure 7, which plots, in the top row for the current generation, and in the bottom row for the long-run generation, the response of human wealth and of the (risk-adjusted) returns to saving for laborers and entrepreneurs, against the tax rate of the policy reform. Let's first turn to the immediate implications. The decrease in the capital-income tax from $\tau^K = 0.25$ to $\tau^K = 0$ increases the demand for precautionary saving, and therefore leads to a fall in the interest rate.[40] Roughly speaking, since the capital stock is historically given and cannot change, the fall in the interest rate increases human wealth. For poor agents, whether entrepreneurs or laborers, human wealth constitutes a significant part of total wealth, and hence they benefit from the elimination of the tax. Furthermore, poor agents do not benefit much from insurance directly, since they invest little or nothing in the risky asset. Therefore, in the short run, poor agents prefer the zero capital-income tax regime, mainly because the elimination of the tax increases their safe income, and safe income is a big part of their total wealth. Turning to the long run, the elimination of the capital-income tax increases the demand for precautionary saving, and it therefore leads to a fall in the interest rate. But now, the general-equilibrium implications of the interest rate adjustment for capital accumulation become relevant. In particular, the fall in the interest rate reduces steady-state wealth and capital accumulation. It turns out that the fall in the steady-state capital stock dominates the fall in the interest rate, so that in the end steady-state human wealth falls. This adversely affects poor agents of all types, since human wealth represents a big part of their total wealth. Because the risk-adjusted return for entrepreneurs, $\hat{\rho}$, increases when the capital-income tax is eliminated, the cost of the policy change is not as high for an entrepreneur as it is for a laborer at any given level of wealth.

---

[40]Immediately after a reform that reduces the capital tax from a very high level, the net interest rate may actually increase. This possibility, which does not emerge in the long run, is due to the usual distortionary effect of big tax increases on investment.
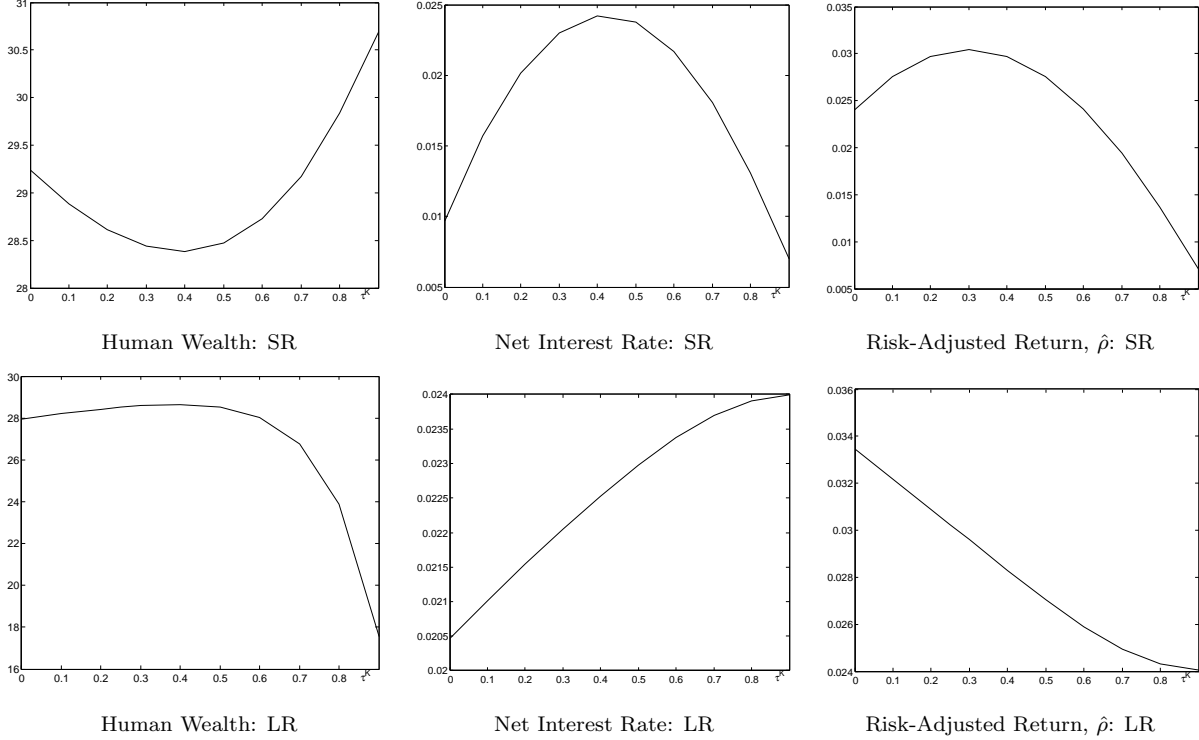
Figure 7. *Immediate (SR) vs Long Run (LR): Human Wealth and Saving Returns*

In conclusion, the elimination of the capital-income tax has welfare implications that differ across time and in the cross-section of the population. These differences are due to the general-equilibrium effects of the interest rate adjustment on capital accumulation. In particular, they operate mainly through the different response of human wealth: immediately after the elimination of the tax, when the capital stock cannot adjust, human wealth increases, whereas in the long run, when capital accumulation changes endogenously, human wealth falls. Therefore, current-generation poor agents prefer a zero capital-income tax in the short run, whereas future-generation poor agents prefer a positive capital-income tax. Rich agents always prefer a positive tax, but less forcefully in the long-run, because in the long run the elimination of the tax increases the mean entrepreneurial portfolio return.

8   Conclusions

This paper studies the aggregate and welfare effects of capital-income taxation in an environment where agents face uninsurable idiosyncratic entrepreneurial risk. The counter-intuitive result emerging is that an increase in the capital-income tax may actually, due to its general-equilibrium insurance aspect, stimulate capital accumulation. This result stands in stark contrast to the effects of capital-income taxation in either complete-markets models, or in Bewley-type incomplete-markets

models, since in those models capital-income taxation necessarily discourages capital accumulation. Furthermore, the result is quantitatively significant: for the preferred calibration of the model, the steady-state levels of the capital stock, output, and employment are all maximized for a positive value of the capital-income tax, at which point output per work-hour is 2.65% higher than it would have been had the tax rate been zero.[41]

Although the present paper provides some useful guidance about the direction of optimal policy, it does not solve for the fully optimal policy. An interesting direction for future research is the formal study of optimal policy, either in the Ramsey tradition (though allowing for lump-sum taxes, as in the present model), or in the Mirrlees tradition of endogenizing the source of market incompleteness and having no ad hoc restrictions placed on the set of available instruments.

This paper focuses on the effects of uninsurable entrepreneurial risk, and abstracts from labor-income risk, borrowing constraints, and decreasing returns to scale at the individual level. Extending the model to include these relevant aspects of the data and revisiting the effects of capital taxation in this richer setting is important, not only to get a better quantitative evaluation of the implications of capital taxation, but also to examine whether the general-equilibrium effects identified here might interact with other sources of market incompleteness in an interesting way. For example, after an increase in the capital-income tax, the increase in steady-state wealth documented here could make borrowing constraints less binding. At the same time, the increase in the steady-state interest rate could also increase the cost of borrowing. Further investigating these rich general-equilibrium interactions will greatly facilitate a better theoretical and quantitative assessment of the implications of fiscal policy in dynamic heterogeneous-agent environments.

---

[41]Under complete markets, and starting from a steady state with $\tau^K = 0.25$, the capital-income tax would have to be reduced to approximately $\tau^K = 0.15$, in order for output per work-hour to increase by 2.65%.

9 Appendix: Proofs

Lemma 1. *Let preferences be described by:*

$$J_t = \{(1 - e^{-\beta \Delta t})(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta} + e^{-\beta \Delta t}(E_t[J_{t+\Delta t}^{1-\gamma}])^{\frac{1-1/\theta}{1-\gamma}}\}^{\frac{1}{1-1/\theta}},$$

where c is consumption and n is leisure. Then, given the processes for c and n, the utility process is defined as the solution to the following integral equation:

$$U_t = E_t \int_t^{\infty} z(c_s, U_s)\, ds, \tag{31}$$

where

$$z(c, U) \equiv \frac{\beta}{1 - 1/\theta} \left[\frac{(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta}}{((1-\gamma)U)^{\frac{-1/\theta+\gamma}{1-\gamma}}} - (1-\gamma)U \right]. \tag{32}$$

Proof of Lemma 1. Define the functions:

$$g(x) = \frac{((1-\gamma)x)^{\frac{1-1/\theta}{1-\gamma}}}{1 - 1/\theta},$$

$$U_t = \frac{J_t^{1-\gamma}}{1 - \gamma}.$$

Then:

$$g(U_t) = \frac{J_t^{1-1/\theta}}{1 - 1/\theta} = (1 - e^{-\beta \Delta t})\frac{(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta}}{1 - 1/\theta} + e^{-\beta \Delta t}g(E_t[U_{t+\Delta t}]).$$

Take a first order Taylor expansion in Δt:

$$g(U_t) = g(U_t) + \beta\frac{(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta}}{1 - 1/\theta}\Delta t - \beta g(U_t)\Delta t + g'(U_t)E_t[\Delta U_t].$$

Then:

$$E_t[\Delta U_t] = -\frac{\beta\frac{(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta}}{1-1/\theta} - \beta g(U_t)}{g'(U_t)}\Delta t,$$

where:

$$\frac{g(U_t)}{g'(U_t)} = \frac{(1-\gamma)U_t}{(1-1/\theta)}.$$

Hence:

$$E_t[\Delta U_t] = -z(c_t, n_t, U_t)\Delta t,$$

where:

$$z(c_t, n_t, U_t) \equiv \frac{\beta}{1 - 1/\theta}\left[\frac{(c_t^{1-\psi} n_t^{\psi})^{1-1/\theta}}{((1-\gamma)U_t)^{\frac{-1/\theta+\gamma}{1-\gamma}}} - (1-\gamma)U_t\right].$$

36

For a more general proof of the above and for a proof of existence and uniqueness of the solution to the integral equation (31) see Duffie and Epstein (1992).

Proof of Proposition 1. Because of the CRRA/CEIS specification of preferences, guess that the value function for an entrepreneur is:

$$J(w^E, t) = B_t^E \frac{w^{E\,1-\gamma}}{1-\gamma},$$

where the term B_t^E captures the time dimension. The Bellman equation for an entrepreneur is:

$$0 = \max_{c^E, n^E, \phi} \; z(c^E, n^E, J^E(w^E, t)) + \frac{\partial J^E}{\partial w^E}(w^E, t)[(\phi(1-\tau_t^K)r_t + (1-\phi)(1-\tau_t^K)R_t)w^E - c^E - (1-\tau_t^L)\omega_t n^E]$$

$$+ \frac{\partial J^E}{\partial t}(w^E, t) \; + \frac{1}{2}\frac{\partial^2 J^E}{\partial w^{E\,2}}(w^E, t)\sigma^2(1-\tau_t^K)^2\phi^2 w^2 \; + \; p_{EL}[J(w^L, t) - J(w^E, t)],$$

where the function z is given by (32), and where the last term shows that the entrepreneur might switch into being a worker with probability p_{EL}. Because of the homogeneity of J^E in w^E, the marginal propensity to consume and the portfolio choice will be the same for all entrepreneurs. The first order condition for the optimal portfolio allocation gives the condition for ϕ_t in (16). Combining the first order conditions for consumption and leisure we get the optimal leisure choice:

$$n_t^i = \frac{\psi}{1-\psi}\frac{1}{(1-\tau_t^L)\omega_t}c_t^i. \tag{33}$$

From the envelope condition we get:

$$m^E \equiv B^{E\frac{1-\theta}{1-\gamma}}\left(\frac{\psi}{1-\psi}\frac{1}{(1-\tau^L)\omega}\right)^{-\psi(1-\theta)}(1-\psi)^\theta\beta^\theta.$$

Similarly, guess that the value function for a laborer is:

$$J(w^L, t) = B_t^L \frac{w^{L\,1-\gamma}}{1-\gamma},$$

The Bellman equation for a laborer is:

$$0 = \max_{c^L, n^L} \; z(c^L, n^L, J^L(w^L, t)) \; + \frac{\partial J^L}{\partial w^L}(w^L, t)[R_t w^L - c^L - (1-\tau_t^L)\omega_t n^L]$$

$$+ \frac{\partial J^L}{\partial t}(w^L, t) \; + \; p_{LE}[J(w^E, t) - J(w^L, t)].$$

37

Following similar steps, we get from the envelope condition that:

$$m^L \equiv B^{L\frac{1-\theta}{1-\gamma}} (\frac{\psi}{1-\psi} \frac{1}{(1-\tau^L)\omega})^{-\psi(1-\theta)} (1-\psi)^\theta \beta^\theta .$$

It follows that:

$$\frac{B^E}{B^L} = (\frac{m^E}{m^L})^{\frac{1-\gamma}{1-\theta}} .$$

Using this, the first order conditions, the envelope conditions, and plugging back into the Bellman equation we get (18) and (19).

Proof of Proposition 2. Let $\tilde{R}_t$ be the effective risk-free rate. The human wealth for each individual $i = E, L$ in the economy is $h_t^i = \int_t^\infty e^{-\int_t^s \tilde{R}_j dj} ((1-\tau_s^L)\omega_s + T_s) ds$. The human wealth of the measure-χ_t group of entrepreneurs is $H_t^E = \chi_t \int_t^\infty e^{-\int_t^s \tilde{R}_j dj} ((1-\tau_s^L)\omega_s + T_s) ds$, and the human wealth of the measure-$(1-\chi_t)$ group of laborers is $H_t^L = (1-\chi_t) \int_t^\infty e^{-\int_t^s \tilde{R}_j dj} ((1-\tau_s^L)\omega_s + T_s) ds$. Hence total human wealth is $H_t = H_t^E + H_t^L = \int_t^\infty e^{-\int_t^s \tilde{R}_j dj} ((1-\tau_s^L)\omega_s + T_s) ds = h_t^i$. Using the Leibniz rule, and substituting in from the government budget constraint (7), we get that the evolution of total human wealth is described by (23). Since only entrepreneurs invest in capital, the aggregate capital stock in the economy is given by $K_t = \phi_t W_t^E$. For an agent in the E and L group respectively, $b_t^E + h_t^E = (1-\phi_t)w_t^E$ and $b_t^L + h_t^L = w_t^L$. Aggregating over each group, we get $B_t^E + \chi_t H_t = (1-\phi_t)W_t^E$ and $B_t^L + (1-\chi_t)H_t = W^L$. Adding up and using the fact that $B_t^E + B_t^L = 0$, we get $H_t = (1-\phi_t)W_t^E + W_t^L$. Now using $W_t = W_t^E + W_t^L$ and $K_t = \phi_t W_t^E$, we get $W_t = K_t + H_t$. Combining $H_t = (1-\phi_t)W_t^E + W_t^L$, $K_t = \phi_t W_t^E$, and $\lambda_t = W_t^E/W_t$, we get (24). Aggregating across leisure choices we get $(\psi/(1-\psi))(1/((1-\tau_t^L)\omega_t))C_t + L_t = 1$, where $C_t = m_t^E W_t^E + m_t^L W_t^L$, $W_t^L = W_t - W_t^E$, and $W_t = K_t + H_t$. Aggregating across (12) and (13), and adding up, using $B_t^E + B_t^L = 0$, $H_t = H_t^E + H_t^L$, and labor market clearing, we get:

$$W_t = [(1-\tau_t^K)r_t K_t + (1-\tau_t^K)R_t H_t - \frac{1}{1-\psi}C_t]dt .$$

Using $H_t = (1-\phi_t)W_t^E + W_t^L$, $K_t = \phi_t W_t^E$, $\mu_t = (1-\tau_t^K)r_t - (1-\tau_t^K)R_t$, and $C_t = m_t^E W_t^E + m_t^L W_t^L$, and dividing through with W_t we get:

$$\frac{\dot{W}_t}{W_t} = (1-\tau_t^K)r_t \phi_t \mu_t + (1-\tau_t^K)R_t - \frac{1}{1-\psi}(\lambda_t m_t^E + (1-\lambda_t)m_t^L) ,$$

which gives (21) when we use $\rho_t = \phi_t \mu_t + (1-\tau_t^K)R_t$. Aggregating across (12), and subtracting from (21), we get (22).

Proof of Proposition 3. Consider first the case with $\lambda = 1$ and $v = 0$ (and labor is exogenous, so $\psi = 0$). Combining (18) or (19) in steady state, with (21) in steady state, and using the definitions of ρ and $\hat{\rho}$, we get equation (25). Combining (24) with (23) in steady state , we get equation (26).

Now, let $\mu(R)$ and $\phi(R)$ denote, respectively, the risk premium and the fraction of effective wealth held in capital, when K is given by (25):

$$\mu(R) \equiv \sqrt{\frac{2\theta\gamma\sigma^2}{1+\theta}(\beta - (1 - \tau^K)R)} \quad \text{and} \quad \phi(R) \equiv \sqrt{\frac{2\theta}{\gamma\sigma^2(1+\theta)}(\beta - (1 - \tau^K)R)} \quad . \tag{34}$$

Note that $\mu'(R) < 0$ and $\phi'(R) < 0$. Next, let $K(R)$ denote the solution to (25), or equivalently:

$$K(R) = \left[\frac{\mu(R) + \delta + R}{\alpha}\right]^{\frac{1}{\alpha - 1}} . \tag{35}$$

Finally, for $\tau^K \simeq 0$, $\tau^L \simeq 0$, $\delta \simeq 0$, $G = gY$, and $Y = f(K) = K^\alpha$, we can write equation (26) as:

$$D(R; g) \equiv (1 - \alpha - g)\frac{K(R)^{\alpha - 1}}{R} - \frac{1 - \phi(R)}{\phi(R)} , \tag{36}$$

where $\alpha + g < 1$. To establish existence and uniqueness of the steady state , it suffices to show that there exists a unique R that solves $D(R; g) = 0$. For a given g, consider the limits of D as $R \to 0^+$ and $R \to \beta^-$. Note that $\mu(0) = (\frac{2\theta\gamma\sigma^2}{1+\theta}\beta)^{1/2}$ is finite and hence both $\phi(0)$ and $K(0)$ are finite. It follows that:

$$\lim_{R \to 0^+} D(R; g) = (1 - \alpha - g)K(0)^{\alpha - 1} \lim_{R \to 0^+} \frac{1}{R} - \frac{1}{\phi(0)} + 1 = +\infty .$$

Furthermore, $\mu(\beta) = 0$, implying $\phi(\beta) = 0$ and $K(\beta) = K_{compl} \equiv (f')^{-1}(\beta)$ is finite. Hence:

$$\lim_{R \to \beta^-} D(R; g) = (1 - \alpha - g)K(\beta)^{\alpha - 1}\frac{1}{\beta} - \lim_{R \to \beta^-} \frac{1}{\phi(R)} + 1 = -\infty .$$

These properties, together with the continuity of $D(R)$ in R, ensure the existence of an $R \in (0, \beta)$ such that $D(R) = 0$. If $D(R; g)$ is strictly decreasing in R, then we also have uniqueness. To show this, note that, from (36):

$$\frac{\partial D}{\partial R} = (1 - \alpha - g)\frac{K(R)^{\alpha - 1}}{R^2}\left[(\alpha - 1)R\frac{K'(R)}{K(R)} - 1\right] + \frac{\phi'(R)}{\phi(R)^2} . \tag{37}$$

In addition:

$$K^{\alpha - 1} = \frac{f'(K)}{\alpha}, \quad \frac{K'}{K} = \frac{1}{\alpha - 1}\frac{\mu' + 1}{f'(K)}, \quad \text{and} \quad \frac{\phi'}{\phi^2} = \frac{\gamma\sigma^2\mu'}{\mu^2} ,$$

39

where the dependence of K, μ, and ϕ on R has been dropped for notational simplicity. Hence:

$$\begin{aligned}\frac{\partial D}{\partial R} &= \frac{1-\alpha-g}{\alpha}\frac{f'(K)}{R^2}\left[R\frac{\mu'+1}{f'(K)}-1\right]+\frac{\gamma\sigma^2\mu'}{\mu^2} = \\ &= \frac{1-\alpha-g}{\alpha}\frac{R\mu'+R-f'(K)}{R^2}+\frac{\gamma\sigma^2\mu'}{\mu^2}\,.\end{aligned}$$

Since $\mu'(R)<0$ and $R<f'(K(R))$ for all $R\in(0,\beta)$, it follows that $\partial D/\partial R<0$ for all $R\in(0,\beta)$, which completes the argument. When $v>0$, an extension of the proof above shows that there is a unique R solving $D(R)=0$, where $R\in(-v,\beta)$, and where:

$$D(R)\equiv\frac{(1-g\tau^L)(1-\alpha)K(R)^{\alpha-1}+(1-g)\tau^K f_K(R)}{((1-\tau^K)R+v)}-\frac{1}{\phi(R)\lambda(R)}+1\,.$$

However, in that case, uniqueness has not been proved, although simulations suggest that the steady state is always unique.

In order to see how the existence of two types of agents modifies the characterization of the steady state, consider next the case where $\lambda\neq1$, and take $\theta=1$ for simplicity. Then, the marginal propensity to consume is always constant and equal to β, for both types of agents. Equation (21) in steady state yields $\lambda=(\beta-(1-\tau^K)R)/(\phi\mu)$. Combining this with (22) in steady state gives:

$$\lambda=\frac{\beta-(1-\tau^K)R+p_{LE}}{\beta-(1-\tau^K)R+p_{LE}+p_{EL}}\,,\tag{38}$$

which verifies that $\lambda<1$. Plugging this back into (21) in steady state, we get:

$$\phi\mu=\frac{(\beta-(1-\tau^K)R)(\beta-(1-\tau^K)R+p_{LE}+p_{EL})}{(\beta-(1-\tau^K)R+p_{LE})}\,,$$

from which, if we use the definition of μ, we get:

$$F_K(K)-\delta=R+\sqrt{\frac{1}{\lambda(R)}\gamma\sigma^2\left(\beta-(1-\tau^K)R\right)}\,,\tag{39}$$

as the relevant version of (25). Finally, combining (23) in steady state with (24), we get:

$$K=\frac{\phi(K,R)\,\lambda(R)}{1-\phi(K,R)\,\lambda(R)}\frac{(1-\tau^L)\,\omega(K)+(1-g)\left(\tau^L\,\omega(K)+\tau^K\left(F_K(K)-\delta\right)K\right)}{(1-\tau^K)\,R+v}\,,\tag{40}$$

which is the relevant version of (26). Hence, the steady state is characterized by equations (38), (39), and (40). So, (38) expresses λ as a function of R, (39) expresses K as a function of R, and (40) solves for the equilibrium R, using $K(R)$ and $\lambda(R)$.

When labor is endogenous and $\theta=1$, then $m^E=m^L=(1-\psi)\beta$, and the proofs above carry through the same way, only now with $f_K(K/L)$ and $\omega(K/L)$. So for characterization of the steady

40

state we need to add the labor market clearing condition, and the steady state system will be in K, L, R. In particular, labor market clearing, combined with $C = (1 - \psi)\beta W$, $\lambda = W^E/W$, and $W^E = K/\phi$ gives:

$$L = (\frac{\psi\beta}{(1 - \tau^L)\omega(K, L)} \frac{1}{\lambda(R)} \frac{K/L}{\phi(K, L, R)} + 1)^{-1}.$$

Finally, when labor is endogenous and $\theta \neq 1$ then:

$$\lambda = \frac{\frac{1}{1-\psi}m^L - (1 - \tau^K)R + p_{LE}}{\frac{1}{1-\psi}m^L - (1 - \tau^K)R + p_{LE} + p_{EL}},$$

and

$$\mu = \sqrt{\frac{\gamma\sigma^2(1 - \tau)^2}{\lambda}(\frac{1}{1 - \psi}(m^E\lambda + m^L(1 - \lambda)) - (1 - \tau^K)R)} \quad .$$

Here we need to add two more equations to characterize the steady state, namely the Euler conditions for the marginal propensities to consume. This will be a system of two equations in two unknowns to be solved as a function of steady state prices.[42]

Lemma 2. *When the interest rate is exogenous, $W'(R) > 0 \Leftrightarrow R > \underline{R}$.*

Proof of Lemma 2. Let $v = 0$, $\lambda = 1$, $\tau^K \simeq 0$, $\tau^L \simeq 0$, $G = gY$, and $Y = K^\alpha$. Then, from (23) in steady state, we have that $H(R) = (1 - \alpha - g)K(R)^\alpha/R$, while K as a function of R is given by (35). Hence, we can write $W(R) = K(R) + H(R)$. Differentiating with respect to R, we get that:

$$W'(R) > 0 \iff \mu'(R) < (\alpha - 1) \frac{\alpha \, K^{2(\alpha-1)}}{R^2} \frac{\phi}{\phi + \alpha(1 - \phi)} - 1,$$

which means that the interest rate has to be higher than a given threshold, i.e. $R > \underline{R}$, since from (34) it is easy to show that $\mu''(R) < 0$.

Proof of Proposition 4. Let the newborn household receive a weighted average $aW_t + (1-a)w_t^i$, where $0 < a < 1$, upon birth.[43] Let d_t be the indicator function, where $d_t = 1$ for entrepreneurs and $d_t = 0$ for laborers. The dynamic system for the state vector (ξ_t^i, d_t), where , $\xi_t^i \equiv w_t^i/W_t$ as in the text, is:

$$\dot{\xi}_t^i = \mu(\xi_t^i, d_t) + \sigma(\xi_t^i, d_t) \, dz_t^i - (\xi_t^i - 1)dN_t^1$$
$$\dot{d}_t = s(d_t) \, dN_t^2,$$

---

[42]The conditions needed for establishing that $\lambda > 0$ are satisfied in simulations.

[43]One could rationalize this through the existence of an estate tax on the agent: if the agent dies, the government takes away $a(w_t^i - W_t)$ from his descendants. The idea is to keep the aggregate wealth unaffected. Here, expected wealth for the agent at any point in time is $v \cdot (aW_t + (1 - a)w_t^i) + (1 - v) \cdot w_t^i$, and aggregating across agents yields the desired result. The special case $a = 1$ implies that each newborn agent enters the economy endowed with the sum of the mean economy wealth plus his human wealth.

where dN_t^1 is the Poisson process denoting death with arrival rate vdt, and where dN_t^2 is the Poisson switching process with arrival intensity $p(I)\,dt$:

$$
\begin{aligned}
p(d) &= p_{LE} \quad \text{if } d = 0 \\
p(d) &= p_{EL} \quad \text{if } d = 1\,,
\end{aligned}
$$

where:

$$
\begin{aligned}
s(d) &= 1 \quad \text{if } d = 0 \\
s(d) &= -1 \quad \text{if } d = 1\,,
\end{aligned}
$$

and:

$$
\begin{aligned}
\mu(\xi_t, 1) &= [\frac{1-\lambda}{1-\psi}(m_t^L - m_t^E) + \phi_t(1-\lambda_t)(1-\tau_t^K)(r_t - R_t)]\xi_t \\
\mu(\xi_t, 0) &= [\frac{\lambda}{1-\psi}(m_t^E - m_t^L) - \phi_t\lambda_t(1-\tau_t^K)(r_t - R_t)]\xi_t \\
\sigma(\xi_t, 1) &= \phi_t\sigma_t(1-\tau_t^K)\xi_t \\
\sigma(\xi_t, 0) &= 0\,.
\end{aligned}
$$

Let $\Phi_E \equiv \Phi(\xi, 1)$ and $\Phi_L \equiv \Phi(\xi, 0)$ be the conditional distributions for entrepreneurs and laborers respectively. In steady state the conditional distribution Φ_L satisfies the forward Kolmogorov equation:

$$
0 = -\frac{\partial(\mu(\xi, 0)\,\Phi_L)}{\partial\xi} - p(0)\,\Phi_L + (p\Phi_L)(\xi, 0 - \eta(0)) - v\Phi_L + \frac{v}{1-a}\Phi_L(\frac{\xi - a}{1-a})\,,
$$

and the conditional distribution Φ_E satisfies the forward Kolmogorov equation:

$$
0 = \frac{1}{2}\frac{\partial^2(\sigma(\xi, d)^2\Phi_E)}{\partial\xi^2} - \frac{\partial(\mu(\xi, 1)\,\Phi_E)}{\partial\xi} - p(1)\,\Phi_E + (p\Phi_E)(\xi, 1 - \eta(1)) - v\Phi_E + \frac{v}{1-a}\Phi_E(\frac{\xi - a}{1-a})\,.
$$

In the two equations above we need to calculate:

$$
(p\Phi)(\xi, d - \eta(d)) = p(d - \eta(d))\Phi(\xi, d - \eta(d))\,.
$$

To that end, let the old state be d, and the new state be d'. They are related through $d' = d + s(d)$, and we need to compute $\eta(d') = s(d)$. For $d = 0$, we have $d' = 0 + s(0) = 0 + 1 = 1$, and $\eta(d') = \eta(1) = s(0) = 1$, hence $\eta(1) = 1$. For $d = 1$, we have $d' = 1 + s(1) = 1 - 1 = 0$, and $\eta(d') = \eta(0) = s(1) = -1$, hence $\eta(0) = -1$. Therefore:

$$
p(0 - \eta(0))\Phi(\xi, 0 - \eta(0)) = p(1)\Phi(\xi, 1) = p_{EL}\Phi_E\,,
$$

and:
$$p(1 - \eta(1))\Phi(\xi, 1 - \eta(1)) = p(0)\Phi(\xi, 0) = p_{LE}\Phi_L.$$

Substituting for $\mu(\xi_t, 0)$, $\mu(\xi_t, 1)$, $\sigma(\xi_t, 0)$, $\sigma(\xi_t, 1)$ and using the above, we can write the system of the two Kolmogorov equations as:

$$0 = c_1\xi^2\frac{\partial^2\Phi_E}{\partial\xi^2} + c_2\xi\frac{\partial\Phi_E}{\partial\xi} + c_3\Phi_E + p_{LE}\,\Phi_L + \frac{v}{1-\alpha}\Phi_E(\frac{\xi-a}{1-a})$$

$$0 = c_4\xi\frac{\partial\Phi_L}{\partial\xi} + c_5\Phi_L + p_{EL}\,\Phi_E + \frac{v}{1-a}\Phi_L(\frac{\xi-a}{1-a}),$$

where:
$$c_1 = \frac{\phi^2\sigma^2(1-\tau)^2}{2}$$

$$c_2 = 2\phi^2\sigma^2(1-\tau)^2 - [\frac{1}{1-\psi}(\bar{m} - m^E) + \phi\mu(1-\lambda)]$$

$$c_3 = \phi^2\sigma^2(1-\tau)^2 - [\frac{1}{1-\psi}(\bar{m} - m^E) + \phi\mu(1-\lambda)] - p_{EL} - v$$

$$c_4 = \lambda\phi\mu - \frac{1}{1-\psi}(\bar{m} - m^L)$$

$$c_5 = \lambda\phi\mu - \frac{1}{1-\psi}(\bar{m} - m^L) - p_{LE} - v$$

Now, the Laplace transform for any variable y is defined as:

$$Y(s) = \int_0^\infty e^{-st}y(t)dt,$$

and therefore:
$$Y'(s) = -\int_0^\infty e^{-st}ty(t)dt = -L[ty(t)] \;\Rightarrow\; L[ty] = -\frac{d}{ds}Y(s),$$

and:
$$Y'(s) = -\int_0^\infty e^{-st}ty(t)dt \;\Rightarrow\; Y''(s) = \int_0^\infty e^{-st}t^2y(t)dt = L[t^2y(t)].$$

Hence, we have that:
$$L[ty'] = \int_0^\infty e^{-st}ty'dt = -sY'(s) - Y(s),$$

and:
$$L[t^2y''] = \int_0^\infty e^{-st}t^2y''dt = s^2Y''(s) + 4sY'(s) + 2Y(s).$$

Let $c = \frac{1}{1-a}$, $k = \frac{a}{1-a}$, and $\tau = ct - k$, then $d\tau = cdt$ and $t = \frac{\tau+k}{c}$. So we have:

$$L[y(\frac{t-a}{1-a})] = \int_{k/c}^\infty e^{-st}y(ct - k)dt = \frac{1}{c}e^{-s\frac{k}{c}}L[y(t)]_{s\to s/c} = (1-a)e^{-k(1-a)}L[y(t)]_{s\to s(1-a)}.$$

43

Therefore, when $a = 1$:

$$L[y(\frac{t-a}{1-a})] = (1-1)e^{-k(1-1)} \int_0^\infty y(t)dt = 0 \cdot 1 \cdot 1,$$

if y is a probability density function. Hence, the last term in both Kolmogorov equations will drop out when $a = 1$. After changing variables to $\xi = e^x$, and defining $\partial \Phi_E / \partial x \equiv \Phi_2$ we get:

$$
\begin{pmatrix} \Phi'_L \\ \Phi'_E \\ \Phi'_2 \end{pmatrix} = \begin{pmatrix} 0 & c_5/c_4 - 1 & p_{EL}/c_4 \\ 1 & 0 & 0 \\ c_2/c_1 - 3c_1 & -p_{LE}/c_1 & -2 + c_2/c_1 - c_3/c_1 \end{pmatrix} \begin{pmatrix} \Phi_L \\ \Phi_E \\ \Phi_2 \end{pmatrix}
$$

Since all coefficients are constant, and ξ is bounded, a Lipschitz condition is satisfied, hence the solution to the system exists and is unique.[44] The conditional densities can be recovered by inverting the Laplace transforms.

Proof of Corollary 2. Consider the case with $\lambda = 1$ and $v = 0$. Then, using (18) or (19) and (21) in steady state, using the definition of $\hat{\rho}$, and taking the total differential with respect to K and R, gives:
$$\frac{\partial K}{\partial R} = \frac{\phi - \theta(1-\phi)}{\phi(\theta+1)} \frac{1}{F_{KK}},$$
which proves that:
$$\frac{\partial K}{\partial R} > 0 \Leftrightarrow \theta > \frac{\phi}{1-\phi}.$$

Lemma 3. *The steady state measure of entrepreneurs is given by* $p_{LE}/(p_{LE} + p_{EL})$.

Proof of Lemma 3. Call χ the measure of entrepreneurs today, and χ' their measure tomorrow. Then $\chi' = \chi(1 - p_{EL}) + (1 - \chi)p_{LE}$. But in steady state $\chi = \chi'$ hence $\chi = p_{LE}/(p_{LE} + p_{EL})$.

---

[44] For proper (and non-restrictive) initial conditions, the solution is also stable.

References

Ahsan, S. M. (1974), "Taxation in a Two-Period Temporal Model of Consumption and Portfolio Allocation," *Journal of Public Economics* 5, 337-352.

Aït-Sahalia, Y., Parker, J. A., and M. Yogo (2001), "Luxury Goods and the Equity Premium," NBER, Working Paper 8417.

Aiyagari, S. R. (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics* 109, 659-684.

Aiyagari, S. R. (1995), "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy* 103, 1158-1175.

Alan, S., and M. Browning (2008), "Estimating Intertemporal Allocation Parameters using Synthetic Residual Estimation," Cambridge and Oxford University mimeo.

Albanesi, S., and C. Sleet (2005), "Dynamic Optimal Taxation of Productive Assets," 2005 Society for Economic Dynamics Meeting Papers.

Altig, D., Auerbach, A. J., Kotlikoff, L. J., Smetters, K. A., and J. Walliser (2001), "Simulating Fundamental Tax Reform in the United States," *American Economic Review* 91, 574-595.

Alvarez, Y., Burbidge, J., Farrell, T., and L. Palmer (1992), "Optimal Taxation in a Life-Cycle Model," *Canadian Journal of Economics* 25, 111-122.

Angeletos, G.-M. (2007), "Uninsured Idiosyncratic Investment Risk and Aggregate Saving," *Review of Economic Dynamics* 10, 1-30.

Angeletos, G.-M. , and V. Panousi (2009), "Revisiting the Supply-Side Effects of Government Spending," forthcoming in *Journal of Monetary Economics*, February.

Angeletos, G.-M. , and I. Werning (2006), "Pecuniary Externality and Efficiency with Uninsurable Investment Risk," MIT mimeo.

Atkeson, A., Chari, V. V., and P. J. Kehoe (1999), "Taxing Capital Income: A Bad Idea ," Federal Reserve Bank of Minneapolis Quarterly Review 23, 3-17.

Attanasio, O. P., and G. Weber (1993), "Consumption Growth, Interest Rate, and Aggregation," *Review of Economic Studies* 60, 631-649.

Attanasio, O. P., Banks, J., and S. Tanner (2002), "Asset Holding and Consumption Volatility ," *Journal of Political Economy* 110, 771-793.

Barsky, R. B., Juster, F. T., Kimball, M. S., and M.D. Shapiro (1997), "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study," *Quarterly Journal of Economics* 112, 537-579.

Benabou, R. (2002), "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency," *Econometrica* 70, 481-517.

Benhabib, J., and S. Zhu, R. (2008), "Age, Luck, and Inheritance," NBER Working Paper 14128.

Bitler, M. P., Moskowitz, T. J., and A. Vissing-Jørgensen (2005), "Testing Agency Theory with Entrepreneur Effort and Wealth," *Journal of Finance* 60, 539-576.

Bisin, A., and A. Rampini (2008), "A Mirrleesian Theory of Ramsey Taxation," Working Paper.

Bustos, A., Engel, E. M. R. A., and A. Galetovic (2004), "Could Higher Taxes Increase the Long-Run Demand for Capital? Theory and Evidence from Chile," *Journal of Development Economics* 73, 675-697.

Cagetti M., and M. De Nardi (2006), "Entrepreneurship, Frictions and Wealth," *Journal of Political Economy* 114, 835-870.

Cagetti M., and M. De Nardi (2007), "Estate Taxation, Entrepreneurship, and Wealth," NBER Working Paper 13160.

Campbell, J. Y. (1996), "Understanding Risk and Return," *Journal of Political Economy* 104, 298-345.

Campbell, J. Y., Lettau, M., Malkiel, B. G., and Y. Xu (2001), "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk ," *Journal of Finance* LVI, 1-41.

Carroll, C. D. (2000), "Portfolios of the Rich," NBER Working Paper 7826.

Chamley C. (1986), "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica* 54, 607-622.

Chari, V. V., Christiano, L. J., and P. J. Kehoe (1994), "Optimal Fiscal Policy in a Business Cycle Model," *Journal of Political Economy* 102, 617-652.

Christiano, L. J., and M. Eichenbaum (1992), "Current Real-Business-Cycle Theories and Aggregate Labor Market Fluctuations," *American Economic Review* 82, 430-450.

Cohen, A., and L. Einav (2005), "Estimating Risk Preferences from Deductible Choice ," NBER Working Paper 11461.

Conesa, C., and D. Krueger (2006), "On the Optimal Progressivity of the Income Tax Code," *Journal of Monetary Economics* 53, 1425-1450.

Davila, J., Hong, J. H., Krusell, P., and J.-V. Ríos-Rull (2007), "Constrained Efficiency in the Neoclassical Growth Model with Uninsurable Idiosyncratic Shocks," Working Paper.

Davis, S. J., Haltiwanger, J., Jarmin, R., and J. Miranda (2006), "Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms," NBER Working Paper 12354.

Desai, M. A., Dyck, A., and L. Zingales (2007), "Theft and Taxes," *Journal of Financial Economics*.

Dohmen, T., Falk, A., Huffman, D., Sunde, U., Schupp, J., and G. G. Wagner (2005), "Individual Risk Attitudes: New Evidence from a Large, Representative, Experimentally-Validated Survey," IZA Discussion Paper 1730.

Domeij, D., and J. Heathcote (2004), "On the Distributional Effects of Reducing Capital Taxes," *International Economic Review* 45, 523-554.

Domar, E. D., and A. M. Musgrave (1944), "Proportional Income Taxation and Risk Taking," *Quarterly Journal of Economics* 58, 388-422.

Duffie, D., and L. G. Epstein (1992), "Stochastic Differential Utility," *Econometrica* 60, 353-394.

Erosa, A., and M. Gervais (2002), "Optimal Taxation in Life Cycle Economies," *Journal of Monetary Economics* 105, 338-369.

Erosa, A., and T. Koreshkova (2007), "Progressive Taxation in a Dynastic Model of Human Capital," *Journal of Monetary Economics* 54, 667-685.

Garriga, C. (2003), "Optimal Fiscal Policy in Overlapping Generations Models," Working Paper.

Gentry, W. M., and G. Hubbard (2004), "Entrepreneurship and Household Saving," *Advances in Economic Analysis and Policy* 4, 1053-1108.

Golosov, M., Kocherlakota, N, and A. Tsyvinski (2003), "Optimal Indirect and Capital Taxation," *Review of Economic Studies* 70, 569-588.

Guiso, L., and M. Paiella (2005), "The Role of Risk Aversion in Predicting Individual Behavior," Banks of Italy Working Paper 546.

Himmelberg, C. P., Hubbard, R. G., and I. Love (2002), "Investor Protection, Ownership, and the Cost of Capital," Working Paper.

Jokisch, S. and L. J. Kotlikoff (2006), "Simulating the Dynamic Macroeconomic and Microeconomic Effects of the Fair Tax".

Judd, K. L. (1985), "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics* 28, 59-83.

King, R. G., Plosser, C., and S. Rebelo (1988), "Production, Growth and Business Cycles: I. the Basic Neoclassical Model," *Journal of Monetary Economics* 21, 195-232.

Kitao, S. (2007), "Entrepreneurship, Taxation, and Capital Investment," *Review of Economic Dynamics* 11, 44-69.

Li, W. (2002), "Entrepreneurship and Government Subsidies: A General Equilibrium Analysis," *Journal of Economic Dynamics and Control* 26, 1815-1844.

Malloy, C. J., Moskowitz, T. J., and A. Vissing-Jørgensen (2006), "Long Run Stockholder Consumption Risk and Asset Returns," Working Paper.

Meh, C.A. (2005), "Entrepreneurship, Wealth Inequality, and Taxation," *Review of Economic Dynamics* 8, 688-719.

Moskowitz, T.J., and A. Vissing-Jørgensen (2002), "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?," *American Economic Review* 92, 745-778.

Pálsson, A. - M. (1999), "Does the Degree of Relative Risk Aversion Vary with Household Characteristics?," *Journal of Economic Psychology* 17, 771-787.

Panousi, V., and D. Papanikolaou (2008), "Investment, Idiosyncratic Risk, and Ownership, " Working Paper.

Quadrini, V. (2000), "Entrepreneurship, Saving, and Social Mobility,"*Review of Economic Dynamics* 3, 1-40.

Reiter, M. (2004), "The Optimal Nonlinear Taxation of Capital in Models with Uninsurable Income Risk," Working Paper.

Roussanov, N. L. (2009), "Diversification and its Discontents: Idiosyncratic and Entrepreneurial Risk ain the Quest for Social Status," Working Paper.

Sandmo, A. (1977), "Portfolio Theory, Asset Demand and Taxation: Comparative Statics with Many Assets," *Review of Economic Studies* 44, 369-379.

Stiglitz, J. E. (1969), "The Effect of Income, Wealth, and Capital Gains Taxation on Risk-Taking," *Quarterly Journal of Economics* 83, 263-283.

Uhlig, H., and N. Yanagawa (1996), "Increasing the Capital Income Tax May Lead to Faster Growth," *European Economic Review* 40, 1521-1540.

Vissing-Jørgensen, A., and O. P. Attanasio (2003), "Stock Market Participation, Intertemporal Substitution, and Risk Aversion," *American Economic Review* 93, 383-391.

Werning, I. (2007), "Optimal Fiscal Policy with Redistribution," *Quarterly Journal of Economics* 122, 925-967.