Fundamental Tax Reform: The Growth and Utility Effects of a Revenue-Neutral Flat Tax

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ABSTRACT

We estimate the growth and utility effects of switching from a graduated-rate federal income tax to a flat tax along the lines of Hall-Rabushka (1995). We, furthermore, calculate the post-reform transition dynamics for a number of variables, including the economic growth rate, the representative household's utility – using consumption equivalents as suggested by Lucas (2003) – , the allocation of time to education and market work, as well as the interest and wage rates. To achieve these goals, we rely on a dynamic equilibrium model proposed by Cassou and Lansing (2003), and calibrated to fit historical data about the U.S. economy and the Internal Revenue Service (IRS) tax return statistics for the 2005 tax year. In the process, we specify a step-by-step calibration procedure for the model – a non-trivial undertaking left largely unexplained in Cassou and Lansing (2003).

We find that the flat tax reform increases long-term economic growth, and that the magnitude of this effect depends on the U.S. economy's intertemporal elasticity of substitution in labor supply (IES). For values of IES that range from 0.25 to 1, the introduction of a Hall-Rabushka flat tax increases the long-term economic growth rate by 0.003 - 0.255 percentage points. Although the flat tax reform has clear benefits in the long run, we find that it decreases economic growth during the first post-reform year, and lowers utility for several years after its implementation. Politicians concerned about their re-election prospects may, as a result, be inclined to carefully consider the political consequences of the flat tax reform in the timing of its adoption.

I. INTRODUCTION

In March 1981, Robert E. Hall and Alvin Rabushka, economists at the Hoover Institution at Stanford University, first proposed the idea of a flat tax in a *Wall Street Journal* article entitled “The Attractions of a Flat-Rate Tax System.” Having sparked debate about simplifying the federal income tax and introducing a single marginal rate, Hall and Rabushka published, in December 1981, another Wall Street Journal essay, “A Proposal to Simplify Our Tax System,” in which they elaborated on the details of their suggestion (Rabushka, 2002). In 1983, they expanded the essay into a book, *Low Tax, Simple Tax, Flat Tax*, followed by the best-selling *The Flat Tax* in 1985 and its second edition in 1995.

Hall and Rabushka (1985) wrote that their flat tax proposal “[met] the tests of efficiency, equity and simplicity better than the other politically popular plans.” They believed that the flat tax would improve the performance of the U.S. economy. Increased take-home wages, they argued, would stimulate work effort, and raise total output. Rational investment incentives, furthermore, would spur additional investment and channel it into its most productive uses.

The flat tax has attracted some attention in American politics, especially during the early 1990s. Former California governor Gerald Brown, for instance, endorsed the idea during his 1992 run in the Democratic primaries for the U.S. presidency.[1] Most notably, Steve Forbes, a businessman and a Republican presidential candidate, made it the centerpiece of his 1996 campaign. Despite the enthusiasm that it created in some parts

[1] See Hall and Rabushka (1995), pp. 49.

of the political spectrum, the flat tax – whether in the Hall-Rabushka or any other form – has not been adopted in the United States.

The flat income tax has, however, been implemented in a number of Central and Eastern European countries. In 1995, Estonia introduced it, and was followed by several other countries that had formerly been part of the Soviet Union - most notably Russia, which instituted a single marginal rate of 13 percent in 2001. In 2004, Slovakia introduced a 19-percent flat-rate income tax, and, in 2005, Romania switched to a 16-percent flat tax on personal income and corporate profits (Moore, 2005; Grecu, 2004). Although these reforms differed, to some extent, from the Hall-Rabushka proposal, they all retained its central feature: Income was taxed at a single marginal tax rate.

In this paper, we consider the effects of replacing the current graduated-rate federal income tax with a Hall-Rabushka flat tax, as specified in Hall and Rabushka (1995). We, furthermore, calculate the post-reform transition dynamics for variables such as the economic growth rate, the representative household's utility expressed in consumption-equivalent variations following Lucas (2003), the allocation of time to education and market work, or the interest and wage rates. In doing so, we rely on a dynamic equilibrium model proposed by Cassou and Lansing (2003), calibrated to fit historical data about the U.S. economy and the Internal Revenue Service (IRS) tax return statistics for the 2005 tax year. Finally, we comment on the political implications of our findings, and suggest potential avenues for further research.

II. THE FEDERAL INCOME TAX

A. Individual Income Tax

The federal government levies a tax on the taxable income of individuals. The rate structure of the federal income tax is graduated with a number of brackets, in which income is subject to increasing marginal tax rates. During the 2006 fiscal year, marginal tax rates ranged from 10 to 35 percent.

The calculation of tax liability is a multi-step process, summarized in *Box II.1*.[2] One first has to compute his **gross income** by adding up labor income (wages and salaries), capital income (interest, rents and dividends), and other business income. To obtain the **adjusted gross income (AGI)**, the taxpayer subtracts certain business expenses incurred in earning his income. Some forms of income are excluded from the AGI: These include state and local bond interest, unrealized capital gains, employers' contributions to retirement funds and health insurance plans, saving into tax-preferred individual accounts,[3] alimonies paid to a former spouse and, since 2003, some dividends which are now taxed using the capital gains rate schedule.

[2] For detailed instructions on filling out federal income tax returns, see IRS, Pub. 17.
[3] These tax-favored savings options include individual retirement accounts (IRAs), 401(k) plans, Self-Employed Retirement plans and Education Savings Accounts (Rosen, 2005).

BOX II.1 – COMPUTATION OF THE FEDERAL INDIVIDUAL INCOME TAX

Income from taxable sources

- Includes wages, interest, rents, profits, dividends, realized capital gains, etc.

= Gross income

- Certain business expenses

= Adjusted gross income (AGI)

- Personal exemptions

- Deductions
  - either **itemized**: charitable contributions, some medical expenses, property and state income taxes, some interest payments, etc.
  - or **standard deduction**

= Taxable income (tax base)

► Apply income tax rate schedule

- Tax credits

= Tax liability (total tax payment)

- Withholding

= Final payment (or refund) due

Source: Adapted from Rosen (2005) and Gruber (2005).

One's **taxable income**, also called the tax base, is then computed by subtracting exemptions and deductions from the AGI.

A family is allowed an exemption, adjusted annually for inflation, for each member. Above certain levels of AGI, however, the personal exemption is phased out, although, since 2006, the phase-out is being gradually eliminated (IRS, Pub. 553).

The taxpayer can then take deductions from his taxable income. He can either choose to subtract a fixed amount, called the standard deduction, or to itemize and deduct for selected expenditures, specified by the tax code. If he opts for the latter, the taxpayer can claim deductions for unreimbursed medical and dental expenses exceeding 7.5 percent of AGI, other taxes paid (state or local income taxes, as well as property and real estate taxes), mortgage interest payments, charitable contributions and some unreimbursed employee expenses, such as union dues or job travel costs (Gruber, 2005; Rosen, 2005).

Having obtained his taxable income, the taxpayer can calculate his **tax liability** by first using the tax rate schedule, and then by applying tax credits, flat amounts subtracted from taxes owed. Most individuals' taxes are withheld directly from their labor income when it is earned (IRS, Pub. 505). A taxpayer's final payment will therefore depend on how much tax has already been withheld. If the government has withheld more in taxes than is due, the taxpayer will receive a refund.

B. Alternative Minimum Tax

The alternative minimum tax (AMT) was enacted in 1969 in response to an uproar about some high-income households' ability to avoid paying income taxes, and further strengthened in 1986 (Gruber, 2005). The tax code's preferential treatment of some types of income, along with its many exemptions, deductions and credits, may allow some high-income individuals to greatly reduce their tax burden. The AMT intends to ensure that individuals who benefit from a variety of tax advantages pay some minimum amount of tax on their incomes.

In order to calculate the AMT tax base, the taxpayer first needs to add *AMT preferences* – items such as personal exemptions, the standard deduction, state and local tax benefits, and others - to her taxable income. After subtracting the AMT exemption, which does not depend on the number of dependents and is phased out for high-income filers, she will obtain her **alternative minimum tax income (AMTI)**. This income is then subject to a marginal tax rate of 26 percent on the first $175,000 and 28 percent above to calculate the taxpayer's **tentative AMT**. If the tentative AMT exceeds tax liability under the individual income tax, the taxpayer must pay the difference (the alternative minimum tax) in addition to her regular income tax (Rosen, 2005).

Unlike the ordinary individual income tax, the AMT brackets are not adjusted for inflation. Consequently, as nominal incomes rise, ever more taxpayers are becoming subject to the alternative minimum tax.

C. Corporation Tax

The corporation tax is levied on the taxable income of corporations.[4] Although it has a graduated schedule of marginal rates (IRS, Pub. 542), most corporate income is taxed at 35 percent. Interest payments to lenders, which are considered to be part of business costs, are excluded from taxable income, but dividends paid out to the shareholders are not. As a result, the corporation tax may discourage firms from raising funds by issuing equity, and instead bias them towards debt financing (Rosen, 2005).

More importantly for the purposes of our analysis, taxing dividends both at the corporate and, after the shareholders receive them, at the individual level, creates a problem of double taxation. Until the changes to the tax code introduced by the *Jobs and Growth Tax Relief Reconciliation Act* of 2003, dividends paid out to the shareholders were taxed at the individual's marginal income tax rate. Today, dividends are subject to the capital gains rate schedule on the individual level, and hence the highest rate that can be applied to them is 15 percent.

[4] Corporations on whose taxable incomes the corporation tax is levied are denoted by the tax code as "C corporations." Some small corporations with no more than a hundred shareholders ("S-corporations") are not subject to the corporation tax.

III. THE HALL-RABUSHKA FLAT TAX

A. Description

Hall and Rabushka (1995) propose an integrated flat tax which would apply a single marginal rate τ to both businesses and individuals.[5] Correspondingly, their proposal consists of two components: the individual wage tax and the business tax.

i) Individual wage tax

The individual wage tax is levied on the income that employees receive as cash. Its base consists of realized payments of wages, salaries and pensions during a given period above a personal or family exemption. There are no deductions for mortgage interest or charitable gifts. Pension contributions are not taxed, and pension income is therefore taxed only once – when the worker receives the payment, but not when his employer sets aside the money. Unless the taxpayer owns a business, the individual wage tax is the only relevant component of the flat tax: there are no separate capital gains, dividend or interest taxes.

To calculate his tax liability, the taxpayer first needs to add up all his cash income – wages, salaries, pensions and retirement benefits. After subtracting the fixed personal or family exception, the individual will arrive at the tax base for the individual wage tax. Finally, the single marginal tax rate τ is applied and, after accounting for taxes already withheld, the taxpayer obtains the final amount he owes. *Box III.1* outlines these computations.

[5] In their book, Hall and Rabushka (1995) propose a single tax rate of $\tau = 19\%$. In our description, however, we consider a more general case, in which we do not specify a particular rate.

BOX III.1 – COMPUTATION OF THE INDIVIDUAL WAGE TAX

Wages and salary
+ Pensions and retirement benefits
- Personal or family exemption
= Tax base for the individual wage tax
►Multiply by single tax rate τ
= Tax liability
- Withholding
= Final payment (or refund) **due**

Source: Adapted from Hall and Rabushka (1995).

ii) Business tax

The business tax is levied on all types of income, except wages, salaries and pensions. It aims to include all income apart from that taxed by the individual wage tax. Since there are no deductions for interest payments or dividends, any income received from business activity has already been taxed, and is therefore not subject to tax on the individual level.

To encourage capital formation, Hall and Rabushka (1995) propose eliminating depreciation deductions, and replacing them by a complete first-year tax write-off of all investment spending. In other words, all investment is expensed.[6]

[6] Judd (1998, 1999) argues, however, that the Hall-Rabushka flat tax proposal is biased against investment in human capital, since individuals cannot deduct their educational spending. *Section C* of this chapter briefly discusses this criticism.

The tax base for the business tax, then, is the firm's total annual revenue less any payments the company has made to its employees and suppliers. To calculate how much the taxpayer owes, she must then multiply the tax base by the single marginal rate τ and account for withholding. *Box III.2* summarizes this process.

BOX III.2 – COMPUTATION OF THE BUSINESS TAX

Revenue from sale of goods and services
- Purchases of inputs
- Payments to employees (wages and pensions)
- Investment in capital equipment
= Tax base for the business tax
►Multiply by single tax rate τ
= Tax liability

Source: Adapted from Hall and Rabushka (1995).

B. Progressiveness

Because it retains personal or family exemptions, the Hall-Rabushka flat tax is progressive in the sense that higher-income taxpayers face higher average tax rates.[7] Individuals whose incomes fall below the level of the exemption do not pay any individual wage taxes.

Consider, for instance, a flat tax regime with a single marginal tax rate τ, and no deductions. *Table III.1* shows the average tax rate faced by individuals whose annual income ranges from 5,000 to 100,000 dollars, as the personal exemption rises from 0 to 20,000 dollars.[8] The progressive nature of the flat tax becomes clear as we examine the rows of the table: For any non-zero exemption, high-income individuals face a higher average tax rate.

TABLE III.1 – FLAT TAX: AVERAGE TAX RATES UNDER THE INDIVIDUAL WAGE TAX WITH SINGLE MARGINAL TAX RATE τ

	Individual's Annual Income ($)					
Exemption ($)	**5,000**	**10,000**	**15,000**	**20,000**	**50,000**	**100,000**
0	τ	τ	τ	τ	τ	τ
5,000	-	0.5 τ	0.667 τ	0.75 τ	0.9 τ	0.95 τ
10,000	-	-	0.333 τ	0.5 τ	0.8 τ	0.9 τ
15,000	-	-	-	0.25 τ	0.7 τ	0.85 τ
20,000	-	-	-	-	0.6 τ	0.8 τ

Note: A blank field indicates that the individual owes no taxes, and his average tax rate therefore equals zero.
Source: Author's calculations.

[7] The marginal tax rate represents the effective tax rate applied on the last (incremental) dollar of income. The average tax rate, on the other hand, is obtained by dividing the total tax liability by an individual's income.

[8] We assume that these individuals are not business owners, and therefore are only affected by the individual wage tax.

C. Relationship to Consumption Taxes

Hall and Rabushka (1995) claim that the flat tax they propose is essentially a consumption tax. By expensing all capital investment at the business level, they argue, the flat tax removes investment spending from the tax base, leaving only consumption. Accordingly, the Hall-Rabushka flat tax is commonly cited as an example of a consumption tax in economic and public policy literature – see, for instance, Ventura (1999), McNulty (2000), or Altig, Auerbach, Kotlikoff, Smetters and Walliser (2001).

A consumption tax would eliminate the bias of the current tax system against investment and saving in any form. Judd (1998, 1999) argues, however, that the Hall-Rabushka flat tax is not a true consumption tax: Their proposal is biased against investment in intangible human capital, as individuals cannot expense or deduct any educational spending, despite Boskin's (1977) early counsel.

IV. ECONOMIC GROWTH

Economic growth is the long-term increase in aggregate per capita output produced by an economy, typically measured in terms of the per capita gross domestic product (GDP). This chapter provides a historical overview of economic growth in the United States, and describes two important models that attempt to explain how economies grow: the Solow growth model and the endogenous growth model. These two well-known models provide us with some insight as to the role of technological progress, as well as of physical and human capital accumulation in economic growth. These important themes also feature in the more complex, dynamic equilibrium model by Cassou and Lansing (2003), which is the workhorse of this paper's analysis.[9]

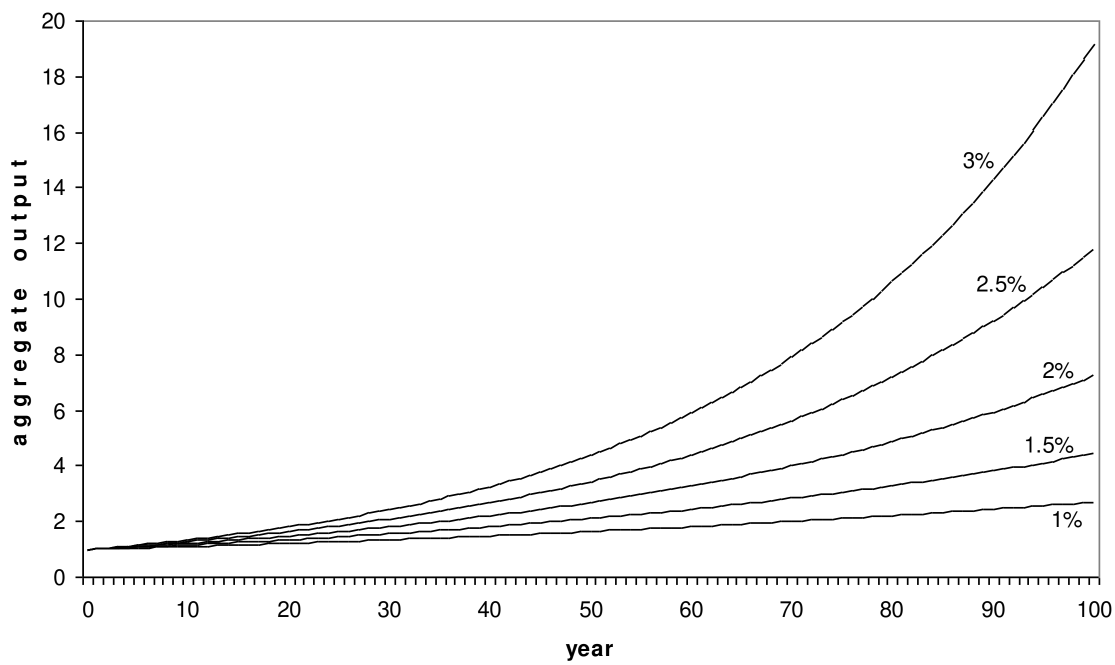
In the long run, sustained economic growth is the most important factor in improving a country's living standards. Because of compounding effects, minor differences in annual growth rates can eventually translate into vast differences in total output and, by extension, in the standard of living. Understanding what government policies have even small positive effects on long-term growth rates can thus go much further in improving living standards than any progress in the macroeconomics of business cycles and countercyclical policy (Barro and Xala-i-Martin, 2004).

It is no wonder then that, as he was contemplating the virtues of high economic growth, the well-known macroeconomist Robert Lucas once famously remarked: "The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else." (Lucas, 1988)

[9] *Chapter V* gives a detailed description of the Cassou-Lansing model.

Figure IV.1 depicts how the level of aggregate output changes depending on the annual growth rate. If the annual growth rate equals 3%, total output will increase almost twenty-fold in 100 years. With an annual growth rate of 2%, however, the increase will only be sevenfold, and with a 1% growth rate, output will less than triple in a hundred years.[10]

FIGURE IV.1 – AGGREGATE OUTPUT DEVELOPMENT WITH VARYING GROWTH RATES



Note: Initially, total output is assumed to be equal to 1 unit. Aggregate output increases at an annual growth rate of 1, 1.5, 2, 2.5 or 3 percent.

For any individual country, economic growth rates typically do not remain constant over time (Jones, 2002). *Figure IV.2* plots the real per capita GDP in the United States during the 1870-2003 time period, compiled by Maddison (2007) and expressed in

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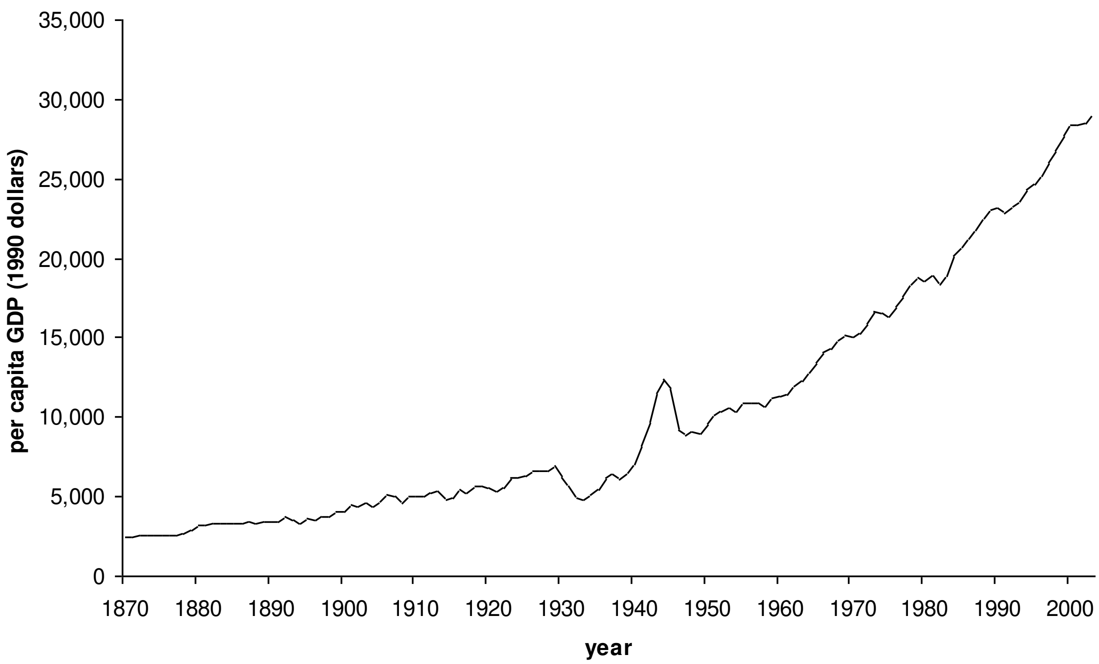
[10] The "rule of 72" can be used to estimate how much time output will take to double at a given growth rate. When the economic growth rate is 3 percent, for example, aggregate output will double in approximately 72/3=24 years.

1990 international Geary-Khamis dollars.[11] Between 1901 and 2000, real per capita GDP rose from 4,464 to 28,403 Geary-Khamis dollars, representing more than a six-fold increase.

FIGURE IV.2 – PER CAPITA GDP IN THE UNITED STATES, 1870-2003



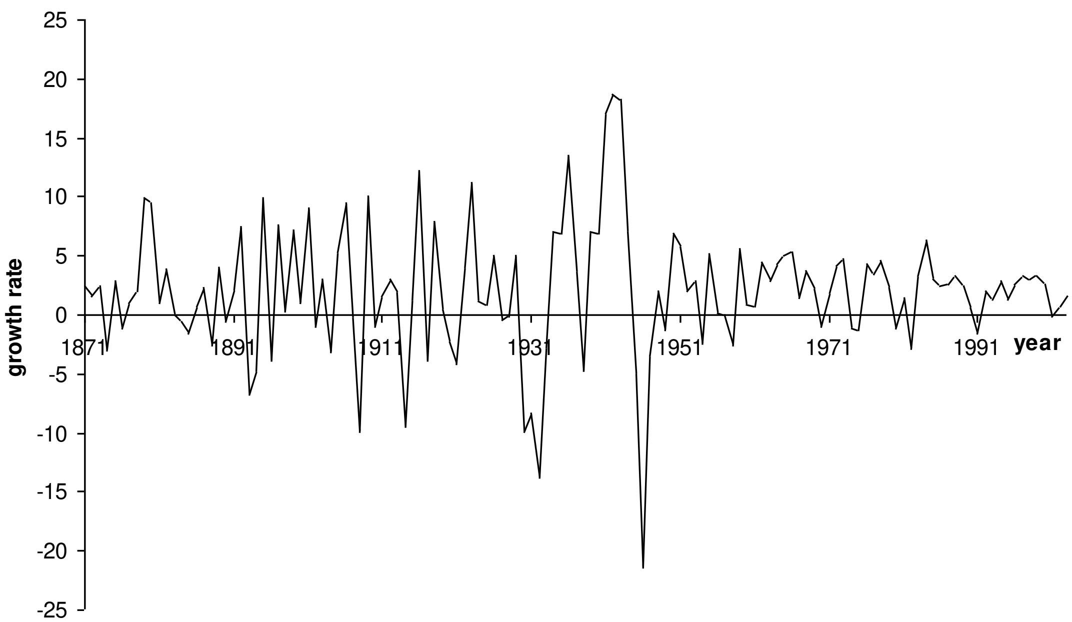
Source: Maddison (2007).

Figure IV.3 shows annual economic growth rates calculated over the same period. We see that, in the course of the 20th century, the U.S. economy saw distinctly negative growth rates during the Great Depression in the 1930s, followed by a pronounced surge in GDP growth during the Second World War. For the rest of the century, periods of

[11] The Geary-Khamis dollar, first suggested by Geary (1958) and later developed by UN statistician Salem Khamis, combines the concepts of category international prices and purchasing power parity into an aggregation method whose properties make it useful in international economic comparisons (UNSD, 1992).

economic growth were interrupted by occasional recessions. Business cycles, however, became milder and less frequent after the Second World War (Temin, 1998).

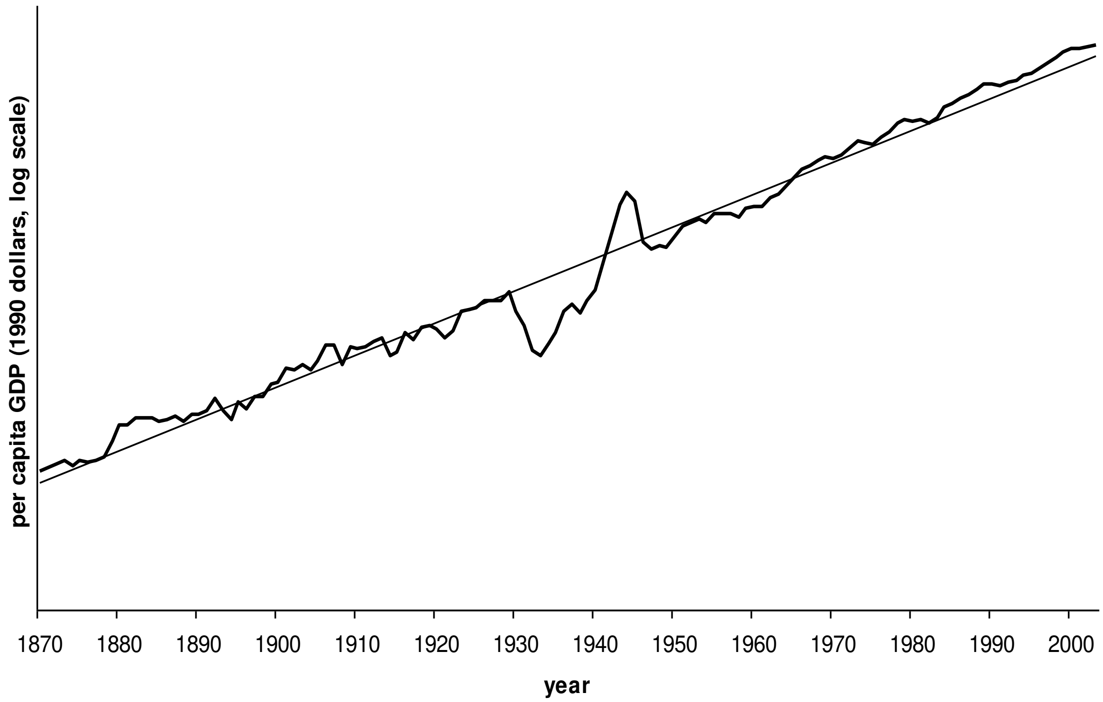
FIGURE IV.3 – ANNUAL ECONOMIC GROWTH RATES IN THE UNITED STATES, 1870-2003



Source: Maddison (2007) and author's calculations.

In advanced countries, long-term economic growth rates tend to be stable, provided one averages over time periods that are long enough to eliminate business cycle effects (Lucas, 1988). Despite the short-term fluctuations in growth rates, the average long-term trend in per capita output in the United States has been positive and relatively constant during the 20th century. *Figure IV.4* displays the 1870-2003 per capita GDP in the United States using a logarithmic scale, along with a linear trend line. When a logarithmic scale is used, the slope of the per capita GDP curve at a given point in time represents the corresponding continuous economic growth rate.

FIGURE IV.4 – PER CAPITA GDP IN THE UNITED STATES, 1870-2003



Source: Maddison (2007) and author's calculations.

The slope of the trend line represents the average long-term growth rate of per capita output in the U.S. economy between 1870 and 2003. A simple, log-linear ordinary least squares (OLS) regression of GDP per capita against the corresponding year[12] yields a slope coefficient of 0.018635, indicating that the long-term growth rate was about 1.86 percent.

[12] Functional form: $\ln\left(\frac{GDP}{capita}\right)_t = \beta_0 + \beta_1 t + e_t$, where t denotes the year in question, and the slope coefficient β_1 represents the average long-term growth rate of per capita GDP.

A. Solow Growth Model

The Solow growth model, also known as the neoclassical or the exogenous growth model, was developed by Robert Solow (1957). It suggests that only improving productivity can sustain economic growth in the long run. With constant growth in total factor productivity, the economy eventually reaches a balanced growth path where output, capital, and consumption per capita all grow at the same rate. The model, however, leaves the source of productivity growth unexplained: Technological progress, in other words, is exogenous. In this section, we examine a variation on the Solow model similar to that described in Williamson (2004) and Williams (2007).

First, we assume that the labor force N grows at a constant rate g_n:

$$g_n = \frac{N_{t+1} - N_t}{N_t}$$

The economy has a Cobb-Douglas aggregate production function:

$$Y_t = z_t\, F(K_t, N_t) = z_t\, K_t^{\theta} N_t^{1-\theta}\ , \quad \theta \in (0,1),$$

where Y_t is total output, z_t represents the state of technology, K_t is the capital stock, and N_t stands for the labor force at time *t*. The exponents θ and $(1-\theta)$ are constants that represent the capital and labor shares in the economy, [13] respectively, and can be interpreted as output elasticities with respect to labor or capital.[14]

[13] If the labor and credit markets are perfectly competitive, the prices of inputs must, in equilibrium, be equal to their marginal products. After imposing this condition, differentiating the aggregate production function and rearranging, we obtain:

$$\theta = \frac{r_t\, K_t}{Y_t} \quad ; \quad 1-\theta = \frac{w_t\, N_t}{Y_t}$$

[14] Output elasticities represent the proportional responsiveness of output to incremental changes in the capital stock or labor force:

First tested against empirical evidence by Cobb and Douglas (1928), this functional form has become standard in macroeconomic research, because of its realistic properties[15] and success at describing the relation between output, capital and labor in the United States (Blanchard, 2003).

In the Solow model, a closed economy produces a single representative good without any government intervention. All factors of production are fully employed.[16] For simplicity, we now assume that technology does not improve over time: total factor productivity (TFP), denoted by z_t, remains constant at 1. Later, we shall consider an economy with technological progress.

Let us now define the following per worker variables:

$$y_t = \frac{Y_t}{N_t} \quad ; \quad k_t = \frac{K_t}{N_t}$$

After plugging the Cobb-Douglas specification for Y_t into the per worker output equation, we obtain:

$$y_t = \frac{Y_t}{N_t} = \frac{K_t^{\theta} N_t^{1-\theta}}{N_t} = K_t^{\theta} N_t^{-\theta} = \left(\frac{K_t}{N_t}\right)^{\theta} = k_t^{\theta}$$

$$\frac{\partial Y_t}{\partial K_t} \times \frac{K_t}{Y_t} = \theta \quad ; \quad \frac{\partial Y_t}{\partial N_t} \times \frac{N_t}{Y_t} = 1 - \theta$$

[15] The Cobb-Douglas production function has intuitively plausible properties. It exhibits constant returns to scale: A doubling of both inputs (labor and capital) will double the output. Each input is essential, since nothing can be produced in the absence of either labor or capital. The law of diminishing returns holds, as marginal productivities of both capital and labor are positive and decreasing:

$$MP_K = \frac{\partial Y_t}{\partial K_t} > 0; \frac{\partial MP_K}{\partial K_t} = \frac{\partial^2 Y_t}{\partial K_t^2} < 0 \qquad MP_N = \frac{\partial Y_t}{\partial N_t} > 0; \frac{\partial MP_N}{\partial N_t} = \frac{\partial^2 Y_t}{\partial N_t^2} < 0$$

The marginal productivity of each factor, furthermore, increases in the other. An additional unit of labor, for instance, yields more output, *ceteris paribus*, if it is combined with a higher capital stock:

$$\frac{\partial MP_K}{\partial N_t} = \frac{\partial MP_N}{\partial K_t} = \frac{\partial^2 Y_t}{\partial K_t \partial N_t} > 0$$

[16] There is no unemployment, and no capital remains idle. Households derive utility only from consumption and do not value leisure, and therefore inelastically supply one unit of labor.

During any given period, the value of total output produced must be equal to the sum of wage and interest incomes received by all households in the economy: $Y_t = r_t K_t + w_t N_t$. Households save a constant fraction s of their incomes, and consume the rest:

$$C_t = (1-s)[r_t K_t + w_t N_t] = (1-s)Y_t$$

In the credit market equilibrium, saving equals investment:

$$sY_t = I_t$$

The law of motion for capital, where δ stands for the depreciation rate:

$$K_{t+1} = (1-\delta)K_t + I_t = (1-\delta)K_t + sY_t$$

$$K_{t+1} - K_t = sY_t - \delta K_t$$

Let us now consider the per capita accumulation of capital:

$$\begin{aligned} k_{t+1} - k_t &= \frac{(K_t - K_{t+1})N_t - (N_t - N_{t+1})K_t}{N_t^2} = \frac{K_{t+1} - K_t}{N_t} - \frac{N_{t+1} - N_t}{N_t}\frac{K_t}{N_t} = \\ &= s\frac{Y_t}{N_t} - \delta\frac{K_t}{N_t} - g_n k_t = sy_t - \delta k_t - g_n k_t = s k_t^{\theta} - \delta k_t - g_n k_t = \\ &= s k_t^{\theta} - (g_n + \delta)k_t \end{aligned}$$

We have thus arrived at the fundamental equation of the Solow growth model:

$$k_{t+1} - k_t = s k_t^{\theta} - (g_n + \delta)k_t$$

The economy eventually settles into a steady state, in which the per capita amount of capital as well as per capita output and consumption remain constant ($k^* = k_{t+1} = k_t$; $k_{t+1} - k_t = 0$):

$$k^* = \left(\frac{s}{g_n + \delta}\right)^{\frac{1}{1-\theta}}$$

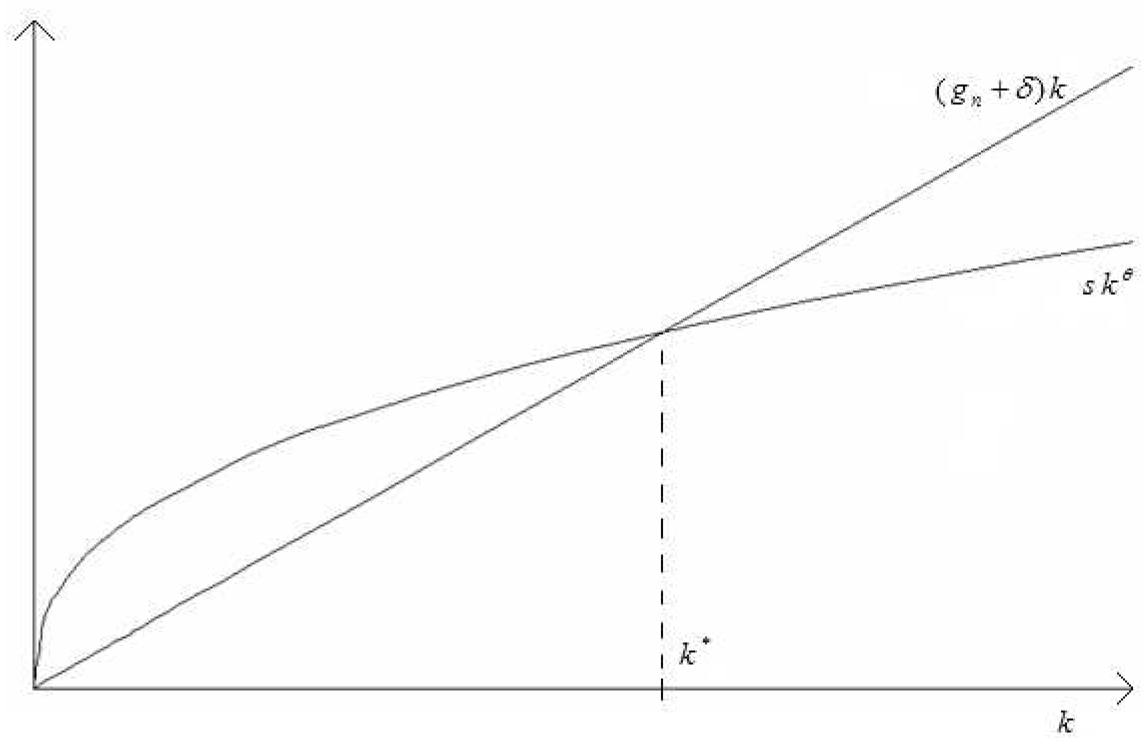
The corresponding values of steady-state per capita output and consumption are:

$$y^* = \left(\frac{s}{g_n + \delta} \right)^{\frac{\theta}{1-\theta}}$$

$$c^* = (1-s)(k^*)^{\theta}$$

In *Figure IV.5*, the steady-state per capita level of capital occurs where the $(g_n + \delta)k$ line, indicating the influence of demographic growth and the depreciation rate, and the sk^{θ} curve which represents the amount of per capita saving in the economy.

FIGURE IV.5 – STEADY-STATE IN THE SOLOW GROWTH MODEL WITHOUT TECHNOLOGICAL GROWTH



Having analyzed an economy without technological progress, we can now introduce productivity growth into the model. For simplicity, we characterize TFP growth as labor-augmenting, or Harrod-neutral, technological change: A unit of labor is more productive when the level of technology is higher (Jones, 2002). The aggregate production function becomes:

$$Y_t = K_t^{\theta}(A_t N_t)^{1-\theta} \ , \quad \theta \in (0,1),$$

which is equivalent to the original definition of $Y_t = z_t \, K_t^{\theta} N_t^{1-\theta}$ with $z_t = A_t^{1-\theta}$.

Let us assume that the rate of technological growth is constant at g_A:

$$g_A = \frac{A_{t+1} - A_t}{A_t}$$

In this case, the economy will ultimately settle on a balanced growth path, where the output, capital and consumption per worker all grow at the same, constant rate g_A. To solve for this equilibrium, we will need to work with variables that remain constant over time:

$$\tilde{y}_t = \frac{y_t}{A_t} = \frac{Y_t}{A_t N_t} \quad ; \quad \tilde{k}_t = \frac{k_t}{A_t} = \frac{K_t}{A_t N_t}$$

We then repeat our earlier analysis with the above variables:

$$\tilde{y}_t = \frac{K_t^{\theta}(A_t N_t)^{1-\theta}}{A_t N_t} = \frac{K_t^{\theta} A_t^{1-\theta} N_t^{1-\theta}}{A_t N_t} = \frac{K_t^{\theta}}{A_t^{\theta} N_t^{\theta}} = \tilde{k}_t^{\theta}$$

The per capita accumulation of the capital stock is described by the equation:

$$\begin{aligned}
\tilde{k}_{t+1} - \tilde{k}_t &= \frac{(K_t - K_{t+1})(A_t N_t) - (A_t(N_t - N_{t+1}) + (A_t - A_{t+1})N_t)K_t}{A_t^2 N_t^2} = \\
&= \frac{K_{t+1} - K_t}{A_t N_t} - \left(\frac{N_{t+1} - N_t}{N_t} + \frac{A_{t+1} - A_t}{A_t} \right) \frac{K_t}{A_t N_t} = s\tilde{k}_t^{\theta} - \delta \tilde{k}_t - (g_A + g_n)\tilde{k}_t = \\
&= s\tilde{k}_t^{\theta} - (g_A + g_n + \delta)\tilde{k}_t
\end{aligned}$$

The fundamental equation of the Solow growth model with technological growth will thus be:

$$s\left(\tilde{k}^*\right)^{\theta} - (g_A + g_n + \delta)\tilde{k}^* = 0$$

By isolating $\tilde{k}^*$, we can solve for the balance growth path equilibrium:

$$\tilde{k}^* = \left(\frac{s}{g_A + g_n + \delta}\right)^{\frac{1}{1-\theta}} \qquad \tilde{y}^* = \left(\frac{s}{g_A + g_n + \delta}\right)^{\frac{\theta}{1-\theta}}$$

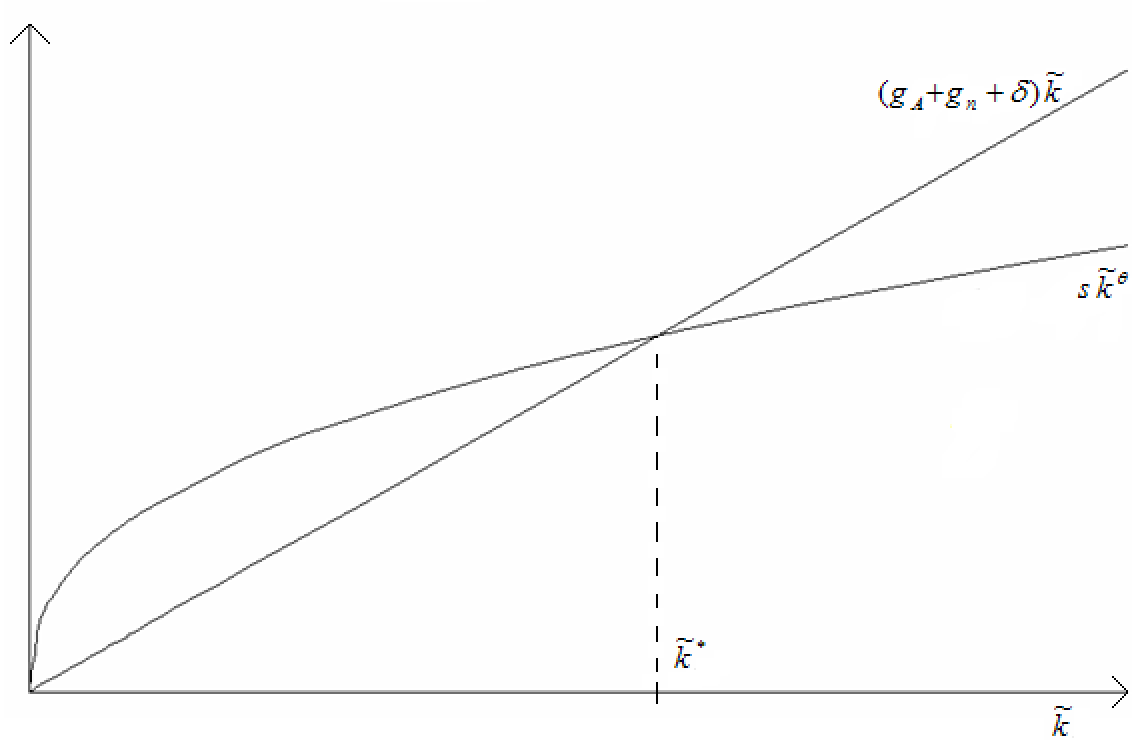
$$k_t = A_t\left(\frac{s}{g_A + g_n + \delta}\right)^{\frac{1}{1-\theta}} \qquad y_t = A_t\left(\frac{s}{g_A + g_n + \delta}\right)^{\frac{\theta}{1-\theta}}$$

$$K_t = N_t A_t\left(\frac{s}{g_A + g_n + \delta}\right)^{\frac{1}{1-\theta}} \qquad Y_t = N_t A_t\left(\frac{s}{g_A + g_n + \delta}\right)^{\frac{\theta}{1-\theta}}$$

A change in the capital stock, labor force growth rate or savings rate can affect the transition dynamics of the economy for some time, but in the long run, continued economic growth will only be driven by sustained technological progress.

Figure IV.6 shows how the balanced growth path level of $\tilde{k}_t$ depends on the amount of saving in the economy, the technological and demographic growth rates, and the depreciation rate.

FIGURE IV.6 – BALANCED GROWTH PATH IN THE SOLOW GROWTH MODEL WITH TECHNOLOGICAL GROWTH



The Cassou-Lansing model, described in *Chapter V*, also settles into a balanced growth path equilibrium in which aggregate output, and the stocks of physical and human capital grow at the same constant rate. Unlike the basic Solow model, however, it does not feature exogenous (and hence unexplained) technological growth, but rather relies on investment in human capital to improve labor productivity. In this respect, the Cassou-Lansing model is similar to the endogenous growth model, whose basic characteristics are outlined in the next section.

B. Endogenous Growth Model

The Solow growth model does not explain the origins of technological progress. During the late 1980s and early 1990s, endogenous growth models that account for productivity growth were developed, most notably by Romer (1986, 1990) and Lucas (1988).

In developing the model,[17] we abstract away from concerns about the labor force, and consider an aggregate production function where total output depends on the total factor productivity, and on the use of human and physical capital:[18]

$$Y_t = z_t H_t^{\theta} K_t^{1-\theta} \ , \quad \theta \in (0,1)$$

If we assume that human capital is a constant fraction j of physical capital $(H_t = jK_t)$ and let $A = zj^{\theta}$, we can reformulate the production function as follows:

$$Y_t = zH_t^{\theta} K_t^{1-\theta} = z(jK_t)^{\theta} K_t^{1-\theta} = [zj^{\theta}]K_t = AK_t$$

Human capital includes the education, skills and training that workers possess, and which increase their productivity.[19] In addition, we could interpret human capital as the stock of knowledge that results from continuing research and development, or as the degree to which physical capital is being put to better use as a result of learning-by-doing.

Our production function indicates that, as the economy expands, the stock of human capital increases proportionally with the amount of physical capital.

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[17] As in the section on the Solow growth model, lecture notes by Williams (2007) form the basis of our description.

[18] This specification of the production function has, much like the one in the Solow growth model, the Cobb-Douglas form. In this endogenous growth model, however, we do not consider the input of labor, and instead focus on human capital.

[19] Without the gradual accumulation of human capital, the law of diminishing returns would set in for physical capital. Investment in education, skills and training offsets this decrease in the marginal product of physical capital, as the marginal products of human and physical capital are both increasing in the other factor.

As in the exogenous model, households save a constant fraction s of the output and, in the credit market equilibrium, total savings equal investment. We obtain the following law of motion for physical capital:

$$K_{t+1} = K_t + I_t - \delta K_t$$

$$K_{t+1} - K_t = I_t - \delta K_t = sAK_t - \delta K_t = (sA - \delta) K_t$$

We can now solve for the balanced growth path equilibrium, in which per worker output and physical capital grows at the same rate $sA - \delta$:

$$\frac{Y_{t+1} - Y_t}{Y_t} = \frac{K_{t+1} - K_t}{K_t} = sA - \delta$$

The long-run rate of economic growth therefore depends largely on the savings rate. Higher savings mean more investment in physical capital, which in turn translates into a higher stock of human capital (more education, research and development, or learning-by-doing). A higher savings rate thus yields greater productivity and higher growth.

Neither the basic Solow growth model nor the endogenous growth model takes into account the government's fiscal policy – its taxing and spending decisions. The Cassou-Lansing model, explained in the next chapter, incorporates the basic insights and approaches from these two models – their focus on the role of capital accumulation, for instance, in explaining economic growth – into a complex dynamic equilibrium framework along the lines of Kydland and Prescott (1982), while also accounting for the effects of changes in the tax code.

V. THE CASSOU-LANSING MODEL

Cassou and Lansing (2003) construct a dynamic equilibrium model to simulate the effects that changes in the tax code have on the long-term growth rate. The model economy consists of a representative household, the government, and a firm that encompasses the entire productive economy. Its parameters are calibrated to fit empirical facts about the U.S. economy. Functional forms are chosen to allow for a closed-form solution.

A. Basic Framework

The representative household maximizes the following utility function:

$$\sum_{t=0}^{\infty} \beta^t \ln\left[c_t - V\left(h_t, 1-l_t\right)\right], \qquad \beta \in (0,1), \tag{1}$$

where t indexes time, β is the discount factor, c_t is private consumption, h_t represents the household's stock of human capital, and l_t stands for leisure as a proportion of the overall time endowment. Total time available is normalized to equal 1, and $(1-l_t)$ thus represents the proportion of time spent in non-leisure activities – either at work or in education. Function V quantifies the disutility associated with non-leisure time:

$$V(h_t, 1-l_t) = B\,h_t(1-l_t)^{\gamma}\ , \qquad B > 0, \gamma > 0\,, \tag{2}$$

where h_t adjusts for the quality of foregone leisure, along the lines of Heckman (1976) and Becker (1965). Heckman (1976) notes that human capital can be assumed to be a direct source of consumption benefits as it augments a household's effective consumption time. The exponent γ suggests that the disutility from non-leisure increases exponentially as the household spends more time at work or in education. The intertemporal elasticity of substitution in labor supply, furthermore, equals $\sigma = (\gamma - 1)^{-1}$.[20]

Time can be allocated to leisure, work or education. If the overall time endowment is 1 and the proportion of time spent in education is denoted by e_t, $(1 - l_t - e_t)$ represents the proportion of time spent working.[21]

The representative household consumes part of its after-tax income (c_t), and devotes the rest to investment into either human or physical capital (i_{ht} and i_{kt}, respectively). In any given period *t*, the household receives a rental rate r_t for each unit of physical capital used in production, and earns a wage w_t for each unit of effective labor, measured by $h_t(1 - l_t - e_t)$, it supplies. In maximizing its utility, the representative household must thus conform to the within-period budget constraint:

$$c_t + i_{kt} + i_{ht} = r_t k_t + w_t h_t (1 - l_t - e_t) - T_t \qquad \textbf{(3)}$$

Taxes T_t paid to the government are given by the equation:

$$T_t = \tau_{pt}\left[w_t h_t (1 - l_t - e_t) - D_t + \eta(1 - \bar{\tau}_b)(r_t k_t - \phi_k i_{kt} - \phi_h i_{ht})\right] + \bar{\tau}_b\left[r_t k_t - \phi_k i_{kt} - \phi_h i_{ht}\right], \qquad \textbf{(4)}$$

where τ_{pt} is the personal tax rate, $\bar{\tau}_b$ is the business (corporate) tax rate. In every period, personal taxable income consists of labor income less the standard deduction D_t and the

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[20] Since $\lim_{\gamma \to \infty} (\gamma - 1)^{-1} = 0$, as γ gets larger (i.e., labor supply σ becomes less elastic across time), $(1 - l_t)^{\gamma}$ approaches unity and the model reduces to one with a fixed allocation of time.

[21] Compared to Cassou and Lansing (2003), we have changed some of the notation to make it more intuitive. In the original paper, for instance, l_t denoted time spent in non-leisure activities, which is somewhat confusing.

after-tax business income, assumed to be paid out as dividends. The extent to which there is double taxation of business income is denoted by $\eta \in [0,1]$. The parameters $\phi_k, \phi_h \in [0,1]$ represent the fractions of physical and human capital, respectively, that can be expensed.[22]

The aggregate production function in the model economy has a Cobb-Douglas form and is given by:

$$y_t = zk_t^{\theta}\left[h_t\left(1-l_t-e_t\right)\right]^{1-\theta}, \qquad z>0,\ \theta \in (0,1), \tag{5}$$

where per capita output y_t is an increasing function of the state of technology, of the stock of physical capital and of the amount of effective labor supplied. The constant θ represents the capital share in the economy, while $(1-\theta)$ is the labor share.

In a perfectly competitive, profit-maximizing environment, factors of production earn their marginal products. The equilibrium rental rate r_t and wage w_t will therefore be:

$$r_t = \frac{\theta\, y_t}{k_t}, \qquad w_t = \frac{(1-\theta)\, y_t}{h_t\left(1-l_t-e_t\right)} \tag{6), (7}$$

The laws of motion for physical and human capital are as follows:

$$k_{t+1} = A_k k_t^{1-\delta_k} i_{kt}^{\delta_k}, \qquad A_k > 0,\ \delta_k \in (0,1] \tag{8}$$

$$h_{t+1} = A_h h_t^{1-\delta_h} i_{ht}^{\delta_h} e_t^{v}, \qquad A_h > 0,\ \delta_h \in [0,1],\ v \geq 0, \tag{9}$$

The nonlinear functional form reflects adjustment costs, as suggested in Lucas and Prescott (1971). Whereas physical capital only accumulates through investment i_{kt},

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[22] An expenditure that is expensed can be immediately deducted from business taxable income.

households can build up their human capital stocks either through direct investment i_{ht}, or by allocating some of their time to education (e_t).

B. Taxes and Government Spending

The personal tax rate is given by the equation:

$$\tau_{pt} = \bar{\tau}_p \left[\frac{w_t h_t (1 - l_t - e_t) - D_t + \eta (1 - \bar{\tau}_b)(r_t k_t - \phi_k i_{kt} - \phi_h i_{ht})}{w_t H_t (1 - L_t - E_t) - D_t + \eta (1 - \bar{\tau}_b)(r_t K_t - \phi_k I_{kt} - \phi_h I_{ht})} \right]^n, \qquad \textbf{(10)}$$

where capital letters denote averages across all households in the economy. An individual household's decisions cannot affect the values of these economy-wide averages. Parameters $\bar{\tau}_p \in [0,1)$ and $n \geq 0$ represent the level and slope of the tax schedule, respectively, and are estimated by regression from empirical data.

In a graduated tax-rate schedule, $n > 0$ as households with an above-average taxable income pay higher tax rates than those whose income is below the average. With a flat tax, on the other hand, marginal tax rates are the same regardless of taxable income, and hence, $n = 0$, and $\tau_{pt} = \bar{\tau}_p$.

The standard deduction D_t, whose level is set by the government, is assumed to be a constant fraction α of the aggregate output Y_t:

$$D_t = \alpha Y_t, \qquad \alpha \geq 0 \qquad \textbf{(11)}$$

Average tax rates are calculated by dividing the amount of tax revenue by taxable income. Marginal rates, on the other hand, can be thought of as the tax rates paid on the last dollar of income earned, and are computed by differentiating tax revenue with respect to taxable income.

To calculate the marginal personal tax rate, we first multiply equation (10) by personal taxable income $\left[w_t h_t(1-l_t-e_t)-D_t+\eta(1-\bar{\tau}_b)(r_t k_t-\phi_k i_{kt}-\phi_h i_{ht})\right]$:

$$\tau_{pt}\times pers.taxable\ \ income=\bar{\tau}_p\,\frac{\left[w_t h_t(1-l_t-e_t)-D_t+\eta(1-\bar{\tau}_b)(r_t k_t-\phi_k i_{kt}-\phi_h i_{ht})\right]^{n+1}}{\left[w_t H_t(1-L_t-E_t)-D_t+\eta(1-\bar{\tau}_b)(r_t K_t-\phi_k I_{kt}-\phi_h I_{ht})\right]^{n}}$$

Differentiating this expression with respect to personal taxable income yields:

$$\bar{\tau}_p\,(n+1)\frac{\left[w_t h_t(1-l_t-e_t)-D_t+\eta(1-\bar{\tau}_b)(r_t k_t-\phi_k i_{kt}-\phi_h i_{ht})\right]^{n}}{\left[w_t H_t(1-L_t-E_t)-D_t+\eta(1-\bar{\tau}_b)(r_t K_t-\phi_k I_{kt}-\phi_h I_{ht})\right]^{n}}$$

Finally, we impose aggregate consistency conditions $h_t=H_t$, $k_t=K_t$, $i_{kt}=I_{kt}$, $i_{ht}=I_{ht}$, and $(1-l_t-e_t)=(1-L_t-E_t)$. Hence, the marginal personal tax rate **(12)** is equal to $\bar{\tau}_p(n+1)$ **.** The average personal tax rate **(13)** simply equals $\bar{\tau}_p$.

The average business tax rate can be computed by dividing the total amount of tax collected on business income, whether through the corporate tax or by the double taxation of dividends in the personal income tax, by business taxable income $(r_t k_t-\phi_k i_{kt}-\phi_h i_{ht})$:

$$\frac{\bar{\tau}_b\,(r_t k_t-\phi_k i_{kt}-\phi_h i_{ht})+\eta(1-\bar{\tau}_b)\bar{\tau}_p\,(r_t k_t-\phi_k i_{kt}-\phi_h i_{ht})}{(r_t k_t-\phi_k i_{kt}-\phi_h i_{ht})}=\bar{\tau}_b+\eta(1-\bar{\tau}_b)\bar{\tau}_p \qquad \textbf{(14)}$$

Each additional dollar of business income is taxed at the statutory corporate tax rate $\bar{\tau}_b$ and, with double taxation of business income, also at the marginal personal tax rate $\bar{\tau}_p(n+1)$. The marginal business tax rate **(15)** thus becomes $\bar{\tau}_b+\eta(1-\bar{\tau}_b)\bar{\tau}_p(n+1)$.

The government uses tax receipts to finance its spending G_t. In their model, Cassou and Lansing (2003) assume that government expenditures are unproductive and do not provide any utility to the households.

A period-by-period balanced budget is imposed (**16**) – during each time period, government expenditures must equal the sum of personal and business income taxes collected:

$$G_t = \tau_{pt}\left[w_t H_t (1 - L_t - E_t) - D_t + \eta(1-\bar{\tau}_b)(r_t K_t - \phi_k I_{kt} - \phi_h I_{ht})\right] + \bar{\tau}_b\left[r_t K_t - \phi_k I_{kt} - \phi_h I_{ht}\right]$$

Over time, government spending is modeled to increase in fixed proportion with the aggregate income in the economy, and thus remains a significant portion of output:

$$G_t = \psi Y_t\,, \quad \psi \geq 0 \tag{17}$$

C. Optimal Decision Rules

In *Appendix A*, we show how Cassou and Lansing (2003) derive the following closed-form decision rules for the household's choice of $\{i_{kt}, i_{ht}, c_t, 1-l_t-e_t, e_t\}$ at any given time *t*:

$$i_{kt} = a_0(1-\bar{\tau}_k)y_t\,, \quad \bar{\tau}_k \equiv \frac{(1-\phi_k)\left[\bar{\tau}_b + \eta(1-\bar{\tau}_b)\bar{\tau}_p(n+1)\right]}{1-\phi_k\left[\bar{\tau}_b + \eta(1-\bar{\tau}_b)\bar{\tau}_p(n+1)\right]} \tag{18}$$

$$i_{ht} = b_0(1-\bar{\tau}_h)y_t\,, \qquad \bar{\tau}_h \equiv \frac{(n+1)\bar{\tau}_p - \phi_h\left[\bar{\tau}_b + \eta(1-\bar{\tau}_b)\bar{\tau}_p(n+1)\right]}{1-\phi_h\left[\bar{\tau}_b + \eta(1-\bar{\tau}_b)\bar{\tau}_p(n+1)\right]} \tag{19}$$

$$\begin{aligned} c_t = \big\{ & 1 - \bar{\tau}_p(1-\theta-\alpha) - \theta\left[\bar{\tau}_b + \eta(1-\bar{\tau}_b)\bar{\tau}_p\right] \\ & - a_0(1-\bar{\tau}_k)\left[1-\phi_k\left[\bar{\tau}_b + \eta(1-\bar{\tau}_b)\bar{\tau}_p\right]\right] \\ & - b_0(1-\bar{\tau}_h)\left[1-\phi_h\left[\bar{\tau}_b + \eta(1-\bar{\tau}_b)\bar{\tau}_p\right]\right]\big\} y_t \end{aligned} \tag{20}$$

$$(1 - l_t - e_t) = p_1 \left\{ \left[1 - \bar{\tau}_p \,(n+1)\right] \left(\frac{h_t}{k_t} \right)^{-\theta} \right\}^{\frac{1}{\theta+\gamma-1}} \tag{21}$$

$$e_t = p_1 p_2 \left\{ \left[1 - \bar{\tau}_p \,(n+1)\right] \left(\frac{h_t}{k_t} \right)^{-\theta} \right\}^{\frac{1}{\theta+\gamma-1}} = p_2 \left(1 - l_t - e_t\right), \tag{22}$$

where a_0, b_0, p_1 and p_2 are combinations of deep parameters, as specified in the appendix.

We see that private investment in physical and human capital depends on the effective marginal tax rates $\bar{\tau}_k$ and $\bar{\tau}_h$, which are derived by combining tax code parameters $\bar{\tau}_p, \bar{\tau}_b$, η and n with the expensing parameters ϕ_k and ϕ_h.[23] The labor and education decision rules imply that households spend less time in non-leisure activities, as they accumulate more human capital relative to their stock of physical capital. Intuitively, this happens because the opportunity cost of foregone leisure increases with the stock of human capital a household possesses, as suggested by Heckman (1976).

A close examination of the optimal decision rules reveals how investment in human and physical capital, and the amount of time spent in market work and education react to changes in the personal ($\bar{\tau}_p$) and business ($\bar{\tau}_b$) tax rate levels, and in the slope of the personal income tax rate schedule n. In examining these reactions, we assume that all other parameters are within ranges we would reasonably expect to see in real world economies.

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[23] With a pure consumption tax, $\phi_h = \phi_k = 1$, $\bar{\tau}_h = \bar{\tau}_k = 0$

The effective marginal tax rate on physical capital ($\bar{\tau}_k$) is an increasing function of $\bar{\tau}_b$ and, as long as there is at least some double taxation of dividends (η > 0), of $\bar{\tau}_p$ and n as well. In the absence of double taxation of dividends (when η = 0), $\bar{\tau}_k$ is unresponsive to changes in the personal tax rate or in the tax schedule slope. As a result, we can conclude that lowering the personal and business tax rates, or making the tax rate schedule flatter, in general, increases the level of investment in physical capital (i_{kt}), which is decreasing in $\bar{\tau}_k$. In the special case of no double taxation of dividends, however, lowering the personal tax rate or changing the slope of the tax rate schedule has no effect on physical capital investment.

The effective marginal tax rate on human capital ($\bar{\tau}_h$) is also increasing in the personal tax level $\bar{\tau}_p$ and in the tax rate schedule slope n, but decreases with higher business tax rates $\bar{\tau}_b$. Unlike in the case of $\bar{\tau}_k$, the direction of $\bar{\tau}_h$'s responsiveness to tax rate changes does not depend on the extent of the double taxation of dividends, or on changes in the slope of the tax rate schedule. Consequently, since i_{ht} increases with lower values of $\bar{\tau}_h$, we infer that the level of human capital investment rises with lower personal tax rates $\bar{\tau}_p$, a flatter tax schedule n, and higher business tax rates $\bar{\tau}_b$. This result makes intuitive sense, since taxing physical capital more heavily by increasing business tax rates may induce households to substitute more human capital investment.

The proportion of time households spend engaged in market work $(1-l_t-e_t)$ is a decreasing function of the personal tax rate $\bar{\tau}_p$ and of the slope of the tax rate schedule n, as is the proportion of time spent in education (e_t). In other words, taxing personal income at lower rates – a policy which can be achieved either by reducing the base level of personal tax rates, or by flattening the tax rate schedule - induces households to work more, and spend more time building up their human capital. The business tax rate $\bar{\tau}_b$, however, has no effect on the way households allocate their time.

After substituting the decision rule for the proportion of time spent at work (21) into the aggregate production function (5), we obtain an expression for the equilibrium output per capita:

$$y_t = zp_1^{1-\theta} k_t^{\frac{\theta\gamma}{\theta+\gamma-1}} h_t^{\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}} \left[1-\bar{\tau}_p(n+1)\right]^{\frac{1-\theta}{\theta+\gamma-1}} \qquad \textbf{(23)}$$

Given an initial stock of human (h_0) and physical (k_0) capital, we can characterize the model economy's dynamic transition path. To obtain the equilibrium laws of motion, we substitute investment decision rules (18), (19) and (22), as well as the equilibrium per capita output relation (23), into the basic laws of motion (8) and (9):

$$k_{t+1} = A_k \left(a_0 zp_1^{1-\theta}\right)^{\delta_k} \left[1-\bar{\tau}_p(n+1)\right]^{\frac{\delta_k(1-\theta)}{\theta+\gamma-1}} \left(1-\bar{\tau}_k\right)^{\delta_k} \left(\frac{h_t}{k_t}\right)^{\frac{\delta_k(1-\theta)(\gamma-1)}{\theta+\gamma-1}} k_t \qquad \textbf{(24)}$$

$$h_{t+1} = A_h \left(p_1 p_2\right)^{v} \left(b_0 zp_1^{1-\theta}\right)^{\delta_h} \left[1-\bar{\tau}_p(n+1)\right]^{\frac{\delta_h(1-\theta)+v}{\theta+\gamma-1}} \left(1-\bar{\tau}_h\right)^{\delta_h} \left(\frac{h_t}{k_t}\right)^{\frac{-\theta(\gamma\delta_h+v)}{\theta+\gamma-1}} h_t \qquad \textbf{(25)}$$

The specification of the Cassou-Lansing model implies that, in the balanced growth path, the values of y_t, c_t, k_t, h_t, i_{kt} and i_{ht} all grow at the same constant rate μ. Under these circumstances, the ratio *R* of the stock of human and physical capital will not change over time. To derive *R* (**26**), we divide (25) by (24), and impose the balanced growth path condition $R = \frac{h_t}{k_t} = \frac{h_{t+1}}{k_{t+1}}$:

$$R=\frac{h_t}{k_t}=\left\{(p_1p_2)^v\left(zp_1^{1-\theta}\right)^{(\delta_h-\delta_k)}\left(\frac{A_h b_0^{\delta_h}}{A_k a_0^{\delta_k}}\right)\frac{(1-\bar{\tau}_h)^{\delta_h}\left[1-\bar{\tau}_p(n+1)\right]^{\frac{(\delta_h-\delta_k)(1-\theta)+v}{\theta+\gamma-1}}}{(1-\bar{\tau}_k)^{\delta_k}}\right\}^{\frac{\theta+\gamma-1}{\delta_k(1-\theta)(\gamma-1)+\theta(\gamma\delta_h+v)}}$$

The balanced growth path ratio *R* of the stock of human capital to that of physical capital increases as business income is taxed at a higher rate $\bar{\tau}_b$, and as the tax rate schedule becomes flatter (n goes down). The effect of changes in $\bar{\tau}_p$ is theoretically ambiguous, and depends – in practice – on the current tax code parameters ($\bar{\tau}_p$, $\bar{\tau}_b$ and n), and the relative size of the elasticities δ_k and δ_h.

To calculate the growth rate μ (**27**) on the balanced growth path, we take logarithms of equations (24) and (25) and plug in (26) for *R*:

$$\begin{aligned}\mu&=\ln\frac{k_{t+1}}{k_t}=\ln\frac{h_{t+1}}{h_t}=\ln\frac{y_{t+1}}{y_t}=\ln\frac{c_{t+1}}{c_t}\\&=\ln\left[A_k\left(a_0 zp_1^{1-\theta}\right)^{\delta_k}\right]+\frac{\delta_k(1-\theta)(\gamma-1)}{\theta+\gamma-1}\ln(R)+\frac{\delta_k(1-\theta)}{\theta+\gamma-1}\ln\left[1-\bar{\tau}_p(n+1)\right]+\delta_k\ln(1-\bar{\tau}_k)\\&=\ln\left[A_h(p_1p_2)^v\left(b_0 zp_1^{1-\theta}\right)^{\delta_h}\right]-\frac{\theta(\gamma\delta_h+v)}{\theta+\gamma-1}\ln(R)+\frac{\left[\delta_h(1-\theta)+v\right]}{\theta+\gamma-1}\ln\left[1-\bar{\tau}_p(n+1)\right]+\delta_h\ln(1-\bar{\tau}_h)\end{aligned}$$

The above expression implies – provided that all parameters have realistic values - that the economic growth rate on the balanced growth path increases as the personal tax rate schedule flattens (in other words, as n decreases), and as $\bar{\tau}_p$ and $\bar{\tau}_b$ decrease.[24] A relatively flat tax rate schedule with low tax rates, then, will be more conducive to faster economic growth than one with high and steeply increasing marginal tax rates.

[24] A simple way to find out how changes in the tax code parameters affect the economic growth rate is to combine all the individual terms in equation (27) into a single logarithm, and then examine how the changing values of $\bar{\tau}_p$, $\bar{\tau}_b$ and n affect μ.

D. Revenue Neutrality

If we substitute equations (6), (7), (11), (18) and (19) into equation (16), and impose aggregate consistency conditions, we can rearrange equation (17) to obtain the following relationship among the various tax code parameters:

$$\psi = \bar{\tau}_p\left(1-\theta-\alpha\right)+\left[\bar{\tau}_b+\eta\left(1-\bar{\tau}_b\right)\bar{\tau}_p\right]\left\{\theta-a_0\phi_k\left(1-\bar{\tau}_k\right)-b_0\phi_h\left(1-\bar{\tau}_h\right)\right\} \qquad \textbf{(28)}$$

A tax reform can affect the growth rates of a number of variables, and the concept of revenue neutrality in the model must therefore be a relative one – specifically, the expected tax revenue at the time of reform must constitute the same fraction of output as the pre-reform tax receipts did. Since all government spending is paid for using tax revenues, to achieve revenue neutrality, the value of ψ in the government spending equation (17) will have to remain unchanged as the government changes the tax schedule parameters.

To maintain revenue neutrality, therefore, we choose the flat tax rate $\bar{\tau}=\bar{\tau}_p=\bar{\tau}_b$ to satisfy equation (28) at the pre-reform value of ψ.

E. Calibration of the Model: General Procedure

To yield meaningful predictions, the Cassou-Lansing model must first be calibrated to fit empirical data about the U.S. economy up until the time of reform. In this appendix, we describe a systematic way, obtained by reverse-engineering Cassou and Lansing (2003), to calibrate the parameters of the model.

To calibrate the model, we need estimates, based on the econometric analysis of empirical data or on the findings of previous studies, of the following variables on the pre-reform balanced growth path:

$(1-\hat{l}_t-\hat{e}_t)$ - the proportion of time a typical household spends working

$\hat{e}_t$ - the average fraction of time households spend in education

$\dfrac{\hat{h}_t}{\hat{k}_t}$ - the proportion of the stock of human capital to that of physical capital

$\dfrac{\hat{k}_t}{\hat{y}_t}$ - the average amount of physical capital as a proportion of total output

$\dfrac{\hat{i}_{kt}}{\hat{y}_t}$, $\dfrac{\hat{i}_{ht}}{\hat{y}_t}$ - the ratios of physical and human capital investment to output

$\hat{\mu}$ - the average rate of long-run economic growth

In addition to estimates of the above variables, in calibrating the model we make use of variables describing the fundamental features of the economy – such as the discount factor β, and the labor and capital shares ($1-\theta$ and θ, respectively) – and of the pre-reform characteristics of the tax system: the level $\hat{\tau}_p$ and slope $\hat{n}$ of the personal income tax schedule estimated through a regression analysis of government data, the statutory corporate tax rate $\hat{\tau}_b$, and the extent $\hat{\eta}$ of double taxation of business income.

First, we rearrange the optimal decision rule for the proportion of time spent at work (21) to estimate p_1:

$$\boxed{p_1 = \frac{(1-\hat{l}_t - \hat{e}_t)}{\left\{\left[1-\hat{\tau}_p\,(\hat{n}+1)\right]\left(\frac{\hat{h}_t}{\hat{k}_t}\right)^{-\theta}\right\}^{\frac{1}{\theta+\gamma-1}}}}$$

We can now use the decision rule for time spent in education (22) to derive an estimate for p_2:

$$\boxed{p_2 = \frac{\hat{e}_t}{p_1\left\{\left[1-\hat{\tau}_p\,(\hat{n}+1)\right]\left(\frac{\hat{h}_t}{\hat{k}_t}\right)^{-\theta}\right\}^{\frac{1}{\theta+\gamma-1}}}}$$

To find z, we rearrange the aggregate production function (23) in the following manner:

$$y_t = zp_1^{1-\theta} k_t^{\frac{\theta\gamma}{\theta+\gamma-1}} h_t^{\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}} \left[1-\bar{\tau}_p(n+1)\right]^{\frac{1-\theta}{\theta+\gamma-1}}$$

$$y_t = zp_1^{1-\theta} k_t^{\frac{\theta\gamma}{\theta+\gamma-1}} \left(\frac{h_t}{k_t} k_t\right)^{\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}} \left[1-\bar{\tau}_p(n+1)\right]^{\frac{1-\theta}{\theta+\gamma-1}}$$

$$y_t = zp_1^{1-\theta} k_t^{\frac{\theta\gamma}{\theta+\gamma-1}+\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}} \left(\frac{h_t}{k_t}\right)^{\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}} \left[1-\bar{\tau}_p(n+1)\right]^{\frac{1-\theta}{\theta+\gamma-1}}$$

$$\frac{y_t}{y_t^{\frac{\theta\gamma}{\theta+\gamma-1}+\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}}} = zp_1^{1-\theta}\left(\frac{k_t}{y_t}\right)^{\frac{\theta\gamma}{\theta+\gamma-1}+\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}}\left(\frac{h_t}{k_t}\right)^{\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}}\left[1-\bar{\tau}_p(n+1)\right]^{\frac{1-\theta}{\theta+\gamma-1}}$$

For simplicity, we assume that, at the time of reform, the per capita output equals one unit. After plugging in, the above expression thus becomes:

$$1 = zp_1^{1-\theta}\left(\frac{k_t}{y_t}\right)^{\frac{\theta\gamma}{\theta+\gamma-1}+\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}}\left(\frac{h_t}{k_t}\right)^{\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}}\left[1-\bar{\tau}_p(n+1)\right]^{\frac{1-\theta}{\theta+\gamma-1}}$$

After rearranging, we obtain an expression that will allow us to calibrate z:

$$\boxed{z = \frac{1}{p_1^{1-\theta}\left(\frac{\hat{k}_t}{\hat{y}_t}\right)^{\frac{\theta\gamma}{\theta+\gamma-1}+\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}}\left(\frac{\hat{h}_t}{\hat{k}_t}\right)^{\frac{(1-\theta)(\gamma-1)}{\theta+\gamma-1}}\left[1-\hat{\tau}_p(\hat{n}+1)\right]^{\frac{1-\theta}{\theta+\gamma-1}}}}$$

To estimate δ_k, we plug in the expression for a_0, as defined in *Appendix A*,[25] into the decision rule for physical capital investment (18):

$$\frac{i_{kt}}{y_t} = a_0(1-\bar{\tau}_k) = \frac{\theta\delta_k}{\rho+\delta_k}(1-\bar{\tau}_k)$$

We then isolate δ_k:

$$\boxed{\delta_k = \frac{\rho\frac{\hat{i}_{kt}}{\hat{y}_t}}{\theta(1-\hat{\tau}_k)-\frac{\hat{i}_{kt}}{\hat{y}_t}} = \frac{\left(\frac{1}{\beta-1}\right)\frac{\hat{i}_{kt}}{\hat{y}_t}}{\theta(1-\hat{\tau}_k)-\frac{\hat{i}_{kt}}{\hat{y}_t}}}$$

[25] $a_0 = \frac{\theta\delta_k}{\rho+\delta_k}$, where $\rho = \frac{1}{\beta}-1$

To calibrate v and δ_h, we need to solve two simultaneous equations, one based on the optimum decision rule for human capital investment (19) and the other on an expression for p_2 from *Appendix A*:

$$\begin{cases} \dfrac{i_{ht}}{y_t} = b_0\left(1-\bar{\tau}_h\right) = \dfrac{(1-\theta)\delta_h}{\left(\rho+\delta_h+\dfrac{v}{\gamma}\right)}\left(\dfrac{\gamma+1}{\gamma}\right)\left(1-\bar{\tau}_h\right) \\ p_2 = \dfrac{v}{\left(\rho+\delta_h+\dfrac{v}{\gamma}\right)}\left(\dfrac{\gamma-1}{\gamma}\right) \end{cases}$$

Let us rearrange the second equation to bring v to the left-hand side, and plug the resulting expression into the human investment decision rule (19):

$$v = \frac{p_2\gamma(\rho+\delta_h)}{\gamma-p_2-1}$$

$$\frac{i_{ht}}{y_t} = \frac{(1-\theta)\delta_h}{\left(\rho+\delta_h+\dfrac{p_2\;(\rho+\delta_h)}{\gamma-p_2-1}\right)}\left(\frac{\gamma+1}{\gamma}\right)\left(1-\bar{\tau}_h\right)$$

We now solve for δ_h:

$$\boxed{\delta_h = \frac{\gamma\rho\dfrac{\hat{i}_{ht}}{\hat{y}_t}}{(1-\theta)(\gamma-p_2-1)(1-\hat{\tau}_h)-\dfrac{\hat{i}_{ht}}{\hat{y}_t}\gamma} = \frac{\gamma\left(\dfrac{1}{\beta-1}\right)\dfrac{\hat{i}_{ht}}{\hat{y}_t}}{(1-\theta)(\gamma-p_2-1)(1-\hat{\tau}_h)-\dfrac{\hat{i}_{ht}}{\hat{y}_t}\gamma}}$$

To obtain an estimate for v, we plug δ_h back into the expression:

$$\boxed{v = \frac{p_2\gamma(\rho+\delta_h)}{\gamma-p_2-1}}.$$

To calibrate B, we now rearrange the expression for p_1, given in *Appendix A*:

$$p_1 = \left\{ \frac{z(1-\theta)}{B\gamma} \left[1 + \frac{v}{\left(\rho + \delta_h + \frac{v}{\gamma}\right)} \left(\frac{\gamma - 1}{\gamma}\right) \right]^{1-\gamma} \right\}^{\frac{1}{\theta+\gamma-1}}$$

$$\boxed{B = \frac{z(1-\theta)}{\gamma\, p_1^{\theta+\gamma-1}} \left[1 + \frac{v}{\left(\rho + \delta_h + \frac{v}{\gamma}\right)} \left(\frac{\gamma - 1}{\gamma}\right) \right]^{1-\gamma} = \frac{z(1-\theta)}{\gamma\, p_1^{\theta+\gamma-1}} \left[1 + \frac{v}{\left(\frac{1}{\beta-1} + \delta_h + \frac{v}{\gamma}\right)} \left(\frac{\gamma - 1}{\gamma}\right) \right]^{1-\gamma}}$$

Finally, we make use of the long-term growth equation (27) to calibrate A_k and A_h, respectively:

$$\mu = \ln A_k + \delta_k \ln\left(a_0 z p_1^{1-\theta}\right) + \frac{\delta_k (1-\theta)(\gamma-1)}{\theta+\gamma-1} \ln(R) + \frac{\delta_k (1-\theta)}{\theta+\gamma-1} \ln\left[1-\bar{\tau}_p (n+1)\right] + \delta_k \ln(1-\bar{\tau}_k)$$

$$\ln A_k = \mu - \delta_k \ln\left(a_0 z p_1^{1-\theta}\right) - \frac{\delta_k (1-\theta)(\gamma-1)}{\theta+\gamma-1} \ln\left(\frac{\hat{h}_t}{\hat{k}_t}\right) - \frac{\delta_k (1-\theta)}{\theta+\gamma-1} \ln\left[1-\bar{\tau}_p (n+1)\right] - \delta_k \ln(1-\bar{\tau}_k)$$

$$\boxed{A_k = e^{\ln A_k} = e^{\mu - \delta_k \ln\left(a_0 z p_1^{1-\theta}\right) - \frac{\delta_k (1-\theta)(\gamma-1)}{\theta+\gamma-1} \ln\left(\frac{\hat{h}_t}{\hat{k}_t}\right) - \frac{\delta_k (1-\theta)}{\theta+\gamma-1} \ln\left[1-\bar{\tau}_p (n+1)\right] - \delta_k \ln(1-\bar{\tau}_k)}}$$

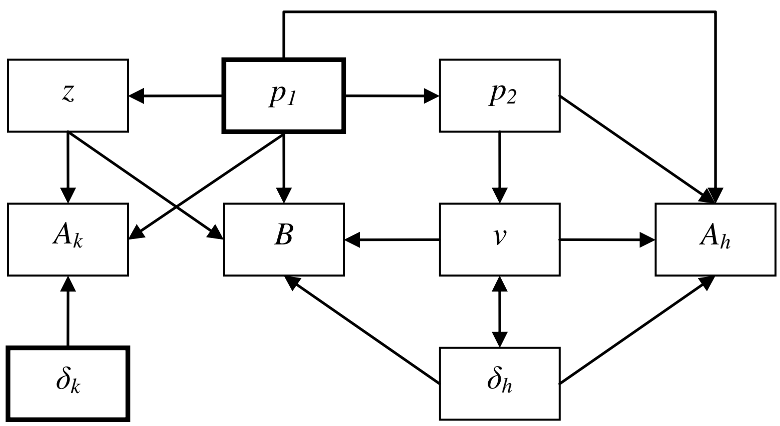
$$\mu = \ln A_h + v \ln(p_1 p_2) + \delta_h \ln\left(b_0 z p_1^{1-\theta}\right) - \frac{\theta(\gamma\delta_h + v)}{\theta+\gamma-1} \ln(R) + \frac{\left[\delta_h (1-\theta) + v\right]}{\theta+\gamma-1} \ln\left[1-\bar{\tau}_p (n+1)\right] + \delta_h \ln(1-\bar{\tau}_h)$$

$$\ln A_h = \mu - v \ln(p_1 p_2) - \delta_h \ln\left(b_0 z p_1^{1-\theta}\right) + \frac{\theta(\gamma\delta_h + v)}{\theta+\gamma-1} \ln\left(\frac{\hat{h}_t}{\hat{k}_t}\right) - \frac{\left[\delta_h (1-\theta) + v\right]}{\theta+\gamma-1} \ln\left[1-\bar{\tau}_p (n+1)\right] - \delta_h \ln(1-\bar{\tau}_h)$$

$$\boxed{A_h = e^{\ln A_h} = e^{\mu - v \ln(p_1 p_2) - \delta_h \ln\left(b_0 z p_1^{1-\theta}\right) + \frac{\theta(\gamma\delta_h + v)}{\theta+\gamma-1} \ln\left(\frac{\hat{h}_t}{\hat{k}_t}\right) - \frac{\left[\delta_h (1-\theta) + v\right]}{\theta+\gamma-1} \ln\left[1-\bar{\tau}_p (n+1)\right] - \delta_h \ln(1-\bar{\tau}_h)}}$$

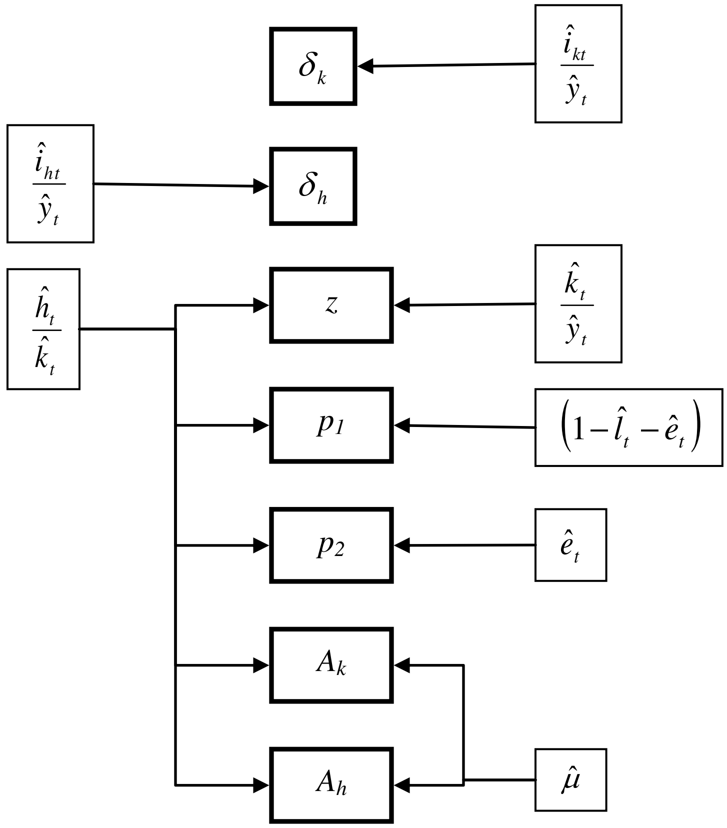
Figure V.1 shows how the variables to be calibrated relate to one another. An arrow pointing from one variable to another signifies that one cannot calibrate the latter until the value of the former has been estimated. *Figure V.2* describes the manner in which empirical facts about the U.S. economy, assumed to hold on the pre-reform balanced growth path, are used as inputs into the calibration process.

FIGURE V.1 – RELATIONSHIPS AMONG VARIABLES TO BE CALIBRATED



Note: Arrows denote how the estimated values of individual variables are used as inputs into the calibration of other variables. If a variables is not pointed at by any arrow, it is "exogenous" in the sense that it can be estimated directly from empirical facts about the U.S. economy, without using any of the other variables in the figure.

FIGURE V.2 – EMPIRICAL FACTS ABOUT THE U.S. ECONOMY AS CALIBRATION INPUTS



Note: The middle column denotes the variables that need to be calibrated. The constants in the left and right columns represent empirical facts about the U.S. economy, generally obtained from previous economic studies. The arrows indicate which empirical facts are used in the calibration of individual variables.

VI. QUANTITATIVE CALIBRATION AND RESULTS

A. Calibration

I. Characteristics of the U.S. Economy

a.) Discount Factor β

We calibrate the discount factor β to be 0.9615, which implies an annual real pre-tax interest rate of 4 percent.[26] This estimate is similar to that used by Kydland and Prescott (1982) and by Greenwood, Rogerson and Wright (1993), who use a quarterly discount factor of 0.99, corresponding to a real interest rate of 1 percent per quarter and, hence, approximately 4 percent per year. Our calibrated value is somewhat lower than the 0.979 estimate used by Lansing and Cassou (2003), designed to achieve an after-tax rate of 4 percent based on Poterba (1997), and Paez-Farrell's (2005) estimate of 0.9801, which assumed a quarterly discount factor of 0.995. On the other hand, our value of β exceeds the discount factor of 0.9433 used by Gomme, Kydland and Rupert (2001), who wanted to approximate a real annual interest rate of 6 percent in the steady state. In light of the sizeable variation in discount factor values in the economic literature, our conservative estimate of 0.9615 appears to be appropriate.

b.) Long-Term Growth Rate $\hat{\mu}$

During the time period from 1870 until 2003, the long-term growth rate of the U.S. economy $\hat{\mu}$ was about 0.0186, or 1.86 percent, as the regression analysis in *Chapter IV* has shown.

[26] $\beta = \dfrac{1}{1+r} = \dfrac{1}{1.04} = 0.9615$

c.) Labor $(1-\theta)$ and Capital (θ) Shares

We estimate the average long-run share of labor income in the economy from the *Economic Report of the President* (CEA, 2007).[27] For any given year *t*, the corresponding labor share $(1-\theta)_t$ is the proportion of national income that employees receive as their compensation.[28] In a traditional Cobb-Douglas setting, similar to that employed by Cassou and Lansing (2003) where the only aggregate inputs are labor and capital, the capital share during the year *t* will simply equal $\theta_t = 1-(1-\theta)_t$. *Figures VI.1* and *VI.2* show the annual labor and capital shares, respectively, for the time period from 1959 to 2005. Consistent with the stylized facts about capital accumulation outlined by Kaldor (1961), the labor and capital shares in the U.S. economy have remained approximately constant over time.[29]

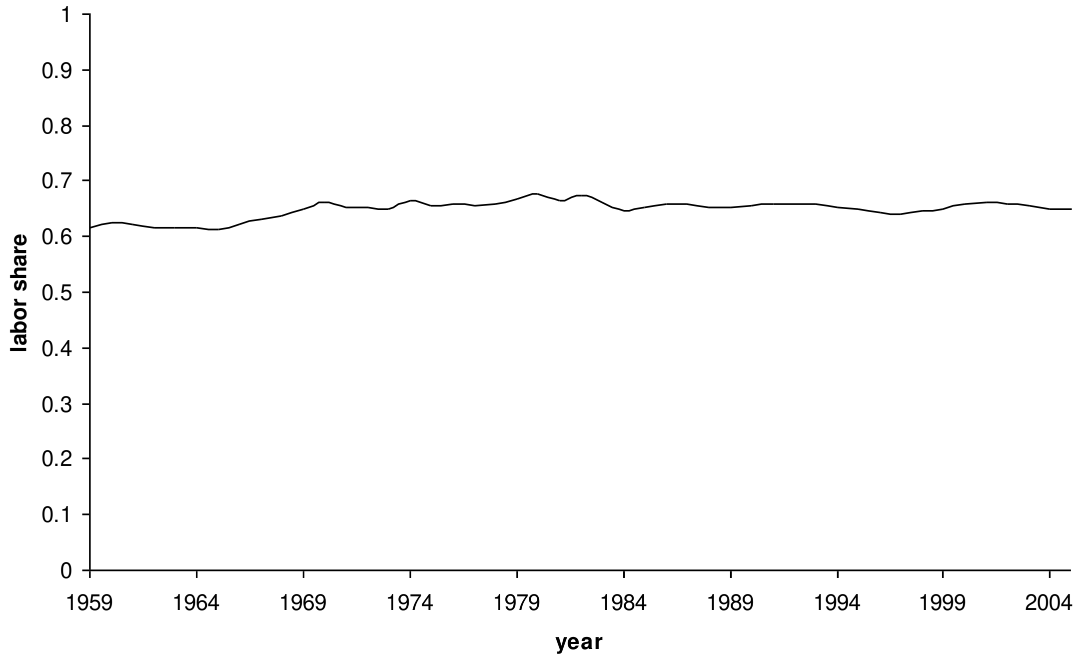
In our analysis, we use the average long-run share of labor and capital income in the U.S. economy, computed by taking the arithmetic mean of annual income shares from 1959 to 2005:

$$(1-\theta) = \frac{1}{(2005-1959)+1}\sum_{t=1959}^{2005}(1-\theta)_t = 0.649$$

$$\theta = \frac{1}{(2005-1959)+1}\sum_{t=1959}^{2005}\theta_t = 0.351$$

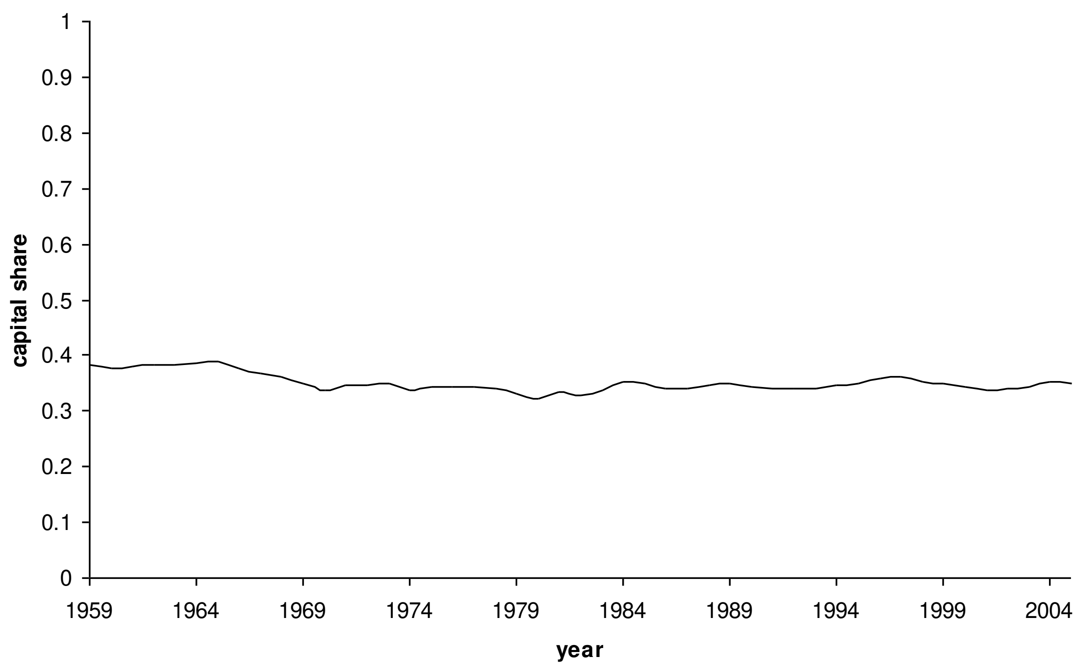
[27] See "Statistical Table B-28: National Income by Type of Income," (CEA, 2007).
[28] In the report, employee compensation consists of wages and wage accruals, complemented by employer contributions to pension funds, insurance funds and government social insurance.
[29] In our sample, the minimum annual value of the labor share is 0.323, whereas the maximum is 0.389.

FIGURE VI.1 – LABOR SHARE IN THE U.S. ECONOMY, 1959-2005



Source: Council of Economic Advisers (2007) and author's calculations.

FIGURE VI.2 – CAPITAL SHARE IN THE U.S. ECONOMY, 1959-2005



Source: Council of Economic Advisers (2007) and author's calculations.

d.) Intertemporal Elasticity of Substitution in Labor Supply $\sigma = (\gamma - 1)^{-1}$

A number of studies have tried to estimate the intertemporal elasticity of substitution (IES) in labor supply, and their conclusions vary widely. Hall (1988) and Ball (1990) have found estimates close to zero. Similarly, MaCurdy (1981) found an IES between 0.1 and 0.3. Altonji's (1986) estimates indicate a value between 0 and 0.35. For French (2004), a conservative range for the IES would run from -0.5 to 0.6.

Some economists, however, argue that the intertemporal elasticity of substitution in labor supply may be significantly higher. Ham and Reilly (2006), for instance, consider an implicit contracts model, in which workers bargain over state-contingent contracts denominated in terms of consumption and hours of work, and find an IES of either 0.9 or 1.[30] Rupert, Rogerson and Wright (2000) argue that IES estimates obtained from traditional life cycle models exhibit a large downward bias, as they neglect changes in work done at home over time. Beaudry and van Wincoop (1996) find that the IES is significantly different from zero, and probably close to 1.

In light of the varied findings, we estimate the Cassou-Lansing model for three different IES values σ = 0.25 (low), 0.5 (intermediate) and 1 (high). The corresponding values of γ are 5, 3 and 2, respectively.

[30] Ham and Reilly's (2006) analysis of the *Panel Study of Income Dynamics* (PSID) yields an estimate of 0.9, whereas their examination of the *Consumer Expenditure Survey* (CES) yields a value of 1.

e.) Proportion of Time Spent in Market Work $(1-\hat{l}_t-\hat{e}_t)$ and Educational Activities $(\hat{e}_t)$

Since our representative household has three members – two employed adults, and one child who attends school[31] – its daily time endowment is 3 x 24 = 72 hours.[32] In calibrating our model, we normalize this time endowment to one.

According to the *American Time Use Survey* (BLS, 2007), in 2006, employed Americans spent an average of 7.6 hours in market work every day. Consequently, in the representative household with two working adults, about 21.11 percent of time will be spent in market work.[33]

The same survey indicates that, during 2006, about 9 percent of people in the United States engaged in educational activities. Those who attended school spent a daily average of 4.5 hours in class, the survey finds, and those who did homework or research spent about 2.4 hours on it every day. We assume that, in the representative household, the child's studying habits conform to these findings, and estimate that education takes up 9.583 percent of the household's time.[34] This estimate is probably conservative, as it does not include time spent in on-the-job training, or any other education the adults may pursue.

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[31] We describe the representative household in *Part II.a* of this section.

[32] The 2006 *American Time Use Survey* (BLS, 2007) finds that, on an average day, the typical American over the age of fifteen slept for about 8.6 hours, bringing his effective daily time endowment to 24 – 8.6 = 15.4 hours. In our model, however, we assume that production is uninterrupted, and therefore each person's time endowment is 24 hours. A three-member household, therefore, will have an endowment of 72 hours.

[33] $$1-\hat{l}_t-\hat{e}_t=\frac{2(\textit{daily time in employment per employed member})}{3(\textit{daily time endowment per member})}=\frac{2(7.6)}{3(24)}=\frac{15.2}{72}=0.2\overline{1}$$

[34]

$$\hat{e}_t=\frac{1\,(\textit{daily class per student}+\textit{daily hw \& research per student})}{3(\textit{daily time endowment per member})}=\frac{4.5+2.4}{3(24)}=0.0958\overline{3}$$

Given the above findings, we estimate $\hat{e}_t$ to equal to 0.09583, and $(1-\hat{l}_t-\hat{e}_t)$ to be 0.2111.

f.) Investment in Physical and Human Capital: $\frac{\hat{i}_{kt}}{\hat{y}_t}$ and $\frac{\hat{i}_{ht}}{\hat{y}_t}$

We use data on the U.S. national accounts from the *Economic Report of the President* (CEA, 2007) to estimate $\frac{\hat{i}_{kt}}{\hat{y}_t}$, the long-term investment in physical capital as a proportion of GDP.[35] As in Cassou and Lansing (2003), our definition of investment includes consumer purchases of durable goods, residential fixed investment, changes in private inventories, and investment in non-residential structures, as well as in equipment and software.[36] For the 1959-2005 time period, the mean value of $\frac{\hat{i}_{kt}}{\hat{y}_t}$ was 0.2453.

Our measure of the long-term human capital investment as a proportion of GDP, $\frac{\hat{i}_{ht}}{\hat{y}_t}$, includes private sector expenditures on education, and on research and development (R&D). We obtained R&D data from *National Patterns of R&D Resources* (NSF, 2007), and private education expenditure figures from *OECD Statistics* (OECD, 2006). Between 1997 and 2004, $\frac{\hat{i}_{ht}}{\hat{y}_t}$ averaged 0.0384.

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[35] See "Statistical Table B-1: Gross Domestic Product, 1959-2006," (CEA, 2007).
[36] To obtain total investment for a given year using *Table B-1* in CEA, 2007, we must therefore combine the total amount of gross private domestic investment and the private consumption of durable goods.

g.) Stock of Physical and Human Capital, and Their Ratio: $\frac{\hat{k}_t}{\hat{y}_t}$, $\frac{\hat{h}_t}{\hat{y}_t}$ and $\frac{\hat{h}_t}{\hat{k}_t}$

To gauge $\frac{\hat{k}_t}{\hat{y}_t}$, the stock of physical capital as a proportion of total output, we use data collected by Turner, Tamura, Mulholland and Baier (2007), whose time series covers the time period from 1840 to 2000 at ten-year intervals. Turner et al. (2007) use Gallman (1960) to derive the physical capital stock during the 1840-1920 period, and then rely on the *Fixed Reproducible Tangible Wealth* series (BEA, 1999) for the period until 2000.[37] *Figure IV.3* depicts the historical development of the $\frac{\hat{k}_t}{\hat{y}_t}$ ratio. We estimate the long-term value of $\frac{\hat{k}_t}{\hat{y}_t}$ to be 3.1424.[38]

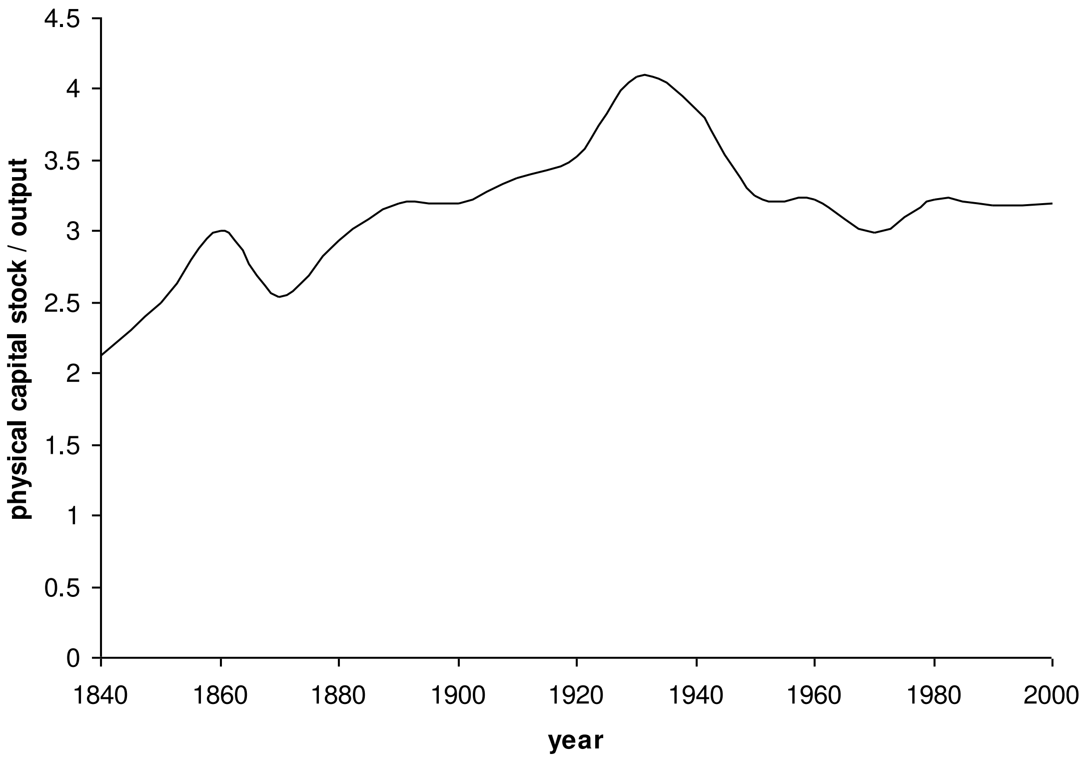
Compared to results obtained by other studies, this figure appears conservative: Cassou and Lansing (2003), for instance, used a value of 2.61, and data from the Fiscal Year 2007 edition of *Analytical Perspectives – Budget of the United States* (OMB, 2006) suggest an estimate of about 4.28.[39]

To estimate the ratio of human to physical capital, $\frac{\hat{h}_t}{\hat{k}_t}$, we use Jorgenson and Fraumeni's (1992) lifetime labor-income based estimates of human wealth, which

[37] Katz and Herman (1997) revisit and improve the *Fixed Reproducible Tangible Wealth* estimates by the Bureau of Economic Analysis.
[38] This value is obtained by taking the arithmetic mean of the observations in Turner, Tamura, Mulholland and Baier (2007).
[39] To estimate total capital from *Analytical Perspectives* (OMB, 2006), we take the sum of publicly and privately owned physical capital assets as given by *Table 13-4* in the report. We use Johnston and Williamson's (2007) GDP estimates to obtain an approximate ratio of the physical capital stock to aggregate output.

encompass both market and non-market labor activities.[40] *Figure IV.4* depicts how the stock of human capital compared to the gross domestic product between the years 1947 and 1986.[41] The mean value of $\frac{\hat{h}_t}{\hat{y}_t}$ during this period was 64.11, indicating the ratio of human to physical capital can be estimated to be $\frac{\hat{h}_t}{\hat{k}_t} = 20.4$.[42]

FIGURE VI.3 – PHYSICAL CAPITAL AS A PROPORTION OF OUTPUT IN THE U.S. ECONOMY, 1840-2000



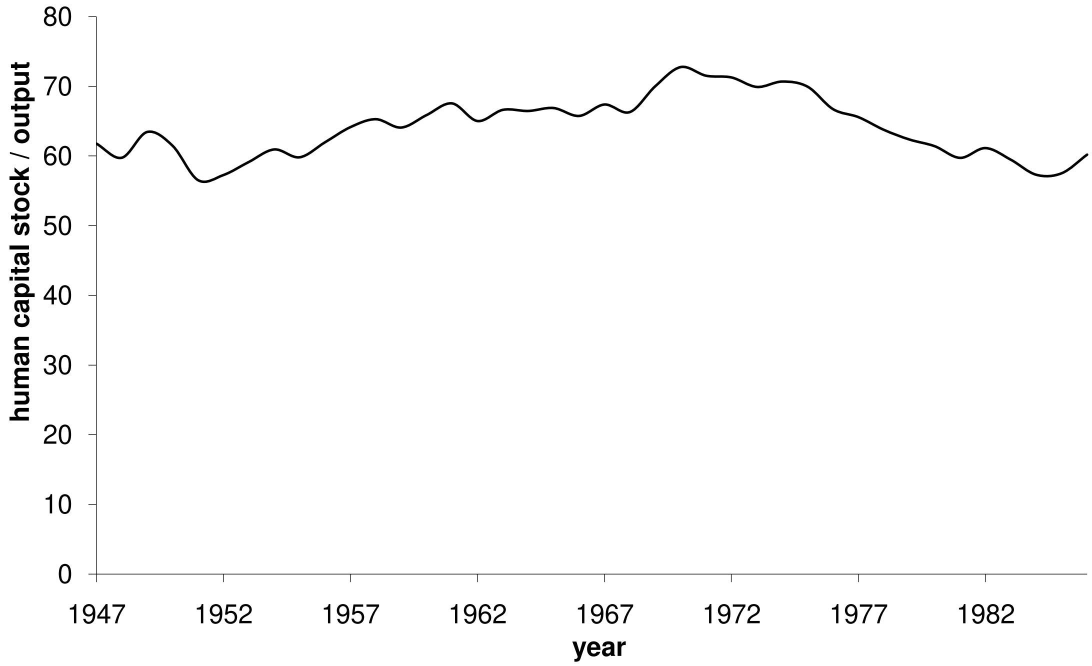
Source: Turner, Tamura, Mulholland and Baier (2007) and author's calculations.

[40] The lifetime labor income-based approach yields much higher estimates of the human capital stock than do studies which employ a cost-based approach, such as Kendrick (1976). Cassou and Lansing (2003) point to Davies and Whalley (1989) for a comprehensive overview of previous attempts to estimate the stock of human capital.

[41] Gross domestic estimates in current dollars were obtained from the Bureau of Economic Analysis (2007).

[42] $$\frac{\hat{h}_t}{\hat{k}_t} = \frac{\frac{\hat{h}_t}{\hat{y}_t}}{\frac{\hat{k}_t}{\hat{y}_t}} \cong \frac{64.11}{3.1424} \cong 20.4$$

FIGURE VI.4 – HUMAN CAPITAL AS A PROPORTION OF OUTPUT IN THE U.S. ECONOMY, 1947-1986



Source: Jorgenson and Fraumeni (2007), Bureau of Economic Analysis (2007), and author's calculations.

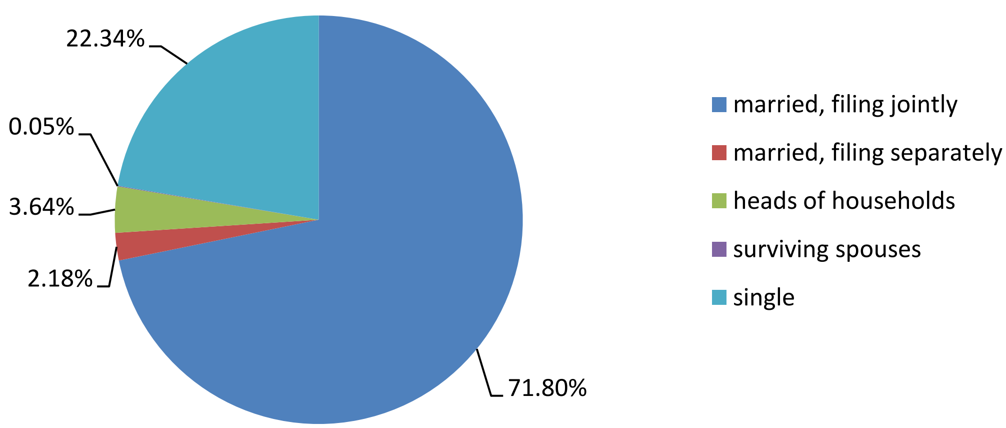
II. Tax Code Parameters

a.) Representative Household for the Individual Income Tax

To derive a typical marginal tax rate schedule for the individual income tax, whose parameters we later estimate, we first construct a representative household based on U.S. demographic and tax revenue data.

We assume that the representative household consists of two married parents who file joint tax returns. According to the Internal Revenue Service, during the 2005 tax year, taxpayers who filed jointly as married couples paid over $671 billion in income taxes, which amounts to 71.8 percent of the total $935 billion for all taxpayers (IRS, Pub. 1304). For a proportional breakdown of individual income taxes paid according to filing status, see *Figure IV.5*.

FIGURE VI.5 – PROPORTIONAL BREAKDOWN OF INDIVIDUAL TAXES PAID BY FILING STATUS



Source: Internal Revenue Service, *Publication 1304 (Tax Year 2005*) and author's calculations.

Based on U.S. women's fertility data, the representative household is assumed to have only one dependent child. This estimate is obtained by taking the average number of children ever had by women between the ages of 15 and 44, as given by the Current Population Survey (CPS, 2005).[43]

b.) Level ($\bar{\tau}_p$) and Slope (n) of the Tax Code for the Individual Income Tax

Figure IV.6 shows a typical family's marginal and average tax rate schedule during the 2005 tax year. It only considers effective tax rates that originate from the federal individual income tax, and does not consider other redistribution programs. For marginal tax rate schedules that take into account liability and benefits created by, for instance, the Earned Income Tax Credit (EITC), child and education tax credits, or FICA taxes,[44] see Hassett (2005), or Hassett and Moore (2005).

Figure IV.7 plots the average tax rate schedule against the income ratio, calculated by dividing a representative household's taxable income[45] by the mean 2005 taxable income of $67,595.[46] Furthermore, the regression line in *Figure IV.7* indicates that the level and slope of the pre-reform tax code is $\bar{\tau}_p = 0.105$, and n = 0.646,

---

[43] According to the Current Population Survey's report *Fertility of American Women - June 2004* (CPS, 2005), 17.2 percent of women between the ages of 15 and 44 had one child, 21.9 percent had two children, 10.8 had three, and 44.6 percent did not have any. 3.6 percent of women in this age group had four children, 1.5 percent had five or six, and 0.3 percent had seven or more. We calculate the approximate number of children a typical woman has by taking a weighted average as follows:

average number of children per woman = (44.6%) (0) + (17.2%) (1) + (21.9%) (2) + (10.8%) (3) +
+ (3.6%) (4) + (1.5%) (5.5) + (0.3%) (7) = 1.1815, which we round off to 1 child per woman

[44] Federal Insurance Contributions Act (FICA) taxes, also known as payroll taxes, are imposed by the federal government on both employees and employers, and finance Social Security and Medicare.

[45] Because of the standard deduction ($10,000 for a married couple filing jointly), personal exemptions ($3,200 per person) and the deduction for dependents ($800 per dependent), the first 20,400 dollars of family income are tax-free.

[46] During the 2005 tax year, there were 52,505,729 tax returns filed jointly by married couples, which accounted for $3,549,102,642,000 in taxable income. We obtain the mean taxable income by dividing the latter amount by the number of tax returns filed.

respectively. To obtain these estimates, we take logarithms of the following relationship between the income ratio and the corresponding average individual tax rate, both indexed by taxable income level:

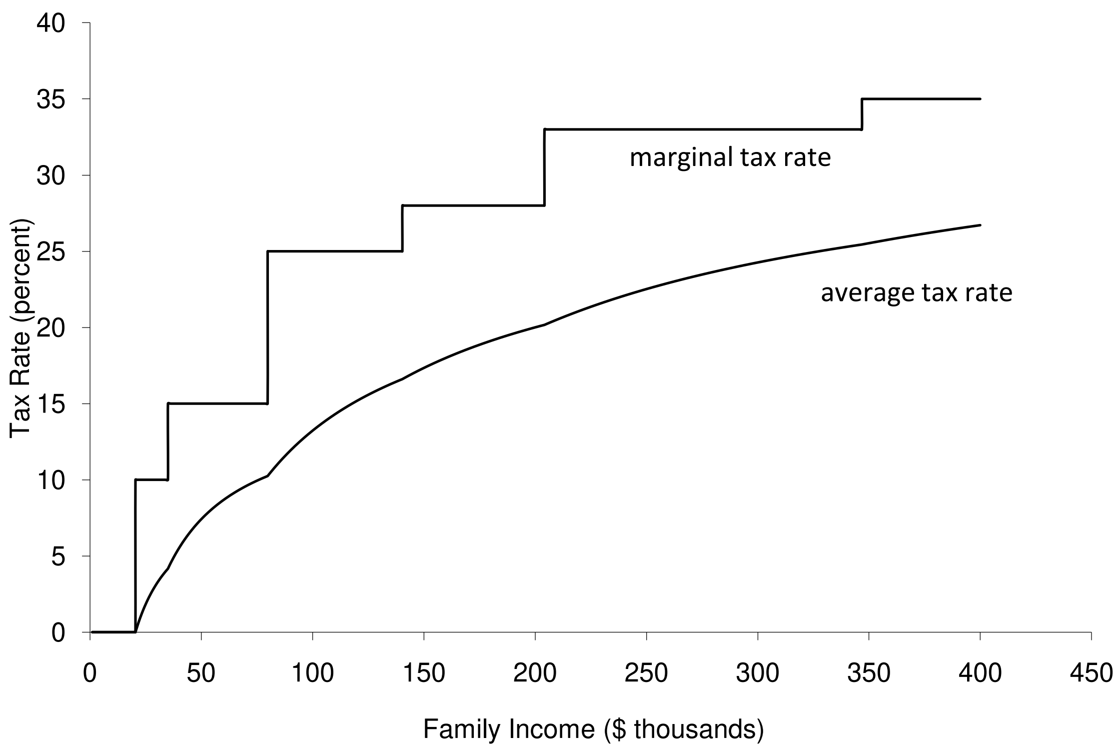
$$Average\ Individual\ Tax\ Rate_i = \bar{\tau}_p \left(Income\ Ratio_i\right)^n$$

$$\ln\left(Average\ Individual\ Tax\ Rate_i\right) = \ln\left[\bar{\tau}_p \left(Income\ Ratio_i\right)^n\right]$$

$$\ln\left(Average\ \ Individual\ \ Tax\ Rate_i\right) = \ln \bar{\tau}_p + n\ \ln\left(Income\ \ Ratio_i\right)$$

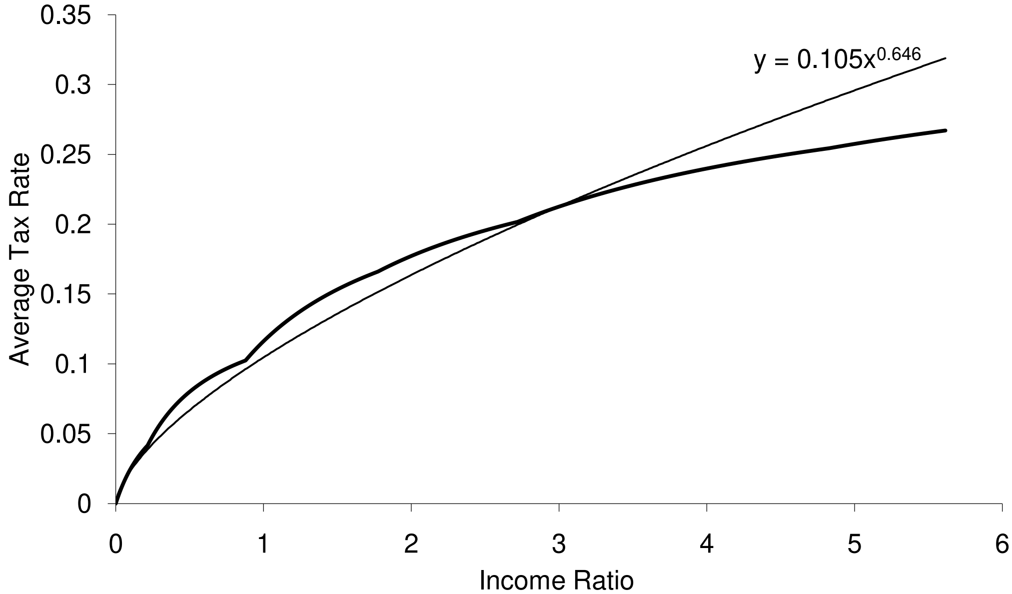
To obtain estimates of the tax code parameters, we run a simple ordinary least squares (OLS) log-log regression, as specified by the expression above.

FIGURE VI.6 – MARGINAL AND AVERAGE TAX RATE SCHEDULE FOR A REPRESENTATIVE HOUSEHOLD, 2005



Source: Author's calculations.

FIGURE VI.7 –AVERAGE TAX RATE SCHEDULE FOR A REPRESENTATIVE HOUSEHOLD, 2005



Source: Author's calculations.

c.) Business Tax Rate ($\bar{\tau}_b$)

The corporation tax, described in *Section C* of *Chapter II*, has a graduated marginal tax schedule. Most corporate income, however, is taxed at a 35 percent rate. We therefore use the statutory 35 percent as the business tax rate for calibration purposes.

d.) Double Taxation of Dividends (η)

Since the passage of the *Jobs and Growth Tax Relief Reconciliation Act (JGTRRA)* in 2003, dividends are no longer taxed at the individual's marginal income tax rate, but rather are subject to the capital gains rate schedule with the highest applicable rate of 15 percent. Since the mean taxable income is $67,595,[47] and according to *Figure VI.6* would be taxed at the 15 percent marginal rate, we can, for simplicity, assume that there is a pre-JGTRRA double taxation of dividends: $\eta = 1$.

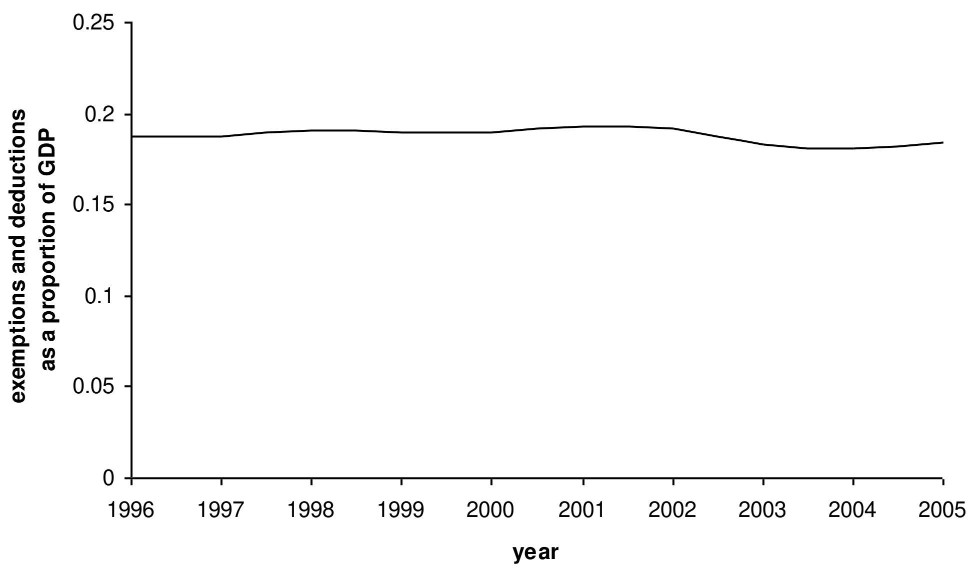
[47] See footnote 46.

e.) Exemptions and Deductions as a Proportion of GDP (α)

We use Internal Revenue Service (IRS) tax return data for the 1996-2005 time period to calibrate α, a long-term measure of the total amount of exemptions and deductions taken by the average taxpayer in proportion to GDP. For each year, we first subtract the amount of taxable income (*TaxInc*) from the total adjusted gross income (*AGI*) to derive an implied total amount of exemptions and deductions.[48] We then divide this number by the nominal gross domestic product, as given by the Bureau of Economic Analysis, to obtain an annual proportion. These have been fairly constant over time, as can be seen in *Figure IV.8*. Finally, α is the arithmetic mean of the annual proportions:

$$\alpha = \frac{1}{(2005-1996)+1}\sum_{t=1996}^{2005}\frac{(AGI_t - TaxInc_t)}{GDP_t^{nom}} = 0.188$$

FIGURE VI.8 – EXEMPTIONS AND DEDUCTIONS AS A PROPORTION OF THE U.S. GDP, 1996-2005



Source: Internal Revenue Service (2007), Bureau of Economic Analysis (2007) and author's calculations.

[48] This number differs from the total amount of exemptions and deductions taken as given by the IRS. In some cases, total exemptions and deductions can exceed gross adjusted income, and therefore cannot be fully applied in the calculation of taxable income.

f.) Expensing of Physical and Human Capital Investment: ϕ_k and ϕ_h

To calibrate ϕ_k, the proportion of physical capital investment that firms can expense, we employ a calibration strategy suggested by Cassou and Lansing (2003). In particular, we choose ϕ_k such that the amount of expensed investment, given by $\phi_k i_{kt}$, equals $\hat{\delta} k_t$, a measure of total capital depreciation:

$$\phi_k i_{kt} = \hat{\delta} k_t$$

$$\phi_k = \frac{\hat{\delta}}{\frac{i_{kt}}{k_t}}$$

For simplicity, to obtain a value for $\hat{\delta}$, we use a standard linear law of motion:

$$k_{t+1} = i_{kt} + \left(1 - \hat{\delta}\right) k_t$$

Let us now divide both sides by k_t :

$$\frac{k_{t+1}}{k_t} = \frac{i_{kt}}{k_t} + 1 - \hat{\delta}$$

In the balanced growth path, physical capital stock and aggregate output grow at the same rate $\mu = \ln \frac{y_{t+1}}{y_t} = \ln \frac{k_{t+1}}{k_t}$. We can therefore express $\frac{k_{t+1}}{k_t}$ as e^{μ}, and continue to isolate $\hat{\delta}$:

$$e^{\mu} = \frac{i_{kt}}{k_t} + 1 - \hat{\delta}$$

$$\hat{\delta} = 1 + \frac{i_{kt}}{k_t} - e^{\mu}$$

We use our previous estimates of $\frac{\hat{i}_{kt}}{\hat{y}_t}$ and $\frac{\hat{k}_t}{\hat{y}_t}$ to estimate $\frac{i_{kt}}{k_t}$ and $\hat{\delta}$:

$$\frac{i_{kt}}{k_t} = \frac{\frac{\hat{i}_{kt}}{\hat{y}_t}}{\frac{\hat{k}_t}{\hat{y}_t}} = \frac{0.2453}{3.1424} = 0.07806$$

$$\hat{\delta} = 1 + \frac{i_{kt}}{k_t} - e^{\mu} = 1 + 0.07806 - e^{0.0186} = 0.0593$$

Given these estimates, we can now calibrate ϕ_k :

$$\phi_k = \frac{\hat{\delta}}{\frac{i_{kt}}{k_t}} = \frac{0.0593}{0.07806} = 0.7597$$

According to the *National Patterns of R&D Resources* (NSF, 2007), industry expenditures on research and development averaged 1.78 percent of the gross domestic product during 1959-2005. Privately-funded R&D investment is largely tax-deductible, while private education expenditures are not. We can use our estimate of $\frac{\hat{i}_{ht}}{\hat{y}_t}$ from the previous section to approximate ϕ_h, the proportion of human capital investment that can be expensed:

$$\phi_h = \frac{0.0178}{\frac{i_{ht}}{\hat{y}_t}} = \frac{0.0178}{0.0384} = 0.4626$$

g.) Calibration Summary

Tables VI.1 and *VI.2* summarize the characteristics of the U.S. economy, as estimated earlier in this chapter, and the parameters of the tax code, respectively. We use the calibration procedure specified in *Section E* of *Chapter V* to calculate the endogenous parameters of the model for the three different values of the intertemporal elasticity of substitution (IES) in labor supply σ – 0.25 (low), 0.5 (intermediate) and 1 (high). *Table VI.3* summarizes the results.

The middle column, which contains endogenous parameters for the intermediate IES value, is highlighted in bold, as we will use these estimates to calculate the balanced growth path characteristics and the transition dynamics, associated with replacing the graduated income tax with a revenue-neutral flat tax. In *Appendix B*, we perform a sensitivity analysis in which we recalculate the results for the low and high values of IES.

TABLE VI.1 – CALIBRATION: CHARACTERISTICS OF THE U.S. ECONOMY

Characteristics of the U.S. Economy		
discount factor	β	0.9615
long-term growth rate	$\hat{\mu}$	0.0186
labor share	$1-\theta$	0.649
capital share	θ	0.351
time in market work	$1-\hat{l}_t-\hat{e}_t$	0.2111
time in education	$\hat{e}_t$	0.09583
leisure time	$\hat{l}_t$	0.69306
investment in physical capital	$\frac{\hat{i}_{kt}}{\hat{y}_t}$	0.2453
investment in human capital	$\frac{\hat{i}_{ht}}{\hat{y}_t}$	0.0384
physical capital stock	$\frac{\hat{k}_t}{\hat{y}_t}$	3.1424
human-to-physical capital ratio	$\frac{\hat{h}_t}{\hat{k}_t}$	20.4

TABLE VI.2 – CALIBRATION: TAX CODE PARAMETERS

Tax Code Parameters		
level of tax code for the individual income tax	$\bar{\tau}_p$	0.105
slope of marginal tax rate for the individual income tax	n	0.646
business tax rate	$\bar{\tau}_b$	0.35
double taxation of dividends	η	1
exemptions and deductions as a proportion of GDP	α	0.188
expensing of physical capital investment	ϕ_k	0.7597
expensing of human capital investment	ϕ_h	0.4626

TABLE VI.3 – CALIBRATION: ENDOGENOUS PARAMETERS

intertemporal elasticity of substitution in labor supply (σ)	0.25 (low)	**0.5 (medium)**	1 (high)
γ	5	**3**	2
z	0.123366	**0.123366**	0.123366
A_k	1.764785	**1.764785**	1.764785
A_h	1.115687	**1.159327**	1.339508
p_1	0.28124	**0.358975**	0.531785
p_2	0.453955	**0.453955**	0.453955
δ_k	0.21544	**0.21544**	0.21544
δ_h	0.003448	**0.004904**	0.010386
v	0.027837	**0.039591**	0.083846
B	0.893989	**0.140365**	0.064623
ψ	0.10984	**0.10984**	0.10984

B. Flat Tax: Balanced Growth Path

a.) Parameters of a Hall-Rabushka Revenue-Neutral Flat Tax

The most salient feature of the Hall-Rabushka (1995) flat tax proposal is its introduction of a single marginal tax rate $\bar{\tau}$ on both labor and business income: $\bar{\tau} = \bar{\tau}_p = \bar{\tau}_b$. The marginal tax schedule is not graduated: $n = 0$.

We assume that the tax reform is revenue-neutral: The expected tax revenue at the time of reform constitutes the same proportion of output as the pre-reform tax receipts did. Immediately after the implementation of the flat tax, parameter ψ remains constant at its pre-reform value $\psi = 0.10984$. Using equation (28), we calculate that, to achieve revenue neutrality, the single marginal tax rate $\bar{\tau} = \bar{\tau}_p = \bar{\tau}_b$ must equal 21.909 percent.

Business income is taxed only once, and is not subject to either the individual income tax or the capital gains tax. As a consequence, the Hall-Rabushka proposal eliminates the double taxation of dividends: $\eta = 0$. All physical investment expenditures are written off during the first year on the individual level. Investment in physical capital, in other words, is fully expensed: $\phi_k = 1$. No changes, however, are made to the expensing of human capital: $\phi_h = 0.4626$. Finally, we assume that personal deductions and exemptions make up the same proportion of aggregate output as they did before the reform: $\alpha = 0.188$.

Table VI.4 summarizes the above tax code parameters for the revenue-neutral Hall-Rabushka flat tax reform.

TABLE VI.4 – FLAT TAX REFORM: TAX CODE PARAMETERS

Tax Code Parameters for the Flat Tax		
level of tax code for the individual income tax = single marginal tax rate	$\bar{\tau} = \bar{\tau}_p$	0.21909
slope of marginal tax rate for the individual income tax	n	0
business tax rate = single marginal tax rate	$\bar{\tau} = \bar{\tau}_b$	0.21909
double taxation of dividends	η	0
exemptions and deductions as a proportion of GDP	α	0.188
expensing of physical capital investment	ϕ_k	1
expensing of human capital investment	ϕ_h	0.4626

b.) Characteristics of the Balanced Growth Path (Intermediate IES)

Table IV.5 summarizes the characteristics of the post-reform balanced growth path for an intermediate value of the intertemporal elasticity of substitution (IES) in labor supply (σ = 0.5). For the balanced growth path characteristics of a flat tax model with a low and high IES (σ = 0.25 and σ = 1, respectively), see the sensitivity analysis in *Appendix B*.

After the flat tax is introduced, the long-term growth rate increases to 1.911 percent. The representative household spends 21.6 percent of its time endowment engaged in market work, 9.8 percent in education, and the remaining 68.6 percent in leisure activities. In the balanced growth path, consumption makes up 56.2 percent of the gross domestic product. Investment in physical capital comprises 29.6 percent of GDP, whereas human capital investment accounts for 3.2 percent. Consequently, in the long run, the ratio of physical capital stock to aggregate output tends to 3.783, and the human-to-physical capital stock ratio approaches 15.004.

Compared to the pre-reform balanced growth path with a graduated-rate federal income tax, calibrated in *Section A* of this chapter, the long-term growth rate rises slightly from 1.86 to 1.911 percent – a difference of 0.05 percentage points. We can therefore conclude that the introduction of a revenue-neutral Hall-Rabushka flat tax has, in the long run, a mildly positive effect on the rate of economic growth. This is consistent with some previous studies, such as Lucas (1990), Stokey and Rebelo (1995) and the original Cassou and Lansing (2003), that indicated that switching to a flat tax would lead to only a slight increase in the economic growth rate. These results are less in line with studies that suggest a quantitatively large influence of tax policies on economic growth, such as King and Rebelo (1990), or Jones, Manuelli and Rossi (1993).

After the flat tax is implemented, furthermore, the representative household spends slightly more time in market work and education, with a corresponding decrease in the proportion of time spent in leisure activities. As a proportion of GDP, the reform increases investment in physical capital, but reduces human capital investment. Accordingly, in the new balanced growth path, the ratio of physical capital stock to output increases, while the ratio of human-to-physical capital stock falls.

To explain how these changes in balanced growth path values occurred, we consider how optimal decision rules react to changes in tax code parameters.[49] Compared to the pre-reform situation, the flat tax reform – by introducing a single marginal tax rate of 21.909% ($\bar{\tau} = \bar{\tau}_p = \bar{\tau}_b =$ 0.21909; n = 0) - decreased the business tax rate (from $\bar{\tau}_b$ = 0.35), increased the base level of personal tax rates (from $\bar{\tau}_p$ = 0.105), and completely flattened the personal tax rate schedule (from $n =$ 0.646).

In the case of the level of physical capital investment, as well as time spent in market work and education, the positive effects of lower business tax rates and of the rate schedule flattening outweigh the negative effect of higher base rate for the personal tax rate. For human capital investment, the positive effect of switching to a flat tax schedule is outweighed by the negative effects of the decrease in the business tax rate and of the increase in the personal tax base rate.

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[49] See *Section C: Optimal Decision Rules* in *Chapter V*.

TABLE VI.5 – FLAT TAX REFORM: BALANCED GROWTH PATH CHARACTERISTICS (INTERMEDIATE IES)

Characteristics of the Balanced Growth Path After the Flat Tax Reform (intermediate IES = 0.5)			**Comparison**	
			Before the reform	**Difference**
long-term growth rate	μ	**0.01911**	0.0186	+ 0.00051
time in market work	$1-l_t-e_t$	**0.216**	0.2111	+ 0.0049
time in education	e_t	**0.098**	0.09583	+ 0.002
leisure time	l_t	**0.686**	0.69306	- 0.007
consumption / GDP	$\frac{c_t}{y_t}$	**0.562**	0.606	- 0.044
investment in physical capital	$\frac{i_{kt}}{y_t}$	**0.296**	0.2453	+ 0.0507
investment in human capital	$\frac{i_{ht}}{y_t}$	**0.032**	0.0384	- 0.006
physical capital stock	$\frac{k_t}{y_t}$	**3.783**	3.1424	+ 0.641
human-to-physical capital ratio	$\frac{h_t}{k_t}$	**15.004**	20.4	- 5.396

C. Flat Tax: Transition Dynamics

After the introduction of a revenue-neutral Hall-Rabushka flat tax, the model economy takes some time to settle into the new balanced growth rate. *Figures VI.9* to *VI.27* depict the transition dynamics of variables such as the economic growth rate, time in market work and education, leisure time, utility, and others. Some of the diagrams depict long-term time series of 100 years, while others focus on the short run and only display the variable dynamics for ten years following the tax reform.

Figures VI.9 and *VI.10* show the long- and short-term transition dynamics of the expected economic growth rate, respectively. With an intermediate IES, the model economy's growth slows down slightly during the first post-reform year: The growth rate falls from the pre-reform value of 1.86% to 1.821%.[50] The next year, however, growth rebounds to an impressive 3.219%. In the years that follow, the growth rate gradually decreases until it reaches, after approximately 55 years, the new balanced growth path value of 1.911 percent.

To understand what drives the changes in economic growth rates is, we examine the production function (5):

$$y_t = zk_t^{\theta}\left[h_t\left(1-l_t-e_t\right)\right]^{1-\theta}$$

The stock of physical capital k_t grows faster after the reform is implemented (since $\frac{i_{kt}}{y_t}$ rises), while the accumulation of human capital h_t slows down somewhat, as $\frac{i_{ht}}{y_t}$ decreases. The changing rates of physical and human capital accumulation, then,

[50] We assume that the economy is on a balanced growth path before the flat tax is implemented.

appear to have countervailing effects on the rate of economic growth. An examination of the transition dynamics for the proportion of time the representative household spends in market work $(1 - l_t - e_t)$ suggests,[51] however, that time allocation decisions may, in fact, be the driving force behind the changes in the economic growth rate. Like the growth rate, the proportion of time spent in market work at first falls fairly significantly, before it rebounds, and then gradually drops off to its new balanced growth path level, which is still higher than the pre-reform value.

We see, therefore, that the introduction of a flat tax is likely to accelerate economic growth in the long run. The long-term benefits, however, come at the cost of a temporary slowdown or even recession during the first post-reform year – a politically unpalatable consequence that is likely to make the policy a less attractive option to politicians. During the second year, however, the economic growth rate rebounds to a high level, and remains relatively high, albeit decreasing, afterwards. The high economic growth rates after the first year are likely to be popular with voters.

These results suggest that the timing of the flat tax reform may have important political consequences for incumbents. Due to the risk of an economic slowdown in the very short term, an incumbent would be unlikely to introduce a flat tax one year before the election, fearing that the low economic growth during the first post-reform year could lead to his electoral defeat. The incumbent, however, might find it politically expedient to introduce the reform two or more years before the election.

Lower economic growth could make a politician's re-election less likely for a variety of intuitively plausible reasons. Most obviously, lower economic growth is

[51] See *Figures VI.14* and *VI.15*, below.

associated with greater unemployment, and with relatively sluggish increases in living standards. Dissatisfied voters may, as a result, decide to vote for the incumbent's opponent. Alternatively, voters may consider economic growth to be a good proxy for the incumbent's ability to govern – a valued trait that may be difficult to observe directly (Persson and Tabellini, 1990). For high economic growth rates, the reverse of the above considerations applies: Politicians may find it easier to win re-election as voters enjoy improved living standards, a lower unemployment rate, and if they perceive the incumbent to be a competent public servant.

Empirical evidence on the effect of economic conditions on the probability of re-election, however, is somewhat mixed. A recent study of voting behavior in a sample of 74 countries over the 1960-2003 time period by Brender and Drazen (2005) found that higher growth rates of GDP per capita raised the incumbent's probability of re-election only in less democratic countries and new democracies. Using regularly updated data from U.S. presidential elections, on the other hand, Fair (2006) estimates that higher economic growth has consistently had a positive effect on the likelihood that the incumbent will get re-elected.

One should note, also, that the model does not take into account business cycles. It may well be that a robust, business cycle-related economic expansion could outweigh, or at least mitigate, the effects of a tax reform-induced slowdown, and thus make the post-implementation transition period less problematic.

FIGURE VI.9 – FLAT TAX REFORM: TRANSITION DYNAMICS – ECONOMIC GROWTH RATE (INTERMEDIATE IES)

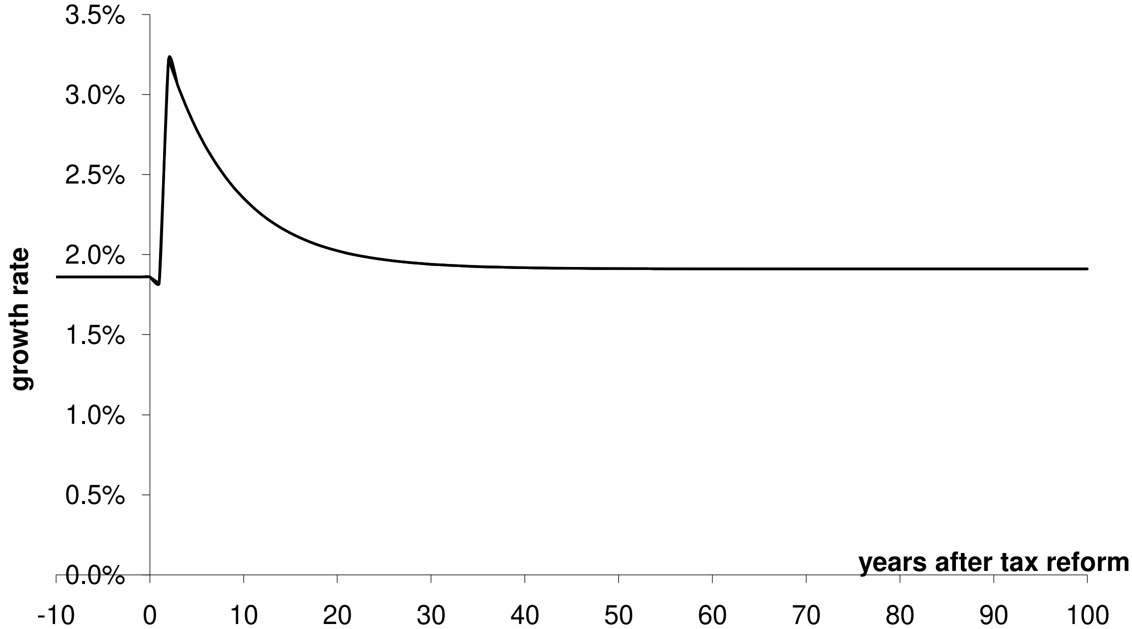


FIGURE VI.10 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – ECONOMIC GROWTH RATE (INTERMEDIATE IES)

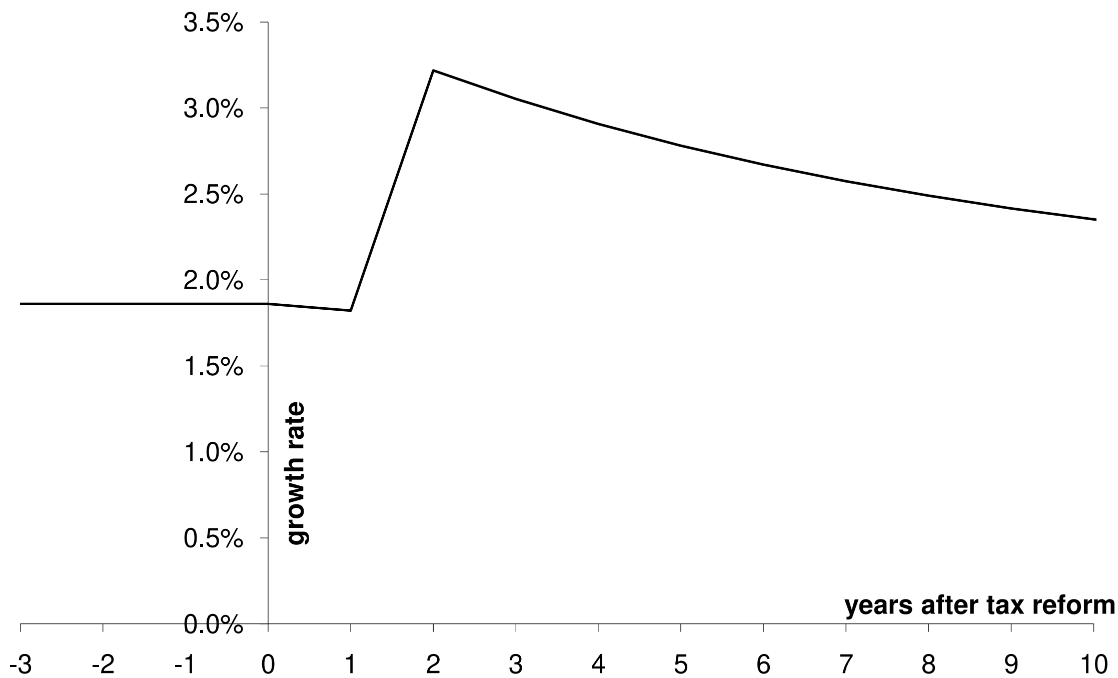


FIGURE VI.11 – FLAT TAX REFORM: TRANSITION DYNAMICS – INDEX OF AGGREGATE OUTPUT (INTERMEDIATE IES)

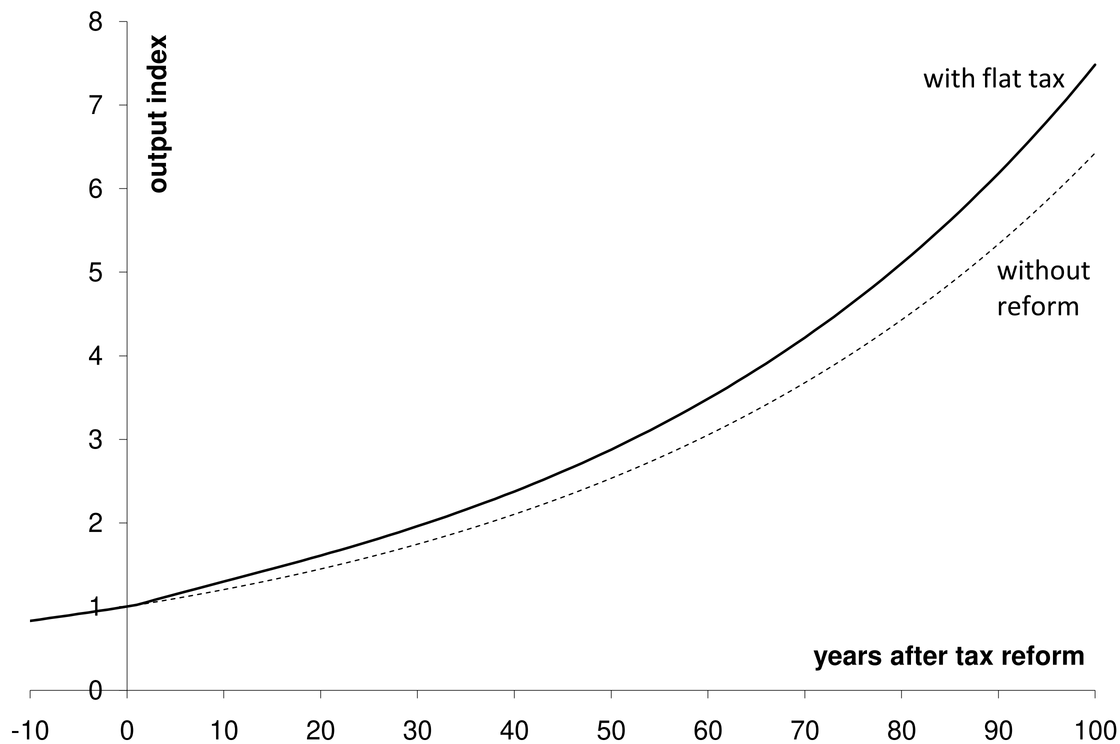


Figure VI.11 tracks the dynamic of the gross domestic product, or aggregate output, of the model economy. We index aggregate output to equal 1 at the time of reform. Values on the vertical axis therefore reflect how many times larger output is at any given time, given by the horizontal axis, than it was when the flat tax was introduced. By examining the solid line, which represents GDP dynamics after the introduction of the flat tax, we see that output doubles after approximately 31 years,[52] and that, 100 years after the tax reform, the economy is expected to be almost 7.5 times larger.[53] The dashed line represents how gross domestic product would have evolved in the absence of tax reform: output would have doubled after 37 years, and, 100 years after the moment the

[52] More specifically, the index of aggregate output equals 1.9989 during the 31st year after the flat tax was implemented, and 2.0379 during the 32nd year.
[53] In the 100th year after the tax reform, the aggregate output index equals 7.4829.

flat tax would otherwise have been adopted, the economy would be only 6.4 times larger. If the United States adopts the flat tax, then, a hundred years after the reform its aggregate output would be expected to be about 17 percent larger that it would have been in the absence of any tax reform.

Figures IV.12 and *IV.13* depict what proportion of its time the representative households spends in education, expressed as a value between 0 and 1. During the first year, this proportion drops from the initial value of 0.096583 down to about 0.094, and then it gradually rises until it reaches a new balanced growth value of almost 0.098.

The proportion of time spent in market work follows a similar trajectory, as can be seen in *Figures IV.14* and *IV.15*. The first year brings a decrease from 0.2111 to a little more than 0.207, and then it grows until it eventually stabilizes at 0.216.

Since both the proportion of time spent in education and in market work declines one year after the reform, and then rebounds to higher than initial values, the proportion of time the representative household can spend on leisure activities must increase at first, before falling to lower values. Indeed, *Figures IV.16* and *IV.17* show that the proportion of leisure time spikes in the first year at a value of 0.699, and that it then gradually falls to about 0.686 on the new balanced growth path.

Figures IV.18 and *IV.19* focus on the transition dynamics for the wage and interest rates. We see that the wage rate gradually rises from approximately 0.048 to 0.053, whereas the interest rate falls from 0.112 to 0.093. Both the wage and the interest rate assume their new values smoothly, without any significant departures during the first year after the tax reform.

FIGURE VI.12 – FLAT TAX REFORM: TRANSITION DYNAMICS – TIME IN EDUCATION (INTERMEDIATE IES)

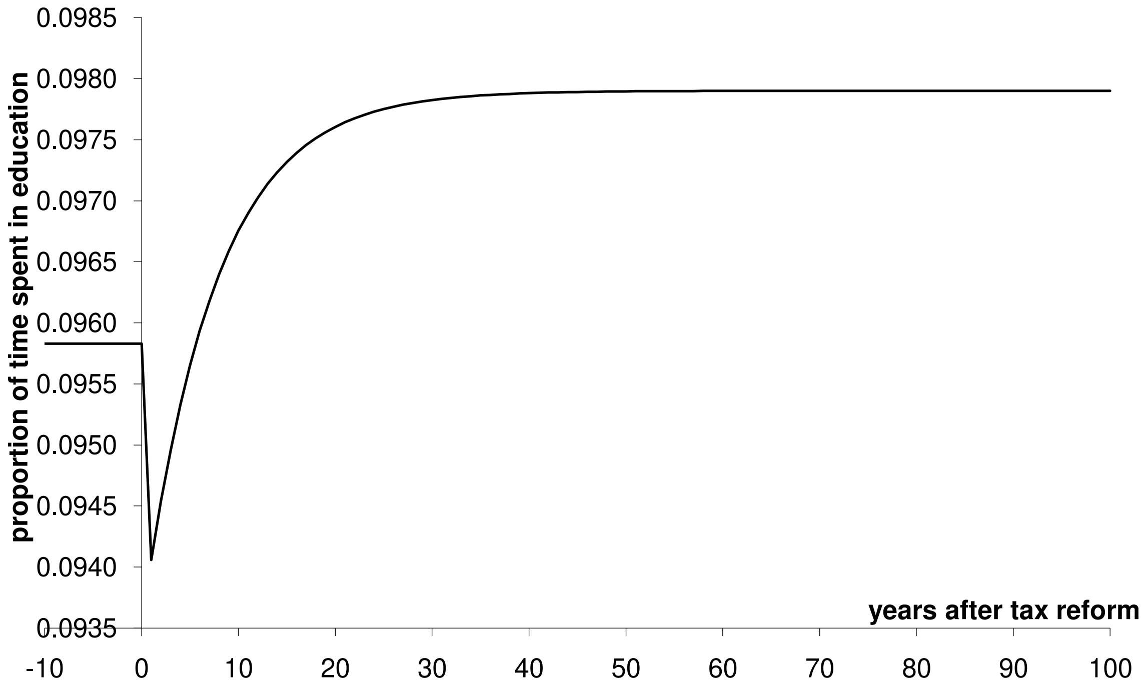


FIGURE VI.13 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – TIME IN EDUCATION (INTERMEDIATE IES)

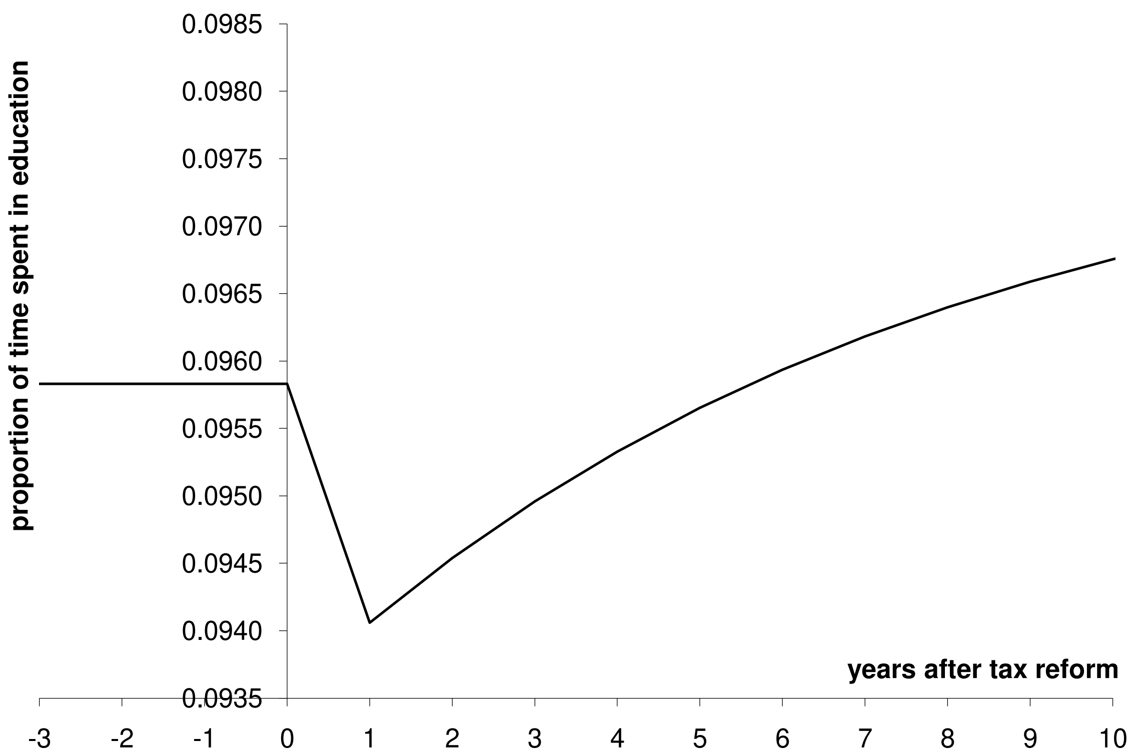


FIGURE VI.14 – FLAT TAX REFORM: TRANSITION DYNAMICS – TIME IN MARKET WORK (INTERMEDIATE IES)

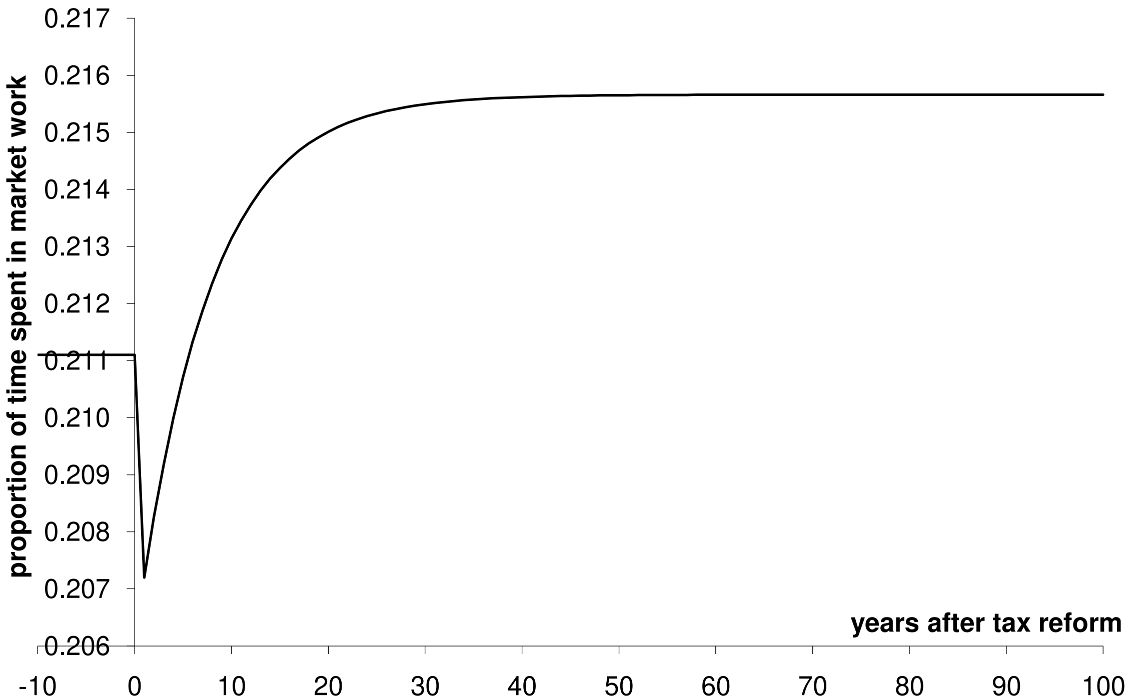


FIGURE VI.15 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – TIME IN MARKET WORK (INTERMEDIATE IES)

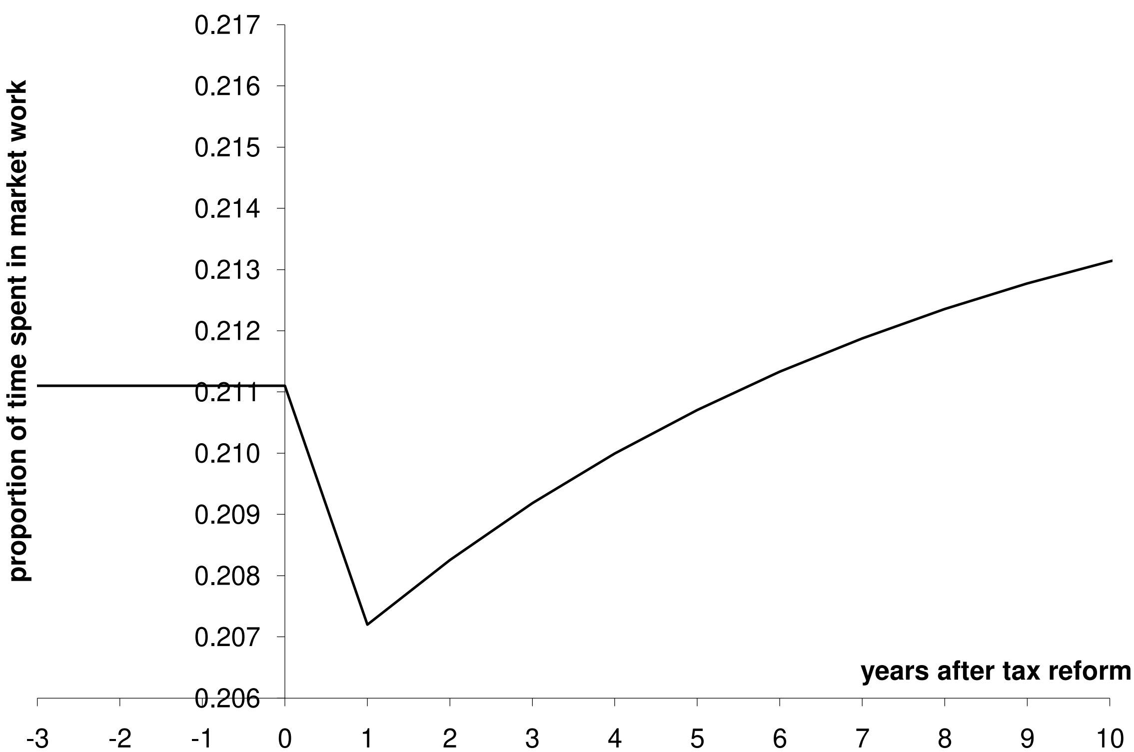


FIGURE VI.16 – FLAT TAX REFORM: TRANSITION DYNAMICS – LEISURE TIME (INTERMEDIATE IES)

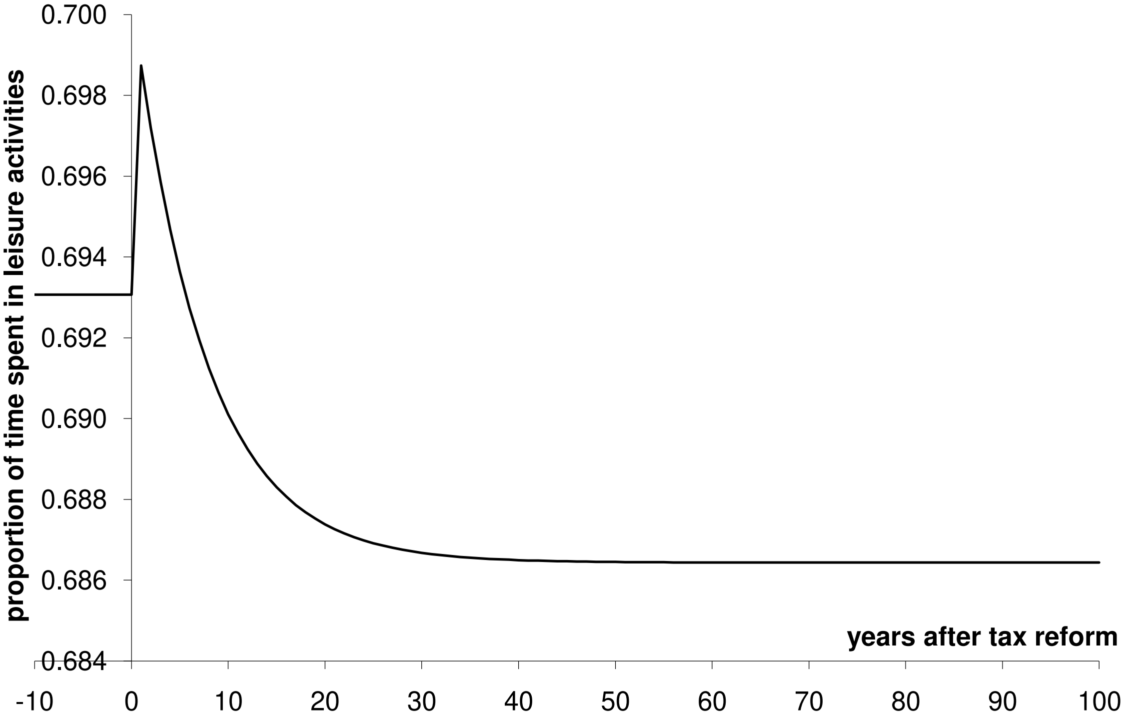


FIGURE VI.17 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – LEISURE TIME (INTERMEDIATE IES)

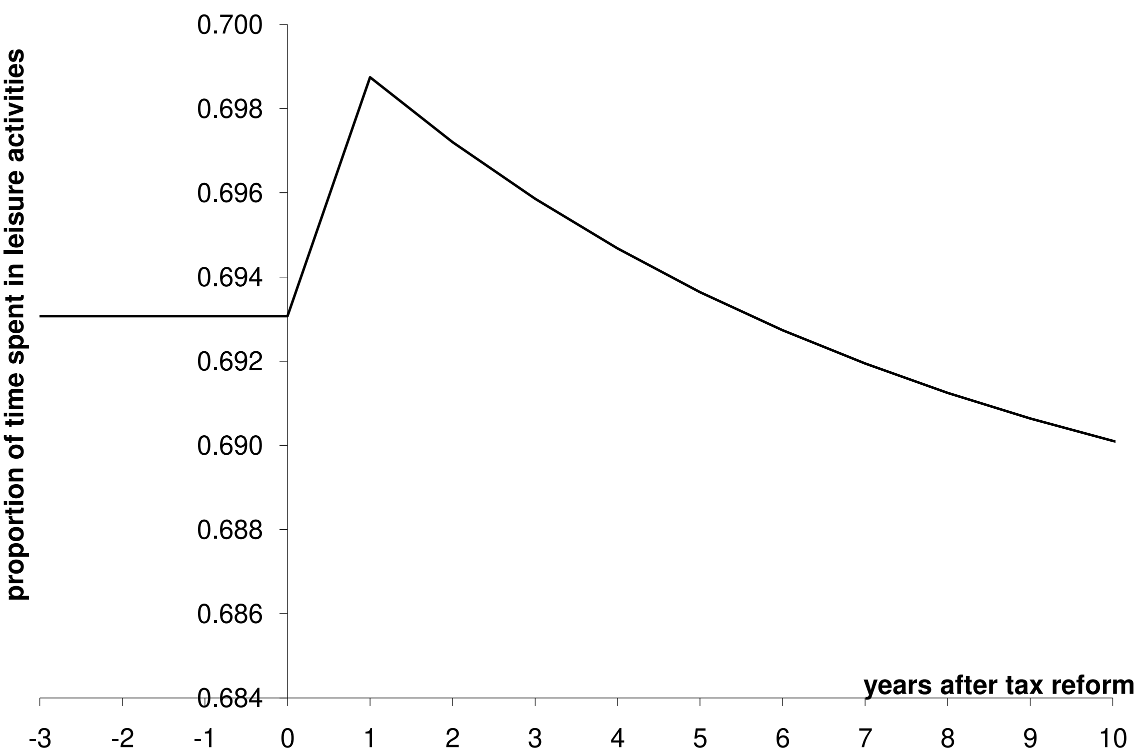
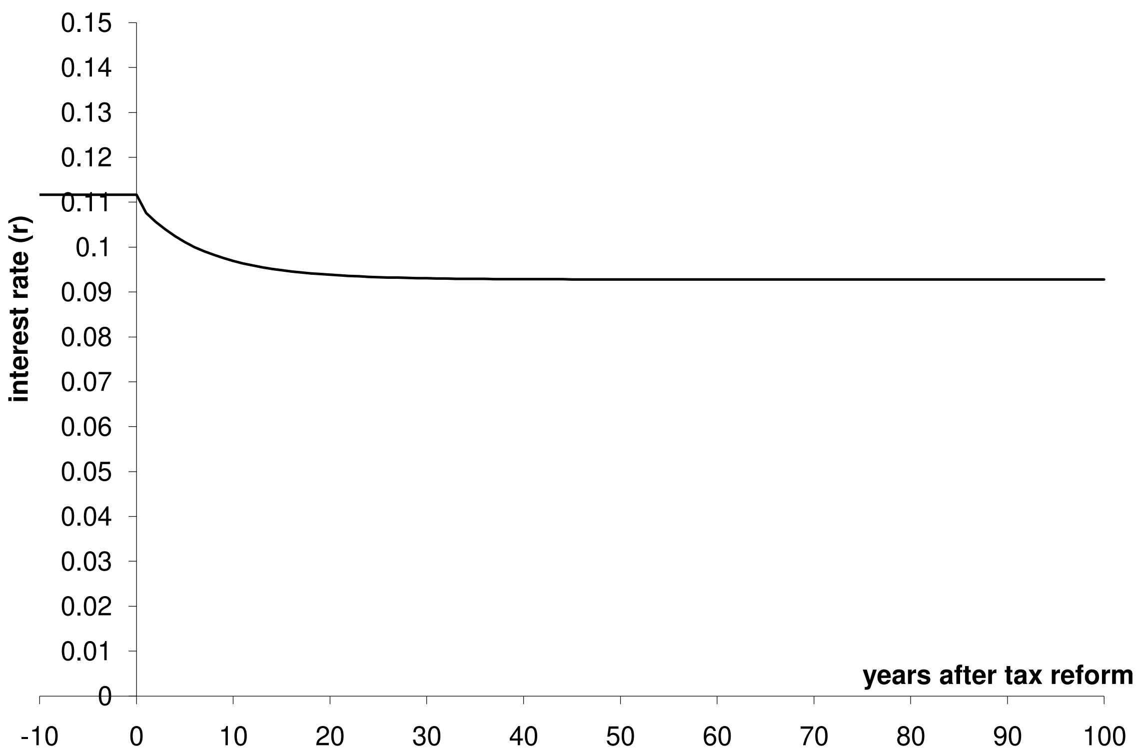


FIGURE VI.18 – FLAT TAX REFORM: TRANSITION DYNAMICS – WAGE RATE (INTERMEDIATE IES)

FIGURE VI.19 – FLAT TAX REFORM: TRANSITION DYNAMICS – INTEREST RATE (INTERMEDIATE IES)



Figures IV.20 and *IV.21* trace the representative household's utility, which – as can be seen from the utility function given by (1) – depends positively on consumption and leisure, and negatively on time spent in market work or education. In these diagrams and ones that follow, the solid line depicts the post-reform transition dynamics, while the dashed line shows transition dynamics in the absence of tax reform.

The diagrams measure utility in *utils*, a numerical unit whose values, however, represent ordinal, rather than cardinal, utility:[54] A household whose utility equals 8 utils, for instance, is clearly more satisfied than one with a utility of 4 utils, but is not necessarily twice as happy. Assuming that a household's preferences over consumption and leisure meet the assumptions of completeness, transitivity and continuity, any numerical utility ranking (U) can be transformed into another set of numbers by the function F(U), as long as it is order-preserving (Nicholson, 2005).[55]

Following the introduction of the flat tax, the representative household's utility falls, and does not reach its original level until between three or four years later. *Figures IV.22* and *IV.23* depict – on an annual basis - changes in the representative household's utility over time.

[54] In other words, utility values record the relative desirability of consumption-leisure bundles.
[55] Function F(x) is order preserving, for instance, if its first derivative is greater than zero for all values of x. (In other words, the function slopes upwards everywhere.)

FIGURE VI.20 – FLAT TAX REFORM: TRANSITION DYNAMICS – UTILITY (INTERMEDIATE IES)

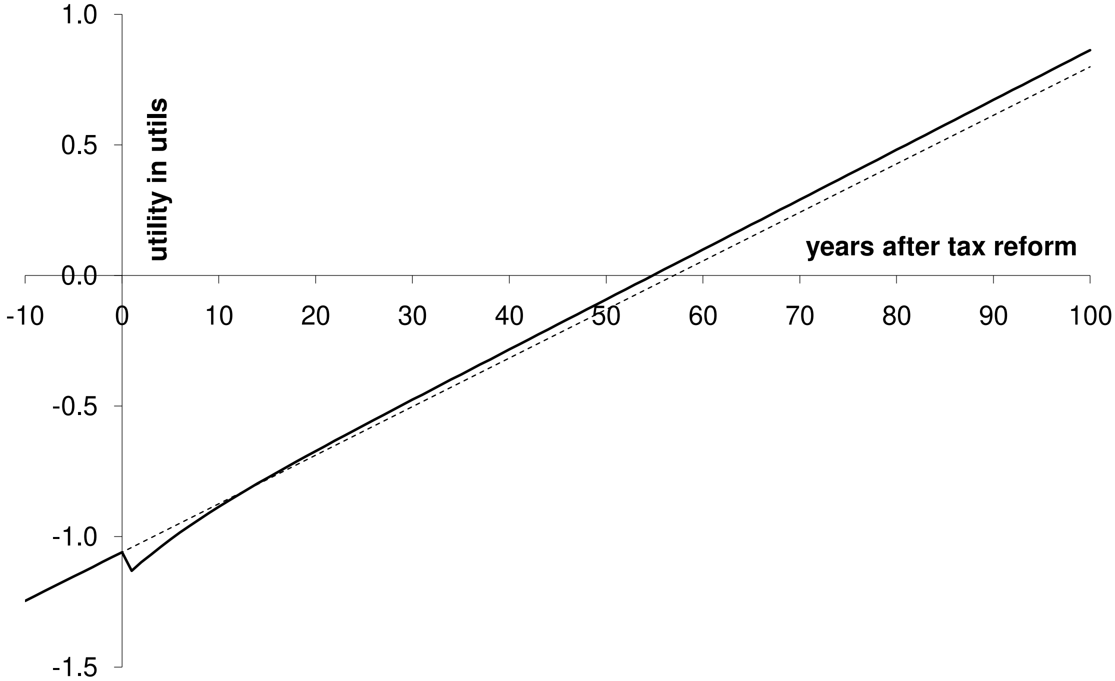


FIGURE VI.21 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – UTILITY (INTERMEDIATE IES)

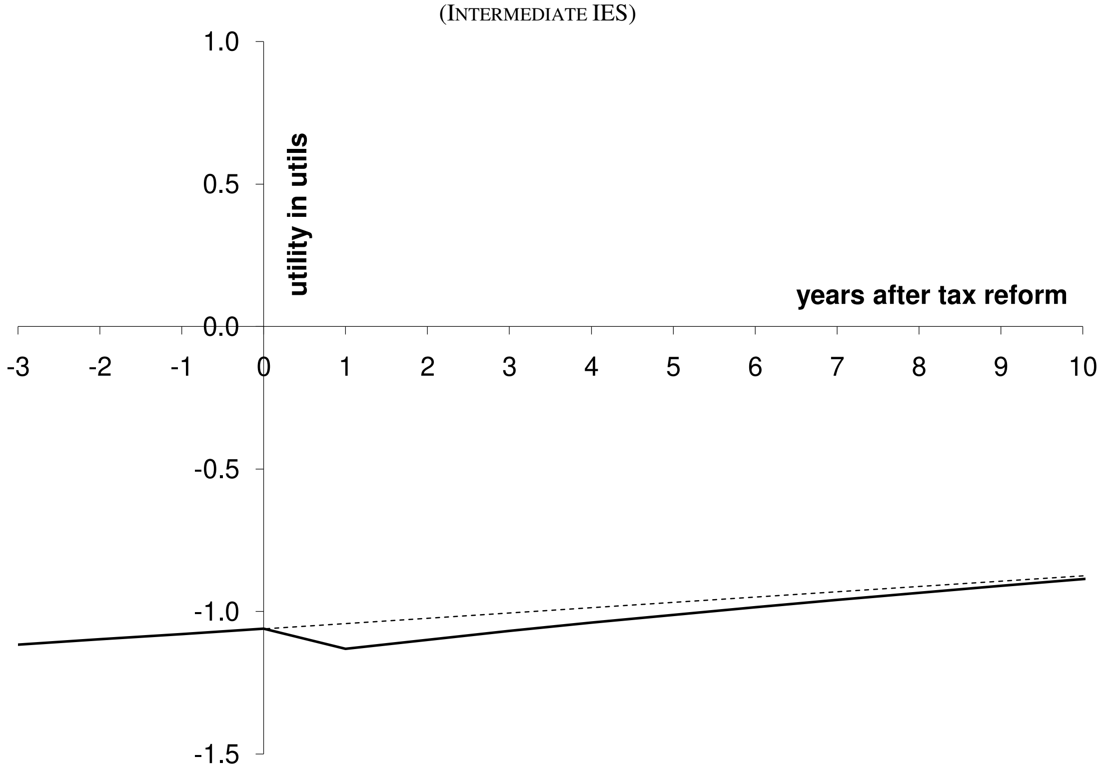


FIGURE VI.22 – FLAT TAX REFORM: TRANSITION DYNAMICS – UTILITY CHANGE (INTERMEDIATE IES)

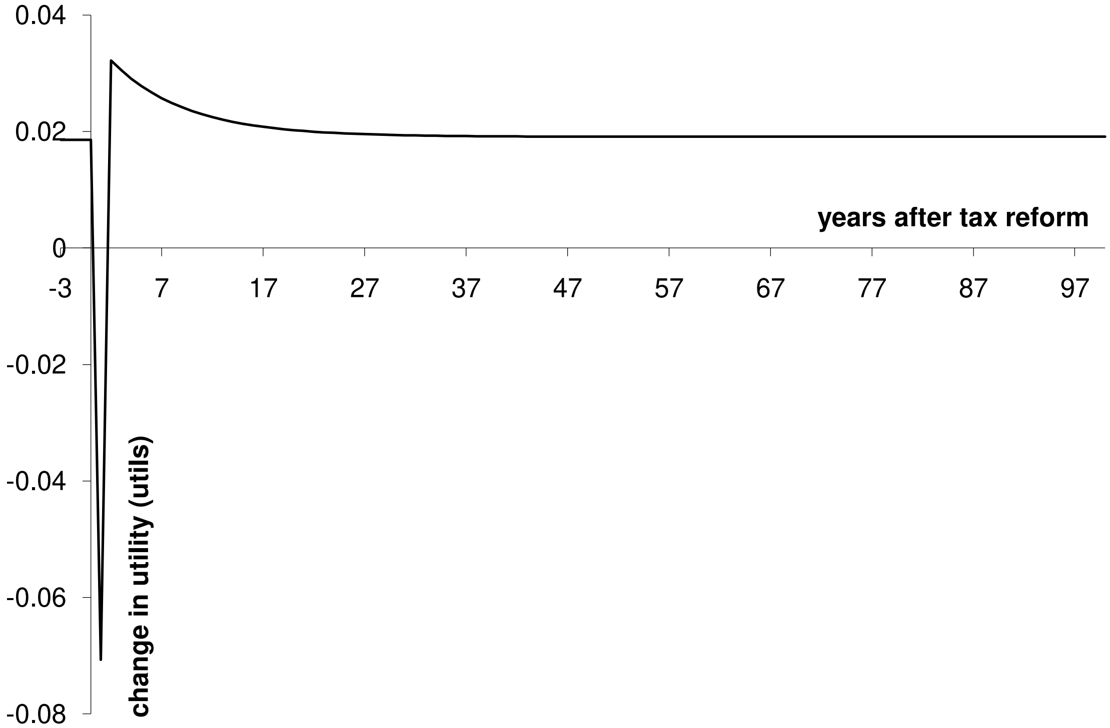
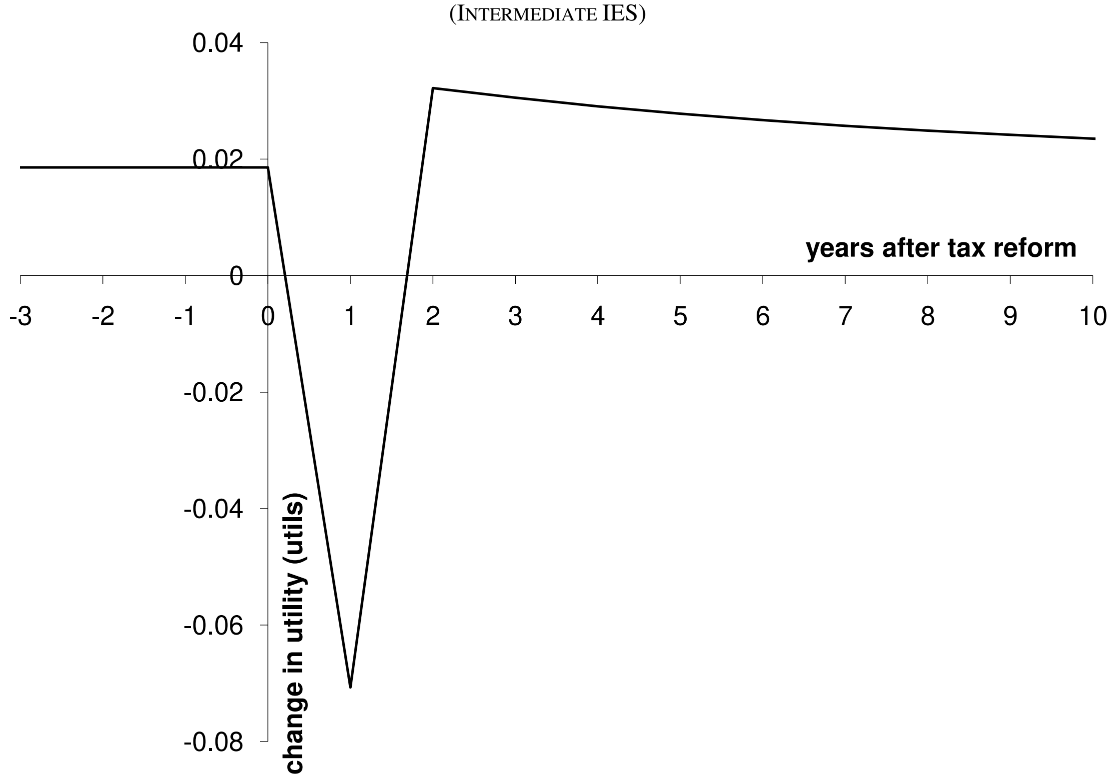


FIGURE VI.23 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – UTILITY CHANGE (INTERMEDIATE IES)



Since measures of welfare denominated in utils can only represent relative utility differences and do not have an absolute interpretation, economists often use consumption-equivalent variations to express utility differences in more tangible terms. The consumption-equivalent variation is the amount of additional consumption that, at a given point in time, would make a household as well off in the absence of a policy change as it would be if the change were implemented. Consumption equivalents have been used as a method of quantitative welfare analysis in a variety of economic applications, including Social Security and pension reform (Conesa and Krueger, 1999; Bütler, 2000), studies on income inequality (Krueger and Perri, 2003), the distributional effects of child labor legislation (Krueger and Donohue, 2005), and tax code progressiveness (Conesa and Krueger, 2006).

In this paper, we translate utility changes – whether these represent how household utilities change over time after the implementation of the flat tax, or express the difference between the post-reform utilities and what they would have been in the absence of tax reform at a given point in time - into changes in the representative household's consumption as a proportion of total output, and also use the concept of a multiplicative consumption-equivalent welfare gain, as described in Lucas (2003). We express equivalent consumption as a proportion of aggregate gross domestic product to take into account our earlier simplifying assumption that, at the time of reform, total output equaled 1, and to provide a measure of welfare changes that would be intuitively easy to grasp.

First, we examine how the well-being of the representative household changes over time after the introduction of a Hall-Rabushka flat tax. Equation (1) suggests that the representative household's utility at time *t* can be expressed as:

$$u_t = \ln\left[c_t - V\left(h_t, 1 - l_t\right)\right] \tag{29}$$

The consumption-equivalent variation Δc_t denotes the extra utility that the representative household enjoys at time *t* expressed in terms of consumption, compared to $u_{t=0}$, its utility at the time of reform. The following must, therefore, be true:

$$\ln\left[c_t - \Delta c_t - V\left(h_t, 1 - l_t\right)\right] = u_{t=0} \tag{30}$$

To isolate Δc_t, we first exponentiate both sides of (30) with the base of *e*, and then rearrange the expression:

$$c_t - \Delta c_t - V\left(h_t, 1 - l_t\right) = e^{u_{t=0}}$$

$$\Delta c_t = c_t - V\left(h_t, 1 - l_t\right) - e^{u_{t=0}} \tag{31}$$

We now plug (2) into (31) to replace $V\left(h_t, 1 - l_t\right)$:

$$\Delta c_t = c_t - Bh_t\left(1 - l_t\right)^{\gamma} - e^{u_{t=0}} \tag{32}$$

Finally, we divide (32) by y_t, the aggregate output at time *t*, to express the consumption-equivalent variation as a proportion of the gross domestic product:

$$\frac{\Delta c_t}{y_t} = \frac{c_t - Bh_t\left(1 - l_t\right)^{\gamma} - e^{u_{t=0}}}{y_t} \tag{33}$$

Figures IV.24 and *IV.25* show how representative household's utility, expressed in terms of consumption-equivalent variations as a proportion of aggregate output, changes over time in the short and the long run, respectively. Compared to its well-being at the time of reform, the representative household becomes, during the first post-reform year, worse off by an amount of consumption equivalent to 2.32 percent of the gross domestic product. Over time, this utility gap closes, and the representative household reaches the initial level of well-being after a little more than three years.

The temporary decrease in household utility following the introduction of a Hall-Rabushka flat tax can, along with the initial slowdown in economic growth, detract from the reform's political acceptability. As long as the citizens' satisfaction influences how they vote, an incumbent may be reluctant to enact a flat tax for fear of losing an election. To the extent that utility considerations outweigh concerns about economic growth in the voters' minds, incumbents would find it politically very risky to implement a flat tax reform less than three or four years before an election. Reforming the tax system closer to the election might mean that the typical household would find itself worse off on election day that it was at the time of time reform, and would be more likely to vote for the opposition party or candidate. In the context of the four-year political cycle that applies to presidential politics in the United States, these utility effects may well make the adoption of a Hall-Rabushka flat tax politically infeasible.

FIGURE VI.24 – FLAT TAX REFORM: LONG-RUN TRANSITION DYNAMICS
PATH OF UTILITY OVER TIME: CONSUMPTION-EQUIVALENT VARIATION AS A PROPORTION OF AGGREGATE OUTPUT
(INTERMEDIATE IES)

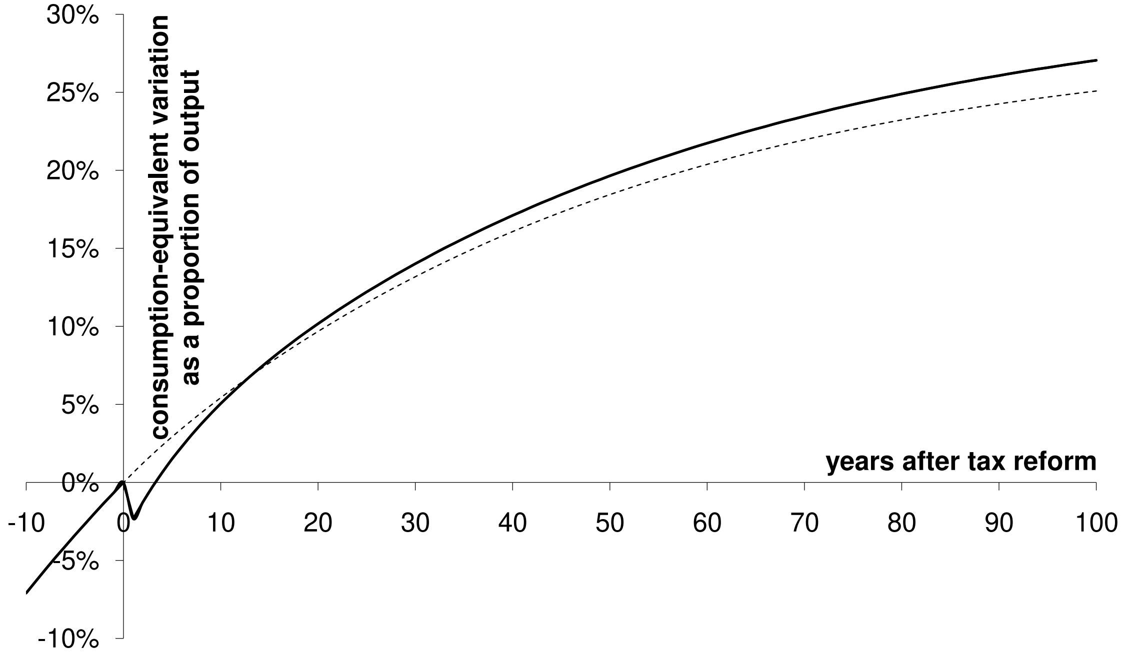
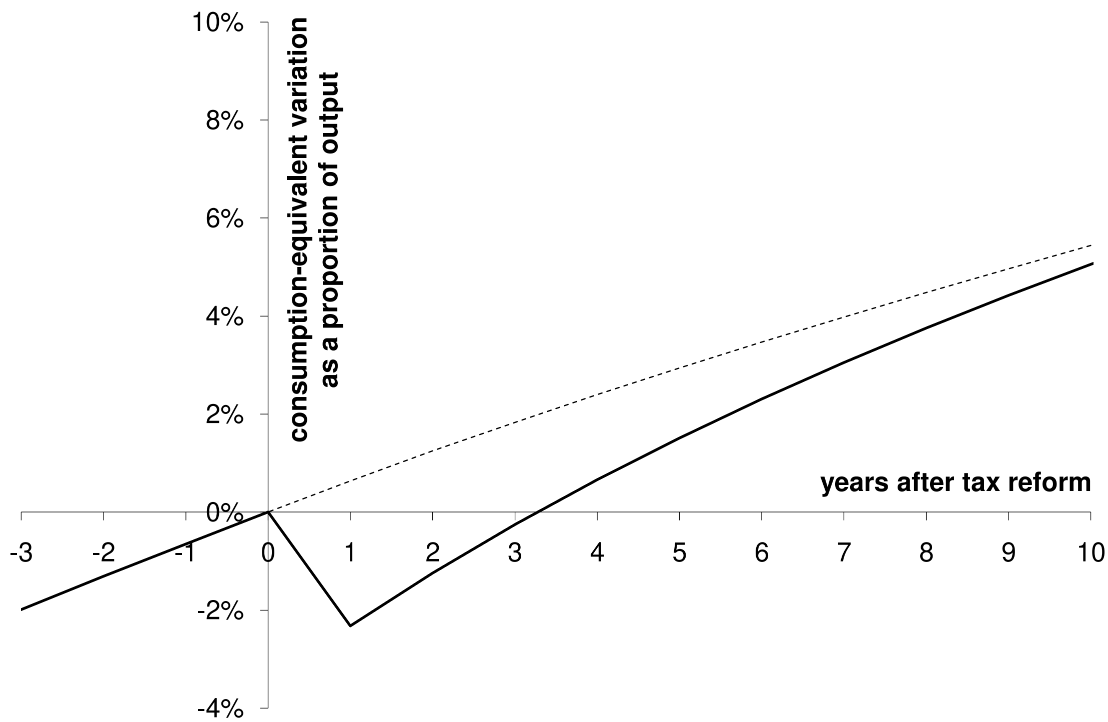


FIGURE VI.25 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS
PATH OF UTILITY OVER TIME: CONSUMPTION-EQUIVALENT VARIATION AS A PROPORTION OF AGGREGATE OUTPUT
(INTERMEDIATE IES)



In addition to examining how utility changes over time, we might wish to consider what the effects of introducing a flat tax on the representative household's well-being would be, when compared to how well off the household would have been in the absence of any tax reform. We perform this analysis, first, by computing consumption-equivalent variations as a proportion of aggregate output, and then consider an alternative method that relies on the multiplicative welfare gain as proposed in Lucas (2003).

Here, the consumption-equivalent variation Δc_t represents the additional utility that the representative household enjoys at time *t* expressed in terms of consumption, compared to the utility the household would have enjoyed at the same point in time in the absence of tax reform. More formally:

$$\ln\left[c_t^1 - V\left(h_t^1, 1-l_t^1\right)\right] = \ln\left[c_t^0 + \Delta c_t - V\left(h_t^0, 1-l_t^0\right)\right], \qquad (34)$$

where the superscript index 1 indicates post-reform values, while an index of 0 denotes values that would have been attained on the original balanced growth path, in the absence of any tax reform.

We find Δc_t by getting rid of the logarithms on both sides of (34), and then shuffling the terms:

$$c_t^1 - V\left(h_t^1, 1-l_t^1\right) = c_t^0 + \Delta c_t - V\left(h_t^0, 1-l_t^0\right)$$

$$\Delta c_t = c_t^1 - c_t^0 + V\left(h_t^0, 1-l_t^0\right) - V\left(h_t^1, 1-l_t^1\right) \qquad (35)$$

We plug (2) into (35):

$$\Delta c_t = c_t^1 - c_t^0 + B\left[h_t^0\left(1-l_t^0\right)^\gamma - h_t^1\left(1-l_t^1\right)^\gamma\right] \qquad (36)$$

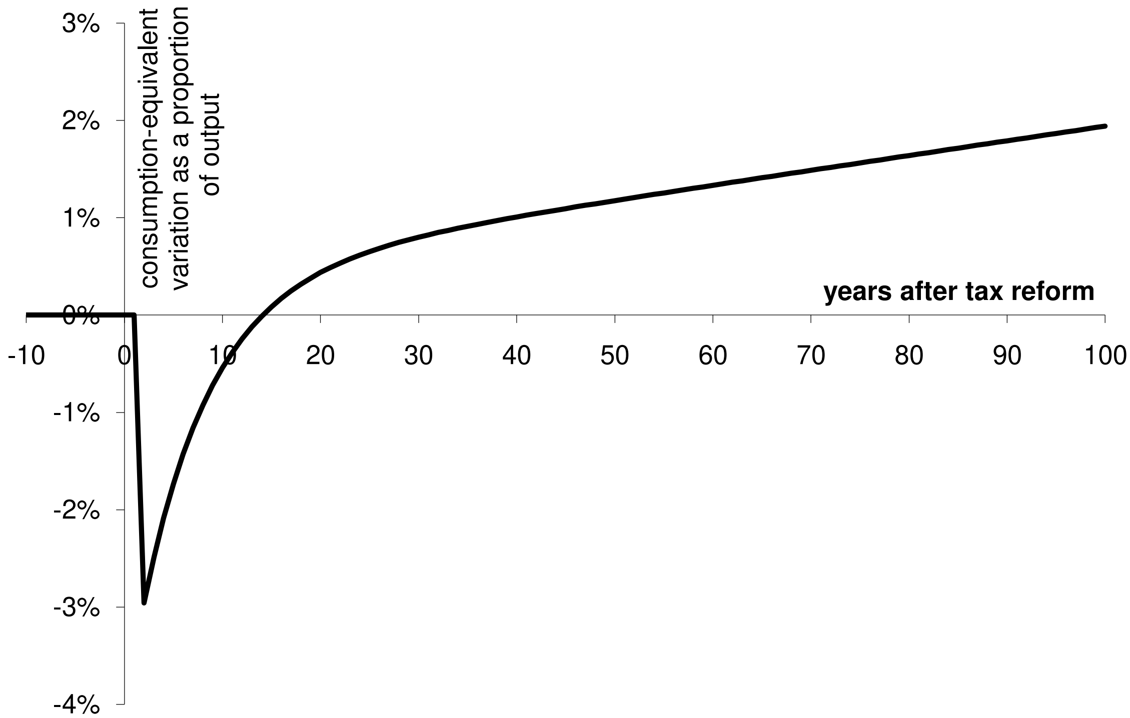
Finally, we divide (36) by y_t, the aggregate output at time t, to express the consumption-equivalent variation as a proportion of the gross domestic product:

$$\frac{\Delta c_t}{y_t} = \frac{c_t^1 - c_t^0 + B\left[h_t^0\left(1-l_t^0\right)^\gamma - h_t^1\left(1-l_t^1\right)^\gamma\right]}{y_t} \qquad \textbf{(37)}$$

Figure VI.26 shows the dynamics of the consumption-equivalent variation $\frac{\Delta c_t}{y_t}$ as a proportion of aggregate income when we compare the post-reform and no-reform scenarios. Positive values of $\frac{\Delta c_t}{y_t}$ indicate that, at time *t*, the representative household is better off than it would have been had the tax reform not taken place. Negative values, on the other hand, suggest that – at the given point in time - the household would have been better off without the changes in the tax code. *Figure VI.26*, then, suggests that, compared to a no-reform alternative, the introduction of the flat tax does not pay off – in terms of the utility opportunity cost for the representative household – until the thirteenth post-reform year.

The relatively long time period that elapses before the introduction of the flat tax raises the representative household's utility above the level where it would have been had the reform not taken place suggests that, while the tax code change has benefits in the long run, it may lead to a short- to medium-term decrease in well-being. If voters' concerns about their perceived well-being are an important consideration, and if they realize that they may have foregone utility in the years following the implementation of the Hall-Rabushka flat tax, they may be less likely to re-elect incumbents. As a result, the flat tax reform would be difficult to pass, as politicians would fear that it might cost them at the polls.

FIGURE VI.26 – FLAT TAX REFORM: LONG-RUN TRANSITION DYNAMICS
UTILITY WITH VS. WITHOUT REFORM: CONSUMPTION-EQUIVALENT VARIATION AS A PROPORTION OF AGGREGATE OUTPUT
(INTERMEDIATE IES)



Alternatively, we can express utility differences between the post-reform and no-reform states in terms of a multiplicative welfare gain, as outlined in Lucas (2003).[56] We begin with the following expression::

$$\ln\left[c_t^1 - V\left(h_t^1, 1-l_t^1\right)\right] = \ln\left[\left(1+\lambda\right)c_t^0 - V\left(h_t^0, 1-l_t^0\right)\right], \quad \textbf{(38)}$$

where, again, the superscript indices 0 and 1 denote no-reform and post-reform values, respectively. The multiplicative welfare gain λ can be interpreted as the proportion by which post-reform consumption exceeds what consumption would have been on the original, no-reform balanced growth path. After isolating λ on the left-hand side of the equation, we obtain:

$$c_t^1 - V\left(h_t^1, 1-l_t^1\right) = \left(1+\lambda\right)c_t^0 - V\left(h_t^0, 1-l_t^0\right)$$

$$\left(1+\lambda\right)c_t^0 = c_t^1 + V\left(h_t^0, 1-l_t^0\right) - V\left(h_t^1, 1-l_t^1\right)$$

[56] In his paper, Lucas (2003) simply uses the term "welfare gain."

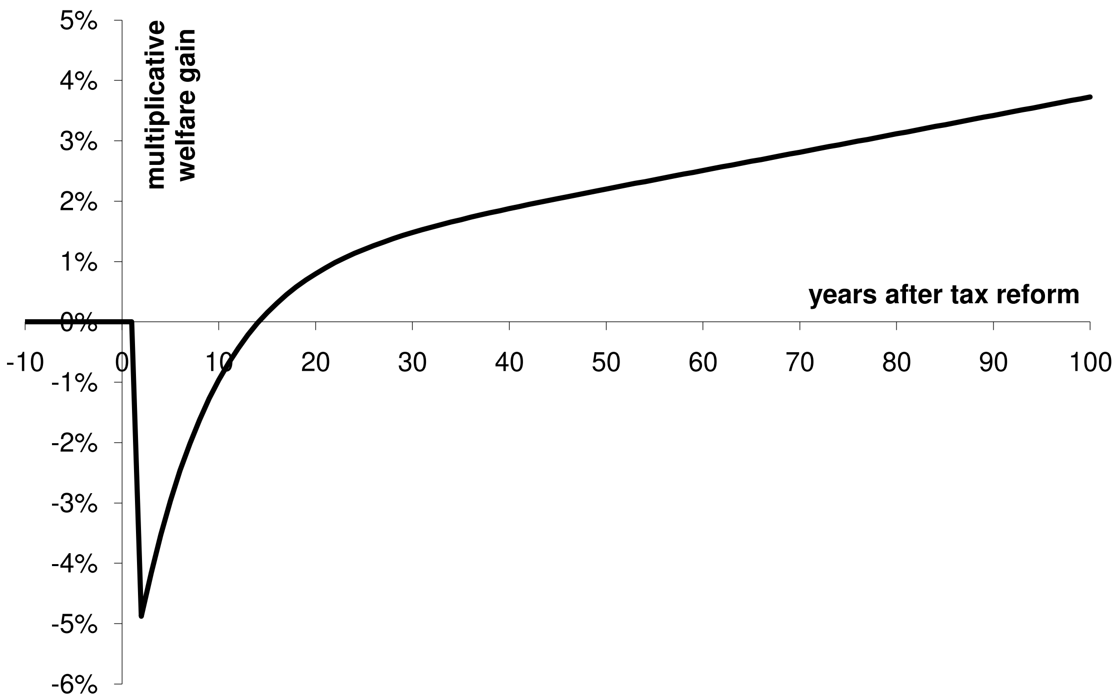
$$\lambda = \frac{c_t^1 + V\left(h_t^0, 1 - l_t^0\right) - V\left(h_t^1, 1 - l_t^1\right)}{c_t^0} - 1 \qquad \textbf{(39)}$$

Finally, we plug (2) into (39) to find an expression that can be evaluated using our calibrated parameters:

$$\lambda = \frac{c_t^1 + B\left[h_t^0\left(1 - l_t^0\right)^\gamma - h_t^1\left(1 - l_t^1\right)^\gamma\right]}{c_t^0} - 1 \qquad \textbf{(40)}$$

Because the underlying utility is the same, *Figure IV.27* - which depicts the multiplicative welfare gain that results from the introduction of the Hall-Rabushka flat tax, as compared to a situation without any tax reform – indicates, like *Figure IV.26* does, that the tax reform does not pay off, when considering the opportunity cost in terms of the representative household's utility, until 13 years after its implementation.

FIGURE VI.27 – FLAT TAX REFORM: LONG-RUN TRANSITION DYNAMICS
UTILITY WITH VS. WITHOUT REFORM: MULTIPLICATIVE WELFARE GAIN (INTERMEDIATE IES)



VII. CONCLUSION

Using a dynamic equilibrium model proposed by Cassou and Lansing (2003), and calibrated to fit empirical data about the U.S. economy, we have estimated the growth and utility effects of replacing the current graduated-rate federal income tax by a revenue-neutral Hall-Rabushka flat tax.

We find that the flat tax reform increases long-term economic growth, and that the magnitude of this effect depends on the U.S. economy's intertemporal elasticity of substitution in labor supply (IES). For values of IES that range from 0.25 to 1, the introduction of a Hall-Rabushka flat tax increases the long-term economic growth rate by 0.003 - 0.255 percentage points.

A more intertemporally elastic labor supply results in a higher long-term economic growth rate and – due largely to compounding effects – a higher level of aggregate output in the decades following the reform. In the short run, however, higher IES may lead to greater fluctuations in economic growth rates and a longer-lasting temporary decrease of household utility. Such reactions are understandable, as higher IES means that households reconsider their intertemporal consumption and labor allocation more dramatically in response to changes in the tax code, leading to increased long-term productivity but also to a period of significant adjustment in the short run.

The transition dynamics exhibited similar patterns for all three values of IES that we examined (0.25, 0.5 and 1), as can be seen in *Appendix B*. When IES is assumed to be intermediate or high (0.5 or 1), the economic growth rate falls during the first year, rebounds rapidly, and then gradually decreases to its balanced growth path value. In the scenario with a low IES (0.25), there is no decrease in the economic growth rate during the first post-reform year.

Regardless of the value of IES, after the implementation of the flat tax, the representative household's utility falls, and does not reach its pre-reform value until a few years later. The combination of a possible economic slowdown and temporarily decreased household satisfaction may make the flat tax reform seem unpalatable to some politicians, especially those worried about their re-election. Incumbents concerned about an upcoming election may, as a result, be inclined to carefully consider the political consequences of the flat tax reform in the timing of its adoption.

Immediately after the enactment of the flat tax reform, the representative household spends less time in education and market work. As the economy approaches the new balanced growth path, the proportion of time spent in education and work increases, and eventually exceeds its pre-reform values. The wage rate increases, and the interest rate falls.

The long-term results of our simulation are consistent with Hall and Rabushka's (1995) conjectures that their proposal, if implemented, would improve the performance of the U.S. economy, increase take-home wages, and stimulate work effort.

The analysis presented in this paper has a number of limitations, which represent, in our view, an opportunity for further research. First and foremost, our results are based on a theoretical model which relies on assumptions about the form of utility and production functions, as well as on the assumption that government expenditure is unproductive. Evidence from other empirical studies could give us some indication about the appropriateness of these assumptions.

Although the *Jobs and Growth Tax Relief Reconciliation Act*, passed in 2003, made dividends subject to the capital gains rate schedule on the individual level, we used decision rules derived from the assumption that there is full double taxation of dividends. One could bring the model closer to the current version of the tax code by deriving optimal decision rules that take into account the new treatment of dividends.

In an influential essay, Friedman (1953) has argued that the usefulness of theoretical models should be judged not by the realism of their assumptions, but rather on the basis of the accuracy of their predictions. In the light of this proposition, one could attempt to determine the model's predictive power by estimating it retrospectively for economies – for instance, in Central or Eastern Europe - that have adopted a flat tax comparable to the Hall-Rabushka proposal.

The Cassou-Lansing model estimates the effects of a flat tax reform by looking at the behavior of a representative household. An interesting extension of this model could consider the distributional effects of tax reform. Instead of modeling the reactions of a single representative household, a modified model could consider how changes in the tax code would affect the work effort and investment decisions of various income groups. Equity considerations are important in deciding what tax policy to pursue, and extending our model to estimate the possible differential effects of a flat tax on several income groups could help policy-makers reach a judgment about the fairness of the reform.

Finally, one could combine the modified Cassou-Lansing model with one that aims to explain business cycles. Since the revolutionary innovations introduced in Lucas and Prescott (1971) and built upon in Kydland and Prescott (1982), dynamic equilibrium models have been commonly used to analyze business cycles (Rebelo, 2005; Royal Academy of the Science, 2004). In light of these advances, combining the Cassou-Lansing model with a dynamic equilibrium model that explains short-term economic fluctuations should not be an insurmountable task. Such a combined model could improve our understanding of the interactions between tax reforms and the business cycles, and would shed more light on the possible political consequences of fundamental tax reform.

Appendix A: Derivation of Optimal Decision Rules

This section summarizes the derivation of optimal decisions rules, as detailed in Cassou and Lansing (2003). To derive equilibrium decision rules, we solve the household's problem – maximizing utility (1) given the budget constraint (3) specified in the basic framework of the model – using the Lagrangian:

$$L=\sum_{t=0}^{\infty}\beta^t\left\{\ln\left[c_t-Bh_t(1-l_t)^\gamma\right]+\lambda_t\left[w_th_t\left(1-l_t-e_t\right)-T_t+r_tk_t-i_{kt}-i_{ht}-c_t\right]\right\}$$

After substituting (4) for T_t:

$$\begin{aligned}L=\sum_{t=0}^{\infty}\beta^t\Big\{&\ln\left[c_t-Bh_t(1-l_t)^\gamma\right]+\lambda_t\Big[w_th_t\left(1-l_t-e_t\right)-\tau_{pt}\Big[w_th_t\left(1-l_t-e_t\right)-D_t+\\&+\eta\left(1-\bar{\tau}_b\right)\left(r_tk_t-\phi_ki_{kt}-\phi_hi_{ht}\right)\Big]-\bar{\tau}_b\left(r_tk_t-\phi_ki_{kt}-\phi_hi_{ht}\right)+r_tk_t-i_{kt}-i_{ht}-c_t\Big]\Big\}\end{aligned}$$

We collect terms that contain r_tk_t, i_{kt} and i_{ht}:

$$\begin{aligned}L=\sum_{t=0}^{\infty}\beta^t\Big\{&\ln\left[c_t-Bh_t(1-l_t)^\gamma\right]+\lambda_t\Big[w_th_t\left(1-l_t-e_t\right)-\tau_{pt}\Big[w_th_t\left(1-l_t-e_t\right)-D_t+\\&+\eta\left(1-\bar{\tau}_b\right)\left(r_tk_t-\phi_ki_{kt}-\phi_hi_{ht}\right)\Big]+\left(1-\bar{\tau}_b\right)r_tk_t-\left(1-\phi_k\bar{\tau}_b\right)i_{kt}-\left(1-\phi_h\bar{\tau}_b\right)i_{ht}-c_t\Big]\Big\}\end{aligned}$$

After plugging in equation (10) for τ_{pt}, we obtain:

$$\begin{aligned}L=\sum_{t=0}^{\infty}\beta^t\Big\{&\ln\left[c_t-Bh_t(1-l_t)^\gamma\right]+\lambda_t\Big[w_th_t\left(1-l_t-e_t\right)-\\&-\bar{\tau}_p\frac{\left[w_th_t\left(1-l_t-e_t\right)-D_t+\eta\left(1-\bar{\tau}_b\right)\left(r_tk_t-\phi_ki_{kt}-\phi_hi_{ht}\right)\right]^{n+1}}{\left[w_tH_t\left(1-L_t-E_t\right)-D_t+\eta\left(1-\bar{\tau}_b\right)\left(r_tK_t-\phi_kI_{kt}-\phi_hI_{ht}\right)\right]^{n}}+\\&+\left(1-\bar{\tau}_b\right)r_tk_t-\left(1-\phi_k\bar{\tau}_b\right)i_{kt}-\left(1-\phi_h\bar{\tau}_b\right)i_{ht}-c_t\Big]\Big\}\end{aligned}$$

By differentiating the Lagrangian, we obtain the first-order conditions (FOCs) with respect to variables k_{t+1}, h_{t+1}, c_t, $1-l_t$, and e_t:

- FOC with respect to k_{t+1}: [57]

$$\frac{\partial L}{\partial k_{t+1}} = 0 \quad \Leftrightarrow$$

$$\lambda_t\left[1-\phi_k\bar{\tau}_b - \eta\left(1-\bar{\tau}_b\right)\phi_k\tau_{pt}\left(n+1\right)\right]\frac{i_{kt}}{\delta_k k_{t+1}} = \beta\lambda_{t+1}\left\{\left[1-\bar{\tau}_b - \eta\left(1-\bar{\tau}_b\right)\tau_{pt}\left(n+1\right)\right]r_{t+1}\right. \quad \textbf{(A.1a)}$$
$$\left. + \left[1-\phi_k\bar{\tau}_b - \eta\left(1-\bar{\tau}_b\right)\phi_k\tau_{pt+1}\left(n+1\right)\right]\frac{\left(1-\delta_k\right) i_{kt+1}}{\delta_k k_{t+1}}\right\}$$

- FOC with respect to h_{t+1}: [58]

$$\frac{\partial L}{\partial h_{t+1}} = 0 \quad \Leftrightarrow$$

$$\lambda_t\left[1-\phi_h\bar{\tau}_b - \eta\left(1-\bar{\tau}_b\right)\phi_h\tau_{pt}\left(n+1\right)\right]\frac{i_{ht}}{\delta_h h_{t+1}} = \frac{-\beta B\left(1-l_{t+1}\right)^{\gamma}}{c_{t+1} - Bh_{t+1}\left(1-l_{t+1}\right)^{\gamma}} + \quad \textbf{(A.1b)}$$
$$+\beta\lambda_{t+1}\left\{\left[1-\tau_{pt+1}\left(n+1\right)\right]w_{t+1}h_{t+1}\left(1-l_{t+1}-e_{t+1}\right)\right.$$
$$\left. + \left[1-\phi_h\bar{\tau}_b - \eta\left(1-\bar{\tau}_b\right)\phi_h\tau_{pt+1}\left(n+1\right)\right]\frac{\left(1-\delta_h\right) i_{ht+1}}{\delta_h h_{t+1}}\right\}$$

---

[57] $i_{kt} = \left[\frac{k_{t+1}}{A_1 k_t^{1-\delta_k}}\right]^{\frac{1}{\delta_k}}$; $\frac{\partial i_{kt}}{\partial k_{t+1}} = \frac{1}{\delta_k}\left[\frac{k_{t+1}}{A_1 k_t^{1-\delta_k}}\right]^{\frac{1}{\delta_k}-1}\frac{1}{A_1 k_t^{1-\delta_k}} = \frac{1}{\delta_k k_{t+1}}\left[\frac{k_{t+1}}{A_1 k_t^{1-\delta_k}}\right]^{\frac{1}{\delta_k}} = \frac{i_{kt}}{\delta_k k_{t+1}}$

$i_{kt+1} = \left[\frac{k_{t+2}}{A_1 k_{t+1}^{1-\delta_k}}\right]^{\frac{1}{\delta_k}}$; $\frac{\partial i_{kt+1}}{\partial k_{t+1}} = \frac{1}{\delta_k}\left[\frac{k_{t+2}}{A_1 k_{t+1}^{1-\delta_k}}\right]^{\frac{1}{\delta_k}-1}\frac{k_{t+2}\left(\delta_k-1\right)}{A_1 k_{t+1}^{1-\delta_k} k_{t+1}} = -\frac{\left(1-\delta_k\right) i_{kt+1}}{\delta_k k_{t+1}}$

[58] $i_{ht} = \left[\frac{h_{t+1}}{A_2 h_t^{1-\delta_h} e^{v}}\right]^{\frac{1}{\delta_h}}$; $\frac{\partial i_{ht}}{\partial h_{t+1}} = \frac{1}{\delta_h}\left[\frac{h_{t+1}}{A_2 h_t^{1-\delta_h} e_t^{v}}\right]^{\frac{1}{\delta_h}-1}\frac{1}{A_2 h_t^{1-\delta_h} e_t^{v}} = \frac{i_{ht}}{\delta_h h_{t+1}}$

$i_{ht+1} = \left[\frac{h_{t+2}}{A_2 h_{t+1}^{1-\delta_h} e_t^{v}}\right]^{\frac{1}{\delta_h}}$; $\frac{\partial i_{ht+1}}{\partial h_{t+1}} = \frac{1}{\delta_h}\left[\frac{h_{t+2}}{A_1 h_{t+1}^{1-\delta_h} e_t^{v}}\right]^{\frac{1}{\delta_h}-1}\frac{h_{t+2}\left(\delta_h-1\right)}{A_1 h_{t+1}^{1-\delta_h} e_t^{v} h_{t+1}} = -\frac{\left(1-\delta_h\right) i_{ht+1}}{\delta_h h_{t+1}}$

- FOC with respect to c_t :

$$\frac{\partial L}{\partial c_t} = 0 \quad \Leftrightarrow \quad \lambda_t = \frac{1}{c_t - Bh_t l_t^{\gamma}} \qquad \textbf{(A.1c)}$$

- FOC with respect to $1 - l_t$:

$$\frac{\partial L}{\partial (1-l_t)} = 0 \quad \Leftrightarrow \quad \beta^t \left[-\frac{B\gamma h_t (1-l_t)^{\gamma-1}}{c_t - Bh_t (1-l_t)^{\gamma}} + \lambda_t w_t h_t - \lambda_t \bar{\tau}_{pt} (n+1) w_t h_t \right] = 0$$
$$\Leftrightarrow \quad \lambda_t \left[1 - \bar{\tau}_{pt} (n+1) \right] w_t h_t = \frac{B\gamma h_t (1-l_t)^{\gamma-1}}{c_t - Bh_t (1-l_t)^{\gamma}} \qquad \textbf{(A.1d)}$$

- FOC with respect to e_t :

$$\frac{\partial L}{\partial e_t} = 0 \quad \Leftrightarrow$$
$$\left[1 - \tau_{pt} (n+1)\right] w_t h_t = \left[1 - \phi_h \bar{\tau}_b - \eta (1 - \bar{\tau}_b) \phi_h \tau_{pt} (n+1)\right] \frac{v i_{ht}}{\delta_h e_t} \qquad \textbf{(A.1e)}$$

To obtain the optimal decision rules, we use the method of undetermined coefficients. Cassou and Lansing (2003) conjecture that the decision rules take the following form:

$$i_{kt} = a_0 \left[\frac{1 - \left[\bar{\tau}_b + \eta (1 - \bar{\tau}_b) \bar{\tau}_p (n+1)\right]}{1 - \phi_k \left[\bar{\tau}_b + \eta (1 - \bar{\tau}_b) \bar{\tau}_p (n+1)\right]} \right] y_t \equiv a_0 (1 - \bar{\tau}_k) y_t \qquad \textbf{(A.2)}$$

$$i_{ht} = b_0 \left[\frac{1 - \bar{\tau}_p (n+1)}{1 - \phi_h \left[\bar{\tau}_b + \eta (1 - \bar{\tau}_b) \bar{\tau}_p (n+1)\right]} \right] y_t \equiv b_0 (1 - \bar{\tau}_h) y_t \qquad \textbf{(A.3)}$$

$$\lambda_t = \frac{1}{d_0 y_t} \qquad \textbf{(A.4)}$$

$$(1 - l_t - e_t) = f_0 l_t , \qquad \textbf{(A.5)}$$

where the constants a_0, b_0, d_0 and f_0 need to be determined. After plugging in the decision rules and equations (8), (9) into the first-order conditions for k_{t+1} and h_{t+1}, they obtain expressions for a_0 and b_0:

$$a_0 = \frac{\theta \delta_k}{\rho + \delta_k} \quad ; \qquad b_0 = \frac{(1-\theta)\delta_h}{\rho + \delta_h}\left(1 - \frac{1}{\gamma f_0}\right), \qquad \textbf{(A.6), (A.7)}$$

where $\rho = \frac{1}{\beta} - 1$ denotes the household's rate of time preference. To derive an expression for f_0, they substitute the profit-maximizing condition (7) for w_t into the FOC with respect to e_t, and also use the conjectured decision rules and the above expression for b_0:

$$f_0 = \frac{\rho + \delta_h + \frac{v}{\gamma}}{\rho + \delta_h + v} \qquad \textbf{(A.8)}$$

Substituting this result into (A.7) yields:

$$b_0 = \frac{(1-\theta)\delta_h}{\left(\rho + \delta_h + \frac{v}{\gamma}\right)}\left(\frac{\gamma - 1}{\gamma}\right) \qquad \textbf{(A.9)}$$

One can now plug equation (7) into the FOC for $1 - l_t$ (A.1d), and then use relations (A.1c), (A.5) and (A.8):

$$1 - l_t = \left\{ \frac{A_0(1-\theta)}{B\gamma}\left[1 + \frac{v}{\left(\rho + \delta_h + \frac{v}{\gamma}\right)}\left(\frac{\gamma - 1}{\gamma}\right)\right]^{\theta}\left[1 - \bar{\tau}_p(n+1)\right]\left(\frac{h_t}{k_t}\right)^{-\theta}\right\}^{\frac{1}{\theta+\gamma-1}} \qquad \textbf{(A.10)}$$

Using this result and given the aforementioned conjectured forms, Cassou and Lansing (2003) derive the optimal decision rules for $(1-l_t-e_t)$ and e_t,:

$$(1-l_t-e_t)=p_1\left\{\left[1-\bar{\tau}_p\,(n+1)\right]\left(\frac{h_t}{k_t}\right)^{-\theta}\right\}^{\frac{1}{\theta+\gamma-1}} \quad \textbf{(A.11)}$$

$$p_1\equiv\left\{\frac{z(1-\theta)}{B\gamma}\left[1+\frac{v}{\left(\rho+\delta_h+\frac{v}{\gamma}\right)}\left(\frac{\gamma-1}{\gamma}\right)\right]^{1-\gamma}\right\}^{\frac{1}{\theta+\gamma-1}} \quad \textbf{(A.12)}$$

$$e_t=p_1p_2\left\{\left[1-\bar{\tau}_p\,(n+1)\right]\left(\frac{h_t}{k_t}\right)^{-\theta}\right\}^{\frac{1}{\theta+\gamma-1}}=p_2\left(1-l_t-e_t\right) \quad \textbf{(A.13)}$$

$$p_2\equiv\frac{v}{\left(\rho+\delta_h+\frac{v}{\gamma}\right)}\left(\frac{\gamma-1}{\gamma}\right) \quad \textbf{(A.14)}$$

To find the decision rule for equilibrium consumption, they plug the profit-maximizing rules (6) and (7), as well as the human and physical capital investment rules (A.2) and (A.3) into the within-period budget constraint (3):

$$\begin{aligned} c_t=\{&1-\bar{\tau}_p\left(1-\theta-\alpha\right)-\theta\left[\bar{\tau}_b+\eta\left(1-\bar{\tau}_b\right)\bar{\tau}_p\right]\\ &-a_0\left(1-\bar{\tau}_k\right)\left[1-\phi_k\left[\bar{\tau}_b+\eta\left(1-\bar{\tau}_b\right)\bar{\tau}_p\right]\right]\\ &-b_0\left(1-\bar{\tau}_h\right)\left[1-\phi_h\left[\bar{\tau}_b+\eta\left(1-\bar{\tau}_b\right)\bar{\tau}_p\right]\right]\}y_t \end{aligned} \quad \textbf{(A.15)}$$

Finally, Cassou and Lansing (2003) verify that the conjectured forms of the equilibrium decision rules are correct by showing that d_0 is constant. To do that, they rearrange the FOC for consumption:

$$\lambda_t=\frac{1}{d_0y_t}=\frac{1}{c_t-Bh_t\left(1-l_t\right)^{\gamma}}$$

$$d_0 y_t = c_t - Bh_t\left(1-l_t\right)^{\gamma} \tag{A.16}$$

They rewrite (A.16) after plugging in equations (A.1c) and (A.1d):

$$d_0 y_t = c_t - \frac{1-\theta}{\gamma f_0}\left[1-\bar{\tau}_p\left(n+1\right)\right] y_t \tag{A.17}$$

By plugging in the decision rule for equilibrium consumption (A.15) into the above expression and isolating d_0 on the left-hand side, one obtains a constant expression that validates the conjecture:

$$\begin{aligned} d_0 = {} & 1-\bar{\tau}_p\left(1-\theta-\alpha\right)-\theta\left[\bar{\tau}_b+\eta\left(1-\bar{\tau}_b\right)\bar{\tau}_p\right]-a_0\left(1-\bar{\tau}_k\right)\left\{1-\phi_k\left[\bar{\tau}_b+\eta\left(1-\bar{\tau}_b\right)\bar{\tau}_p\right]\right\} \\ & -b_0\left(1-\bar{\tau}_h\right)\left\{1-\phi_k\left[\bar{\tau}_b+\eta\left(1-\bar{\tau}_b\right)\bar{\tau}_p\right]\right\}-\frac{1-\theta}{\gamma f_0}\left[1-\bar{\tau}_p\left(n+1\right)\right] \end{aligned} \tag{A.18}$$

Appendix B: Sensitivity Analysis

Table B.1 summarizes the post-reform balanced growth path characteristics, if we estimate the intertemporal elasticity of substitution (IES) in labor supply to be relatively low at σ = 0.25. As shown in *Figures B.1* to *B.19*, the transition dynamics of most variables such as the index of aggregate output, the proportion of time spent in education/market work/leisure, the wage and interest rates, and utility (expressed in utils, consumption-equivalent variations, and as a multiplicative welfare gain) follow the same general pattern as they do with an intermediate value of IES (σ = 0.5).

With a low IES, one variable whose dynamics differ from those that would prevail with an intermediate IES is the economic growth rate. Unlike in the case where σ = 0.5, the economic growth rate does not decrease during the first post-reform year when σ = 0.25, making tax reform somewhat more palatable for politicians in the short-run.

Household utility falls during the first post-reform year, but then gradually rebounds until it reaches its balanced growth path level. As a result, the political implications of a flat tax reform, when labor supply is relatively inelastic over time and if households' perceived well-being outweighs the economic growth rate as a factor in their voting decisions, may remain the same: Politicians might be unwilling to reform the tax system for fear of being punished at the polls by votes who had suffered adverse short-term consequences.

Figure B.3 shows how aggregate output evolves after the Hall-Rabushka flat tax is adopted (solid line) relative to how large it would have been in the absence of tax reform (dashed line), if IES is low (σ = 0.25). If the flat tax is adopted, 100 years after the reform we expect aggregate output to be about 11.4% higher than it would have been without changes in the tax code. *Figure B.17* shows that, three years after the reform, household utility attains the value it had at the time of reform. *Figures B.18* and *B.19* suggest that, compared to a no-reform alternative, the flat tax does not pay off – in household utility terms – until about 12 years after its implementation.

Table B.2, on the other hand, provides the balanced growth path characteristics after the flat tax reform was enacted in an economy whose IES is estimated to be relatively high at $\sigma = 1$. *Figures B.20* to *B.38* show that, for this value, the transition dynamics – in this case, including the economic growth rate - have, yet again, similar characteristics as those we saw with an intermediate value of IES. Even with a high IES, therefore, politicians will be wary about implementing the reform because of its potential short-term political implications.

Figure B.22 depicts the output dynamics for a high level of IES ($\sigma = 1$). According to the Cassou-Lansing model, 100 years after the adoption of the flat tax aggregate output will have increased by as much as 38.2 percent. *Figure B.36* shows that, after the initial decrease, household utility reaches its reform-time value after a little more than four years. *Figures B.37* and *B.38* suggest that, when compared to the alternative of staying on the original, no-reform, balanced growth path, the flat tax will not yield higher household utility until as late as 19 years after its adoption.

TABLE B.1 – FLAT TAX REFORM: BALANCED GROWTH PATH CHARACTERISTICS (LOW IES)

Characteristics of the Balanced Growth Path After the Flat Tax Reform (low IES = 0.25)			**Comparison**	
			Before the reform	**Difference**
long-term growth rate	μ	**0.01863**	0.0186	+ 0.00003
time in market work	$1-l_t-e_t$	**0.213**	0.2111	+ 0.0019
time in education	e_t	**0.097**	0.09583	+ 0.001
leisure time	l_t	**0.690**	0.69306	- 0.003
consumption / GDP	$\frac{c_t}{y_t}$	**0.562**	0.606	- 0.044
investment in physical capital	$\frac{i_{kt}}{y_t}$	**0.296**	0.2453	+ 0.0507
investment in human capital	$\frac{i_{ht}}{y_t}$	**0.032**	0.0384	- 0.006
physical capital stock	$\frac{k_t}{y_t}$	**3.791**	3.1424	+ 0.649
human-to-physical capital ratio	$\frac{h_t}{k_t}$	**15.110**	20.4	- 5.29

FIGURE B.1 – FLAT TAX REFORM: TRANSITION DYNAMICS – ECONOMIC GROWTH RATE (LOW IES)

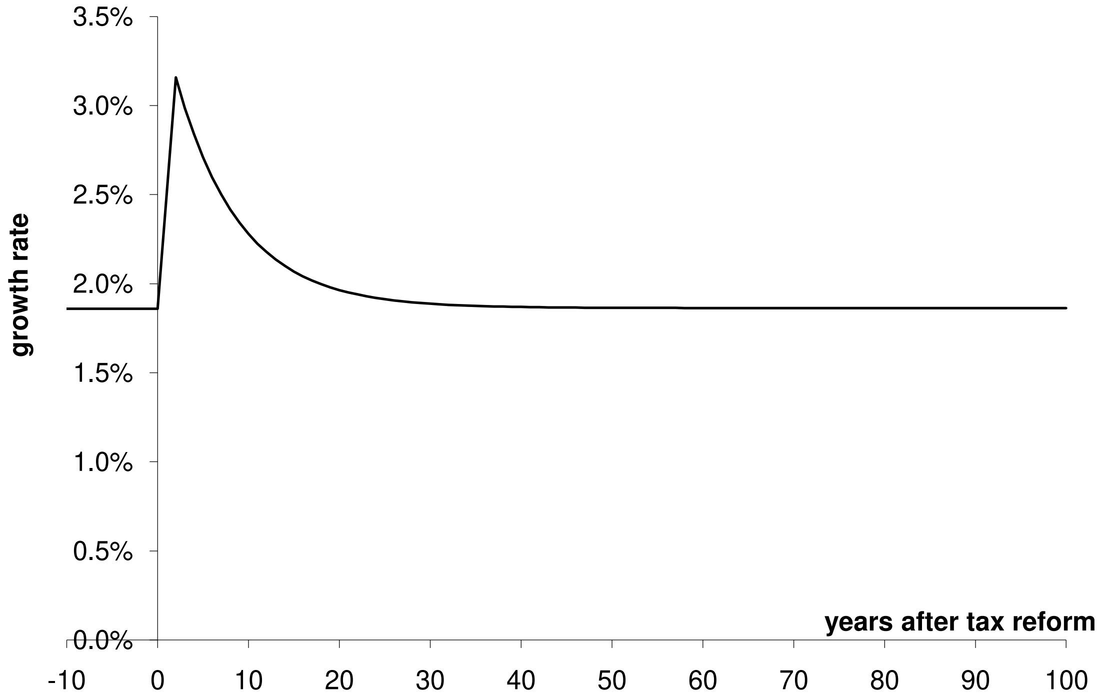


FIGURE B.2 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – ECONOMIC GROWTH RATE (LOW IES)

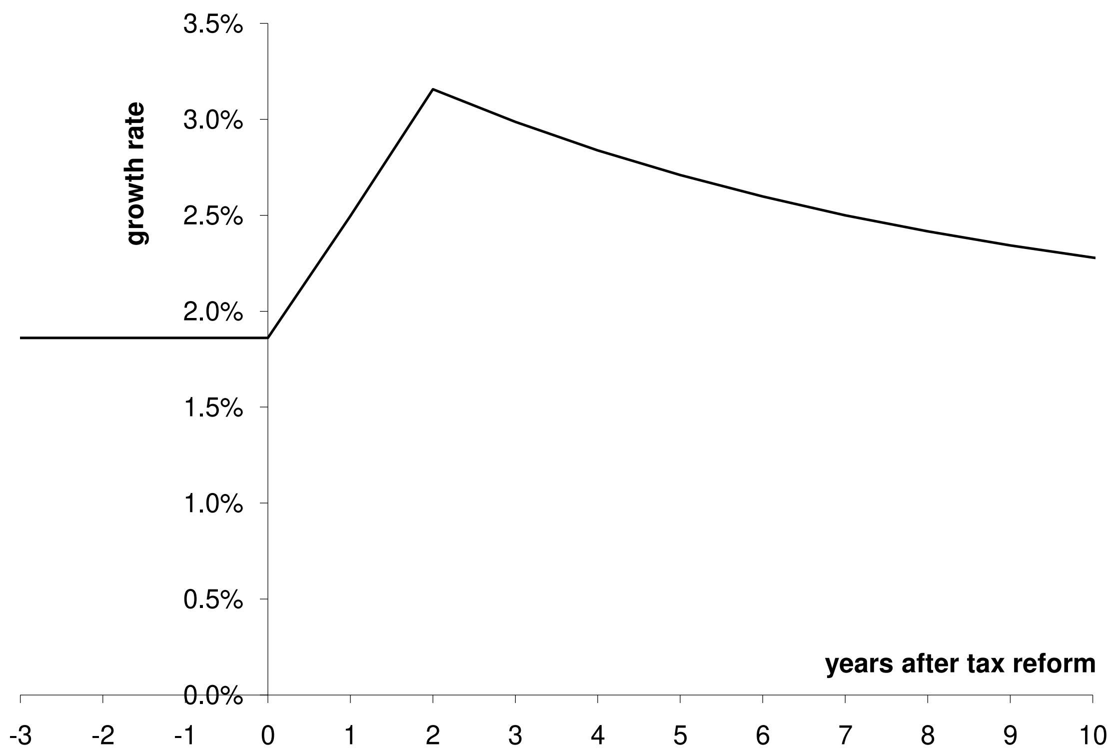


FIGURE B.3 – FLAT TAX REFORM: TRANSITION DYNAMICS – INDEX OF AGGREGATE OUTPUT (LOW IES)

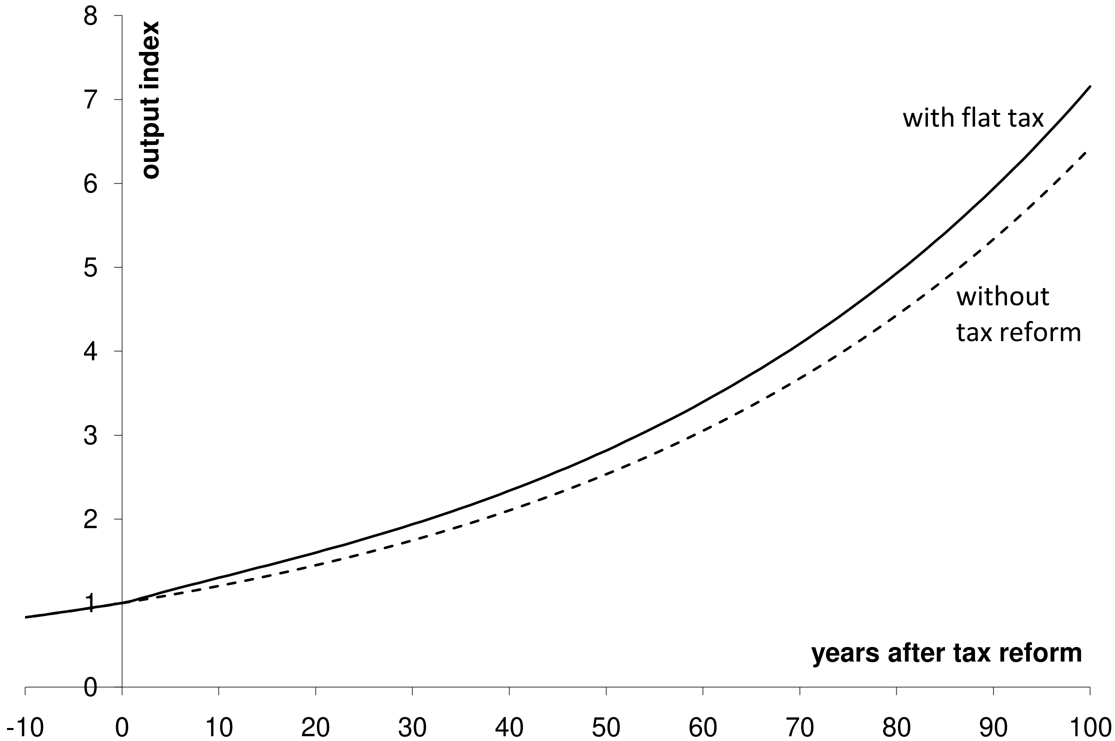


FIGURE B.4 – FLAT TAX REFORM: TRANSITION DYNAMICS – TIME IN EDUCATION (LOW IES)

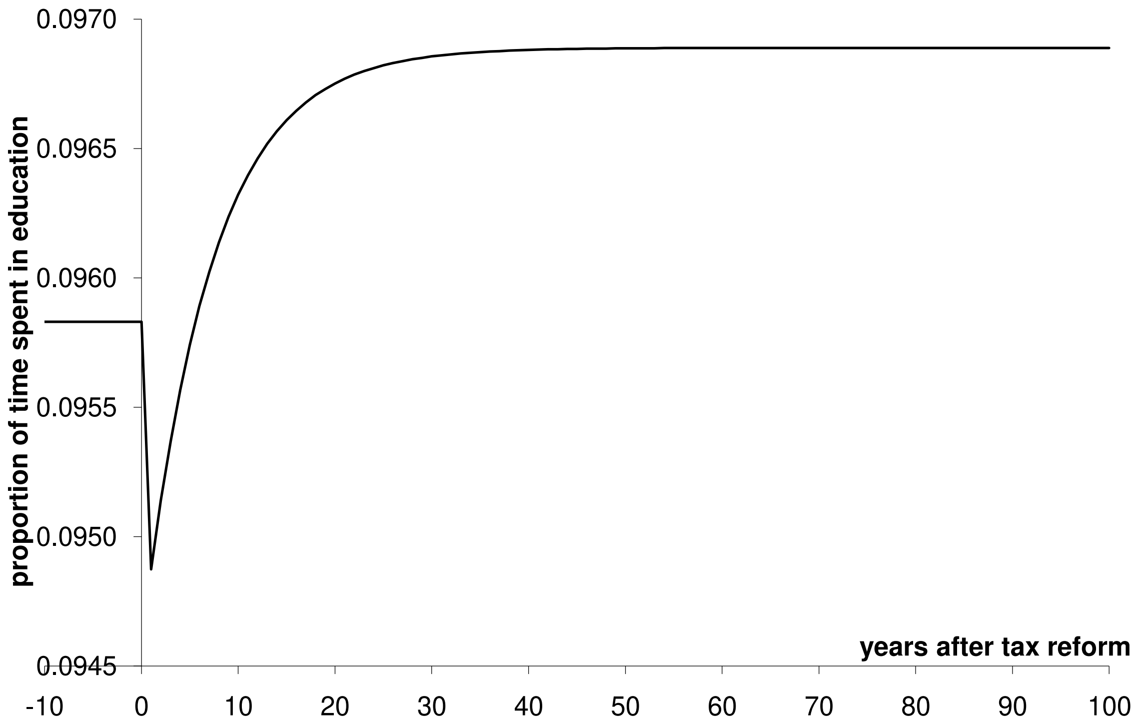


FIGURE B.5 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – TIME IN EDUCATION (LOW IES)

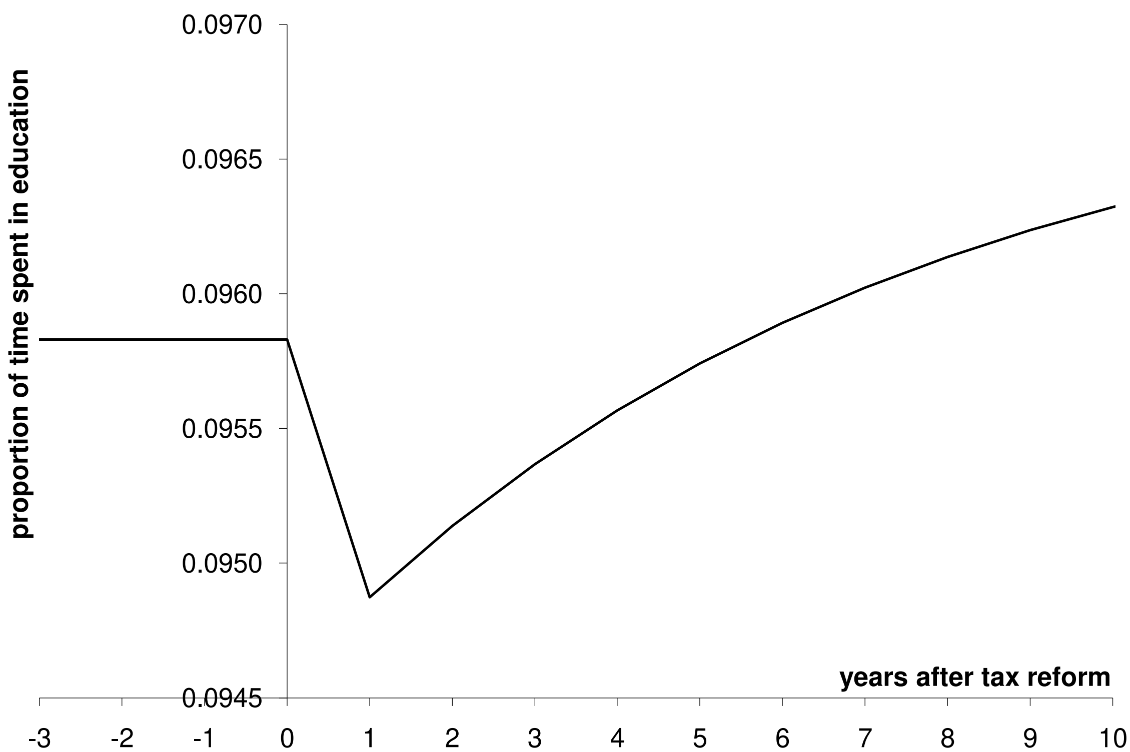


FIGURE B.6 – FLAT TAX REFORM: TRANSITION DYNAMICS – TIME IN MARKET WORK (LOW IES)

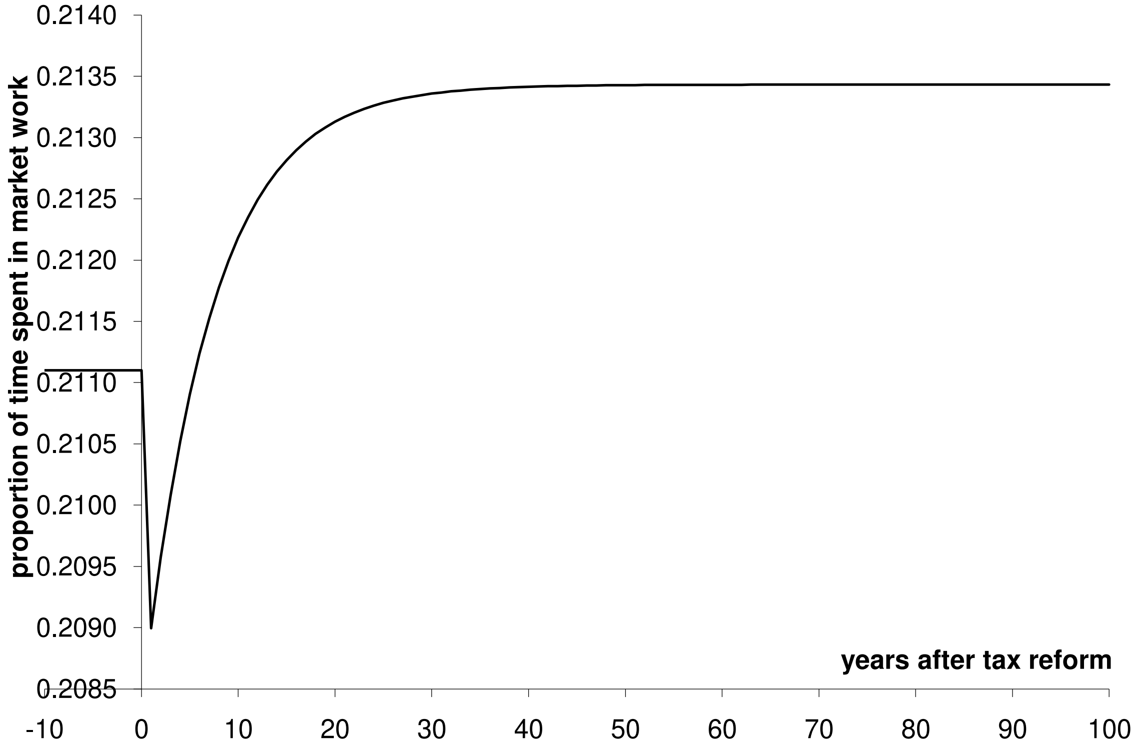


FIGURE B.7 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – TIME IN MARKET WORK (LOW IES)

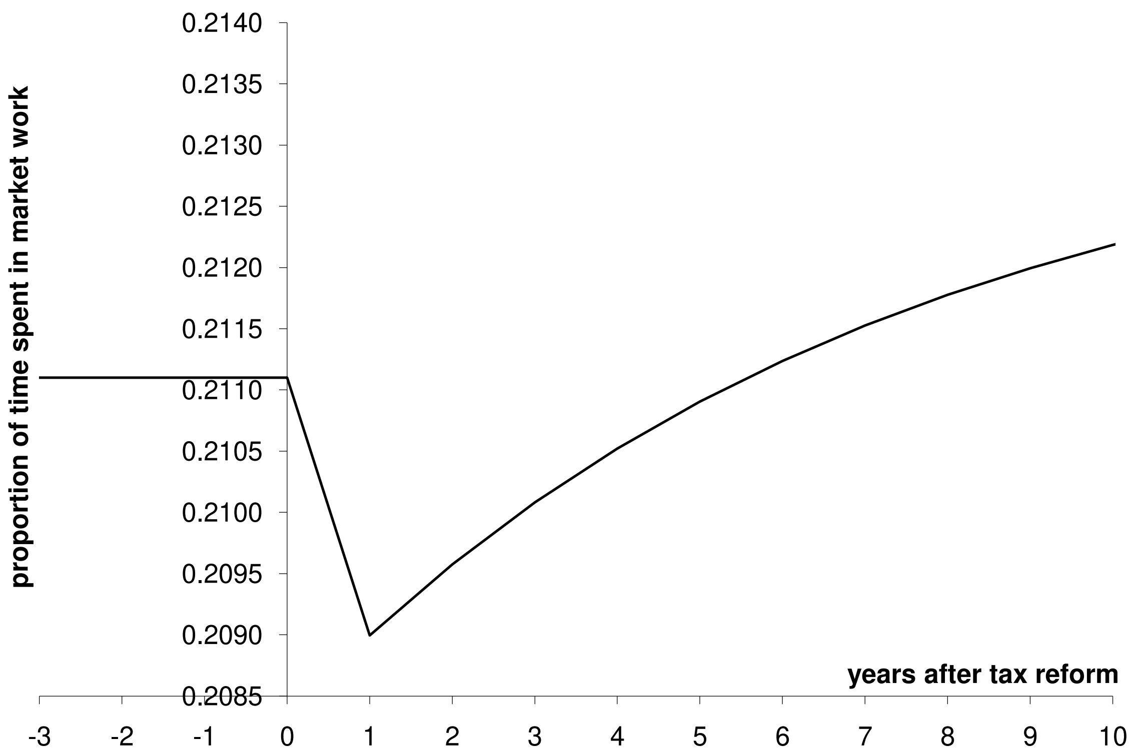


FIGURE B.8 – FLAT TAX REFORM: TRANSITION DYNAMICS – LEISURE TIME (LOW IES)

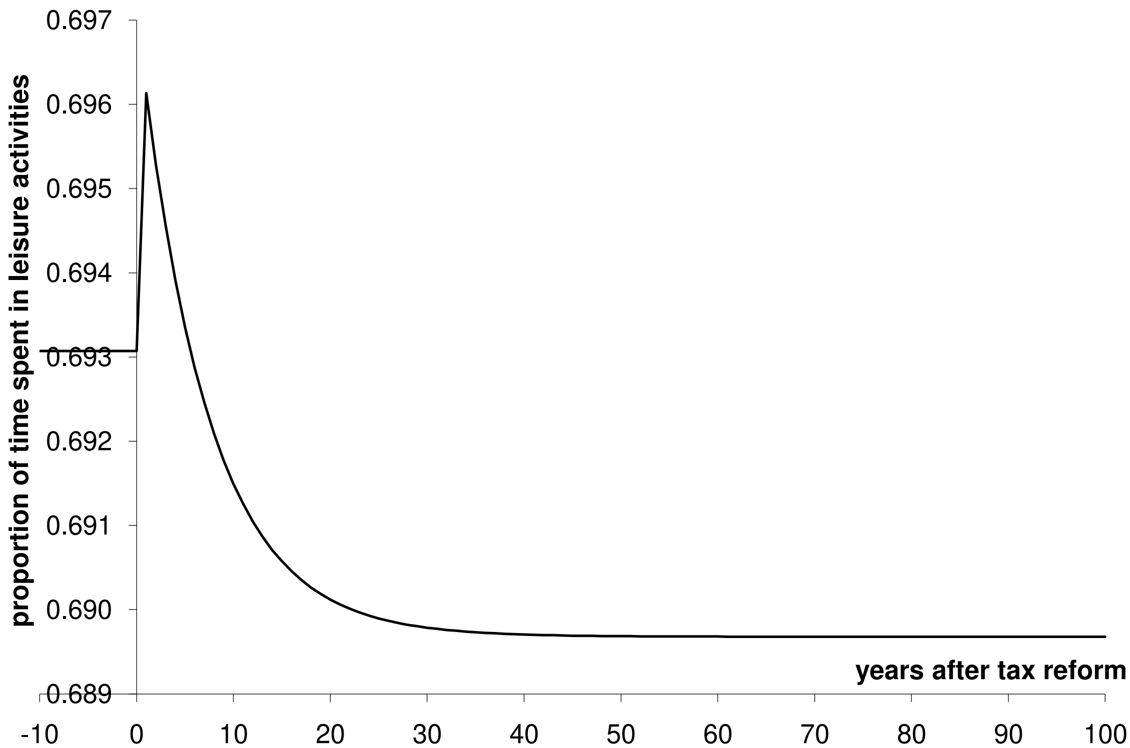


FIGURE B.9 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – LEISURE TIME (LOW IES)

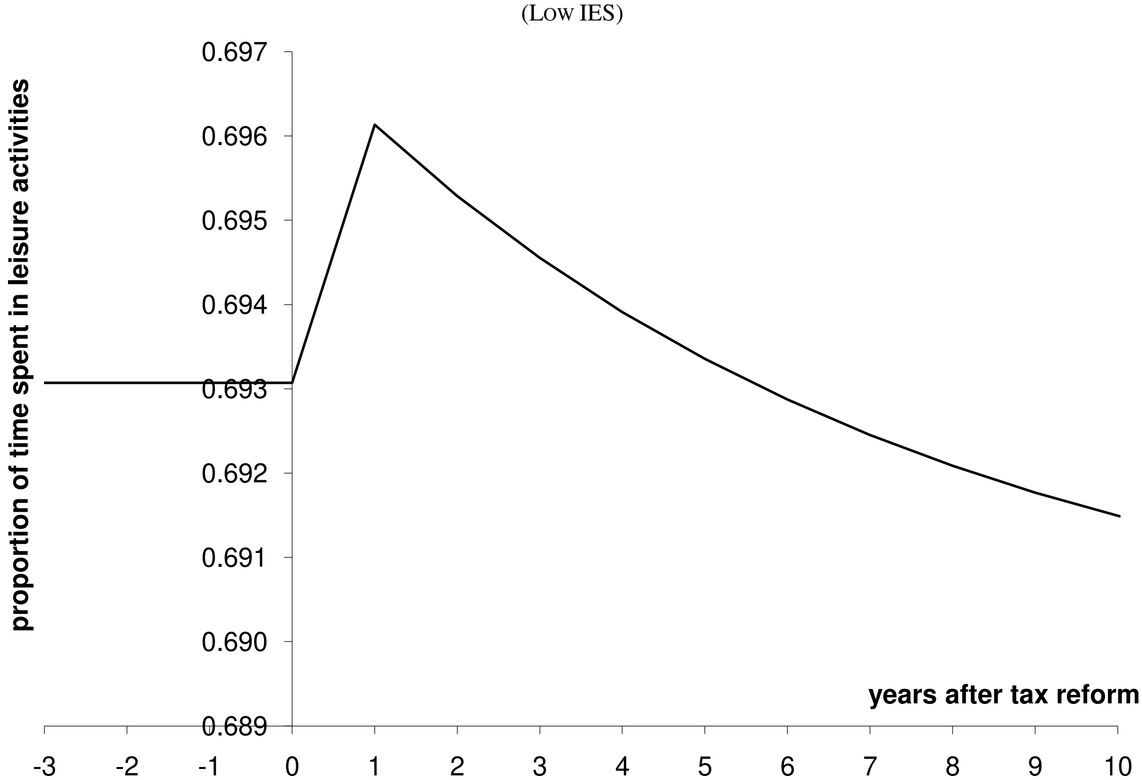


FIGURE B.10 – FLAT TAX REFORM: TRANSITION DYNAMICS – WAGE RATE (LOW IES)

FIGURE B.11 – FLAT TAX REFORM: TRANSITION DYNAMICS – INTEREST RATE (LOW IES)

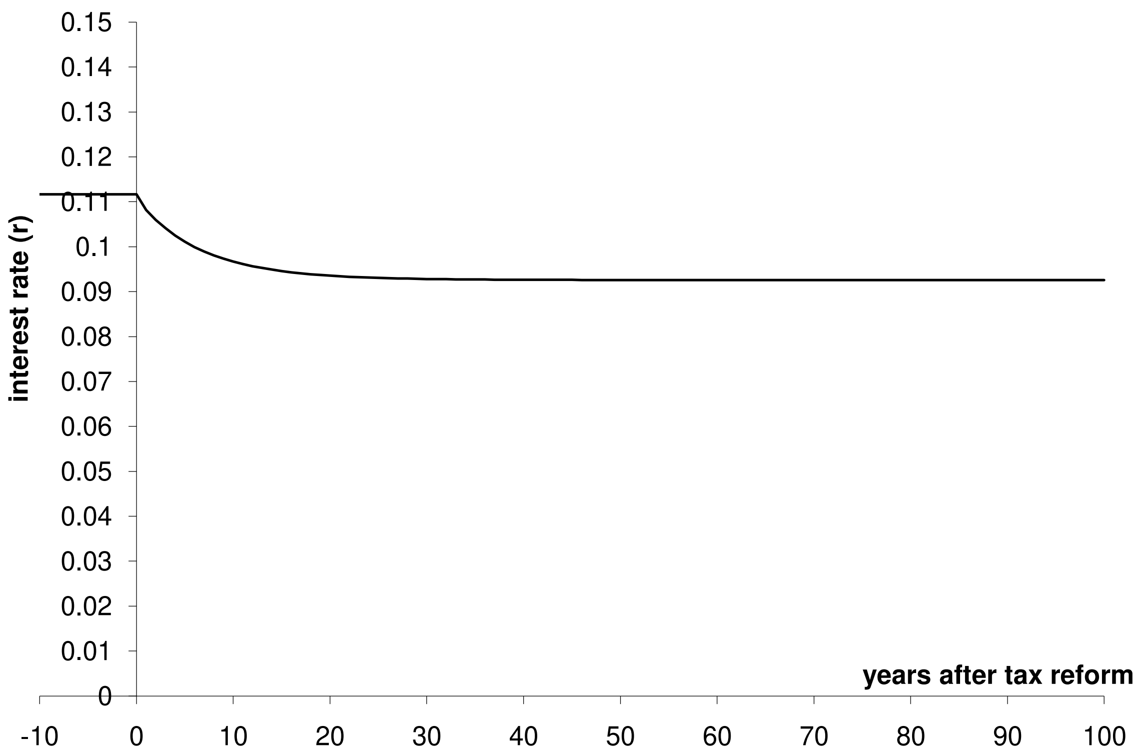


FIGURE B.12 – FLAT TAX REFORM: TRANSITION DYNAMICS – UTILITY (LOW IES)

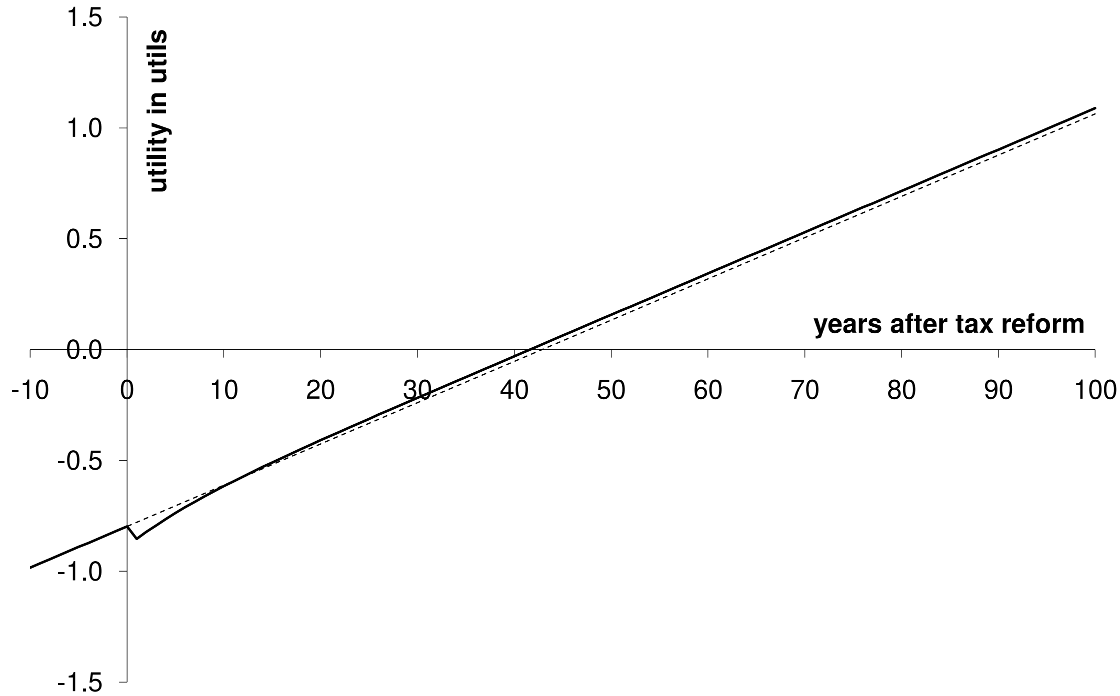


FIGURE B.13 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – UTILITY (LOW IES)

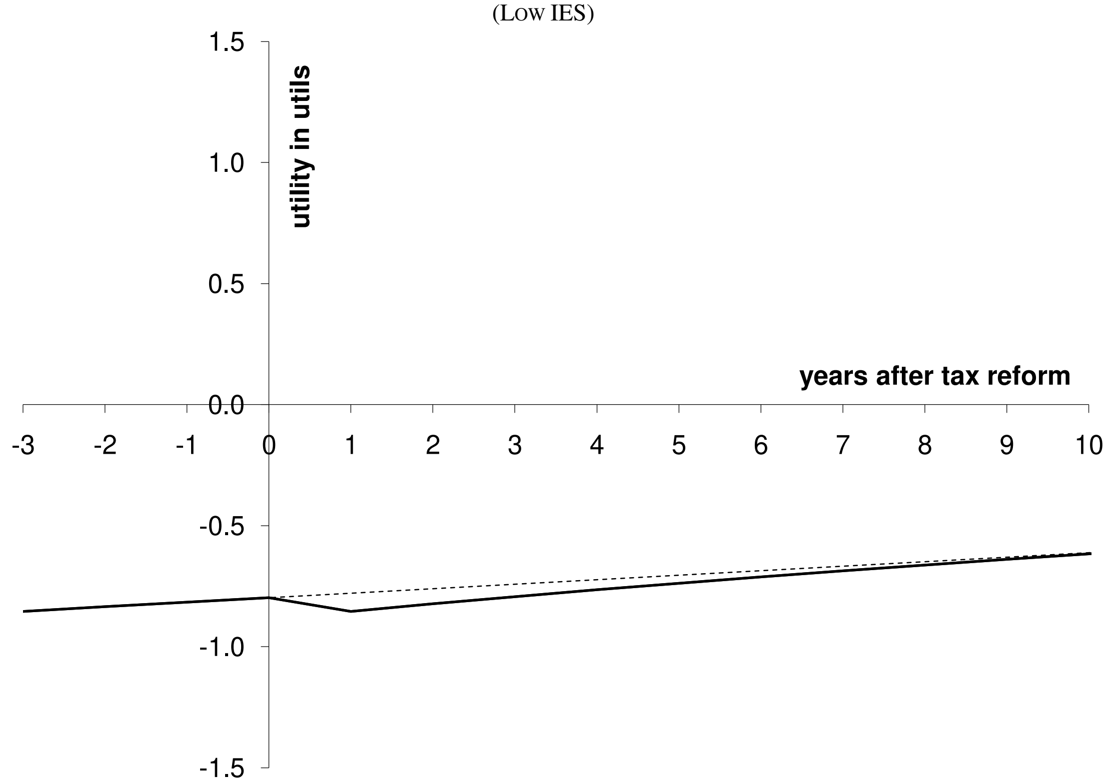


FIGURE B.14 – FLAT TAX REFORM: TRANSITION DYNAMICS – UTILITY CHANGE (LOW IES)

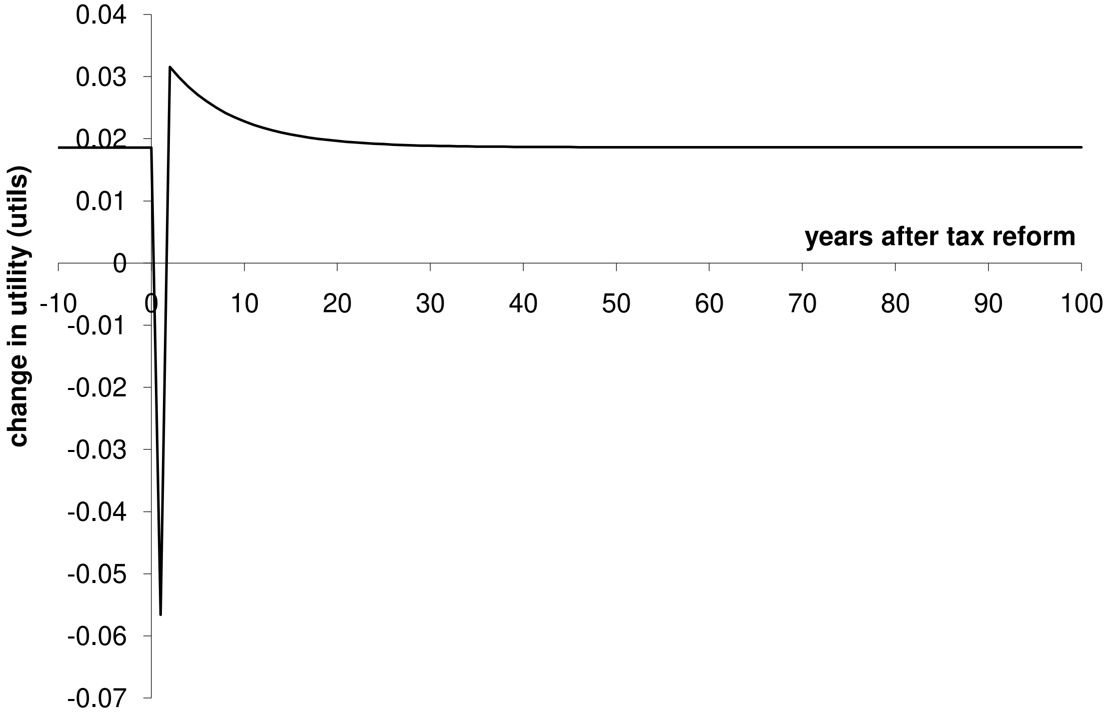


FIGURE B.15 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – UTILITY CHANGE (LOW IES)

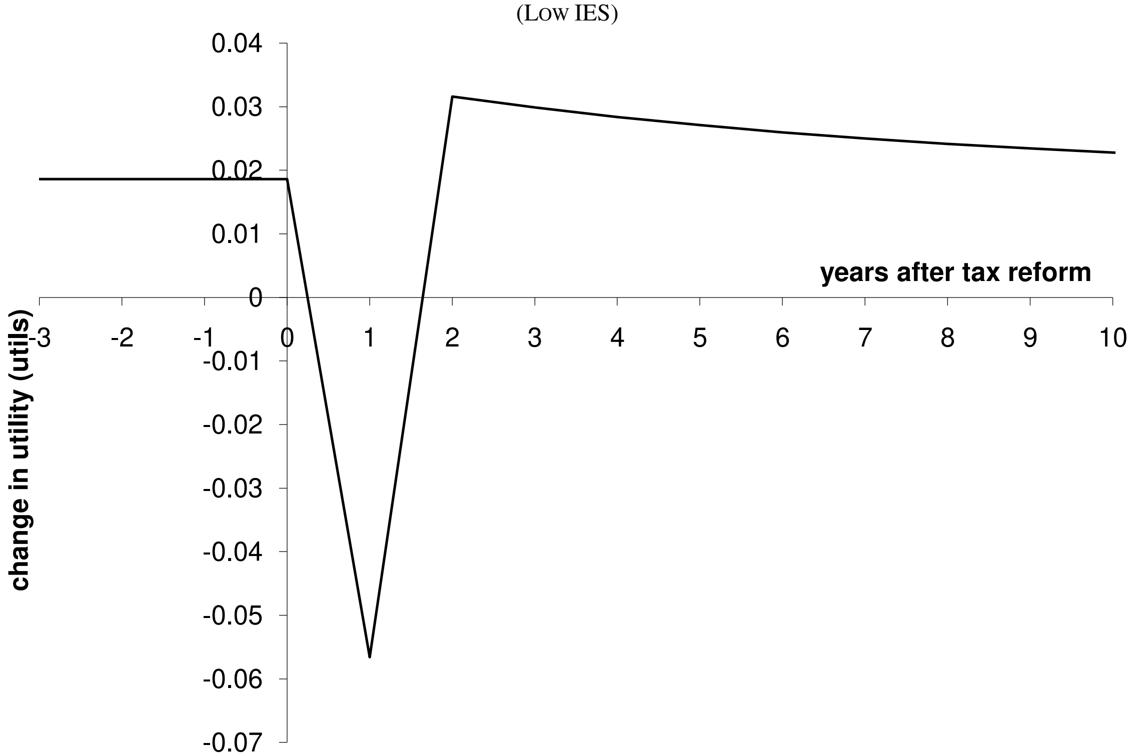


FIGURE B.16 – FLAT TAX REFORM: LONG-RUN TRANSITION DYNAMICS
PATH OF UTILITY OVER TIME: CONSUMPTION-EQUIVALENT VARIATION AS A PROPORTION OF AGGREGATE OUTPUT
(LOW IES)

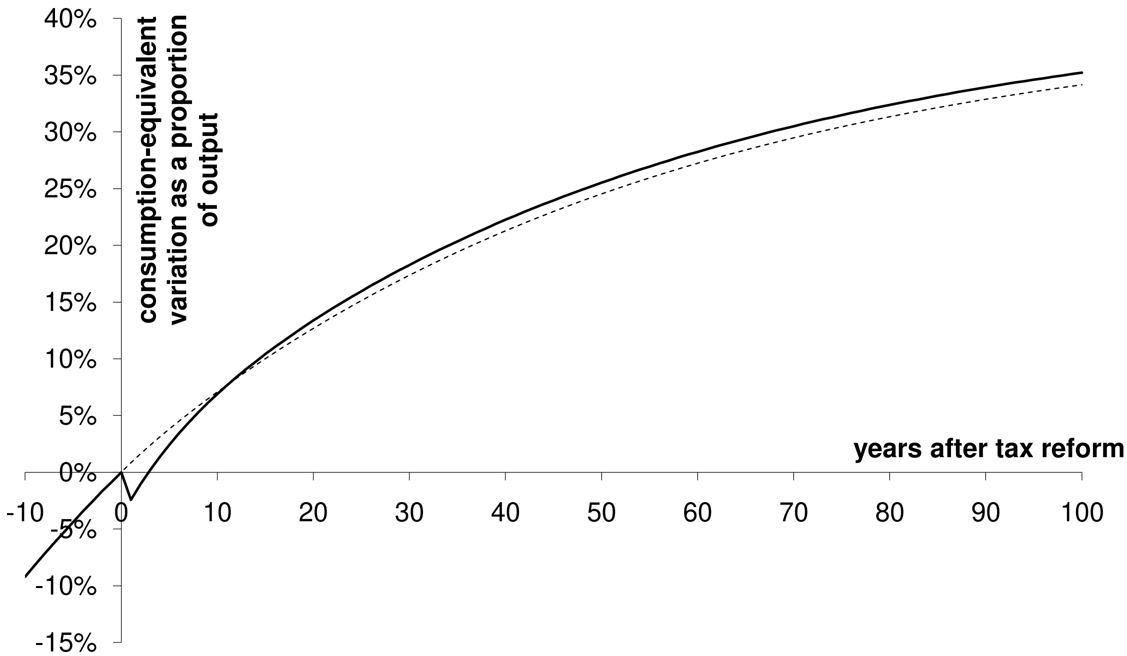


FIGURE B.17 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS
PATH OF UTILITY OVER TIME: CONSUMPTION-EQUIVALENT VARIATION AS A PROPORTION OF AGGREGATE OUTPUT
(LOW IES)

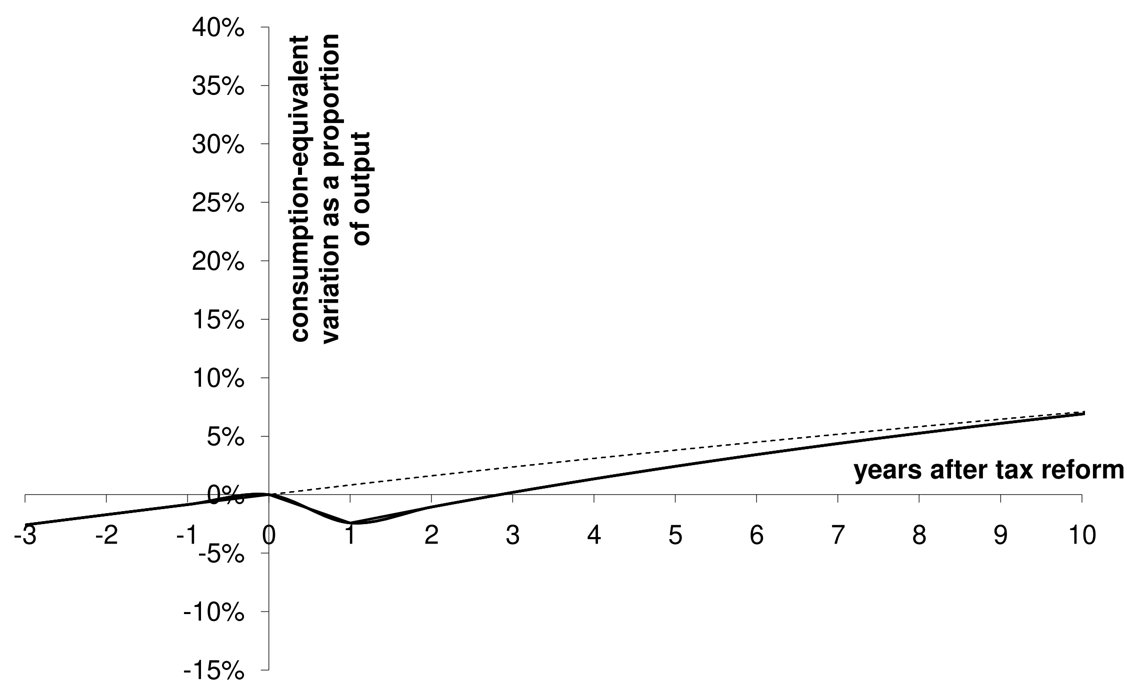


FIGURE B.18 – FLAT TAX REFORM: LONG-RUN TRANSITION DYNAMICS
UTILITY WITH VS. WITHOUT REFORM: CONSUMPTION-EQUIVALENT VARIATION AS A PROPORTION OF AGGREGATE OUTPUT (LOW IES)

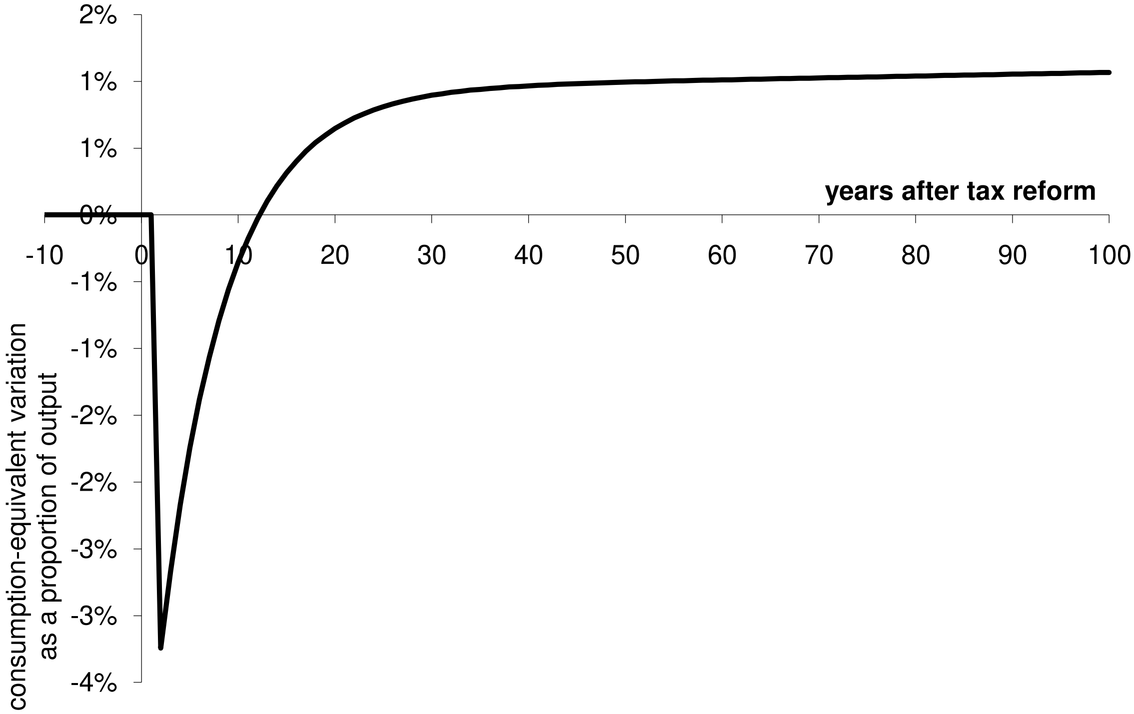


FIGURE B.19 – FLAT TAX REFORM: LONG-RUN TRANSITION DYNAMICS
UTILITY WITH VS. WITHOUT REFORM: MULTIPLICATIVE WELFARE GAIN (LOW IES)

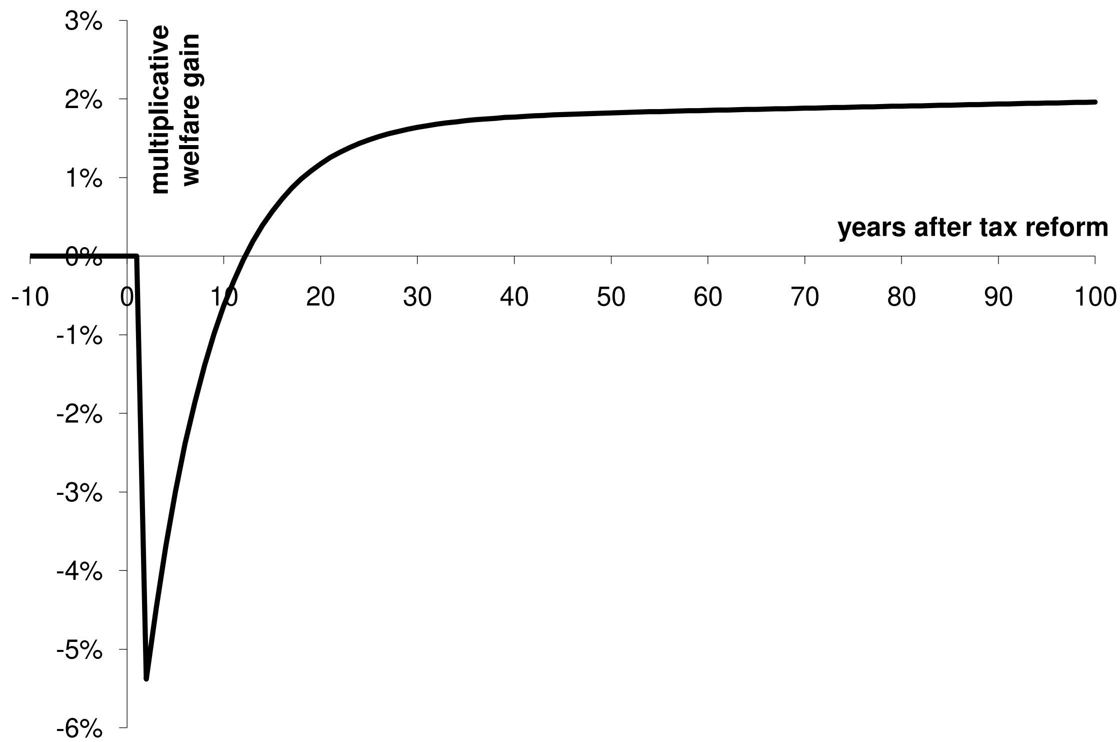


TABLE B.2 – FLAT TAX REFORM: BALANCED GROWTH PATH CHARACTERISTICS (HIGH IES)

Characteristics of the Balanced Growth Path After the Flat Tax Reform (high IES = 1)			**Comparison**	
			Before the reform	**Difference**
long-term growth rate	μ	**0.02115**	0.0186	+ 0.00255
time in market work	$1-l_t-e_t$	**0.219**	0.2111	+ 0.0079
time in education	e_t	**0.100**	0.09583	+ 0.004
leisure time	l_t	**0.681**	0.69306	- 0.012
consumption / GDP	$\frac{c_t}{y_t}$	**0.562**	0.606	- 0.044
investment in physical capital	$\frac{i_{kt}}{y_t}$	**0.296**	0.2453	+ 0.0507
investment in human capital	$\frac{i_{ht}}{y_t}$	**0.032**	0.0384	- 0.006
physical capital stock	$\frac{k_t}{y_t}$	**3.747**	3.1424	+ 0.605
human-to-physical capital ratio	$\frac{h_t}{k_t}$	**14.980**	20.4	- 5.42

FIGURE B.20 – FLAT TAX REFORM: TRANSITION DYNAMICS – ECONOMIC GROWTH RATE (HIGH IES)

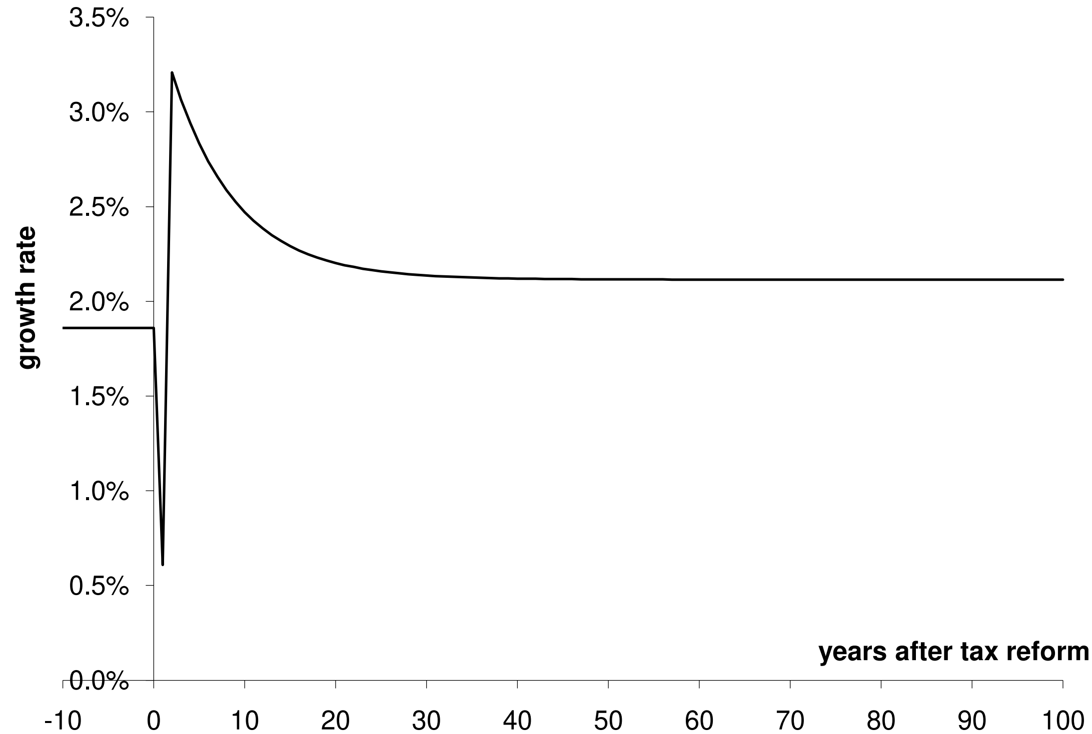


FIGURE B.21 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – ECONOMIC GROWTH RATE (HIGH IES)

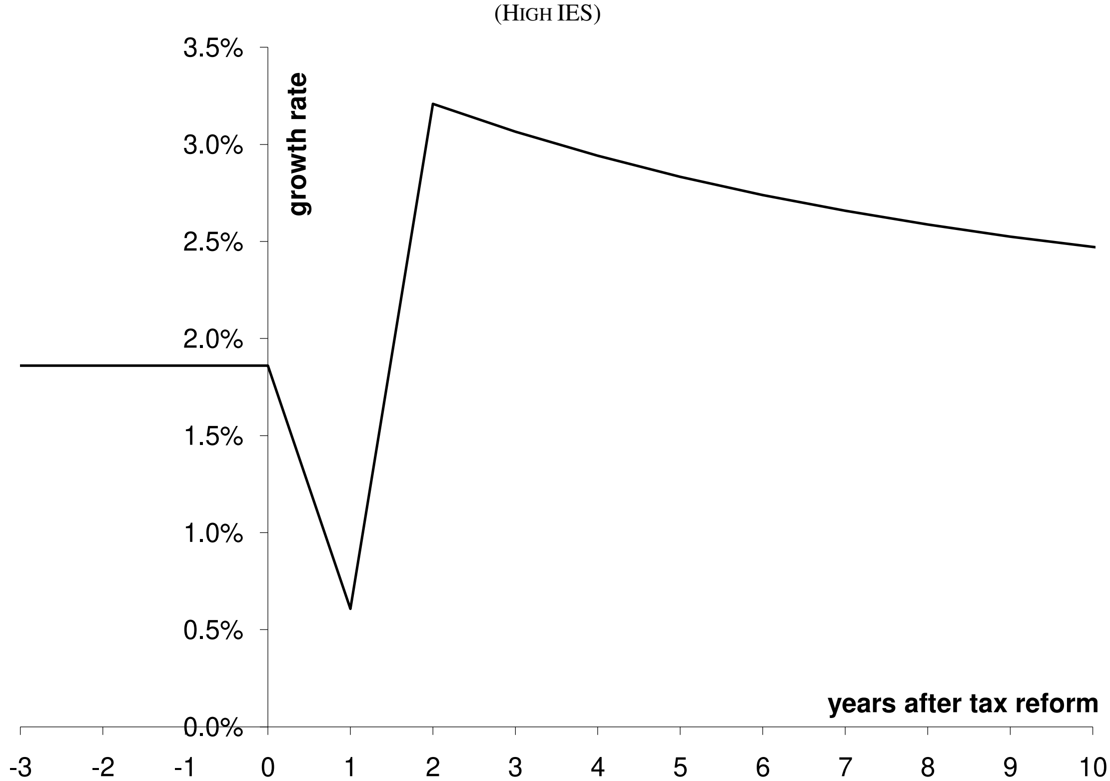


FIGURE B.22 – FLAT TAX REFORM: TRANSITION DYNAMICS – INDEX OF AGGREGATE OUTPUT (HIGH IES)

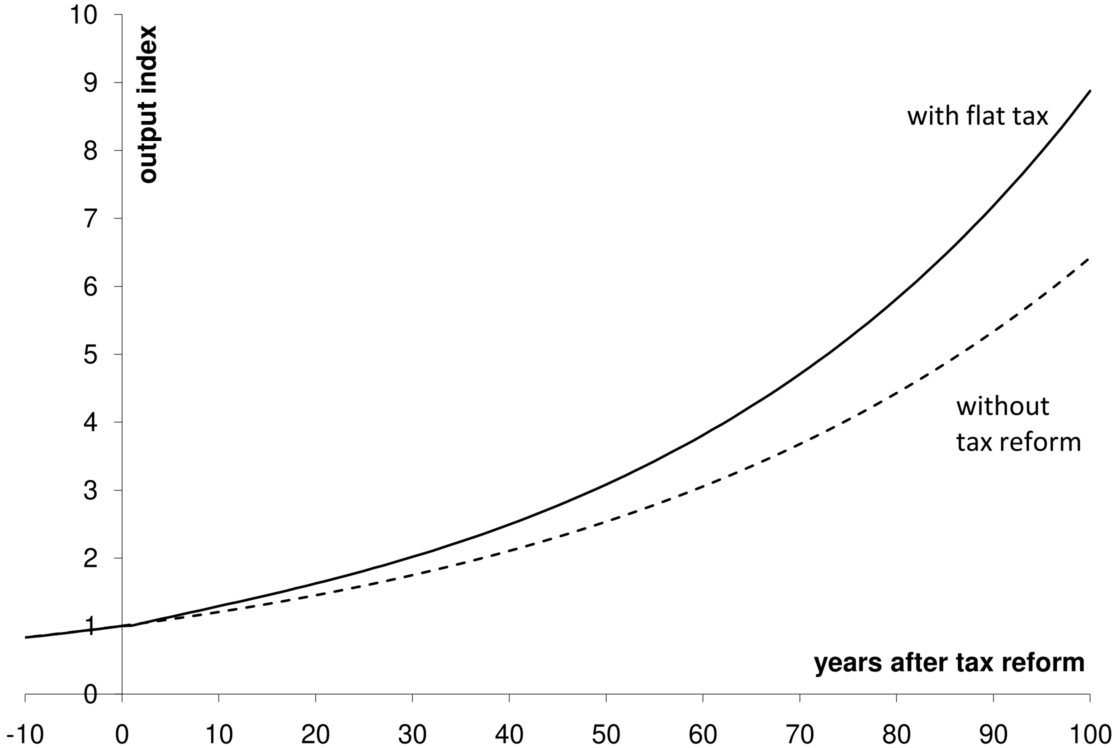


FIGURE B.23 – FLAT TAX REFORM: TRANSITION DYNAMICS – TIME IN EDUCATION (HIGH IES)

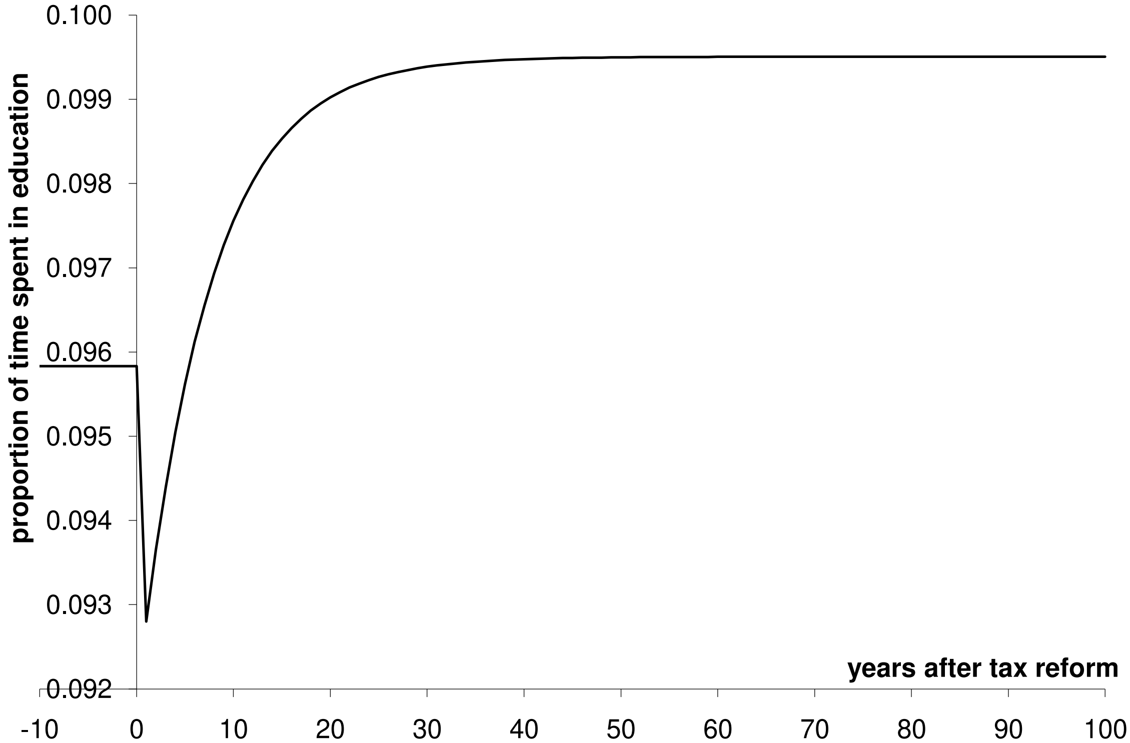


FIGURE B.24 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – TIME IN EDUCATION (HIGH IES)

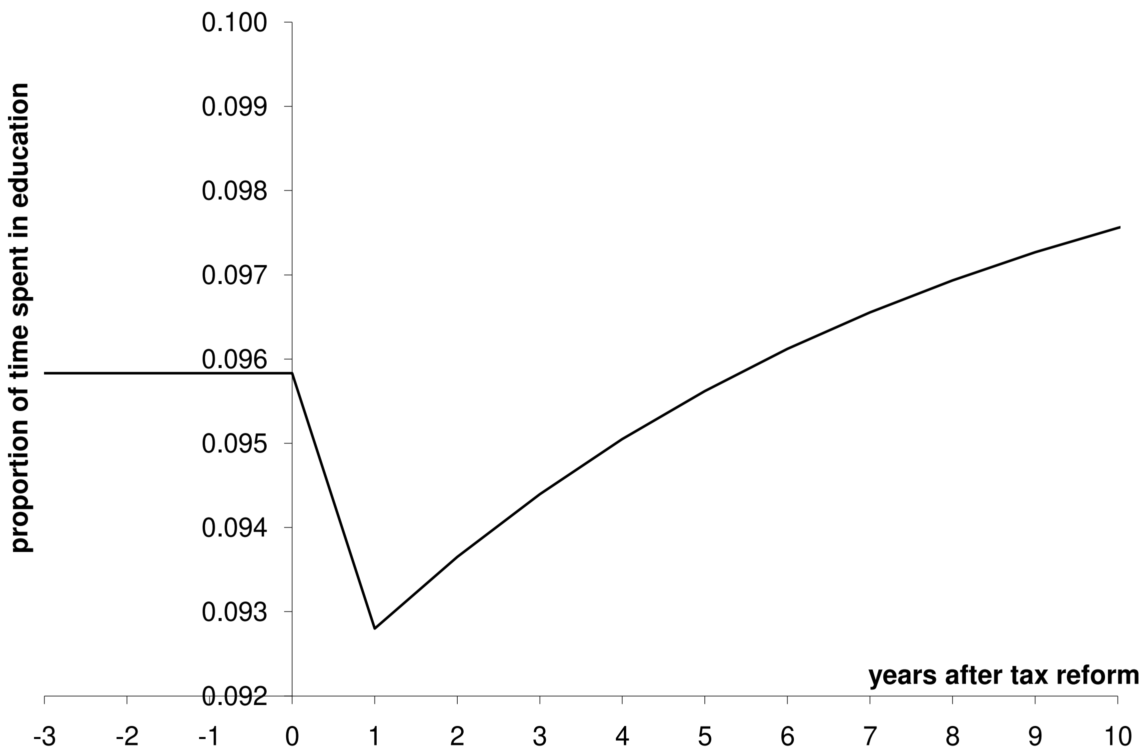


FIGURE B.25 – FLAT TAX REFORM: TRANSITION DYNAMICS – TIME IN MARKET WORK (HIGH IES)

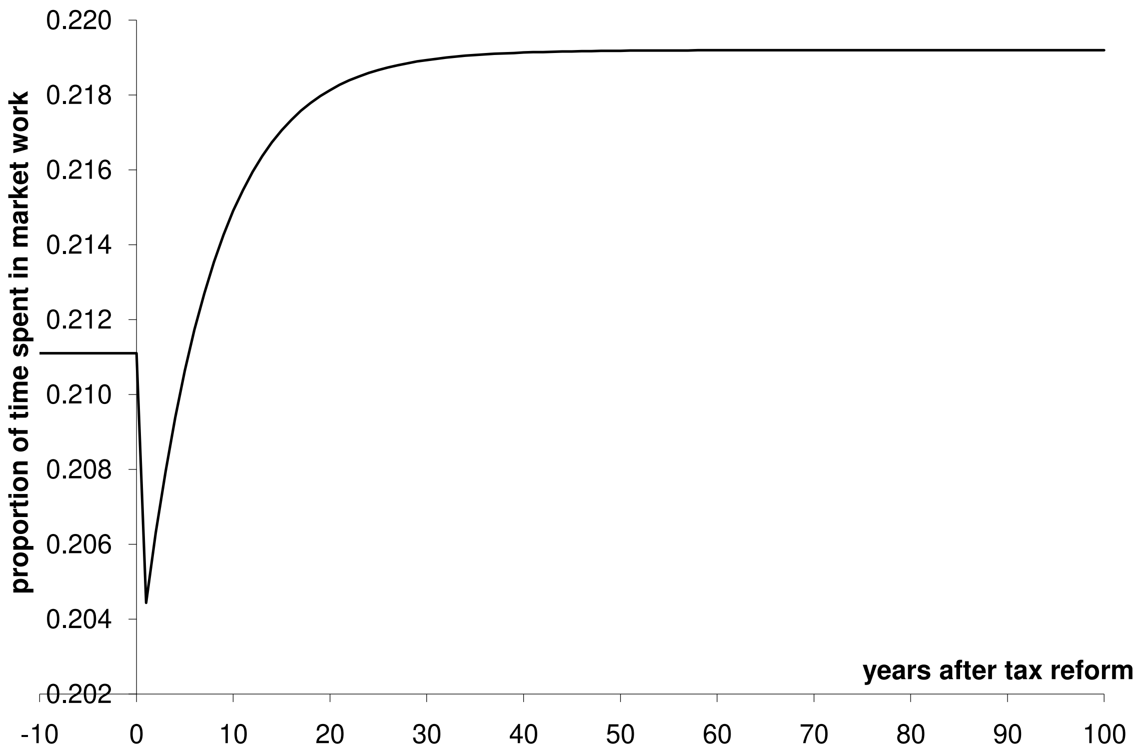


FIGURE B.26 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – TIME IN MARKET WORK (HIGH IES)

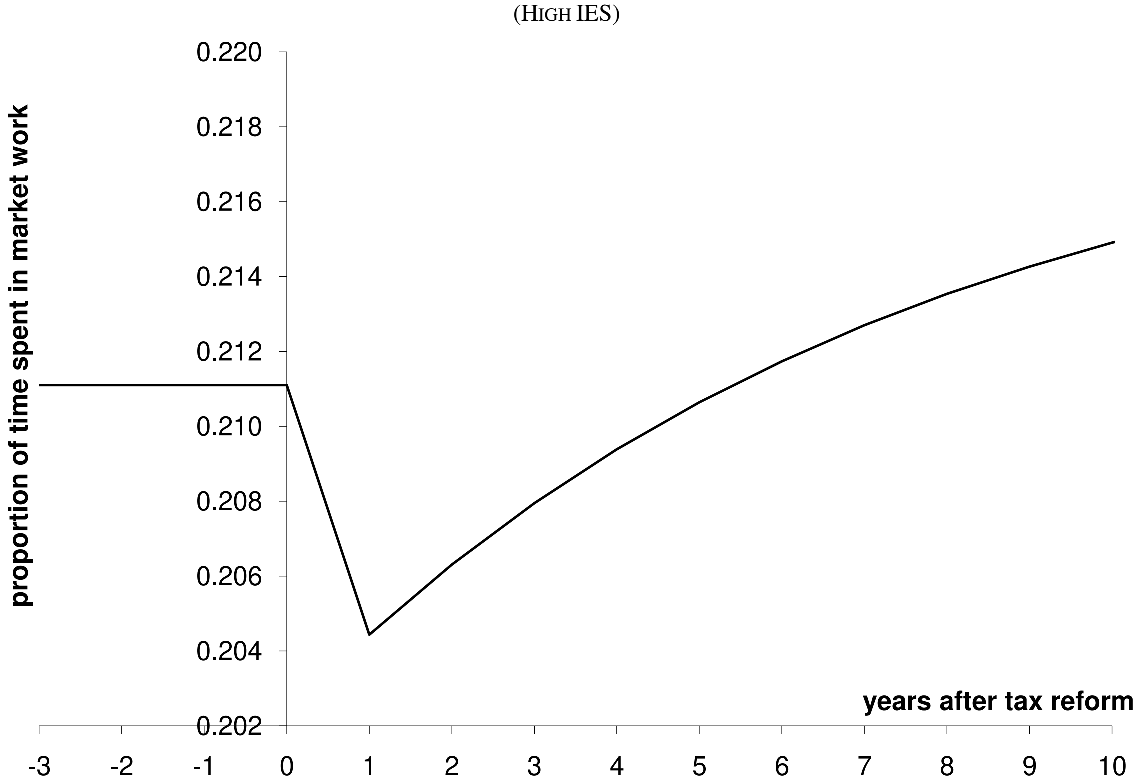


FIGURE B.27 – FLAT TAX REFORM: TRANSITION DYNAMICS – LEISURE TIME (HIGH IES)

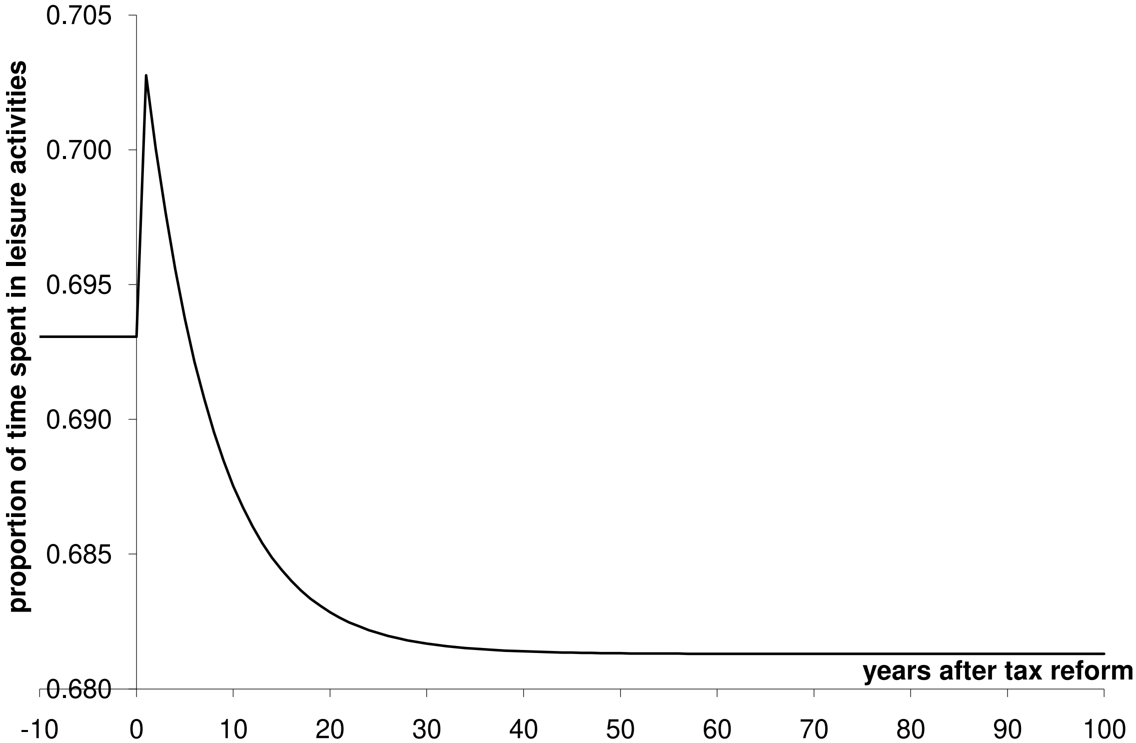


FIGURE B.28 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – LEISURE TIME (HIGH IES)

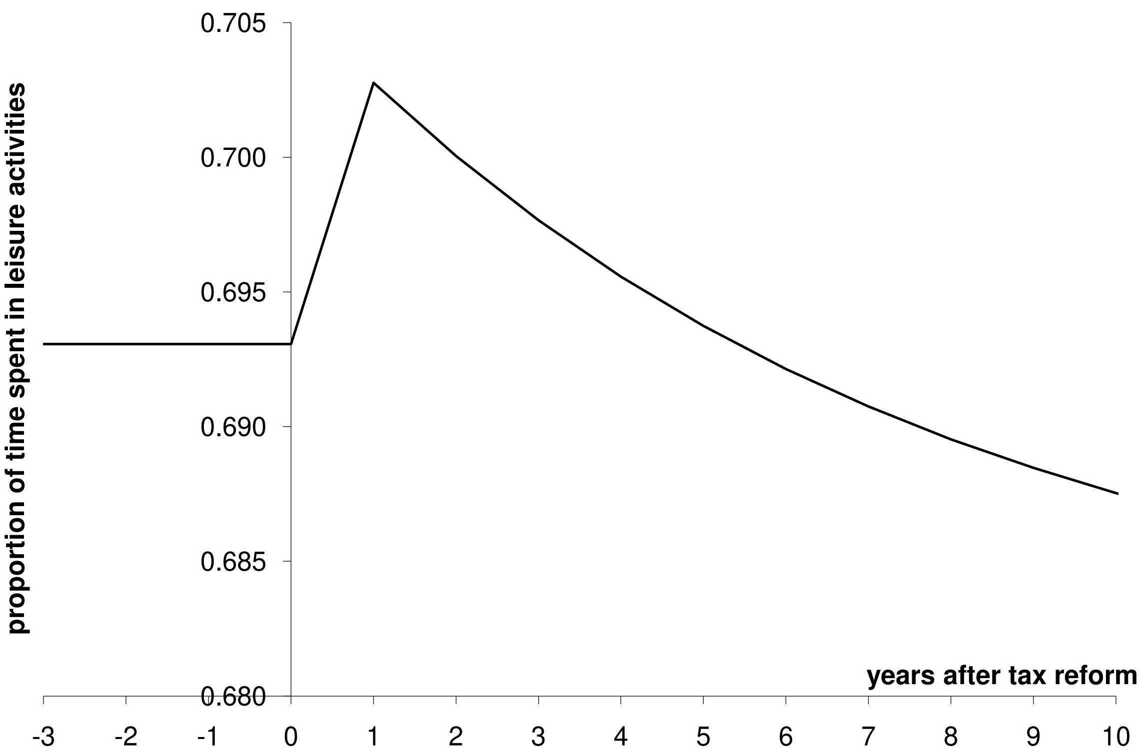


FIGURE B.29 – FLAT TAX REFORM: TRANSITION DYNAMICS – WAGE RATE (HIGH IES)

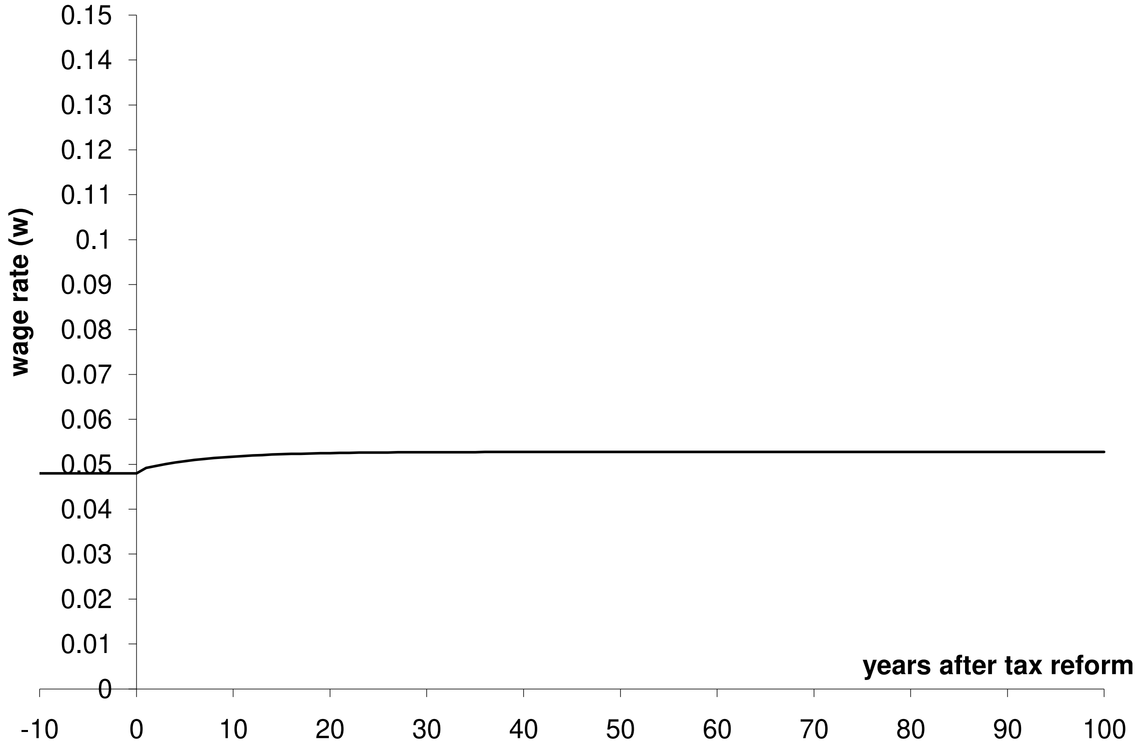


FIGURE B.30 – FLAT TAX REFORM: TRANSITION DYNAMICS – INTEREST RATE (HIGH IES)

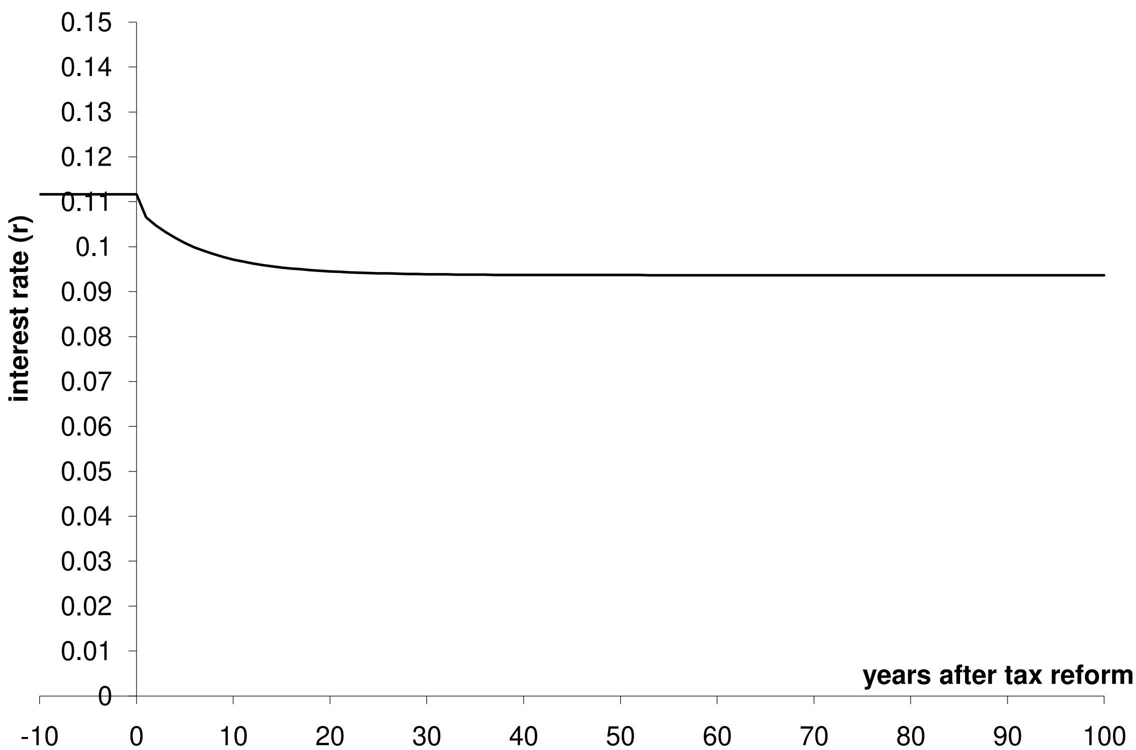


FIGURE B.31 – FLAT TAX REFORM: TRANSITION DYNAMICS – UTILITY (HIGH IES)

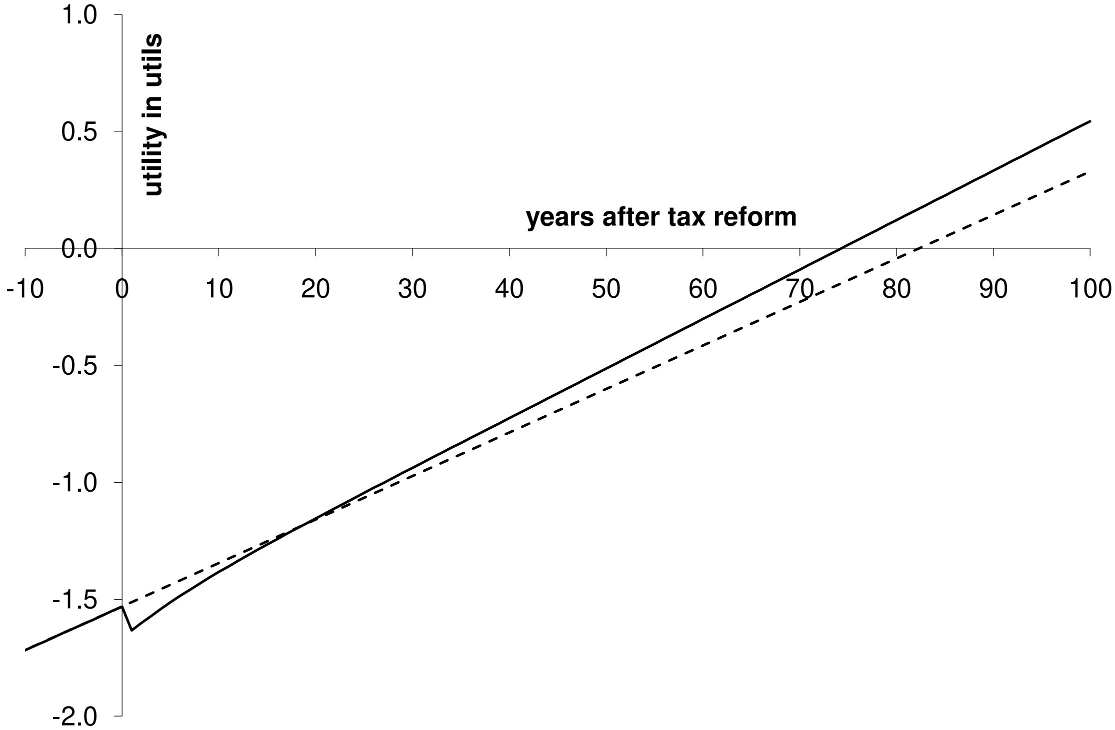


FIGURE B.32 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – UTILITY (HIGH IES)

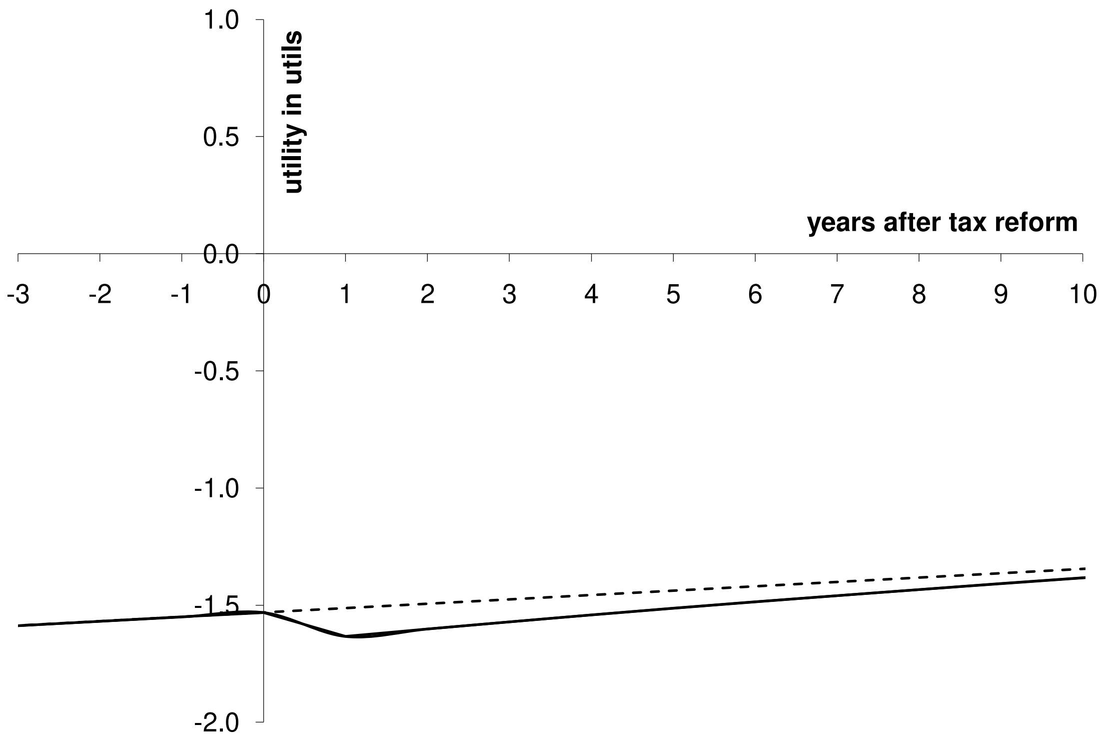


FIGURE B.33 – FLAT TAX REFORM: TRANSITION DYNAMICS – UTILITY CHANGE (HIGH IES)

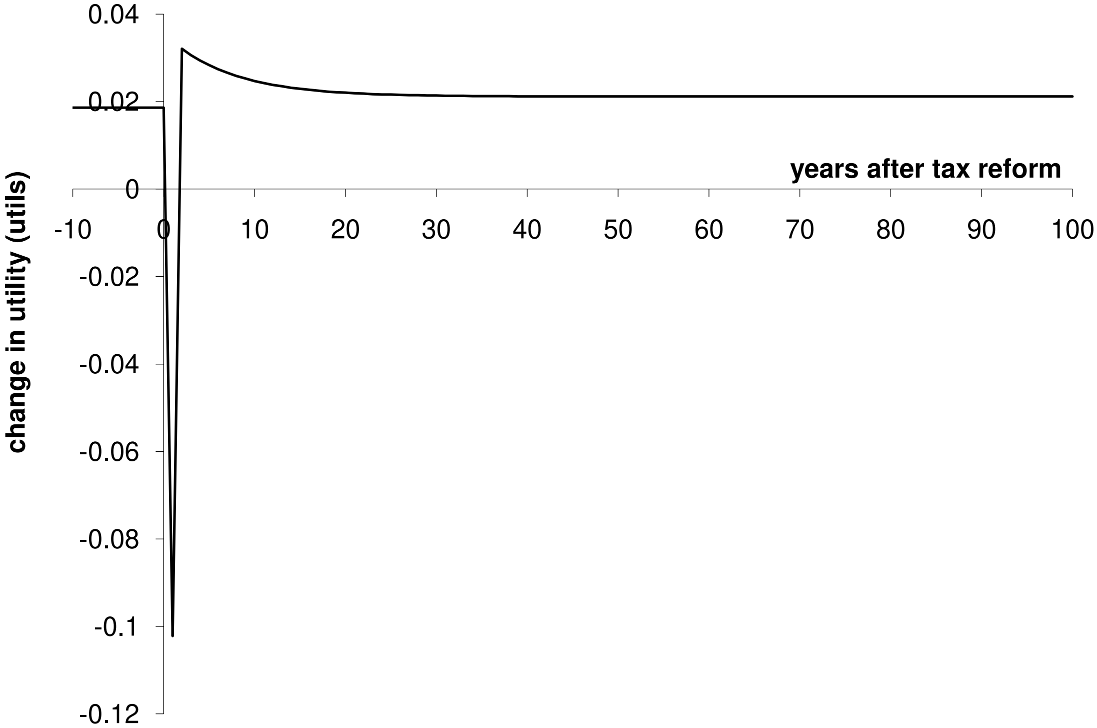


FIGURE B.34 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS – UTILITY CHANGE (HIGH IES)

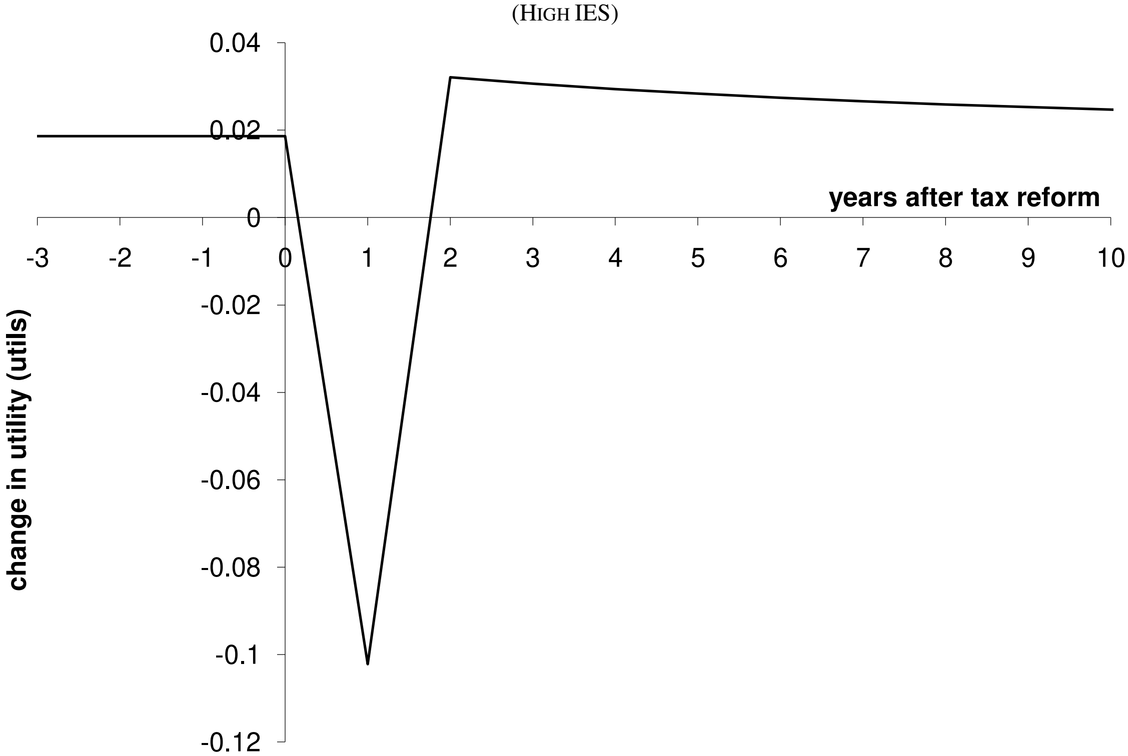


FIGURE B.35 – FLAT TAX REFORM: LONG-RUN TRANSITION DYNAMICS
PATH OF UTILITY OVER TIME: CONSUMPTION-EQUIVALENT VARIATION AS A PROPORTION OF AGGREGATE OUTPUT
(HIGH IES)

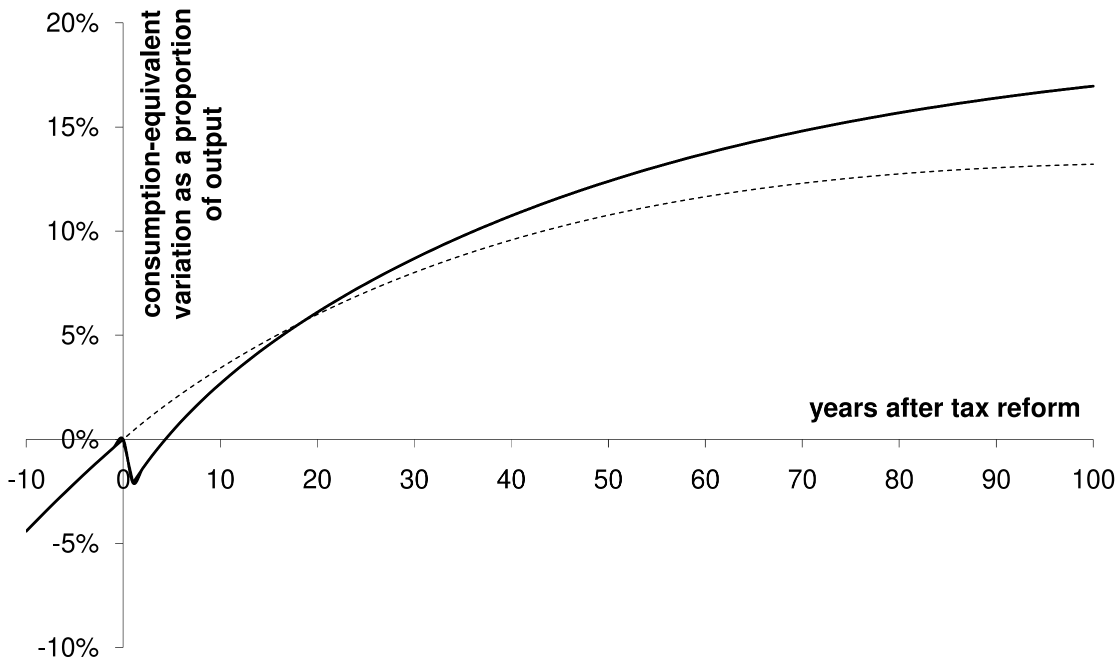


FIGURE B.36 – FLAT TAX REFORM: SHORT-RUN TRANSITION DYNAMICS
PATH OF UTILITY OVER TIME: CONSUMPTION-EQUIVALENT VARIATION AS A PROPORTION OF AGGREGATE OUTPUT
(HIGH IES)

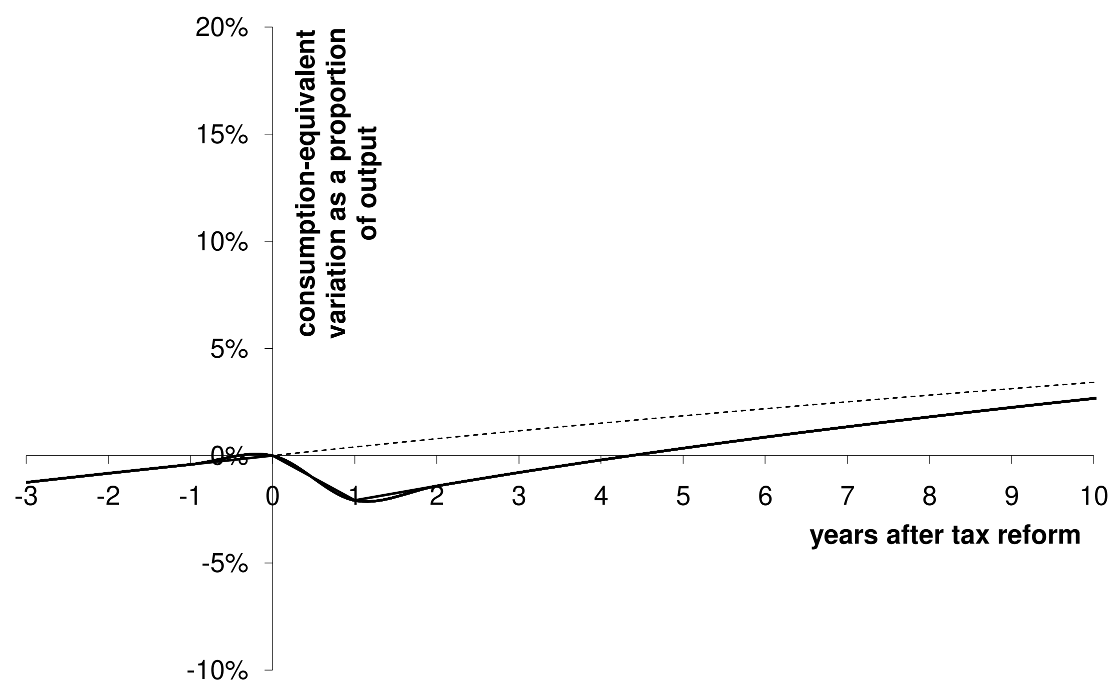


FIGURE B.37 – FLAT TAX REFORM: LONG-RUN TRANSITION DYNAMICS
UTILITY WITH VS. WITHOUT REFORM: CONSUMPTION-EQUIVALENT VARIATION AS A PROPORTION OF AGGREGATE OUTPUT (HIGH IES)

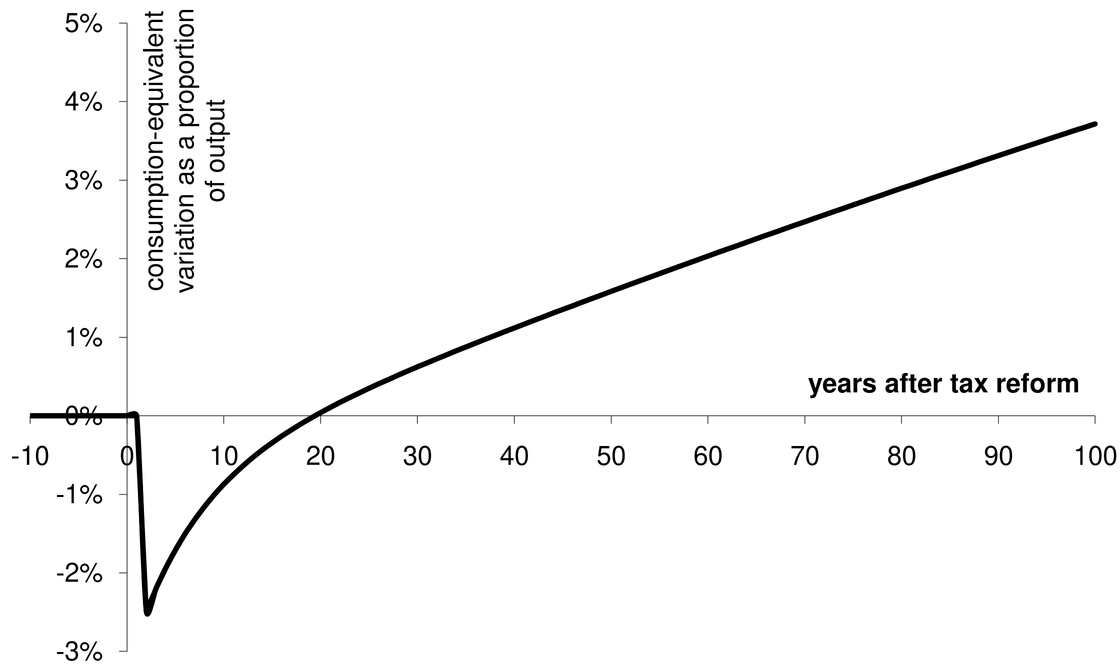
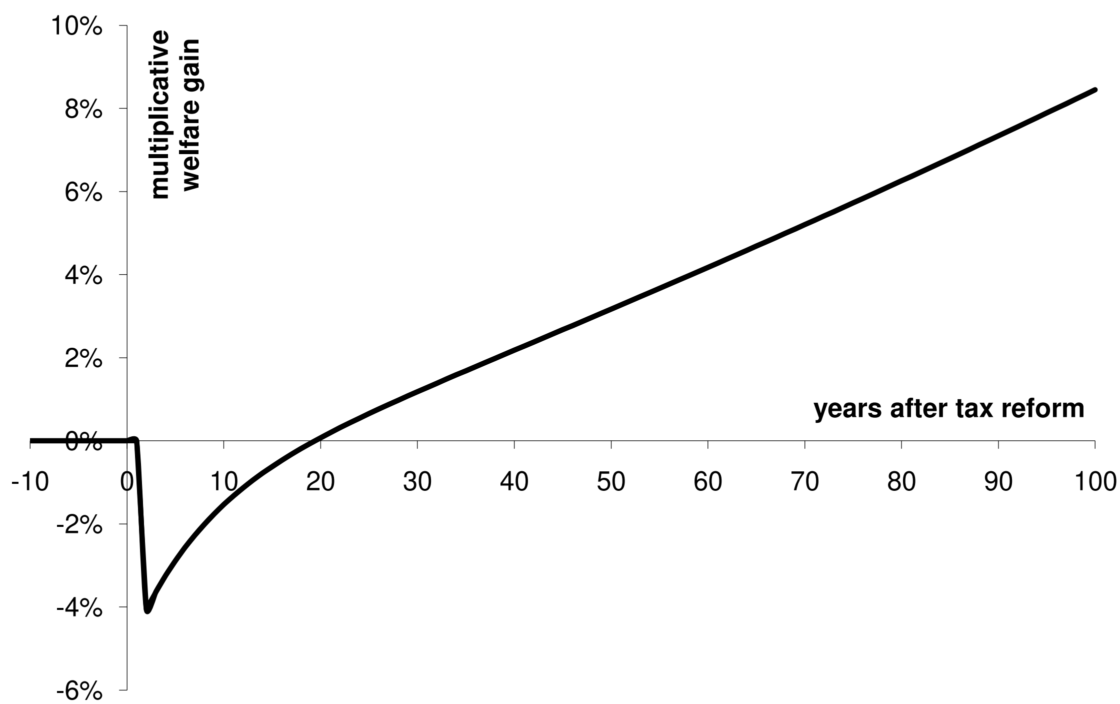


FIGURE B.38 – FLAT TAX REFORM: LONG-RUN TRANSITION DYNAMICS
UTILITY WITH VS. WITHOUT REFORM: MULTIPLICATIVE WELFARE GAIN (HIGH IES)



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