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INTRODUCTION*

Structural analysis is the main topic of this paper and structural change is a dominant theme of the present work, since it is mainly related to changes in the structure of the production system caused by the evolution of technical progress and marked organizational changes.

The analysis of structural models and of theories of structural changes carried out in this paper has a double meaning. On the one hand, it allows to pick up several essential principles that characterize these models, on the other hand, it should allow us to reconsider some important methodological issues under a new light, such as different methods of decomposition of the productive systems, the problem of complexity and the strategies to reduce complexity.

The economics of structural change has a long story which goes back to the Classical Economists, as Hagemann, Landesmann and Scazzieri (2003) pointed out:

« Classical Economists... concentrated their attention upon distinct phases of economic development: in their view, each particular phase is characterized by a distinct pattern of interaction of fundamental variables, and may be associated with a distinct representation of economic structure »[1].

« A characteristic feature of the classical approach is the classification of economic activities into a simple scheme, and the reduction of economic dynamics to the interaction among a few critical variables »[2].

In the following pages, I limit my analysis to the models of Leontief, von Neumann, Sraffa and Pasinetti, since these economists, similarly to Classical Economists, focus their analysis on economic structure as the crucial factor in understanding the working of economic systems. Indeed, their models may be connected to the theoretical line that is centred on the analysis of the conditions of reproduction of the economic system. The foremost economists of this line of research are Quesnay and Classical Economists like Marx, but also some economists of the Russian-German School, such as Dmitriev and von Borkiewicz, who are at the root of von Neumann's and Leontief's contributions. The choice of Leontief's, von Neumann's, Sraffa's and Pasinetti's models is relevant because it is possible to single out analytical principles and discuss methodological issues, which are at the root of structural change analysis, with the aim of contributing towards a further epistemic advance.

In general, the analysis of structural models brings out a clear distinction between the description of the production system in terms of material flows reflecting inter-industry relationships and the description of the same system in terms of a set of vertically integrated sectors. This methodological aspect is pointed out in the paper, and the models are also distinguished in relation to it.

Moreover, the paper tries to compare Quesnay's *Tableau,* taken as a benchmark model, with Leontief's, von Neumann's and Sraffa's models to pick up the different features of these models with respect to his theoretical framework and also to identify their characteristics for structural analysis and structural change.

The paper is organized as follows: the first section examines the input-output method trying to focus on complexity, on the method of decomposition by industries and on the hypothesis of limited variety which marks Leontief's structural analysis. The second section focuses on von Neumann's "balanced growth" growth model, which is characterized by a path of proportional dynamics, based on the circularity of production process and on the assumption of non-decomposability. The third section discusses two different methodological approaches, which characterize structural models,

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[1] Hagemann, Landesmann, Scazzieri (2003), pp. XIII.

[2] Hagemann, Landesmann, Scazzieri (2003), pp. XIV.

based on circular process and on vertical integrated process respectively; in particular, it analyzes Sraffa's *subsystems* and Pasinetti's *vertically integrated sectors*. The fourth section adopts Quesnay's *Tableau* as a benchmark model for structural analysis. Conclusions end up the paper.

1. THE INPUT-OUTPUT METHOD OF ANALYSIS: COMPLEXITY, DECOMPOSITION AND THE ASSUMPTION OF LIMITED VARIETY

The *input-output* method considers the quantitative interdependence among different productive activities of an economic system. The interdependence among single sectors or 'industries' of an economic system is described by a system of linear equations. The specific structural characteristics of this system are thus reflected in the coefficients of the equations and these coefficients are determined empirically.

In his *Structure of American Economy 1919-1939*[3], Leontief presents frameworks of inter-industry relationships based on two systems of linear equations, one is referred to physical quantities, the other to prices. There is a first framework which is 'closed' with respect to final demand, since final demand is considered to be one among the industries of the system. Here, the system of physical quantities determines the *structure* of the economy but not its *scale* of operation, as the model assumes constant returns to scale in each industry.

Leontief means by *structure*:

« the interdependence between the quantities of the inputs absorbed and the amounts of the product or products turned out in a given process »[4].

These relationships reflect the internal structure of each individual sector of the economy, and

« these relationships are anything but simple »[5].

This 'closed' model does not entail a dynamic analysis, as the analysis carried out on its basis is limited to a virtual state of simple reproduction, in which there are no savings neither investment *(stationary state)*.

Leontief goes further towards a dynamic analysis and introduces another model[6], which he calls 'the open model with respect to final demand'. In this model, the matrix of technical coefficients does not include the consumption coefficients and the labour coefficients, whereas there is a separate vector of direct labour coefficients. This matrix and this vector together represent the technology of the system. The model also has a vector of final demand coefficients that are exogenously determined and known.

In these models, productive relationships are of circular type. This is because the analytical representation of economic structure is based on a circular description of the flows of goods from one industry to another: each industry delivers goods and services to other industries. Conversely, in order to be able to carry out production, each industry needs goods and services that are produced in other industries. These *input-output* models thus record the circulation of goods and services going on in the course of the production process[7].

Actually, the Leontief's framework of inter-industry relationships points out the role of intermediate goods, which are also factors of production used in one or several production

[3] Leontief (1951).
[4] Leontief (1951), p.204.
[5] Ibid.
[6] Leontief (1951), pp.205-207; Leontief (1987), p.861. For an analytical representation of this 'open' model see Appendix 1.
[7] Bortis (1990), p.67.

processes. The circular concept of economic structure used by Leontief lends itself to a utilization for 'direct structural analysis'[8], that is for empirical assessment.

Leontief's models in the *Structure of American Economy* make the assumption of fixed technical coefficients, disregarding both variations in returns to scale and technical progress. At the background of the fixed coefficients is the notion, in the absence of technical change, that the system should economize on the use of the primary (i.e. non-produced) factors or resources. The mechanism which selects the techniques is substantially grounded on the non-substitution theorem[9]. In these models, Leontief focuses upon the real economy, being aware that his static *input-output* analysis is constrained by the *relative invariance* of the structural characteristics of the *input-output* system, in which the strategic factor that fixes over time the structure is technology, whereas institutions and behavioural patterns (of firms and consumers) are not taken into account. Thus, the actions of the producers are strictly determined with respect to the quantities of intermediate products needed in order to produce a given quantity of some other good. There is no free choice in this type of frameworks once the technique of production has been selected.

As a matter of fact, Wassily Leontief had already introduced the concept and the analytical framework of *circular flow* in a contribution of 1928[10]. He followed the idea, already existing in the Quesnay's *Tableau economique,* of the economic activity as a *circular process* that reproduces all the material goods used up in the process of production, so that this process can continue in the same way over the next period. The *circular flow* is considered as a process of rotation, which is indefinitely repeated. Thus a model of a system of economic flows in a stationary state economy is represented. In this model of 1928 Leontief went beyond the simple representation of the economic process and identified the objective technological framework as the basis on which to construct, together with the relevant economic factors, a theory of the economic system.

From Leontief's analysis of *circular flow* economy (1928) we can draw some observations. First, the theory of the circular flow system emphasizes the important role of technology in determining the structure of the economy, but it also introduces hypotheses of behaviour. This is especially clear in the analysis of exchange, in which Leontief sets the problem concerning the relation between income distribution and the determination of prices and shows that the solution of this problem may be found in the different possible institutional set-ups related to the social organization of ownership. In doing so Leontief introduces *degrees of freedom* in his analytical system, which thus becomes an 'open' one. Second, the theory of *circular flow* is more flexible than his *input-output* analysis, because it considers not only technology, but also other elements (i.e. possible different income distribution solutions), whereas in the latter case it is technology that thoroughly determines the structure of economic system. Third, this theory of *circular flow*, despite all the attempts made by Leontief to consider the case in which technical coefficients may change unevenly, is associated with the absence of structural change in the stationary state. Finally, it is worth stressing that the analytical framework of the *circular flow* introduces a pattern of interrelationship among production activities, and that these activities are, in any case, limited in number.

In his later work *Studies in the Structure of the American Economy*, Leontief undertakes the analysis of structural change[11]. One important difference with respect to the *Structure of the American Economy* is that in the previous *input-output* frameworks he had considered only *flows* of goods and services, whereas in the *Studies* he also considers commodity *stocks.* Hence he allows the empirical analysis of the investment process. With the introduction of stock-flow relationships Leontief overcomes the *stationary state* and moves towards a dynamic analysis, in which the assumption that a *constant proportion* of input flows is allocated to investment is dropped (as it is

[8] Leontief (1966), p.39.

[9] Following this theorem, if there would be another technique to produce good *i*, the mechanism eliminates the technique in use, if incorporating it would result in a higher social or macroeconomic use of the primary resources.

[10] Leontief (1991[1928]). This is Leontief's PhD thesis initially published in German "Die Wirtschaft als Kreislauf" in 1928 and later translated and published in English in the journal *Structural Change and Economic Dynamics"(* 1991).

[11] Leontief (1953) chp. 2.

possible to analyse an economy whose sectors *do not* grow in a uniform way). It is important to stress that the dynamics of stock-flow relationships accounts for one aspect of economic change only: what can be explained in terms of invariant structural constants. The other, more deep-rooted causes of transformation are to be found in the variation of the basic structural relationships themselves, that is, in changes in consumers' tastes and in the structure of productive processes.
Leontief also gives a definition of statics and dynamics in terms of his *input-output* frameworks and of the structural characteristics of the system[12]. In particular, his definition of statics is not associated with the concept of equilibrium, as in traditional analysis. This is because Leontief associates the change in the variables of any given economic system with the observed variations in the basic structural relationships (such as changes in the structure of productive processes), As a result, Leontief's conception of statics highlights both the aspect of structure and that of variation. This means that in Leontief's static analysis the structural relationships show a relative invariance, making the change limited in extent. Also Leontief's definition of dynamics highlights both aspects of structure and change. In this case, the change in the value of variables over time is explained in terms of a fixed *empirical law of change.* Such a law is an invariant structural characteristic of the system[13].

The analysis of structural change, in particular, embodies both aspects of structure and change. Such an analysis is developed by Leontief in a static context, by comparing the empirical relationships of the (American) economy observed at different points of time, and trying to identify possible substitutions of new combinations of inputs with respect to the old ones.

«The usefulness of the static input-out approach in the study of an actual economy is conditioned by the relative invariance of its structural characteristics»[14].

Hence structural change in his *input-output* framework is defined as

« a change in the structural matrix of the system »[15],

where the investigation of the causes that have determined the change in the structural matrix (such as technical progress) is not carried out.

It is worth noting that *input-output* analysis is based on disaggregation and therefore on the decomposition of the productive system into a *limited number* of sub-units, such as the 'industries' or sectors, which identify the productive processes. This approach makes the analysis of structural change easier, because technical change is observed in each industry as the change of one or few coefficients, where the relative persistence of certain relationships and/or of certain elements is explicitly taken into account[16]. This shift in the focus of analysis from the *continuum* of heterogeneous activities (as being asserted by the neo-classical theory) to a relatively small number of sectors is typical of the economic analysis of structural change. Thus, a *finite variety* of features and activities is envisaged. Moreover, the notion of 'relative structural invariance', which may be implicitly derived from the analysis of technological change, becomes a distinctive feature in the analysis of structural change[17].

In Leontief's analysis, economic change may alternatively be explained as structural change, or as a dynamic process. In the former case, the variation of dependent variables is related «to the

[12] Leontief (1953), p.53.
[13] Ibid. p.53: « Dynamic theory thus enables us to derive the empirical law of change of a particular economy from information obtained through the observation of its structural characteristcics at one single point of time ».
[14] Leontief (1953), p.19.
[15] Leontief (1953), p.19. «Economic systems with identical sets of input coefficients can be said to be structurally identical, and systems with unlike technical matrices structurally different. Structural change, in other words, is a change in the structural matrix of the system»˙.
[16] Leontief (1987), p.863.
[17] On the 'relative structural invariance' see Landesmann e Scazzieri (1996), pp.6-8; se also Hagemann, Landesmann and Scazzieri (2003), pp.XI-XIII.

underlying changes in some of the basic data»[18], in the latter case the law of change itself is considered to be given, that is, as 'built within' the structure of the explanatory framework[19]. Also the *empirical law of change* could change over time. This is the case of structural change in a dynamic system, or *structural dynamics*, which leads to a much more complex type of analysis.

Hence in the *Studies,* Leontief makes an explicit distinction between three levels of analysis: structural change, dynamics and structural dynamics. Such a distinction lends itself to differences in theories, that is to different methods for describing and explaining the observed facts, rather than to some intrinsic property of the same observed world. This is why Leontief believes in a hierarchic relationship among these theories, which joins them together, rather than in a contrast that would make such theories alternative and therefore mutually exclusive. Leontief is also convinced that the generalization of any given theoretical approach (such as the one required if one wished to move from structural change analysis to dynamic analysis) presupposes the enlargement and deepening of its empirical basis.

A fundamental observation with respect to Leontief's analytical frameworks concerns the problem of complexity. As a matter of fact, one of the central questions in Leontief's analysis concerns the complexity of the real productive system and the fact that this system is undergoing continuous processes of change.

« In contrast to most physical science, we study a system that is not only exceedingly complex …but it is also in a state of constant flux »[20].

To face the complexity of interactions among economic units of the productive system and to identify both the direction and character of flows in the productive system, and the changes in the basic structural relationships, according to Leontief's strategy the following steps are necessary: (i) to extend and deepen the direct empirical knowledge of data and measurable parameters,(ii) to identify a *partition* of the set of productive activities into a *finite number* of industrial sectors.

The decomposition of the productive system into a finite number of separate but interdependent industries is a first step to reduce complexity. The process of decomposition is also a pre-condition for the analysis of dynamic properties.

The identification of invariant structural relationships is another essential analytical step to reduce complexity. This is because it takes explicitly into account the *relative persistence* of some elements or some relationships. Such *persistence* comes out in all its evidence when time is considered as a discrete variable. Moreover, the identification of invariant structural relationships depends on the type of empirical analysis to be carried out and on the problem context to be faced, as all that determines the structural specification of the model.

When a limited number of separate but horizontally interdependent industries are associated with a *finite variety* of activities, an assumption *of limited heterogeneity* is introduced. Such hypothesis has a strategic role in the analysis of structural change and allows a selective description of the productive system. This description is associated with a particular structural representation of the economy, which determines the set of relevant causal relationships. Thus the analysis of structural change which can be derived from Leontief's models is defined within specific methodological features and analytical boundaries.

The analysis of Leontief's frameworks suggests that there is a clear link between them and Quesnay's *Tableau Economique.* Both Quesnay and Leontief consider the economic activity as a circular process, yet in the *input-output* framework the structure is determined only by the *state of technology* and not by the assumptions concerning economic agents' or social classes' behaviour, as it happens in Quesnay. Naturally, Leontief in his 1928 "The Economy as a Circular Flow" had considered the system to be 'open' with respect to other economic factors like, for instance, agents'

[18] Leontief (1953), p.17.
[19] Leontief examines the dynamic model in the *Studies* (1953), chp.3, and in the *Dynamic Inverse* (1972).
[20] Leontief (1971), p.1. See also the definition and the complex nature of structure in Leontief (1951), p.204.

behaviors and institutions. Another crucial difference between the two economists is that Quesnay proposes his model as a normative model[21], that is, as a kind of benchmark model that is not aimed to observe and measure the working of a real economy characterized by a circular structure. On the contrary, Leontief develops a method for empirical analysis and proposes analytical instruments such as the inverse matrix in the static *input-output* model and the dynamic inverse for the dynamic analysis, which enables him to measure the direct and indirect effects of a change in the structure of a real system of production.

APPENDIX 1.

A1. LEONTIEF'S 'OPEN' MODEL

(1) $(\mathbf{I} - \mathbf{A})\ \mathbf{x} = \mathbf{y^*}$

where $\mathbf{A}$ is the coefficient matrix in which the technical coefficients of production are represented; $\mathbf{I}$ is the identity matrix of (n-1) order; $\mathbf{x}$ is the column vector of the (n-1) physical quantities of the goods to be produced; $\mathbf{y^*}$ is the given final demand vector. The square matrix $(\mathbf{I} - \mathbf{A})$ of (n-1) order represents in its columns the (n-1) production processes, which are given by the technologies in use.

For the properties on the rank of the matrix, the system (1) is a determined system and it has only one solution. Since $(\mathbf{I} - \mathbf{A})$ is nonsingular, the inverse of $(\mathbf{I} - \mathbf{A})$ exists. If we multiply both sides of the system (1) for the inverse $(\mathbf{I} - \mathbf{A})^{-1}$, we obtain

(2) $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\, y^*$

which represents the solution for the system[22].

To give an economic meaning to the expression (2) it is necessary to identify those values of $\mathbf{x}$ which are non-negative. If all the elements of $(\mathbf{I} - \mathbf{A})^{-1}$ would be non-negative the non-negativity of $\mathbf{x}$ should be determined for all the possible non-negative vectors y^*.

Applying the theorems of Perron-Frobenius regarding the maximun eigenvalue of $\mathbf{A}$[23] we can determine the necessary and sufficient condition to state that none of the elements of the matrix $(\mathbf{I} - \mathbf{A})^{-1}$ and, therefore, none of the elements of $\mathbf{x}$ are negative.

The economic interpretation of the condition on the eigenvalue, such that the maximum eigenvalue of $\mathbf{A}$ must be less than 1, is that the technical characteristics of our economic system, in which each sector functions by absorbing directly and indirectly outputs from some other sectors, must assume certain values to allow the production of *at least* some goods beyond those goods necessary to sustain itself and thus to replace the means of production used during the production process. It is the *viable condition* of the system and it is known as the Hawkins – Simon's condition[24].

The total labour requirement can be computed in the following way:

(3) $L = \mathbf{l'x} = \mathbf{l'}\,(\mathbf{I} - \mathbf{A})^{-1}\, \mathbf{y^*}$

where $\mathbf{l'}$ is the row vector of technical labour coefficients representing the technologically determined amounts of labour that each industry employs per unit of its total output.

[21] See section 4. Steenge (2001).

[22] It is possibile to formulate a similar system of equations dual to system (1) with (n-1) prices as unknowns, with the same matrix $(\mathbf{I} - \mathbf{A})$ and with a vector of (n-1) added values, which are assumed to be known.

[23] Pasinetti (1989), pp. 363-366.

[24] For an analytical expression of such condition Leontief (1987), p.862.

In this Leontief's model the quantities of goods produced, which are the unknown of the model, are determined starting from levels of final demands. Thus in the model it is *given* that which is logically at the end of the production process[25]; this view is consistent with the observation that the *input-output* analysis " is only a *practical* instrument for programming"[26] rather than a theory of economic process.

Indeed, it is important to understand the economic meaning of each element of the matrix $\mathbf{(I - A)^{-1}}$. Whereas in the matrix A each coefficient a_{ij} represents the physical quantity of the goods i required *in the industry* j for the production of one physical unit of goods j, therefore these coefficients represent *the direct requirements* of goods for the production of other goods. In the matrix $\mathbf{(I - A)^{-1}}$ each coefficient represents the physical quantity of goods i that is necessary to produce *within the whole economic system* to obtain one physical unit of goods j that will be available as final commodity, these coefficients represent *the total requirements* (that is the direct and indirect requirements) for the production of goods to satisfy the final demand, that is for consumption and investments.

The Leontief's model just described is referred to the physical quantities. It is possible to define a similar system of equation, which is dual to the system (1) and which is referred to prices. This dual system has (n-1) prices as unknowns, the same square matrix of coefficients $\mathbf{(I - A)}$ and a vector of (n-1) values added, that are assumed as given.

2. VON NEUMANN'S "BALANCED GROWTH" MODEL: CIRCULARITY AND THE INDECOMPOSABILITY ASSUMPTION.

If Leontief's framework based on a circular economy resembles Quesnay's *Tableau,* another model which is based on the circularity of production process is the *Model of General Economic Equilibrium*[27] worked out in 1937 by John von Neumann where:

> « Goods are produced not only from "natural factors of production", but in the first place from each other. These processes of production may be circular, i.e. good G_1 is produced with the aid of good G_2 and G_2 with the aid of G_1 »[28].

von Neumann formalizes in an elegant way a multi-sector growth model characterized by a path of proportional dynamics. He elaborates a framework of extended reproduction, like in Marx, adopting the assumption of structural invariance for the analysis of interdependence within each accounting period, thus showing some features of his model of semi-stationary growth[29]. In particular, the problem addressed by von Neumann is to establish conditions under which a circular economy may grow over time at a maximum rate. To find the solution and to prove the existence of an equilibrium in his model von Neumann devices a generalization of 'Brouwer's Fixed-Point Theorem', making use of topology. Moreover, he utilizes the minimax and maximin solution methods[30].

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[25] Napoleoni (1961).

[26] Napoleoni (1961), p.109.

[27] von Neumann (1945), [1937]. The paper was read for the first time in the winter of 1932 at the mathematical seminar of Princeton University. It was first published in German. After it was translated into English by G. Morgenstern. The model is also known as von Neumann's "Expanding Economic Model". It represented a controversial source of many strands of research in economic theory: from general equilibrium models, to linear models of production based on input-output analysis, to structural models of reproduction. However, David G. Champernowne (1945) in his *Note* was the first to underline the 'classical' features of von Neumann's Expanding Economic Model.

[28] von Neumann (1945), p.1.

[29] In Leontief's circular framework, instead, the notion of 'relative structural invariance' constitutes a typical characteristic of his analysis of structural change, which is carried within an environment of comparative static.

[30] In the model, maximum growth implies the existence of a dynamic equilibrium, namely, the existence of a saddle point of a function relating the input-output matrices.

The model presented by von Neumann is, therefore, the first complete and mathematically rigorous formulation of a fully circular view of the economic system, in which neither original resources nor final consumption play a crucial role.

The model reflects an economic system, within a given period of time, that allows the transformation of certain goods into others. The model works with *n* goods and *m* production processes and it represents a closed economy in which the production requirements of certain goods in any given period, cannot come from outside the economy, but must proceed from the production of the previous period of the same economy.

The model is based on the following assumptions: A) There are constant returns to scale. This assumption means that any economic process can be carried out at *x* times its given scale, without any increase in costs per unit output. B) The natural factors of production, including labour, can be expanded in unlimited quantities. This assumption implies that there are no limits on natural resources needed for expansion. Moreover, conditions of perfect competition in the long run are assumed. C) Consumption of goods takes place only through the processes of production which include necessities of life consumed by workers and employees. This assumption implies that all income from property in excess of necessities of life is saved *and* reinvested.

In this model prices depend on supply conditions alone and are determined exclusively by the minimum cost of the goods obtained from the other goods, whereas the distribution of income is completely exogenously determined: wages are at the subsistence level and the residual net output is represented by the von Neumann's "interest factor" β, which is the same in all the economic system and it is determined by the expansion rate of the system, which, in turn, is equal to the least rate of expansion of any good involved in the system.

In equilibrium, the system of production actually used will have the greatest rate of expansion of all possible productive systems.

Von Neumann provides a linear system of reproduction, which is later used by Leontief and Sraffa. Moreover, he introduces the idea of incorporating fixed and circulating capital via joint production. He is convinced that the joint production method is the correct method to deal with durable instruments of production. According to him, the "activity" (the act of production) should, in fact, be multi-product, so the products and the capital goods (at the appropriate stages of wear and tear) should be jointly produced.

Von Neumann seeks, in particular, a "balanced growth" that justify a continuous and permanent growth of the output, thus he needs to establish complex and restrictive input-output relationships. David G. Champernowne (1945) adopts the term 'quasi-stationary state' to indicate von Neumann's approach to economic equilibrium:

« He is concerned not with short period problems but with the properties of the economic system when it has settled down to an equilibrium position which may be described as a quasi-stationary state. In such a state, all prices remain constant, the production of all goods remains in the same proportion although a uniform geometric rate of growth is allowed to the whole system. ... Thus in equilibrium there is no progress or change in production per head of population: growth merely consists of replication and the economic system expands like a crystal suspended in a solution of its own salt. The composition of any given volume of the crystal is at all times the same. To describe a system with uniform expansion of this kind we have introduced the term quasi-stationary state »[31].

The analysis of the determinants of the surplus accruing in a particular accounting period and used to expand capacities and production levels in the next period characterizes the path of growth of von Neumann's model. The determination of that surplus is, in fact, the element that provides the principle that allows von Neumann to extend the view of a productive system from a single accounting period to a whole sequence of circular-flow descriptions of an economic system. In the

[31] Champernowne (1945), p.11.

model the growth rate is determined endogenously and it depends on the technical production processes which are available, and all outputs expand at the same rate throughout the economy.

Von Neumann's view of the growth pattern implies no structural change:

« We are interested in those states where the whole economy expands without change of structure... »[32].

His model, in fact, implies that, in each period of time, there will have to be a series of production processes that are capable of generating the necessary output in terms of relative *intensities* and relative proportions which, when they are used as inputs in the following period, would guarantee the production of an output that maintains the same structure in terms of relative proportions and *intensities* as the output produced during the previous period. In such a case it is possible to speak of "balanced growth".

Von Neumann's definition of "growth in equilibrium" or "balanced growth" is quite similar to that of *steady state*, in the sense that, in equilibrium, an uniform expansion of the entire system is possible. This definition of equilibrium implies the inclusion of certain highly restrictive assumptions about technology in the model. In particular, the implicit assumption of constant relative intensities of inputs for all goods in all time periods determines von Neumannn's choice to work with the assumption of constant technology, without considering any technological progress.

The lack of structural change in von Neumann' growth model is equivalent to the lack of structural change in a stationary state as in Quesnay or in Leontief (1928). At the same time, the description of economic structure in von Neumann has common elements with that developed by Sraffa (1960)[33].

A first observation about von Neumann's model concerns a paradox of the complexity problem. In fact, his model is based on the simplifying idea of a uniform growth for all sectors of the economic system. To make this possible, von Neumann needs to assume a complex and restrictive hypothesis about technology, which must remain constant over time.

Another important observation concerning the method of analysis and the complexity problem is that von Neumann assumes the *non-decomposability* of the system. He postulates that the system does not break up into separate, or independent, sub-systems – processes that can be carried on quite independently of one another: such independent sub-systems must be excluded[34]. This assumption is, instead, consistent with von Neumann's definition of "balanced growth" and his exclusion of any change of structure of the economy in the model. Thus, Von Neumann's dynamic growth model, characterized by the circularity of production processes, is a semi-stationary growth model, which excludes the existence of different growth rates for each sector of the system, and it is strictly dependent on the assumption of *non-decomposability* of the system. This feature can sound peculiar, if it is compared to Sraffa's stationary-state model, where the *decomposability* of the system in sub-systems, as I will show in the next chapter, is possible.

Finally, the treatment of capital goods in terms of wear and tear is a step towards simplifying the complexity of the system, even if the adoption of joint production together with the introduction of fixed capital is *per se* a complication[35].

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[32] ibid., p.2.

[33] However, in Sraffa (1960) the conditions for the reproduction of the economic system are analyzed within a stationary-state framework. Moreover, von Neumann derives a unique rate of profit in equilibrium, while Sraffa discusses shifts in the income distribution. For a further analysis of conceptual equivalences between the two authors see also Kurz and Salvadori (2001), pp.163-168. See section 3 below.

[34] Hicks (1961), p.87.

[35] von Neumann (1945), p.2.

APPENDIX 2.

A2. VON NEUMANN 'S GENERAL ECONOMIC EQUILIBRIUM MODEL

Von Neumann's growth model is expressed in the following analytical way:

The process P_i is :

(1) $\sum^{n}_{j=1} a_{ij}\ G_j \rightarrow \sum^{n}_{j=1} b_{ij}\, G_j$

i) Capital goods are to be inserted on both sides of (1); wear and tear of capital goods are to be described by introducing different stages of wear and tear as different goods, using a separate P_i for each of these.
ii) Each process to be of unit time duration.
iii) (1) can describe the special case where good G_j can be produced only jointly with certain others.

The model tries to answer the question regarding which processes will be used to produce the various goods.

« In the actual economy, these processes P_i, $i = 1,2,..., m$, will be used with certain *intensities* $x_i, i = 1,2, ..., m$. That means that for the total production the quantities of equations (1) must be multiplied by x_i. »[36].

(2) $E = \sum^{m}_{i=1} x_i P_i$ if $x_i = 0$ means that process i is not used.

Since there is no structural change, the whole economy expands without change of structure: the ratios of intensities x_1: ...:, x_m remain unchanged, although $x_1,...,x_m$ themselves may change. Thus they are multiplied by a common factor α per unit of time. This factor α is the *coefficient of expansion of the whole economy.*

Moreover, obviously:

(3) $x_i \geq 0$

(4) $y_j \geq 0$

Since a solution with $x_1 = ... = x_m = 0$, or $y_1 = ... = y_m = 0$ would be meaningless, then

(5) $\sum^{m}_{i=1} x_i > 0$

(6) $\sum^{n}_{j=1} y_j > 0$

The economic equations now become:

(7) $\alpha \sum^{m}_{i=1} a_{ij}\, x_i \leq \sum^{m}_{i=1} b_{ij}\, x_i$

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[36] Sraffa (1960), for instance, preferred to start his analysis with no fixed capital and no joint production.

and if in (7) < applies,

(7’) $y_j = 0$

(8) $\beta \sum^{n}_{j=1} a_{ij} y_j \leq \sum^{n}_{j=1} b_{ij} y_j$

and if in (8) > applies,

(8’) $x_i = 0$

The meaning of (7), (7’) is: it is impossible to consume more of a good G_j in the total process (2) than is being produced. If, however, less is consumed, i.e. if there is excess production of G_j, G_j becomes a free good and its price (y_j) is zero.

The meaning of (8), (8’) is: in equilibrium no profit can be made on any process P_i. If there is a loss, however, (if P_i is unprofitable), then P_i will not be used and its *intensity* x_i is equal to zero.

In this framework a_{ij}, b_{ij} are given, the *intensities* $x_1,\ldots,x_m$, the *prices* $y_1,\ldots,y_n$ of goods $G_1,\ldots, G_n$, the *coefficient of expansion* α and the interest factor β are the numerical unknowns.

Thus, there are $m + n + 2$ unknowns, but since in the case of x_i, y_j only the ratios $x_1:\ldots:x_m$, $y_1:\ldots:y_n$ are essential, they are reduced to $m + n$. Against this, there are $m + n$ conditions (7) + (7’) and (8) + (8’). As these are complicated inequalities, the fact that the number of conditions is equal to the number of unknowns does not constitute a guarantee that the system can be solved.

A remarkable feature of this model is the dual symmetry of equations (3), (5), (7), (7’) of the variables x_i, α and of the concept “unused process” on the one hand, and of equations (4), (6), (8), (8’) of the variables y_j, β and of the concept of “free good” on the other hand.

The system (3) – (8’) can be solved, since several solutions $x_1:\ldots:x_m$, $y_1:\ldots:y_n$ always exist. It is of importance that α, β should have the same value for all solutions (α, β are uniquely determined).

Another assumption of the model is

(9) $a_{ij} + b_{ij} > 0$

a_{ij}, b_{ij} are clearly always ≥ 0. This restriction (9) must be imposed in order to assure uniqueness of α, β.

Von Neumann considers two cases, given a hypothetical solution x_i, α, y_j, β of (3) – (8’).
In the first case, if we had in (7) always <, then we should have always $y_j = 0$ (because of (7’)) in contradiction of (6). In the second case, if we had in (8) always >, then we should always have $x_i = 0$ (because of (8’)) in contradiction to (5). Therefore, in (7) $\leq$ always applies, but = at least once; in (8) $\geq$ always applies, but = at least once.

In consequence:

(10) $\alpha = {}_{j=1,\ldots,n} \text{Min.} \left[\sum^{m}_{i=1} b_{ij} x_i \,/\, \sum^{m}_{i=1} a_{ij} x_i \right]$

(11) $\beta = {}_{i=1,\ldots,m} \text{Max.} \left[\sum^{n}_{i=1} b_{ij} y_j \,/\, \sum^{n}_{j=1} a_{ij} y_j \right]$

Therefore, the x_i, y_j determine uniquely α, β. Von Neumann also specifies that the right hand side of (10) and (11) can never assume the meaningless form 0/0 because of (3) – (6) and (9). Thus it can be stated (7) + (7’) and (8) + (8’) as conditions for x_i, y_j only:

$y_j = 0$ for each j = 1, . . ., n, for which :

$$\sum^{m}_{i=1} b_{ij} x_i \,/\, \sum^{m}_{i=1} a_{ij} x_i$$

does not assume its minimum value (for all j = 1, . . ., n) (7*).

$x_i = 0$ for each i = 1, . . ., m for which:

$$\sum^{n}_{i=1} b_{ij} y_j \,/\, \sum^{n}_{j=1} a_{ij} y_j$$

does not assume its maximum value (for all i = 1, . . ., m) (8*).

The $x_1, \ldots, x_m$ in (7*) and the $y_1, \ldots, y_n$ in (8*) are to be considered as given. The system (3)-(6), (7) and (8) must be solved for x_i , y_j .

Von Neumann formulates the system (3) – (8') in a different and more complex mathematical form to find a saddle point for the variables X' and Y' of a new defined function φ. The question of a possible solution of the saddle point or minimum-maximum type is not simple but it is "far more profound". In fact von Neumann first proves a more general lemma and he needs to use the Brouwer's Fix-point Theorem. After proving the lemma completely von Neumann finds the solution of the system (3) – (8'), thus $\alpha = \beta$ is the value of the saddle point. The interest factor β and the coefficient of expansion of the economy α are equal and uniquely determined by the technically possible processes $P_1, \ldots, P_m$.

3. SRAFFA'S SUBSYSTEMS AND PASINETTI'S VERTICAL INTEGRATED SECTORS.

The analysis of the previous sections shows that Leontief's model of production, characterized by inter-industry relationships, is based on the view of a 'perfect' circular economy, which presupposes that the production of goods is an essential prerequisite for the production of the same or other goods. In this model, all commodities depend on each other's production. A similar approach is followed by von Neumann in his multi-sector model of balanced growth, where an inter-sectoral framework is put forward, in which production is conceived of as a circular flow. This approach can be associated with a view of the economic system that takes into account the whole network of relationships among the productive sectors (the 'industries'). If we consider Leontief's frameworks, in particular, it can be maintained that a model of 'horizontal' integration has been introduced. In the horizontally integrated models, each commodity appears on both sides of the economic system: on the side of the factors of production and on the side of the outputs. Neither primary resource, nor any final consumption commodity has any logically pre-eminent role. The economic literature encompasses many examples of this type of models: the analytical frameworks by Leontief [1951], Sraffa [1960], Hicks [1965], Quadrio Curzio [1986] belong to this class of models.

But there is another approach that diverts attention from the issues concerning the reproduction of the economic system and focuses instead on the relationship between productive resources and their allocation. This is the approach based on the vertical integration of the productive processes. In this case, the productive system does not appear to be characterized any more by circular relationships. Rather, it is the consumption of goods that seems to be the final purpose of the process of production. Here we have a model of 'vertical' integration, in which the production of goods depends in terms of one-way causality on the availability of resources that are themselves independent of productive processes. The models of general economic equilibrium in the modern formulation of the Lausanne School (Allais and Debreu), Hicks's analytical framework of temporary equilibrium, Pasinetti's notion of vertical integration and, finally, the vertical integration along the time dimension set forth by Hicks in *Capital and Time* (1973), although with different ends, all belong to the class of vertically integrated models.

In this section, first, I analyze the notion of *sub-system* proposed by Piero Sraffa (1960) within his theoretical model of production, where the activity of production is represented as a circular process. After, I examine the notion of *vertically integrated sector* set forth by Luigi Pasinetti (1973) who conceives the production process as vertically integrated, but who makes use of the sub-systems to construct his vertically integrated sectors. Although the two notions are devised in different contexts, since the first concept is embedded in the static inter-industry (circular) analysis, whereas the second construct allows dynamic analysis of a multi-sectoral model in a vertical setting straightaway.

In *Production of Commodities by Means of Commodities* Sraffa proposes a model of production in which the economic structure is conceived as a set of relationships among production processes ('industries') as determined by the technology in use. He centres the analysis on the conditions of reproduction of the economic system. Sraffa, like Leontief, tackles the problem of proportions among industries in a classical spirit. The Sraffa's framework assumes that

« commodities are produced by separate industries and are exchanged for one another at a market »[37].

The Sraffa's system overlooks any interdependence among different markets, emphasizes the mutual dependence among techniques in use and maintains that industries must stand in a definite proportion among them. The economic system is characterized, like in Quesnay, by a 'circular' process of production of commodities which are produced by means of the same commodities[38].

The circularity of the production process emerges very clearly in the framework of 'production for subsistence' in chapter I of *Production of Commodities.* In this simplified framework, all commodities are, directly and indirectly, necessary to the production of all other commodities; at the end of the period of production (i.e. one year), all the commodities return to the production process as necessary technical requirements or, indifferently, as necessary means of subsistence for the labourers. In this framework it is also possible to look at the circularity of the production process from the price system view. The framework is, in fact, characterized by a unique set of rates of exchange among commodities, which are determined by technology alone; this is the set of rates of exchange that must be adopted to keep the system in a self-replacing state. Sraffa calls these rates of exchange 'natural prices' or 'values'. In his system, therefore, the relative prices are not the result of the consumers' preferences as the neoclassical analysis would maintain, but simply exchange ratios that allow the system to reproduce itself.

In chapter II of *Production of Commodities* Sraffa put forward a more complex framework of 'production with a surplus[39]. In this second framework

« there is room for a new class of 'luxury' products which are not used, whether as instruments of production or as articles of subsistence, in the production of others »[40].

These 'luxury' products emerge from the production process without re-entering it; they have no part in the determination of the system.

A specific feature of Sraffa's framework is that the domain of structural analysis concerns solely the determination of the system of relative prices and distribution connected with a given set

[37] Sraffa (1960), p. 3.
[38] Sraffa, in the Appendix D, explicitly specifies the meaning of circular process and the connection of his work with Quesnay's *Tableau economique.*
[39] For an analytical representation of Sraffa's circular model of production with a surplus see the Appendix 2.
[40] Sraffa (1960), p.7.

of technique in use. Sraffa in his framework makes a clear distinction between the conditions of reproduction, which are technically founded, and the rules of distribution, which depend on the social conditions of the economic system. Sraffa also works out an analytical device, which he called the method of *sub-systems,* to separate what pertains to the surplus from what pertains to the circular process. The method of *sub-systems* is essentially a *method of decomposition* of the system of production, since the description of the structural properties of the economic system in analytical terms is related to the partition of the same economic system in subunits (the *sub-systems*). Whereas von Neumann, in his model of semi-stationary growth, did not make use of *sub-systems,* since he assumed the non-decomposability of the system, ruling out structural change, Sraffa identifies the *sub-systems* as an implicit application of the descriptive-analytical approach to structural change[41] , adopting a stationary state framework.

Sraffa devices in the Appendix A of his *Production of Commodities* the *sub-systems* 'to see at glance' the amount of labour which directly and indirectly goes into producing each commodities. He considers a system of industries, each producing a different commodity, which is in a self-replacing state, arguing that:

« The commodities forming the gross product....can be unambiguously distinguished as those which goes to replace the means of production and those which together form the net product of the system. Such a system can be subdivided into as many parts as there are commodities in its net product, in such a way that each part forms a smaller self-replacing system the net product of which consists of only one kind of commodity. These parts we shall call 'subsystems' »[42].

Moreover, in the inter-industry system proposed by Sraffa , long-term structural dynamics does not play a significant role, since, in his original formulation, Sraffa's framework is based on the assumption of a stationary economy, where the term «stationary» is referred to an economy which reproduces itself in the same way over time[43].

The notion of a *vertically integrated sector* is set forth by Pasinetti in his paper "The Notion of Vertical Integration in Economic Analysis"[44]. The author suggests this analytical device for his a multi-sectoral model of structural change characterized by uneven growth, thus treating the growth process in a different way with respect to von Neumann's balanced growth model.

Pasinetti starts from the inter-industry analytical framework of the Leontief type. But he also introduces the synthetic notion of *vertically integrated sector*, that he himself considers to be essential for dynamic analysis. This is done in two steps. First, Pasinetti decomposes his system not into a number of industries (as Leontief has done), but, following Sraffa, into *m sub-systems.* In this way, he stresses the reproducibility of means of production over time through a circular process. Such a notion of subsystem is also relevant to the analysis of uneven growth, since it avoids the "analytical unification" of the economic system through circular relationships [45]. Then, starting from the *sub-systems*, Pasinetti defines the *vertically integrated sector* for the production of a physical unit of a commodity as a final good, which is composed by the *vertically integrated labour coefficient* and a unit of *vertically integrated productive capacity*; in this new analytical construct the circular linkages disappear.

The logical operation of vertical integration is, according to Pasinetti, a device that allows structural analysis through a re-classification of commodity flows and stocks according to *logical units.* This is considered to be a necessary step to go into dynamic analysis. This approach is also especially useful to the identification of one-way causal links within the network of economic relationships. A relevant feature of the concept of *vertically integrated sector* is that it is not affected by technical progress. With regard to this aspect, Pasinetti writes:

[41] Structural change usually implies that under certain conditions a particular structure may break down and a new pattern may emerge. The *sub-systems* can provide a disaggregated representation of a structural change process. Landesmann, Scazzieri (1990).
[42] Sraffa (1960), p.89.
[43] Sylos Labini (2004), pp.336-337.
[44] Pasinetti (1973).
[45] Scazzieri (1990), p.26.

« The vertically integrated sectors seem to belong to that category of synthetic notions that, once obtained, contribute to reduce in many directions the order of magnitude of the analytical difficulties »[46].

Pasinetti also calls attention to the complexity problems arising in the analysis of the structure of the productive system, especially when dynamic analysis has to be developed on the assumption of fixed capital and technical progress. Here, the logical process of vertical integration (with the related decomposition of the economic system into vertically integrated sectors, which allows for a *different unit of measurement* in each sector), becomes a way to reduce the *degree of complexity.*

Pasinetti explicitly acknowledges that there are two different approaches to the analysis of the structure of the economic system[47]. The first approach is the *circular* one and it is referred to the representation and decomposition of the productive system into *sub-systems.* In this case, it is possible to analyse the complex interrelationships among productive processes either in physical terms (when there is production with fixed capital, when there is joint production, when there are natural resources, etc.) or in terms of prices (when we examine the relationships of income distribution, the changes in relative prices, etc.). The second approach is that of *final demand,* relative to *vertically integrated sectors*, in which all complications associated with intermediate processes are left aside and the analysis concentrates on the relationships among final goods, on one side of the process, and their ultimate requirements (the quantities of labour) on the other end of the same process.

It is noteworthy that, in Pasinetti's view, the two approaches are not alternative, but complementary. As a matter of fact, there is a one-to-one correspondence between an economic system expressed in terms of industries and the same economic system expressed in terms of vertically integrated sectors (either for the system of physical quantities or for the price system). But this correspondence is only true for a given technique and at a given period of time. The one-to-one correspondence between the vertical and the horizontal system breaks down in the cases of structural change and structural dynamics.

APPENDIX 3.

A3. SRAFFA'S MODEL OF PRODUCTION WITH A SURPLUS

Sraffa's multi-sector model of production with a surplus, characterized by single product industries, is composed by two systems of equations: the first is referred to the physical quantities and the second to prices. This model of production is also a circular model of production.

(1) $\mathbf{AQ} + \hat{\mathbf{S}}\mathbf{AQ} = \mathbf{Q}$

(2) $\mathbf{lQ} = 1$

In this system of physical quantities: $\mathbf{Q}$ is the vector of the physical quantities, $\mathbf{A} \geq \mathbf{0}$ is the non negative square matrix of the technical coefficients, $\hat{\mathbf{S}}$ is the diagonal matrix with the different physical rates of surplus relative to the single commodity for each industry, $\mathbf{l}$ is the vector of labour coefficients.

[46] Pasinetti (1989), p.405.
[47] Pasinetti (1986).

The price system is:

(i) $(1 + r)\mathbf{Ap} + w\mathbf{l} = \mathbf{p}$

where **p** is the price vector, **l** is the vector of labour coefficients when labour gets a share in the surplus of a surplus wage according to wage rate w. The rule of distribution of the surplus depends on the social conditions of the economy under consideration. The theory of distribution underlying this case is different from the theory based of the assumption that the subsistence of workers is included among the means of production. Here a rate of profit (r) accrues to the class of capitalists and a rate of wage (w), which includes the element of subsistence and a share of the surplus product, accrues to the class of workers. Thus, according to Sraffa, the wage is part of the surplus (S) and the surplus is equal to the sum of all the wages plus the aggregate profits. The system is represented by k equations and k+2 variables: k prices, the wage rate w and the rate of profit r. It is possible to assume as *numeraire* the price of a commodity and, therefore, add an equation. This price system, however, is not able to determine simultaneously all the unknowns: prices, rate of profit and wage rate. Thus there is *a degree of freedom* in this system, hence to find a solution it is necessary to fix one of the two distributive variables (r,w).

4. QUESNAY'S *TABLEAU* AS A BENCHMARK MODEL FOR STRUCTURAL ANALYSIS.

In proposing the models of Leontief, von Neumann, Sraffa and Pasinetti I tried to show different methodological approaches to structural analysis, a range of heterogeneous analytical principles, and also various degrees of openness relative to different institutional conditions and historical set-ups.

In particular, I pointed out that these theoretical models that analyse economic structure face a complexity problem. Thus, a point at issue in this paper has been the need to reduce degree of complexity so to make the analysis of structural change easier and more intelligible. Hence, I have examined different *methods of decomposition* of the system of production, since the description of the structural properties of an economic system in analytical terms is usually related to the partition of the same economic system in subunits as processes of production, industries, subsystems, vertically integrated sectors; these different types of partition enable to make structural analysis and to provide a disaggregated dynamic representation of structural change.

In this section, I consider Quensay's theory of circular flow, expressed in his *Tableau Economique,* as a kind of 'benchmark model' to discuss the relevance of institutional and behavioural factors in the analysis of the structure of the economy and also in the analysis of structural change, and to point out the main differences with respect Leontief's, von Neumann's and Sraffa's models in terms of hypotheses, methods of decomposition of the system of production, etc.

Quesnay's *Tableau Economique* reflects the view of a society made of social classes and also that technological and social linkages together determine the sustainability of any given pattern of social expenditure. Thus, the approach to the decomposition of the economic system in Quesnay goes through the complexity of the society made of social classes and of technological processes. A Quesnay's basic idea is that productivity is localized in a certain part of the economy, and this part is the agricultural class[48]. As Steenge and Van Den Berg (2007) claim, Quesnay expresses the conviction

« that only the primary sector [i.e. agriculture] was capable of producing the 'net product'....

[48] This idea is not far from Sraffa's view that only the industries that produce basic goods are the ones where producitvity is localized.

The productivity of agricultural sector was not at all something that could be taken for granted, but instead was a potential strength that required nurturing through the establishment of a number of technological, economic, and political conditions »[49].

Another important point, underlined by Steenge and Van Den Berg (2007) is that productivity and profitability are no identical in the *Tableau Economique*:

« Productivity, understood as a potential, does not preclude acknowledgement of the fact that net incomes may be earned elsewhere in he economy. Rather, these net incomes, not having a direct relation with the real productive potential of the economy, are considered parasitical and of a transient nature. Hence the distinction, frequently made in the physiocratic literature, between "true" and "false" revenues. … As the acknowledgment of the existence of "false" revenues suggests, the "incidental" fact that a sector becomes "profitable" does not immediately change the judgement about its fundamental unproductivity »[50].

Surpluses income may, in fact, be realized in the less productive places. Thus, if a source of prosperity is identified, it should be protected and stimulated by means of appropriate policies[51]. This leads, according to these authors, to the concept of an ideal or optimal state, where the economy's full potential is realized.

Steenge and Van Den Berg, therefore, pointed out that Quesnay's *Tableau* express two ideas[52]: first, it gives the proportions of the ideal state, where a balance [i.e. an equilibrium] is achieved, that is, the economic system can continue to reproduce a net product at the same level, and also where the size of the net product is considered the highest achievable; second, it is supposed to show that any deviation from this state will mean that the country's surplus will decrease, that is, out of equilibrium means out of the optimal state.

Examining Leontief's models, I have pointed out that there is a link between his frameworks and Quesnay's *Tableau Economique,* but this linkage must not be overstressed. Both Quesnay and Leontief consider the economic activity as a circular process, yet in the *input-output* framework the structure is determined by the *state of technology* and not by the assumptions concerning economic agents' or social classes' behaviour, as in Quesnay[53]. However, it is worth noting that Leontief in his 1928 essay "The Economy as a Circular Flow" takes an 'open' system, which is not strictly determined by technology, but it is open to other factors. This means that individual economic behaviours and institutions have an important role to play, particularly in the exchange system. Moreover, a relevant difference between the two economists is that Quesnay proposes a benchmark model that is a normative model, which is not aimed to observe and measure the working of a real economy characterized by a circular or horizontal structure. The *Tableau* has elements of a system focusing on behaviour and institutions, *and* on technologies in use, and it contains normative aspects[54]. Thus, the concept of optimal state is crucial in Quesnay. Leontief, instead, works out a method for empirical analysis and, adopting fixed coefficents, he focuses on technology as the main factor to explain and measure the direct and the indirect effects (through analytical instruments such as the inverse matrix in the static *input-output* model and the dynamic inverse for the dynamic analysis), of a change in the structure of a real system of production, so Leontief does not need any notion of optimal state.

[49] Steenge and Van Den Berg (2007), p.336.
[50] Steenge and Van Den Berg (2007), pp.348-349.
[51] Ibid., p.352.
[52] Ibid., p.336.
[53] Actually Quesnay never used the term *behavioural hypothesis* rather he was writing about the 'natural order', but in any case the *tableau* was considering the possibility of flexible solutions.
[54] Steenge (2001); Steenge and Van Den Berg (2007).

Furthermore, Quesnay was interested in the investigation of 'natural proportions' when social output is reproduced unchanged from one accounting period to another[55]. This idea in not present in Leontief's frameworks, but it can be found in von Neumann's growth model, in the sense that this latter model shows the conditions that must be satisfied to grow at the maximum rate, following a path of proportional dynamics but, at the same time, under restrictive assumptions on the state of technology, which must remain constant.

Hereunder there is a framework which gives a representation of the *Tableau Economique*[56].

	Farmers	**Artisans**	**Landlords**
Farmers	**2000**	2000	**1000**
Artisans	1000	**-**	1000
Rents	**2000**	**-**	

In this framework, which represents a 'closed system', there are three social classes, the Farmers, who produce goods in the agricultural sector, the Artisans, who produce goods in the 'manufacturing' sector and whose income is irrelevant for the workings of the productive system, the Landlords, who do not produce anything but that own the lands and consume their income (rents) produced by the Farmers. Rents, in turn, are seen as the result of contractual commitments, whose magnitudes are based on agreement about the surplus income that Farmers can reasonably expect to raise. Thus, rents are determined by institutional, economic and technological factors.

The sum of the each column represents the total purchases from that sector (in money terms), while the sum of each of the three rows represents the total expentitures. The numbers in bold character are those that are connected to the agricultural (i.e. "productive") sector, the other numbers belong to the Artisans, which are not "productive", according to Quesnay. This framework, therefore, can be partitioned in small sub-models, which are not the industries or sectors as in Leontief's and von Neumann's models, but are rather similar to Sraffa's sub-systems.

A general observation that can be made from Quesnay's benchmark framework is that any framework for structural analysis must encompass the analysis of the process of reintegration, which allows the system to reproduce itself overtime. This prerequisite can be met by adopting the

[55] Quesnay was not only interested in defining natural proportions between sectors, or the ideal state and the equilibrium position, he was also interested in the adjustment path leading to the new economic structure, following different set of natural proportions. Thus Quesnay analysed also structural change,

[56] This representation of the *Tableau* was that of the *Formule* (1766), the earliest version of 1958 was the "Zigzag" framework. See Steenge and Van Den Berg (2007), pp.332-336.

circular approach *à la Quesnay,* and this is confirmed by the analysis of interdependencies among productive sectors, as in Leontief's, von Neumann's and Sraffa's models.

Another general observation that can be drawn from the foregoing analysis is the need of maintaining a certain degree of flexibility in models that want to give an account of changes in the structure and, therefore, a certain degree of 'openness' relative to different institutional conditions and historical set-ups.

5. CONCLUSIONS.

The main theme of this paper has been to investigate essential characteristics for structural analysis of economic systems that can be identified in the analytical frameworks of Leontief, von Neumann, Sraffa and Pasinetti. These authors can be assimilated, at various degrees, to the Classical Economists with regard to the analysis of production, growth and structural change.

The investigation in the paper was carried out with the aim of contributing towards a further epistemic advance in the analysis of structural change, by examining models which adopt similar analytical frameworks regarding the economic structure, but also different assumptions and methods for the analysis of economic change.

The paper is, therefore, essentially conceptual and methodological, since it tries to reconsider important methodological issues under a new light and to point out fundamental analytical principles mainly from the circular models of Leontief, von Neumann, Sraffa, that should constitute the basis for a deeper analysis of structural change, confronting them with Quesnay's framework as a benchmark model.

A peculiar methodological feature of structural analysis, that comes out from the previous analysis, is the notion of 'relative structural invariance'. A notion explicitly used by Leontief, but which is rooted in the tradition of the analysis of structural change of the Classical economists, who concentrate onto few critical variables and their relations. This means that the economic structure is generally described in a way that some of its elements are considered to be fixed, while other elements may change over time. 'Relative structural invariance', thus, is a distinctive feature of structural change analysis, since it is always relative to a given structural description. This assumption is essential in order to determine the set of possible transformations that any given economic system may undergo. There is a clear relationship between the circular approach and the notion of 'relative structural invariance'. This is because relative structural invariance allows the study of structural change through a 'time differentiated' description of the interrelationships among elements of the economic system.

According to Simon (1962) the analysis of structural change presupposes the ability to get an insight into the "architecture of complexity"[57]. Thus, another important methodological feature is the decomposition of the economic system into sub-units such as *industries* or single productive processes to reduce the *degree of complexity* of the analysis.

Leontief's *input-output* method is consistent with this goal, as it concentrates on a *finite variety* of characteristics and activities of the economic system, but the structure is determined by the *state of technology* and not by the assumptions concerning economic agents' or social classes' behaviour, as in Quesnay. Sraffa, instead, adopts a similar inter-industry framework and a circular approach, he also decomposes the system in sub-units which are the sub-systems, that have a different meaning with respect to the industries in Leontief, and he is not commited only by the state of technology to determine the economic stucture and the price system, but his analytical system is open and it has degrees of freedom. Von Neumann, on the hand, is constrained by his complex and restrictive hypothesis about technology, which must remain constant over time. Another crucial and restrictive assumption in von Neumann's model, concerning his method of analysis and the complexity problem, is that von Neumann assumes the *non-decomposability* of the system. His choice of

[57] Hagemann, Landesmann, Scazzieri (2003), p. XXXVIII.

excluding the partion of the economic structure into independent sub-systems is a consequence of the fact that decomposability is not essential to reduce complexity in a balanced-growth model, and also because there is no structural change in his model, but this approach makes his framework “too rigid”.

Moreover, structural analysis implies that the degree of diversity of the economic environment is reduced by the assumption *of limited heterogeneity*.

The peculiar methodological features of structural change analysis are, therefore, the notion of ‘relative structural invariance’, the adoption of methods of decomposition and the identification of normative conditions of equilibrium. Quesnay, in particular, made use of methods of decomposition and, above all, explicitly considered normative conditions of equilibrium. Leontief, instead, made use of the notion of ‘relative structural invariance’and of methods of decomposition, but he did not take into consideration normative conditions of equilibrium. Von Neumann did not make use of methods of decomposition, because of his assumption of *non-decomposibility*. Von Neumann’s definition of “growth in equilibrium” also implies highly restrictive assumptions on technology in the model that makes his concept of equlibrium independent from the behaviours of the individuals or social classes. Sraffa made an important use of methods of decomposition for his analysis; he also adopted an equilibrium concept which was related with possible different solution about income distribution among social classes.

In conclusion, the analysis of structural change concerns an economy in a state of qualitative and quantitative transformation. The variety of methodological approaches and analytical solutions of theoretical frameworks on structural analysis and structural change confirms the necessity of a theory selection which presupposes both an adequate understanding of the economic structure and the description of relevant features of institutions and of individual or social patterns of behaviour. Indeed, Leontief’s, von Neumann’s and Sraffa’s models can help to sort out this complex task, as we have tried to argue in the present work.

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