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ASSET ALLOCATION APPROACH TO UNDERSTANDING STOCK MARKET DYNAMICS

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ABSTRACT

Equity portfolio managers typically convey instructions to their traders in the form of target portfolio weights for the various shares in their portfolio. We present a set of differential equations that allows the calculation of the share prices, number of shares, and value of each manager's portfolio over time, in terms of share weights. It is also necessary to know the amount of cash flowing into each portfolio and the number of each type of shares outstanding.

We suggest some potentially useful information that might be derived from this formalism, such as a quantitative estimate of the main driver of share price changes, the influence of index investing on the market, and the origin of the equity premium.

We believe that this realistic method could be the basis for a better understanding of how financial markets operate, as compared with the conventional academic approach. In our view standard asset pricing theory makes implausible assumptions about the existence of stochastic processes, the ability of participants to foretell the future, and their capacity to make sound deductions from the information they have. Even an imperfect alternative should be better than that house of cards.

1. INTRODUCTION

A typical manager of an equity portfolio gathers and analyzes information from a variety of sources that is then used to make predictions about the future course of stock prices. From time to time he produces a set of instructions telling the trader what changes to make in the current portfolio. Holdings of some stocks should be increased, others decreased. Similarly, other managers provide instructions to their traders, but the instructions will differ from manager to manager.

The prices of the stocks in the market are continually adjusted as a result of the execution of the instructions by the various traders active at the time, buying and selling stocks between each other. We stress that the resulting prices directly depend only on the nature of the instructions, but not on the procedures that managers use to determine the instructions. Thus, if we knew what all the trading instructions were, we could in principle calculate how all the stock prices would change.

This report describes how such a calculation could be carried out if we use a particular method of representing trading instructions, namely the provision to the trader by the manager of a set of target weights for each of the stocks in the portfolio. To complete the information needed for this method we also need to know the amount of external cash flowing to and from the portfolio, and the number of shares newly issued or retired by each company whose stock is traded.

The calculation requires the solution of a number of coupled differential equations with time as the independent variable. It is easy to solve these equations numerically.

While no single person or organization knows all the information required to perform the calculations, we believe that there may be nevertheless some useful insights to be gained by studying the results of our work. Some possibilities for future study are mentioned below.

Our approach differs in a number of respects from the vast literature on the subject of asset pricing. We question the standard approach that appears to be ill-defined, unrealistic, wrong, or perhaps all three. The standard approach attempts to predict price behavior from an understanding of how market participants think and act. It usually assumes the existence of an underlying universal stochastic process, with little or no discussion of how the process is defined, even formally. For example the term 'expected return' is used frequently but without clear definition. Agents are assumed to have an unrealistic amount of knowledge and unbelievable power to make computations. The obvious diversity among agents is often completely ignored.

In contrast our goals are much less ambitious, but our method can take into account the fact that managers do not all think alike and do not (indeed cannot) follow the fanciful methods assumed in standard asset pricing theory.

Section 2 sets the stage and describes our assumptions. In Section 3 the differential equations are derived, with a summary appearing in Appendix 1. Section 4 shows in a simple case that the results of solving the equations are close to what would occur if the market participants traded in the conventional way. In Section 5 we discuss some of the issues that might be studied on the basis of the framework laid out here. This report is an extension of work done previously [Nu00].

2. MARKET PRICES FROM TRADING RULES

For this report we consider the market to be based on a particular set of shares, such as all, or a specified subset of, those listed on the NYSE. We suppose that there are Q different types of share, e.g. the shares of a number of different companies.

The market consists of just three groups whose actions can directly influence share prices.

- Managers - all portfolio managers and individuals who make decisions to buy and sell shares in the market. Managers may also hold cash;
- Investors - all those who decide when to commit to or withdraw cash from the portfolio managers;
- Companies - all public companies that issue new shares in the defined market to managers, or purchase and retire existing shares from managers.

Stock prices are set entirely by the actions of these three groups, although of course decisions by the groups are influenced by a vast array of external information, including possibly the past behavior of the share prices themselves.

It should be possible to calculate how share prices have varied or will vary over a period of time if we know the following information, all of which varies with time.

- Share prices and the contents of the managers' portfolios at the start of the period in question;
- The rules followed by portfolio managers in deciding when and which shares to buy and sell;
- The rate of flow of investors' cash to or from the portfolios;
- The rate at which companies by sale or purchase change the number of their shares outstanding.

Thus, as time passes, there will be a flow of cash (possibly negative at times) from the investors into each portfolio (e.g. a pension fund receives cash contributions from its members and pays out funds to retirees). These net cash flows to each manager are regarded as given in the model.

Shares might also be redeemed on payment of cash to the managers whose portfolios contain them by the company that issued them, and similarly new shares might be issued for cash, all based on the current market price of the share. The net flow of shares of each type into the system we take as given.

To complete the description we need to specify the rules used by each manager to determine how to adjust the contents of his portfolio at any time. Many professional portfolio managers use optimization software that instructs the manager on the weights of the shares to hold in the portfolio for the next period of time. Here the weight of a share is the fraction of the total portfolio value to be placed in shares of that type. In this report we take the rule to be that the manager must change from the current weights to the weights specified by the optimizer. This will entail buying or selling shares of various types. After the changes have been made the share holdings are not further adjusted until the optimizer is run again, except for rebalancing to maintain the desired weights.

Each manager will execute a succession of trades that move his portfolio towards the required new weights. The other managers will be doing the same thing, and none of them can be sure how many shares of each stock will be needed to produce the desired weight, or what the share price will be at this point. This effect is called market impact, and the equations below calculate its value for each share.

The weights will change with time, and will in general be different for each manager. In the following we assume that the time dependence of all weights is given. In fact an outside observer will not know the weights chosen by the managers, but at least the information does exist. This is in contrast with the situation in standard asset pricing models.

In practice the above ratios and the external flows of cash and shares, the information that determines the behavior of the market in the model, might be discontinuous functions of time, which will lead to erratic changes in share prices. For convenience we assume that the externally determined functions have a sufficiently smooth dependence on time, and that shares are infinitely divisible. The number of shares traded in a given, small time interval will be proportional to the length of the interval. In that interval a manager who trades will exchange small amounts of cash and shares with another manager on the basis of the current market prices for the shares.

With the smoothness assumption, the process is described by a set of coupled, first order, nonlinear, ordinary differential equations with time as the independent variable. The equations are derived in the next section. It is straightforward to write a computer program which will solve these equations numerically with adequate accuracy, using standard techniques of numerical analysis.

3. DERIVATION OF THE EQUATIONS

Let us suppose that there are M managers, and that at a given time t manager j has an amount $C(j)$ of cash and a number $N(j,k)$ of shares of type k. If the price of a share of type k is $P(k)$, we denote the value his portfolio by $V(j)$, so that

$$V(j) = C(j) + \sum_{k=1}^{Q} P(k)N(j,k), \quad j = 1, M$$

For manager j the weight $G(j,k)$, the fraction of the value of his portfolio in shares of type k, is defined by

$$G(j,k) = P(k)N(j,k)/V(j), \text{ i.e. } P(k)N(j,k) = G(j,k)V(j), \quad j = 1, M, \; k = 1, Q \tag{1}$$

In the model trading begins at time $t = 0$ when all the above functions have specified values. We assume that cash from outside sources, or interest and dividends on the portfolio, is added to portfolio j in such a manner that a net amount of cash $R(j)$ has flowed into portfolio j up to time t. We also assume that a net amount $S(k)$ of shares of type k have been issued by the company up to time t to all the managers.

All the above functions depend on time t. The functions G, R and S are specified externally, while C, N, V and P are obtained by solving the differential equations derived below, given their initial values.

We use the symbol G' to represent the time derivative of G. From the notion that a manager will preserve value when trading we obtain

$$C(j)' = R(j)' - \sum_{k=1}^{Q} P(k)N(j,k)', \quad j = 1, M \tag{2}$$

This equation is equivalent to the statement that, for a small interval of time, the increase in the portfolio's cash balance is equal to the amount of cash flowing in from external sources minus the amount spent to buy additional shares.

Differentiating the definition of $V(j)$ we find

$$V(j)' = C(j)' + \sum_{k=1}^{Q} \left[P(k)N(j,k)' + P(k)'N(j,k) \right], \quad j = 1, M$$

which, with (2), gives

$$V(j)' = R(j)' + \sum_{k=1}^{Q} P(k)'N(j,k), \quad j = 1, M\ . \tag{3}$$

Differentiating the definition of $G(j)$ leads to

$$N(j,k)P(k)' + N(j,k)'P(k) = G(j,k)V(j)' + V(j)G(j,k)', \quad j = 1, M;\ k = 1, Q$$

Substituting (3) and rearranging produces

$$N(j,k)' = \frac{1}{P(k)} \begin{bmatrix} V(j)G(j,k)' + G(j,k)R(j)' \\ - P(k)'N(j,k) + G(j,k)\sum_{i=1}^{Q} P(i)'N(j,i) \end{bmatrix}, \quad j = 1, M;\ k = 1, Q \tag{4}$$

The set of equations will be completed with an expression for $P(k)'$. This can be obtained by summing (1) over j, so that

$$\sum_{j=1}^{M} G(j,k)V(j) = P(k)\sum_{j=1}^{M} N(j,k) = P(k)S(k), \quad k = 1, Q\ .$$

Differentiating gives

$$P(k)'S(k)+P(k)S(k)'=\sum_{j=1}^{M}\left[G(j,k)'V(j)+G(j,k)V(j)'\right]$$
$$=\sum_{j=1}^{M}\left[G(j,k)'V(j)+G(j,k)\left(R(j)'+\sum_{i=1}^{Q}P(i)'N(j,i)\right)\right],\quad k=1,Q$$

from (3). Rearranging we have

$$\sum_{i=1}^{Q}H(k,i)P(i)'=\sum_{j=1}^{M}\left[G(j,k)'V(j)+G(j,k)R(j)'\right]-P(k)S(k)',\qquad k=1,Q$$

where

$$H(k,i)=S(k)\delta_{ki}-G(j,k)N(j,i)\qquad k,i=1,Q\,.$$

Thus, with $K(k,i)$ being the $Q\times Q$ matrix that is the inverse of the matrix with elements $H(k,i)$, we have

$$P(k)'=\sum_{i=1}^{Q}K(k,i)\left[\sum_{j=1}^{M}\left[G(j,i)'V(j)+G(j,i)R(j)'\right]-P(i)S(i)'\right],\qquad k=1,Q\,.\qquad (5)$$

The sets of equations (3), (4) and (5) constitute the required set of $MQ+Q+M$ coupled differential equations for V, N and P. Once these are solved, C is found from the definition of V.

4. RELATION TO BID/ASK METHOD OF TRADING

The differential equations that govern time dependence of prices, number of shares and cash are derived under the implied assumption that the various managers collaborate to make changes in these quantities over a small time interval that are consistent with the basic relations between all the variables involved. It is of interest to understand how in a realistic situation this collaboration might occur. Here we examine a simple case involving two managers and one share type with the aim of finding out whether the price change predicted by the differential equations would result from a series bid/ask trades between the two managers. We find little difference between the two methods. This supports that view that the differential equations are a reasonable approximation to what might happen in the real world if managers traded continuously to adjust their portfolios according to specified time-varying weights.

With the previous notation, in this case we have one stock $Q=1$ and two managers $M=2$. Let us assume that at time $t=0$ both portfolios contain cash of 0.5 and 0.5 shares, with a share price of 1.0. Thus we have at time $t=0$

$$C(j)=0.5,\quad N(j,1)=0.5\quad j=1,2\,;\quad P(1)=1.,$$

so that we have share weights

$G(j,1) = 0.5 \quad j = 1,2$.

Let us suppose that Manager 1 wishes to increase the share weight to 0.6, while Manager 2 will keep it constant at 0.5. We assume no external flows of cash or shares, i.e. $R' = S' = 0$ at all times.

DIFFERENTIAL EQUATION METHOD

If we assume that weight $G(1,1)$ increases linearly with time to the value 0.6 at time $t = 1.$, then a numerical solution of the differential equations leads to the following results at time $t = 1.$

$N(1,1) = 0.54796, \quad N(2,1) = 0.45204$	Number of shares in each portfolio
$C(1,1) = 0.44694, \quad C(2,1) = 0.55306$	Cash in each portfolio
$G(1,1) = 0.6, \quad G(2,1) = 0.5$	Weight of shares in each portfolio
$P(1) = 1.22347$	Price of share

BID/ASK METHOD

Now let us assume that each manager does not know the holdings or the intentions of the other manager. Rational strategies for the two managers are

Manager 1 Offer to buy or sell some shares so as to move the share weight in the direction of the target weight 0.6. This will mean that the offer must be at a price different from the current price.

Manager 2 When a price different from the current price is offered, buy or sell an amount of shares just sufficient to return his share weight to 0.5, calculated using the new price.

These actions take place alternately until Manager 1 decides that the portfolio weight is close enough to 0.6.

The process consists of the following steps.

Step 1 Based on the latest price and number of shares in Portfolio 1, Manager 1 calculates his new share weight $G(1,1)$. Unless the latest value of $G(1,1)$ is sufficiently near the target weight, in which case the process terminates, Manager 1 chooses a new price equal to the current price plus an increment DP, and offers to buy/sell shares depending on whether DP is positive/negative. There are two cases

i) If the current price is the original price $P(1) = 1.$, then $DP = DP0$, where $DP0$ is chosen by the manager, preferably as a small percentage of $P(1)$.

ii) Otherwise,

$$DP = DPN \ \text{ if } \ abs(DPN) \le abs(DP0)$$
$$DP = \frac{DPNabs(DP0)}{abs(DPN)} \ \text{ if } \ abs(DPN) > abs(DP0)$$

The quantity DPN is the estimate of DP obtained by Newton's method to the two most recent values of $P(1), \quad G(1,1)$. See Appendix 2.

<u>Step 2</u> Manager 2 sells/buys a number of shares from Manager 1 at the price offered by Manager 1. The number is calculated so that the portfolio weight $G(2,1)$ of Manager 2 returns to its initial value. The share weight of Manager 1 changes accordingly. See Appendix 3. Now Step 1 is repeated.

Numerical results for different values of the initial price change $DP0$ are given in Appendix 4. It will be seen that the two methods give almost identical results for small price change. It would be interesting to study how the two methods compare in more complicated cases.

5. DISCUSSION

There are several possible ways in which the formalism may be extended, and problems where it may be applied. Some of them are discussed below.

EXTENSIONS

Rebalancing

In the procedure used to derive the differential equations each manager continually adjusts his portfolio as stock prices changes, even if his target weights remain the same. Price changes mean that share numbers must change to maintain the same weights. This effect would lead to excessive trading costs, so that, in practice, a manager would not trade in a share until the corresponding weight moved outside a specified range centered on the target weight. The differential equations can be modified to take account of this requirement, but we disregard this possibility in the remarks below.

Extended Definition of Cash and Cash Transfer

We may make the model more realistic by extending the definition of portfolio cash $C(j)$ and cash transferred $R(j)$. In addition to cash we assume that $C(j)$ also includes the value $A(j)$ of other financial assets held in the portfolio. That is assets in addition to the shares previously discussed. Their value is determined externally and not by the market process we described for the shares. If the value of $A(j)$ changes over time, that change is reflected in $R(j)$, which also takes account of investor cash flows as before, as well as dividends plus fees, commissions, expenses, etc. incurred by the manager. With these extended definitions the same equations set out in Appendix 1 govern changes in prices, etc. The equations are a consequence of the assumption that shares are traded so as to maintain specified time-varying share weights, a weight being a fraction of the total portfolio value.

APPLICATIONS

Price Change Drivers - Internal

Let us consider the US stock market, say the largest 1000 stocks by market capitalization together with all the many managers that hold these stocks. In most cases the weights $G(j,i)$ will be small compared to 1, and the number of shares of type i held by manager j will be small compared to $S(i)$, so that it is reasonable to approximate the matrix $H(k,i)$ by its first term $S(k)\delta_{ki}$. This results in the approximation

$$P(k)' \approx \frac{1}{S(k)} \sum_{j=1}^{M} \left[G(j,k)'V(j) + G(j,k)R(j)' \right] - P(k)S(k)', \qquad k = 1, Q$$

This equation shows that share price changes are driven by changes in share weights $G(j,k)$, cash transferred $R(j)$, and number of shares outstanding $S(k)$.We call the first of these internal (endogenous if you are an economist) because share weights are under the control of the managers, and the other two are external (exogenous).

In the US market there is a very wide spread of portfolio values $V(j)$ over managers j, with a few very large ones down to a great many smaller ones. Since we have no reason to suppose that the rates of change of weights $G(j,k)'$ will differ systematically based on portfolio size, it is likely that internal share price changes will normally be dominated by a relatively few large managers because of the factor $V(j)$ in the internal price change formula. Someone who could predict a stock's rate of weight change for a relatively small number of large managers would probably be able to predict price changes, especially if the changes were all in the same direction. Numerical simulation might shed more light on this question.

It is clear that if a high proportion of managers making weight changes, especially the large ones, decided to increase their weights for a particular share, then the price would rise. That rise could get to the point where the approximation above would no longer be justified, and analysis shows that the correct formula would lead to an enhanced price rise. There is no formal limit to how high the price could rise based on internal factors alone, and the same applies in reverse to falls.

Price Change Drivers - External

Note that, in the above scenario, a sufficiently large price rise might induce managers to remove cash and companies to issue more shares, and the external price drivers $R(j)'$ and $S(k)'$ would lead to a compensating tendency for prices to fall.

In general it might be expected that internal price changes would dominate for shorter time periods, but that the external drivers could have a significant effect over long periods. Thus it would be interesting to try to estimate the contributions of the various drivers to the overall increase in stock market prices over lengthy periods in the past. A successful result would provide an explanation for the equity premium.

Numerical Simulations

In addition to the suggestions above, there are other areas where it would be interesting to simulate aspects of market behavior by numerical solution of the differential equations.

An earlier version of this report [Nu00] provides the results of some calculations on simple situations where there is only one type of share. One point to note is the surprisingly large price changes that can result in weight changes (in the earlier version the term 'Asset Allocation Ratio' is used for 'weight') , especially when the initial weights are far from being even. This work could easily be generalized.

A lot of historical information is available about share prices, volumes and market capitalizations. It would be interesting to do experiments with different choices for weight changes in an attempt to discover what types of weight change behavior might be responsible for observed volatilities, etc. It would be even more interesting if some large managers could be persuaded to keep a record of their weight changes for academic study at a later date when the information was no longer of commercial significance.

Index Investing

Index investing has grown in popularity in recent years, and 'closet indexers' effectively add to the number of genuine index funds. We can speculate what would happen if every manager held the market index. Our model suggests that in this situation, not surprisingly, the relative weights of shares would be unchanged, no matter what happened to company fundamentals, although the overall price level of the index could change if cash were added to portfolios.

It would be of interest to examine the situation just before the advent of universal index holdings, when one only small active manager remained.

Efficient Market Hypothesis

The approach that we are advocating stresses the importance of the considerable diversity that is observed among market participants. There is no doubt that different managers use different information sets, analyzed in different ways, to produce different trading instructions. Without knowing the details of the weight changes for at least a group of the most important managers it is not obvious how to predict price changes. The net result is likely to appear to be random to many observers. More accurately we should perhaps regard the outcome as uncertain in the sense of Keynes. We may not have enough information to determine the properties of a probability distribution of returns [Po02].

In this very complicated situation a plausible null hypothesis would be that it is impossible to predict anything useful about future market prices. Thus in this view

'Active investment management adds no value, except by chance.'

This conclusion is much the same as that implied if we assume the validity of the EMH, as does, for instance, Malkiel, who writes, paraphrased,

No method of portfolio management would enable an investor to achieve returns greater than those that could be obtained by holding a randomly selected portfolio of individual stocks with comparable risk. [Ma03], p. 59

However, the reasoning behind the standard justification for the EMH is diametrically opposed to the view of complete ignorance. Thus Malkiel, and probably Fama, believe that the EMH is at least approximately correct, because

When information arises, the news spreads very quickly and is incorporated into the prices of securities without delay [Ma03], p. 59

so that

Prices fully reflect all known information. [Ma03], p. 59

The result is that

News is by definition unpredictable so that resulting price changes must be unpredictable and random. [Ma03], p. 59

Both approaches agree on the futility of active investing, but simplistically the first believes that

'Nobody knows anything'

whereas the second prefers to think that

'Everybody knows everything.'

We tend to favor a position in between the these two extremes, something along the lines of Bernstein [Be99]. Thus we think that

'By taking advantage of the ignorance and incompetence of some of their competitors, a minority of managers may have the skill to pursue successful active management.'

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APPENDIX 1

EQUATIONS - SUMMARY

DEFINITIONS

Q	Number of different types of share
M	Number of managers
$S(k), \quad k=1,Q$	Number of shares outstanding of type k at time t
$P(k), \quad k=1,Q$	Market price of share k at time t
$R(j), \quad j=1,M$	Net cash transferred by investors, etc. to manager j up to time t
$C(j), \quad j=1,M$	Cash in portfolio j at time t
$V(j), \quad j=1,M$	Market value including cash of portfolio j at time t
$N(j,k), \quad j=1,M;\, k=1,Q$	Number of shares of type k in portfolio j at time t
$G(j,k), \quad j=1,M;\, k=1,Q$	Weight of shares of type k in portfolio j at time t

EQUATIONS

The following two equations define the portfolio value $V(j)$ and the share weight $G(j,k)$.

$$V(j) = C(j) + \sum_{k=1}^{Q} P(k)N(j,k), \quad j=1,M$$

$$G(j,k) = P(k)N(j,k)/V(j) \quad j=1,M;\, k=1,Q$$

Define the matrices H and K as follows

$$H(k,i) = S(k)\delta_{ki} - G(j,k)N(j,i) \qquad k,i=1,Q\,.$$

$K(k,i)$ is the $Q \times Q$ matrix that is the inverse of the matrix with elements $H(k,i)$

Below are the formulas for the time derivatives of $P(k)$, $N(j,k)$ and $V(j)$.

$$P(k)' = \sum_{i=1}^{Q} K(k,i)\left[\sum_{j=1}^{M}\left[G(j,i)'V(j) + G(j,i)R(j)'\right] - P(i)S(i)'\right], \qquad k=1,Q\,. \tag{5}$$

$$N(j,k)' = \frac{1}{P(k)}\left[V(j)G(j,k)' + G(j,k)R(j)' - P(k)'N(j,k) + G(j,k)\sum_{i=1}^{Q} P(i)'N(j,i)\right], \quad j=1,M;\, k=1,Q \tag{4}$$

$$V(j)' = R(j)' + \sum_{k=1}^{Q} P(k)'N(j,k), \quad j=1,M \tag{3}$$

APPENDIX 2

Suppose that the two most recent values of weight $G(1,1)$ are ga, gb, with corresponding prices pa, pb, and that the target weight is gt. Then Newton's method estimates that the price pt corresponding to gt is given by

$$pt - pa = \frac{(pa - pb)}{(ga - gb)}(gt - ga).$$

If pa is the most recent price then we define $DPN = pt - pa$.

APPENDIX 3

Suppose that Portfolio 2 contains cash cb and number of shares of nb at the start of this step. Manager 2 sells nd shares to Manager 1 at a price of pa. The problem is to find the value of nd such that the weight of shares in Portfolio 2 becomes $g2 = 0.5$ in this example. To find nd we use the definition of weight

$$g2 = \frac{Value\ of\ \ shares}{Value\ of\ \ shares + Cash} = \frac{pa \times (nb - nd)}{pa \times (nb - nd) + cb + pa \times nd},$$

which may be solved to give

$$nd = \frac{nb(1 - g2)(pa - pb)}{pa},$$

using the fact that the weight in the starting portfolio is $g2$.

APPENDIX 4

The tables show the progress of the 'bid/ask' method for different choices of the initial price change $DP0$. The columns give the number of shares and the share weight for Manager 1 at the end of the step, followed by the starting price and the price change for the step.

$DP0$ 0.002

$N(1)$	$G(1)$	P	DP
0.50050	0.50100	1.00000	0.00200
0.50100	0.50200	1.00200	0.00200
0.50149	0.50299	1.00400	0.00200
0.50199	0.50398	1.00600	0.00200
0.50248	0.50497	1.00800	0.00200
0.50297	0.50596	1.01000	0.00200
0.50346	0.50695	1.01200	0.00200
0.50395	0.50793	1.01400	0.00200
0.50444	0.50892	1.01600	0.00200
0.50492	0.50990	1.01800	0.00200
0.50541	0.51087	1.02000	0.00200
0.50589	0.51185	1.02200	0.00200
0.50637	0.51283	1.02400	0.00200
0.50685	0.51380	1.02600	0.00200
0.50733	0.51477	1.02800	0.00200
0.50781	0.51574	1.03000	0.00200
0.50829	0.51671	1.03200	0.00200
0.50876	0.51767	1.03400	0.00200
0.50923	0.51863	1.03600	0.00200
0.50971	0.51959	1.03800	0.00200
0.51018	0.52055	1.04000	0.00200
0.51065	0.52151	1.04200	0.00200
0.51111	0.52247	1.04400	0.00200
0.51158	0.52342	1.04600	0.00200
0.51204	0.52437	1.04800	0.00200
0.51251	0.52532	1.05000	0.00200
0.51297	0.52627	1.05200	0.00200
0.51343	0.52721	1.05400	0.00200
0.51389	0.52816	1.05600	0.00200
0.51435	0.52910	1.05800	0.00200
0.51481	0.53004	1.06000	0.00200
0.51526	0.53098	1.06200	0.00200
0.51572	0.53191	1.06400	0.00200
0.51617	0.53285	1.06600	0.00200
0.51662	0.53378	1.06800	0.00200
0.51708	0.53471	1.07000	0.00200
0.51753	0.53564	1.07200	0.00200
0.51797	0.53657	1.07400	0.00200
0.51842	0.53749	1.07600	0.00200
0.51887	0.53841	1.07800	0.00200

0.51931	0.53934	1.08000	0.00200
0.51975	0.54025	1.08200	0.00200
0.52020	0.54117	1.08400	0.00200
0.52064	0.54209	1.08600	0.00200
0.52108	0.54300	1.08800	0.00200
0.52152	0.54391	1.09000	0.00200
0.52195	0.54482	1.09200	0.00200
0.52239	0.54573	1.09400	0.00200
0.52282	0.54664	1.09600	0.00200
0.52326	0.54754	1.09800	0.00200
0.52369	0.54844	1.10000	0.00200
0.52412	0.54935	1.10200	0.00200
0.52455	0.55024	1.10400	0.00200
0.52498	0.55114	1.10600	0.00200
0.52541	0.55204	1.10800	0.00200
0.52584	0.55293	1.11000	0.00200
0.52626	0.55382	1.11200	0.00200
0.52669	0.55471	1.11400	0.00200
0.52711	0.55560	1.11600	0.00200
0.52753	0.55649	1.11800	0.00200
0.52795	0.55737	1.12000	0.00200
0.52837	0.55825	1.12200	0.00200
0.52879	0.55913	1.12400	0.00200
0.52921	0.56001	1.12600	0.00200
0.52963	0.56089	1.12800	0.00200
0.53004	0.56177	1.13000	0.00200
0.53046	0.56264	1.13200	0.00200
0.53087	0.56351	1.13400	0.00200
0.53128	0.56438	1.13600	0.00200
0.53169	0.56525	1.13800	0.00200
0.53210	0.56612	1.14000	0.00200
0.53251	0.56698	1.14200	0.00200
0.53292	0.56784	1.14400	0.00200
0.53333	0.56871	1.14600	0.00200
0.53373	0.56956	1.14800	0.00200
0.53414	0.57042	1.15000	0.00200
0.53454	0.57128	1.15200	0.00200
0.53494	0.57213	1.15400	0.00200
0.53535	0.57299	1.15600	0.00200
0.53575	0.57384	1.15800	0.00200
0.53615	0.57468	1.16000	0.00200
0.53654	0.57553	1.16200	0.00200
0.53694	0.57638	1.16400	0.00200
0.53734	0.57722	1.16600	0.00200
0.53773	0.57806	1.16800	0.00200
0.53813	0.57890	1.17000	0.00200
0.53852	0.57974	1.17200	0.00200
0.53891	0.58058	1.17400	0.00200
0.53931	0.58142	1.17600	0.00200
0.53970	0.58225	1.17800	0.00200

0.54009	0.58308	1.18000	0.00200
0.54047	0.58391	1.18200	0.00200
0.54086	0.58474	1.18400	0.00200
0.54125	0.58557	1.18600	0.00200
0.54163	0.58639	1.18800	0.00200
0.54202	0.58721	1.19000	0.00200
0.54240	0.58804	1.19200	0.00200
0.54278	0.58886	1.19400	0.00200
0.54317	0.58967	1.19600	0.00200
0.54355	0.59049	1.19800	0.00200
0.54393	0.59131	1.20000	0.00200
0.54430	0.59212	1.20200	0.00200
0.54468	0.59293	1.20400	0.00200
0.54506	0.59374	1.20600	0.00200
0.54544	0.59455	1.20800	0.00200
0.54581	0.59536	1.21000	0.00200
0.54618	0.59616	1.21200	0.00200
0.54656	0.59697	1.21400	0.00200
0.54693	0.59777	1.21600	0.00200
0.54730	0.59857	1.21800	0.00200
0.54767	0.59937	1.22000	0.00200
0.54797	0.60000	1.22200	0.00159
0.54797	0.60000	1.22359	0.00000

$DP0$ 0.005

$N(1)$	$G(1)$	P	DP
0.50124	0.50249	1.00000	0.00500
0.50248	0.50497	1.00500	0.00500
0.50370	0.50744	1.01000	0.00500
0.50492	0.50989	1.01500	0.00500
0.50613	0.51233	1.02000	0.00500
0.50733	0.51476	1.02500	0.00500
0.50852	0.51718	1.03000	0.00500
0.50970	0.51958	1.03500	0.00500
0.51087	0.52197	1.04000	0.00500
0.51204	0.52435	1.04500	0.00500
0.51319	0.52672	1.05000	0.00500
0.51434	0.52908	1.05500	0.00500
0.51548	0.53142	1.06000	0.00500
0.51661	0.53376	1.06500	0.00500
0.51774	0.53608	1.07000	0.00500
0.51885	0.53839	1.07500	0.00500
0.51996	0.54068	1.08000	0.00500
0.52106	0.54297	1.08500	0.00500
0.52216	0.54524	1.09000	0.00500
0.52324	0.54751	1.09500	0.00500
0.52432	0.54976	1.10000	0.00500

0.52539	0.55200	1.10500	0.00500
0.52646	0.55423	1.11000	0.00500
0.52751	0.55645	1.11500	0.00500
0.52856	0.55865	1.12000	0.00500
0.52961	0.56085	1.12500	0.00500
0.53064	0.56303	1.13000	0.00500
0.53167	0.56520	1.13500	0.00500
0.53269	0.56736	1.14000	0.00500
0.53371	0.56951	1.14500	0.00500
0.53472	0.57165	1.15000	0.00500
0.53572	0.57378	1.15500	0.00500
0.53672	0.57590	1.16000	0.00500
0.53771	0.57801	1.16500	0.00500
0.53869	0.58010	1.17000	0.00500
0.53967	0.58219	1.17500	0.00500
0.54064	0.58427	1.18000	0.00500
0.54161	0.58633	1.18500	0.00500
0.54256	0.58838	1.19000	0.00500
0.54352	0.59043	1.19500	0.00500
0.54446	0.59246	1.20000	0.00500
0.54541	0.59448	1.20500	0.00500
0.54634	0.59650	1.21000	0.00500
0.54727	0.59850	1.21500	0.00500
0.54796	0.59999	1.22000	0.00375
0.54797	0.60000	1.22375	0.00002
0.54797	0.60000	1.22377	0.00000

$DP0$ 0.01

$N(1)$	$G(1)$	P	DP
0.50248	0.50496	1.00000	0.01000
0.50491	0.50988	1.01000	0.01000
0.50732	0.51474	1.02000	0.01000
0.50969	0.51956	1.03000	0.01000
0.51202	0.52432	1.04000	0.01000
0.51432	0.52904	1.05000	0.01000
0.51659	0.53371	1.06000	0.01000
0.51883	0.53834	1.07000	0.01000
0.52104	0.54292	1.08000	0.01000
0.52322	0.54745	1.09000	0.01000
0.52536	0.55194	1.10000	0.01000
0.52748	0.55638	1.11000	0.01000
0.52957	0.56077	1.12000	0.01000
0.53164	0.56512	1.13000	0.01000
0.53367	0.56943	1.14000	0.01000
0.53568	0.57370	1.15000	0.01000
0.53767	0.57792	1.16000	0.01000
0.53963	0.58209	1.17000	0.01000

0.54156	0.58623	1.18000	0.01000
0.54347	0.59032	1.19000	0.01000
0.54536	0.59437	1.20000	0.01000
0.54722	0.59838	1.21000	0.01000
0.54797	0.59999	1.22000	0.00403
0.54797	0.60000	1.22403	0.00002
0.54797	0.60000	1.22406	0.00000

$DP0$ 0.05

$N(1)$	$G(1)$	P	DP
0.51191	0.52409	1.00000	0.05000
0.52300	0.54699	1.05000	0.05000
0.53337	0.56877	1.10000	0.05000
0.54309	0.58948	1.15000	0.05000
0.54783	0.59966	1.20000	0.02541
0.54798	0.60000	1.22541	0.00085
0.54798	0.60000	1.22626	0.00001

$DP0$ 0.1

$N(1)$	$G(1)$	P	DP
0.52273	0.54644	1.00000	0.10000
0.54261	0.58845	1.10000	0.10000
0.54774	0.59946	1.20000	0.02750
0.54799	0.60000	1.22750	0.00136
0.54799	0.60000	1.22886	0.00001
0.54799	0.60000	1.22887	0.00000

$DP0$ 0.5

$N(1)$	$G(1)$	P	DP
0.58333	0.67742	1.00000	0.50000
0.54787	0.59914	1.50000	-0.21818
0.54829	0.60005	1.28182	0.00239
0.54827	0.60000	1.28421	-0.00013
0.54827	0.60000	1.28408	0.00000