The determinants of long-run inequality

Andrea, Canidio

Central European University

May 2009

The Determinants of Long-Run Inequality

Andrea Canidio

Central European University
May 2009

Abstract

I explore the effect of skill-biased technological change on long-run inequality by building a model where the supply of skilled and unskilled workers, the cost of education, and credit rationing are endogenous. In the model, the existence of unequal steady states does not depend on the degree of technological skill bias, but on the credit market, the cost of education, and the growth rate of the economy. However, by building an appropriate measure of inequality, I show that when unequal steady states exist, economies with a higher technological skill bias have a greater long-run inequality. Therefore, the impact of skill-biased technological change on inequality may be permanent.
Keywords: Endogenous Inequality, Skill Bias, Credit Rationing, Growth.
JEL Numbers: J24, J62, O11, O16, O33, O49.

http://www.personal.ceu.hu/staff/Andrea_Canidio/
canidioa@ceu.hu

Acknowledgement. I'm thankful to Dilip Mookherjee and Andrew Newman for their precious help and support.

1 Introduction

The evolution of the wage structure in the United States between the end of the 1970s and the beginning of the 1990s suggests that technology can, in the short run, increase inequality. During this period, the difference between the average wage of workers with a college degree and of workers with a high school degree increased significantly. The reason was the introduction of the personal computer and the unfolding of the information technology era. This wave of innovations was *skill biased*: it increased the productivity of skilled workers (workers with a college degree), leaving unchanged the productivity of unskilled workers.[1]

However, the long-run impact of technology on inequality is not well understood. Contrary to the short run, in the long-run the supply of skilled workers is endogenous: after an increase in the skilled wage the number of skilled workers may increase. The possible reasons are:

- Parents may be willing to spend more on the education of their children when the return on education is higher.
- Student loans have an implicit collateral: the student's future wage. When the wage is higher, more people should be able to access the credit market and finance their education.

In addition, the short-run cost of education is fixed, but since college professors are skilled workers the long-run college tuition is likely to be correlated with the skilled wage. Therefore, the impact of an increase in the skilled wage on the net return of the skilled profession is ambiguous.

Here I explore the effect of skill-biased technological change on long-run inequality by building an overlapping generation model that includes the three mechanisms listed above. The building blocks are:

- Altruistic parents that leave bequests to their children.
- An imperfect credit market where young people can raise resources to finance their education.
- An endogenous cost of education.

Note that each of these building blocks introduces a new potential source of inequality: altruism, credit market imperfections, and the cost of education. In addition, I will include in the analysis an exogenous productivity growth. The reason is that all the mechanisms

[1] See, among many others, Juhn, Murphy, and Pierce (1993) and Autor, Katz, and Kearney (2005).

described above are intertemporal (pay for school today to earn a higher wage tomorrow, borrow today and pay back tomorrow). The increase in productivity across periods may have important consequences for long-run inequality.

I show that, as typical in this class of models, the set of unequal steady states is a continuum. More interestingly, here the set of possible steady-state skill premia (the ratio of skilled and unskilled wage) does not depend on the skill bias of the economy. The existence of a specific steady state (equal or unequal) depends on a set of constraints to be satisfied. For example, in an unequal steady state unskilled workers should not be able to borrow, and the return on the skilled profession should be higher than the return of the unskilled one. I show that these constraints depend on technology only through the skill premium. In the long run, the skill premium is a function of the skill bias and of the skilled-to-unskilled ratio. Therefore, two economies differing only in their skill bias will have different steady-state skilled-to-unskilled ratios so to keep the set of steady-state skill premia constant.

As a consequence, the existence of long-run inequality does not depend on the degree of technological skill bias. Long-run inequality exists when the credit market does not function well, when the growth rate of the economy is low, and when education is costly. Whenever unequal steady states do not exist, the unique steady state is equal: both professions yield the same return.

This first set of results, therefore, is useful when comparing economies with an unknown initial condition. In this sense, skill-biased technology does not matter because the range of possible outcomes (measured in terms of skill premium) does not change with skill bias. However, I also show that if an economy is in an unequal steady state, and this steady state is in the interior of the unequal steady-states set, it is possible to track the convergence of this economy to the new steady state after an arbitrarily small change in skill bias. In this sense, the model allows us to predict what happens to long-run inequality after an increase in skill bias starting from some specific initial conditions.

In order to make a meaningful comparison between economies before and after the technological shock, I build an appropriate measure of long-run inequality that, following Atkinson (1970), depends on the whole distribution of steady-state wealth. I show that after an arbitrarily small increase in skill bias, the long-run skill premium and the long-run inequality both increase. This is true in every steady state except for a zero-measure set. Therefore, in general, short-run inequality and long-run inequality move in the same direction. This implies that the impact of technology on inequality may be permanent: today's inequality may depend on the introduction of the personal computer 30 years ago.

The only other paper exploring the long-run impact of skill-bias technological change is Rigolini (2004). The author develops a model where the cost of education is endogenous and depends on the skilled wage. He shows that an increase in the growth rate of the

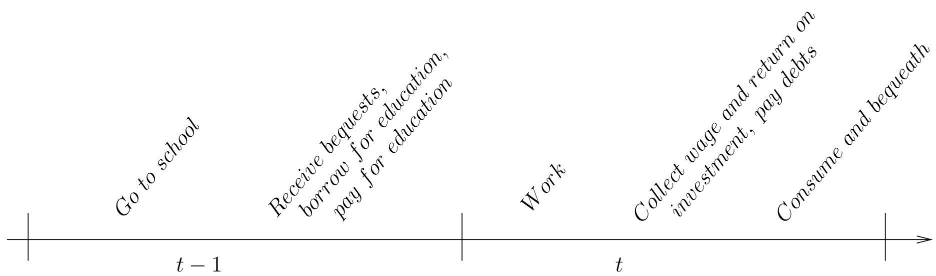

Fig. 1: Timeline

economy may increase or decrease the incentive for unskilled workers to acquire education, measured as the utility difference between an unskilled agent that remains unskilled and an unskilled agent that decides to become skilled. In his model, a higher growth rate increases the cost (to be paid upfront) as well as the return (to be enjoyed in the future) of education. Depending on the utility function and the discount factor, education may become more or less worthwhile. He then claims that the same result applies with respect to skill biased technological change. I show that Rigolini's intuition is incorrect. The difference between an increase in skill bias and an increase in the growth rate is that skill bias also makes unskilled workers poorer relative to skilled workers. As a result, the impact of an increase in skill bias does not depend on the parameters of the utility function.

In the next section I illustrate the model. In the third section I derive the steady state of the economy. I build a measure of inequality in the fourth section. I derive the dynamics and compare steady states before and after a skill-biased technological shock in the fifth section. Finally, in the last section I conclude with a brief summary of my main results.

2 The Model

A small open economy is composed of a measure one of agents, all identical but starting their lives with different levels of wealth.

2.1 The Households.

Each individual is alive for two periods. During the first one, she receives a bequest from her parent and decides whether to get an education or not. If she chooses to go to school, she'll be a skilled type; otherwise she'll be an unskilled type. In the second period she works, earns a wage, consumes and bequeaths to her only child (see figure 1). The end-of-life utility of an agent active at time t depends on her own consumption as well as on the wealth of her child.[2] Define w_t^s and w_t^u as the wage of a skilled and of an unskilled worker active in period t. Define e_t^i as the bequest made by the member of household i active in period $t-1$ (agent $\{i, t-1\}$) to the one active in period t (agent $\{i, t\}$). Define the total wealth of agent $\{i, t\}$ as the sum of the resources available to her before consuming and bequeathing:

$$x_t^i = \begin{cases} (e_t^i - \xi_{t-1})(1+r) + w_t^s & \text{if } \{i,t\} \text{ is high skilled} \\ e_t^i(1+r) + w_t^u & \text{if } \{i,t\} \text{ is low skilled} \end{cases}$$

where ξ_{t-1} is the cost of eduction in period $t-1$. It follows that a parent's utility is given by:

$$U_t^i = u(c_t^i) + \beta v(x_{t+1}^i) \tag{1}$$

I assume that both functions $u(.)$ and $v(.)$ are identical CRRA:

$$U_t^i = \frac{\left(c_t^i\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(x_{t+1}^i\right)^{1-\sigma}}{1-\sigma}$$

As already mentioned in the introduction, the cost of education is endogenous. Children can become unskilled workers for free, but becoming a skilled worker is costly. In each period some skilled workers are employed as teachers, and each teaches to $\frac{1}{\lambda}$ number of students. Therefore the cost of education is given by $\xi_{t-1} = \lambda w_{t-1}^s$.

In order to finance their education, people can borrow on the capital market using the bequest received and their future wage as collateral. However, in case they do not pay back, they will lose only a fraction $1-\theta$ of their total wealth. Thus, they are not fully liable. If a young individual goes to school without borrowing, or if she borrows and repays her loan, her budget constraint is:

$$c_t^i + e_{t+1}^i = w_t^s + (e_t^i - \lambda w_{t-1}^s)(1+r) \tag{2}$$

If she borrows for school and does not repay, her budget constraint is:

$$c_t^i + e_{t+1}^i = \theta w_t^s \tag{3}$$

If she does not go to school, her budget constraint is:

$$c_t^i + e_{t+1}^i = w_t^u + e_t^i(1+r) \tag{4}$$

Given this, banks will lend to agents only if the RHS of 2 is greater than the RHS of 3: access

[2] This form of altruism is called *paternalistic altruism*. It implies that parents care about their direct offspring but not about distant generations. For details, see Mookherjee and Ray (2006).

to the credit market and to school is determined by the bequest received at the beginning of life. More precisely, people can become skilled if:

$$e_t \geq \lambda w_{t-1}^s - w_t^s \left(\frac{1-\theta}{1+r} \right) \tag{5}$$

Note that, although the parameter θ is taken as given, the existence of credit rationing in the economy is determined endogenously. An economy with very severe credit constraints in some periods could evolve toward a perfect credit market. Similarly, an economy with a perfect credit market could later on develop some imperfections. Since inequality and credit market imperfections are strictly interconnected, most of the analysis that follows will focus on the long run evolution of equation 5.

Given the utility function and the budget constraint, it is possible to derive the first order conditions. In principle, there are several different cases, depending on the profession of the parent and on the profession of the child. However, the first order conditions that will be relevant in steady state are: skilled parents with skilled son, interior solution:

$$e_{t+1}^{i,s} = \frac{1}{\rho(1+r)+1} \left((e_t^i - \lambda w_{t-1}^s)(1+r) + w_t^s(1+\rho(1+r)\lambda) - \rho w_{t+1}^s \right) \tag{6}$$

skilled parents with skilled son, corner solution:

$$e_{t+1}^{i,s} = \max \left\{ \lambda w_t^s - w_{t+1}^s \left(\frac{1-\theta}{1+r} \right), 0 \right\} \tag{7}$$

unskilled parent with unskilled son, interior solution:

$$e_{t+1}^{i,u} = \frac{1}{\rho(1+r)+1} \left(e_t^i(1+r) + w_t^u - \rho w_{t+1}^u \right) \tag{8}$$

and unskilled parent with unskilled son, corner solution:

$$e_{t+1}^{i,u} = 0 \tag{9}$$

where $\rho \equiv [\beta (1+r)]^{-\frac{1}{\sigma}}$.

Finally, I will show in the next section that, under fairly weak assumptions, both professions are always employed in production. This implies that becoming a skilled worker must always be at least as profitable as becoming an unskilled worker. In other words, the RHS of 2 is always greater or equal than the RHS of 4:

$$w_t^s - \lambda w_{t-1}^s(1+r) \geq w_t^u \tag{10}$$

2.2 The Production Function.

The final good is produced according to the following function:

$$Y_t = K_t^{\alpha}(a_t S_t^{\epsilon} + b_t U_t^{\epsilon})^{\frac{1-\alpha}{\epsilon}} \tag{11}$$

where K is capital, S and U are skilled and unskilled workers employed in production, and a_t and b_t represent the productivity of the two types of workers. If both a_t and b_t change by the same amount, this translates into a change in the overall productivity on the economy. Instead, variations to $\frac{a_t}{b_t}$ reflect variations in the skill bias of the economy.

Markets are competitive and all inputs receive their marginal products:

$$r = \frac{\partial Y_t}{\partial k_t} = \alpha \left(\frac{(a_t S_t^{\epsilon} + b_t U_t^{\epsilon})^{\frac{1}{\epsilon}}}{K_t} \right)^{1-\alpha} \tag{12}$$

$$w_t^s = \frac{\partial Y_t}{\partial S_t} = K_t^{\alpha}(1-\alpha)(a_t S_t^{\epsilon} + b_t U_t^{\epsilon})^{\frac{1-\alpha}{\epsilon}-1} a_t S_t^{\epsilon-1} \tag{13}$$

$$w_t^u = \frac{\partial Y_t}{\partial U_t} = K_t^{\alpha}(1-\alpha)(a_t S_t^{\epsilon} + b_t U_t^{\epsilon})^{\frac{1-\alpha}{\epsilon}-1} b_t U_t^{\epsilon-1} \tag{14}$$

I assume that $\epsilon < 1$. Under this condition the marginal product of labor at zero is infinity: no matter how high wages are, firms will always demand a strictly positive amount of each type of labor. Note that if $\epsilon \leq 0$ no production can occur unless all inputs are used. If instead $0 < \epsilon < 1$ production can, in principle, occur using only one type of workers. Finally, whenever $1 - \alpha > \epsilon$ the two types of workers are complements, while if $1 - \alpha < \epsilon$ they are substitutes.

Consistent with the literature, let's call the ratio of the two wages *skill premium*:

$$\frac{w_t^s}{w_t^u} = \frac{a_t}{b_t} \left(\frac{U_t}{S_t} \right)^{1-\epsilon} \tag{15}$$

2.3 Market Clearing Conditions.

I assume that the economy is small and that it can freely borrow and lend on the international capital market. Under these assumptions, the domestic market clearing interest rate is equal to the international one, and the capital market is always in equilibrium. In addition, by Walras' law I can ignore the consumption good's market. It follows that the economy is in equilibrium if the two labor markets and the education market clear.

The demand for skilled and unskilled workers is given by equations 13 and 14. The supply depends on whether the returns on the two professions are equal or not. If agents prefer to be skilled, the supply of skilled workers is given by the number of workers with wealth satisfying condition 5. If instead they are indifferent, any agents with wealth satisfying condition 5 can be either skilled or unskilled.

Proposition 1. *Define $F_t(e)$ as the c.d.f of the wealth distribution across the generation active at period t. Whenever*

$$w_t^s - \lambda w_{t-1}^s(1+r) > w_t^u \tag{16}$$

labor market clears if

$$U_t = F_t\left(\lambda w_{t-1}^s - \frac{w_t^s(1-\theta)}{(1+r)}\right) \tag{17}$$

$$1 = U_t + S_t + \lambda S_{t+1} \tag{18}$$

and equations 13 and 14 hold. If instead

$$w_t^s - \lambda w_{t-1}^s(1+r) = w_t^u \tag{19}$$

The labor market clears if there exists a measure μ such that

$$U_t = F_t\left(\lambda w_{t-1}^s - \frac{w_t^s(1-\theta)}{(1+r)}\right) + \mu \tag{20}$$

$$1 = U_t + S_t + \lambda S_{t+1}$$

where $0 \leq \mu \leq 1 - F_t\left(\lambda w_{t-1}^s - \frac{w_t^s(1-\theta)}{(1+r)}\right)$ and, again, 13 and 14 are satisfied.

Finally, in a competitive equilibrium, the education market clears as well. Using the first order conditions of the production sector, it is possible to write

$$w_t^s = a\chi\left(a + b\left(\frac{U_t}{S_t}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}-1}$$

where

$$\chi = \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)$$

In other words, the labor-market clearing wage uniquely determines the skilled-to-unskilled ratio in period t. Call T_{t-1} the stock of teachers available in period $t-1$. The stock of skilled workers in period t is predetermined and equal to $\frac{T_{t-1}}{\lambda}$. Some of these skilled workers will work in production and other will work as teachers.

Proposition 2. *In period t, the education market clears if:*

$$\frac{S_t}{U_t} \leq \frac{\frac{T_{t-1}}{\lambda}}{1 - \frac{T_{t-1}}{\lambda}} \tag{21}$$

Nothing guarantees that condition 21 holds at the wages that clear the labor market. However, in the appendix I show that if a competitive equilibrium exists in period $t+1$, a unique competitive equilibrium exists in period t. This is enough to define a steady state in the usual way.

3 Steady State

Let's assume that the productivity parameters a_t and b_t grow at the common constant rate g.

Definition 3. The economy is in a *steady state* if the number of skilled workers and unskilled workers are constant, and aggregate output, aggregate capital, wages, and consumption grow at a constant rate.

Definition 4. A steady state is *equal* if the return on the two profession is equal. A steady state is *unequal* if the two professions yield different returns.

Since a steady state must also be a competitive equilibrium, it is quite straightforward to see that in an unequal steady state the following properties must hold:

- The return on the skilled profession is higher than the return on the unskilled profession.
- Unskilled workers do not have access to credit (no-credit constraint).
- Either unskilled workers do not want to bequeath enough so that their children can become skilled (no-deviation constraint), or it is impossible for them to do so (no-negative-consumption constraint).

Lemma 5. *In a steady state, if the skilled profession yields a return higher than the unskilled, the descendant of a skilled worker will be skilled, and the descendant of an unskilled worker will be unskilled.*

In other words, in a steady state each child inherits the profession of his parent,[3] and the economy is populated by skilled households and unskilled households.

Lemma 6. *Assume that a_t and b_t grow at a constant rate g. In a steady state, consumption, bequests, wages, and capital grow at a gross rate:*

$$\gamma \equiv 1 + g$$

[3] I assume, without loss of generality, that this is also the case when the two professions yield the same return.

Using lemma 5 and lemma 6, I can rewrite the first order conditions 6, 7, 8, and 9 as:

$$e^s_{t+1,ss} = w^s_{t,ss}\max\left\{\frac{(1-\rho\gamma)(\gamma-\lambda(1+r))}{\gamma-(1+r)(1-\rho\gamma)}, \lambda-\frac{\gamma(1-\theta)}{1+r}, 0\right\} \tag{22}$$

and

$$e^u_{t+1,ss} = w^u_{t,ss}\max\left\{\frac{\gamma(1-\rho\gamma)}{\gamma-(1+r)(1-\rho\gamma)}, 0\right\} \tag{23}$$

In order to guarantee the existence of a steady state, I need to impose some restrictions on the parameters.

Assumption 7. *The net return on the skilled profession is positive:*

$$\gamma > \lambda(1+r) \tag{24}$$

Assumption 7 is equivalent to

$$\frac{w^s_{t+1,ss}}{1+r} > \lambda w^s_{t,ss}$$

where the RHS is the cost of education in a steady state, and the LHS is the skilled wage discounted by one period. This assumption implies that, in order for a steady state to exist, the cost of education shouldn't be too high.

Note that assumption 7 would not be necessary if the cost of education were fixed, as in Mookherjee and Ray (2006). In that case, the skilled wage always adjusts in order to persuade some people to get educated. Here, instead, the steady-state cost of education is endogenous: whether the return on the skilled profession is positive or not does not depend on the skilled wage but on condition 24.

Proposition 8. *Under assumption 7, if the credit market works well enough*

$$\theta < 1 - \frac{\lambda(1+r)}{\gamma}$$

the economy has a unique steady state that is equal.

Proposition 8 is quite intuitive. When it holds, in a steady state everybody is able to borrow and pay for education, even if they have zero wealth. It follows that the only possible steady state is the equal one.

Proposition 9. *Assume the that credit market is imperfect:*

$$\theta > 1 - \frac{\lambda(1+r)}{\gamma}$$

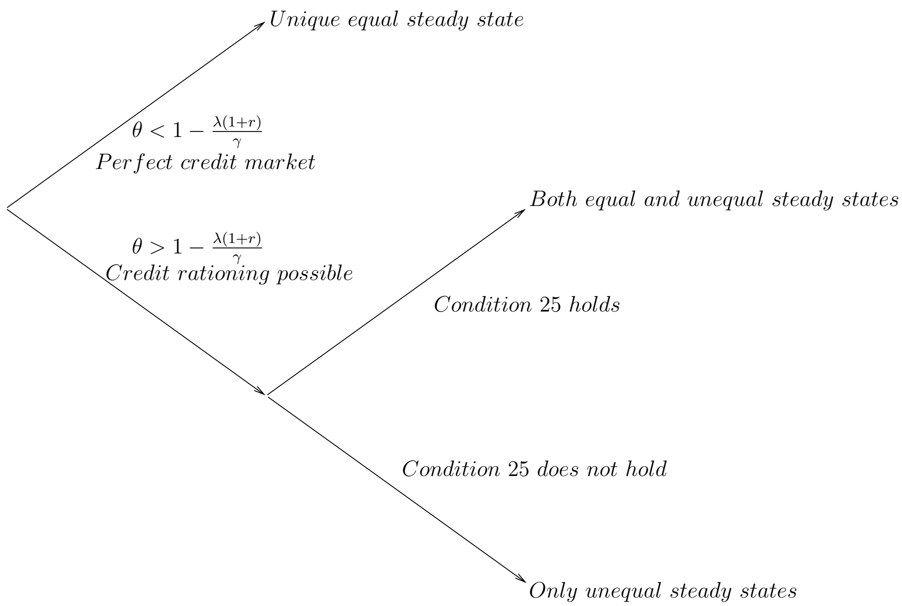

Fig. 2: Steady states.

Under assumption 7, if

$$\frac{1+r-\gamma}{1+r} < \gamma\rho < 1 \tag{25}$$

then the economy has a continuum of unequal steady states and one equal steady state. Otherwise, the economy has a continuum of unequal steady states, but no equal steady state.

The proof of proposition 9 shows that, when credit market is imperfect, there are always steady states where unskilled agents are too poor to borrow and too poor to leave bequests high enough to allow their children to access education. Note that proposition 9 is consistent with Mookherjee and Ray (2006) who find, in a model where there is no credit market, that unequal steady states always exist but the economy may or may not have an equal steady state.

Proposition 9 shows that skill-biased technology has nothing to do with the existence of long-run inequality. Quite intuitively, unequal steady states exist if:

- The credit market functions poorly (high θ).
- The growth rate of the economy γ is low.
- The interest rate r is high.
- The cost of education λ is high.

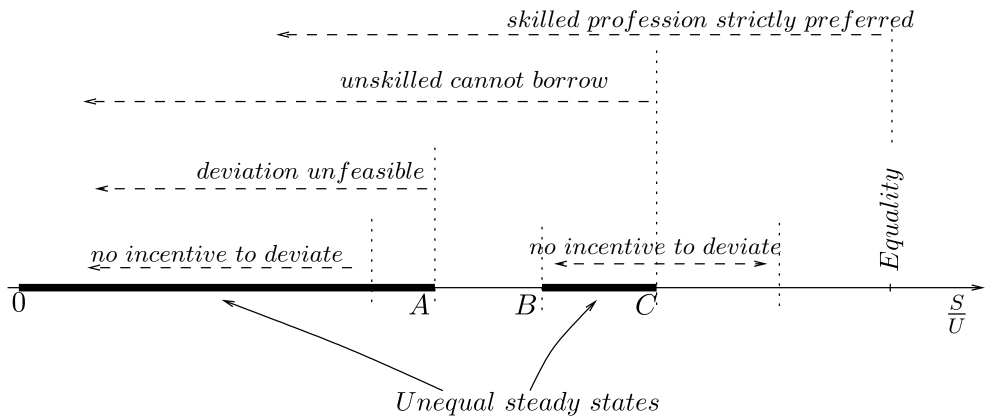

Fig. 3: Unequal steady states.

Therefore, any impact that skill-biased technology may have on inequality is of secondary order: it is relevant only if the other parameters are such that there is inequality in the first place.

Proposition 10. *The set of steady state skill premia does not depend on* $\frac{a}{b}$.

The proof of proposition 10 shows that each of the constraints characterizing an unequal steady state defines a set of skill premia. The reason is that the steady-state cost of education, the steady-state access to credit, and the steady-state bequests are all linear functions of wages. It follows, quite intuitively, that whether education is desirable, whether it is possible to send children to school, or whether it is possible to access credit depends on the skill premium. Since technology affects the constraints only through the skill premium, the steady-state skill premia are independent on the skill bias. Figure 3 depicts one possible set of unequal steady states. When skill bias increases, the points A, B, and C shift to the right and the distance between A and B, and B and C expand, so as to keep the skill premium at the four points constant. Similarly, when skill bias decreases, the set of unequal steady states shrinks, but it never disappears.

Proposition 10 contradicts one of Rigolini's claims. In a model with a continuum of occupations, he shows that the only unequal steady state is the one where parents are indifferent between educating their children or not (as in point B of figure 3). He shows that, after an increase in the overall growth rate of the economy, the steady state skill premium may increase or decrease, depending on the parameters of the utility function. The reason is that the return on education is higher: whether the demand for education increases or not depends on the strength of the income and substitution effects. He then claims that the same holds true for a skill-biased technological change.

Rigolini's claim is not correct because skill-biased technological change has an additional effect on the wealth of unskilled workers. Unskilled parents do not want to bequeath enough so that their children can go to school if

$$\begin{aligned} U(e^u_{t,ss}(1+r) + w^u_{t,ss} - e^u_{t+1,ss}) + \beta U(e^u_{t+1,ss}(1+r) + w^u_{t+1,ss}) &\geq \\ U(e^u_{t,ss}(1+r) + w^u_{t,ss} - e^\star_{t+1,ss}) + \beta U(w^s_{t+1,ss}) \end{aligned}$$

where

$$e^\star_{t+1} = w^s_{t+1,ss}\left(\lambda - \frac{(1-\theta)\gamma}{1+r}\right)$$

is the amount of wealth a young agents needs to access education. The above expression can be rewritten as

$$T \geq \frac{1}{1-\sigma}\left[\left(R(1+r) + 1 - \frac{w^s\gamma}{w^u}O\right)^{1-\sigma} + \beta\left(\frac{\gamma w^s}{w^u}\right)^{1-\sigma}\right] \tag{26}$$

where

$$R = \max\left\{\frac{\gamma(1-\rho\gamma)}{\gamma-(1+r)(1-\rho\gamma)}, 0\right\}$$

$$O = \lambda - \frac{(1-\theta)\gamma}{1+r}$$

$$T = \frac{1}{1-\sigma}\left[((1+r-\gamma)R+1)^{1-\sigma} + \beta\left((1+r)\gamma R+1\right)^{1-\sigma}\right]$$

Therefore, the constraint depends on skill bias only through the skill premium. Also, the RHS of inequality 26 is strictly concave in $\frac{w^s}{w^u}$: the constraint is satisfied for higher skill premium and lower skill premium. It follows that an increase in the skill premium decreases the incentive to acquire education for low $\frac{S}{U}$ and increases it for high $\frac{S}{U}$.

4 Inequality

Most of the papers dealing with skill-biased technological change use skill premium as a measure for inequality. However, the features of this model make skill premium a bad proxy for long-run inequality. For example, the cost of education is endogenous: if skill premium goes up, but so does the cost of education, can we say that the economy is less equal? Skilled workers will earn more but will pay more for becoming skilled workers. They may not gain anything.

I build a measure of inequality based on Atkinson (1970). For a given social welfare function W, define x^{eq}_W as the wealth level that, if equally distributed across all agents, would achieve the current social welfare. Inequality can be measured as

$$\mu = 1 - \frac{x_W^{eq}}{E(x)} \tag{27}$$

where $E(x)$ is the average wealth in a given time period. This measure is always between zero and one, and it's exactly zero when wealth is constant across agents. Furthermore, it is invariant with respect to shifts in the average wealth. The interpretation of this measure is as follows: if $\mu = 0.7$, by distributing wealth equally we could achieve the very same level of social welfare, saving 70% of the current average wealth.

Finally, I specify a particular social welfare function. I use a Rawlsian criterion:

$$W = \min_i U(x^i) \tag{28}$$

This way, inequality measures the amount of wealth relative to the average wealth a social planner can collect if he expropriates the surplus of everybody who owns more than the poorest agent in the economy.

Lemma 11. *In steady state, inequality is given by:*

$$\mu\left(\frac{U}{S}\right) = \begin{cases} \frac{(1+\lambda)\left[\frac{a}{b}\left(\frac{U}{S}\right)^{1-\epsilon}\left(1-\frac{(1+r)\lambda}{\gamma}\right)-1\right]}{(1+\lambda)\frac{a}{b}\left(\frac{U}{S}\right)^{1-\epsilon}\left(1-\frac{(1+r)\lambda}{\gamma}\right)+\frac{U}{S}} & \textit{if } \frac{1+r-\gamma}{1+r} < \gamma\rho < 1 \\ \frac{(1+\lambda)\left[\frac{a}{b}\left(\frac{U}{S}\right)^{1-\epsilon}\left(1+\frac{(1+r)\lambda(\gamma-1)}{\gamma}-\gamma(1-\theta)\right)-1\right]}{(1+\lambda)\frac{a}{b}\left(\frac{U}{S}\right)^{1-\epsilon}\left(1+\frac{(1+r)\lambda(\gamma-1)}{\gamma}-\gamma(1-\theta)\right)+\frac{U}{S}} & \textit{otherwise} \end{cases} \tag{29}$$

For future reference, note that whenever I compare inequality across steady states with the same $\frac{S}{U}$ the use of a Rawlsian social welfare function is without loss of generality. Following Atkinson's intuition, the distribution of wealth can be seen as a lottery, and the measure of inequality can be seen as a normalized risk premium. Using the inequality measure to compare steady states having the same number of skilled and unskilled workers but with different wealth distributions is equivalent to comparing lotteries with two possible outcomes, where the probability of each outcome stays constant but the particular realization varies. Since the normalized risk premium and the inequality measure are invariant to shifts in the mean of the distribution, this is simply a mean-preserving spread. Whereas any risk averse agent has the same preference ranking over all the mean preserving spreads of a given distribution, any concave social welfare function delivers the same ranking of steady states in terms of inequality. However, when using the inequality measure to compare steady states with different skilled-to-unskilled ratios, the results will be specific to the Rawlsian criterion and may not generalize to other social welfare functions.

5 Stability

In models with a continuum of steady states, it is usually very difficult to determine what initial conditions lead to a particular steady state. This is why most of the analysis is typically done in terms of set of steady-state inequality levels or steady-state skill premia. Intuitively, if the set of steady-state inequality levels increases with a parameter, for an unknown initial condition the probability of converging to a more unequal steady state increases as well.

Luckily, here it is possible to derive the dynamics of the economy for some specific initial conditions.

Proposition 12. *Consider an economy in a steady state having a skilled-to-unskilled ratio* m*. If* m *is in the interior of the set of unequal steady states, after a shock the economy will converge again to* m*.*

A shock is, by definition, an arbitrarily small variation to one of the parameters of the economy. If the variation is very small, and m is in the interior, m remains a steady state. The intuition behind proposition 12 is that, after a shock, unskilled workers remain credit constrained. It follows that the number of skilled and unskilled workers remains constant along the transition to the new steady state. Therefore, the economy can only converge back to m.

Proposition 13. *Consider an economy in an unequal steady state. Assume that its skilled-to-unskilled ratio is in the interior of the set of unequal steady states. After an arbitrarily small increase in skill bias, this economy converges to a steady state where skill premium and inequality are higher than before. Similarly, after an arbitrary decrease in skill bias, this economy converges to a steady state where skill premium and inequality are lower.*

Intuitively, keeping $\frac{S}{U}$ fixed, inequality increases when skill bias (and, therefore, skill premium) increases. The steady state wealth is a linear function of the wages of skilled and unskilled workers, and skill bias matters only through the wages. Relative wealth depends on skill bias only through the skill premium, exactly like in the short run case.

Therefore, for the steady states where it is possible to derive the dynamics, skill bias and inequality move together. In addition, if we limit to arbitrarily small increase in skill bias, the only point that is not a steady state anymore after a shock is point B in figure 3. For all the other unequal steady states, proposition 13 holds. Finally, note that this result does not depend on the particular social welfare function chosen, since, as discussed on the preceding page, we are comparing inequality levels across economies having the same $\frac{S}{U}$.

What happen after $\frac{a}{b}$ increases when an economy is at point B in figure 3? My conjecture is that the economy will converge to a new steady state B' having the same skill premium but more skilled workers than B. Note that, before the shock, at point B unskilled parents are indifferent between sending their children to school or not. After the shock, they strictly prefer to purchase education. Some unskilled workers become skilled until the skill premium returns to the original one. At this point, unskilled workers are, again, indifferent between purchasing education or not. When keeping skill premium constant, inequality increases if the number of skilled workers increases. This implies that, if the conjecture is correct, inequality increases following an increase in the skill bias in this case too. However, it is important to remember that the result depends on the specific social welfare function chosen. Since a Rawlsian criterion is a special case, even if the conjecture is correct this result is somehow fragile.

Given the fact that the economy has a continuum of unequal steady states, what happens at point B may seem almost irrelevant. However, in models with an exogenous cost of education by introducing a continuum of occupations (as in Mookherjee and Ray (2006)), or random ability shocks (as in Mookherjee and Napel (2007)), or different fertility rates (as in Mookherjee, Prina, and Ray (2009)), the surviving unequal steady states are the ones where unskilled agents are indifferent between purchasing education or not, exactly like at B. In light of these other works, it is important to fully understand what happens to inequality at point B, using a general class of social welfare functions. Answering this question is left for future work.

6 Conclusion

There is convincing evidence showing that, in the short run, skill-biased technological change can increase inequality. However, very little is known about its long-run effect. My paper aims at filling this gap.

I build an OLG model with several potential sources of inequality: skill-biased technology, imperfect credit market, exogenous productivity growth, altruism, and education technology. I show that long-run inequality exists if the credit market functions poorly, productivity growth is low, altruism is low, or the education technology is inefficient. Surprisingly, the existence of long-run inequality does not depend on the degree of skill bias of the economy.

Whether the degree of skill bias is important in determining the long-run inequality level depends on how inequality is measured. I build a measure of inequality based on Atkinson (1970) and I show that, if inequality exists, after an increase in skill bias almost all the economies will converge to steady states with a higher skill premium and higher inequality. Therefore, short-run and long-run inequality move in the same direction. This result shows that the introduction of a skill biased technological innovation decades ago such as the computer may partly be responsible for today's inequality.

References

Acemoglu, D. (2002, March). Technical change, inequality, and the labor market. *Journal of Economic Literature 40*(1), 7–27.

Aghion, P. and P. Bolton (1997, April). A theory of trickle-down growth and development. *Review of Economic Studies 64*(2), 151–172.

Atkinson, A. B. (1970). On the measurement of inequality. *Journal of Economic Theory 2*, 244–263.

Autor, D. H., L. F. Katz, and M. S. Kearney (2005, October). Trends in u.s. wage inequality: Re-assessing the revisionists. *Harvard Institute of Economic Research Working Papers* (2095).

Banerjee, A. V. and A. F. Newman (1993, April). Occupational chioce and the process of development. *The Journal of Political Economy 101*(2), 274–298.

Becker, G. and N. Tomes (1979). An equilibrium theory of the distribution of income and intergenerational mobility. *The Journal of Political Economy*, 1153–1189.

Galor, O. and O. Moav (2000, May). Ability-biased technological transition, wage inequality, and economic growth. *The Quarterly Journal of Economics 115*(2), 469–497.

Galor, O. and J. Zeira (1993, January). Income distribution and macroeconomics. *Review of Economic Studies 60*(1), 35–52.

Greenwood, J. and B. Jovanovic (1990). Financial development, growth, and the distribution of income. *Journal of Political Economy 98*(5), 1076–1107.

Juhn, C., K. M. Murphy, and B. Pierce (1993, June). Wage inequality and the rise in returns to skill. *The Journal of Political Economy 101*(3), 410–442.

Kuznets, S. (1955). Economic growth and income inequality. *American Economic Association 1*, 1–28.

Lemieux, T. (2006, June). Increasing residual wage inequality: Composition effects, noisy data, or rising demand for skill? *The American Economic Revew 96*(3), 1–64.

Ljungqvist, L. (1993). Economic Underdevelopment: The Case of a Missing Market for Human Capital. *Journal of Development Economics 40*, 219–239.

Loury, G. (1981). Intergenerational transfers and the distribution of earnings. *Econometrica: Journal of the Econometric Society*, 843–867.

Matsuyama, K. (2000, October). Endogenous inequality. *The Revew of Economic Studies 67*(4), 743–759.

Mookherjee, D. and S. Napel (2007). Intergenerational mobility and macroeconomic history dependence. *Journal of Economic Theory 137*(1), 49–78.

Mookherjee, D., S. Prina, and D. Ray (2009). Endogenous fertility, inequality and human capital. *working paper*.

Mookherjee, D. and D. Ray (2003, April). Persistent inequality. *Review of Economic Studies 70*(2), 368–394.

Mookherjee, D. and D. Ray (2006, October). Occupational diversity and endogenous inequality. *Boston University -Department of Economics - The institute for Economic Development Working Papers dp-142*.

Piketty, T. (1997). The dynamics of the wealth distribution and the interest rate with credit rationing. *Review of Economic Studies 64*, 173–189.

Rigolini, J. (2004). Educational technologies, wages and technological progress. *Journal of Development Economics 75*.

Stiglitz, J. E. and A. Weiss (1981, June). Credit rationing in markets with imperfect information. *The American Economic Revew 71*(3), 393–410.

Townsend, R. M. and K. Ueda (2006). Financial deepening, inequality, and growth: a model-based quantitative evaluation. *Review of Economic Studies 73*, 251–293.

A Proofs of section 2.

A.1 Existence of the competitive equilibrium solving backward.

Consider a given sequence of bequests distributions $\{F_t(e)\}_{t=1}^{\infty}$. Suppose that at time $t+1$ there exists a competitive equilibrium. In other words, take the wages w_{t+1}^s and w_{t+1}^u, the number of teachers in period $t+1$ as given. I will show that there exists a competitive equilibrium in period t, therefore the problem can always be solved backward.

In every period, wages are fully determined by the skilled-to-unskilled ratio. Using the first order conditions of the production sector, it is possible to write

$$w_t^s = a\chi\left(a + b\left(\frac{U_t}{S_t}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}-1}$$

where

$$\chi = \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)$$

This implies two things:

- w_{t+1}^s, w_{t+1}^u and the number of teachers in period $t+1$ uniquely determines the number of skilled and unskilled workers in period $t+1$.
- $\frac{S_t}{U_t}$ uniquely determines w_t^s and, therefore, the cost of education in period $t+1$.

Choose $\frac{S_t}{U_t}$ such that the number of agents with access to the credit market is exactly equal to the number of skilled workers in period $t+1$. This is an equilibrium if the return on the skilled profession is greater than the return on the unskilled one in period $t+1$

$$w_t^s \leq \frac{1}{\lambda(1+r)}(w_{t+1}^s - w_{t+1}^u)$$

If the above condition is not satisfied, choose a $\frac{S_t}{U_t}$ such that

$$w_t^s = \frac{1}{\lambda(1+r)}(w_{t+1}^s - w_{t+1}^u)$$

In this case, the number of agents with access to the credit market is higher than the number of skilled agents in period $t+1$. However, agents are indifferent between becoming skilled workers or unskilled workers: I can assume that exactly $S_{t+1} + \frac{1}{\lambda}S_{t+2}$ agents become skilled. Finally, the number of teachers in period t is given by the number of skilled workers in period $t+1$.

Note that the above argument may not work if the distribution $F_{t+1}(e)$ is discontinuous. Suppose that there is a positive measure of agents receiving a particular bequest level $\overline{e}$. Imagine that when w_t^s is such that

$$\overline{e} = \lambda w_t^s - \frac{w_{t+1}^s(1-\theta)}{(1+r)}$$

too many people have access to the credit market:

$$U_{t+1} < F_{t+1}\left(\lambda w_t^s - \frac{w_{t+1}^s(1-\theta)}{(1+r)}\right)$$

but if w_t^s is such that

$$\overline{e} > \lambda w_t^s - \frac{w_{t+1}^s(1-\theta)}{(1+r)}$$

too few people have access to the credit market:

$$U_{t+1} > F_{t+1}\left(\lambda w_t^s - \frac{w_{t+1}^s(1-\theta)}{(1+r)}\right)$$

In this situation, for a competitive equilibrium to exist I need to introduce a tie breaking rule. Note that when

$$\overline{e} = \lambda w_t^s - \frac{w_{t+1}^s(1-\theta)}{(1+r)}$$

banks are indifferent about lending to agents with wealth $\overline{e}$. I assume that the banking sector will lend only to some of the agents with wealth equal to $\overline{e}$. The fraction receiving a loan is such that the number of unskilled workers next period is equal to U_{t+1}. This way, if a competitive equilibrium exists in period $t+1$, a competitive equilibrium exists also in period t, no matter the distribution of wealth.

B Proofs of section 3.

B.1 Proof of lemma 5.

Since the total number of skilled and unskilled workers is a constant in steady state, there can be social mobility only if two households swap occupations. Suppose that there is one household with a skilled father and an unskilled son, and another household with a unskilled father and a skilled son. Since agents prefer to be skilled, it must be the case that the son of the unskilled worker has access to the credit market and the son of the skilled worker does not. This implies that the unskilled father must be wealthier than the skilled father, leading to a contradiction: if the unskilled father had access to the credit market he would have chosen the skilled profession.

B.2 Proof of lemma 6.

Consider equation 12 in steady state at time t. Take the log of both sides and subtract the log of the same equation in $t+1$. The gross growth rate of capital is given by:

$$\gamma = 1 + g$$

Similarly, using equations 13 and 14, wages grow at the same rate γ. Finally, consider the budget constraint of an unskilled agent in a steady state, in period t:

$$e_{t+1,ss} + c_{t,ss} = w_{t,ss} + e_{t,ss}(1+r)$$

divide and multiply each element by itself one period earlier:

$$e_{t,ss}\gamma_e + c_{t-1,ss}\gamma_c = w_{t-1,ss}\gamma + e_{t-1,ss}(1+r)\gamma_e$$

Since the budget constraint must also hold in period $t-1$:

$$e_{t,ss} + c_{t-1,ss} = w_{t-1,ss} + e_{t-1,ss}(1+r)$$

it must be the case that $\gamma = \gamma_c = \gamma_e$.

B.3 Proof of propositions 8, 9 and 10.

Lemma 14. *Under assumption 7, if condition 25 holds, agents are unconstrained:*

$$e^s_{t+1,ss} = w^s_{t,ss}\frac{(1-\rho\gamma)(\gamma-\lambda(1+r))}{\gamma-(1+r)(1-\rho\gamma)}$$

$$e^u_{t+1,ss} = w^u_{t,ss}\frac{\gamma(1-\rho\gamma)}{\gamma-(1+r)(1-\rho\gamma)}$$

Otherwise, the constraints are binding:

$$e^s_{t+1,ss} = w^s_{t,ss} max\left\{\lambda - \frac{\gamma(1-\theta)}{1+r}, 0\right\}$$

$$e^u_{t+1,ss} = 0$$

Proof. Skilled workers are unconstrained when

$$\lambda - \frac{\gamma(1-\theta)}{1+r} < \frac{(1-\rho\gamma)(\gamma-\lambda(1+r))}{\gamma-(1+r)(1-\rho\gamma)}$$

or

$$\theta < \left(\frac{(1-\rho\gamma)(\gamma-\lambda(1+r))}{\gamma-(1+r)(1-\rho\gamma)}\right)\frac{(1+r)}{\gamma} + 1$$

by assumptions 7, when condition 25 holds the LHS is greater than one and the inequality holds. Finally, for unskilled workers the conclusion follows simply by condition 25. □

B.3.1 Proposition 8.

It follows simply because if

$$\theta < 1 - \frac{\lambda(1+r)}{\gamma}$$

everybody can borrow.

B.3.2 Proposition 9.

Assume that credit market is imperfect:

$$\theta > 1 - \frac{\lambda(1+r)}{\gamma}$$

- Case 1: agents are unconstrained (condition 25 holds).

There is an equal steady state if the returns on the two professions is equal:

$$1 - \frac{\lambda(1+r)}{\gamma} = \frac{w^u}{w^s} \tag{30}$$

There is an unequal steady state if the two professions yield different returns:

$$\frac{w^u}{w^s} < 1 - \frac{\lambda(1+r)}{\gamma}$$

unskilled workers cannot access the credit market:

$$\frac{w^u}{w^s} < \left(\lambda - \frac{(1-\theta)\gamma}{1+r}\right)\left(\frac{\gamma-(1+r)(1-\rho\gamma)}{\gamma(1-\rho\gamma)}\right)$$

and the no-negative-consumption constraint holds:

$$\frac{w^u}{w^s}\frac{1}{\gamma} < \left(\lambda - \frac{(1-\theta)\gamma}{1+r}\right)\left(\frac{\gamma-(1+r)(1-\rho\gamma)}{(\gamma-1)(1-\rho\gamma)(1+r)+\gamma}\right)$$

Because of assumption 7, if the credit market is imperfect, there is always some $\frac{S}{U}$ that satisfies the three inequalities above. However, note that the above constraints do not define the set of unequal steady states. The extra constraint missing is the no-deviation constraint: parents should not want to bequeath enough so that their children cannot go to school, whenever this deviation is possible. However, if a $\frac{w^u}{w^s}$ satisfies all the above constraints, this $\frac{w^u}{w^s}$ is an unequal steady state, since the no-negative-consumption constraints requires such deviation not to be feasible.

- Case 2: agents are constrained (condition 25 does not hold).

Unskilled agents bequeath zero wealth and are not able to borrow: the no-credit constraint is always satisfied. Note that there cannot be an equal steady state. This would correspond to a situation where skilled workers bequeath just enough so that their children can to go to school and unskilled workers leave no bequests. However, in this situation a skilled worker's son would be better off not getting an education and saving all the wealth received. Therefore this cannot be a steady state.

There is an unequal steady state if the return on one profession is strictly bigger that the return on the other one:

$$\frac{w^u}{w^s} < 1 - \frac{\lambda(1+r)}{\gamma}$$

and unskilled workers cannot bequeath enough so that their children can go to school:

$$\frac{w^u}{w^s}\frac{1}{\gamma} < 1 - \frac{\lambda(1+r)}{\gamma}$$

Note that by assumption 7, if credit market is imperfect and agents are constrained, there is always some $\frac{S}{U}$ that satisfies both conditions. This $\frac{S}{U}$ is an unequal steady state.

B.3.3 Proof of proposition 10.

All the constraints relevant in an unequal steady state have already been derived in the proof of proposition 9, except for the no-deviation constraint that is discussed in the main text on page 13. All of them are a function of skill bias only through the skill premium.

It is interesting to note here that the set of skill premia satisfying the no-deviation constraint has the shape $(0, x'] \cup [x'', x''']$, while the set of skill premia satisfying the other constraints has shape $(0, y]$. This implies that the set of unequal steady state that is between B and C in figure 3 may not exist.

C Proofs of section 4.

C.1 Proof of lemma 11.

The result follows from simple, but tedious, manipulations. By definition:

$$\mu = \frac{S(1+\lambda)\left[(e^S_{t,ss} - \lambda w^S_{t-1,ss})(1+r) + w^S_{t,ss} - e^u_{t,ss}(1+r) - w^u_{t,ss}\right]}{S(1+\lambda)((e^s_{t,ss} - \lambda w^s_{t-1,ss})(1+r) + w^s_{t,ss}) + U(e^u_{t,ss}(1+r) + w^u)}$$

write $w^s_{t-1,ss} = \frac{w^s_{t,ss}}{\gamma}$ and $e^i_{t,ss} = \frac{e^i_{t+1,ss}}{\gamma}$ for $i = s, u$. Assume that condition 25 holds and plug in the first order conditions 22 and 23 (using the fact that agents are unconstrained):

$$\mu = \frac{S(1+\lambda)\left[w^s_{t,ss}\left(A(\gamma-\lambda(1+r))\frac{(1+r)}{\gamma} - \frac{\lambda(1+r)}{\gamma} + 1\right) - w^u_{t,ss}\left(A(1+r)+1\right)\right]}{S(1+\lambda)w^s_{t,ss}\left(A(\gamma-\lambda(1+r))\frac{(1+r)}{\gamma} - \frac{\lambda(1+r)}{\gamma} + 1\right) + Uw^u_{t,ss}\left(A(1+r)+1\right)}$$

where

$$A = \frac{(1-\rho\gamma)}{\gamma-(1+r)(1-\rho\gamma)}$$

Simplify the expression dividing both numerator and denominator by $w^u\left(A(1+r)+1\right)$:

$$\mu = \frac{(1+\lambda)\left[\frac{w^s}{w^u}\left(1 - \frac{\frac{A\lambda(1+r)^2}{\gamma} + \frac{\lambda(1+r)}{\gamma}}{A(1+r)+1}\right) - 1\right]}{(1+\lambda)\frac{w^s}{w^u}\left(1 - \frac{\frac{A\lambda(1+r)^2}{\gamma} + \frac{\lambda(1+r)}{\gamma}}{A(1+r)+1}\right) + \frac{U}{S}}$$

and note that $\frac{A\lambda(1+r)^2}{\gamma} + \frac{\lambda(1+r)}{\gamma} = \left(A(1+r)+1\right)\left(\frac{(1+r)\lambda}{\gamma}\right)$.

In case condition 25 does not hold following the same steps, the solution is:

$$\mu = \frac{(1+\lambda)\left[\frac{w^s}{w^u}\left(1+\lambda(1+r)\left(1-\frac{1}{\gamma}\right) - \gamma(1-\theta)\right) - 1\right]}{(1+\lambda)\frac{w^s}{w^u}\left(1+\lambda(1+r)\left(1-\frac{1}{\gamma}\right) - \gamma(1-\theta)\right) + \frac{U}{S}}$$

D Proofs of section 5.

D.1 Proof of proposition 12.

If the shock is small, unskilled workers will be unable to borrow also after the shock. For as long as unskilled workers cannot borrow, wages remain constant at their post-shock level and bequests converge monotonically to the steady state bequests corresponding to the same m and the new parameters. Since in the new steady state they cannot borrow, and because the convergence is monotonic, there was no moment during the transition when unskilled workers had access to the credit market. The economy must converge to the steady state corresponding to the new parameters and the old m.

D.2 Proof of proposition 13.

Differentiating expression 29, it is possible to show that a higher $\frac{w^s}{w^u}$ increases inequality for given $\frac{S}{U}$.