Entrepreneurship and the Hidden Economy: An Extended Matching Model

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Abstract

This paper develops a labour market matching model in order to address the problem of the persistence of the hidden sector and of its regional concentration, as in Italy and in the enlarged Europe. The main novel features of the model are that entrepreneurial ability affects job productivity, and that regular firms receive negative externalities from the hidden sector, which may capture the pressure typically exerted by corruption and organized crime, and positive externalities from the other regular firms. At least one interior equilibrium emerges, thus providing an explanation for the so-called "shadow puzzle", with the possibility that tougher monitoring may reduce both the hidden sector and unemployment. If externalities are non-linear, two equilibria may emerge, thus accounting for regional dualism. The "better" equilibrium is in fact characterised by a smaller hidden sector, higher levels of overall productivity, output, entrepreneurial ability used, extra-profits, relative wages, and more favourable externalities.

JEL classification: E26, J23, J24, J63, J64, L26

Keywords: entrepreneurship, hidden economy, shadow economy, underground economy, matching models

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1 – Introduction

This paper addresses the problem of the hidden sector as a persistent and backward component of the economy, and as relatively concentrated in specific regions. The typical example is the Italian hidden sector, which consists of many small firms and productive activities framed in an advanced economic and institutional setting but also localised and linked to the specific socio-economic context of the country's southern regions (ISTAT, 2005, 2008; Daniele and Marani, 2008). Another example is the enlarged Europe, where the hidden sector is concentrated in the Eastern countries, which are also especially characterised by organised crime, corruption, and low law enforcement (Van Dijk, 2006; Johnson et al., 2000), but not necessarily by a heavy tax burden (Johnson et al., 1999).

The paper adopts a matching model à la Pissarides (2000) extended to the hidden sector, to heterogeneous entrepreneurial ability, and to sectoral externalities. Although the extension of matching models to the hidden sector is not new in the literature (see Boeri and Garibaldi, 2002, 2006; Bouev, 2002, 2005; Kolm and Larsen, 2003; Fugazza and Jacques, 2004; Albrecht et al., 2009), their additional extension to entrepreneurship is somewhat novel. Only Fonseca et al. (2001) and Pissarides (2002) have pursued this kind of analysis, but they have ignored the fact that entrepreneurship affects job productivity. If this fact is taken into consideration, interesting analytical consequences follow. The first is that the zero-profit condition, which usually applies to all firms in matching models because perfect competition prevails, only holds for the firm employing the minimum level of entrepreneurial ability. The other abler entrepreneurs earn extra-profits in posting vacancies, because entrepreneurial ability is a non-tradeable input for firms. The second consequence is that firms become heterogeneous in productivity, thus providing a new solution for the problem of finding an interior equilibrium, where vacant jobs are allocated to both the regular and the hidden sector.

The extension of the model to sectoral externalities is based on the idea that entrepreneurial ability is embedded in a socio-economic context which may be unfavourable because of high transaction costs. The entry to regular production may be hindered by various forms of rent stemming from corruption and criminal activity, but also because market connections are substituted with family rent-seeking connections.[1] By contrast, regular firms

[1] The southern regions of Italy constitute a typical case in which the socio-economic context of organized crime (Peri, 2004; Daniele and Marani, 2008), and of "amoral familism" (Banfield, 1958) has heavily burdened the economy.

may find a favourable socio-economic context when networking with the other firms is easy and trust prevails.

The paper is thus able to give theoretical account for a number of facts: at the macroeconomic level, the persistence of a substantial proportion of the hidden sector with detrimental effects on overall output and productivity; at the microeconomic level, some key characteristics of irregular firms, such as their relatively lower entrepreneurial ability, lower profits and relative wages. In particular, the model is able to determine the conditions under which a reduction of the hidden sector increases or reduces unemployment.

When the analysis concentrates on the role of externalities, it yields other results by recognising the particular non-linearity of externalities in diffusing themselves (Minniti, 2005; Ormerod, 2005; Puga and Venables, 1996; Krugman, 1991). In this case, two macroeconomic equilibria may emerge within the same institutional structure and with the same economic potential. The "bad" equilibrium consists of a relatively large hidden sector, important negative externalities, and reduced positive externalities; the "good" equilibrium consists of a relatively small hidden sector, important positive externalities, and reduced negative externalities.

This approach to the problem of the hidden economy makes it possible to extend the opportunity of policy actions from the fine tuning of institutional duties (Kolm and Larsen, 2003), from larger individual benefits of participating in the regular sector (Fugazza and Jacques, 2004), and from labour-market liberalisation (Boeri and Garibaldi, 2002, 2006; Bouev, 2002, 2005), to actions intended to increase positive externalities and to reduce negative ones.

The paper is organised as follows: section 2 presents the benchmark model; section 3 extends the model to endogenous externalities; while section 4 concludes with some remarks on policy implications. The appendices set out the relevant proofs and math details.

2 – The benchmark model

The paper proposes a general model of equilibrium unemployment (Mortensen and Pissarides, 1994; Pissarides, 2000). The economic environment is characterised by a non-competitive labour market with wage bargaining. Numerous firms competitively produce a homogeneous product, but adopt different institutional and technological set-ups. They may be registered, and therefore pay a production tax and adopt a relatively advanced technology; or they may not be registered, and therefore evade taxes and adopt a less efficient technology.

Hence non-registered firms form the hidden sector of the economy, which is illegal because of the process employed, not because of the good being produced.[2]

As is usual in matching-type models (Pissarides, 2000; Petrongolo and Pissarides, 2001), the meeting of vacant jobs and unemployed workers is regulated by an aggregate matching function $m_i = m(v_i, u)$, where $i \in \{r, s\}$ denotes the sector (*r* = *regular*, *s* = *shadow*), v_i is the number of vacancies in the sector and u is the number of unemployed (who are the only job-seekers). By assumption, the matching function is non-negative, increasing, concave in both arguments and performs constant returns to scale, so that the job-finding rate, $g(\theta_i) = m(v_i, u)/u = m(\theta_i, 1)$, is positive, increasing and concave in the ratio of vacancies to unemployment, $\theta_i = v_i / u$. Analogously, the rate at which vacancies are filled, $f(\theta_i) = m(v_i, u)/v_i = m(1, \theta_i^{-1})$, is a positive, decreasing and convex function of market tightness, θ_i. Further, the *Inada*-type conditions hold: $\lim_{\theta_i \to 0} f(\theta_i) = \lim_{\theta_i \to \infty} g(\theta_i) = \infty$; $\lim_{\theta_i \to \infty} f(\theta_i) = \lim_{\theta_i \to 0} g(\theta_i) = 0$, with $i \in \{r, s\}$.

The Bellman equations specified to find infinite horizon steady-state solutions are:[3]

Value of …	***Hidden sector***	***Official sector***
a vacancy	$r \cdot V_s = -c_s + f(\theta_s) \cdot [J_s - V_s]$	$r \cdot V_r = -c_r + f(\theta_r) \cdot [J_r - V_r]$
a filled job	$rJ_s = x_s y - w_s + (\delta + \rho) \cdot [V_s - J_s]$	$rJ_r = (px_r y + k - s) - w_r - \tau + \delta \cdot [V_r - J_r]$
searching a job	$r \cdot U_s = z + g(\theta_s) \cdot [W_s - U_s]$	$r \cdot U_r = z + g(\theta_r) \cdot [W_r - U_r]$
being employed	$r \cdot W_s = w_s + (\delta + \rho) \cdot [U_s - W_s]$	$r \cdot W_r = w_r + \delta \cdot [U_r - W_r]$

where V_i is the value of a vacancy; J_i is the value of a filled job; U_i is the value for seeking a job;[4] W_i is the value for being employed; c_i is the start-up cost; z is the opportunity cost of employment; x_i is entrepreneurial ability; y is labour productivity; $p > 1$ is the exogenous productivity premium in the regular sector; w_i is the wage rate; τ is an exogenous production tax; ρ is the exogenous instantaneous probability of a firm being discovered (and destroyed) as unregistered; δ is the exogenous destruction rate. The symbols *k* and *s* denote the specific advantages and disadvantages for regular firms, such as the benefits of participating in a

[2] The general equilibrium character is particularly stressed, because the model considers two types of firms, thus forming two sectors, and because each firm is affected by the sectoral composition.
[3] Time is continuous, and individuals are risk neutral, live infinitely, and discount the future at the rate *r*.
[4] The unemployed cannot search for a job in both sectors at the same time (i.e. there is directed search). However, irrespective of the sector, if an unemployed person fails to find a job, s/he falls back into the same pool of unemployment.

larger information network and of receiving specific public services, and conversely, of paying bureaucratic and administrative costs, including bribes and money protection if imposed by criminal organisations.[5]

As usual, wages are assumed to be the outcome of a Nash bargaining problem:

$$w_i = \arg\max\left\{(W_i - U_i)^{\beta} \cdot (J_i - V_i)^{1-\beta}\right\} \Rightarrow (W_i - U_i) = \frac{\beta}{(1-\beta)} \cdot (J_i - V_i) \qquad \text{with } i \in \{r,s\}$$

where $\beta \in (0,1)$ is the surplus share for labour. Simple manipulations thus yield:

$$w_r = (1-\beta) \cdot rU_r(\theta_r) + \beta \cdot (px_r y + k - s - \tau - rV_r(\theta_r))$$

$$w_s = (1-\beta) \cdot rU_s(\theta_s) + \beta \cdot (x_s y - rV_s(\theta_s))$$

with $w_i{}'(\theta_i) > 0 \ \ \forall i$, since $V_i{}'(\theta_i) < 0$, and $U_i{}'(\theta_i) > 0 \ \ \forall i$.

The surplus of a job in each sector (divided between one entrepreneur and one worker by the wage) is defined as the sum of the worker's and firm's value of being on the job, net of the respective outside options, so that $S_i = J_i - V_i + W_i - U_i$, with $i \in \{r,s\}$. Using the Bellman equations, we get:

$$S_s = \frac{x_s \cdot y - z + c_s}{r + \delta + \rho + (1-\beta) \cdot f(\theta_s) + \beta \cdot g(\theta_s)}; \qquad S_r = \frac{p \cdot x_r \cdot y + k - s - \tau - z + c_r}{r + \delta + (1-\beta) \cdot f(\theta_r) + \beta \cdot g(\theta_r)}.$$

Note that both the surplus and wages are heterogeneous within the two sectors, besides being different between them. This is because of the overall heterogeneity of entrepreneurial ability. Since $(J_s - V_s) = (1-\beta) \cdot S_s$ and $(J_r - V_r) = (1-\beta) \cdot S_r$, it is straightforward to get:

$$rV_s(x) = \frac{f(\theta_s) \cdot (1-\beta) \cdot (x_s \cdot y - z) - c_s \cdot (r + \delta + \rho + \beta \cdot g(\theta_s))}{r + \delta + \rho + (1-\beta) \cdot f(\theta_s) + \beta \cdot g(\theta_s)} \qquad [1]$$

$$rV_r(x) = \frac{f(\theta_r) \cdot (1-\beta) \cdot (p \cdot x_r \cdot y + k - s - \tau - z) - c_r \cdot (r + \delta + \beta \cdot g(\theta_r))}{r + \delta + (1-\beta) \cdot f(\theta_r) + \beta \cdot g(\theta_r)} \qquad [2]$$

As in Fonseca et al. (2001), we ignore the range beyond which θ_i is large enough to turn rV_i negative. Hence, it must be that $\theta_i \in [0, \tilde{\theta}_i) \ \ \forall i$, where $\tilde{\theta}_i < \infty$ is the value such that $V_i(\tilde{\theta}_i) = 0$. Furthermore, since for $\theta_i = 0$ the vacancy would be always filled, the relevant interval for θ_i becomes $\theta_i \in (0, \tilde{\theta}_i) \ \ \forall i$, which implies $u \neq 0$, $v_i \neq 0 \ \ \forall i$.

2.1 Entrepreneurial ability and the career choice

A key feature of the model is that the comparison between the expected profitability of posting vacancies in the two sectors depends on the entrepreneurial ability of individuals (x).

[5] Both *s* and *k* are assumed as parameters in this section, but they will be treated as variables in section 3.

In this model, indeed, job productivity depends on both entrepreneurial ability and labour productivity, whereas in Fonseca et al. (2001) and Pissarides (2002) entrepreneurial ability does not affect job productivity, and in Albrecht et al. (2009) and Boeri and Garibaldi (2006) job productivity only depends on labour productivity. Each individual is assumed to be endowed with a specific entrepreneurial ability, and all individuals are heterogeneous with respect to this ability. Formally, entrepreneurial ability x is distributed over a continuum of infinitely-living individuals who expect to enter the labour market, and it can be measured in continuous manner, $x \in [0, x_{max}]$, following the known *c.d.f.* $F : [0, x_{max}] \rightarrow [0,1]$.

The minimum ability required to open a vacancy in the hidden sector can be obtained very simply from the zero-profit condition, i.e. from $V_s = 0$ in equation [1]:

$$\lim_{v_s \to 0}\left[\frac{c_s}{f(\theta_s)} = \frac{(1-\beta_s)\cdot(x\cdot y - z)}{(r+\delta+\rho+\beta_s\cdot g(\theta_s))}\right] \Rightarrow x_{min} = \frac{z}{y} > 0$$

Therefore, the zero-profit condition can be used to distinguish entrepreneurs from workers.[6] The minimum level of ability x_{min} is not only the threshold for individuals to become entrepreneurs in the hidden sector, it is also the threshold to become an entrepreneur generally, because the level of ability required to enter the regular sector is even higher, as will shortly be made clear. Since ability is not tradeable, all the individuals endowed with $x > x_{min}$ will earn extra-profit as a rent in posting vacancies. Accordingly, for an equal or lower level of ability, individuals become workers and then do not post any vacancy.

Let us now define a threshold level of entrepreneurial ability $T \in]x_{min}, x_{max}]$ such that two entrepreneurs drawn from the two sectors yield equal expected profitability, i.e.:

$$V_r(x = T) = V_s(x = T) \tag{3}$$

T can therefore be derived in a straightforward way from equations [1], [2], and [3]:

$$T = \frac{\dfrac{(\tau+z+s-k)+c_r\cdot A}{A+1} - \dfrac{z+c_s\cdot B}{B+1}}{\dfrac{py}{A+1} - \dfrac{y}{B+1}} \tag{4}$$

with $A \equiv \dfrac{r+\delta+\beta\cdot g(\theta_r)}{(1-\beta)\cdot f(\theta_r)}$ and $B \equiv \dfrac{r+\delta+\rho+\beta\cdot g(\theta_s)}{(1-\beta)\cdot f(\theta_s)}$.

In order to have a positive expression on the *r.h.s* of [4], the following restrictions are sufficient: $(\tau+s-k) > 0$, $(\tau+z+s-k) > c_s$, $c_r > z$, and p must be sufficiently great (see

[6] In a framework in which the number of firms is fixed, the zero-profit condition is no longer used to determine the labour market tightness (see Fonseca et al., 2001, and Pissarides, 2002).

Appendix A for the details). The first three restrictions are realistic,[7] the fourth restriction is necessary for the regular sector to be able to survive. An interesting result can be obtained from these restrictions, given that it has been observed that the intercept of $V_r(x)$ is more negative than the intercept of $V_s(x)$, and that the slope of $V_r(x)$ is steeper than the slope of $V_s(x)$.

========== *Figure 1 about here (now at the end)* ==========

Remark 1. *Official jobs are manage by the relatively more able entrepreneurs.*

This is one of the key results of the benchmark model, and it runs counter to the argument that the hidden sector is an incubator of infant industries: in fact, regular firms are more productive because they are run by more able entrepreneurs (see also Pugno, 2000a; Carillo and Pugno, 2004; Rauch, 1991, Levenson and Maloney, 1998).

From the macroeconomic point of view, the entrepreneurs' indifference condition [3] implies that the share of entrepreneurs who open a vacancy in the hidden sector is $F(T)-l=v_s$, while the share $1-F(T)=v_r$ opens a vacancy in the official sector. Entrepreneurs may thus post a vacancy and then fill the job, or fail to fill it, in one of the two sectors, so that it can be simply stated that $v_r = 1-(v_s + l)$.[8] Hence, equation [4] can be re-written in a more general form as follows:

$$T = T(v_s) \qquad [4']$$

since u is given to the entrepreneurs. The property that $\partial T / \partial v_s < 0$ follows from the restrictions $c_r > (\tau + z + s - k) > c_s$, which include the previous ones (see again *Appendix A*).

Equations $T = T(v_s)$ can be coupled with the equation $v_s = v_s(T)$, which depends on the distribution of ability across entrepreneurs and is monotonically rising in *T* from x_{min} up to x_{max}. Both equations can be represented in the diagram with axes [v_s, T], as in fig. 2. Equation [4'] has been built for $T \in]\, x_{\min}, x_{max}\,]$, so that its vertical start-point is higher than the intercept of $v_s = v_s(T)$.

========== *Figure 2 about here (now at the end)* ==========

Remark 2. *A unique couple of (v_S , T) exists in the model.*

[7] The value of the start-up cost in the hidden sector c_s should be very low, since ease of entry is often one of the criteria used to define the informal sector (Gërxhani, 2004). By contrast, the start-up cost c_r is often very heavy because of excessive regulations, administrative burdens, licence fees, bribery (Bouev, 2005).

[8] In this model the number of incumbent entrepreneurs is exogenous and outside the population, i.e. $v_r + v_s + u + 2(n_r + n_s) = 1 + n_r + n_s$. Matters thus become simpler without loss of generality.

This second key result rules out the possibility of a perverse equilibrium whereby the more able entrepreneurs enter the hidden sector.

2.2 The unemployment equation

Although the economy has two sectors, empirically we observe a unique rate of unemployment. Since the total share of workers in the population is *l*, the unemployment identity requires:

$$u_r + u_s \equiv u = l - n_r - n_s \qquad [5]$$

where n_r and n_s represent steady-state employment in the official and hidden sectors, respectively. Since jobs arrive to unemployed workers at the rate $g(\theta_i)$, with $i \in \{r, s\}$, and regular and irregular filled jobs are destroyed at the rate δ and $(\delta + \rho)$, respectively, then in the steady state equilibrium it must be that:

$$\delta \cdot n_r = u \cdot g(\theta_r) \qquad [6]$$

$$(\delta + \rho) \cdot n_s = u \cdot g(\theta_s) \qquad [7]$$

Steady-state unemployment is thus given by [5], [6] and [7]:

$$u = \frac{l}{\frac{g(\theta_r)}{\delta} + \frac{g(\theta_s)}{\delta + \rho} + 1} \qquad [8]$$

Equation [8] closes the model, since *u*, which has been previously given to the entrepreneurs, can now be determined, and the following result can be drawn:

Proposition. *An aggregate equilibrium with positive u exists and is unique. The qualitative results obtained in partial equilibrium, where u is given, also hold in general equilibrium, where u is endogenous (see Appendix B for proofs).*

Hence, the equilibrium of the model can be defined thus:

Definition. *The solutions for the four key variables* v_s, v_r, T *and* u *are obtained by considering: 1) the Bellman equations; 2) the entrepreneur's indifference condition between running firms in the two sectors, given their entrepreneurial ability distribution; 3) the unemployment identity and the equilibrium condition of the transition flows on the supply side of the labour market.*

2.3 Discussion

The main result is that an interior solution exists whereby both the hidden sector and the regular sector survive in equilibrium (see also Pugno, 2000a, and Carillo and Pugno, 2004). This may explain the so-called “shadow puzzle”, i.e. the persistence of the hidden sector despite advances in detection technologies and in organisation by public authorities to reduce irregularities (Boeri and Garibaldi, 2006).

A number of other important results can be drawn from comparative statics exercises. A general exercise concerns the effects of the shift of the *T*-curve [4] due to changes in some parameters. Its downward shift decreases both the (partial) equilibrium of v_s in fig. 2, and the model’s (general) equilibrium of v_s. Therefore, the downward shift of the *T*-curve [4] squeezes the proportion of the hidden sector and expands the proportion of the regular sector, as clearly emerges from the definitions of v_s and v_r, and as can be easily derived from equations [5], [6] and [7] jointly.

The downward shift of the *T*-curve [4] can thus increase overall output, because it increases the proportion of the most productive sector. The regular sector is in fact more productive than the hidden sector for two reasons: the regular sector exhibits the premium *p*, which captures its higher technological level, and the most able entrepreneurs prefer this sector.

The downward shift of the *T*-curve [4] also increases the shadow wage gap, i.e. the wage differentials between the two sectors. This effect is due to the rise of the equilibrium level of v_r, since the wages are increasing functions with respect to the vacancies level.

The main policy implications of the benchmark model can be drawn from the effects of the changes in the policy parameters on *T*, and hence on the proportion of the hidden sector, i.e.:

$\frac{\partial T}{\partial \rho} < 0$, since $\frac{\partial B}{\partial \rho} > 0$; $\frac{\partial T}{\partial \tau} > 0$; $\frac{\partial T}{\partial c_r} > 0$.

In words, closer monitoring, lower taxation and lower start-up costs reduce the hidden sector. This is in line with the conclusions of other models (see e.g. Friedman et al., 2000; Johnson et al., 2000; Sarte, 2000; Bouev, 2005).

A new important contribution of the benchmark model regards a much more controversial question, i.e. the intricate relationship between unemployment and hidden economy. Indeed, according to the matching model of Bouev’s (2002, 2005) scaling down the unofficial sector can lead to a decrease in the level of unemployment; whereas according to the matching model of Boeri and Garibaldi (2002, 2006) attempts to reduce, in the first place, shadow employment will result in higher open unemployment. This model says that if $v_s > v_r$ a reduction of the hidden sector decreases unemployment, whereas with $v_r > v_s$ a reduction of the hidden sector increases unemployment if the monitoring parameter ρ is sufficiently low or even zero, and it decrease unemployment if ρ is sufficiently great (see the *Appendix C*). This is an interesting result from the policy implications point of view. In fact, in the more usual case where $v_r > v_s$, the role of the monitoring parameter is strengthened: in fact, if ρ is sufficiently great, any policy directed to reduce the irregular sector may also reduces the unemployment rate.[9]

3 – The model with endogenous externalities

The performances of regular and irregular firms differ not only because of their technological level and other specific economic features but also because of the socio-economic contexts in which they operate. Indeed, if regular firms are diffused and pervasive in the economy with respect to irregular firms, information flows more easily, trust is more widespread, networking is more frequent, and a more efficient use of public services, including information and assistance from the public authorities and agencies, becomes possible.[10] By contrast, if the hidden sector is widespread, large negative externalities on the regular firms may be at work. The unfortunate cases of the southern Italian regions and the eastern European countries provide the clearest example of these externalities, because in those regions the hidden sector is likely linked to the illegal sector and to criminal organisations. Transaction costs become greater in this case, market networking becomes distorted, and tax morality worsens.[11]

Both positive and negative externalities can be characterised by a non-linearity which is typical of contagion-type diffusion. In the case of positive externalities, the diffusion of information and trustful entrepreneurial behaviour typically exhibits the bandwagon effect,

[9] Bosch and Esteban-Pretel (2009) focus on the role of job destruction rate. According to their matching model, policies that reduce the cost of formality (or those that increase the cost of informality) produce an increase in the share of formal employment while also reducing unemployment because the reallocation between formal and informal jobs has non-neutral effects on the unemployment rate, since informal jobs report much higher separation rates.

[10] There is a large body of evidence for the spillover effects on productivity. See Cooper and Haltiwanger (1996) for a survey on this literature. For the importance of social networks for entrepreneurship see Aldrich and Zimmer (1986), and Granovetter (1985).

[11] Cross-section analysis of developed and developing countries shows that the size of the hidden sector is significantly negatively correlated with generalised trust (D'Hernoncourt and Méon, 2008), and that generalised trust is negatively correlated with corruption. Although the connection between trust and corruption is reciprocal, the effect of trust on corruption seems greater than the reverse (Uslaner, 2002). Further, hidden activity is larger in countries where managers are more likely to pay bribes, where managers pay for mafia-type protection, where managers have less faith in the legal system (Johnson et al., 2000), and where corruption is generally more widespread (Buehn and Schneider, 2009).

which characterises the acceleration of the central phase of the diffusion process (Minniti, 2005). A similar pattern seems to be exhibited by criminal behaviour (Glaeser et al., 1996) and criminal enterprises (Pugno, 2000b), which exert negative externalities on regular firms. The *S-shaped* pattern of diffusion is based on Schelling's argument (1978: ch.3) of critical mass in imitative behaviour on the spatial dimension (see also Granovetter 1978). The non-linear diffusion also emerges if imitation simply follows costs reduction because of strategic complementarities on the spatial dimension, thus explaining geographical concentration (Krugman, 1991; Puga and Venables, 1996).

Our model is able to capture these phenomena with interesting results. Let us cease considering *s* and *k* as fixed parameters and treat them as logistic functions of v_r and v_s:

$$s = s(v_s) \qquad [9]$$

$$k = k(v_r) \qquad [10]$$

The key property of [9], which is monotonically increasing with respect to v_s, is convexity in the first phase and then concavity. Function [10] has the same properties with respect to v_r, but opposite properties with respect to v_s, so that their algebraic sum reinforces the non-linear effect in the same direction. Both functions are bounded.

If the functions $s(v_s)$ and $k(1-l-v_s)$ as in [9] and [10] are plugged into [4], then the relationship between *T* and v_s can change significantly because a "hump" can arise in the representation on the (v_s, T)–axes. The threshold value of entrepreneurial ability *T* is in fact declining when v_s remains low, but it can rise when the density of the irregular firms accelerates the negative externalities and decelerates the positive externalities, since greater entrepreneurial ability is required (after these effects of the externalities, the usual forces that reduce *T* once again prevail). This captures two distinct facts: that a widespread hidden sector discourages the establishment of regular firms, thus reducing the proportion of the regular sector; and that efficient networking requires numerous regular partner firms.

If accelerations and decelerations are significant and externalities diffuse themselves roughly, then three intersections become possible, as depicted in fig. 2 (dotted line), and as it can be checked by simulations with parameter values drawn from the literature (see *Appendix D*).[12] The two extreme equilibria, which are the relevant ones, may be labelled as "good" and "bad" because they define two different conditions where the proportion of the hidden sector

[12] Also Minniti's (2005) model of entrepreneurship and non-linear externalities, but without the hidden sector, exhibits multiple equilibria.

is small and, respectively, large; production is high and, respectively, low; the entrepreneurial ability is used efficiently and, respectively, inefficiently; shadow wage gap is high, and, respectively, low; negative externalities are limited, and, respectively, widespread; positive externalities are exploited, and, respectively, scarce.

This result is interesting because it can represent an economy characterised by a uniform structure, including the institutional structure, as captured by the same parameters of the model, but with two regional economies that differ in their histories alone. The region starting with a greater proportion of the hidden sector may converge towards the "bad" equilibrium, while the region starting with a smaller proportion of the hidden sector may converge towards the "good" equilibrium. Distortions, both costly and beneficial, develop differently, and eventually establish a dualism in both economic and social aspects. The Italian North-South divide, which is special but not unique in the world, can thus find an explanation.

4 – Final remarks

This paper has proposed a model able to account for the persistence and the localisation of the hidden sector. The persistence is captured by the interior equilibrium, where the hidden sector coexists with the regular sector. The key assumption yielding this result is a new and also natural one, i.e. the heterogeneous ability of entrepreneurs that affects job productivity. The localisation of the hidden sector due to the socio-economic context is captured by the possibility of two equilibria, given the same structure of parameters, where the hidden sector may be substantial and negligible respectively, depending on the starting conditions, i.e. on history. The key assumption yielding this result is again a new one, i.e. sufficient negative externalities from the hidden sector, and positive externalities from the regular sector, on regular firms.

The model also suggests some policy measures besides the more usual ones, although it is not designed to determine the optimal policy. Any policy action that discourages the profitability of irregular firms will improve the overall production level and productivity through the composition effect. Entrepreneurs take advantage of their abilities to "go over-ground", while tax morality is strengthened.

The extended model yields a further result, since it suggests policy actions from the sectoral perspective, rather than from the firm perspective alone, with possible powerful effects. Policy measures may be directed at changing the externalities. In the case of negative externalities, the contagion effect should be combatted, for example, by supporting those firms which pledge not to pay bribes and protection money, and by building a virtuous network of customer, creditors, etc. for them. In the case of positive externalities, infrastructure, network facilities and specific public services for regular firms should be provided. These policy measures may be especially effective in that they can trigger an endogenous change from the equilibrium where the hidden sector is substantial to the equilibrium where the hidden sector is negligible.

Appendices

Appendix A: Properties of equation [4]

The threshold *T* is a special *x*, so that it must be positive since $x > x_{min} \geq 0$. Hence, also the *r.h.s* of [4] must be positive. Sufficient conditions for the positivity of the *r.h.s* of [4] are:

$$\frac{p}{A+1} > \frac{1}{B+1} \qquad \text{[A.1]}$$

$$\frac{(\tau + z + s - k) + c_r \cdot A}{A+1} > \frac{z + c_s \cdot B}{B+1} \qquad \text{[A.2]}$$

Let us examine the limit of these conditions for v_r (and v_s) which goes to zero.

- If $v_r \to 0$, then $A \to 0$ and $B \to \{0 < \overline{B} < \infty\}$, so that:

  $p > \frac{1}{\overline{B}+1}$, which is always true since $p > 1$, and

  $(\tau + z + s - k) > \frac{z + c_s \cdot \overline{B}}{\overline{B}+1} \Rightarrow \overline{B} > \frac{z - (\tau + z + s - k)}{(\tau + z_r + s - k) - c_s}$, which requires as sufficient conditions that: $(\tau + s - k) > 0$, and $(\tau + z + s - k) > c_s$.

- If $v_s \to 0$, then $B \to 0$ and $A \to \{0 < \overline{A} < \infty\}$, so that:

  $\frac{p}{\overline{A}+1} > 1 \Rightarrow p > (\overline{A}+1)$ which requires that *p* is sufficiently greater than 1,

  $\frac{(\tau + z + s - k) + c_r \cdot \overline{A}}{\overline{A}+1} > z \Rightarrow \overline{A} > \frac{z - (\tau + z + s - k)}{c_r - z}$, with requires $c_r > z$ as a sufficient condition to hold.

The proof that $\partial T / \partial v_s < 0$ in [4] thus becomes straightforward, having recalled that $1 - l = v_s + v_r$, and that $\theta_i = v_i / u$. Since $\frac{\partial A}{\partial v_s} < 0$ and $\frac{\partial B}{\partial v_s} > 0$, the main denominator of [4] is rising in v_s, i.e. $\frac{\partial}{\partial v_s}\left(\frac{p}{A+1} - \frac{1}{B+1}\right) > 0$, while, the main numerator of [4] is decreasing in v_s:

$$\frac{\partial}{\partial A}\left(\frac{(\tau+z+s-k)+c_r\cdot A}{A+1}\right)=\frac{c_r-(\tau+z+s-k)}{(A+1)^2}>0 \qquad \text{if } c_r>(\tau+z+s-k)$$

$$\frac{\partial}{\partial B}\left(\frac{z+c_s\cdot B}{B+1}\right)=\frac{c_s-z}{(B+1)^2}>0 \qquad \text{if } c_s>z\,.$$

The complete restriction set of the parameters can thus be reduced: $c_r>(\tau+z+s-k)>c_s$. Note that these are sufficient but not necessary conditions to obtain $\partial T/\partial v_s<0$.

Appendix B: Proof of the proposition in section 2.2

In order to prove the existence and uniqueness of the solution for *u*, let us rewrite equation [8] as follows:

$$u=\frac{l}{\frac{g(v_r/u)}{\delta}+\frac{g(v_s/u)}{\delta+\rho}+1}=\Gamma(u) \qquad [8']$$

It can be observed that *u* ranges between 0 and *l* (where $l<1$), and that the *r.h.s.* of [8'] is a rising and concave function in *u* for given v_r and v_s, because $\partial g(\theta_i)/\partial u<0$ and $\partial^2 g(\theta_i)/\partial u^2>0$. Since $\lim_{u\to 0}\Gamma(u)=0$ and $\lim_{u\to l}\Gamma(u)<l$, because of the Inada conditions, a unique intersection exists between the *l.h.s.* and the *r.h.s.* of [8'].

Since v_r and v_s vary with *u*, equation [8'] can be rewritten as follows:

$$u=\frac{l}{\frac{g(v_r(u)/u)}{\delta}+\frac{g(v_s(u)/u)}{\delta+\rho}+1} \qquad [8'']$$

Note that induced changes of $v_r(u)$ and $v_s(u)$, through changes in *T*, cannot cumulate because v_r and v_s are complementary, being $1-l=v_r+v_s$. Further, since the properties of $\Gamma(u)$ hold even if either v_r or v_s goes to zero, a unique intersection also exists between the *l.h.s.* and the *r.h.s.* of [8''].

In order to prove that the qualitative results which are obtained in partial equilibrium also hold in general equilibrium, it is sufficient to prove that:

$\text{sign}\left(\frac{\partial T(v_s)}{\partial v_s}\right)=\text{sign}\left(\frac{\partial T(v_s,u(v_s))}{\partial v_s}\right)<0$, where *u* is fixed in the first term, while in the second term $u=u(v_s)$ is the explicit general form of [8'].

This inequality follows from the conditions:

- $\left(\frac{\partial T(v_s)}{\partial v_s}\right)<0$, as obtained in the *Appendix A*;
- $\left(\frac{\partial A}{\partial u}\right)<0$, $\left(\frac{\partial B}{\partial u}\right)<0$, and to $\left(\frac{\partial T}{\partial A}\right)>0$, $\left(\frac{\partial T}{\partial B}\right)<0$, as obtained in the *Appendix A* under the stated restrictions on the parameters.

Finally, when $\left(\frac{\partial(u(v_s))}{\partial v_s}\right)$<0, and $\left(\frac{\partial B}{\partial u}\right)<\left(\frac{\partial A}{\partial u}\right)$<0, or when $\left(\frac{\partial(u(v_s))}{\partial v_s}\right)$>0, and $\left(\frac{\partial A}{\partial u}\right)<\left(\frac{\partial B}{\partial u}\right)$<0, then in both cases $\left(\frac{\partial T(u(v_s))}{\partial v_s}\right)$<0 if ρ is sufficiently low. The former case takes place when $v_r > v_s$, while the latter case takes place when $v_s > v_r$. If ρ is sufficiently high, then $\left(\frac{\partial u}{\partial v_s}\right)$>0 irrespective of the proportion of vacancies, but the ensuing condition $\left(\frac{\partial B}{\partial u}\right)<\left(\frac{\partial A}{\partial u}\right)$<0, does not guarantee that $\left(\frac{\partial T(u(v_s))}{\partial v_s}\right)$<0.

Appendix C: Beveridge Curves analysis

From equation [8], it is straightforward to get the Beveridge Curve of both sectors:[13]

$$\frac{\partial u}{\partial v_r}=-\left\{\frac{l\cdot\delta\cdot(\delta+\rho)^2\cdot \overset{>0}{g'(\theta_r)}}{[(\delta+\rho)\cdot g(\theta_r)+\delta\cdot g(\theta_s)+\delta\cdot(\delta+\rho)]^2}\right\}<0$$

$$\frac{\partial u}{\partial v_s}=-\left\{\frac{l\cdot\delta^2\cdot(\delta+\rho)\cdot \overset{>0}{g'(\theta_s)}}{[(\delta+\rho)\cdot g(\theta_r)+\delta\cdot g(\theta_s)+\delta\cdot(\delta+\rho)]^2}\right\}<0$$

Assuming as in Boeri and Garibaldi's (2006) calibrations that $\theta_r>\theta_s$, i.e. $v_r>v_s$ (which is a realistic situation also in the developing and transition countries), and knowing that $g'(\theta_i)>0$, $g''(\theta_i)<0$, we obtain $g'(\theta_s)>g'(\theta_r)$. Hence, if there is no monitoring ($\rho=0$), the unemployment rate increases when the irregular vacancies decreases, because the *Beveridge Curve* of the hidden sector is steeper than the *Beveridge Curve* of the official sector, i.e. $\partial u/\partial v_s>\partial u/\partial v_r$.

However, a positive level of monitoring is a necessary condition to preserve legal jobs. Indeed, there is an efficient level of monitoring which reverses the previous result:

$$\rho>\{\delta\cdot[g'(\theta_s)/g'(\theta_r)-1]\}\equiv\sigma \qquad \text{[C.1]}$$

which is a positive value since $[g'(\theta_s)/g'(\theta_r)]>1$. If $\rho>\sigma$, then the unemployment rate increases when the irregular vacancies increases, because now it is the *Beveridge Curve* of the official sector that is steeper.

Note that in the inverse case ($\rho<\sigma$) we cannot ensure that the monitoring rate is positive, since σ may be a very small value.

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[13] Indeed, equation [8], like the standard *Beveridge Curve*, is a decreasing and convex function with respect to both v_r and v_s:

$$\frac{\partial^2 u}{\partial v_r^{\,2}}=\frac{-l\cdot\delta\cdot(\delta+\rho)^2\cdot \overset{<0}{g''(\theta_r)}\cdot H^2-\left\{-l\cdot\delta\cdot(\delta+\rho)^2\cdot g'(\theta_r)\cdot 2\cdot H\cdot(\delta+\rho)\cdot g'(\theta_r)\right\}}{H^4}>0$$

$$\frac{\partial^2 u}{\partial v_s^{\,2}}=\frac{-l\cdot\delta^2\cdot(\delta+\rho)\cdot \overset{<0}{g''(\theta_s)}\cdot H^2-\left\{-l\cdot\delta^2\cdot(\delta+\rho)\cdot g'(\theta_s)\cdot 2\cdot H\cdot\delta\cdot g'(\theta_s)\right\}}{H^4}>0$$

where $H\equiv[(\delta+\rho)\cdot g(\theta_r)+\delta\cdot g(\theta_s)+\delta\cdot(\delta+\rho)]$.

Appendix D: Multiple equilibria

The baseline specification of the model's parameters has been drawn from Boeri and Garibaldi (2006), and it is the following:[14]

parameter	***notation***	***regular sector***	***hidden sector***
workers' surplus share	β	0.50	
discount rate	r	0.03	0.03
monitoring rate	ρ	–	0.06
destruction rate	δ	0.15	0.15
unemployed income	z	0.10	0.00
production tax	τ	0.20	–
unemployment rate (unweighted sectors average)	u	0.0981	
search cost	c	0.40	0.40
matching elasticity	a	0.50	0.50
matching function constant	A	0.50	0.50

We follow the bulk of the existing literature by assuming a Cobb-Douglas matching function (Petrongolo and Pissarides, 2001; Stevens, 2007): $m_i = A \cdot v_i^{1-a} \cdot u^a$, with $i \in \{r, s\}$. Regarding function $v_s(T)$, we use a distribution for the entrepreneurial ability x that is negative exponential.[15] The simulation considers logistic functions, i.e.:

$$s = \frac{\Phi_1}{1 + e^{\Phi_2 - \Phi_3 v_s}} \qquad \text{[D.1]}$$

$$k = \frac{\Omega_1}{1 + e^{\Omega_2 - \Omega_3 v_r}} \qquad \text{[D.2]}$$

the parameter values in equations [D.1] and [D.2] are calibrated so as to ensure that $T > 0$.[16] The simulation's result is depicted in fig. 3.

========== *Figure 3 about here (now at the end)* ==========

In short, the simulation shows the special role played by the parameters which regulate the acceleration/deceleration of the externalities: in fact, the greater is Φ_3 and the lower is Ω_3, the higher is the "hump" of the extended function [4], because the negative externalities rise faster and the positive ones end up quicker.

[14] The productivity premium is calibrated so as to ensure $T > 0$ (see *Appendix A*).
[15] A negative exponential distribution is used by Boeri and Garibaldi (2006) for the distribution of productivity.
[16] Greek capital letters denote the horizontal position of the inflection point, if numbered with 2, and the slope of the function, if numbered with 3. The parameter Φ_1 captures the administrative and bureaucratic burdens and the maximum burden imposed by the criminal context, while Φ_3 denotes the acceleration effect when the critical density of the criminal activity has been approached. Similarly, Ω_1 captures the maximum possible effect of the positive externalities arising from the diffusion of regular firms, while Ω_3 denotes the acceleration effect of these externalities.

References

Albrecht, James, Lucas Navarro, and Susan Vroman, (2009), "The Effects of Labour Market Policies in an Economy with an Informal Sector," *Economic Journal*, 119(539), July, 1105-1129.

Aldrich, Howard E., and Catherine Zimmer, (1986), "Entrepreneurship Through Social Networks," (pp.3-23) in Donald Sexton and Raymond Smilor (eds.), *The Art and Science of Entrepreneurship*, New York: Ballinger.

Banfield C. Edward, (1958), *The Moral Basis of a Backward Society*. New York: Free Press.

Boeri, Tito, and Pietro Garibaldi, (2002), "Shadow Activity and Unemployment in a Depressed Labour Market," *CEPR Discussion Papers*, 3433, June.

Boeri, Tito, and Pietro Garibaldi, (2006), "Shadow Sorting," *Fondazione Collegio Carlo Alberto Working Paper Series*, 10, May.

Bosch, Mariano, and Julen Esteban-Pretel, (2009), "Cyclical Informality and Unemployment," *CIRJE Discussion Papers*, 613, February.

Bouev, Maxim, (2002), "Official Regulations and the Shadow Economy: A Labour Market Approach," *William Davidson Institute Working Paper*, 524, December.

Bouev, Maxim, (2005), "State Regulations, Job Search and Wage Bargaining: A Study in the Economics of the Informal Sector," *William Davidson Institute Working Paper*, 764, April.

Buehn, Andreas, and Friedrich Schneider, (2009), "Corruption and the Shadow Economy: A Structural Equation Model Approach," *IZA Discussion Papers*, 4182, May.

Carillo, Maria Rosaria, and Maurizio Pugno, (2004), "The underground economy and underdevelopment," *Economic Systems*, 28(3), September, 257-279.

Cooper, Russell, and John Haltiwanger, (1996), "Evidence on Macroeconomic Complementarities," *The Review of Economics and Statistics*, 78(1), February, 78-93.

Daniele, Vittorio, and Ugo Marani, (2008), "Organized Crime and Foreign Direct Investment: The Italian Case," *CESifo Working Paper*, 2416, October.

D'Hernoncourt, Johanna, and Pierre-Guillaume Méon, (2008), "The not so dark side of trust: Does trust increase the size of the shadow economy ?," *Centre Emile Bernheim (CEB) Working Paper*, 08/030.

Fonseca, Raquel, Lopez-Garcia Paloma, and Christopher A. Pissarides, (2001), "Entrepreneurship, start-up costs and employment," *European Economic Review*, 45(4-6), May, 692-705.

Friedman, Eric, Simon Johnson, Daniel Kaufmann, and Pablo Zoido-Lobaton, (2000), "Dodging the grabbing hand: the determinants of unofficial activity in 69 countries," *Journal of Public Economics*, 76(3), June, 459-493.

Fugazza, Marco, and Jean-Francois Jacques, (2004), "Labor market institutions, taxation and the underground economy," *Journal of Public Economics*, 88(1-2), January, 395-418.

Gërxhani, Klarita, (2004), "The Informal Sector in Developed and Less Developed Countries: A Literature Survey," *Public Choice*, 120(3-4), 09, 267-300.

Glaeser, L. Edward, Bruce Sacerdote, and Jose A. Scheinkman, (1996), "Crime and Social Interactions," *The Quarterly Journal of Economics*, 111(2), May, 507-548.

Granovetter, Mark, (1978), "Threshold Models of Collective Behavior," *American Journal of Sociology*, 83(6), May, 1420-1443.

Granovetter, Mark, (1985), "Economic Action and Social Structure: the Problem of Embeddedness," *American Journal of Sociology*, 91(3), November, 480–510.

Istituto Nazionale di Statistica, ISTAT, (2005), "La misura dell'economia sommersa secondo le statistiche ufficiali," *Statistiche in breve*, Settembre.

Istituto Nazionale di Statistica, ISTAT, (2008), “La misura dell’occupazione non regolare nelle stime di contabilità nazionale,” *Statistiche in breve*, Febbraio.

Johnson, Simon, Daniel Kaufmann, and Pablo Zoido-Lobaton, (1999), “Corruption, public finances, and the unofficial economy,” *Policy Research Working Paper Series*, 2169, August.

Johnson, Simon, Daniel Kaufmann, John McMillan, and Christopher Woodruff, (2000), “Why do firms hide ? Bribes and unofficial activity after communism,” *Journal of Public Economics*, 76(3), June, 495-520.

Kolm, Ann-Sofie, and Birthe Larsen, (2003), “Wages, Unemployment, and the Underground Economy,” *CESifo Working Paper*,1086, November.

Krugman, Paul, (1991), *Geography and Trade*, Cambridge, MA: The MIT Press.

Levenson, R. Alec, and William F. Maloney, (1998), “The informal sector, firm dynamics, and institutional participation,” *Policy Research Working Paper Series*, 1988, September.

Minniti, Maria, (2005), “Entrepreneurship and Network Externalities,” *Journal of Economic Behaviour and Organization*, 57(1), May, 1–27;

Mortensen, T. Dale, and Christopher A. Pissarides, (1994), “Job Creation and Job Destruction in the Theory of Unemployment,” *Review of Economic Studies*, 61(3), July, 397-415.

Ormerod, Paul, (2005), *Crime: economic incentives and social networks*, London: The Institute of Economic Affairs.

Peri, Giovanni, (2004), “Socio-Cultural Variables and Economic Success: Evidence from Italian Provinces 1951-1991,” *Berkeley Electronic Journal, Topics in Macroeconomics*, 4(1), article 12, September.

Petrongolo, Barbara, and Christopher A. Pissarides, (2001), “Looking into the Black Box: A Survey of the Matching Function,” *Journal of Economic Literature*, 39(2), June, 390-431.

Pissarides, Christopher A., (2000), *Equilibrium Unemployment Theory*, The MIT Press.

Pissarides, Christopher A., (2002), “Company Start-Up Costs and Employment,” *CEP Discussion Papers*, dp0520, March.

Puga, Diego, and Anthony J. Venables, (1996), “The Spread of Industry: Spatial Agglomeration in Economic Development,” *Journal of the Japanese and International Economies*, 10(4), December, 440–464.

Pugno, Maurizio, (2000a), “Economia sommersa e disoccupazione: un modello per l'analisi e per le politiche di intervento,” *Rivista Italiana degli Economisti*, 5(2), August, 269-290.

Pugno, Maurizio, (2000b), “Rent Seeking e questione meridionale,” *Rivista di Politica Economica*, 3, 387-419.

Rauch, James E., (1991), “Modelling the informal sector formally,” *Journal of Development Economics*, 35(1), January, 33-47.

Sarte, Pierre-Daniel G., (2000), “Informality and Rent-seeking Bureaucracies in a Model of Long-run Growth,” *Journal of Monetary Economics*, 46(1), August, 173-197.

Schelling, Thomas, (1978), *Micromotives and Macrobehavior*, New York: Norton.

Stevens, Margaret, (2007). “New Microfoundations For The Aggregate Matching Function,” International Economic Review, 48(3), 08, 847-868.

Uren, Lawrence, (2007), “Entrepreneurship and Labour Market Fluctuations,” *Economics Bulletin*, 10(13), 1–11.

Uslaner, Eric M., (2002), *The Moral Foundations of Trust*, New York: Cambridge University Press.

Van Dijk, Jan, (2006), “Organized Crime and Collective Victimization,” *Centre for the Study of Democracy*, Sofia, Bulgaria (International Conference on *Corruption and Organized Crime: Bridging Criminal and Economic Policies*, June 23-24, 2006).

Figures

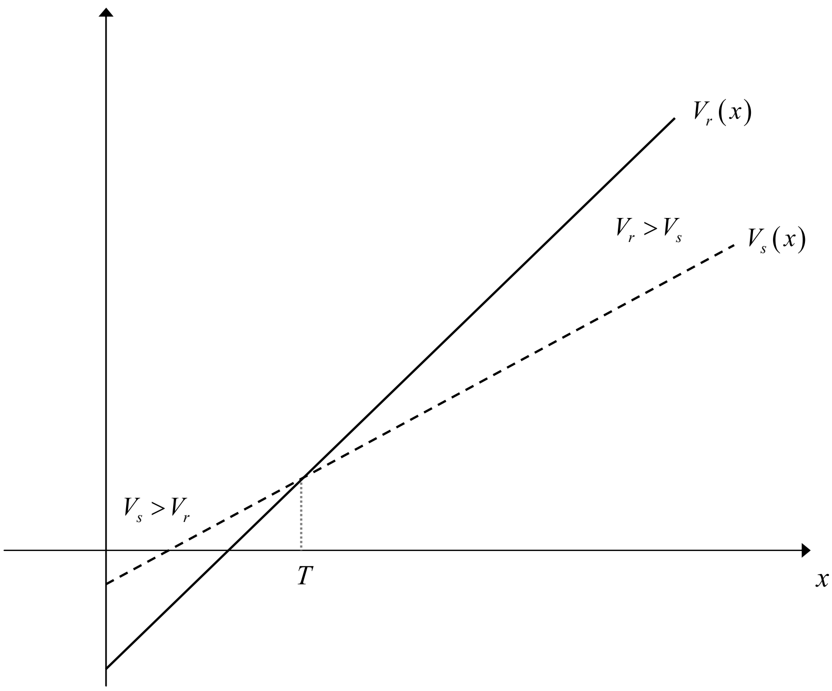

Figure 1. Entrepreneurs' indifference condition

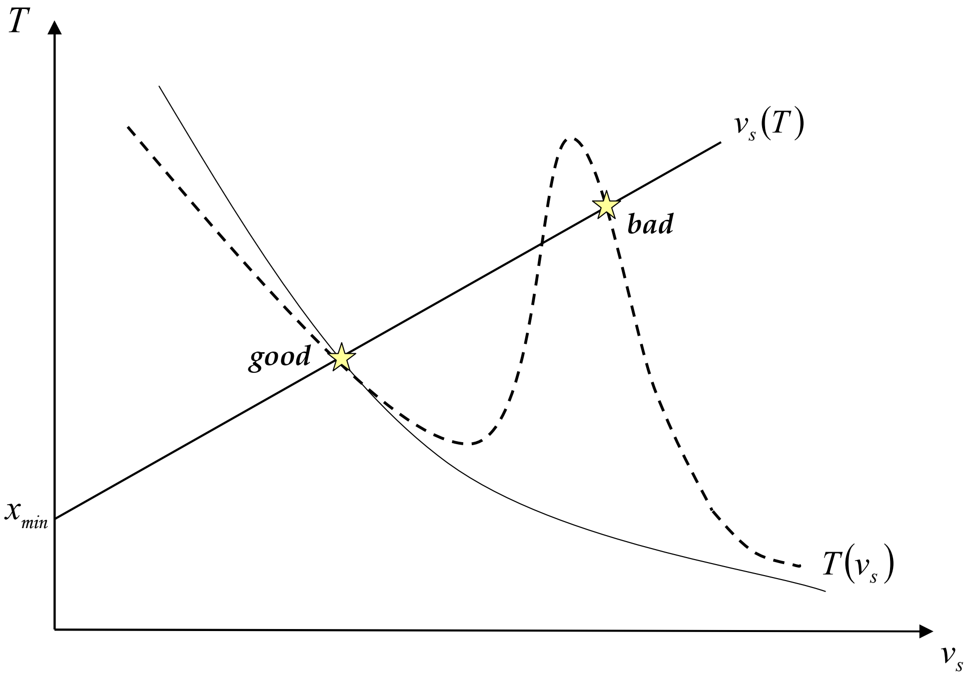

Figure 2. Interior equilibrium and multiple equilibria

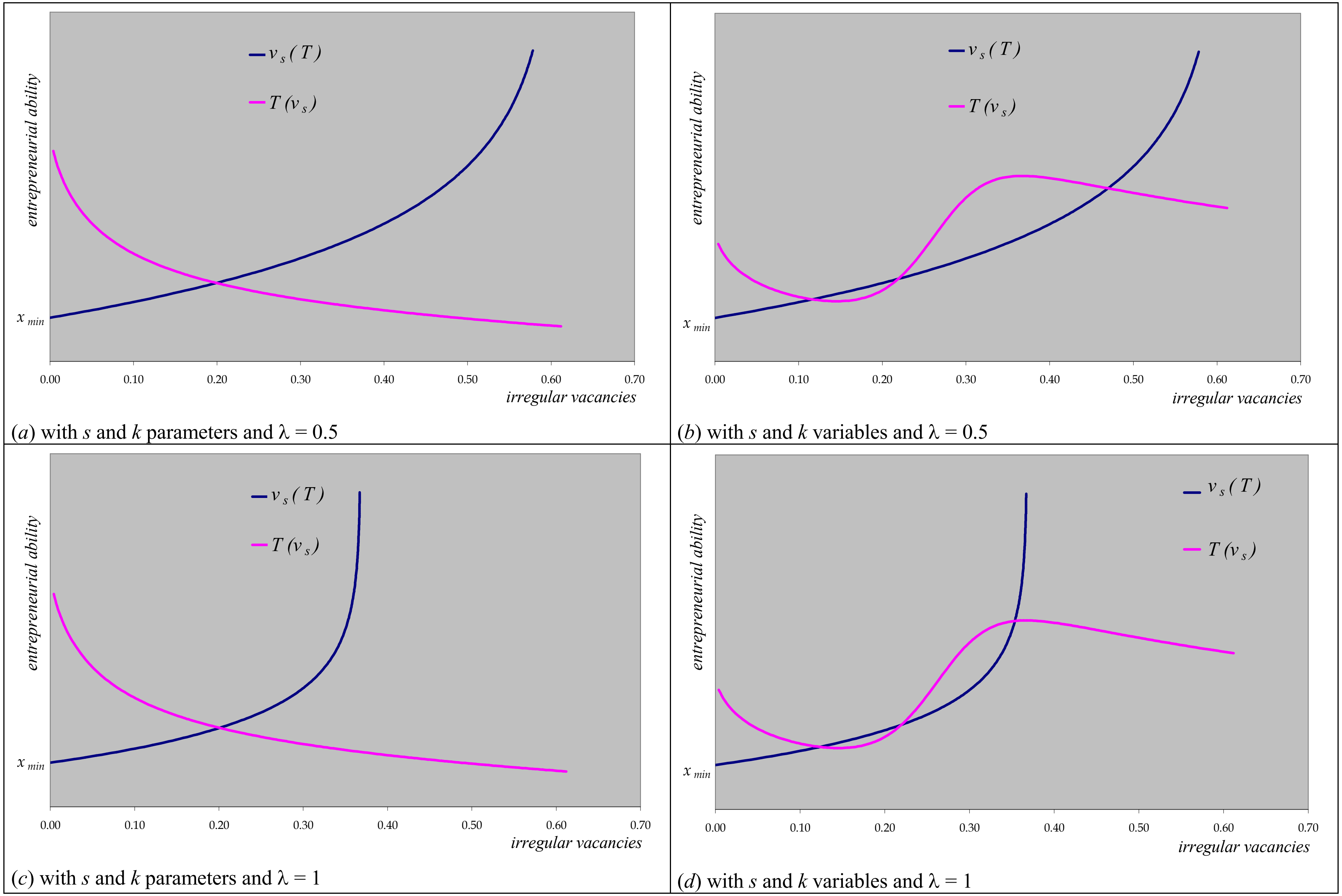

Figure 3. Simulation with Exponential distribution and Logistic functions
Source: Authors' calculations