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Abstract

This paper considers the problem of optimal unemployment insurance in a moral hazard framework. Unlike existing literature, unemployed workers can secretly participate in a *hidden labor market*; as a consequence, an endogenous lower bound for promised utility preventing "immiserization" arises. Moreover, the presence of a hidden labor market makes possible an extra deviation and therefore hardens the provision of incentives. Under linear cost of effort, we show that the optimal contract prescribes no participation in the hidden labor market and a decreasing sequence of unemployment payments until the lower bound for promised utility is reached. At that moment, participation jumps and unemployment payments *drop down to zero.* For the case of non-linear effort cost we calibrate the model to Spain. As in the linear cost of effort, this exercise reproduces no participation and decreasing payments during the initial phase of unemployment. After around three years of unemployment, the contract prescribes a *jump* in participation and an *abrupt decline* in unemployment payments. To the best of our knowledge, this is the first paper justifying an abrupt drop in unemployment payments. In addition, the quantitative analysis suggests that in an environment in which agents differ in separation rate, the hidden labor market reinforces the benefits from a type-dependent unemployment system.

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1 Introduction

Unemployed workers face a trade-off between search effort and leisure. Since effort is unobservable, the design of unemployment insurance systems balances insurance and incentives. This moral hazard problem has been the object of study since the pioneering work of Shavell and Weiss (1979). In this literature it is assumed that employment status is observable and hence individuals cannot cheat by working and asking for unemployment payments. But what if there is a shadow economy that allows unemployed individuals to secretly work and, simultaneously, ask for unemployment payments? This paper explores the implications of a hidden labor market in the design of optimal unemployment insurance. Hopenhayn and Nicolini (2001) stress the importance of this contribution:

> Our focus in discussing optimal unemployment insurance has been the group of workers whose unemployment risk is higher, namely, the less educated and less experienced workers. Often, the members of this group find job opportunities mostly in the informal sector of the economy. Therefore, it may be very hard to monitor the employment status of the worker. Our investigation assumed that while search effort is not observable, employment status is. A strong incentive problem arises when the latter does not hold, since an unemployed worker receiving benefits would not have incentives to disclose having found a job.

Table 1 reports three facts suggesting that incorporating a hidden labor market in the study of optimal unemployment insurance might be crucial[1]:

- The hidden labor market or the shadow economy is important in many countries, ranging from 13-30 percent of GDP in industrialized economies to 39-76 for African countries.

- In economies with a sizeable shadow economy, unemployment is a likely event. On average, unemployment rates range from 6.8 in industrialized economies to 11.6 for African countries.

[1]We interpret the informal sector as a hidden labor market in which the worker can always find a job but his productivity is lower than in the formal sector.

- With different coverage, replacement rate, and benefits duration, most of the countries protect the workers against unemployment risk.[2] Replacement rates are around 45-75 percent during 1-2 years.

In light of these facts, it is reasonable to think about economies where the hidden labor market is important, workers face high unemployment risk, and the government provides unemployment insurance. In the current paper we study the optimal unemployment insurance in economies with these features.

Incorporating a hidden labor market modifies the standard unemployment insurance problem. Methodologically, the presence of a hidden labor market imposes an endogenous lower bound for promised utilities that has to be incorporated as an explicit restriction in the planner's problem. In addition, participation in the hidden labor market hardens the encouragement of job search effort. In fact, unless the cost of effort is linear, an extra incentive compatibility constraint must be added to keep participation at the desired level.

Important insights can be obtained in the case of linear cost of effort, when the unemployment insurance system can be characterized analytically. Initially, consumption is strictly decreasing during unemployment and workers do not participate in the hidden labor market. For sufficiently large unemployment spells, the lower bound for promised utilities is eventually reached. Only at that moment, the optimal contract prescribes positive participation in the hidden labor market together with zero unemployment payments. Also, once the lower bound for promised utilities binds, planner's instruments—taxes and benefits—and prescriptions—search effort and hidden labor market participation—remain constant over time.

[2] Also, the number of countries providing unemployment insurance programs increased from 21 in 1940 to 68 in 1997. Social Security Programs Throughout the World, 1997 (Social Security Administration, US).

Table 1: Shadow economy size, unemployment rate, and unemployment insurance

	Shadow economy % GDP	Unemployment rate %	Unemployment insurance		
			Coverage	Replacement rates %	Benefit duration
Industrialized economies	Vary between 13 and 30.	6.8	Most of the countries provide and cover all the employed.	Vary between 40 and 75. Higher in Sweden and Denmark.	Most of the countries limit the length between 8 and 36 weeks.
Transition economies	Vary between 9 and 43.	9.2	Most of the countries provide and cover all the employed.	Vary between 50 and 75. Some countries flat benefits.	Most of the countries limit is 26 weeks. Exception Hungary, 2 years.
Latin America and the Caribbean	Vary between 25 and 60.	8.0	Most of the countries provide and cover all the employed.	Vary between 50 and 60. Some countries flat benefits.	Maximum entitlement 1 year.
Asia	Vary between 13 and 70.	4.8	Differs significantly. Bangladesh only commerce and industry. South Korea: all firms.	Vary between 50 and 55.	Bangladesh 30-120 days. China 1-2 years. South Korea 90-240 days.
Africa	Vary between 39 and 76.	11.6	Egypt excludes public sector workers. South Africa excludes highly paid workers.	Vary between 45 and 75.	Vary between 26 weeks and 3 years.

Sources: Schneider and Enste (2000), ILO (2004) and Vodopivec and Raju (2002).

For the case of non-linear effort cost, we calibrate the model to Spain. The quantitative results resemble those obtained in the linear case. For the benchmark calibration, participation is zero during the first three years and unemployment payments decrease gradually. Just after that moment, participation in the hidden labor market jumps to a fairly constant level, and unemployment payments drop down to zero.

To the best of our knowledge, this is the first paper justifying an *abrupt decline* in unemployment payments. The intuition goes as follow. Initially, participation remains at zero to facilitate the encouragement of search effort. More specifically, zero participation helps to encourage search effort because: (i) the marginal cost of search effort remains low, and (ii) decreasing payments translates to decreasing consumption. Eventually, when the consumption delivered by payments is too low, zero participation is not longer incentive compatible. At that moment, unemployment payments decrease abruptly. This variation in payments does not imply such variation of consumption because workers participate in hidden labor market to smooth consumption. Nevertheless, this drop provides dynamic incentives guaranteeing that long run unemployment remains a serious threat.

In addition, the results indicate that a hidden labor market reinforces the importance of a type-dependent unemployment insurance system for economies in which labor mobility differs among workers. In particular, payments duration should be shorter for workers with higher separation rates. The intuition behind this result goes as follow. When separation rates are lower, jobs are more "valuable" and therefore the planner is willing to delay participation in the hidden labor market. Naturally, this requires more unemployment payments to workers with lower separation rates.

Unemployment insurance systems have been studied from two different perspectives. One approach uses quantitative general equilibrium models to study the influence of unemployment insurance systems on macroeconomics variable and welfare—see for example Hansen and Imrohoroglu (1992) and Wang and Williamson (1996). The other approach, followed by this paper, uses contract theory in a partial equilibrium environment to study the optimal design of unemployment insurance. In a moral hazard framework in which the planner can only choose unemployment payments, early contributions find that payments must decrease over time in order to encourage search effort—Shavell and Weiss (1979). A subsequent work allows the planner to choose, together with unemployment payments, an after re-employment tax, and numerically solves for the optimal contract using a recursive representation of the problem. The paper finds that extending the planner's sets of instruments does not modify the previous result of a decreasing path for unemployment payments—Hopenhayn and Nicolini (1997).

A controversial result from this early literature is that the optimal contract leads unemployed workers toward "immiserization". More precisely, "if the workers utility function is unbounded below then efficiency requires that workers expected discounted utility falls, with positive probability, below any arbitrary negative level"—Pavoni (2003). To prevent this result, Pavoni (2003) imposes an arbitrary lower bound for promised utility and characterizes the resulting optimal unemployment insurance.[3] Our work is related to this idea because it justifies the existence of a lower bound in life-time utility and connects it with the structure of the economy through the presence of a hidden labor market. However, our problem goes beyond a justification for the lower bound for promised utility imposed in Pavoni (2003); in particular, the presence of a hidden labor market makes possible an extra deviation and therefore hardens the provision of incentives.

[3]There are several related environments where "immiserization" does not hold. For instance, see Wang (1995) and Phelan (1995).

The policy implications of a hidden labor market are studied in different environments and with different goals. Among others, labor regulations, wage controls, migration policies, and taxation are analyzed in frameworks with hidden labor markets by Banerjee (1983), Fortin, Marceau, and Savard (1997), and Johnson, Kaufmann, and Zoido-Lobaton (1998). Surprisingly, there is not paper in this strand of literature considering the optimal design of unemployment insurance. Our work fills this gap studying the implications of a hidden labor market for the optimal unemployment insurance.

Two recent papers are related to our work because of their interest in the informal sector or bad jobs and unemployment insurance. Hopenhayn and Nicolini (2001) study the effects of heterogeneity on the design of optimal unemployment insurance. Importantly, they comment that one way to cope the incentive problem generated by an informal sector is to design programs that require those receiving benefits to appear at the insurance office. As they mention, this monitoring schemes may come at a significant cost. We designed an incentive compatible contract that requires no monitoring. Likewise, Hopenhayn and Nicolini (2005) focus on workers' incentives to accept bad jobs when layoffs and quits are undistinguishable. In particular, their interest is about workers accepting and then quitting those jobs just to upgrade their unemployment insurance benefits. Instead, we study the possibility that those jobs are non-observable, what implies that workers could accept them and ask for unemployment payments.

2 Model

2.1 Environment

The economy is populated by a continuum of infinitely-lived risk averse *workers*. At each period, a worker's labor status is denoted by m_t. The variable $m_t \in \{u, e\}$, where u stands for unemployment and e for employment. The employment history of a worker up to time t is represented by $h^t = (m_0, ..., m_t)$.

In contrast to previous studies, this environment includes a secondary labor market in which workers obtain a wage, ϖ, for unit of participation, s. Henceforth, we refer to this market as the *hidden labor market*.

Workers employed in the formal sector—employed workers—exert a constant work effort $\bar{a}$, have productivity of ω, and receive wages w. Employment relationships exogenously end with probability δ. By assumption, we rule out participation in the hidden labor market while employed as well as on-the-job search. Productivity in the hidden labor market is lower than in the formal sector, $\varpi < \omega$.

Unemployed workers search for a job and can, simultaneously, participate in the hidden labor market. When exerting search effort a, they find a job next period with probability $p(a)$. The total effort while unemployed, x, is restricted to lie in the compact set $[0, \vartheta]$. The function p is strictly increasing, strictly concave, twice differentiable and satisfies standard Inada conditions.

The instantaneous felicity function is separable in consumption and total effort

$$u(c) - v(x). \tag{1}$$

The function u reflects the utility from consumption. It is strictly increasing and bounded above by $\overline{u}$. It is also strictly concave, twice differentiable, and satisfies Inada conditions. Similarly, the function v measures the cost of effort exerted either when searching for a job, when participating in the hidden labor market or when employed. We assume this function is convex, strictly increasing and twice differentiable. Notice that total effort, x, equals $\bar{a}$ if the worker is employed and $s + a$ otherwise. Finally, workers can neither save nor borrow and they discount future with the factor $\beta \in (0, 1)$.

We suppose there is a risk neutral *planner* who provides unemployment insurance and affects workers' earnings by levying a history dependent after-reemployment tax. The planner cannot observe either hidden labor market participation, s, or job search effort, a. He can borrow and save at a constant interest rate $r = 1/\beta - 1$.

2.2 Optimal Contract Problem

The planner has to decide unemployment payments, after reemployment wages, search effort, and hidden labor market participation. A contract, precisely described in the next definition, collects those decision for each possible history node.

Definition 1 *A* ***contract*** *or* ***unemployment insurance system****, $\mathcal{W}$, is a collection of functions specifying unemployment payments, b, wages after reemployment, w, search effort, a, and hidden labor market participation, s, for each possible history, h^t, at each period; i.e., $\mathcal{W} = \{b_t(h^t), w_t(h^t), a_t(h^t), s_t(h^t)\}$.*

Notice that a contract can be divided into two components: those representing planner's direct instruments, $\mathcal{I} = \{b_t(h^t), w_t(h^t)\}$, and those representing planner's prescriptions for non-observable workers' actions, $\mathcal{P} = \{a_t(h^t), s_t(h^t)\}$.

Feasibility imposes some restrictions on the set of contracts. In particular, planner's prescriptions, $\mathcal{P}$, and instruments, $\mathcal{I}$, are required to belong to the following sets:

$$\mathbf{A} = \{\mathcal{P} : \text{ for each } h^t \text{ and each } t,\ (a_t(h^t), s_t(h^t)) \in [0,\vartheta]^2, a_t(h^t) + s_t(h^t) \leq \vartheta$$
$$\text{if } m_t = u, \text{ and } a_t(h^t) = s_t(h^t) = 0 \text{ if } m_t = e\},$$

$$\mathbf{T} = \{\mathcal{I} : b_t(h^t) \geq 0 \text{ for each } h^t, \text{ with equality if } m_t = e, \text{ and } w_t(h^t) > 0\ \}.$$

Thus, a contract $\mathcal{W} = (\mathcal{P}, \mathcal{I})$ is said to be *a feasible contract* if $(\mathcal{P}, \mathcal{I}) \in \mathbf{A} \times \mathbf{T}$. Notice that each contract prescribes sequences for consumption, c, total effort, x, and one period planner's payoffs, y, as follows:

$$c_t(h^t) = \begin{cases} b_t(h^t) + s_t(h^t)\varpi & \text{if } m_t = u \\ w_t(h^t) & \text{if } m_t = e \end{cases}$$
$$x_t(h^t) = \begin{cases} a_t(h^t) + s_t(h^t) & \text{if } m_t = u \\ \bar{a} & \text{if } m_t = e \end{cases} \tag{2}$$
$$y_t(h^t) = \begin{cases} -b_t(h^t) & \text{if } m_t = u \\ \omega - w_t(h^t) & \text{if } m_t = e. \end{cases}$$

Define $\mathcal{U}_t(\mathcal{W}; h^t)$ as the workers' *promised utility* associated with the contract $\mathcal{W}$, from a particular history h^t. Likewise, define $\mathcal{V}_t(\mathcal{W}; h^t)$ as the planner's *continuation value.* That is,

$$\mathcal{U}_t(\mathcal{W}; h^t) \equiv E_t \left[\sum_{n=t}^{\infty} \beta^{n-t} (u(c_n(h^n)) - v(x_n(h^n))) | \mathcal{W}, h^t \right], \tag{3}$$
$$\mathcal{V}_t(\mathcal{W}; h^t) \equiv E_t \left[\sum_{n=t}^{\infty} \beta^{n-t} y_n(h^n) | \mathcal{W}, h^t \right]. \tag{4}$$

In (3)-(4) expectations are taken conditional on the contract $\mathcal{W}$ and on the initial node h^t. This is because the actions prescribed by the contract affect the probability measure on histories.

Since search effort and hidden labor market participation are not observable, the unemployment insurance scheme has to be *incentive compatible.* That is, the levels of search effort and hidden labor market participation prescribed by the optimal contract have to be consistent with workers' best responses.

Definition 2 *A contract $\mathcal{W} = (\mathcal{P}, \mathcal{I})$ is* ***incentive compatible*** *if for each history and each period, there is not an alternative feasible action $\widetilde{\mathcal{P}}$ providing a higher lifetime utility for the worker than the one induced by the contract; i.e., for each h^t and t,*

$$\mathcal{U}_t((\mathcal{P}, \mathcal{I}); h^t) \geq \mathcal{U}_t((\widetilde{\mathcal{P}}, \mathcal{I}); h^t) \text{ for each } \widetilde{\mathcal{P}} \in \mathbf{A}. \tag{5}$$

In the standard dynamic moral hazard model—without hidden labor markets, Pavoni (2003) shows that the constrained efficient contract "implies a weaker form of what is known as the immiserization result: if the workers utility function is unbounded below then efficiency requires that workers expected discounted utility falls, with positive probability, below any arbitrary negative level".[4] In the environment presented in this paper, we show that the contract cannot punish the worker with a promised utility below the critical value $\underline{U}$, where

$$\underline{U} \equiv \frac{u(s^* \varpi) - v(s^*)}{1 - \beta}, \tag{6}$$

and s^* solves

$$u'(s^* \varpi)\varpi = v'(s^*). \tag{7}$$

We call $\underline{U}$ the *lower bound for promised utilities.* Notice it is the life-time utility the worker attains when she does not search for a job at any period, receives no unemployment payments, and chooses participation in the hidden labor market optimally. The proposition 1 formally states this result. Before presenting the formal proof, we remark that optimality requires

$$m_{t+1} = m_t = e \Rightarrow \mathcal{U}_t(\mathcal{W}; h^t) = \mathcal{U}_{t+1}(\mathcal{W}; h^{t+1}). \tag{8}$$

As we argue next, this is because the cheapest way to provide certain life-time utility in absence of incentive friction is to guarantee the worker a constant stream of consumption.

Proposition 1 (No "immiserization") *In the optimal contract there is not "immiserization"; i.e., the promised utilities provided by the constrained efficient contract $\mathcal{W}^*$ are bounded below by $\underline{U}$.*

Proof: See Appendix. ■

[4]See proposition 3 in Pavoni (2003). Green (1987) and Thomas and Worral (1990) in repeated unobservable endowments economies, prove a similar but stronger result.

The intuition for this result is transparent: the hidden labor market provides a protection for the workers that prevents "immiserization". Whenever the planner pretends to deliver a promised utility below $\underline{U}$, the worker deviates to guarantee herself at least $\underline{U}$.

A direct implication of the last result is that problem (11)-(13) is not well defined whenever $U < \underline{U}$ because the set of incentive compatible contracts is empty. In contrast, we prove that for each $U \geq \underline{U}$ there is always an incentive compatible contract providing U to the workers. This result is formalized in lemma 1.

Lemma 1 *Define* $\mathbf{C}(U; h^0)$ *as the set of incentive compatible contracts providing promised utility greater or equal than U given initial state* h^0*; i.e.,*

$$\mathbf{C}(U; h^0) \equiv \{\mathcal{W} \in \mathbf{C} : \mathcal{U}_0((\mathcal{P}, \mathcal{I}); h^0) \geq U\}. \tag{9}$$

Then, for any $U \geq \underline{U}$*, the set* $\mathbf{C}(U)$ *is not empty.*

Proof: See Appendix. ■

Recall that u is bounded above $\overline{u}$. As a consequence, the lifetime utilities the contract can deliver is bounded above by $\overline{U} \equiv \frac{\overline{u}}{1-\beta}$. Define $\mathbf{P}_m$ as the *set of promised utilities* lower than $\overline{U}$ such that $\mathbf{C}(U; h^0) \neq \emptyset$ when $h^0 = m$. From proposition 1 and lemma 1 it transpires

$$\mathbf{P}_u = \mathbf{P}_e = \mathbf{P} \equiv \{U \in \mathbb{R} : \overline{U} \geq U \geq \underline{U}\}. \tag{10}$$

The set $\mathbf{P}$ is the set of promised utility the contract can deliver. In order to define the contractual problem, let $\mathbf{F}$ be the set of all *feasible contracts* and U be the utility the contract has to deliver to the worker. When the planner offers a contract to a worker whose initial labor status equals m_0, it solves

$$V^{m_0}(U) = \max_{\mathcal{W}\in\mathbf{F}} \mathcal{V}_0(\mathcal{W}; h^0) \tag{11}$$

$$s.t$$

$$\mathcal{W} \in \mathbf{C}(U, m_0), \tag{12}$$

$$\mathcal{U}_0(\mathcal{W}; h^0) \geq U. \tag{13}$$

$$\mathcal{U}_t(\mathcal{W}; h^t) \in \mathbf{P} \forall h^t, t \tag{14}$$

Equations (12) and (13) are, respectively, the incentive compatibility and the promise keeping constraints. The contract solving problem (11)-(13) is called the *constrained efficient contract* and is denoted by $\mathcal{W}^*$.

2.3 Recursive Representation

Due to its history dependence, the problem (11)-(13) is not tractable for quantitative purposes. Fortunately, following the lines of Spear and Srivastava (1987) and Abreu, Pearce, and Stacchetti (1990) the problem has a recursive representation in which the history is completely encoded in two single variables, namely, promised utility and labor status. As implied by proposition 1 and lemma 1, the planner's problem is well defined if and only if promised utility is not smaller than $\underline{U}$. Therefore, the recursive representation must explicitly impose that promised utilities belong to $\mathbf{P}$.

Before presenting the recursive planner's problem we introduce the definition of a *recursive contract.*

Definition 3 *A* ***recursive contract*** *or* ***unemployment insurance system****,* $\mathcal{W}^R$*, is a collection of functions specifying unemployment payments,* b*, wages after reemployment,* w*,* *search effort, a, hidden labor market participation, s, and promised utilities in any state m, U^m, for each U in $\mathbf{P}$; i.e., $\mathcal{W}^R = \{b(U), w(U), a(U), s(U), U^u(U), U^e(U)\}$.*

A recursive contract can be divided into two components: those representing planner's direct instruments, $\mathcal{I}^R = \{b(U), w(U), U^u(U), U^e(U)\}$, and those representing planner's prescriptions for workers' actions, $\mathcal{P}^R = \{a(U), s(U)\}$.

In order to define the planner's problem recursively, let $\mathbf{E}$ be the set of one-period feasible actions and $\mathbf{M}$ the set of one-period best response of workers, taking as given unemployment payments and promised utilities. That is,

$$\mathbf{E} \equiv \{(a,s) : a \geq 0, s \geq 0 \text{ and } s + a \leq \vartheta\},$$

$$\mathbf{M} \equiv \{\text{argmax}_{(a,s)\in\mathbf{E}}\ u(b + s\varpi) - v(a+s) + \beta[p(a)U^e + (1-p(a))U^u]\}.$$

Then, the planner's problem is represented by the following functional equations system:

$$V^u(U) = \max_{b,a,s,U^u,U^e} -b + \beta[p(a)V^e(U^e) + (1-p(a))V^u(U^u)], \tag{15}$$

$s.t.$

$$u(b + s\varpi) - v(s+a) + \beta[p(a)U^e + (1-p(a))U^u] - U = 0, \tag{16}$$

$$(a,s) \in \mathbf{M}, \tag{17}$$

$$(U^u, U^e) \in \mathbf{P}^2, \tag{18}$$

With

$$
\begin{aligned}
V^e(U) = \max_{\tilde{U}^u, \tilde{U}^e, w} \quad & \omega - w + \beta[\delta V^u(\tilde{U}^u) + (1-\delta)V^e(\tilde{U}^e)] \\
s.t. & \\
& u(w) - v(\overline{a}) + \beta[(1-\delta)\tilde{U}^e + \delta \tilde{U}^u] - U = 0, \\
& (\tilde{U}^u, \tilde{U}^e) \in \mathbf{P}^2,
\end{aligned}
$$

Equation (16) is the promise keeping constraint in recursive form, while equation (17) is the incentive compatibility constraint. Finally, (18) restricts the value of promised utilities to the set $\mathbf{P}$. We call this constraint the *feasibility of promised utilities constraint* (FPUC).

Pavoni (2003) also introduces a lower bound for promised utility in his recursive representation of the optimal unemployment insurance problem. However, the framework we propose here differs from that studied by Pavoni (2003) in some crucial aspects. First, our lower bound is explicitly connected with the structure of the economy and it is not arbitrarily imposed but arises due to incentive compatibility. More importantly, the participation in the hidden labor market affects the incentives provision due to its effect on the marginal cost of effort and the marginal utility of consumption. In other words, the presence of a hidden labor market involves not only a lower bound for promised utilities, but also an extra constraint when providing incentives.

The inconvenience with the previous representation is that constraint (17) is a complicated object which makes the problem non-tractable. A common strategy is the so called *first order condition approach*. To describe this strategy, consider an unemployed worker who takes as given (b, U^u, U^e) and chooses $(a, s) \in \mathbf{E}$ to maximize utility.[5] For such a problem

[5]Under the assumption that p satisfies Inada conditions and provided that in equilibrium $U^e > U^u$, we

the first order condition are:

$$\beta p'(a)[U^e - U^u] - v'(s+a) = 0, \tag{ICC1}$$

$$-u'(b + s\varpi)\varpi + v'(s+a) \geq 0. \tag{ICC2}$$

Now define $\mathbf{M}^{foc}$ as the set of all $(a, s) \in \mathbf{E}$ such that equations (ICC1)-(ICC2) are satisfied. The first order condition approach basically replaces constraint (17) in the recursive problem (15)-(18) by the condition that $(a, s) \in \mathbf{M}^{foc}$.

Since $\mathbf{M} \subseteq \mathbf{M}^{foc}$, the solution to the problem using the first order condition approach may differ from the solution to the original problem. A recent example is provided in Kocherlakota (2004). In our environment, given (b, U^u, U^e), the worker's problem is convex. This implies that the first order conditions are both necessary and sufficient and hence, the first order approach is valid.[6]

Using the first order condition approach, the recursive planner's problem is

$$V^u(U) = \max_{b,a,s,U^u,U^e} -b + \beta[p(a)V^e(U^e) + (1 - p(a))V^u(U^u)], \tag{PP}$$

$s.t.$

$$u(b + s\varpi) - v(s+a) + \beta[p(a)U^e + (1 - p(a))U^u] - U = 0, \tag{PKC}$$

$$\beta p'(a)[U^e - U^u] - v'(s+a) = 0, \tag{ICC1}$$

$$-u'(b + s\varpi)\varpi + v'(s+a) \geq 0, \tag{ICC2}$$

$$(U^e, U^u) \in \mathbf{P}^2, \tag{FPUC}$$

$$s \geq 0,$$

can guarantee that the non-negativity constraint for a is never binding. Likewise, we can always set ϑ big enough to avoid $a + s = \vartheta$. Hence, we only focus on non-negativity constraint for participation in the hidden labor market.

[6]First notice that E is a convex set. Second, the function f, defined as $f(a, s) = u(b + s\varpi) - v(a) + \beta(p(a)U^e + (1 - p(a))U^u)$, is a strictly concave function provided that $U^e \geq U^u$.

With

$$
\begin{aligned}
V^e(U) = \max_{\tilde{U}^u,\tilde{U}^e,w} \quad & \omega - w + \beta[\delta V^u(\tilde{U}^u) + (1-\delta)V^e(\tilde{U}^e)] \\
s.t. \quad & \\
& u(w) - v(\overline{a}) + \beta[(1-\delta)\tilde{U}^e + \delta\tilde{U}^u] - U = 0, \\
& (\tilde{U}^u, \tilde{U}^e) \in \mathbf{P}^2,
\end{aligned}
$$

Henceforth, we refer to the previous problem as (PP). We use this problem to analytically characterize the first best contract—the optimal contract without information frictions—and the constrained efficient contract with linear cost of effort. We also use it to numerically characterize the constrained efficient contract with non-linear cost of effort. [7]

2.4 First best contract

As a benchmark, we characterize the optimal contract when search effort and participation in the hidden labor market are both observable. The problem is the same as in (PP) but now there is no need to introduce constraints (ICC1), (ICC2), and (FPUC).

Let λ be the multiplier associated with constraint (PKC). Then the first order conditions

[7] As commented before, since there is not moral hazard during employment, optimality requires that promised utility during employment remains constant; that is $\tilde{U}^e = U$. To see this, notice that the first order condition with respect to $\tilde{U}^e$ together with envelope condition yield

$$V'^e(\tilde{U}^e) = V'^e(U),$$

which implies $\tilde{U}^e = U$ provided concavity of V^e. The functions V^u, V^e turn out to be strictly concave in all the numerical experiments we performed.

satisfied by first best contract are

$$(s:) \quad -u'(b+s\varpi)\varpi + v'(s+a) \geq 0, \tag{19}$$

$$(b:) \quad \frac{1}{u'(c)} = \lambda, \tag{20}$$

$$(U^u:) \quad \lambda + V'^u(U^u) = 0, \tag{21}$$

$$(U^e:) \quad \lambda + V'^e(U^e) = 0, \tag{22}$$

$$(a:) \quad \beta p'(a)[V^e(U^e) - V^u(U^u)] = \lambda[v'(a+s) - \beta p'(a)(U^e - U^u)]. \tag{23}$$

Envelope condition implies

$$V'^u(U) = -\lambda \Longrightarrow V'^u(U^u) = -\lambda'. \tag{24}$$

Combining (20),(21) and (24),

$$\frac{1}{u'(c)} - \frac{1}{u'(c')} = 0 \Longrightarrow c = c'. \tag{25}$$

That is, the first best allocation implies a constant stream of consumption during unemployment.[8]

Notice that under hidden effort, the first best allocation is not incentive compatible when planner values employment state and unemployment state differently. This is clear since (23) is inconsistent with (ICC1). However, if the planner has control over search effort, observability regarding participation in the hidden labor market becomes immaterial. This property is formally stated in lemma 2.

Lemma 2 *If search effort is observable, the first best allocation is achievable even if the planner cannot observe participation in the hidden labor market.*

[8]Using a similar argument, it can be shown that there is also constant consumption across labor status.

Proof: See Appendix. ■

The reason for the last result is that once search effort is guaranteed at the social optimum level, planner and worker's positions regarding the optimal level of participation in the hidden labor market are aligned. However, it does not imply we can discard (ICC2) when solving the problem with asymmetric information. When search effort is not observable, the cost and the benefit of an additional unit of participation in the hidden labor market are valued differently by the planner and by the worker. In particular, the planner internalizes the marginal effect of a unit of hidden labor market participation in the cost of providing incentives to make compatible the desired search effort. In contrast, with linear cost of effort, there is no marginal change in the cost of effort and hence this situation does not arise. This result is presented in lemma (3).

Lemma 3 *If v is linear, we can solve the problem (PP) without considering (ICC2).*

Proof: See Appendix. ■

2.5 Constrained efficient contract

Now we turn to the case in which the planner does not observe either search effort or hidden labor market participation. As mentioned, we characterize this contract through the problem labeled as (PP).

Let μ_1 be the multiplier associated with (ICC1), μ_2 the multiplier associated with (ICC2) and ϕ the one associated with non-negativity constraint for s. As before, let λ be the multiplier associated with (PKC). Finally, let ξ^u and ξ^e be the multipliers associated with constraints (FPUC) for U^u and U^e, respectively.

The first order conditions for the planner's problem are

$$(s:)\ \ \lambda[u'(b+s\varpi)\varpi - v'(s+a)] - \mu_1 v''(s+a) = \tag{26}$$

$$\mu_2[u''(b+s\varpi)\varpi^2 - v''(s+a)] + \phi,$$

$$(b:)\ \ \lambda = \frac{1}{u'(c)} + \mu_2\frac{u''(c)\varpi}{u'(c)} = \frac{1}{u'(c)} - \mu_2\rho_a(c)\varpi, \tag{27}$$

$$(U^u:)\ \ \lambda + V'^u(U^u) + \frac{\xi^u}{\beta(1-p(a))} = \mu_1\frac{p'(a)}{1-p(a)}, \tag{28}$$

$$(U^e:)\ \ \lambda + V'^e(U^e) + \frac{\xi^e}{\beta p(a)} = -\mu_1\frac{p'(a)}{p(a)}, \tag{29}$$

$$(a:)\ \ p'(a)[V^e(U^e) - V^u(U^u)] +$$

$$\mu_1[\beta p''(a)(U^e - U^u) - v''(s+a)] + \mu_2 v''(s+a) = 0, \tag{30}$$

where ρ_a represents the coefficient of absolute risk aversion. These conditions together with complementary slackness and the corresponding constraints completely characterize the contract solving (PP).

Using envelope condition and equations (27)-(28) we can get the expression that governs the profile of consumption during unemployment spell,

$$\frac{1}{u'(c)} - \frac{1}{u'(c')} + \underbrace{\varpi[\mu_2'\rho_a(c') - \mu_2\rho_a(c)]}_{\varrho} + \frac{\xi^u}{\beta(1-p(a))} = \mu_1\frac{p'(a)}{1-p(a)}. \tag{31}$$

There, we can see that the presence of a hidden labor market affects the profile of consumption during unemployment through three ways: (1) a direct effect represented by the term ϱ which does not show up in the standard case, (2) an indirect effect through the term μ_1 and, (3) other direct effect through the term ξ^u.[9]

In order to provide a further characterization of the problem, we must either add assumptions about functional forms, or rely on numerical solution. In the remaining of the

[9] Notice that the term ϱ in (31) is a function of the absolute coefficient of risk aversion. A similar result is found in an environment with moral hazard and hidden savings. See Abraham and Pavoni (2005).

paper we develop both strategies.

3 Qualitative analysis

In this section the cost of effort is defined by

$$v(x) = \alpha x. \tag{32}$$

With this additional assumption, we can provide a detailed characterization of the constrained efficient contract.[10]

When the cost of effort is defined as in (32), the dynamic of consumption is characterized by a decreasing path, until a period when it becomes flat. This result is formalized through a set of lemmas. The first of them shows that consumption is *strictly* decreasing while the promised utility is above its lower bound $\underline{U}$.

Lemma 4 *When the constraint for promised utility if unemployed (FPUC) does not bind— i.e., $U^u > \underline{U}$—the consumption path during unemployment is strictly decreasing.*

Proof: First observe that $\mu_2 = 0$.[11] To see this, notice that if $s = 0$, constraint (ICC2) does not bind and, by complementary slackness, $\mu_2 = 0$. On the other hand, if $s > 0$, equation (26) implies $\mu_2 u''(b + s\varpi)\varpi = 0$ and then $\mu_2 = 0$. Now, manipulating first order conditions, and letting $\xi^u = 0$, we find

$$\frac{1}{u'(c_t)} - \frac{1}{u'(c_{t+1})} = \mu_1 \frac{p'(a)}{1 - p(a)} > 0. \tag{33}$$

[10]Some results also hold under a more general specification of v. Consider $v(a, s) = g(a) + q(s)$ with $g'(a) > 0$ and $q'(s) > 0$. Then, the sequence of consumption during unemployed is still decreasing and bounded below. Likewise, the lower bound for promised utility is eventually reached. Moreover, when lower bound for promised utility is reached, the contracts prescribes zero payments and the lower bound for consumption. However, now it is not the case that participation in the hidden labor market occurs only when the lower bound for promised utility is reached.

[11]This is consistent with lemma 3 which indicates that with linear cost of effort incentive compatibility constraint (ICC2) is not needed to formulate the constrained planner's problem.

Finally, by strict concavity of u we have $c_t > c_{t+1}$. ■

Due to the presence of a hidden labor market, the consumption path is bounded below. This result is formally established in the next lemma.

Lemma 5 *The sequence of consumption during unemployment is bounded below by*

$$\underline{c} \equiv u'^{-1}(\alpha/\varpi).$$

This consumption level is achieved whenever unemployed workers participate in the hidden labor market.

Proof: The lower bound may be violated only if $b < \underline{c}$. In that case, $s = 0$ is not incentive compatible. Since participation in the hidden labor market must be optimal from the agent's point of view, it will solve $u'(b + s\varpi)\varpi = \alpha$. This implies that whenever the unemployed worker participates in the hidden labor market, $c = b + s\varpi = \underline{c}$. ■

The following lemma deals with the question of whether the lower bound for utility $\underline{U}$ is eventually reached for a worker with a sufficiently large unemployment spell.

Lemma 6 *For sufficiently large unemployment spell, the lower bound for utility is reached; i.e., there exist a period t^* such $U^u_{t^*} = \underline{U}$.*

Proof: Suppose by contradiction that constraint (FPUC) never binds. Then, we can write the problem (PP) without it. Moreover, by corollary (1), $s_t = 0$ for each t. As a consequence, the problem (PP) collapses into the problem studied by Hopenhayn and Nicolini (1997) with no hidden labor market participation and no relevant bound for promised utilities. As known, in this environment $c_t = b_t$ decreases toward "immiserization". This implies that lower bound for consumption is eventually violated, which contradicts lemma 5. ∎

The next lemma shows that once the lower bound for utility is reached, promised utility if unemployed remains constant at its minimum level.

Lemma 7 *If V^u is concave, the lower bound for utility is a fixed point of the function U^u; i.e., $U^u(\underline{U}) = \underline{U}$.*

Proof: Let $U = \underline{U}$, using first order condition with respect to U^u and envelope condition we get

$$V'^u(U^u) - V'^u(\underline{U}) = \mu_1 \frac{p'(a)}{(1-p(a))} - \frac{\xi^u}{\beta(1-p(a))}.$$

Clearly $U^u < \underline{U}$ is not possible because it violates (FPUC). Suppose then that $U^u > \underline{U}$. This implies (FPUC) does not bind and hence $\xi^u = 0$. However if $\xi^u = 0$, the concavity of V^u implies $U^u \leq \underline{U}$, which contradicts our assumption that $U^u > \underline{U}$. Therefore, it must be that $U^u = \underline{U}$ ∎

Putting these results together we can affirm that initially consumption decreases over the unemployment spell until the time when the lower bound for utility is reached. From that period on, consumption remains constant. Denote the level at which consumption stabilizes by $\bar{c}$. We show in the next lemma that consumption stabilizes at its lower bound.

Lemma 8 *If V^u is concave, at the promised utility's lower bound, the optimal contract prescribes the consumption's lower bound; i.e., $c(\underline{U}) \equiv \bar{c} = \underline{c}$.*

Proof: By lemmas 6 and 7, there exist a t^* such that $U_t^u = \underline{U}$ and $c_t = \bar{c}$ for each $t > t^*$. By lemma 5, $\bar{c} \geq \underline{c}$ which rules out $\bar{c} < \underline{c}$. We prove $\bar{c} > \underline{c}$ is not a solution. Suppose by contradiction that $\overline{c} > \underline{c}$. Since $u'(\overline{c})\varpi < \alpha$, at this point $s = 0$. Let a be the search effort prescribed in the optimal contract. We can write

$$u(\overline{c}) - v(a) + \beta[p(a)U^e + (1-p(a))\underline{U}] \geq$$
$$u(\overline{c}) + \beta\underline{U} > u(\underline{c}) - v(s^*) + \beta\underline{U} \equiv \underline{U}.$$

The first inequality arises form the fact that incentive compatibility requires that the utility delivered by the contract is not lower than the one associated with any feasible deviation, in particular with $s = a = 0$. The second *strict* inequality arises form our assumption that $\overline{c} > \underline{c}$ and from the fact that v and u are strictly increasing. However, this implies that in the contract the planner is delivering a lifetime utility strictly higher than the one promised, which is not optimal. Hence $\overline{c} = \underline{c}$. ■

Corollary 1 *When lower bound for promised utility is not binding, hidden labor market participation is zero.*

Proof: Suppose by contradiction that at certain period $s_t \neq 0$ but lower bound for promised utility has not been reached. By lemma (5), $c_t = \underline{c}$, but since $U^u > \underline{U}$, by lemma 4, $c_{t+1} < c_t = \underline{c}$ which is a contradiction. ■

Previous corollary highlights a key property of the optimal contract in the case of linear cost of effort, namely: *the planner initially does not encourage participation in the hidden labor market.* The explanation for this somehow unexpected results is that, once the planner induces participation, it completely looses ability to punish the worker through reduction in consumption. Moreover, any further reduction in unemployment payments motivates more participation in the hidden labor market and hence, increases the marginal cost of effort, which make harder to encourage search effort. As we will see in the numerical exercise, this property is still present, to some extent, in the case of nonlinear cost of effort .

With a hidden labor market consumption does not necessarily equal unemployment payments. Hence, the characterization of consumption is not enough to identify the optimal path for unemployment payments. Therefore, we characterize unemployment payments under linear cost of effort. The main result is presented in the following proposition.

Proposition 2 *If V^u is concave, $b_{t+1} < b_t$ If constraint (FPUC) does not binds. If constraint (FPUC) binds, $b_t = 0$. In other words, unemployment insurance payments decrease until the lower bound for promised utility is reached. After that moment they become zero.*

Proof: When the lower bound for U does not bind, consumption is strictly decreasing and above $\underline{c}$. Since in this phase $s_t = 0$, $b = c$ and therefore, unemployment payments also decrease. When $U = \underline{U}$ and b is constant at $\underline{b}$ because $U^u(\underline{U}) = \underline{U}$. Suppose, by contradiction, $\underline{b} > 0$. By incentive compatibility the contract must deliver an utility consistent with worker's best response, that is worker's utility is

$$\tilde{U} \equiv \max_{a,s} u(b + \varpi s) - v(a + s) + \beta[p(a)U^e + (1 - p(a))\underline{U}].$$

Moreover, this utility must be as good as any deviation, in particular one in which search intensity is set to zero and s is set optimally. That is

$$\tilde{U} \geq \max_{s} u(b + \varpi s) - v(s) + \beta\underline{U} \equiv \hat{U}.$$

Since u is strictly increasing,

$$\hat{U} > \max_{s} u(\varpi s) - v(s) + \beta\underline{U} \equiv \underline{U}.$$

This implies that there exist a feasible reduction in b which increase planner profit. Therefore $b > 0$ is not optimum. ■

Corollary 2 *When lower bound for promised utility is binding, hidden labor market participation equals $s^* > 0$.*

Proof: The proof is trivial. When $U = \underline{U}$, b=0. Then, by incentive compatibility for s, $s = s^*$. ■

In summary, there a two different phases during unemployment. During the first phase, payments (consumption) decrease smoothly without reaction of hidden labor market participation. It is because the payments are relatively high and hence the marginal cost of participation is higher than its marginal benefit. However, as payments decline, the marginal benefit of participation increases, and participation is eventually optimal. At that point the second phase of unemployment starts. In this phase, with positive participation, linearity and incentive compatibility uniquely determined the consumption level ($\underline{c}$). At that level of consumption, the promised utility can be satisfied with zero payments (any positive payments deliver a higher life-time utility). Thus, during this phase, zero payments become optimal.

A feature of the optimal unemployment insurance is certain lack of smoothness. In particular, at a certain period, payments drop to zero and hidden labor market participation jumps away from zero. This lack of smoothness should be striking just if it is reflected on consumption. This is not the case here since in this phase participation compensates payments drops, and consumption is perfectly smooth.

4 Quantitative analysis

In this section we carry out a quantitative analysis for the case of non-linear cost of effort. First, we present the calibration strategy. Then, we describe the dynamics of unemployment benefits, job search effort and informal sector participation under the optimal contract. Finally, we derive welfare implications associated with the implementation of the optimal contract. The algorithm is presented in the appendix.

4.1 Calibration

Functional forms are standard. Preferences over consumption goods are described by a constant relative risk aversion function with parameter σ, while the cost of effort function is $v(a) = a^\gamma/\gamma$. Finally, the job finding probability is $p(a) = 1 - \exp(-\rho a)$.

A key preliminary question regarding the calibration is, Which economy should be considered as benchmark for our calibration? Quantitative research usually uses the U.S. economy. However, our environment naturally applies to developing countries, where unemployment rate and informal sector participation are high. Working with developing countries involves certain problems. First, it is hard to find reliable data for the informal sector. More importantly, there is not accurate information about unified and explicitly designed unemployment insurance systems. We calibrate our model using data for Spain due to the importance of its informal sector and the availability of information. In particular, we use data for the end of the 90's, when unemployment and the size of the informal sector were relatively high. We aware that in the last 25 years unemployment rate, informal sector participation, and labor markets regulations have changed significantly in Spain. Therefore, the conclusions of this exercise may not apply to the current Spanish economy where the labor market performance has substantially improved. However, these results may still be valid for developing countries, whose current labor market structure is similar to that used in the calibration.

Most of the parameters are taken from previous literature. We interpret each period as a month and set the discount factor $\beta = 0.994$ (annual interest rate of 7.5%). We normalized workers' productivity, ω, and effort in the formal labor market, $\bar{a}$ to one. Data about average tenure in Spain prescribes $\delta = 0.015$.[12]. The relative risk aversion coefficient, σ, is set equal to 2, which is a common value in the real business cycle literature.[13]

The calibration strategy uses the "actual" unemployment insurance scheme instead of the optimal contract. That is, for plausible combinations of ρ and ϖ, we compute the stationary distribution of an economy in which agents take optimal decisions considering the "actual" unemployment insurance scheme. From this distribution, we calculate unemployment and informal sector participation. These outcomes are then compared with those in the data. The calibration consists in finding values of ρ and ϖ compatible with our targets. This strategy is natural since unemployment and informal sector participation are outcomes of the optimal reaction of individuals to the actual system.

The calibration matches unemployment rate and informal sector size in Spain for 1997. Unemployment rate, μ, was 18.5 percent while production in the informal sector as a proportion of the GDP, ζ, was 23 percent—Schneider and Enste (2000).

In Spain, unemployed workers qualify for benefits if they have contributed to social security over a minimum period in the previous years. Depending on their employment duration, unemployed workers receive payments from 4 months to a maximum of 2 years. After that period, they qualify for unemployment subsidy. A typical worker receives 70 percent replacement ratio during the first 6 months, and 60 percent for the next 18 months.

[12] According to Arranz and Garca-Serrano (2004) the average tenure in the private sector is around 6 years.

[13] Here we depart from the traditional values in the literature of optimal unemployment insurance. Pavoni (2003) works with log preferences ($\sigma = 1$) while Hopenhayn and Nicolini (1997) set $\sigma = 0.5$.

Unemployment subsidy is 80 percent of the minimum wage. This implies a replacement ratio of 30 percent after two years of unemployment.[14] Taxes financing the system are set at 5 percent.

Workers decide effort and hidden labor market participation to maximize their life-time utility, taking as given the "actual" unemployment insurance scheme. This scheme is represented by $(\tau, \{b_n\}_{n=1}^{\infty})$, where subindex n refers to unemployment duration. Since b_n is constant for $n \geq n^*$, the agent's problem has the following recursive representation

$$\hat{V}_n^u = \max_{a,s} u(b_n + s\varpi) - v(a+s) + \beta[p(a)\hat{V}^e + (1-p(a))\hat{V}_{n+1}^u], \forall n \in \{0, 1, ..., n^* - 1\}$$

$$\hat{V}_{n^*}^u = \max_{a,s} u(b_n + s\varpi) - v(a+s) + \beta[p(a)\hat{V}^e + (1-p(a))\hat{V}_{n^*}^u], \forall n \geq n^*$$

$$\hat{V}^e = u(\omega(1-\tau)) - v(\bar{a}) + \beta[(1-\delta)\hat{V}^e + \delta\hat{V}_0^u],$$

where $\hat{V}_n^u$ is life-time utility of an unemployed agent with unemployment spell n and $\hat{V}^e$ corresponds to life-time utility of an employed agent.

The backward solution to the previous problem provides policy functions a_n and s_n that can be used to compute the stationary distribution of agents over unemployment spell, $\bar{\Omega}$. The stationary distribution is a fixed point of the transition law

$$\Omega_0' = \left(1 - \sum\nolimits_n \Omega_n\right)\delta$$

$$\Omega_n' = (\Omega_{n-1}(1 - p(a_{n-1}))\, \forall n < n^*,$$

$$\Omega_{n^*}' = [\Omega_{n^*-1}(1 - p(a_{n^*-1})) + \Omega_{n^*}(1 - p(a_{n^*}))].$$

In the stationary distribution, we measure the unemployment rate and the informal sector size by

$$\mu = \sum\nolimits_n \bar{\Omega}_n(1 - s_n),$$

[14] This number is obtained dividing 80 percent of the minimum wage by the income per capita.

$$\zeta = \frac{\varpi\chi}{\varpi\chi + (1-\sum_n \bar{\Omega}_n)\omega},$$

where $\chi = \sum_n \bar{\Omega}_n s_n$ is the average participation in the hidden labor market. Table 2 summarizes our calibration. With these parameters the artificial economy reproduces $\mu = 0.1787$ and $\zeta = 0.2326$, very close to our targets, 0.185 and 0.23, respectively.

Table 2: Calibration strategy

Parameter	Value	Description	Basis and Targets
β	0.994	Discount factor	Interest rate
σ	2	Relative risk aversion coefficient	Standard
γ	2	Search cost elasticity	Standard
ω	1	Productivity in formal sector	Normalization
$\bar{a}$	1	Effort when employed	Normalization
δ	0.015	Separation rate	Average tenure
ρ	0.045	Parameter in probability of finding a job	Unemployment rate
ϖ	0.415	Wage in informal sector	Informal sector size

4.2 Optimal policy

The dynamic of the hidden labor market participation is crucial in determining the dynamic of optimal payments. Figure 1 shows the path for search effort and hidden labor market participation while Figure 2 shows optimal unemployment payments and wages. In both cases initial promised utility is set such that the planner breaks even.[15]

Figure 1 indicates that the optimal participation is initially zero and search effort increases with unemployment spell, as it does in the economy without hidden labor market. After approximately 3 years of unemployment, when effort peaks, participation jumps and search effort drops. After that point, participation increases and search effort decreases

[15] Planner's surplus in our contract does not consider any administrative cost. According to data from the "Ministerio de Trabajo y Asuntos Sociales", available at www.mtas.es, slightly above 10 percent of the expenditures on social assistance went to administrative costs during 1999. To account for these costs, we subtract 0.07 for each period from the surplus functions before we set them equal to 0.

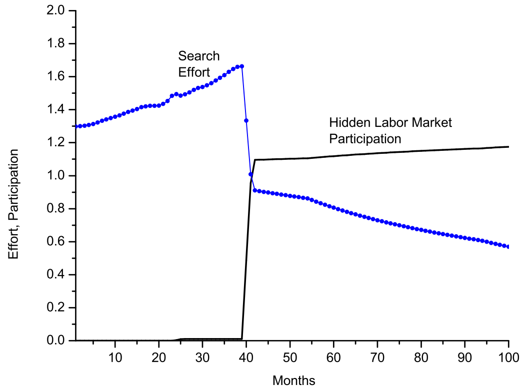

Figure 1: Search effort and hidden labor market participation.

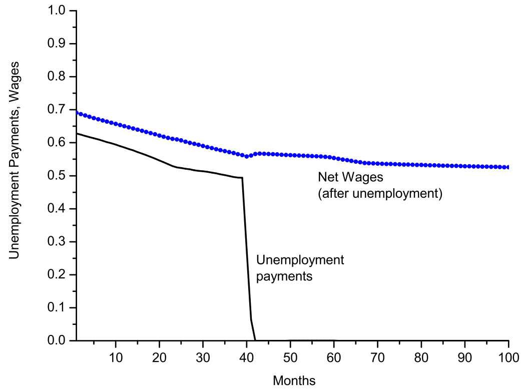

Figure 2: Unemployment payments and net wages dynamics.

gradually toward fairly steady levels. Similarly, Figure 2 shows that unemployment payments and wages decrease smoothly while the agent is not participating in the hidden labor market. When participation jumps, payments drop to zero but the dynamic of wages is not significantly affected.

Some aspects are remarkable from this behavior. First, required effort is not monotonically increasing in the unemployment spell. The reason is that in order to induce an increase in participation, it is optimal to reduce the search effort such that the marginal cost of inducing participation falls. Second, in the early phase of unemployment, workers are fundamentally searchers with no participation in the hidden labor market. It is worth noting that our contract does not imply hidden labor market participation during the first 3 years of unemployment. Thus, it is very likely that unemployed agents find a job before they start to participate in the hidden labor market. Finally, hidden labor market participation increases. This is because at this point consumption has decreased implying a higher marginal benefit of participation. It is also remarkable that once participation is induced, unemployment payments drop sharply down to zero. That is, with a hidden labor market, it is optimal to give no payment for workers with a sufficiently large unemployment spell. The reason is simple, if workers obtain payments and participate in the hidden labor market, large unemployment spells are not strong enough threats to encourage search effort.

Finally, notice the qualitative similarities with the linear cost of effort case. For instance, participation is initially zero and then increases sharply up to a fairly constant level. Likewise, unemployment payments reduce gradually during the initial phase, but then, when workers participate in the hidden labor market, they drop fast to zero.[16]

4.3 Budget Savings

In this section we quantify the budget savings of switching from the stylized Spanish scheme presented in the previous section to the optimal program considering participation in the hidden labor market.

As described above, during the calibration we calculate the stationary distribution over unemployment spell, $\bar{\Omega}$. With this distribution at hand, we can compute the present value of providing unemployment insurance forever using the Spanish system (C_S),

$$C_S = \frac{1}{r}\left[\sum\nolimits_n \bar{\Omega}_n b_n - \left(1 - \sum\nolimits_n \bar{\Omega}_n\right)\omega(1-\tau)\right].$$

Similarly, using the solution to the recursive problem of an unemployed agent in Spain, $(\hat{V}_n^u, \hat{V}^e)$, we transform the distribution over n to a distribution over U. Then, we calculate the cost of providing the same distribution of life-time utility with the optimal contract (C_N),

$$C_N = -\left[\sum\nolimits_n \bar{\Omega}_n V^u(\hat{V}_n^u) + \left(1 - \sum\nolimits_n \bar{\Omega}_n\right) V^e(\hat{V}^e)\right)\right].$$

[16]A similar reasoning to the one used in the linear case can be adopted here. First, participation also increases consumption linearly in this framework. Second, even though $v'()$ is no longer constant, it is different from zero for any positive effort a. These aspects explain why no participation in the initial phase of unemployment is possible. In addition, consumption may be smooth even with drops in the payments as long as there exist a corresponding drop in search intensity that stimulate higher participation.

Important inter-temporal budget savings are associated with the implementation of the optimal contract. Given the stationary distribution of life-time utility, the cost under Spanish scheme is 42.18. On the other hand, providing the same life-time utility distribution with the optimal contract costs 21.65. These savings come from two sources. First, the standard savings arising from the Hopenhayn and Nicolini (1997) type of contract; that is, from a contract considering deviation form search effort toward leisure. Hopenhayn and Nicolini (1996) calibrate this optimal contract for the Spanish economy and find savings ranging between 20%-37%. We believe the additional savings are strictly related with the implementation of a contract considering deviation form search effort toward hidden labor market participation.

5 A type-dependent unemployment insurance?

Separation rates may differ across workers. Therefore, it may be important to design a δ-dependent unemployment insurance. Wang and Williamson (2002) perform an exercise in these lines. In the case of fully experience rating, in the sense that taxes in each "industry" fully fund unemployment benefits for workers in that industry, they find that the replacement ratio is lower for agents in industries where the nature of production implies frequent transitions between employment and unemployment. However, they find it is optimal to extend benefits indefinitely for each type.

In this section we answer a quantitative question: Do hidden labor market*s* reinforce the importance of a δ-dependent unemployment insurance? To that end, we compute the optimal contract for three values of δ (0.01, 0.015 and 0.02) for an economy with an important hidden labor market ($\varpi = 0.415$)—as calibrated for Spain—and for an economy with a very small one ($\varpi = 0.05$). For the remaining parameters we use the values in the benchmark calibration in both economies.

This experiment fits in the environment of Wang and Williamson (2002) in which there are workers employed in different industries, with different separation rates, and a worker who is re-employed after an unemployment spell always returns to employment in the same industry. Following the case of fully experience rating in Wang and Williamson (2002), we start the experiment at the level of payments for which promised utility results in a balanced budget for each δ considered.

Figure 3 shows that without an important informal sector the optimal payments' dynamics is not significatively affected by the separation rate. In fact, as in Wang and Williamson (2002), it just changes the level at which the payments start without affecting payments durations. In contrast, when we consider an economy with an important hidden labor market—see Figure 4, the dynamics of payments varies with the separation rate. In particular, the duration of payments increases from 29 to 55 months when the separation rate increases from 0.01 to 0.02. This result clearly prescribes an easy-to-implement contract for the optimal δ-dependent unemployment insurance in an economy with a hidden labor market. That is, the duration of payments should be shorter for workers with higher separation rates. Thus, *considering the existence of a hidden labor market is also relevant because it reinforces the importance of an δ-dependent unemployment insurance system.*

The intuition behind this result goes as follow. The planner considers employment of workers with lower separation rates more "valuable". Therefore, it is willing to delay participation in the hidden labor marketextending the initial phase of unemployment promoting high search intensity for longer periods.

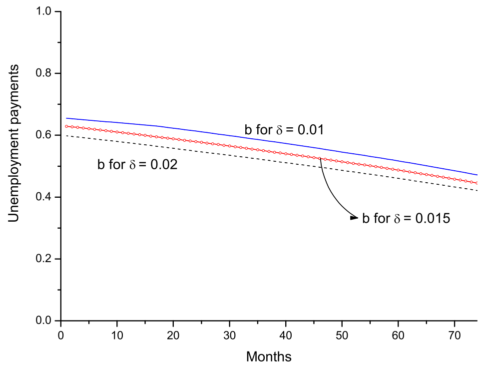

Figure 3: The dynamics of optimal payments, $\varpi = 0.05$.

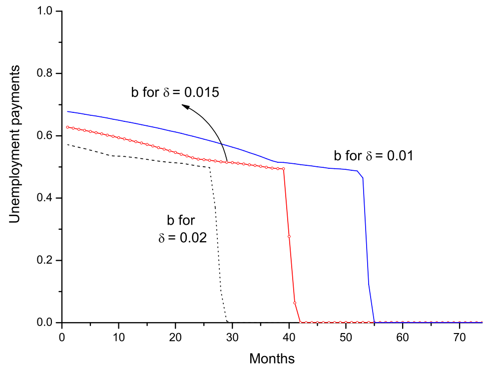

Figure 4: The dynamics of optimal payments, $\varpi = 0.415$.

6 Conclusions

The problem of the optimal unemployment insurance has been addressed from the perspective of a principal-agent framework since the pioneering work of Shavell and Weiss (1979). Early literature mainly focuses on the moral hazard problem arising under non-observability of search effort. More recent works emphasize also the non-observability of assets. However, the implications of the existence of a hidden labor market were unknown. This was an important omission in the unemployment insurance literature because the incorporation of a hidden labor market has non-trivial consequences in the treatment of the problem. Moreover, understanding the role of a hidden labor market is highly relevant given that the informal labor market, a hidden market by nature, represents a sizeable segment of the labor market in many countries. This paper fills this gap by providing an environment in which unemployed workers can simultaneously search for a job and participate in a hidden labor market.

The main conclusions are

1. The presence of a hidden labor market imposes an endogenous lower bound for promised utilities and, unless the cost of effort is linear, an extra incentive compatible constraint must be added.

2. With linear cost of effort, when the optimal unemployment insurance can be characterized, there are two important phases. First, consumption decreases during unemployment and workers do not participate in the hidden labor market. Second, for sufficiently large unemployment spells, the lower bound for promised utilities is reached. After that period, promised utility if unemployed remains constant at the lower bound. Moreover, during this phase consumption remains constant, payments become zero and participation jumps to a positive constant.

3. In an economy calibrated to Spain, the initial phase of unemployment spell has workers searching for a job with no participation in the hidden labor market and unemployment payments gradually decreasing. Around the third year of unemployment, the participation jumps away from zero and payments drop to zero.

4. Also in the numerical exercise, fairly large budget savings—compared with the Spanish system—are obtained when the unemployment insurance system takes into account the existence of a hidden labor market.

This paper suggests two main policy lessons. The first one has to do with the inconvenience of providing unemployment payments indefinitely. This is a very important difference with the previous literature, where efficiency requires that unemployment payments last indefinitely. The existence of a hidden labor market together with payments forever, guarantee sufficiently high consumption even after a very long period of unemployment. As a consequence, long unemployment spells are not serious threats to encourage search effort.

The second lesson is associated with the provision of an *type-dependent unemployment system*. When the hidden labor market is not important, the optimal payments' dynamic does not change significatively with the separation rate. However, the profile of unemployment payments—in particular payments duration—dramatically changes with the separation rate when the hidden labor market is important. Since the separation rates are usually different among worker groups, this result suggests that an type-dependent contract may be optimal. Specifically, the numerical exercise indicates that the duration of payments should be shorter for the high-separation-rate group.

The current work induces interesting directions for future research. A natural extension is to incorporate different sources of heterogeneity among workers. For instance, we could consider an environment with human capital depreciation and two type of workers, namely, skilled and unskilled. The worker's type and human capital are both observable. It is plausible that the depreciation rate while not employed in the formal sector differs across workers' type. This introduces interesting consequences in the design of the optimal contract. For instance, it is possible that the contract prescribes a higher search effort and a lower hidden labor market participation for workers whose human capital depreciates faster—presumably more skilled workers. Alternatively, we could consider an environment with non-observable types. Types can play a role either in the productivity in the hidden labor market or in the cost of effort. In any case, it could be interesting to analyze incentive provision under such frameworks.

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Appendix

Proof of Proposition 1

Proof: Suppose by contradiction that at any arbitrary period of time T, the constrained efficient contract $\mathcal{W}^* = (\mathcal{P}^*, \mathcal{I}^*)$ delivers a promised utility $\mathcal{U}_T(\mathcal{W}^*; h^T) < \underline{U}$. We prove this violates incentive compatibility by constructing a feasible deviation for any possible labor status m_T providing a utility higher than $\underline{U}$.

- If $m_T = u$, consider the following feasible deviation $\widetilde{\mathcal{P}}$, with $(\widetilde{a}_t(h^t), \widetilde{s}_t(h^t)) = (0, s^*)$ if $m_t = u$, and $(\widetilde{a}_t(h^t), \widetilde{s}_t(h^t)) = (0, 0)$ if $m_t = e$. Clearly, $\widetilde{\mathcal{P}} \in \mathbf{A}$ and $\mathcal{U}_T((\widetilde{\mathcal{P}}, \mathcal{I}); h^T) = \underline{U} > \mathcal{U}_T((\mathcal{P}, \mathcal{I}); h^T)$. Hence, the contract $\mathcal{W}^*$ is not incentive compatible and therefore, cannot be the constrained efficient contract.

- If $m_T = e$, define $\widehat{T}$ as the first period of that employment spell. By property (8), $\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}) = \mathcal{U}_T(\mathcal{W}^*; h^T)$. Now consider the situation in $\widehat{T}-1$. By definition, $m_{\widehat{T}-1} = u$. Moreover, since $m_{\widehat{T}} = e$, it has to be the case that the contract prescribes $a_{\widehat{T}-1}(\cdot) >$

0. However, if $\mathcal{U}_T(\mathcal{W}^*; h^T) < \underline{U}$, it is the case that $a_{\widehat{T}-1}(\cdot) > 0$ is not compatible incentive because a deviation with $\widetilde{a}_{\widehat{T}-1}(\cdot) = 0$ implies higher utility than $\underline{U}$. To see this notice that since v is strictly increasing,

$$\mathcal{U}_{\widehat{T}-1}(\mathcal{W}^*; h^{\widehat{T}-1}) = u(c_{\widehat{T}-1}) - v(s_{\widehat{T}-1} + a_{\widehat{T}-1}) +$$
$$\beta[p(a_{\widehat{T}-1})\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = e) + (1 - p(a_{\widehat{T}-1}))\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = u)]$$
$$< u(c_{\widehat{T}-1}) - v(s_{\widehat{T}-1}) +$$
$$\beta[p(a_{\widehat{T}-1})\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = e) + (1 - p(a_{\widehat{T}-1}))\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = u).]$$

We showed in the first part of this proof that during unemployment, promised utility must be at least $\underline{U}$; i.e.,

$$\underline{U} \leq \mathcal{U}_{\widehat{T}-1}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = u).$$

Then if $\mathcal{U}_{\widehat{T}-1}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = e) < \underline{U}$, it is the case that

$$\mathcal{U}_{\widehat{T}-1}(\mathcal{W}^*; h^{\widehat{T}-1}) < u(c_{\widehat{T}-1}) - v(s_{\widehat{T}-1}) + \beta\mathcal{U}_{\widehat{T}}(\mathcal{W}^*; h^{\widehat{T}}|m_{\widehat{T}} = u). \tag{34}$$

The last expression indicates that $\mathcal{W}^*$, which prescribes $a_{\widehat{T}-1} > 0$, is not incentive compatible because a deviation with $\widetilde{a}_{\widehat{T}-1} = 0$ provides higher utility to the worker. Hence, $\mathcal{W}^*$ cannot be the constrained efficient contract.

■

Proof of lemma 1

Proof: Take $U \geq \underline{U}$ and consider the contract $\widetilde{\mathcal{W}}$. For each t and h^t it assigns $b_t(h^t) = \tilde{b}$, $s_t(h^t) = \tilde{s}$, $a_t(h^t) = 0$, and $w_t(h^t) = \tilde{w}$. Define $(\tilde{b}, \tilde{w}, \tilde{s})$ by

$$u'(\tilde{b} + \tilde{s}\varpi)\varpi = v'(\tilde{s}), \tag{35}$$

$$U = \frac{u(\tilde{b} + \tilde{s}\varpi) - v(\tilde{s})}{1 - \beta}, \tag{36}$$

$$u(\tilde{b} + \tilde{s}\varpi) - v(\tilde{s}) = u(\tilde{w}) - v(\bar{a}). \tag{37}$$

By construction $u(c_t(h^t)) - v(x_t(h^t))$ is constant across time and histories and $\mathcal{U}_t(\widetilde{\mathcal{W}}; h^t) = U$. It remains to prove that $\widetilde{\mathcal{W}} \in \mathbf{C}$. Since deviations are only possible during unemployment and a contract is incentive compatible against any possible deviation if and only if it is incentive compatible against one period deviation, incentive compatibility implies that for each t and each h^t

$$\begin{aligned} U \geq \max_{(s,a)} u(\tilde{b} + s\varpi) - v(s + a) + \beta U. \\ \text{s.t } a \geq 0, s \geq 0, \text{ and } s + a \leq \vartheta. \end{aligned} \tag{38}$$

In (38) we impose the fact that for each history and period, $\mathcal{U}_t(\widetilde{\mathcal{W}}; h^t) = U$. Clearly, (38) is satisfied because the solution of the maximization problem implies $a = 0$ and $s = \tilde{s}$. Hence, $\widetilde{\mathcal{W}} \in \mathbf{C}(U; h^0)$. ■

Proof of lemma 2

Proof: The proof is straight forward. The first best contract is characterized by equations (19)-(23). Under observability of effort, (ICC1) is not relevant while (ICC2) is satisfied by (19). ■

Proof of lemma 3

Proof: Consider the following *Relaxed Problem*:

$$V_R^u(U) = \max_{b,e,s,U^e,U^u} -b + \beta[p(a)V^e(U^e) + (1-p(a))V_R^u(U^u)] \tag{RP}$$

$s.t.$

$$u(b + s\varpi) - v(s+a) + \beta[p(a)U^e + (1-p(a))U^u] - U = 0 \tag{PKC}$$

$$\beta p'(a)[U^e - U^u] - v'(s+a) = 0 \tag{ICC1}$$

$$(U^e, U^u) \in \Phi^2 \tag{FPUC}$$

$$s \geq 0.$$

We need to prove that if v is linear, the solution for (RP) also solves (PP). To that end, denote $D(U)$ the set of all (b, a, s, U^u, U^e) satisfying (ICC1),(ICC2),(PKC), (FPUC), and non negativity for s. Likewise, denote $D_R(U)$ the set of all (b, a, s, U^u, U^e) satisfying previous restriction but (ICC2). Clearly $D(U) \subseteq D_R(U)$. Hence, $V_R^u(U) \geq V^u(U)$. Notice however that

$$(\tilde{e}, \tilde{s}, \tilde{U}^e, \tilde{U}^u) \in \text{argmax}_{D_R(U)} V_R^u(U)$$

satisfies (ICC2). To see this, take first order condition with respect to s and notice that $\lambda > 0$ and $\phi \geq 0$. In consequence, $(\tilde{e}, \tilde{s}, \tilde{U}^e, \tilde{U}^u) \in D(U)$ and solves (PP); otherwise, it will not be true that $V_R^u(U) \geq V^u(U)$. ∎

Algorithm

1. Define a grid for promised utilities: $\mathbf{U} = \{U_1, ..., U_N\}$. The range of U is set such that $U_1 = \underline{U}$ and U_N is big enough to guarantee that the value functions become negative.

2. Define search effort grids for effort and participation: $\mathbf{E_f} = \{a_1, ..., a_M\}$ and $\mathbf{S} =$

$\{s_1, ..., s_M\}$, where $a_1 = s_1 = 0$, and a_M and s_M are big enough to avoid grid-dependent corner solutions.

3. Make initial guess $V^{u,0}(U) = V^{u,FB}(U)$ for each $U \in \mathbf{U}$, where $V^{u,FB}(U)$ is the first best planner's surplus. For values out of the grid, use piece-wise linear interpolation.

4. Iteration i starts. Get $V^{u,i+1} = TV^{u,i}$ as follow:

   (a) Suppose $s = 0$. Use equation (27) to obtain b. Given b, for each $a \in \mathbf{E_f}$ use (ICC1) and (PKC) to solve for the values of U^e and U^u. Choose the combination $\{a, b, U^e, U^u\}$ maximizing (PP). Check if (ICC2) is satisfied for $s = 0$. If it is satisfied, the maximization ends. If not, go to the next step.

   (b) Look for the best solution $\{a, s\} \in \mathbf{E_f} \times \mathbf{S}$, using (ICC1), (ICC2) and (PKC) to solve for the corresponding b, U^e and U^u.

5. Check convergency. That is, given a metric d, evaluate $d(V^{u,i}, V^{u,i+1}) > \epsilon$? If so, let $V^{u,i} = V^{u,i+1}$ and go to (4), otherwise END.