Optimal State-Contingent Unemployment Insurance

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Abstract

Since the probability of finding a job is affected not only by individual effort but also by the aggregate state of the economy, designing unemployment insurance payments conditional on the business cycle could be valuable. This paper answers a fundamental question related to this issue: How should the payments vary with the aggregate state of the economy?

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1 Introduction

The effort dedicated to job search affects the probability of success. Because effort is not observable, research on optimal unemployment insurance, such as that of Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), studies this issue as a repeated moral hazard problem. In addition to individual searching effort, the aggregate conditions of the economy (business cycles) determine the probability of finding a job. During the last 50 years, the US monthly job finding rate was below 20 percent in the more severe recessions and above 40 percent in times of economic prosperity (see Figure 1). Existing research on optimal unemployment insurance ignores this fact and considers a setup without business cycles.[1] By incorporating aggregate fluctuations to the previous framework, this paper contributes to the literature characterizing the *optimal state-contingent unemployment insurance*.

This insurance problem can be characterized using standard recursive contracts techniques. In particular, by applying the promised utility approach, this problem can be reduced to a system of two functional equations whose first order conditions describe the dynamics of benefits and taxes. This analysis provides two key results. First, business

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[1] Although he does not include aggregate fluctuations in his model, Kiley (2003) analyzes the relationship between unemployment insurance and business cycles. His work is based in the comparison of the optimal unemployment insurance for different probabilities of finding a job.

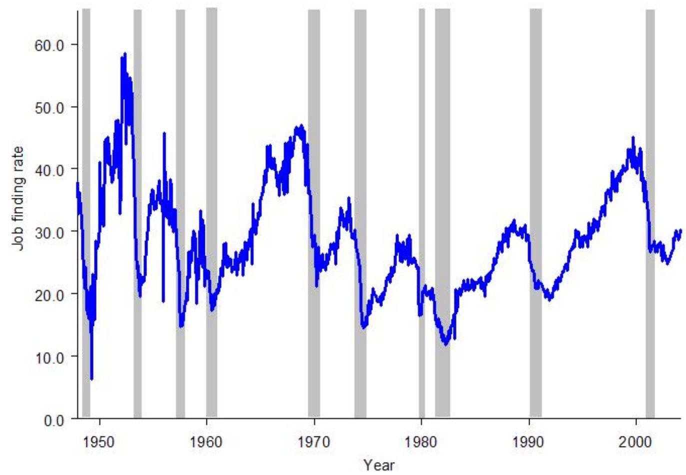

Figure 1: Probability of finding a job in the US.

Note: The job finding rate was obtained from Hall (2005). Shadow areas are recessions according to the NBER.

cycles do not affect the most important finding in this literature. That is, unemployment insurance payments still decrease with unemployment duration and the taxes financing this system are decreasing in tenure. Second, under some reasonable restrictions on the probability functions, unemployment benefits should decrease faster during economic booms than during recessions.

2 The Model

The unemployment insurance contract is studied looking at the relationship between an agent and a planner. The former maximizes her life-time utility subject to the contract offered by the latter and to the constraints imposed by the environment. Given that he has to provide the agent with a certain level of lifetime utility, the planner designs a cost minimizing contract that is compatible with the agent behavior. All the elements needed to formally state this problem are described below.

2.1 Environment

Time is discrete and denoted by $t = 0, 1, 2, ...$ There is a single perishable good whose consumption is denoted by c. individuals or agents are infinitely-lived and risk averse. At each period t, an agent's *employment status* $\varsigma \in \mathbf{S} = \{e, u\}$ takes one of two states: employed, e or unemployed, u. The *aggregate state of the economy* is indexed by $i \in \mathbf{N} = \{1, ..., N\}$. Let $s_t = (\varsigma, i) \in \mathbf{S} \times \mathbf{N}$ and $s^t = \{s_1, s_2, ..., s_t\}$ denotes the history of s_t up to date t.

Since the agent is not allowed to save, his consumption equals his output at each period. The agent's output depends just on her employment status.[2] Formally,

$$\varpi(s_t) = \begin{cases} \omega \text{ if } s_t = (e, i) \text{ for each } i \\ 0 \text{ if } s_t = (u, i) \text{ for each } i. \end{cases} \quad (1)$$

If the agent has a job at t, he will be employed at $t+1$ with probability $q(h)$, where $h \in \mathbb{R}_+$ is the *level of effort* that the agent exerts in period t. *If the agent is unemployed, he will find a job next period with probability* $p_i(h)$.[3] The functions $p_i : \mathbb{R}_+ \to [0,1]$ and $q : \mathbb{R}_+ \to [0,1]$ are strictly increasing, strictly concave and twice differentiable in $h \in \mathbb{R}_+$ for each $i \in \mathbf{N}$ and satisfy standard Inada conditions so that effort h is interior. Also, $p_i(\cdot)$ is increasing in i. This means that the aggregate conditions are better (it is easier to find a job) for higher values of j.

The aggregate state of the economy follows a discrete-time Markov Chain. Let π_{ij} denotes the probability of transition from the aggregate state i to j, where $i, j \in \mathbf{N}$. The agent discounts the future by the factor $\beta \in (0,1)$. The utility function for consumption is represented by $u : \mathbb{R}_+ \to \mathbb{R}$ and is strictly increasing, strictly concave and differentiable with $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. Likewise, the cost of effort function is represented by $v : \mathbb{R}_+ \to \mathbb{R}$ and is strictly increasing, convex and differentiable with $\lim_{h \to 0} v'(h) = 0$ and $\lim_{h \to \infty} v'(h) = \infty$.

In the next section, the recursive formulation of the optimal contract problem is carefully defined. Its sequential representation can be found in the appendix.

2.2 Recursive problem

The planner's goal is to maximize profits or *surplus*, given that he must provide lifetime utility $U \geq \tilde{U}$. This exercise is the dual problem for the maximization of lifetime utility U subject to an intertemporal budget constraint. Following Spear and Srivastava (1987) lifetime utility U is used as a state variable. This variable is sufficient to summarize information about an individual total employment history.

In order to formally specify what a recursive contract is, some notation will be now introduced. Let w denote *net wages* and b *unemployment insurance payments*. The *promised lifetime utility* from the state $s = (\varsigma, i)$ to $s' = (\varsigma', j)$ is $U_{i,\varsigma}^{\varsigma',j}$. Then, a mapping $\boldsymbol{C} : \mathbb{R} \times \mathbf{S} \times \mathbf{N} \to \mathbb{R}^{2+2N}$ is a *recursive contract*.

Additional assumptions are standard. The planner can borrow or lend resources without affecting the interest rate r which satisfies $(1+r)\beta = 1$. He can observe the agent's employment status history but he cannot observe individual's effort.

Two conditions restrict the planner's maximization. The *incentive compatibility constraint* assures that the agent cannot be better off by deviating from the recommended sequence of effort. The *promise keeping constraint* guarantees a given level of lifetime utility to the agent.

[2] This is a simplification. If output ϖ depends on the state of the economy, but the agent cannot affect it, all results still hold.

[3] Notice that we have assumed that the aggregate state does not affect the job-keeping probability q while it does affect the job-finding probability p. This assumption is supported by the empirical evidence: Hall (2005) and Shimer (2005) find that fluctuations in the separation rate over the business cycles are very small.

Recalling that the agent decides effort after he sees the unemployment insurance contract is a major element in identifying the planner's problem. Thus, the problem of an unemployed agent is

$$\max_{h\geq 0}\left\{u\left(b\right)-v\left(h\right)+\beta\sum_{j=1}^{N}\pi_{ij}\left[p_{j}\left(h\right)\left(U_{u,i}^{e',j}-U_{u,i}^{u',j}\right)+U_{u,i}^{u',j}\right]\right\}. \tag{2}$$

The first order condition is

$$\beta\sum_{j=1}^{N}\pi_{ij}\left[p_{j}'\left(h\right)\left(U_{u,i}^{e',j}-U_{u,i}^{u',j}\right)\right]=v'\left(h\right). \tag{3}$$

This will be the incentive compatibility constraint of the planner's problem with an agent starting the contract unemployed. Similarly, the problem of an employed agent is

$$\max_{h\geq 0}\left\{u\left(w\right)-v\left(h\right)+\beta\sum_{j=1}^{N}\pi_{ij}\left[q\left(h\right)\left(U_{e,i}^{e',j}-U_{e,i}^{u',j}\right)+U_{e,i}^{u',j}\right]\right\}. \tag{4}$$

In this case, the first order condition,

$$\beta\sum_{j=1}^{N}\pi_{ij}\left[q'(h)\left(U_{e,i}^{e',j}-U_{e,i}^{u',j}\right)\right]=v'(h), \tag{5}$$

is the incentive compatibility constraint of the planner's problem with an agent starting the contract employed.

Using first order conditions as incentive compatibility constraints simplifies the analysis. However, the validity of this approach was questioned in some frameworks. This is not troublesome here, since the above assumptions guarantee that first order conditions are necessary and sufficient.

Now, the *discounted lifetime surplus of a planner providing insurance to an agent starting the contract unemployed*, with promised utility U when the aggregate state is i, can be represented by

$$\begin{aligned} V_i^u(U) = & \max_{h\geq 0, b\geq 0, \left\{U_{u,i}^{u',j}, U_{u,i}^{e',j}\right\}_{j=1}^{N}} -b \\ & + \tfrac{1}{1+r}\sum_{j=1}^{N}\pi_{ij}\left\{p_j(h)\left[V_j^e(U_{u,i}^{e',j})-V_j^u(U_{u,i}^{u',j})\right]+V_j^u(U_{u,i}^{u',j})\right\}, \end{aligned} \tag{6}$$

subject to the incentive compatibility constraint (3) and the promise keeping constraint

$$u(b)-v(h)+\beta\sum_{j=1}^{N}\pi_{ij}\left[p_j(h)\left(U_{u,i}^{e',j}-U_{u,i}^{u',j}\right)+U_{u,i}^{u',j}\right]=U. \tag{7}$$

Likewise, the *discounted lifetime profits of a planner providing insurance to an agent starting the contract employed*, with promise utility U when the aggregate state is i, are

$$\begin{aligned} V_i^e(U) = & \max_{h\geq 0, w\geq 0, \left\{U_{e,i}^{e',j}, U_{e,i}^{u',j}\right\}_{j=1}^{N}} \omega - w \\ & + \tfrac{1}{1+r}\sum_{j=1}^{N}\pi_{ij}\left\{q(h)\left[V_i^e(U_{e,i}^{e',j})-V_i^u(U_{e,i}^{u',j})\right]+V_i^u(U_{e,i}^{u',j})\right\}, \end{aligned} \tag{8}$$

subject to the incentive compatibility constraint (5) and the promise keeping constraint

$$u(w) - v(h) + \beta \sum_{j=1}^{N} \pi_{ij} \left[q(h) \left(U_{e,i}^{e',j} - U_{e,i}^{u',j} \right) + U_{e,i}^{u',j} \right] = U. \tag{9}$$

3 How should the payments vary?

As proved by Pavoni (2003) in a similar setup, the interior solution to this problem can be characterized by its first order conditions.[4] Thus, there are two key equations to consider.[5] The first one uses the first order conditions with respect $U_{u,i}^{u',j}$ and b, together with the envelope condition. It is

$$\frac{1}{u'(b)} = \frac{1}{u'(b'_j)} + \mu \frac{p'_j(h)}{(1 - p_j(h))}, \tag{10}$$

where μ is the multiplier of the incentive compatibility constraint, which is always positive. To find the second important equation the first order condition with respect to $U_{u,i}^{e',j}$ is now considered. This gives

$$\frac{1}{u'(b)} = \frac{1}{u'(w'_j)} - \mu \frac{p'_j(h)}{p_j(h)}. \tag{11}$$

Adding equations (10) and (11) and using Jensen's inequality implies

$$u'(b) \leq p_j(h) u'(w'_j) + (1 - p_j(h))\, u'(b'_j). \tag{12}$$

This inequality provides the intuition for the dynamics of benefits: the optimal way to provide incentives in the next period is to punish the agent so severely when he is unemployed that in the current period he would like to save.[6]

At this point, some important result can be established. First, notice that the optimal unemployment insurance is decreasing over time, a result that also holds in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997).

Lemma 1 *Unemployment insurance payments b decrease with unemployment spell for each j.*

Proof. By inspecting equation (10), it follows that $1/u'(b'_j) < 1/u'(b)$. Then, by strict concavity of the utility function it transpires that $b'_j < b$ for each j. That is, unemployment benefits decrease from the current period to the next one for each possible state of the aggregate economy. ■

The next result shows that net wages increase with tenure. In a closely related environment Zhao (2000) finds the analogous result.

Lemma 2 *Net wages w increase with tenure for each j.*

[4] See propositions 3 to 7 in Pavoni (2003), where it is proved that a unique solution to the recursive problem exist, the value functions are concave and the maximum exist. The extension to this setup is direct.

[5] The complete system of first order equations is provided in the appendix.

[6] This intuition is provided by Kocherlakota (2004).

Proof. This proof uses the analogous equation to (10) for the case of the planner's problem with an agent starting the contract employed. It is

$$\frac{1}{u'(w)} = \frac{1}{u'(w'_j)} - \mu_e \frac{q'(h)}{q(h)}, \tag{13}$$

where μ_e is the multiplier of the incentive compatibility, which is always positive.

Then, inspecting (13) it is clear that $1/u'(w'_j) > 1/u'(w)$. Thus, by strict concavity of the utility function, it transpires $w'_j > w$ for each j. That is, the optimal way to give incentives is to increase net wages (decrease taxes) of employed agents that keep their job for the next period. ■

Before the changes in net wages across the aggregate states of the economy are characterized, it is useful to define the *elasticity of the probability of staying unemployed with respect to effort*

$$\xi_h^j \equiv \frac{\partial(1-p_j(h))}{\partial h} \frac{h}{(1-p_j(h))} = -\frac{p'_j(h)h}{(1-p_j(h))}. \tag{14}$$

The main result of this paper is presented in the following proposition.

Proposition 1 *$b - b'_j$ is increasing in j if and only if ξ_h^j is decreasing in j.*

Proof. It is required to show that $b'_{j+1} < b'_j$ occurs if and only if $\xi_h^j > \xi_h^{j+1}$. First notice that the latter condition holds if and only if

$$\frac{p'_{j+1}(h)}{1-p_{j+1}(h)} > \frac{p'_j(h)}{1-p_j(h)}. \tag{15}$$

Then, the first direction of the proposition is to show that $b'_{j+1} < b'_j$ holds if $\frac{p'_{j+1}(h)}{1-p_{j+1}(h)} > \frac{p'_j(h)}{1-p_j(h)}$ is true. For this propose, notice that equation (10) implies

$$\frac{1}{u'(b'_j)} + \mu \frac{p'_j(h)}{(1-p_j(h))} = \frac{1}{u'(b'_{j+1})} + \mu \frac{p'_{j+1}(h)}{(1-p_{j+1}(h))}. \tag{16}$$

Therefore,

$$\frac{1}{u'(b'_j)} - \frac{1}{u'(b'_{j+1})} = \mu \left[\frac{p'_{j+1}(h)}{(1-p_{j+1}(h))} - \frac{p'_j(h)}{(1-p_j(h))} \right]. \tag{17}$$

Hence, if $\frac{p'_{j+1}(h)}{1-p_{j+1}(h)} > \frac{p'_j(h)}{1-p_j(h)}$ we have $b'_{j+1} < b'_j$ by strict concavity of the utility function.

For the proof in the other direction, notice that by the same reasoning if $\frac{p'_{j+1}(h)}{1-p_{j+1}(h)} \leq \frac{p'_j(h)}{1-p_j(h)}$ we have that $b'_{j+1} \geq b'_j$. This completes the proof. ■

The previous results completely characterized the *optimal state-contingent unemployment insurance*. Payments decrease during the unemployment spell, as prescribed by Hopenhayn and Nicolini (1997). Moreover, the rate at which these payments decrease vary across the business cycles. In particular, unemployment benefits decrease faster during booms than recessions (see Figure 2).

Figure 2: Unemployment payments dynamics.

The intuition for the result in the last proposition is appealing. If an agent did not find a job during a boom, it is more likely that he was not exerting enough effort, compared to someone looking for a job during a recession. Therefore, to provide incentives, the former should receive a tougher punishment than the latter. This is in fact what occurs if payments decrease faster during booms than recessions.

Similarly, this result can be interpreted in terms of investing effort in looking for a job. Job search effort during a boom has a higher return than during a recession. To take advantage of this higher return, the optimal contracts gives incentives for higher effort during booms providing lower unemployment benefits.

The last result is particularly interesting because the *decreasing elasticity condition* $\xi_h^j > \xi_h^{j+1}$ is intuitive and natural. If effort increases, the probability of staying unemployed decreases proportionally more during a boom than a recession. Thus, this condition is natural if recessions are thought as periods when there is less return on dedicating effort to looking for a job. Moreover, in terms of specific functional forms, notice that this condition is satisfied for reasonable specifications of $p_j(h)$. For example, the most common specification could be $p_j(h) = 1 - e^{-\rho_j h}$ with ρ_j increasing in j, which clearly satisfies the decreasing elasticity condition.

4 Related policies

Analyzing the implementation of this system requires thinking about simplified versions of this contract. However, without a quantitative exercise, it is hard to know if these systems are really close to the optimal contract.

In this vein, it is natural to consider the unemployment insurance scheme in the US. The current system contains triggers for the extension of emergency benefits. It extends benefits from 26 to 39 weeks during periods of high unemployment. Moreover, in economic downturns, the federal government has traditionally supplemented regular state unemployment insurance with additional benefits. The *Emergency Unemployment Compensation* (EUC) following the 1990-91 recession was in effect from November 1991 through February 1994. The program following the 2001 recession, *Temporary Extended Unemployment Compensation* (TEUC), was implemented from March 2002 through December 2003. These two periods are lagged with respect to the business cycles defined by the NBER but seem to coincide with the cycles in unemployment rate and in the job finding rate (see Figure 3).

Is the US unemployment insurance really close to the optimal contract? This question requires a quantitative answer that is not provided here. However, it seems clear that other simple schemes are closer to the optimal contract. For example, consider a system providing benefits according to a decreasing step function. In this case, the duration or level of each step could vary over the business cycles.

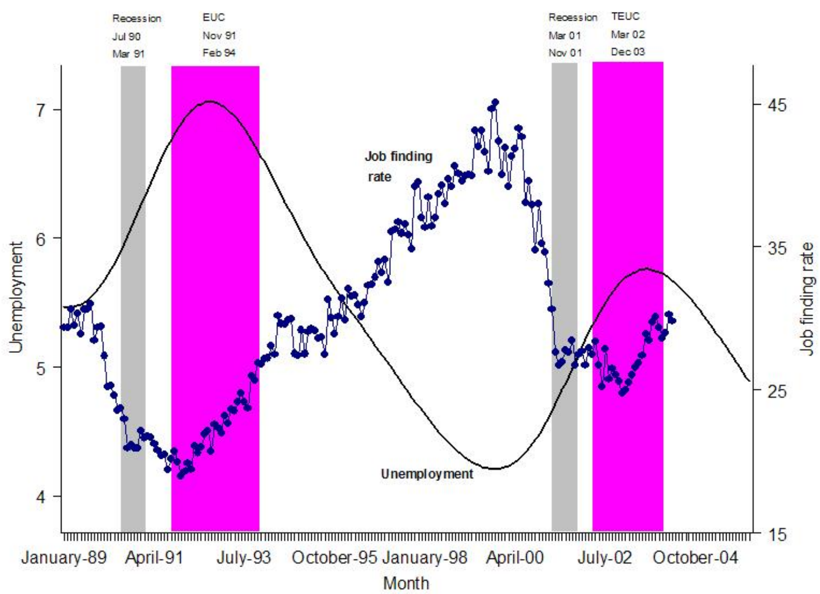

Figure 3: Unemployment and exceptional programs in the US.

Note: Unemployment data corresponds to Civilian Unemployment Rate from BLS. It was seasonally adjusted and HP filtered. The job finding rate was obtained from Hall (2005).

5 Concluding remarks

A generalization of Hopenhayn and Nicolini (1997) was developed, allowing for fluctuations in the probability of finding a job across aggregate states of the economy and studying the *optimal state-contingent unemployment insurance*. The model here has two key features. First, it is closely related to previous literature on unemployment insurance. Therefore, the model remains effective for standard questions on this issue. Second, it incorporates aggregate fluctuations in a natural way, allowing for a new characterization of the payments dynamics.

The most important findings correspond to the key features of the model. First, standard results in the literature still hold: insurance payments decrease with unemployment duration and net wages increase with tenure. Second, under specifications of $p_j(h)$ satisfying an intuitive and natural condition, unemployment benefits should decrease faster during economic booms than during recessions. The study of quantitative implications of this contract is beyond the scope of this paper, and is left for future research.

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Appendix

Sequential planner's problem

The *optimal dynamic contract* will be a history-dependent earnings scheme $\{c(s^t)\}_{t=1}^{\infty}$ and effort recommendations $\left\{h(s^{t-1})\right\}_{t=1}^{\infty}$ that maximize the ex-ante expected surplus of the planner, subject to *incentive compatibility constraint* and *promise keeping constraint* .

Let $h^t \in H^t$ be the history of effort level. Denote by $\mu(s^{t+1}; s_0, h^t)$ to the probability of being in the publicly observed node s^{t+1} given the initial state and the history of effort h^t.

Then, we can formally state the sequential incentive compatibility constraint and promised keeping constraint, respectively:

$$\begin{aligned}&\sum_{t=\tau}^{\infty}\sum_{s^t}\beta^t\mu(s^t;s_0,h^{t-1})\left[u\left(c(s^t)\right)-v\left(h(s^t)\right)\right]\geq\\&\sum_{t=\tau}^{\infty}\sum_{s^t}\beta^t\mu(s^t;s_0,\overline{h}^{t-1})\left[u\left(c(s^t)\right)-v\left(\overline{h}(s^t)\right)\right]\forall\tau,s^t,\overline{h}^t,\end{aligned}\tag{18}$$

and

$$\sum_{t=1}^{\infty}\sum_{s^t}\beta^t\mu(s^t;s_0,h^{t-1})\left[u\left(c(s^t)\right)-v\left(h(s^t)\right)\right]\geq U.\tag{19}$$

To simplify notation, call a *dynamic contracts* $\{c(s^t),h(s^t)\}_{t=1}^{\infty}=(c,h)$. Then, the sequential problem is

$$\max_{(c,h)}\sum_{t=1}^{\infty}\sum_{s^t}\frac{\mu(s^t;s_0,h^{t-1})}{(1+r)^{t-1}}\left[\varpi(s^t)-c(s^t)\right],\tag{20}$$

subject to (18) and (19).

Planner's problems and their first order conditions

The planner's problem with an agent starting the contract unemployed is:

$$\begin{aligned}V_i^u(U)=&\sup_{h\geq0,b,\left\{U_{u,i}^{u',j},U_{u,i}^{e',j}\right\}_{j=1}^{N}}\ \inf_{\lambda,\mu\geq0}-b\\&+\tfrac{1}{1+r}\sum_{j=1}^{N}\pi_{ij}\left\{p_j(h)\left[V_j^e(U_{u,i}^{e',j})-V_j^u(U_{u,i}^{u',j})\right]+V_j^u(U_{u,i}^{u',j})\right\}\\&+\lambda\left\{u(b)-v(h)+\beta\sum_{j=1}^{N}\pi_{ij}\left[p_j(h)\left(U_{u,i}^{e',j}-U_{u,i}^{u',j}\right)-U_{u,i}^{u',j}\right]-U\right\}\\&+\mu\left[\beta\sum_{j=1}^{N}\pi_{ij}p_j'(h)\left(U_{u,i}^{e',j}-U_{u,i}^{u',j}\right)-v'(h)\right].\end{aligned}\tag{21}$$

The interior solution of this problem can be described by the following system of equations:

$$\begin{aligned}&(b:)\quad-1+\lambda u'(b)=0,\\&(h:)\quad\tfrac{1}{1+r}\sum_{j=1}^{N}\pi_{ij}\left\{p_j'(h)\left[V_j^e(U_{u,i}^{e',j})-V_j^u(U_{u,i}^{u',j})\right]\right\}\\&+\lambda\left\{-v'(h)+\beta\sum_{j=1}^{N}\pi_{ij}\left[p_j'(h)\left(U_{u,i}^{e',j}-U_{u,i}^{u',j}\right)\right]\right\}\\&+\mu\left[\beta\sum_{j=1}^{N}\pi_{ij}p_j''(h)\left(U_{u,i}^{e',j}-U_{u,i}^{u',j}\right)-v''(h)\right]=0,\\&(U_{u,i}^{u',j}:)\quad\tfrac{1}{1+r}\pi_{ij}\left(1-p_j(h)\right)V_j'^u(U_{u,i}^{u',j})+\lambda\beta\pi_{ij}\left(1-p_j(h)\right)-\mu\beta\pi_{ij}p_j'(h)=0,\\&(U_{u,i}^{e',j}:)\quad\tfrac{1}{1+r}\pi_{ij}p_j(h)V_j'^e(U_{u,i}^{e',j})+\lambda\beta\pi_{ij}p_j(h)+\mu\beta\pi_{ij}p_j'(h)=0.\end{aligned}\tag{22}$$

Likewise, the planner's problem with an agent starting the contract unemployed is:

$$
\begin{aligned}
V_i^e(U) = & \sup_{h\geq 0, w, \left\{U_{e,i}^{u',j}, U_{e,i}^{e',j}\right\}_{j=1}^{N}} \inf_{\lambda,\mu_e\geq 0} \omega - w \\
& + \tfrac{1}{1+r}\sum_{j=1}^{N}\pi_{ij}\left\{q(h)\left[V_j^e(U_{e,i}^{e',j}) - V_j^u(U_{e,i}^{u',j})\right] + V_j^u(U_{e,i}^{u',j})\right\} \\
& + \lambda_e\left\{u(w) - v(h) + \beta\sum_{j=1}^{N}\pi_{ij}\left[q(h)\left(U_{e,i}^{e',j} - U_{e,i}^{u',j}\right) - U_{e,i}^{u',j}\right] - U\right\} \\
& + \mu_e\left[\beta\sum_{j=1}^{N}\pi_{ij}q'(h)\left(U_{e,i}^{e',j} - U_{e,i}^{u',j}\right) - v'(h)\right].
\end{aligned} \tag{23}
$$

Now, the interior solution of this problem can be described by the following system of equations:

$$
\begin{aligned}
& (w:) \quad -1 + \lambda_e u'(w) = 0, \\
& (h:) \quad \tfrac{1}{1+r}\sum_{j=1}^{N}\pi_{ij}\left\{q'(h)\left[V_j^e(U_{e,i}^{e',j}) - V_j^u(U_{e,i}^{u',j})\right]\right\} \\
& + \lambda_e\left\{-v'(h) + \beta\sum_{j=1}^{N}\pi_{ij}\left[q'(h)\left(U_{e,i}^{e',j} - U_{e,i}^{u',j}\right)\right]\right\} \\
& + \mu_e\left[\beta\sum_{j=1}^{N}\pi_{ij}q''(h)\left(U_{e,i}^{e',j} - U_{e,i}^{u',j}\right) - v''(h)\right] = 0, \\
& (U_{e,i}^{u',j}:) \quad \tfrac{1}{1+r}\pi_{ij}\left(1-q(h)\right)V_j'^u(U_{e,i}^{u',j}) + \lambda_e\beta\pi_{ij}\left(1-q(h)\right) - \mu_e\beta\pi_{ij}q'(h) = 0, \\
& (U_{e,i}^{e',j}:) \quad \tfrac{1}{1+r}\pi_{ij}q(h)V_j'^e(U_{e,i}^{e',j}) + \lambda_e\beta\pi_{ij}q(h) + \mu_e\beta\pi_{ij}q'(h) = 0.
\end{aligned} \tag{24}
$$