A small victorious war: domestic revolution and international conflict

Libman, Alexander

Frankfurt School of Finance Management and Russian Academy of Sciences

September 2010

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Alexander Libman

September 22, 2010

Abstract

The paper provides an extremely simple model of the interaction of international and internal conflicts. Specifically, it analyzes the situation when investments in both domestic and international wars are complementary. Therefore it predicts a possibility of inefficiently high investments in international conflict due to the threat of the domestic revolution or - in a modified setting - of purely wasteful investments in the military capacity without any international war at all. The results are debated from the point of view of several cases of military conflicts caused by he domestic regime stability considerations.

Keywords: domestic conflict, international conflict, political bias, diversionary conflict

JEL classification: D74; F51

Contact details: Alexander Libman, Frankfurt School of Finance & Management, Sonnemannstrasse 9-11, 60314 Frankfurt/Main, Germany, and Russian Academy of Sciences, Moscow, Russia

1 Introduction

"We need a small, victorious war to avert a revolution" is a phrase often attributed to Vyacheslav Plehve, minister of interior of the Russian Empire at the edge of the 20th century and one of the strongest proponents of the military engagement against Japan. A similar idea of a "small victorious war" seems to be present in many non-democratic political systems, when the dictators use military conflicts (even against superior enemy) as an instrument to ensure the survival of the government. Ironically, many autocrats overestimated the power of their armed forces, and the resulting defeat actually triggered regime changes; however, in some cases "small victorious wars" indeed turn out to be a good instrument of regime stabilization. Moreover, this type of conflicts is often initiated by the democratic political systems, where military success in a small and fast conflict can become an effective tool of influencing the behavior of the voters.

The idea underlying the "small victorious war" logic is in fact very simple: if a country experiences a domestic political conflict, investments in armed forces and initiation of an external war *can* be used to achieve the victory "at home" as well. Basically, investments in an international and in a domestic war can be treated as complementary in this case. In particular, three factors could be driving this complementarity.

First, in non-democracies army is often used to suppress internal political dissidents and rebels, and hence, creating strong military forces may be used to fight opposition. Even regimes with very strong internal political police were forced to rely on military in case of major political turbulences, like the Soviet Union for the Novocherkassk workers' riots in 1962 (see Kozlov, 1999). However, when the strong army exists, the involvement in international conflicts becomes ceteris paribus more attractive (and, in addition, sustaining large army as a combat-ready force without wars is often difficult) - so, creating a large army to fight internal opposition is likely to induce incentives to engage in "small victorious wars". This case is probably the less explored in the literature, and the model presented in this paper focuses explicitly on this scenario.

Second, investments in army may be a hidden form of bribes to the military, preventing it from taking over the power through a coup. These investments, however, are not so harmless, since the military is a well-known source of political threats for any regime: as Finer (2002: 5) puts it, "instead of asking why the military engages in politics, we ought surely ask why they ever do otherwise". Investing in army therefore constitutes a threat for the autocrat;[1] however, *not* investing in army can be even more dangerous. So, one possible alternative can be to create a military force and then let it engage in an external conflict, satis-

[1] In fact, the coup can turn out to be "the only successful operation of the army", as the president of Bolivia Daniel Salamanca supposedly said to the generals, who removed him from office after the long and completely unsuccessful Chaco War in the first half of the 20th century.

fying the ambitions of the generals and distracting them from domestic politics. In addition, if a country experiences strong domestic political instability, the military turns into an autonomous actor in decisions over war and peace, and, as Dassel and Reinhardt (1999) show, is likely to initiate an international war.

Finally, the success in international conflicts seems to act as a sort of "multiplier" for the government's position in any sort of internal disputes - from elections to conflicts within ruling elite - simply because public opinion is often in favor of the victorious warlord. This is a well-known scenario of the "diversionary war", systematically analyzed in the political science literature. While the previous two scenarios are rather typical for non-democracies or very weak democracies, "diversionary wars" constitute an important element of the democratic policy-making as well. The empirical research on the "diversionary wars" so far produced highly inconclusive results (Levy, 1989; Leeds and Davis, 1997), which seem to be dependent on a variety of factors, including domestic political structure (Miller, 1995), level of support from critical domestic groups (Morgan and Bickers, 1992), access to domestic repressions (Gelpi, 1997) and presence of unresolved issues between countries (Mitchell and Tyne, 2009).

The aim of this paper is to provide an extremely simple theoretical model predicting the possible "small victorious war". It looks at a standard two-country setup used in the economics of conflict, but modifies it by including political bias and domestic struggle for redistribution of rents. From that point of view it is related to two literatures. First, in economics a number of theoretical studies examine the interaction of international and domestic conflicts (Garfinkel, 1994, 2004; Muenster, 2007; Muenster and Staal, 2010). In this literature an external and an internal conflict are treated as substitutes: external conflict diminishes the value of domestic prize parties compete for or reduces the resources at hand to successfully fight the domestic war. This paper also uses the tools of the economics of conflict to predict the "small victorious war" outcomes and does rely on fully rational agents without any information asymmetry - so, it can be treated as complementary to these papers. However, the driving mechanism of my model is different: spending for an international conflict is not a substitute for the domestic warfare, but complementary to it and increases the probability of victory in the domestic war. It has an important normative implication: in the model presented in this paper it is possible to straightforwardly show that the existence of the domestic conflict results in an overspending for international warfare, which is in fact inefficient from the point of view of the overall revenue at disposal of the nation.

The second literature is mostly associated with the political science and deals specifically with the "diversionary wars" (Richards et al., 1993; Smith, 1996; Tarar, 2006). The idea of these studies is to show that the external war can be attractive for the incumbent political leader to gather domestic support. The model of this paper is different from this literature in several respects. First, almost all papers of the "diversionary war" studies do not model wars

as such, concentrating on the domestic policy (and describing the international war as a simple lottery rather than an outcome of the strategic interaction of two players) - hence, potential changes in the behavior of foreign opponents are ignored (see Chiozza and Goemans, 2004). A recent exception is Arena (2007), who, however, looks primarily on the problem of escalation of potential military disputes. Second, the "diversionary wars" literature heavily relies on the information asymmetry between politicians and electorate. There are some recent exceptions (Goemans and Fey, 2009), but they still treat the war as a purely random lottery. In my model no uncertainty or informational problems are required. Finally, the "diversionary wars" models usually concentrate on the democratic political regimes (see Levy, 1998:153). However, although the diversionary behavior is indeed shown to be more typical for democracies (Miller, 1999; Heldt, 1999; Mitchell and Prins, 2004), the examples above demonstrate that autocracies often engage in "small victorious wars" as well (Pickering and Kisangani, 2010). Gent (2009) does incorporate different regimes and an explicit model of the attacking and defending states in the model, but still focuses on the incomplete information setup. In addition, the "diversionary wars" models always look just at the third reason for the "small victorious wars" as it has been introduced above (as it in fact follows from the very label of the literature). The model of this paper concentrates on non-democratic regimes and introduces a number of different driving forces for the political behavior: for instance, the political bias for the rents obtained by the autocrat from the external war and the peacetime production.

The structure of the paper is the following. The next section introduces the reference model without domestic conflict. The third section looks at the effect of domestic conflicts on military spending and probability of war. The fourth section distinguishes between small and large wars and looks at potential differences of the impact of domestic conflict for these two settings. The fifth section offers a discussion of the main findings of the model using a number of anecdotal case studies of "small victorious wars" initiated by the non-democratic regimes. The last section concludes.

2 Reference model without domestic conflict

Assume that the world consists of two countries A and B, fighting for a given resource V. The probability of country B to gain control over resources is

$$p^w = \frac{m_b}{m_a + m_b} \tag{1}$$

if the investments of at least one party in the conflict are non-zero and .5 otherwise. The war happens if both parties invest a non-zero effort in warfare; if only one party makes military investments and the other does not, it is assumed that the second party surrenders. m_i for $i = A, B$ denotes the investments in warfare. Both countries are endowed with identical resources R. The resources

spent for the war could be alternatively used for productive activity, and both countries have linear production functions; so, each unit of resource R invested in productive activity yields β_a units of output in country A and respectively β_b units in country B (this is actually the main driving force for obtaining peaceful equilibria in the model; see also Bennour, 2009). In the reference model let me ignore the internal conflict in countries A and B and focus exclusively on the international warfare. The payoffs of both conflict parties are denoted by

$$\pi_a = (1 - p^w)V + \beta_a(R - m_a) \tag{2}$$

and

$$\pi_b = p^w V + \beta_b(R - m_b) \tag{3}$$

In this case one can immediately show that

Proposition 1: (i) The war takes place if and only if at least one of the following cases holds: (a) $\frac{\beta_a}{\beta_b} \in (.5, 2)$, (b) $\frac{\beta_a}{\beta_b} \leq .5$ and $\frac{V}{\beta_b} \geq R$ and (c) $\frac{\beta_a}{\beta_b} \geq 2$ and $\frac{V}{\beta_a} \geq R$; (ii) if $V > \beta_i R$, zero investments in war are never optimal for the country i.

Proof: See Appendix

Hence the probability of war is increasing if countries are sufficiently poor (as opposed to the size of V) and just cannot afford investing too much in preventing military confrontation or if countries are rich enough, but relatively similar to each other in terms of relative productivity of alternative use of resources averted from military spending to peaceful production.[2] I will assume for the time being that both opponents are able to invest sufficiently large resources in war and restrict attention to case (a) of Proposition 1. The second result of the Proposition follows from the symmetricity of countries in terms of their endowment: since any country cannot send more on war than R, then if the peacetime production revenue is smaller than the expected prize, even if the opponent spends everything on war, it is still optimal not to surrender.

3 Dictator as aggressor

Now consider the case when the country B is ruled by a dictator, who takes away an exogenous share of rents d produced in the country. The population keeps the rest of the rents. Analogously, the dictator receives a share d^w of the revenue from war against A. The ratio $\frac{d^w}{d}$ is the political bias (Jackson and Morelli, 2007), i.e. relative gain from war as opposed to the gains from the peacetime production. For the country B I assume $d^w \in [d; 1]$, that is, the ruler receives at least as much from the external conflicts as from the peacetime

[2] This is not surprising: the claim of Hirshleifer (1989) that the conflict is unavoidable in a ratio success function is made exactly for the case when $\beta_a = \beta_b = 1$.

production at home and the political bias is larger or equal to one.[3] Obviously, the public dislikes the oppression by the dictator and is willing to rebel. The rebellion is modeled as an additional (internal) conflict: so, there are two sequential conflicts: an external (war) and an internal (rebellion). The rebellion is also described with a ratio conflict success function, where both dictator and public invest fraction of their revenue in weapons.[4] The timing of events is the following: first the countries engage in war. After the war, the revolution takes place in both countries. In the country A $d = d^w = 0$, and hence, the public always spends zero on the rebellion (because there is nothing to gain) - so, its leadership basically does not differ from that of the reference case model. I ignore the collective action problem for the revolution: there is no "opting out" for individuals not participating in the revolt. Moreover, while in the first stage the question is how to distribute the resources between military capacity and production, in the second stage production has already been implemented; the sides just struggle for the division of the pie.

Denoting the spending for means of internal conflict in country B as a_p and a_d for the public and for the dictator respectively, one can write the payoffs of the public and of the dictator in the country B as follows:

$$\pi_p = (1-d)\beta_b(R-m_b) + (1-d^w)Vp^w + p^r(d\beta_b(R-m_b) + d^wVp^w) - a_p \quad (4)$$

$$\pi_d = (1-p^r)(d\beta_b(R-m_b) + d^wVp^w) - a_d \quad (5)$$

If both dictator and public invest zero effort in conflict the dictator keeps the share of the national revenue with certainty. For the payoff of the population the first two terms represent the share of the revenue from productive activity and war the public receives in case the dictatorship remains in power. The third term is the expected gain from revolution, which expropriates the revenue of the dictator if successful (with p^r being the probability of successful rebellion). The first term in the payoff of the dictator is her revenue if the revolution is unsuccessful; in case the rebellion is able to overthrow the current regime, the revenue of the dictator is assumed to be zero.[5] The key assumption of the model

[3] Of course, there are cases when the share of the revenue obtained from the conquered territories is smaller than from the domestic ones: if there are significant costs of maintaining control over the occupied land in presence of strong domestic opposition or if conquests are implemented by a quasi-independent warlord, who then gets control ver the conquered land and just delivers a fraction of the revenues to his supposed sovereign (for example, Russian conquest of Siberia or early Spanish colonialism). The overall revenues from the conquered territories may still be large, but because of extremely large V, which makes even smaller d^w attractive for the dictator. However, in order to properly model these cases one has to introduce a third party (warlord or by the occupation forces) acquiring part of the resources from the conquest, which go neither to the population of the homeland nor to the dictator. If one assumes these costs to be exogenous, one could equivalently model the interaction I am interested in just by reducing the size of V.

[4] Although for simplicity I refer to weapons, the means of internal conflict may as well be propaganda or repressions; important is that the dictator has to spend some part of her revenue to prevent a revolution.

[5] One can speculate whether this assumption is adequate, as I will do in what follows.

is the definition of p^r. Probability of success for the revolution is given by

$$p^r = \frac{a_p}{a_p + a_d\theta(m_b)} \tag{6}$$

where $\theta(m_b)$ is assumed to be a continuous, monotonous and (for technical convenience) twice differentiable function, mapping m_b on $\Re^0_+$. This term represents the *complementarity* between spending on international and on domestic conflict. By investing in the first conflict the dictator changes the probability of the success in the second one. If $\theta'(m_b) < 0$, the investments in army actually reduce the probability of success of the ruler in the revolution: for example, army may become an alternative power center potentially dangerous for the dictator (since it could stage a coup). If $\theta'(m_b) > 0$, military spending for external security increases the probability of success in internal conflict. Finally, $\theta'(m_b) = 0$ means that there is no effect of external army on internal security; this case is uninteresting and therefore left aside. Moreover, I assume $\theta(m_b) \geq 0$ and $\theta(0) = 1$, i.e. if for zero military expenditures for the external war the probability of the dictator and of the public to win in the domestic revolution is identical. Considering $\theta(0) < 1$ would imply the virtually powerless domestic police,[6] which forces the dictator to create an otherwise useless army.

The case of $\theta'(m_b) > 0$ can be used to describe all three scenarios of the "small victorious war" introduced in this paper. For the direct use of the military against opposition, the interpretation is straightforward. If one considers the opportunity of the military coup, increasing θ indicates that by providing funding to the military the dictator reduces the opportunity of its involvement in the domestic revolution on the side of the rebels. Finally, in the "diversionary war" case external aggression through large investments in army and/or associated propaganda (which is also necessary as part of the military investments themselves to boost morale) can be used to "distract" the population from the revolution: it means that the effectiveness of the population's investments in the rebellion go down, so, one can write $p^r = \frac{a_p/\theta(m_b)}{a_p/\theta(m_b)+a_d}$, what is equivalent to the expression (6). There is also another way of re-labeling the variables, which I will discuss in section 5.2.

Solving the model by backward induction implies that one has to start with the internal conflict. The first-order condition for the population is given by:

$$\frac{\partial \pi_p}{\partial a_p} = -1 + \frac{a_d\theta(m_b)}{(a_p + a_d\theta(m_b))^2}(d\beta_b(R - m_b) + d^w p^w V) = 0 \tag{7}$$

which yields the reaction curve of

$$a_p = \sqrt{a_d\theta(m_b)(d\beta_b(R - m_b) + d^w p^w V)} - a_d\theta(m_b) \tag{8}$$

[6] Once again, the term "police" here is used just for simplicity: it can as well mean that the propaganda machine of the dictator collapses or that any other measures implemented to fight the domestic rebellion do not work.

if $a_d\theta(m_b) < d\beta_b(R - m_b) + d^w p^w V$ and zero otherwise. Now the dictator's problem for non-zero a_p gives

$$\frac{\partial \pi_d}{\partial a_d} = -1 + \frac{1}{2}\sqrt{\frac{d\beta_b(R - m_b) + d^w p^w V}{a_d}\theta(m_b)} = 0 \tag{9}$$

and therefore the optimal choice of the dictator is

$$a_d = \frac{d\beta_b(R - m_b) + d^w p^w V}{4}\theta(m_b) \tag{10}$$

The dictator's investments are large enough to make a revolution impossible if $\theta(m_b) \geq 2$, i.e. the positive effect from spending on army for internal security is large enough (recall, however, that m_b is a choice variable which is set in the first stage of the game). Otherwise the revolution takes place. On the other hand, by similar calculations one shows that the condition for the dictator to spend zero on internal security is $a_p \geq d\beta_b(R - m_b) + d^w p^w V$; however, the public always chooses a_p weakly smaller than the prize to win, and hence, the dictator always makes some investments in internal security. To conclude, the revenue of the dictator for a given level of m_b is

$$\pi_d = \frac{\theta(m_b)}{4}(d\beta_b(R - m_b) + d^w p^w V) \tag{11}$$

if $\theta(m_b) < 2$. The rest is spent for internal conflict.

Now, if $\theta(m_b) \geq 2$, the dictator, simply following (10), might "over-invest" in domestic conflict: it is sufficient for her just to invest enough to ensure that $a_p = 0$. The condition is then $a_d = \frac{1}{\theta(m_b)}(d\beta_b(R - m_b) + d^w p^w V)$. One can immediately see that for $\theta(m_b) = 2$ this condition and optimal response (10) yield identical investments in domestic conflict; for $\theta(m_b) > 2$ the investments in domestic conflict under (10) were higher. Thus, it is reasonable for the dictator to invest the smallest amount possible in the revolution ensuring the surrender of the opposition. Given $a_d = \frac{1}{\theta(m_b)}(d\beta_b(R - m_b) + d^w p^w V)$, and also $p^r = 0$, equation (5) transforms into

$$\pi_d = \frac{\theta(m_b) - 1}{\theta(m_b)}(d\beta_b(R - m_b) + d^w p^w V) \tag{12}$$

representing the revenue of the dictator for a given level of m_b for $\theta(m_b) \geq 2$. From the results of internal conflict, the reaction curve of m_b for given m_a is thus described by the following equations

$$\frac{\theta'(m_b)}{4}(d\beta_b(R - m_b) + d^w p^w V) + \frac{\theta(m_b)}{4}(-d\beta_b + d^w V\frac{m_a}{(m_a + m_b)^2}) = 0 \tag{13}$$

if $\theta(m_b) < 2$ and

$$\frac{\theta'(m_b)}{\theta^2(m_b)}(d\beta_b(R - m_b) + d^w p^w V) + \frac{\theta(m_b) - 1}{\theta(m_b)}(-d\beta_b + d^w V\frac{m_a}{(m_a + m_b)^2}) = 0 \tag{14}$$

otherwise.

Unfortunately, there is no reason to assume that the equilibrium is unique: both equations can have multiple solutions, meeting the constraints. It is also possible that the equations have no solutions; in this case the equilibrium expenditures are either strictly zero or R (depending upon the sign of the left-hand side expression). However, one can still make several conclusions about the spending on military purposes of the country B even for these general assumptions. I just restrict my attention to the equilibria when in case there are no solutions to the equations satisfying the constraint on $\theta(m_b)$, the optimal choice is R and not 0.

In this model there are two effects influencing the conflict behavior. First, there is a *political bias effect*. Assume $p^r = 0$. The dictator cares only about a fraction of national revenue, which is attributed to her own budget; from the point of view of the dictator the nation is just less productive, because from each unit of internal output a portion is "wasted" as revenue of the population. If the dictator gets a higher share of her domestic revenue, the difference between the "real country" and the "part of the country generating utility for the dictator" becomes smaller, and the decision of dictator is closer to that of the reference model leaders.[7] Similarly, she cares only about a fraction of revenue from conflict: if d^w decreases, the dictator gets a smaller share of the rent V which is contested, and does not want to attack any more. The effect of political bias depends upon the relation between the ratio of peacetime productivities and the ratio of dictator's revenue from external conflict and production of the peaceful times, as it is shown in the *Figure 1*. Assume that in the reference model war was possible only in the dotted interval (remember, that I disregard the problem of potentially lacking resources for the dictator). Now if one considers the pure political bias effect in one country, the conflict zone moves to the shaded area. For the political bias equal to 1 (i.e. share of peacetime production and of revenue from war is exactly identical), there is no difference between the decision of the dictator and the society without political bias; if the political bias goes up, the lower bound of the war zone shifts to the x-axis (because dictator will still attack even for relatively large investments of her opponent), but, on the other hand, the upper bound also shifts downwards (because soon enough dictator invests so much in warfare that country A surrenders).

If the rebels can win in the internal conflict with a positive probability, there is also a second effect of domestic conflict on military spending, which I will refer to as *complementarity effect*. First assume $\theta'(m_b) > 0$, i.e. armed forces support the police in restoring internal peace. In this case, intuitively, the dictator is likely to invest more in external conflict, than the leadership of country B in the reference case; even if external conflict is not attractive per se, the dictator

[7] In fact, the idea is from a certain point of view similar to the McGuire and Olson (1996): the desire of the dictator to engage in wasteful policies is smaller since the size of the pie also goes down.

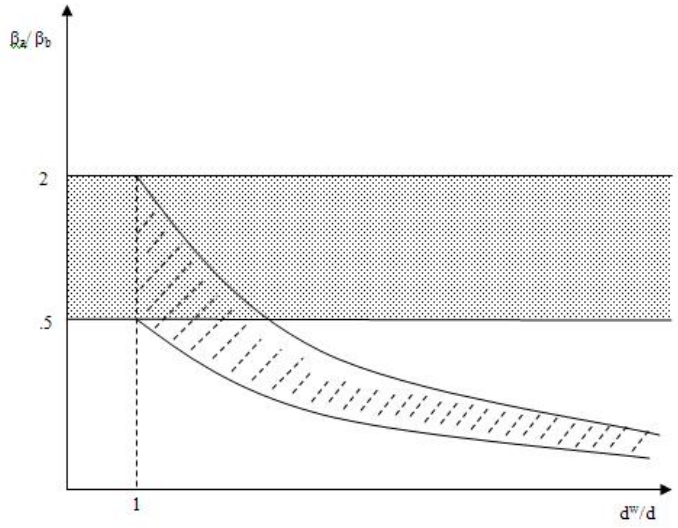

Figure 1: Peaceful settlement and war equilibria if B is ruled by the dictator and A is not: political bias effect

is still interested in providing army for internal purposes. For the country B it is possible to claim that

Proposition 2: For an increasing $\theta(m_b)$ in country B (i) even if A sets m_a large enough to ensure surrender of B in the reference case, B will not sureneder (ii) investments in warefare of B are always stricktly larger than in the reference case; (iii) there exists political bias

$$b^* = \frac{R\beta_b}{V}[1 - R\theta'(0)] \tag{15}$$

such that for $\frac{d^w}{d} > b^*$ the zero level of m_b is never optimal; (iv) for any $\hat{\theta}(.)$ function such that

$$\hat{\theta}'(0) > \frac{1}{R}[1 - \frac{d^w}{d}\frac{V}{\beta_b R}] \tag{16}$$

the zero level of m_b is never optimal.

Proof : See Appendix

The Proposition suggests that there are two forces driving the military expenditures - high political bias and low effectiveness of internal police without military support, which act as substitutes. If the political bias is high enough, then even a relatively small marginal increase of the strength of domestic police through the military support at $m_b = 0$ ensures non-zero investments. If the

marginal increase of power of internal police is large enough, even small political bias suffices to ensure non-zero investments. One can therefore predict that a "small victorious war" is to be expected either if the political bias is very large ("greedy dictator" - or dictator strongly constraint by the domestic institutions at home, but unconstraint abroad and able to get large rents from foreign conquests[8]) or if the gains for domestic police from the military support are very large ("army as the main power basis of the dictator"), or if both conditions hold simultaneously. However, if the greed is too small and the internal police does not gain a lot in terms of efficiency from the military support, dictator can decide not to go to war.

Consider the case of $\theta'(m_b) < 0$. In this case the dictator faces a problem: investing in military capacities she makes her demise through an internal revolution more likely. There is a trade-off between the chances to get the prize in external war and the probability to lose power - a problem often faced by dictators with strong military forces. One can show that under these conditions investments of A sufficient to ensure surrender of B in the reference case are "not enough" for the case B is ruled by the dictator only if $\frac{d^w}{d} > 1 - \theta'(0)R$.[9] However, once again, there exists a political bias large enough so that the dictator invests non-zero effort in warfare for a given $\theta(.)$. It reflects the "greed" of the dictator: if the political bias effect is very large, the dictator becomes too greedy and is ready to risk an international war and to build up her military even knowing that it will cause trouble in internal conflict (where large military actually reduces her ability to fight the rebels). In the same way, if the reduction of power of the domestic police through investing in the military is not very large, even moderate political bias can suffice to engage in war. In what follows I will consider only increasing $\theta(m_b)$ throughout the paper.

As a final remark, one should notice, that both the political bias and the complementarity of military expenditures and the internal police (increasing $\theta(.)$) make the conflict expenditures inefficient in the sense that the overall revenue of the country after war (described by expression (3)) is not maximized. Hence, the countries might engage in a war which actually reduces the overall size of the pie available for the redistribution between the dictator and the population (through the complementarity effect and the political bias effect). For some parameters the predictions of the model are even more extreme. As Jervis (1988) conjectures, domestic politics (or other factors, like the willingess to impress the third countries, which are not covered by this paper) may support the decision of leaders to fight wars they are likely to lose. This is exactly what can happen

[8] One example is that of the late Roman Republic, where successful warlords could benefit from conquering new provinces, but had to deal with strong countervailing power centers at home. Another example is the German Empire of the late 19 century, which received direct rule over the territories conquered from France, but had very limited access to the fiscal resources of the member states.

[9] By replicating the results of Proposition 2 and noticing that in expression (28) the first term is now negative.

in the model presented above. Intuitively, while for fairly small probabilities of success in the reference model the country decided to surrender, in the model with political bias and complementarity, given a very high level of "greed" and very low efficiency of domestic police without military support, it can be attractive for the government to still make non-zero investments in army and hence to go to war.[10]

Larger military expenditures of country B do not automatically imply, however, that there is going to be war according to the definition introduced above. The optimal level of military expenditures may exceed $\frac{V}{\beta_a}$, thus making country A surrender. However, one can show that this "over-investments" are also more likely to occur for regimes with very high political bias and with particular $\theta(.)$ functions. More specifically:

Proposition 3: A sufficient condition for optimal investments of country B to ensure surrender of country A is $\frac{\beta_a}{\beta_b} \geq \frac{2}{\frac{d^w}{d}}$, if $m_b = V/\beta_a$ is feasible given the resource constraint R.

Proof: See Appendix.

The result of the Proposition suggests that if the political bias gets larger, the lower boundary of the β_a/β_b area sufficient to ensure unilateral surrender of A goes down (this is, by the way, also evident from *Figure 1*). For any political bias exceeding 1 it is also larger than in the reference case discussed in the previous section. Hence, *war actually is more likely to happen if the dictatorship has an intermediate level of political bias.*

It is, however, more difficult to make a general claim with respect to the impact of $\theta(.)$ function on the investments ensuring the surrender of A. The reason is

[10] Consider for example an equilibrium associated with $p^w = .25$ (when the chances of winning the war are too small). Knowing that $m_a = \sqrt{\frac{V m_b}{\beta_a}} - m_b$, as I have shown in the proof for Proposition 1, one can easily find that $m_b/(m_a + m_b) = .25$ if $m_b = \frac{V}{16\beta_a}$. From that follows, knowing the best response of A to any policy of B, that $m_a = \frac{3V}{16\beta_a}$. However, substituting this expression into $m_b = \sqrt{\frac{V m_a}{\beta_b}} - m_a$, one obtains that $m_b = \frac{V}{16\beta_a}$ only if $\frac{\beta_a}{\beta_b} = \frac{1}{3}$ - but, as it has been shown above, for this combination of β's war does not take place in the reference model (of course, I once again disregard the resource constraints for the investments in war). However, in case of an autocrat with the political bias and the complementarity between investments in army and fight against revolution, the combination of $m_a; m_b$ equal to $\frac{3V}{16\beta_a}; \frac{V}{16\beta_a}$ can, as a matter of fact, be an equilibrium if, for instance, $\beta_b = 3\beta_a \frac{d^w}{d} + \frac{\theta'(\frac{V}{16\beta_a})}{\theta(\frac{V}{16\beta_a})}[\beta_b(R - \frac{V}{16\beta_a}) + \frac{d^w}{d}\frac{V}{4}]$ (assuming in this case $\theta(\frac{V}{16\beta_a}) < 2$; for larger values of $\theta(.)$ result follows analogeously from (14)). The ratio $\frac{\beta_a}{\beta_b}$ is actually even larger, then in the reference case, yet if the political bias is large enough, even for this combination B makes non-zero investments in war, and hence, war happens with the probability of victory equal to .25.

that while for the purpose of Proposition 3 there is no difference between analyzing the first order conditions derived from (11) and (12), it is not the case for the impact of the complementarity function. As a matter of fact, from (11) one can easily show that the sufficient condition (once again, assuming the existence of the equilibrium) for the optimal m_b to be high enough to ensure the unilateral surrender of A is that the first term of (29) is larger than the absolute value of the second one for any $m_b < V/\beta_a$, what is achieved if $\frac{\theta'(m_b)}{\theta(m_b)}$ is large enough. However, for the first-order condition derived from (12) the same requirement of the absolute value of the negative term being smaller than that of the positive term is achieved for $\frac{\theta'(m_b)}{\theta(m_b)(\theta(m_b)-1)}$ being large enough.

In order to make the result more intuitive, consider the simplest possible complementarity function satisfying conditions set up in the main part of the paper: $\theta(m_b) = cm_b + 1$, where $c > 0$. The advantage of this function is that it has a constant derivative c which can be thus interpreted as the quantiative measure of complementarity: if c increases, additional military investments provide stronger support for the internal police. Then the condition $\frac{\theta'(m_b)}{\theta(m_b)}$ reads as $\frac{c}{cm_b+1}$, which is decreasing in m_b, but increasing in c. Thus, the level of investments ensuring unilateral surrender of the opponent is more likely to be achieved for high c. However, $\frac{\theta'(m_b)}{\theta(m_b)(\theta(m_b)-1)}$ (which is relevant if the optimal $m_b \geq 1/c$, i.e. $\theta(m_b) \geq 2$) now reads as $\frac{1}{m_b(cm_b+1)}$, which is decreasing in c. To conclude, one can state that *for $\theta(m_b) = cm_b + 1$ increasing the complementarity between domestic and international warfare does not necessarily lead to military investments ensuring the surrender of A*. As a matter of fact, in the discussion I will present several cases of very high complementarity (i.e. when domestic police was obviously inadequate to fight the rebellion), where investments in war were too small not just to ensure surrender, but even to win the war against another country. But, on the other hand, if the benefits of the dictator from the war are much larger than from the peacetime production, the over-investment is more probable.

4 Large vs. small war

The analysis of the paper so far has been based on a simplified assumption, which is usually made in the literature: if both sides invest in war, I assumed that there actually is going to be a military conflict. The problem with this approach in the setting of this paper is that, if military expenditures are at least partly motivated by the internal considerations, it is hard to believe that the government will automatically use the accumulated military potential for warfare. This issue does not exist in the model of the previous section by construction. To show that, let me assume that the government makes two decisions: first, the size of expenditures on armed forces, and second, whether it attacks or not. It is easy to see that if investments are positive, the attack strictly dominates the decision to withdraw from conflict (since the expenditures

have already been made, and the only difference is whether there is a chance to obtain the prize or not). Obviously in the reality the situation is different: first, war always means additional expenditures (even if the army is well trained and prepared to fight), and second, there is not only a prize to win, but there may be transfers from the losing party to the winner. In this section I will examine the second option, assuming that there are going to be transfers to the winner if the war takes place. One can roughly make the distinction between "small" and "large" wars: a small local conflict usually does not imply significant costs for the economy in general and also does not require countries to make serious concessions - the conflict influences only the relatively small prize. However, in case of a large war the costs will be overwhelming for the whole country.

Reference model with large war. In order to model a large war, assume that there is not a prize V, but a transfer, such that V can be obtained in case of victory, but has to be paid in case of defeat. In this case the payoffs of two countries without domestic conflict (the reference case) can be written as

$$\pi_a = (1 - p^w)V - p^w V + \beta_a(R - m_a) \tag{17}$$

$$\pi_b = p^w V - (1 - p^w)V + \beta_b(R - m_b) \tag{18}$$

The optimal choice of B is either

$$m_b = \sqrt{\frac{2V}{\beta_b} m_a} - m_a \tag{19}$$

for $m_a < 2V/\beta_b$ or zero otherwise (since underinvesting is costly, the threshold for peaceful settlement is higher). However, computing first derivative of π_a and evaluating it at the threshold m_a still yields the same conditions as in case without transfers for peaceful settlement. To conclude, war is possible if $\frac{\beta_a}{\beta_b} \in (.5, 2)$ or the investments guaranteeing the surrender of the opponent are unfeasible (what I will, as above, ignore for simplicity).

Reference model with large war and choice of military engagement. In the next step let me introduce an additional choice variable: let the choice set of both countries be $\{A_i; S_i\} \times m_i$, where A_i stands for "attack" for the country i and S_i for "surrender". The war happens if both countries choose A_i, otherwise one of the countries is assumed to surrender and has to pay the transfer V with certainty. Moreover, assume that no war is preferred to a war with zero revenue; this assumption is reasonable if one thinks of additional costs of the war which can occur. In case both countries surrender simultaneously, they both receive zero revenue from war. The game has the following timing: first, both countries simultaneously decide on m_a and m_b; second, both countries simultaneously select $\{A_i; S_i\}$.

Solve the game by backward induction and start the second stage. From the conflict success function it follows that the country i receives a positive payoff

only if $m_i > m_{-i}$. Otherwise payoff is negative. Hence, for any combination of $m_a \neq m_b$ at least one country will prefer to attack. Does it mean that the other country prefers to surrender? One can easily see that surrender is never an equilibrium, if the country made non-zero investments in army, since under this condition investments are wasteful. Thus, in stage 2 there is no war either if one of the countries made zero investments in the first stage, or if investments of both countries were equal to each other. Basically, the introduction of the additional choice of $\{A_i; S_i\}$ does not change the payoffs of the parties for given m_i combination: for investments in m_i equal to zero payoff is always equal to $-V$, regardless of whether A_i or S_i will be chosen in the subsequent stage; and the unique case when both parties surrender (obviously, investing zero in army and simultaneously surrendering for both sides is never an equilibrium) is when $m_a = m_b$; however, if both countries went to war under these conditions, they would still have the same payoff from the military confrontation.

Now consider the first stage of the game. Since payoffs for given m_i are the same regardless of the choice of A_i or S_i, I can use the same reaction curves as in the reference case for large war above. First, consider the case when both countries make equal investments in army. I can find the condition for the equilibrium $m_b = m_a$ by using the best responses from the system of three equations: $m_b = \sqrt{\frac{2V}{\beta_b} m_a} - m_a$; $m_a = \sqrt{\frac{2V}{\beta_a} m_b} - m_b$ and $m_a = m_b$. Then both sides make equal investments in military capacity and do not go to war only if $\beta_a = \beta_b = \beta$ (and then $m_a = m_b = \frac{V}{2\beta}$). This is indeed an equilibrium: if a country i decides to deviate from the equilibrium by investing less then $\frac{V}{2\beta}$, the opponent will attack and the payoff will decrease (because the expected revenue from war is negative); if a country i decides to invest more than $\frac{V}{2\beta}$, the payoff of war is positive, but it is smaller than the non-realized peacetime production. Moreover, analogeously to Proposition 1, one can show that zero investments are never made for $\frac{\beta_a}{\beta_b} \in (.5, 2)$.

To conclude, the only difference between this model and the previous one (where no option of military investments without warfare was given) is for identical countries, which have not just the same resources R, but also the same productivity β: in this case there is "perfect deterrence", since both sides create equal armies and do not go to war. Otherwise introducing the opportunity to "opt out" for positive investments in army does not change anything.

Country B ruled by a dictator. Now consider the case when the country B is ruled by a dictator. The interaction of the countries is determined through a sequence of three simultaneous games:

1. Both countries make investments, i.e. decisions on m_a and m_b
2. Both countries make decision on $\{A_i; S_i\}$
3. Dictator and population in country B make decisions on effort they put

in domestic revolution

Payoffs are realized after the domestic revolution. Once again, I solve the game through the backward induction.

1. The problem of stage 3 (domestic revolution) for given decisions on m_i and $\{A_i; S_i\}$ is almost identical to that of the analysis above of the previous section, with the exception of the introduction of a different payoff from war (because of the transfer setup), which is just a "constant" for the optimization problem of this game (as, once again, it has been in the previous section). Then the problems of the dictator and the population in country B for given military investments and choice of attack or surrender (set at the previous stage of the game) and their solutions are analogous to the previous section.

2. Consider then the second stage of the game. If $m_i > m_{-i}$, country i has incentives to attack. Once again, surrender is never optimal if opponent attacks and own military investments are different from zero. Payoff from war is equal to zero only if $m_a = m_b$, and therefore this stage of the game is not affected by the existence of the dictator. Once again, in equilibrium if country i has made military investments different from m_{-i} and not equal to zero, it will not surrender. Once again, as in the previous case, the choices of stage 2 do not affect the payoffs for given m_b,

3. Finally, consider the first stage of the game. The analysis of zero investments in war is not really interesting, since it is very similar to that performed in the previous section: once again, for very high political bias or very high complementarity dictator will never decide to abolish the army and to surrender. A more interesting point is the "balance-of-power" case, when armies' strength for both countries is the same - this case was absent in the "small war" scenario. The reaction function of A did not change, and thus one can show that A sets $m_a = m_b$ if $m_b = \frac{V}{2\beta_a}$. Consider, once again, completely symmetric countries with identical peacetime productivity β, to compare the results with and without the dictator. The best response of the dictator is given by:

$$\frac{\theta'(m_b)}{4}(d\beta(R-m_b)+d^w\frac{m_b-ma}{m_b+m_a}V)+\frac{\theta(m_b)}{4}(-d\beta+d^wV\frac{2m_a}{(m_a+m_b)^2})=0 \tag{20}$$

if $\theta(m_b) < 2$ and

$$\frac{\theta'(m_b)}{\theta^2(m_b)}(d\beta(R-m_b)+d^wp^w\frac{m_b-ma}{m_b+m_a}V)+\frac{\theta(m_b)-1}{\theta(m_b)}(-d\beta+d^wV\frac{2m_a}{(m_a+m_b)^2})=0 \tag{21}$$

otherwise. For $m_b = m_a = \frac{V}{2\beta}$ the conditions can be re-written as:

$$\frac{\theta'(\frac{V}{2\beta})}{4}(d\beta(R-\frac{V}{2\beta}))+\frac{\theta(\frac{V}{2\beta})}{4}(-d\beta+d^w\beta)=0 \tag{22}$$

if $\theta(\frac{V}{2\beta}) < 2$ and

$$\frac{\theta'(\frac{V}{2\beta})}{\theta^2(\frac{V}{2\beta})}(d\beta(R - \frac{V}{2\beta})) + \frac{\theta(\frac{V}{2\beta}) - 1}{\theta(\frac{V}{2\beta})}(-d\beta + d^w\beta) = 0 \tag{23}$$

otherwise. For both equations the first term is strictly positive, and the second term is positive if $d^w > d$, thus, equalities do not hold and the military investments of B exceed $\frac{V}{2\beta_b}$ in case of perfectly symmetric countries and political bias exceeding unity.

The presence of the "greedy dictator" prevents the "perfect deterrence" for two identical countries: intuitively, the dictator will "over-invest" in army, because his share from the spoils of war is larger than that from the peacetime production, and therefore the war still happens. Anyway, the general conclusion seems to be that introduction of a "large war" does not change much: the only result where equilibrium is indeed very different is that for equal countries, if there is no dictatorship ruling them.

There is, however, a further issue of interest if one introduces the possibility of a large war for a country ruled by the dictator. It is clear that the conditions derived by Proposition 2 to ensure non-zero investments in war also change. For example, political bias ensuring the non-zero military investments is now

$$b^{**} = \frac{\beta_b R}{V}\frac{1 - \theta'(0)R}{2 - \theta'(0)R} \tag{24}$$

which for $\theta'(0)R > 2$ is larger than in the Proposition 2. The reasons are straightforward: now the dictator has also more to loose in case of war, and even benefits from being stronger in internal revolution do not offset the costs of military defeat. In case of surrender the revenue from internal revolution is just zero; however, if the war takes place, it can be negative. Once again, this observation is not completely weird, if one thinks of real-world experience: the dictators can just prefer to surrender and escape from their country, rather than to fight a war (of course, if one sets the payoff from defeat in the revolution to be $-\infty$ (if the dictator is caught and executed) at least with some positive probability, things do change) - as a matter of fact, the case of Uganda-Tanzania war I discuss briefly in the next section is very instructive from this point of view.

Unilateral aggression. Finally, consider a variation of the initial model. Assume country A is unable to attack, so its investments are purely defensive in the sense of Grossman and Kim (1995). Now war happens only if B chooses to attack; if B chooses not to go to war, no war is possible and gain of both countries from the military conflict is zero. There are two situations when this is indeed the case. First, it is possible that country A's political system includes numerous veto-players preventing it from developing an unambiguous foreign policy agenda, or that the war is somehow unacceptable for the majority of the population (for example, because it associates extremely high costs with

the military confrontation). The actions of the UK, France and the US before World War II provide a good example of extreme reluctance of going to war or the presence of domestic political constraints preventing the first strike (even at the very early stage of the war, when the military advantage against Germany or Japan seemed realistic). Multiple veto-players may occur also in case of multiple countries participating in a coalition, which then requires unanimous support to start a war. Second, it is possible that the resources of A are different for an aggressive and for a defensive war - if, for instance, they are supplied by foreign allies. For example, Taiwan can count on the US support in case of a defensive war against China, but not necessarily in case of a military aggression - while it is the US support which makes out the decisive advantage of the island in any military confrontation with the mainland China.

Once again, solve the game by backward induction. The third stage does not change. There is a significant change in the second stage: B will attack only if $m_b > m_a$ in the first stage. However, the most interesting changes are in the first stage. If B were not ruled by the dictator, in case it does not attack it makes zero investments in war effort. However, if B is ruled by the dictator, the situation is different: now B may choose non-zero investments even if it does decide not to go to war. It happens, specifically, if for zero revenues from war zero investments in military effort are suboptimal *and* equilibrium investments will guarantee a larger response from A. Given the reaction function of A in the first-stage game shown above, $m_b < m_a$ if $m_b < \frac{V}{2\beta_a}$. On the other hand, for zero revenue from war and $\theta(m_b) < 2$ profits of the dictator are given by $\theta(m_b)(d\beta_b(R - m_b))$. Then, from the first order condition evaluated at 0, country B never attacks, but makes positive investments in war if $\theta'(0) > \frac{1}{R}$ and the optimal $m_b < \frac{V}{2\beta_a}$. It means that if the first derivative of the complementarity function is large enough, country A creates a purely wasteful army, which never goes to war.

5 Examples

5.1 Army and domestic repressions

The major novelty of the model presented above is that investments in army to be devoted to an external war and the strength of the ruler vis-a-vis the domestic rebellion can be complementary. In what follows I will provide several examples of the instances similar to those described in the main model of the paper in terms of timing and outcomes. I am looking at dictatorships, which increased the strength of their military in order to combat rebellion and to start an international war, however, lost their "small victorious wars" and afterwards actively used the army in the domestic struggle. As I will show, this exact scenario has indeed been observed several times in non-democratic political systems, although these cases, to my knowledge, have so far been rarely discussed in the literature.

Syria: Yom Kippur War and Hamah rebellion. Hafez al-Asad's accession to power in Syria during the "Corrective Revolution" of 1970 was strongly supported by the military. Although the new regime refrained from establishing a pure military dictatorship, Asad continuously expanded the army, relying on the military strength in both domestic and international matters. Between 1967 and 1973 the military forces of Syria increased from 50,000 to 170,000 men and were re-equipped using i.a. the Soviet support and help from other Arab countries. Hence, Syria was among the most aggressive members of the coalition launching an attack on Israel in 1973 (the Yom Kippur War), hoping to retake the Golan Heights through purely military means. Although the early stage of the war was characterized by the relative success of the armies of the Arab countries, the Yom Kippur war ended by a serious military defeat of this coalition. In addition, the war triggered the chain of events resulting in the signing of the Israeli-Egyptian peace agreement in 1979, thus preventing the formation of a stable alliance of Arab countries Asad preferred. All in all, the Yom Kippur war resulted in serious weakening of Syria's position in the region and thus can be treated as yet another example of the lost "small victorious wars".

However, in the aftermath of the war Syrian army was much more successfully involved in fighting the domestic rebellion in 1976-1981, initiated by the Muslim Brotherhood. Although the struggle between the Brotherhood and the ruling Ba'ath party can be attributed already to the 1960s, the conflict escalated significantly in the second half of the 1970s. The revolt posed a serious threat to the Syrian leadership, so that in June 1980 Asad himself barely escaped death after a terrorist attack. However, the revolt was put to an end through active use of the Syrian army against the opposition to the regime, with the most prominent example of 1982 Hamah massacre. After the Brotherhood took control of the city of Hamah in the northern part of the country, the Syrian army besieged it for 27 days and used artillery, tanks and combat helicopters to restore governmental control, resulting in significant causalities among the civilian population. Some of the 12,000 men troops were withdrawn from Lebanon. Thus, the strengthening of the military forces, although not successful in the war against the main external opponent of the Asad regime (Israel), demonstrated its relative efficiency in suppressing the internal threat to the government represented by the Muslim Brotherhood (Zisser 2001; Brownlee, 2002; Wiedl 2006). The timing and the structure of events seem to be almost identical to the model of this paper.

Jordan: Six Day War and Black September. Jordan faced a somewhat similar situation to Syria a decade earlier. As Syria, Jordan was a member of the coalition of Arab states in the Six Day War in 1967, which, as the Yom Kippur war, ended with a victory of Israel. There are of course, several differences in this case. To start with, unlike the Yom Kippur war, the Six Day War was started with the Israeli pre-emptive attack (against Egypt - but not against Jordan). However, one can claim that the Arab countries at least did plan an

offensive against Israel, thus the case is still relevant for the discussion of this paper (I will describe a similar situation in the next subsection discussing the Russo-Japanese war). It is also the case that Jordan, unlike Syria, has been rather a reluctant member of the coalition, although ultimately deciding to join Egypt. However, the decision to attack Israel, even if based on misinterpretation of the early results of combat between Israel and Egypt, was made by the government of Jordan, also taking domestic political considerations into account (for Jordan they were probably even more pronounced than for Syria: military success over Israel was crucial for the nation-building project pursued by the leadership of the country at that moment). Hence, with certain caveats, the Six Day War fits the "small victorious wars" profile, and, as the "small victorious wars" we are interested in, was lost the autocratic government.

And, as Syria, Jordan faced a threat of domestic uprising almost immediately after the end of the war. In this case the key reason for the domestic political disturbance was the situation in the Palestinian refugee camps, where autonomous political and military institutions began to emerge. The activity of Palestinian organizations in Jordan, as of the Muslim Brotherhood in Syria, has been a serious problem for the government already for at least a decade (since early 1960s), though the primary concern was not internal (the threat to the regime) but the external (the measures implemented by the Palestinian organizations against Israel were out of control of the Jordanian government and therefore problematic for the country's foreign policy). After the war Palestinian resistance movements, however, turned into an issue in the domestic affairs as well. After Karameh Operation in 1968 their prestige in the country increased significantly; they received substantial internal and international support. Palestinian organizations claimed quasi-fiscal and regulatory authority in the camps, and the left wing of the resistance movements criticized the monarchy in Jordan. All in all, after the assassination attempts on the king and the planes hijackings in September 1970, the martial law was declared and the military forces were used to attack the camps and the headquarters of Palestinian organizations in Amman. Jordainan army became the key tool of fighting off the Palestinian organizations, which went on until mid 1971, when the latter were expelled from Jordan and king Hussein successfully restored his power (Fruchter-Ronen 2008; Nevo 2008).

In case of Jordan the strategic motive behind the establishing the army for the purpose of the domestic conflict is not as clear as in the case of Syria: the spread of the Palestinian refugee camps and the emergence of alternative power centers in Jordan could have been avoided if the fight against the "external" enemy were more successful. However, somewhat more successful (but not decisively victorious - which was, however, neither planned nor intended by Jordan) war against Israel could as well just strengthen the Palestinian organizations without removing the camps from Jordan, thus even encouraging the threat to the king. Anyway, this conjecture remains purely speculative. In addition, one could consider the Palestinian rebels an "external" threat for Jordan as

well - this claim, however, is likely to be inconsistent with the ideology of the country itself, during the period in question rather intending to represent the Palestinians as part of its own nation. It is thus not surprising that the events of 1970-1971 are often perceived as a "civil war". Anyway, the key lesson from the Black September conflict is that military forces unsuccessful in fighting the "true" external enemy were quite instrumental in suppressing rebellion at home.

Further examples. While the examples so far closely followed the model presented in the main part of the paper, several wars demonstrate a somewhat similar pattern, confirming the main message of this paper (complementarity between fight against the rebel and international war), but either deviate from the timing of the model or provide somewhat inconclusive results regarding the efficiency of the fight against the rebels at home. I start by considering two further "small victorious wars". The fact that army has been created in order to fight primarily the domestic opponents rather than foreign nations can be clearly demonstrated by the case of Idi Amin regime in Uganda. The very reason for the start of the Uganda-Tanzania war in 1978-1979 was associated with the regime's attempt to combat the rebellion, with some of the rebels fleeing across the Tanzanian border.[11] However, unlike the wars discussed above, the Uganda-Tanzania conflict resulted in the deposition of Amin - thus the military was ultimately unsuccessful in the fight against both domestic and foreign opponents.

Similar situation was observed in Somalia after the Ogaden War against Ethiopia of 1977-1978. The Ogaden War fits the pattern of the "small victorious wars" I described so far, although in this case the key reason for the Somalian defeat was the ultimate decision of the USSR to withdraw its support (the Soviet Union happened to be allied with both sides of the conflict). Immediately after the war the Somalian military has been used in suppressing the rebellion against the regime in 1978. This time the initial involvement of troops has been successful; however, the massacre of the Majeerteen clan seems to have unraveled the chain of events, which several years later resulted in the deposition of the regime of Siad Barre (although probably delayed it by half a decade). Hence, in both cases the army, although used against domestic opponents (and probably designed to be used against domestic opponents), was ultimately not successful - however, in both cases countries used this army to wage external wars.[12]

[11] Starting war with foreigners because of the fight of domestic opposition is by far not a unique feature of this case: for example, the relations between Venezuela, Ecuador and Colombia in the last decades have been severely influenced by the military operations against rebels, sometimes crossing national borders.

[12] One should bear in mind though, that the success of the conflict (especially the rebellion) observed in the real world is of binary nature (either win or loose). The model, however, makes predictions based on expected payoffs of the conflicts. So, ultimately there is no contradiction to the model of this paper. One should, however, acknowledge yet another common feature of two conflicts described above, which is different from the model presented in this paper: that the rebellion was also initiated by the military, which, however, has been combated by the troops loyal to the dictators. In Syria and Jordan, however, military has been mostly loyal to

Finally, other non-democratic regimes also applied their strong army against domestic protests, although in this case a link between international wars and domestic conflicts is less pronounced. The use of army against the domestic opposition seems to be rather rule than an exception in Africa (see Herbst 2004). The Chinese People's Liberation Army performance in the post-World War II military conflicts has been mixed: it was successful in the Sino-Indian war in 1962 and Korean War in 1950, but ultimately unsuccessful in the Sino-Vietnamese war in 1979 and in the border disputes with the Soviet Union in 1969. On the other hand, military involvement was a key element of suppressing the Tiananmen Square protests in 1989 - thus, on the domestic front army actually performed better than on the international one. However, this case is somewhat less interesting: while it demonstrates that (given proper governmental control) army can be used as the ultimate argument in both domestic and international disputes, there was no unambiguous "small victorious war" (ultimately lost by the dictator) present - and therefore the case is of lesser importance for this paper.

5.2 Diversionary war

As we have discussed above, the model can be applied in a somewhat more general setting, also analyzing the diversionary wars, where governments initiate an international conflict in order to gain internal political support. In this case, however, some re-definition of parameters is required. Assume, for instance, that the "rebellion" is initiated not by the population, but by a different political group ("revolutionary party" or "civil rights movement"), which requires public support for its success. Assume further that m_i represents not (only) the quantitative increase of the military forces and equipment, but also the investments in propaganda, boosting morale and increasing the chances of military success. Then one can assume that there is once again a complementarity between propaganda investments directed to the military forces and the general population, which is then "diverted" from the support of the revolutionaries, which is reflected by the $\theta(.)$ in the conflict success function (6). However, the effects can be similar to those increasing the "quantitative" strengthening of the army - the well motivated forces should optimally, under the assumptions of a small victorious war, be used for international war as well.

While the diversionary wars have been well studied in the literature (unlike the case I have presented above), there is still at least one prediction of the model, which is of interest in this scenario: the complementarity between domestic and international conflicts and the political bias. In order to start the war, either political bias should be high enough (if the complementarity is not so obvious), or the complementarity should be significant (for small political bias). In what follows I will consider three examples of often cited "small victorious

the incumbent ruler.

wars" (see also Levy, 1988), which allow me to provide at least some anecdotal illustrations for the interplay of these two factors in the logic of aggression of non-democracies.

The existence of the complementarity between the international conflict and the domestic success of the dictatorships seems to be supported in a number of cases of potential "small victorious wars". The probably most famous example, the Falklands War of 1982 between Argentina and the UK, seems to be strongly influenced by the interests of the military and the "diversionary" motives (for a detailed discussion see Levy and Vakili, 1992, and Oakes, 2006). First, the war could reinforce the unity between individual groups within the military in Argentina and re-shape the balance between the factions of the military elite, which at that moment was ruling the country. Specifically, it seems to be the navy which was expecting an overproportional benefit from the war and could probably shift the army as the traditionally dominant service. It is even claimed that the support that Leopoldo Galtieri, the head of the military government since 1981, received from the navy was conditional on the involvement in Falklands. Second, Argentina experienced an increasing pressure from the domestic civilian opposition requesting return to the democracy and changes in the economic policy-making. Some examples include scheduled strikes organized by the General Confederation of Labor and the multi-party coalition (Multipartidaria) established to support reforms. It is claimed that Galtieri expected to combine the "successful" war with general elections, which aimed to "transfer" the military victory into the domestic success. To conclude, the complementarity between the domestic and the international war seems to be relatively high.

In the same way, the first war in Chechnya in Russia of 1994-1996 also seems to be motivated by the political instability of the central Russian government.[13] Once again, two factors contributed to the emergence of war. On the one hand, the parliament elections of 1993 demonstrated an enormous popularity of the nationalist movements among the Russian population; the administration of president Boris Yeltsin faced the accelerating loss of public confidence due to the hardships of the economic crisis and the deep transition recession in Russia. So, a "hardliner" position for Chechnya would allow Yeltsin to position himself as a proponent of a "stronger" Russia and partly beat the potential nationalist opposition on its own territory, as well as increase the public support of his government (see e.g. Bowker, 2004). Second, however, the war in Chechnya was associated with an attempt to prevent further secessions of other regions, or at least further increase of their autonomist aspirations, which at that moment were considered at least to be possible (though in the final account the secession threats turned out to be non-credible and even the claims of higher autonomy were never institutionalized and easily removed by the central government under Vladimir Putin, see Libman, 2009). In the same way, the second Chechen

[13] Although de-jure the conflict represents an internal war between the central government and one of the regions, de-facto it seems to fit the logic of this paper quite well.

war in 1999-2000 had clear implications for the electoral success of Vladimir Putin, who was "appointed" the Yeltsin's successor exactly during this period and immediately concentrated on solving the problem of Chechnya. This war constitutes, by the way, one of the examples when "small victorious wars" indeed turned out to be victorious - at least in the short run. Putin's success in the presidential elections of 2000 is often clearly attributed to the military success in the Caucasus (see McFaul, 2000).

The Russo-Japanese war of 1904-1905, though an also often-cited example of the diversionary wars initiated by autocracies (specifically, by the Russian Empire), is not so easy to interpret. To start with, there is no clear conclusion as to who actually started the war (the first military strikes seem to have been implemented by the Japanese, but since no international law on the official declaration of wars existed at that moment, the interpretation of individual actions is not so simple), and what is even more important, which intentions did the Russian political leadership actually have. On the one hand, the imperial government of Nicholas II expresses a clear and significant interest in the expansion in the East Asia, which is claimed to be particularly promoted by the emperor himself - he is sometimes described to have developed an "obsession" with the region (Geller, 1997). Hence, the clash of interests between Japan and Russia was inevitable due to the purely external political logic. On the other hand, Russia indeed experienced a strong increase of the revolutionary activity in the first half of the 20 century associated with the spread of the Social Democratic ideology and the emergence of the Socialist Revolutionary movement. From that point of view a war against Japan could come in handy. Even in this case it is questionable whether the war could increase the popularity of the government - whether it was successful or not .[14] Finally, there seems to be conflicting evidence whether the Russian government indeed was interested in the war or simply did not put enough effort to avoid it due to the misperception of the Japan's ability to attack (Perrie, 2000). Anyway, at least for some groups of the Russian political elite the diversionary war logic seemed to be highly attractive (see Geller, 1997; Airapetov, 2004).

Hence, the internal complementarity between the international and the domestic conflicts was either straightforward (Chechen war, Falklands war) or at least possible and expected by some factions of the elites (Russo-Japanese war). In the next step let me consider the political bias. The model predicts, that exactly in the case with the most questionable complementarity one should observe the

[14]There is some anecdotal evidence that the intellectual part of the society was extremely critical towards the war from the very beginning: for instance, there have been cases of enthusiastic greetings to the Japanese prisoners of war in the European provinces of Russia by the local population. On the other hand, there have been several cases of patriotic manifestations of students in St. Petersburg in the beginning of the war, so the assessment of the perception of war is once again inconclusive. As for the uneducated masses of the Russian rural population, the only group indeed showing some support towards the war were the Siberian peasants, expecting the access to the land resources of the newly conquered territories, see Airapetov, 2004.

highest political bias, given there was indeed a war started. As for the Falklands islands, the economic gains from the war can be considered negligible: basically, the islands are claimed to be potentially attractive from the point of view of the oil deposits, but their commercial value is yet to be determined even as of 2010. Chechnya does have well-recognized oil deposits, and also possesses strategically important location from the point of view of the pipeline routs, but during the Yeltsin rule the central government had extremely limited control over both private companies dominating in the oil extraction and, what is even more important, state-owned corporations like Gazprom and Rosneft, which basically were run almost independently by their management with the support of particular groups in the elite. So, the rents to be obtained from the Chechen war are also questionable. The situation is different for the Russo-Japanese war. In the case of the Far East expansion of the Russian Empire, there have been significant rents, at least for some groups of the elite involved in the exploration of economic resources of Manchuria and Korea, where large economic projects were initiated. In this case it is also likely that for these groups of the elite (associated with the emperor himself) the gains from exploring these projects were significantly larger than those from investing resources in the peacetime production. It is a different problem though that Far East was not the only priority for the Russian foreign politics,[15] so that even in this case the political bias could not have been extremely high - but at least it seems to be larger than in two other cases considered in this section.

To conclude, political bias and complementarity effects indeed seem to be present in all three cases. If the complementarity effect is high enough, even a very low political bias suffices to start the war. For lower complementarity effects political bias required, as predicted by the model, should be higher. In addition, however, one should notice that in both Russian cases (Chechen war and Japanese war), the political bias, either relatively high or relatively low, seems to never reach the level sufficient to generate military investments ensuring the unilateral surrender of the opponent - once again, as discussed in the model. In both cases the initial military forces mobilized for the war were rather limited as compared to the overall military potential of the country (although gradually increased). In the case of the Russo-Japanese war the initial stage of the conflict was indeed clearly focused on the attempts to wait until the reinforcements from the European part of Russia arrive (though this strategy turned out to be disastrous from the military point of view). On the other hand, the second Chechen

[15] In Russia this multitude of goals reflected itself in a multitude of decision-making centers involved in development the overall strategy of the country and the political decisions regarding Far East in particular. To provide one example, when the Russian military minister A. Kuropatkin was informed about the re-deployment of troops in Korea in 1903, he was surprised to find out that any Russian troops were located in the region at all. There was a permanent competition between different directions of policy - Port-Artur and Dalny harbors, Manchuria and Korea as main priorities etc., ee Airapetov 2004. One should notice that this multiplicity of goals seems to be present in almost all cases of military conflicts between large and small states (Mack, 1975).

war with an extremely high level of complementarity[16] was also associated with significant military investments for this conflict - finally, ensuring at least the short-term military victory.[17]

6 Conclusion

This paper examined the problem of a "small victorious war", looking at two factors potentially influencing the military expenditures of the countries: degree of complementarity between external security and internal peace and political bias of the dictator. First, it shows that in case of a conflict for an exogenous prize, regimes with high complementarity and with high political bias tend to overinvest in military effort, even if the expected payoff for the entire nation goes down (and thus the war is inefficient from the point of view of the overall rent obtained by the nation). Of course, the investments may become so high, that the opponent simply surrenders, and there is once again no war (and it is more likely to be the case for a dictator with a very high political bias effect). In some cases, a fully rational and well-informed dictator will start the "small victorious wars" she is doomed to lose, although without the presence of the political bias and complementarity this war would not have been started. The anecdotal evidence, first, provides examples of (more or less) successful application of the army against the domestic enemy after the unsuccessful international war, and of the interplay between the political bias and the complementarity effect in terms of the initiation of war. Finally, in case of a large war the model shows that it is possible to observe the development of large military forces, which, however, are purely wasteful and never engage the external enemy, being created exclusively for the internal purposes - if the aggression, however, is unilateral.

The study acknowledges its limitations. In particular, even in the empirical cases discussed in the paper (both of diversionary wars and application of military against the domestic rebellion after lost wars), the countries seem to make decisions for the "small victorious war" scenario, in fact being in the "large war" world. As I have stated in the introduction, following the approach of the economics of conflict, this paper developed a model, where inefficient overinvestments in the military capacity happen without any informational asymmetries. However, information and misperceptions can matter in terms of distinguishing between the small and the large war: from that point of view an attempt to start a "small victorious war" in a real world is very likely to be associated with significant miscalculations of the opponent's strength, ability

[16] The public perception of the importance of victory was much higher after the terrorist attacks in Moscow in 1999 than in 1994, when Chechnya constituted just one, and probably not the most important, example of autonomist aspirations of the regions, and seemed to be perceived by the population as a problem less important than, for example, the ongoing economic crisis.

[17] The long-term stability in the region of Northern Caucasus and Chechnya is, of course, still questionable.

and readiness to fight (see also Fearon, 1995). Nevertheless, although the issue of information is of course highly relevant for explaining the causes of war, it still seems to be important to show that the inefficient military investments in the "small victorious wars" can be predicted even in an extremely simple model with full information.

7 Literature

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8 Appendix

Proof of Proposition 1: (i) Since the first order condition for the country B is

$$\frac{\partial \pi_b}{\partial m_b} = \frac{m_a V}{(m_b + m_a)^2} - \beta_b = 0 \tag{25}$$

one can immediately find that the country B sets its investments in warfare equal to

$$m_b = \sqrt{\frac{V}{\beta_b} m_a} - m_a \tag{26}$$

if $m_a < \frac{V}{\beta_b}$ and zero otherwise. The derivative of the payoff of country A given the reaction of country B is

$$\frac{\partial \pi_a}{\partial m_a} = \frac{1}{2}\sqrt{\frac{\beta_b V}{m_a}} - \beta_a \tag{27}$$

Evaluating the derivative at $m_a = \frac{V}{\beta_b}$ one can see that it is non-negative if and only if $\frac{\beta_a}{\beta_b} \leq \frac{1}{2}$. Hence, if this condition holds, the country A in equilibrium chooses the level of military expenditures such that the country B invests zero in warfare and "surrenders". Symmetrically, if $\frac{\beta_a}{\beta_b} \geq 2$, country B in equilibrium chooses the level of military spending ensuring the surrender of country A. The conflict therefore takes place if $\frac{\beta_a}{\beta_b} \in (.5, 2)$. One should bear in mind that the level of $m_i = \frac{V}{\beta_j}$, $i \neq j$ may exceed countries resources. It is true, for instance, if $\frac{V}{\beta_i} > R$. If this inequality holds for $i = a$, the lower bound of the "peaceful interval" disappears, since A can never invest sufficient amount of resources to ensure surrender of B. Obviously, for $i = b$ the opposite holds, and the proof is complete.

Furthermore, one has to examine the case when both sides invest zero in warfare. According to (26), optimal response for opponents zero investments is also zero, and hence, the probability of winning ought to be .5. However, (0; 0) is never a Nash equilibrium: assume that the opponent invests zero effort in war - than by investing an infinitely small amount of effort in war one wins with probability 1, and hence, for any positive V there is an incentive to deviate from (0; 0). It is therefore also not considered further.

(ii) Consider equation (25) for $m_b = 0$. Then the first derivative is larger 0 (and hence, non-zero investment in war for B is optimal) if $V > \beta_b m_a$. But $m_a \leq R$ by definition, and therefore if $V > \beta_b R$, the condition holds always. The result for A is derived symmetrically.

Q.E.D.

Proof of Proposition 2: (i) Consider the reaction curve (13) given $m_b = 0$. Then one has to check the condition

$$\theta'(0) d\beta_b R + \frac{d^w V}{m_a} - d\beta_b > 0 \tag{28}$$

(reaction curve (14) is irrelevant, since it is calculated at $\theta(m_b) > 2$, while by assumption $\theta(0) = 1$). Substitute $m_a = \frac{V}{\beta_b}$ (condition ensuring surrender in the reference case). Then one obtains $\theta'(0)d\beta_b R + \frac{d^w V \beta_b}{V} - d\beta_b = \theta'(0)d\beta_b R + \beta_b(d^w - d) > 0$ (the first term is positive, the second is positive since $d^w > d$. For smaller m_a the second (positive) term of (28) is larger, while other terms do not change, and hence the expression is also positive. Thus, investing zero in external warfare is never optimal.

(ii) Consider equation (13) and compare it with the first-order condition (25). One can see that if $d^w = d$ an m_b setting (25) to be equal to zero makes the second term of (13) equal to zero. However, the first term of (13) is still positive, and hence, the root of (13) should be larger than that of (25). For (14) the result can be established analogeously. Now, if $d^w > d$, for $m_b = \sqrt{\frac{V m_a}{\beta_b}} - m_a$ the second term of (13) (and (14)) is positive, while the first term is positive anyway. Therefore, investments optimal in the reference case are suboptimally low for the case of a dictator.

(iii) Now assume $m_a = R$ (maximal investment possible for country A). Substituting it in the condition (28) and re-arranging terms one obtains the result of the Proposition.

(iv) In the same way, result follows from re-arranging terms in (28) for $m_a = R$.

Q.E.D.

Proof of Proposition 3: Substitute into (11) the optimal response of A $m_a = \sqrt{\frac{V m_b}{\beta_a}} - m_b$ and calculating the first order condition yielts

$$\theta'(m_b)[\beta_b(R - m_b) + \frac{d^w}{d}\sqrt{V\beta_a m_b}] + \theta(m_b)[-\beta_b + \frac{\frac{d^w}{d}\sqrt{V\beta_a}}{2\sqrt{m_b}}] = 0 \tag{29}$$

Assume the equation does have a root. The first term is always positive. The second term can be positive or negative. It is clear then that the equilibrium can exist only in the range where the second term is negative (and its absolute value is equal to that of the first term). Consider $m_b \leq \frac{V}{\beta_a}$ and analyze the sign of the second term for this range of m_b. For this purpose consider $-\beta_b + \frac{\frac{d^w}{d}\sqrt{V\beta_a}}{2\sqrt{V/\beta_a}} > 0$. The condition holds if $\frac{\beta_a}{\beta_b} \geq \frac{2}{\frac{d^w}{d}}$. If this condition does hold, for all $m_b < \frac{V}{\beta_a}$ the second term of the equation (29) is positive. Thus, the root of (29), if $\frac{\beta_a}{\beta_b} \geq \frac{2}{\frac{d^w}{d}}$, ought to be not smaller than $\frac{V}{\beta_a}$, and therefore ensure unilateral surrender of A. Considering (12) in the analogeuous manner yields identical results. If the first-order conditions do not have a root, B invests the maximal amount possible in warfare by assumption, and therefore the only case when surrender of A is not ensured is if the amount of investments necessary is not feasible. The proof is complete

Q.E.D.