A Single-Mindedness model with n generations

Emanuele, Canegrati
Catholic University of Sacred Heart

April 2007

A Single-minded model with n generations

Emanuele Canegrati
Catholic University of Sacred Heart; e-mail address: emanuele.canegrati@unicatt.it

April 3, 2007

Abstract

In this paper I will analyse the redistribution of income amongst n generations using the Single-mindedness Theory. I will introduce a new expression for the balanced-budget constraint, no longer based on lump-sum transfers as in the traditional literature, but rather on more realistic labour income taxation. Since the Government has to clear the budget, some generations obtain a benefit, whilst some other must pay the entire cost of social secutiry systems. I will demonstrate that generations which are more single-minded on leisure are the most better off since they are more able to capture politicians in the political competition. Furthermore, it could be the case that candidates are not forced to undertake the same policies in equilibrium and I will demonstrate that this result holds only once an endogenous density function for individual preferences for politicians is considered.

1 Introduction

The effects of income redistribution amongst different social groups or generations has always been one of the most important topics in the Public Economics. The redistribution of income does not involve only considerations on efficiency; also equity must be considerated in the evaluating the goodness of public policies. Every time a Government passes the budget law, economic analysts and journalists try to assess which groups are better off and which ones worse off. Since in the economic science the concept of equity is very close to the concept of fairness and justice, it seems universally accepted that budget laws should favour the most disadvantaged groups in society and disfavour the more advantaged. Especially the vertical equity is associated with the idea that people with a greater ability to pay taxes should pay more. Unfortunatelly, this is only what a normative approach would suggest. Once we take political economy considerations into account we realize that most of the time, politicians who only aim to win elections tailor public policies to the most powerful and influencial social groups of society. As a consequence, it could be perfectly possible that the strongest groups are the rich, who usually have more *voice*. The redistribution of wealth takes place via a redistribution of resources amongst social groups; usually the economic literature has considered lump-sum transfers (see Mulligan & Sala-i-Martin [ ], Lindbeck & Weibull); nevertheless, this redistribution mechanism does not reflect reality where, almost always, social security programs are financed via labour income (payroll) taxation (see Diamond [ ]). Thus, a new kind of budget must be considered, which is based on distortive taxation. Of course a distortive taxation on labour income produces a distortion of individuals' choices on labour supply, which is also an imperfection for the labour market. This is for sure a market failure that model based on lump-sum transfers are not able to catch.

Things worsen once we consider that there is also a political failure that politicians have to face, which refers to the necessity of favour the strongest groups in society which prevents the Government to undertake fair policies.

In this paper I will unify both the market and the political failure in one model.

The paper is organized as follows: section 1 Introduces, section 2 describes the new redistribution scheme I am going to introduce with respect to the previous literature, section 3 describes the basic model, section 4 states conditions for the existence and uniqueness of an equilibrium, section 5 solve the model and section 6 concludes.

2 A new redistribution scheme

The core idea of this model is to explain how the early retirement phenomenon derives from features of social security systems. Nevertheless, social security systems are not something of exogenously given but are based on passed laws and, more in general, on the political process. If we consider Leviathan politicians, we must admit that political choices about the social security systems reflect the willingness and preferences of the electorate. Furthermore, in society no every group has the same power to capture politicians; there exist some groups, the single-minded groups, which due to their ability to be focused on a very few number of issues are able more than others to influence political candidates. In this model I will assume that social groups are generations and the issue they are focuses on is leisure. Hence, workers are perfectly able to choose their labour supply which is a peculiarity which differentiates my model from previous models such as Lindbeck & Weibull's (1987), which do not allow workers for choosing the amount of work and leisure. But preventing this possibility, the early retirement phenomenon cannot be explained, since the fixed labour supply hypothesis contraddicts the early retirement. Thus a model with flexible labour supply, such as in Mulligan and Sala-i-Martin (1999) and Profeta (2002) is required.

This model is also willing to bring something new for what concerns the redistribution scheme. Mulligan and Sala-i-Martin suggest a model where both the intra-group and the inter-group redistributions are feasible. The former takes place amongst members of the the same group and it is achieved via a labour income distortive taxation, whilst the latter takes place amongst different groups via lump-sum transfers. To justify the existence of intra-group transfer, although memebers of the same cohort are all alike, Mulligan and Sala-i-Martin affirm that in distorting decisions on labour supply, the elder, which aim to get more leisure to use a fraction of it in lobbying politicians, are able to solve a free-riding problem. This problem arises, since every old would like to have more leisure, but without an incentive to retire nobody voluntarely would retire. As a consequence, they are more than happy to auto-tax themselves. Otherwise, Lindbeck and Weibull do not allow for the possibility to redistribute resources amongst individuals of the same cohort. In both the models, inter-group redistribution takes place according to a lump-sum transfers. With respect to these two models I do not support the idea by Mulligan and Sala-i-Martin, since I do not find any sense in distorting individual decisions about labour supply, just in order to redistribute the same (distorted) resources amongst the same (all alike) individuals. I do not support the idea of inter-group transfer realized via lump-sum transfers neither, since it is simply unrealistic. In real world social security programs are financed mostly via labour income, or payroll, taxation, according to Diamond (1997). Table 1 syntetizes the main differences amongst the three models

TABLE 1 HERE

3 The basic model

I consider a model with n overlapping generations. Let $I = \{\tau - n + 1, ..., \tau\}$, where $\tau - n + 1$ denotes the elder generation and τ the younger generation. Each generation represents a fraction of the population equal to n^i, with $\sum_{i+1}^{n} n^i = 1$. n^i also represents the number of voters of generation i. I assume that there is no within-generation heterogeneity, which means that all the members within the same group are alike. Each individual maximizes a quasi-linear function:

$$U^i = C(c_t^i, \beta) + \psi^i G(l_t^i, \beta) \tag{1}$$

where C represents the actual value of the flow of consumption c_t^i discounted by a discount factor equal to $\beta = \frac{1}{1+r}$, and where G is a concave function ($G' > 0$, $G'' < 0$) which represents the actual value of the flow of leisure l_t^i, weighted by a parameter $\psi^i \in [0, 1]$, which denotes an idionsyncratic component representing preferences of individuals for leisure. This parameter is equal for every memeber within the same generation. An important assumption is that this parameter is increasing with respect to age; that is, the elder generations have a greater level of preferences for leisure than the younger generations. Thus, $\psi^{\tau-n+1} > ... > \psi^1$. This assumption is supported by both the economic theory and the empirical evidence, even though a unique consensus on this assumption still does not exist.[1]

The budget constraint which individuals must respect is given by

$$C(c_t^i, \beta) = B(w, l_t^i, \beta, a_t^i, \tau, \bar{t}) \tag{2}$$

where B represents the actual value of the labour income flow, given by the net-of-taxes wage rate $w(1-\widetilde{\tau})$, with $\widetilde{\tau} = \tau(1-a_t^i)$, where w is the wage rate and $\widetilde{\tau}$ is the effective marginal tax rate given by the nominal marginal tax rate τ and the tax credit a_t^i. Without loss of generality I will assume that the wage rate is the same for every generation and I will normalize it to the unity. Individuals have a labour supply equal to $\bar{t} - l_t^i$, where $\bar{t}$ represents the total endowment of time which must be divided between leisure and work. Hence, at time t, the labour income of a worker is given by:

$$w(1-\widetilde{\tau})(\bar{t} - l_t^i) \tag{3}$$

I consider also the existence of a political competition amongst two candidates, say D and R. To quote Buchanan, I will assume that candidates are Leviathans and only aim to maximize the probability to win elections or, equivalently, the number of votes. They have to choose an optimal vector of policies

$$\overrightarrow{q} = (a_t^{\tau-n+1}, ..., a_t^{\tau}) \tag{4}$$

Political equilibrium does not change over time, so that there are no differences in calculating the equilibrium in any period of time t. Furthermore, in maximizing the probability of winning elections, candidates must also respect a *balanced-budget constraint* in the $\mathbb{R}^n$ space, given by:

$$\sum_{i=1}^{n} n^i(1-\widetilde{\tau})(\bar{t} - l_t^i) = \overbrace{\sum_{i=1}^{n} \overbrace{n^i(1-(1-a_t^i))(\bar{t} - l_t^i)}^{tax\ revenue\ generation\ i}}^{Total\ tax\ revenues} \tag{5}$$

The probability of winning is given by a traditional Probabilistic Voting Model and, for candidate j, is equal to:

$$\pi^j = \frac{1}{2} + \frac{s}{d}\sum_{i=1}^{n} n^i s(l_t^i)\left(V^i(\overrightarrow{q}^j) - V^i(\overrightarrow{q}^{-j})\right) \tag{6}$$

where $\forall i$ the utility the worker gets from the policy chosen by canditate j, $V^i(\overrightarrow{q}^j)$ is given by the observable component represent by the Indirect Utility Function obtained by the resolution of the maximization problem of the individual, plus two further components which define the so-called *stochastic heterogeneity.* ξ^i represent an idiosyncratic variable which captures the preferences of individuals for different candidates; it is uniformly distributed on the

[1] For a complete review of the literature on individuals' preferences for leisure see Canegrati (2006)

interval $\left[-\frac{1}{2s(l^i)};\frac{1}{2s(l^i)}\right]$, with $E(\xi^i) = 0$ and density function equal to $s(l^i)$. Notice that, unlike the traditional PVMs, the density function is endogenous and monotonically increasing in the level of leisure, meaning that the broadness of the interval, and thus the tickness of the distribution is a function of leisure; the higher the leisure, the thicker the distribution or the more compact the group. This result captures the concept of single-mindedness: those generations which are more focused or prefer more a single issue (leisure in our case) are more compact in terms of distribution. In figure 1 it is represented an illustrative example with only three generations which differs for the level of leisure they get (High, Medium and Low). It can be seen that the distribution of the generation which get more leisure is ticker than the other two generations which get less leisure. From candidates point of view, those groups represent the greater threat, since, due to their particular distribution, they are politically powerful. In figure 2 I depicted the effects of a change in the policy vector chosen by a candidate. In order to evaluate whether a change in a policy is feasible, a candidate must count the number of voters is going to gain once the policy has changed and compare them with the number of voters is going to lose. Of course, he will be willing to change the policy if and only if the numeber of voters he gains is greater than the number of voters he loses. Suppose now that he is willing to adopt a policy which intends to favour the Medium leisure group. The question is: would the candidate undertake this policy? The answer is no, since even though the change in the policy enables him to gain voters from the Medium leisure group (figure 2.b), voters in both High leisure group and in Low leisure group would be worse off and then they would never vote him (Figure 2.a and 2.c). Since the sum of the number of lost voters is greater than the number of gained voters, a rational candidate will not undertake the policy. Notice that the greatest lost in voters is provided by the most single-minded group, that one which gets the greatest level of leisure. Hence, candidates realize that single-minded groups are the more powerful and the most able to influence the electoral outcome, making difficult to allow for the possibility to change a policy forgetting about these social groups.

The other variable ς represents the initial advantage of one of the two candidates; it is not idiosyncratic and it is again uniformly distributed on the interval $\left[-\frac{1}{2d};\frac{1}{2d}\right]$, with $E(\varsigma) = 0$ and density function equal to d.

A voter votes for candidate D if and only if $V^i(\overrightarrow{q}^D) > V^i(\overrightarrow{q}^R)$, votes for candiate R if and only if $V^i(\overrightarrow{q}^R) > V^i(\overrightarrow{q}^D)$ and toss a coin if $V^i(\overrightarrow{q}^R) = V^i(\overrightarrow{q}^D)$. Voters who found in the third case are defined as 'swing voters', since they do not have any particular preference for any candidate. A little change in the policy makes them swing from a candidate to another.

The timing of the game is as follows: in the first period candidates choose (and commit to) their optimal policy vectors; thus I do not take dynamic inconsistency problems into account. Furthermore, no cooperation between candidates is allowed. In the second period elections take place. In the first period all the workers of each generation choose their optimal leisure. I solve the game by backwar induction.

4 Existence and Uniqueness of an equilibrium

First of all I analyse the existence of an equlibrium. From this point of view is fundamental to consider the existence of uncertainty in the political competition.

Proposition 1 *With certainty, $j = 2$ and $i \geq 3$ an equilibrium does not exist.*

Proof. Suppose there exists no party bias and only one individual per generation. Then, for every proposed policy vector by candidate $-j$ $\overrightarrow{q}^{-j}$, candidate j may obtain $n-1$ votes by choosing a vector:

$$\overrightarrow{q}^{j} = \left\{ \overrightarrow{q}^{1-j} - \varepsilon, \overrightarrow{q}^{2-j} + \frac{\varepsilon}{n-1}, ..., \overrightarrow{q}^{n-j} + \frac{\varepsilon}{n-1} \right\}, \forall \varepsilon \in (0, \overrightarrow{q}^{1-j}) \qquad (7)$$

that is subtracting an amount ε from one generation (i.e.1) and equally redistribute this amount amongst the other $n-1$ generations which would be better off and thus they would vote for him. But this cannot be an equilibrium since candidate j may choose the following vector:

$$\overrightarrow{q}^{-j} = \left\{ \overrightarrow{q}^{1j} - \varepsilon, \overrightarrow{q}^{2j} + \frac{\varepsilon}{n-1}, ..., \overrightarrow{q}^{nj} + \frac{\varepsilon}{n-1} \right\}, \forall \varepsilon \in (0, \overrightarrow{q}^{1j}) \qquad (8)$$

. But then again this cannot be an equilibrium since candidate $-j$ would have an incentive to choose another vector of policies and so on *ad infinitum.* ■

With uncertainty infinitesimal shifts in policies give rise to infinitesimal, and not discrete, shifts in votes;

Proposition 2 *In a zero-sum game, if (i) Individual strategy sets are compact (ii) and convex, (iii) pay-off functions are continuous (iv) and convex, then a pure-strategy Nash Equilibrium exists.*
Proof. *(see Rosen, 1965, Th.1 or Owen 1982, Th.IV,6.2)* ■

Unfortunately, as demonstrated by Lindbeck and Weibull (1987), in PVMs only assumptions (ii) and (iii) are fulfilled, meaning that the existence of an equilibrium should be verified case by case.

Proposition 3 *If condition A1 holds, then there exists a unique NE of the game*

$$\text{A1: sup} \quad \frac{\overbrace{\left| s^{i'} \right|}^{uncertainty\ threshold}}{s^{i}} \leq \inf \overbrace{\frac{|Vi''(\overrightarrow{q})|}{(Vi'(\overrightarrow{q}))^2}}^{concavity\ index} \ \forall i$$

Proof. *(see Lindbeck & Weibull, 1987)* ■

Condition A1 states that environments characterized by high levels of uncertainty, the uniqueness of an equilibrium is more likely to be found. In fact if the lower bound of the set defined by the concavity index measure is not lower than the upper bound of the set defined by the uncertainty threshold an equilibrium always exists.

5 Resolution of the game

In the third stage individuals choose their amount of leisure, solving 1 under 2. The optimal value is given by:

$$l_t^{i*} = l(\psi^i, a_t^i, \tau) = \arg\max U^i \tag{9}$$

With a log-linear function 9 becomes:

$$l_t^{i*} = \frac{\psi^i}{1-\widetilde{\tau}} \tag{10}$$

Comparative statics shows that

$$\frac{\partial l_t^{i*}}{\partial a_t^i} = -\left(\frac{\tau\psi^i}{(1-\widetilde{\tau})^2}\right) < 0 \tag{11}$$

and thus an increase in the tax credits entails a decrease in the optimal level of leisure.

Substituting 10 in 1 we obtain an expression for the indirect utility function, which for a log-linear function is equal to

$$V_t^{\tau-n+1} = \overbrace{\bar{t}((1\text{-}\tau) + \tau a_t^{\tau-n+1})}^{income\ effect} - \psi^{\tau-n+1} + \tag{12}$$

$$+\psi^{\tau-n+1}\log\psi^{\tau-n+1} - \psi^{\tau-n+1}\log\overbrace{((1-\tau)+\tau a_t^{\tau-n+1})}^{leisure\ effect} \tag{13}$$

The question is now, what is the effect of an increase in the optimal tax credit on the wealth of an individual? Immediately, one may prone to answer that an increase in the tax credits entails an increase in the utility of the individual, since the effective marginal tax rate is reduced and the net-of-taxes labour income increase. But 12 states that this effect (called income effect) is not the only effect which play a role. The leisure effect says that an increase in the tax credits reduces leisure, which is something that increases the utility. Thus the total effect on the welfare on an individual depends on which effect prevails.

Proposition 4 *There exists a threshold such that:*

$$\forall a_t^{\tau-n+1} < \widehat{a}_t^{\tau-n+1} \Longrightarrow \frac{\partial V_t^{\tau-n+1}}{\partial a_t^{\tau-n+1}} < 0 \wedge a_t^{\tau-n+1} > \widehat{a}_t^{\tau-n+1} \Longrightarrow \frac{\partial V_t^{\tau-n+1}}{\partial a_t^{\tau-n+1}} > 0 \tag{14}$$

where $\widehat{a}_t^{\tau-n+1} \equiv \min V_t^{\tau-n+1}$

Proof. Calculate the total differential and obtain

$$\frac{\partial V_t^{\tau-n+1}}{\partial a_t^{\tau-n+1}} = \bar{t}\tau - \frac{\tau\psi^{\tau-n+1}}{(1-\tau)+\tau a_t^{\tau-n+1}} \tag{15}$$

which entails the existence of $a_t^{\tau-n+1}$such that $\frac{\partial \mathrm{Vt}^{\tau-n+1}}{\partial a_t^{\tau-n+1}} = 0$. ■

Proposition 5 *In the second stage the political equilibirum is a tie.*

Proof. The set of First Order Condition may be written as follows:

$$\lambda^{ij} = \sum_{i=1}^{n} \frac{\partial \mathrm{s}_t^i}{\partial l_t^{i*}} \frac{\partial l_t^{i*}}{\partial a_t^i} \left(V^i(\overrightarrow{q}^j) - V^i(\overrightarrow{q}^{-j}) \right) + \sum_{i=1}^{n} \frac{\partial V_t^i}{\partial a_t^i} s_t^i \tag{16}$$

$$\lambda^{i-j} = \sum_{i=1}^{n} \frac{\partial s_t^i}{\partial l_t^{i*}} \frac{\partial l_t^{i*}}{\partial a_t^i} \left(V^i(\overrightarrow{q}^j) - V^i(\overrightarrow{q}^{-j}) \right) + \sum_{i=1}^{n} \frac{\partial V_t^i}{\partial a_t^i} s_t^i \tag{17}$$

and this must be equal for every i. This is because in a PVM Lagrange multipliers represent the per capita marginal gain in expected votes, w.r.t. marginal shifts in transfers should be equal for all groups. But it is evident that in an equilibrium this value must be equal for every generation. Suppose not; if so, the expected number of votes on a party could be improved without violation of the public budget constraint. This last statements affirms that there would exist an incentive for the candidates to increase transfers towards those groups which promise a greater increase in the expected number of votes. ■

Proposition 6 *(Policy Convergence) Assume (i) $\overrightarrow{q}$ is a convex set, (ii) for each I and j π_a is strictly monotonic, (iii) for each j V_i is concave. If (q_j^*, q_{-j}^*) is a pure strategy electoral equilibrium, then $q_j^* = q_{-j}^*$.*

Proof. see the Mathematical Appendix. ■

In the first stage of the game candidates chose the optimal vector of policies.

Proposition 7 *A pair of reaction functions for candidate j and -j if policies are not convergent.*

$$R_t^j(\overrightarrow{q}^{-j}) \equiv \overrightarrow{q}^j = a\left(s(l_t^{i*}), s(l_t^{-i*}), n^i, n^{-i}, \tau_L, \bar{t}, \psi^i, \psi^{-i}, \overrightarrow{q}^{-j}\right) \tag{18}$$

$$R_t^j(\overrightarrow{q}^j) \equiv \overrightarrow{q}^{-j} = a\left(s(l_t^{i*}), s(l_t^{-i*}), n^i, n^{-i}, \tau_L, \bar{t}, \psi^i, \psi^{-i}, \overrightarrow{q}^j\right) \tag{19}$$

Write the first order conditions in explicit form for both the candidates and notice that in the case of policy convergence

Proposition 8 *If policies are convergent then reaction functions are independent from the policy chosen by the other candidate; that is $V^i(\overrightarrow{q}^j) = V^i(\overrightarrow{q}^{-j})$.*

Proof. Equations 21 and 22 reduce to:

$$\lambda^{ij} = \sum_{i=1}^{n} \frac{\partial V_t^i}{\partial a_t^i} s_t^i \tag{20}$$

$$\lambda^{i-j} = \sum_{i=1}^{n} \frac{\partial V_t^i}{\partial a_t^i} s_t^i \tag{21}$$

which are of course independent from the other candidate's policy vector. ■

Proposition 9 *Optimal tax credits are a function of the number and density of both groups, of the marginal tax rate, of the total endowment of time and of the parameters representing preferences of groups for leisure:*

$$a_t^{i*} = a\left(s(l_t^{i*}), s(l_t^{-i*}), n^i, n^{-i}, \tau_L, \bar{t}, \psi^i, \psi^{-i}\right) \tag{22}$$

Proof. *Solve the system of equations 20 and 21.* ■

6 Conclusions

7 Mathematical Appendix

From Proposition 5 we know that λ^j and λ^{-j} must be equal for every i. This entails that the ratio between the two Lagrange multiplier of the different candidates must be equal for every i. We call this ratio as

$$\rho^i = \frac{\lambda^{ji}}{\lambda^{-ji}} = \frac{\sum_{i=1}^{n} \frac{\partial s^i}{\partial l^{i*}} \frac{\partial l^{i*}}{\partial a^{ij}} \left(V^i(\overrightarrow{q}^j) - V^i(\overrightarrow{q}^{-j})\right) + \sum_{i=1}^{n} \frac{\partial V_t^i}{\partial a^{ij}} s_t^i}{\sum_{i=1}^{n} \frac{\partial s_t^i}{\partial l_t^{i*}} \frac{\partial l_t^{i*}}{\partial a_t^i} \left(V^i(\overrightarrow{q}^{-j}) - V^i(\overrightarrow{q}^j)\right) + \sum_{i=1}^{n} \frac{\partial V_t^i}{\partial a_t^{i-j}} s_t^i} \tag{23}$$

The problem now is to evaluate whether this condition may be achieve under a policy differentiation or may only be achieved under a policy convergence. To prove this, we start assuming that candidates chose different policy vectors, that is $\overrightarrow{q}^j = \overrightarrow{q}^{-j}$. Since candidates must also clear the balanced-budget constraint, there must exist a set of generations which get higher tax credits under candidate j than under candidate $-j$ and I will denote this set with $I^+ = \{i \in I | a^{j+} > a^{-j+}\}$ and a set of generations which get higher tax credits under candidate $-j$ than under candidate j, $I^- = \{i \in I | a^{j-} > a^{-j-}\}$. We have to evaluate if $\rho^+ = \rho^-$. If we verify that this is true, it will mean that an equilibrium may be achieved via different policies; otherwise, we must conclude that the only possibility to achieve the equilibrium is via convergent policies. The problem becomes to find a monotonicity condition on both the numerator and the denominator. If so, then it would mean that either $\overrightarrow{q}^j > 1 > \overrightarrow{q}^{-j}$ or $\overrightarrow{q}^j < 1 < \overrightarrow{q}^{-j}$ and thus an equilibrium cannot never be achieved via different policies.

The simplest tecnique is to impose the ratio equal to one, subtract the denominator from the numerator and see whether a monotonicity is reached. Denoting $z = V^i(\overrightarrow{q}^j) - V^i(\overrightarrow{q}^{-j})$ expressions we get are the following:

$$\sum_{i=1}^{n} \frac{\partial s^+}{\partial a^{j+}} z - \sum_{i=1}^{n} \frac{\partial s^+}{\partial a^{-j+}} z + \left(\sum_{i=1}^{n} \frac{\partial V^+}{\partial a^{j+}} - \frac{\partial V^+}{\partial a^{-j+}}\right) s^+ \tag{24}$$

for groups +, and

$$\sum_{i=1}^{n}\frac{\partial s^-}{\partial a^{-j-}}z-\sum_{i=1}^{n}\frac{\partial s^-}{\partial a^{j-}}z+\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}}-\frac{\partial V^+}{\partial a^{-j+}}\right)s^+ \tag{25}$$

for groups -.

Now the sign of z changes according to the interval where tax credits find. Denoting with $a_{V=0^+}$ and $a_{V=0^-}$ points where the IUF intersects the axis (respectively at the right and at the left hand side) representing the tax credit we may easily see that we have to study 6 cases:

$$a^{-j+}<a^{j+}<a_{V=0}\Longrightarrow z<0 \tag{26}$$

$$a^{-j+}<a_{V=0}<a^{j+}<\widehat{a}\Longrightarrow z<0 \tag{27}$$

$$a_{V=0}<a^{-j+}<a^{j+}<\widehat{a}\Longrightarrow z>0 \tag{28}$$

$$a_{V=0}<a^{-j+}<\widehat{a}<a^{j+}<a_{V(a^j)<V(a^{-j})}\Longrightarrow z>0 \tag{29}$$

$$a_{V=0^+}<a^{-j+}<\widehat{a}<a_{V(a^j)<V(a^{-j})}<a^{j+}<a_{V=0^-}\Longrightarrow z>0 \tag{30}$$

$$a_{V=0^+}<a^{-j+}<\widehat{a}<a_{V(a^j)<V(a^{-j})}<a_{V=0^-}<a^{j+}\Longrightarrow z>0 \tag{31}$$

As a consequence we may study the sign of expression...

First case:

$$\overbrace{\overbrace{\sum_{i=1}^{I^+}\overbrace{\frac{\partial s^+}{\partial a^{j+}}}^{<0}\overbrace{z}^{<0}}^{>0}-\overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^+}{\partial a^{-j+}}}^{<0}\overbrace{(-z)}^{>0}}^{<0}}^{>0}+\overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}}-\frac{\partial V^+}{\partial a^{-j+}}\right)}^{<0}\overbrace{s^+}^{>0} \tag{32}$$

$$\overbrace{\overbrace{\sum_{i=1}^{I^-}\overbrace{\frac{\partial s^-}{\partial a^{-j-}}}^{<0}\overbrace{(-z)}^{>0}}^{<0}-\overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^-}{\partial a^{j-}}}^{<0}\overbrace{z}^{<0}}^{>0}}^{<0}+\overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}}-\frac{\partial V^+}{\partial a^{-j+}}\right)}^{>0}\overbrace{s^+}^{>0} \tag{33}$$

Second case:

$$\overbrace{\overbrace{\sum_{i=1}^{I^+}\overbrace{\frac{\partial s^+}{\partial a^{j+}}}^{<0}\overbrace{z}^{<0}}^{>0}-\overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^+}{\partial a^{-j+}}}^{<0}\overbrace{(-z)}^{>0}}^{>0}}^{>0}+\overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}}-\frac{\partial V^+}{\partial a^{-j+}}\right)}^{<0}\overbrace{s^+}^{>0} \tag{34}$$

$$\overbrace{\overbrace{\sum_{i=1}^{I^-}\overbrace{\frac{\partial s^-}{\partial a^{-j-}}}^{<0}\overbrace{(-z)}^{>0}}^{<0}-\overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^-}{\partial a^{j-}}}^{<0}\overbrace{z}^{<0}}^{>0}}^{<0}+\overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}}-\frac{\partial V^+}{\partial a^{-j+}}\right)}^{>0}\overbrace{s^+}^{>0} \tag{35}$$

In this case it easy to see that we have not found a monotonicity condition and thus it could exist an equilibrium achieved via a differentiation of policies.

Third case:

$$\overbrace{\overbrace{\sum_{i=1}^{I^+}\overbrace{\frac{\partial s^+}{\partial a^{j+}}}^{<0}\overbrace{z}^{>0}}^{<0}-\overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^+}{\partial a^{-j+}}}^{<0}\overset{<0}{(-z)}}^{>0}}^{<0}+\overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}}-\frac{\partial V^+}{\partial a^{-j+}}\right)}^{<0}\overbrace{s^+}^{>0} \tag{36}$$

$$\overbrace{\overbrace{\sum_{i=1}^{I^-}\overbrace{\frac{\partial s^-}{\partial a^{-j-}}}^{<0}\overbrace{(-z)}^{<0}}^{>0}-\overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^-}{\partial a^{j-}}}^{<0}\overbrace{z}^{>0}}^{<0}}^{>0}+\overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}}-\frac{\partial V^+}{\partial a^{-j+}}\right)}^{>0}\overbrace{s^+}^{>0} \tag{37}$$

Fourth case:

$$\overbrace{\overbrace{\sum_{i=1}^{I^+}\overbrace{\frac{\partial s^+}{\partial a^{j+}}}^{<0}\overbrace{z}^{>0}}^{<0}-\overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^+}{\partial a^{-j+}}}^{<0}\overset{<0}{(-z)}}^{>0}}^{<0}+\overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}}-\frac{\partial V^+}{\partial a^{-j+}}\right)}^{>0}\overbrace{s^+}^{>0} \tag{38}$$

$$\overbrace{\overbrace{\sum_{i=1}^{I^-}\overbrace{\frac{\partial s^-}{\partial a^{-j-}}}^{<0}\overbrace{(-z)}^{<0}}^{>0}-\overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^-}{\partial a^{j-}}}^{<0}\overbrace{z}^{>0}}^{<0}}^{>0}+\overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}}-\frac{\partial V^+}{\partial a^{-j+}}\right)}^{<0}\overbrace{s^+}^{>0} \tag{39}$$

Fifth case:

$$\overbrace{\overbrace{\sum_{i=1}^{I^+}\overbrace{\frac{\partial s^+}{\partial a^{j+}}}^{<0}\overbrace{z}^{<0}}^{>0}-\overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^+}{\partial a^{-j+}}}^{<0}\overset{>0}{(-z)}}^{<0}}^{>0}+\overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}}-\frac{\partial V^+}{\partial a^{-j+}}\right)}^{>0}\overbrace{s^+}^{>0} \tag{40}$$

$$\overbrace{\overbrace{\sum_{i=1}^{I^-}\overbrace{\frac{\partial s^-}{\partial a^{-j-}}}^{<0}\overbrace{(-z)}^{>0}}^{<0} - \overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^-}{\partial a^{j-}}}^{<0}\overbrace{z}^{<0}}^{>0}}^{<0} + \overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}} - \frac{\partial V^+}{\partial a^{-j+}}\right)}^{<0}\overbrace{s^+}^{>0} \tag{41}$$

Sixth case:

$$\overbrace{\overbrace{\sum_{i=1}^{I^+}\overbrace{\frac{\partial s^+}{\partial a^{j+}}}^{<0}\overbrace{z}^{>0}}^{<0} - \overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^+}{\partial a^{-j+}}}^{<0}(-z)}^{>0}}^{<0} + \overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}} - \frac{\partial V^+}{\partial a^{-j+}}\right)}^{>0}\overbrace{s^+}^{>0} \tag{42}$$

$$\overbrace{\overbrace{\sum_{i=1}^{I^-}\overbrace{\frac{\partial s^-}{\partial a^{-j-}}}^{<0}\overbrace{(-z)}^{<0}}^{>0} - \overbrace{\sum_{i=1}^{n}\overbrace{\frac{\partial s^-}{\partial a^{j-}}}^{<0}\overbrace{z}^{>0}}^{<0}}^{>0} + \overbrace{\left(\sum_{i=1}^{n}\frac{\partial V^+}{\partial a^{j+}} - \frac{\partial V^+}{\partial a^{-j+}}\right)}^{<0}\overbrace{s^+}^{>0} \tag{43}$$

Hence, a strict monotonicity condition was found only in the third and fifth case, meaning that $\rho^+ \neq \rho^-$ and thus an equilibrium is only possible to achieve under policy convergence.

8 Aknowledgements

I am particularly grateful to Massimo Bordignon and Federico Etro for useful suggestions; to the London School of Economics and Political Science where this work was written and to all the participants at the Ph.D. work in progress seminar, especially to Leonardo Felli and Michele Piccione, Torsten Persson, Asser Lindbeck, Jorgen Weibull, Ronnie Razin, Gilat Levy, Frank Cowell and Hollie Sutherland. All remaining errors are mine.

References

[1] Banks, J. & Duggan, J. Probabilistic Voting in the Spatial Model of Elections: The Theory of Office-motivated Candidates, (2003), mimeo

[2] Dixit, A. & Londregan J. Redistributive Politics and Economic Efficiency, (1994) NBER Working Papers 1056

[3] Allan M. Feldman (1987). "equity," The New Palgrave: A Dictionary of Economics, v. 2, pp. 182-84

[4] Hinich M.J.: Equilibrium in Spatial Voting: The Median Voter Theorem is an Artifact, (1977) Journal of Economic Theory, Vol. 16, pp. 208-219

[5] Lindbeck A. & Weibull J. W.: Balanced-Budget Redistribution as the Outcome of Political Competition, (1987), Public Choice 52, pp. 273-297

[6] Mulligan C. B. & Sala-i-Martin: Gerontocracy, Retirement and Social Security, (1999) NBER Working Paper 7117

[7] Mulligan C. B. & Sala-i-Martin: Social Security in Theory and Practice (I): Facts and Political Theories, (1999) NBER Working Paper 7118

[8] Mulligan C. B. & Sala-i-Martin: Social Security in Theory and Practice (II): Efficiency Theories, Narrative Theories and Implications for Reforms, (1999) NBER Working Paper 7119

[9] Mulligan C. B. & Sala-i-Martin: Social Security, Retirement and the Single-Mindness of the Electorate, (1999) NBER Working Paper 9691

[10] Profeta, P.: Retirement and Social Security in a Probabilistic Voting Model, (2002) International Tax and Public Finance, 9, pp 331-348

[11] Persson T. & Tabellini G.: Political Economics: Explaining the Economic Policy, (2000) MIT Press

	Mulligan & Sala-i-Martin	**Lindbeck & Weibull**	**Canegrati**
Political Competition	No	Yes	Yes
Intra-generational redistribution	Labour-income taxation	No	No
Inter-generational redistribution	Lump-Sum	Lump-sum	Labour-income taxation
Labour Supply	Variable	Fixed	Variable
Early-retirement	Yes	No	Yes

Table 1