Coefficient of Structural Concordance and an Example of its Application: Labour Productivity and Wages in Slovenia

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Summary

The article presents the underlying principles, derivation and properties of a simple descriptive measure of concordance between two analogous rank structures that we call the coefficient of structural concordance. It is based upon the idea of Kendall's coefficient of concordance, which we extend to two rank structures. As the coefficient of structural concordance is a pure intergroup measure of concordance, it is designed to complement the Kendall's intragroup coefficient of concordance. We apply this descriptive measure by exploring the relationship between wages and labour productivity in Slovenia for the period 1998–2007. We are able to confirm the hypothesis of high concordance between wages and labour productivity, which indicates a stimulative role of wages in production of market traded goods and services.

Introduction

In science, we often deal with the issue of agreement or concordance in a relationship between phenomena under consideration. In several science disciplines, particularly in natural science, this can usually be measured by very precise and sophisticated methods. The prerequisite for precise measurement of concordance is of course the availability and quality of data on the phenomena under consideration. However, in economics, and especially in social sciences and humanities, this prerequisite is often not met. Either, one does not have cardinal numerical data at his disposal, or the quality of such data is questionable. In such cases it is often convenient to resort to ordinal numerical data, or rank data, where the levels of analysis are indeed limited.

When measuring the concordance between rank orders within an individual rank structure it is common to rely on the work of Maurice G. Kendall and B. Babington Smith (1939) and their successors, using a suitable version of the Kendall's coefficient of concordance. However, when extending the analysis to measure the concordance between two analogous rank structures, there appears to be no simple robust descriptive measure of agreement for rank data. In this article we develop one such measurement based upon the underlying principle of Kendall's coefficient of concordance. We call this measurement the coefficient of structural concordance.

From this point the study applies the coefficient of structural concordance on a relationship between two economic phenomena that is well founded in economic theory. Namely, we explore the linkage between wages and labour productivity, basing our analysis upon industry data on gross wages and value added per employee in Slovenia for the period 1998–2007. Focus is placed on the effects of irregular industry changes in value added per employee, caused by diverse intensity of the technical progress, on deviations from perfect agreement between value added per employee and gross wages.

The outline of the article is as follows. In Section 1, we present the pioneering work of Kendall and Smith (1939), representing the foundation for our work. In Section 2, we discuss particular extensions of measuring the concordance, some being based on the work of Kendall and Smith (1939), while others seeking alternative routes. In Section 3, we derive the coefficient of structural concordance between two rank structures and set out its properties. In Section 4, we present an application of measuring the concordance, based upon the relationship between labour productivity and wages. In the final section we summarize the central findings of the article.

1. Kendall's Coefficient of Concordance

Kendall and Smith (1939) provided a descriptive measure of agreement or concordance for data comprised of M sets of ranks, where $M > 2$. Let us assume an artificial rank structure $\mathcal{A}$ presented by Table 1. We have two variables, X and Y. Variable X consists of values x_i, $i = 1, ..., N$ with N being the number of ranks in each set of ranks. Variable Y consists of values y_j, $j = 1, ..., M$ with M being the number of sets of ranks. Each value of variable X, x_i, has a rank r_{ji}, assigned by the value of the ranking variable Y, y_j. R_i is the rank total for value x_i of the variable X.

< TABLE 1 >

If perfect agreement were observed between the *j* values of the ranking variable, one value of the variable *X* would be assigned a 1 by all *j* values of the ranking variable, and the rank total would be *M*. Another value of the variable *X* would be assigned a 2 by all *j* values of the ranking variable, and the rank total would be 2*M*. Therefore, when perfect agreement exists among ranks assigned by *M* values of the ranking variable, the rank totals are *M*, 2*M*, 3*M*, ..., *NM*. The total sum of ranks for *M* values of the ranking variable is *MN* (*N* + 1) / 2, and the mean rank sum is *M* (*N* + 1) / 2.

The degree of agreement between the values of the ranking variable reflects itself in the variation in the rank totals (George A. Ferguson, 1966: 225–226). When all the values of the ranking variable are in agreement, this variation is at a maximum. Disagreement between the values of the ranking variable reflects itself in a reduction in the variation of rank totals. For maximum disagreement the rank totals tend to be equal.

Since R_i is the rank total for value x_i of the variable *X*, the sum of squared deviations of rank totals from the average rank total for *N* values of variable *X* is:

$$\sum_{j=1}^{M} \left(R_j - \overline{R} \right)^2 ,$$

where $\overline{R} = \dfrac{1}{N} \sum_{j=1}^{M} R_j$ is the average rank total. The maximum value of this expression occurs when perfect agreement exists between the values of the ranking variable. It can easily be shown that this value is equal to:

$$\frac{1}{12} M^2 N (N^2 - 1) .$$

The coefficient of concordance, θ, is defined as the ratio of sum of squared deviations of rank totals from the average rank total to the maximum possible value of the sum of squared deviations of rank totals from the average rank total (Kendall and Smith, 1939; *cf.* Maurice G. Kendall, 1970):

$$\theta = \frac{12 \sum_{j=1}^{M} \left(R_j - \overline{R} \right)^2}{M^2 N (N^2 - 1)} . \tag{1}$$

Alternatively, one can express this nonparametric statistic in terms of the sum of squares of rank totals instead of the sum of squared deviations of rank totals from the average rank total. In that case, the coefficient of concordance θ has the following form (Pierre Legendre, 2005: 229; *cf.* Sidney Siegel and N. John Castellan, 1988: 266):

$$\theta = \frac{12 \sum_{j=1}^{M} R_j^2}{M^2 N (N^2 - 1)} - 3 \frac{N+1}{N-1} . \tag{2}$$

Expressions (1) and (2) are equivalent. The measure of overall (intragroup) concordances, defined by expressions (1) and (2), is commonly referred to as the *Kendall's coefficient of concordance*. When perfect agreement exists between the

values of the ranking variable, $\theta = 1$. When maximum disagreement exists, $\theta = 0$. Kendall's coefficient of concordance does not take negative values and is thus bounded on the interval $0 \leq \theta \leq 1$. Note that with more than two values of the ranking variable (more than two sets of ranks) complete disagreement can not occur.

For N $\leq$ 7, the critical values of this nonparametric statistic at the one and five per cent levels have been tabulated by Milton Friedman (1940), and are reproduced in Sidney Siegel (1956) and Kendall (1970). A useful adaptation of these tables is given by Edwards (1954). Critical values of θ depend both on the number of sets of ranks, *M*, and on the number of ranks in each set, *N*. For $N > 7$, the following χ^2-test may be applied (*cf.* Ferguson, 1966: 227–228; Legendre, 2005: 230–231):

$$\tau = M(N-1)\theta \sim \chi^2_{N-1}, \tag{3}$$

where τ is a test statistic, which is χ^2-distributed with $N-1$ degrees of freedom. One has to be aware that this test provides quite a rough estimate of the required probabilities. There exist other procedures for testing the significance of Kendall's coefficient of concordance (*cf.* Allen L. Edwards, 1954), which are not examined here.

2. Some Extensions of Measuring the Concordance

Several attempts to extend the work of Kendall and Smith (1939) have been made. Based on work of James Durbin (1951), Ben Willerman (1955) provided a formula for computing Kendall's coefficient of concordance when self-ranks were omitted, together with a table of critical values for the test statistic assuming a beta distribution. William R. Schucany and William H. Frawley (1973) constructed a test statistic to test the hypothesis of agreement of several variables on the ranking of items within each of the two groups and between the two groups, which may be unequal in size. They also provided a generalization of the coefficient of concordance. This test statistic was further advanced by Loretta Li and William R. Schucany (1975). Additionally, Myles Hollander and Jayaram Sethuraman (1978) illustrated that the test of agreement between groups has to be used with care if the relevant hypothesis is to be taken as the null hypothesis. They adapted a procedure, proposed by Abraham Wald and Jacob Wolfowitz (1944) in a slightly different context, to furnish a new test for agreement between two groups of variables. James Beckett and William R. Schucany (1979) analyzed the agreement between, and within, more than two groups of variables in the form of an analysis of concordance table.

Finally, Helena C. Kraemer (1981) proposed a coefficient of intergroup concordance, which is consistent with the concept of intragroup concordance as measured by Kendall's coefficient of concordance (Kendall and Smith, 1939). This approach reconciled the approaches of Schucany and Frawley (1973) and Hollander and Sethuraman (1978), to the problem of two-group concordance. The coefficient, also known as the *unconditional measure of concordance*, is calculated as a quotient between Kendall's coefficient of concordance using all respondents, and the average of Kendall's coefficients of concordance calculated separately for the groups and weighted by the sample sizes. Estimation and test procedures for the population were based on jackknife procedures. Extension of the problem of multiple intergroup concordance when groups have factorial structure was also noted.

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In addition, alternative approaches have also been introduced. Let us briefly mention just some of them. Lawrence J. Hubert (1979) proposed a measure of concordance based on a simple nonparametric procedure for comparing proximity matrices, which is appropriate when independent proximity matrices are available. Motivated by the diversity analysis framework of Calyampudi R. Rao (1982), Paul D. Feigin and Mayer Alvo (1986) proposed, a general approach to comparing populations of rankers, developing tests of hypotheses concerning equality of characteristics. Ie-Bin Lian and Wen-Chin Young (2001) proposed two statistics based on restricted principal component and restricted canonical correlation to measure the intragroup and intergroup concordance of variables. Przemysław Grzegorzewski (2006) proposed a generalization of the Kendall's coefficient of concordance that can be used in situations with missing information or noncomparable outputs.

3. Coefficient of Structural Concordance

Despite a number of extensions of measuring the concordance there appears to be no simple robust descriptive measure of agreement or concordance for data in two analogous rank structures. For the purpose of deriving one, let us assume two artificial rank structures, $\mathcal{A}$ and $\mathcal{B}$, of the type presented by the Table 1. In the rank structure $\mathcal{A}$ we have the aforementioned variables X and Y, while in the rank structure $\mathcal{B}$ we analogously have variables W and Z. The remaining notation is the same as in Chapter 2. The two rank structures are equal in dimensions. We are now interested in how one should proceed in order to quantitatively establish the level of concordance between these two rank structures.

Let us demonstrate the derivation of an appropriate measure of concordance between the two rank structures, for the marginal case where perfect disagreement exists between the two rank structures. Evidently, this is possible for any number of values of the ranking variable (any number of sets of ranks). This means that in the rank structure $\mathcal{A}$, each value y_j of the ranking variable Y assigns to the variable X ranks in ascending order, i.e. $r_{j1}^{\mathcal{A}} < ... < r_{ji}^{\mathcal{A}} < ... < r_{jN}^{\mathcal{A}}$. On the other hand, in the rank structure $\mathcal{B}$, each value z_j of the ranking variable Z assigns to the variable W ranks in descending order, i.e. $r_{j1}^{\mathcal{B}} > ... > r_{ji}^{\mathcal{B}} > ... > r_{jN}^{\mathcal{B}}$. In such a case it is easy to illustrate that the values of the Kendall's coefficient of concordance are equal for both rank structures.

Although the Kendall's coefficient of concordance (with its derivatives) presents a useful measure of agreement, or concordance, for data in each separate rank structure, it can not be used as a measure of concordance between two rank structures. However, the rationale behind the Kendall's coefficient of concordance can be applied to derive a new descriptive measure of concordance. For this purpose we shall employ the differences in absolute terms between the ranks of the two rank structures, which we sum up by both the ranks in each set of ranks, i and the sets of ranks, j:

$$\sum_{i=1}^{N} \sum_{j=1}^{M} \left| r_{ji}^{\mathcal{A}} - r_{ji}^{\mathcal{B}} \right|,$$

where $r_{ji}^{\mathcal{A}}$ is a rank in the rank structure $\mathcal{A}$ of the value x_i of the variable X assigned by the value y_j of the ranking variable Y and $r_{ji}^{\mathcal{B}}$ is a rank in the rank structure $\mathcal{B}$ of the value w_i of the variable W assigned by the value z_j of the ranking variable Z.

The sum of differences in absolute terms between the ranks of the two rank structures is compared to the maximum possible sum of differences in absolute terms between the ranks of the two rank structures (this is why the marginal case of perfect disagreement between the two rank structures has been selected for demonstration). Results illustrate that the latter has a different general expression depending on the number of ranks in each set of ranks (N).

For rank structures with an odd number of ranks in each set of ranks the maximum possible sum of differences in absolute terms equals $\frac{1}{2}M(N^2-1)$, while for rank structures with an even number of ranks in each set of ranks the maximum possible sum of differences in absolute terms equals $\frac{1}{2}MN^2$. This is fairly obvious in our marginal case, where the difference in absolute terms in each rank order j starts at the maximum value for $i = 1$, decreases toward the middle of the rank order (where it is 0 for an odd number of ranks and 1 for an even number of ranks in each set of ranks), and then increases towards the maximum value for $i = N$. The maximum possible sum of differences in absolute terms in each rank order j thus equals $\frac{N-1}{2}\left(\frac{N-1}{2}+1\right)$ for an odd number of ranks in each set of ranks, and $2\left(\frac{N}{2}\right)^2$ for an even number of ranks in each set of ranks.

Thus we define a measure of concordance between two rank structures, which we entitle the *coefficient of structural concordance* and denote by ψ, as the ratio of sum of differences in absolute terms between the ranks of the two rank structures to the maximum possible sum of differences in absolute terms between the ranks of the two rank structures. In the case of rank structures with an odd number of ranks in each set of ranks, this is equal to:

$$\frac{2}{M(N^2-1)}\sum_{i=1}^{N}\sum_{j=1}^{M}\left|r_{ji}^{\mathcal{A}}-r_{ji}^{\mathcal{B}}\right|,$$

while in case of rank structures with an even number of ranks in each set of ranks we obtain the following expression:

$$\frac{2}{MN^2}\sum_{i=1}^{N}\sum_{j=1}^{M}\left|r_{ji}^{\mathcal{A}}-r_{ji}^{\mathcal{B}}\right|.$$

It can be readily observed that the above measure of concordance yields value 1 in case when perfect disagreement exists between the two rank structures, and value 0 in case of complete agreement between the rank structures. This results directly from the use of differences between the ranks, while in the case of the Kendall's coefficient of concordance rank totals were used. For the matter of convenience and comparison, we

subtract the above expressions from 1. Thus we formalize the coefficient of structural concordance, ψ, in the following final form, separately for rank structures with an odd number of ranks in each set of ranks:

$$\psi = 1 - \frac{2}{M(N^2-1)}\sum_{i=1}^{N}\sum_{j=1}^{M}\left|r_{ji}^{\mathcal{A}}-r_{ji}^{\mathcal{B}}\right|, \qquad (4)$$

and for rank structures with an even number of ranks in each set of ranks:

$$\psi = 1 - \frac{2}{MN^2}\sum_{i=1}^{N}\sum_{j=1}^{M}\left|r_{ji}^{\mathcal{A}}-r_{ji}^{\mathcal{B}}\right|. \qquad (5)$$

When perfect agreement exists between two rank structures, $\psi = 1$. When perfect disagreement exists, $\psi = 0$. Analogously to the Kendall's coefficient of concordance, the coefficient of structural concordance does not take negative values and is thus also bounded on the interval $0 \le \psi \le 1$.

The essential difference between the approach presented herein, and the work of Schucany and Frawley (1973), Hollander and Sethuraman (1978), and their successors, is in the assumption about what the rank structures represent. The aforementioned articles assume that the rank structures represent different populations of the same phenomenon. As such, these attempts to construct a measure of intergroup concordance are direct extensions of the intragroup concordance idea of Kendall and Smith (1939), often failing to make a clear distinction between intragroup and intergroup concordance and thus posing several difficulties (*cf.* Kraemer, 1981: 642). Conversely, our approach does not put any restrictions on what the rank structures represent, except for those of the theory that leads us to examine the concordance in the first place. The coefficient of structural concordance is thus a pure intergroup measure of concordance designed as a complement to the Kendall's intragroup coefficient of concordance.

However, one issue that is yet to be ascertained is the distribution of this nonparametric statistic. For this purpose one could use the bootstrap method (Bradley Efron, 1979), which is a computation-oriented nonparametric method to construct empirical distribution through resampling from the original sample. Following Bradley Efron and Robert Tibshirani (1986), or Christopher Z. Mooney and Robert D. Duval (1993), one could construct a relative frequency histogram by running an iterative procedure which takes a random sample with replacement from the population sample set and calculates the statistic for that random sample. In addition to examining the empirical distribution one could also calculate the bootstrap confidence intervals (*cf.* James Carpenter and John Bithell, 2000).

4. Labour Productivity and Wages in Slovenia: An Application of Measuring the Concordance

Although we could utilize any two phenomena, from any field of interest, in order to demonstrate the measuring of concordance, we employ a relationship between two economic phenomena that is well founded in economic theory. Namely, we explore the linkage between wages and labour productivity. More precisely, our analysis is based

upon industry data on gross wages and value added per employee (proxy for labour productivity) in Slovenia for the period 1998–2007.

Given (neoclassical) economic theory, differences in productivity are the main generator of the differences in wages, at least in industries that produce tradable goods. An industry with high productivity should therefore also have high rank with respect to wages and *vice versa*. Additionally, an industry that quotes highly with respect to productivity should also have exhibit high wages. However, even though the labour productivity is the main determinant of wages, the inter-industry structure of wages is also affected by various other factors that shape its dynamic over time. Therefore irregular industry changes in labour productivity, caused by diverse intensity of the technical progress (total factor productivity), are reflected in the inter-industry structure of wages. An industry with above-average increase in labour productivity shall improve its relative position in the inter-industry wage structure, while an industry with below-average increase in labour productivity shall deteriorate its relative position.

We are interested in the effects of these processes on deviations from perfect agreement between labour productivity and wages. Or, in other words, in the level of concordance between gross wages and value added per employee. To test this, we first present data on gross wages by industry in Slovenia (Table 2) and its rank order (Table 3) for the period under consideration. We then present data on value added per employee by industry (Table 4) and its rank order (Table 5). We include into our analysis the industries that produce goods and services that are market traded: C – Mining and quarrying; D – Manufacturing; E – Electricity, gas and water supply; F – Construction; G – Wholesale, retail and certain repair; H – Hotels and restaurants; I – Transport, storage and communication; and J – Financial intermediation.

< TABLE 2 >

< TABLE 3 >

At this point we can compute using either expression (1) or expression (2) that the Kendall's coefficient of concordance for gross wages amounted in the period 1998–2007 to 0.9881, while the Kendall's coefficient of concordance for value added per employee in Slovenia in the same period was equal to 0.9395. Both values of the Kendall's coefficient of concordance are highly statistically significant. The value of the test statistic of the χ^2-test, computed using expression (3), amounts to 69.17 in case of gross wages and to 65.77 in case of value added per employee, while the critical value at the 0.01 per cent significance level and 7 degrees of freedom is 20.28. There is therefore a high level of concordance in both the inter-industry structure of gross wages and the inter-industry structure of value added per employee, meaning that the industries are preserving its relative position in the rank order with time.

< TABLE 4 >

< TABLE 5 >

However, as already stressed, we are not interested only in the level of concordance of the two separate rank structures, but also in the level of concordance between the two rank structures. For this purpose, we employ the coefficient of structural concordance,

derived in this article. The expected value of this descriptive measure of concordance given the economic theory is relatively high; heuristically we could place it in the fourth quartile. Since the rank structures of gross wages and value added per employee have an even number of ranks, we shall use expression (5). As it turns out, the value of the coefficient of structural concordance amounts precisely to 0.8. This confirms our hypothesis of high concordance between wages and labour productivity in the period 1998–2007 in Slovenia, and indicates a stimulative role of wages in production of market traded goods and services.

Finally, one should add that in our case we have the cardinal numeric values of the variables of interest at our disposal, so indeed more precise measures of agreement between these two variables could be employed. However, we wanted to demonstrate the use of the coefficient of structural concordance, derived in this article, in a situation that enables the analysis of both actual and rank data for comparative purposes.

Concluding Remarks

The article presents the underlying principles and the derivation of a simple descriptive measure of concordance between two analogous rank structures. Our work is based upon Kendall and Smith (1939), who proposed a measure of agreements between rank orders within an individual rank structure, extending it to two rank structures. While Kendall and Smith (1939) compared the sum of squared deviations of rank totals from the average rank total to the maximum possible value of the sum of squared deviations of rank totals from the average rank total, we compare the sum of differences in absolute terms between the ranks of the two rank structures to the maximum possible sum of differences in absolute terms between the ranks of the two rank structures. We call this descriptive measure the coefficient of structural concordance.

The coefficient of structural concordance can be well employed, especially in economics, social sciences and humanities, where often the availability of data is problematic or the quality of data on the phenomena under consideration is questionable. In such cases one may still resort to rank data. Here, the levels of analysis are indeed limited, but the conclusions based on rank data, although less precise, are often also less problematic. This is particularly true for survey data, where ranking is often the only approach that is due to the availability of alternatives not cognitively too demanding for the examinee. Since the coefficient of structural concordance is a pure intergroup measure of concordance, it is designed to complement the Kendall's intragroup coefficient of concordance.

We applied the coefficient of structural concordance by exploring the relationship between wages and labour productivity in Slovenia for the period 1998–2007. We employed a relationship that is well founded in economic theory and is expected to exhibit a high level of concordance. However, this is also a relationship for which the cardinal numeric values of the variables under investigation are usually available. We thus employed a suboptimal approach to verify our hypothesis in order to demonstrate the use of the coefficient of structural concordance in an environment that allows for future comparison of suitability of different approaches. Indeed, we were able to confirm our hypothesis of high concordance between wages and labour productivity for Slovenia in the period 1998–2007, which indicates a stimulative role of wages in production of market traded goods and services.

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Table 1. Ranks assigned to N values of variable X by M values of variable Y

Variable Y	Variable X				
	x_1	$\cdots$	x_i	$\cdots$	x_N
y_1	r_{11}	$\cdots$	r_{1i}	$\cdots$	r_{1N}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
y_j	r_{j1}	$\cdots$	r_{ji}	$\cdots$	r_{jN}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
y_M	r_{M1}	$\cdots$	r_{Mi}	$\cdots$	r_{MN}
R_i	R_1	$\cdots$	R_i	$\cdots$	R_N

Note: We refer to a table of this kind as a rank structure of two variables.

Table 2. Gross wages by industry, 1998–2007 (current prices, in EUR)

Year	Industry							
	C	D	E	F	G	H	I	J
1998	733.23	551.16	745.35	559.58	630.51	530.10	715.34	970.63
1999	806.08	601.36	836.38	615.55	709.02	580.64	779.71	1064.71
2000	912.64	673.08	914.76	665.75	722.41	628.14	869.71	1239.81
2001	1035.02	745.27	1043.23	722.66	791.22	689.20	970.13	1307.67
2002	1144.22	818.21	1155.02	788.19	863.96	743.09	1049.55	1418.38
2003	1244.04	880.74	1251.09	852.60	926.81	789.64	1136.03	1547.45
2004	1363.46	943.20	1353.46	912.96	988.99	834.81	1212.66	1639.77
2005	1438.28	997.27	1476.53	938.05	1021.87	846.67	1249.28	1727.16
2006	1502.71	1052.25	1559.60	996.07	1078.79	884.13	1293.94	1851.09
2007	1607.72	1123.58	1656.91	1060.89	1161.21	937.19	1367.61	1985.99

Source: Statistical Yearbook of the Republic of Slovenia (SORS, 1998–2007).

Table 3. Rank order of gross wages by industry, 1998–2007

Year	Industry							
	C	D	E	F	G	H	I	J
1998	3	7	2	6	5	8	4	1
1999	3	7	2	6	5	8	4	1
2000	3	6	2	7	5	8	4	1
2001	3	6	2	7	5	8	4	1
2002	3	6	2	7	5	8	4	1
2003	3	6	2	7	5	8	4	1
2004	2	6	3	7	5	8	4	1
2005	3	6	2	7	5	8	4	1
2006	3	6	2	7	5	8	4	1
2007	3	6	2	7	5	8	4	1
Rank total	29	62	21	68	50	80	40	10

Source: Own calculations based on industry gross wage data in current prices.

Table 4. Value added per employee by industry, 1998–2007 (current prices, in EUR)

Year	Industry							
	C	D	E	F	G	H	I	J
1998	14,227	14,168	32,352	13,718	16,686	11,516	20,565	31,163
1999	14,819	15,840	31,752	16,189	17,152	11,736	21,778	35,419
2000	18,004	17,698	37,360	16,386	18,132	12,469	24,180	40,293
2001	17,485	19,751	45,383	17,301	20,470	13,879	26,654	40,201
2002	18,175	21,246	50,660	18,731	22,746	15,268	28,826	44,008
2003	22,808	23,767	53,273	20,526	25,008	16,834	32,353	46,897
2004	29,306	24,990	62,047	21,743	26,916	18,068	36,144	51,614
2005	30,326	25,528	64,943	21,949	27,654	18,093	37,976	51,163
2006	33,759	27,998	69,435	23,726	29,328	19,589	40,357	60,461
2007	35,684	30,060	71,733	25,311	30,534	19,380	39,940	65,904

Source: Statistical Yearbook of the Republic of Slovenia (SORS, 1998–2007); own calculations.

Table 5. Rank order of value added per employee by industry, 1998–2007

Year	Industry							
	C	D	E	F	G	H	I	J
1998	5	6	1	7	4	8	3	2
1999	7	6	2	5	4	8	3	1
2000	5	6	2	7	4	8	3	1
2001	6	5	1	7	4	8	3	2
2002	7	5	1	6	4	8	3	2
2003	6	5	1	7	4	8	3	2
2004	4	6	1	7	5	8	3	2
2005	4	6	1	7	5	8	3	2
2006	4	6	1	7	5	8	3	2
2007	4	6	1	7	5	8	3	2
Rank total	52	57	12	67	44	80	30	18

Source: Own calculations based on industry value added per employee data in current prices.