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May 2010

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September 2010

keywords: financial models, risk, portfolio theory, anthropology, archaeology, maize, grain storage, Fremont Indians
JEL codes: G11, N0, N2, N5, Z13

Comments from many readers are greatly appreciated, including Joel Janetski, David Yoder, Steven Simms, and the participants at the 2010 Workshop of the Quantitative Society for Pensions and Saving.

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Abstract

This paper presents a pair of models of the storage of maize. One is directly based on standard financial models of portfolio choice. Rather than optimally balancing a financial portfolio by choosing from a variety of financial instruments, our agents optimize holdings of maize by choosing from a variety of storage locations. Agents face a tradeoff between the effort of transporting maize to high elevation granaries versus the safety they offer from theft. The second model uses a multi-period framework to look at the costs and benefits of building a granary in the first place. We use our models to extract a perceived probability of maize theft by outsiders among the Fremont Indians that lived in eastern Utah roughly 1700 - 700 years ago. We base our estimates on the caloric content of maize, the caloric cost of transporting it to granaries high above the valley floor where the maize was grown, and the costs of building and maintaining them. Our calculations show that a fairly low level of risk, on the order of 5% to 20%, could easily rationalize the use of cliff granaries.

1. Introduction

The Fremont people were a Native American cultural group that lived in what are now the states of Utah, Nevada & Colorado. Fremont cultivated maize in throughout this region, shifting at various times and places between from part-time horticulture and full-time farming, from roughly AD 300 to 1300[1]. To the south the Anasazi left behind numerous well-preserved pueblos and cliff houses. The Fremont left few such obvious dwellings. Fremont granaries, however, are relatively common and can be found throughout the area. These granaries are often found at higher elevations above the valley floor where the maize was grown. Granaries can be difficult to investigate and frequently require rappelling and rock climbing skills to access.[2]

An obvious question is why the Fremont felt compelled to store food in such inconvenient places. There are several possible answers.

First, it may be that cliffs simply offer better protection from pests such as insects or rodents than subsurface cists in rock shelters or storage pits in houses. Successful storage of maize requires that there be little loss to such pests. This, in turn, requires making the granary virtually airtight. A cliff face offers at least two impenetrable stone barriers in the floor and back wall. Where cracks or fissures compromise the bedrock barrier, these are filled with mud, sometimes still impressed with 1000-year-old fingerprints in the Range Creek granaries. Building a granary on the valley floor requires extra effort to make sure the floor and all walls are secure against pests. If granaries were placed on cliff ledges mainly to protect their contents from small pests, or to save time and labor builders should have always chosen the lowest available cliff. However, often they built granaries in locations that were multiple cliff bands above the maize fields and canyon floor, and which required climbing through or around up to seven separate sets of cliffs, with talus slopes between, to get up to the granaries.

Second, it may be that the visibility of cliff granaries makes internal monitoring easier. This would facilitate monitoring of family and community resources if people within the family, community or band were "stealing" from communal food stores by making the actions of the would-be freeloaders visible to all.

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[1] See Barlow (2006).

[2] See Barlow et al (2008) and Barlow (2010).

A third possibility is that there may have been some external threat of theft or robbery. This could have been from local bands or families of Fremont farmers and horticulturists living in Range Creek and the vicinity of Desolation Canyon, or perhaps Fremont farmers from neighboring areas such as the Uinta/Vernal region. Possibly even people from neighboring cultures, such as Puebloan farmers or Great Basin foragers[3]. Storing maize high on the canyon walls makes it more difficult to steal, particularly in a quick raid, when the owners of the grain may be vulnerable only for a short period of time.

This paper deals only with the third possibility, though the model presented could be modified to incorporate the other two incentives. If external threat was the main cause, how serious must the threat have been in order to justify the effort of building granaries in such locations and transporting maize to them?

Fremont farmers faced an interesting allocation decision. They could either store maize[4] near their dwellings where the cost of transporting it was negligible, but the loss during a raid would be substantial. Or they could store it in hard-to-access granaries high above the valley floor where the transport costs were high, but the loss during a raid would be much smaller. In fact, the decision would be even more complex than this, since there are a variety of places to build granaries offering various tradeoffs between transport cost and loss during a raid.

This paper posits that financial theory can help identify the perceived threat of a raid. Financial theory deals with the behavior of investors who need to allocate a sum of money, rather than maize. They must choose between a variety of investment assets, rather than granary locations. Some of these assets have high returns, but are risky (like storing maize on the valley floor). Others have lower returns, but are less risky (like storing maize in granaries).

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[3] Though this scenario is very unlikely given travel distances to Range Creek of hundreds of kilometers through rather extreme desert terrain, past other, more accessible Fremont farming communities.

[4] We refer to the storage of maize throughout the paper because virtually every granary in Range Creek shows evidence of storing maize. However, we note also that there is direct evidence that other naturally occurring grass seeds from plants such as wild Indian rice-grass and Great Basin wildrye were also stored in some granaries. The type of food and its method of production (i.e. farming or foraging) are irrelevant to our analysis which concentrates solely on storage.

Our approach here is similar in spirit to the work of McCloskey & Nash (1991) and Nielsen (1997) in that we are using general principles of economics in conjunction with data on grain storage to derive inferences about otherwise unobservable variables. In the case of McCloskey & Nash the variable of interest is the interest rate in early modern England. Nielsen is interested in the effect of British government policy on grain prices in the late 16th century. In our case the variable of interest is a perceived threat probability among Fremont Indians. Unlike McCloskey & Nash and Nielsen, we use a theoretic model and assumptions about fundamental parameters to back out threat probabilities. We do not do any econometric estimation. We note that agricultural economist have good theoretical models for the optimal storage of grains in modern market economies[5]. However, they are not well suited for the questions we address.

In this paper we rely on observations of granaries in one small portion of the Fremont cultural area, an area in eastern Utah known as Range Creek. Range Creek offers a unique sampling of Fremont archeological sites that have lain largely undisturbed since being abandoned by the Fremont in the thirteenth century. We construct a measure of caloric cost of transporting maize to and from a typical granary. Using data from modern studies of farmers in developing countries we can obtain estimates of the degree of risk aversion the Fremont would likely have had. With these data we can recover the perceived probability, or expectation of a raid by people from outside the community. For a typical granary in Range Creek this probability lies somewhere between 5% and 20% per year.

The rest of the paper proceeds as follows. Section 2 presents a standard portfolio model from the finance literature and adapts it to the circumstances of Fremont farmers storing grain. We show that optimal storage location(s) depend critically on how safety and transportation costs interact as granaries are built at higher and higher locations. Section 3 presents a special case of the model where only two storage locations exist: the valley floor and the "typical" granary. This model allows us to easily solve for the threat probability. Section 4 considers the costs and benefits of constructing the granary in the first place. We find slightly higher probabilities here, but they are not radically larger than the ones in section 3. Section 5 concludes the paper.

[5] See, for example Monterosso et al (1985) and Benirschka & Binkley (1995)

2. A Simple One-Period Financial Model

We assume that utility is derived from consumption of calories. We abstract from quality and varieties of food for simplicity, though these aspects of utility could be incorporated without altering our fundamental results. We adopt the commonly used mean-variance form of expected utility, originally developed by Markowitz[6] and others in the 50's and 60's. In our model expected utility is a positive function of the expected log of consumption of calories, and a negative function of the variance in the log of consumption of calories (indicating aversion to risk).

$$E\{U\} = E\{\ln C\} - \tfrac{\gamma}{2}V\{\ln C\} \tag{1}$$

Consumption of calories occurs (again, for the sake of simplicity) at the end of the winter after all storage results are known. Calories come from foraging, F, and from storage of a stock of maize, M, from the fall harvest. There are many discrete storage options, indexed by i, and the portion of M stored in location i is denoted, w_i. The net returns on storage will be negative due to spoilage and animal damage, but also due to theft by other humans. Net loss of maize as a percent of the maize stored at a particular location is denoted r_i, and is a random variable. This loss includes spoilage and animal damage as well as the non-random caloric cost of transporting the maize from the field to the storage location and from the storage location to the consumer's dwelling. In addition, it includes the opportunity cost of the time spent transporting, which could be used for foraging or other valuable activities. We denote this portion of the loss as s_i. Loss due to theft varies with the storage location, is a specific non-random value, denoted d_i, when it occurs. However, this loss only occurs if there is an incursion by outsiders which will occur with probability π. Consumption at the end of the winter is thus given by:

$$C = F + M\sum_i w_i(1+r_i)\,;\ \ 0 \le w_i \le 1, \sum_i w_i = 1 \tag{2}$$

The random return on storage at location i is given by:

$$r_i = \begin{cases} -s_i & with\ probability\ 1-\pi \\ -s_i - d_i & with\ probability\ \pi \end{cases} \tag{3}$$

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[6] See Markowitz (1952a, 1952b, & 1959).

We define $f \equiv F/M$ as the ratio of foraging calories to the caloric content of the harvest and treat this as exogenous for simplicity We note that an incursion by outsiders is likely to decrease the amount of food foraged since the members of the community will likely spend time and effort trying to expel them. s_f is the value of f in the absence of an incursion and is assumed to be a random variable. We denote the loss of foraging calories due to an incursion as d_f. Hence f is given by:

$$f = \begin{cases} s_f & with\ probability\ 1 - \pi \\ s_f - d_f & with\ probability\ \pi \end{cases} \quad (4)$$

The various storage options are sorted by difficulty of access from the valley floor, with higher values for i being more costly in terms of transport, but also being safer from theft should an incursion occur. i will be correlated with elevation above the valley floor, but other factors such as the roughness of the terrain and the unique topology of the exact site[7] will also contribute to large values of i.

s_i is the net storage cost for site i inclusive of transportation and because of this its mean (μ_i) is assumed to be strictly increasing in i. d_i is the loss in the event of an incursion for site i and is assumed to be strictly decreasing in i.

Storage losses are location specific and are random variables. They are independent of the probability of an incursion (π):$s_i \sim rv(\mu_i, \sigma_i^2)$.

Foraging income is also a random variable independent of π. Equations (3) and (4) and the assumptions on the distributions of the s_i's and f give the following means, variances, and covariances for $i \in \{1,2,\dots,I,f\}$

$$E\{r_i\} = -\mu_i - \pi d_i \quad (5a)$$

$$V\{r_i\} = \sigma_i^2 + d_i^2 \pi(1-\pi) \quad (5b)$$

$$C\{r_i, r_j\} = \sigma_{ij} + d_i d_j \pi(1-\pi) \quad (5c)$$

Taking the natural logarithm of (2) gives:

$$\ln C = \ln M + f + \sum_i w_i r_i \quad (6)$$

Taking expected values and variances of (6) and substituting various versions of (5) yields:

[7] Many granaries are accessible today only with the use of technical climbing gear.

$$E\{\ln C\} = \ln M + \mu_f - \pi d_f - \textstyle\sum_i w_i(\mu_i + \pi d_i) \quad (7)$$

$$V\{\ln C\} = \textstyle\sum_i w_i\big[\sigma_{if} + d_i d_f \pi(1-\pi)\big] + \sum_i \sum_j w_i w_j[\,\sigma_{ij} + d_i d_j \pi(1-\pi)] \quad (8)$$

Substituting (7) & (8) into (1) gives expected utility as a function of the means, variances and covariances of the returns of the various storage locations and the mean, variance and covariances of foraging income. It also depends on the probability of an incursion.

$$E\{U\} = \ln M + \mu_f - \pi d_f - \textstyle\sum_i w_i(\mu_i + \pi d_i) - \frac{\gamma}{2}\{\sum_i w_i\big[\sigma_{if} + d_i d_f \pi(1-\pi)\big]$$
$$+ \textstyle\sum_i \sum_j w_i w_j[\,\sigma_{ij} + d_i d_j \pi(1-\pi)]\} \quad (9)$$

The economic problem facing the storer/consumer is to maximize (9) by appropriate choice of the amounts stored in each location (the w_i's) subject to the following constraints:

$$0 \le w_i \ \ \forall i \quad (10)$$

$$\textstyle\sum_i w_i \le 1 \quad (11)$$

The typical first-order condition for this maximization problem (with respect to storage site *n*) is:

$$\frac{\partial \mathcal{L}}{\partial w_n} = -(\mu_n + \pi d_n) - \gamma\big[\sigma_{fn} + d_f d_n \pi(1-\pi)\big]$$
$$-\gamma \textstyle\sum_i w_i[\sigma_{in} + d_i d_n \pi(1-\pi)] - \lambda + \lambda_n = 0 \quad (12)$$

Imagine a perfectly safe location. Call this location *s*. This would be one with all the σ's and d_s equal to zero. (12) would reduce to $-\mu_s = \lambda - \lambda_s$ and as long as this location had some storage we would have $\lambda_s = 0$. Hence the value of λ is the negative of the net cost of storage in a completely riskless environment, $\lambda = -\mu_s$.

Now let us compare two locations, *m* and *n* with *n*>*m*.

$$\lambda_m = (\mu_m + \pi d_m) + \gamma\big[\sigma_{fm} + d_f d_m \pi(1-\pi)\big] + \gamma \textstyle\sum_i w_i[\sigma_{im} + d_i d_m \pi(1-\pi)] - \lambda \quad (13)$$

$$\lambda_n = (\mu_n + \pi d_n) + \gamma\big[\sigma_{fn} + d_f d_n \pi(1-\pi)\big] + \gamma \textstyle\sum_i w_i[\sigma_{in} + d_i d_n \pi(1-\pi)] - \lambda \quad (14)$$

We are interested in what conditions must exit so that $\lambda_n > 0$, i.e. so that the non-zero constraint at *n* is binding.

Let us assume that $\lambda_m = 0$, so that some maize is stored at location *m*.

Subtracting (13) from (14) gives:

$$\lambda_n = [\mu_n - \mu_m + \pi(d_n - d_m)] + \gamma\big[\sigma_{fn} - \sigma_{fm} + d_f(d_n - d_m)\pi(1-\pi)\big]$$

$+\gamma \sum_i w_i[\sigma_{in} - \sigma_{im} + d_i(d_n - d_m)\pi(1-\pi)]$

Regrouping terms:

$\lambda_n = (\mu_n - \mu_m) + (d_n - d_m)\pi[1 + \gamma(1-\pi)(d_f + \sum_i w_i\, d_i)]$

$+\gamma\pi(1-\pi)[\sigma_{fn} - \sigma_{fm} + \sum_i w_i(\sigma_{in} - \sigma_{im})]$

We make the following definitions:

$X \equiv \pi[1 + \gamma(1-\pi)(d_f + \sum_i w_i\, d_i)]$ and $Y \equiv \gamma\pi(1-\pi)[\sigma_{fn} - \sigma_{fm} + \sum_i w_i(\sigma_{in} - \sigma_{im})]$

We also define Δf_{ij} as the difference operator for a function f evaluated at locations i and j, i.e. f_i-f_j.

We get:

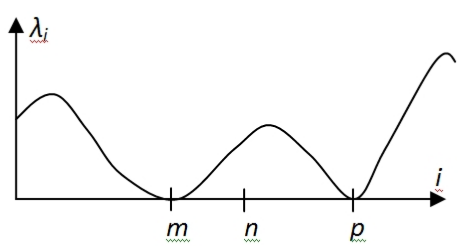
$\lambda_n = \Delta\mu_{nm} + \Delta d_{nm} X + Y$

Since the only difference between the average losses at the two locations is the transport cost, we can rewrite this as:

$$\lambda_n = \Delta t_{nm} + \Delta d_{nm} X + Y \qquad (15)$$

Suppose $n>m$. Since the transport cost is rising in i, $\Delta t_{nm} > 0$. Similarly, we have $\Delta d_{nm} < 0$.

The location of any other optimal storage locations depends on the 2nd differences of the transport and incursion loss functions.



The figure above illustrates a general case. Due to the relative shapes of the t and d functions, two storage locations are used: m and p. n is not used. Without further assumptions about the relative shapes of $t(i)$ and $d(i)$ we cannot make any more general statements.

If the $t(i)$ and $d(i)$ functions are linear, then λ_i will be a linear function of i. There are three possible cases in this situation. 1) λ_i is rising in i, in which case all storage will be at the lowest indexed location (the valley floor). 2) λ_i is falling in i, in which case all storage will be at the highest indexed location. 3) λ_i is constant at zero, in which case all storage locations will be used.

3. A Special Case

As a special case consider what would happen if there were only two locations. 1 is the valley floor with $\mu_1 = s$ and $d_1 = d > 0$ and 2 is the cliffs with $\mu_2 = s + t$ and $d_2 = 0$. Further assume that $d_f = 0$. These assumptions reduce (13) to:

$$t - \pi d = \gamma(w_2 d)\pi(1 - \pi)d \qquad (16)$$

We can solve for π in the cases where $w_2 = 0$ and $w_2 = 1$ to get limits on π. However, in order to do this we need to know the values of γ, t & d.

γ is the coefficient of risk aversion. We calibrate this based on studies of other societies at similar levels of development.[8] Values between 1.0 and 2.0 are fairly common in this literature. We try 2.0 as a ballpark estimate.

d is the proportion of maize stored on the valley floor lost if an incursion occurs. We set this to 50% and view the probabilities of an incursion we back out accordingly.

t is the cost of transporting maize to location n, along with any spoilage costs. This cost is expressed as a proportion of the maize transported. We normalize units using 1 bushel of maize, which is approximately the volume of a typical burden basket used by Native American women to haul plant foods and loads of materials[9]. The use of these baskets by the Range Creek Fremont is certain, as large fragments from a Fremont burden basket were recovered from a cache under a ledge near several granaries in the lower canyon[10]. Barlow (2002) notes that the caloric content of a bushel is 25.2 kg/bu x 3550 Kcal/kg = 89460 Kcal/bu. The caloric cost of transporting 1 bushel will depend on the weight of the individual and the maize as well as the distance and elevation gain. A reasonable range for adult weights of Fremont individuals is 100 – 160 lbs. Studies of calories expended for individuals weighing 130 (the average of our range) for various exercises give: 150/hr for light housecleaning, 650/hr for

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[8] See, for example, Binswanger (1980) and Moscardi & de Janvry (1977).

[9] Barlow et al. (1993).

[10] Barlow et al (2008)

rock climbing and 475 Kcal/hr for rappelling[11]. A good ballpark figure for the additional caloric cost of moving a basket of maize to a granary would be 600 Kcal/hr going uphill loaded with maize[12] and 300 Kcal/hr going downhill with an empty basket[13]. Assuming half an hour to ascend to the granary and 15 minutes to descend the total caloric expenditure would be 350 Kcal beyond that expended in normal activity. We use this as the cost to move the maize up to the granary. The cost to move it back down is calculated as half an hour at 500 Kcal/hr[14] (climbing up with an empty basket) and 15 minutes at 400 Kcal/hr[15] (climbing back down with a full basket), for a total excess caloric expenditure of 375 Kcal. In addition, time spend moving maize is time not available for foraging or other activities. The 1.5 hours required per load (up and down) implies a excess caloric cost of 4275 Kcal given the next best use of time is foraging for seasonally available indigenous foods such as Indian rice-grass, Great Basin wildrye, pine nuts or cattail roots (300 to 6000 Kcal/hr). We know that these were economic activities that Fremont Indians in Range Creek pursued, as we have clear archaeological evidence. Several liters of seeds of Indian rice-grass and Great Basin wildrye were actually found winnowed and parched in one Range Creek granary, radiocarbon dated to 1000 years ago[16]. The total caloric cost is 5000 Kcal, which implies a cost per calorie stored of 5.6%.

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[11] http://www.nutristrategy.com/activitylist4.htm

[12] 650 Kcal/hr from rock climbing effort, plus an additional 100 Kcal/hr from the extra weight of the maize, less 150 Kcal/hr that would've been expended in some alternative activity.

[13] 450 Kcal/hr from walking downhill, less 150 Kcal/hr.

[14] 100 Kcal/hr less than going uphill with a full load.

[15] 100 Kcal/hr more than going downhill with an empty basket.

[16] See Simms (1987) and Barlow (2006) for details. Figures there are reported as post-encounter caloric net gains, and do not include search costs. Post-encounter gains are very high for large mammals such as deer and bighorn sheep. We can be close to certain these were hunted from depictions of such hunts in local rock art of Fremont origin. In addition, archaeologists have recovered 1200 to 1000 year-old bones of bighorn sheep, cottontail and jackrabbits from Fremont pithouse villages, as well as bones of elk and mountain lion. The time spent finding such prey was much higher than that spent finding other forms of food. We have chosen caloric gains for foods that would have required relatively little search time. We compute our model with higher and lower values of foraging yields as well and report these results in Table 1.

Hence, we set $\gamma = 2.0$, $d = .5$, $t = .056$ and solve (16) for π. If maize is stored both on the valley floor and in granaries on the cliffs, the value of π must lie somewhere between 5.0% and 8.3%. The decision on where to store can be summarized as:

- For $\pi < 5.0\%$, store only on the valley floor.
- For $5.0\% < \pi < 8.3\%$, store both on the valley floor and in granaries on the cliffs.
- For $8.3\% < \pi$, store only in granaries on the cliffs.

These numbers depend crucially on the values of γ, d & t chosen. Table 1 shows the results of sensitivity analysis. The rows correspond to three different values of γ: 0.5, 1.0 & 2.0. The first column is our baseline where d=.5 and t=1.5 hours. The second column assumes a higher transport cost of t=3.0. The third column assumes a higher incursion loss of 95%. The 4th column assumes both a higher transport cost and a higher incursion loss. We also check for sensitivity to the opportunity cost of time by using a foraging return of 1500 Kcal/hr in column 5 and 6000 Kcal/hr in the final column.

The probabilities associated with at least some storage in granaries range from a low of 0.3% (with a high incursion loss, a low transport cost, and high risk aversion) to a high of 13.3% (with a low incursion cost, high transport cost, and low risk aversion). The probabilities associated with exclusive storage in granaries range from 1.9% to 17.3% for the same cases as above, though risk aversion does not have any effect on these values.

With the possible exception of the case of high transport costs and low incursion losses, and the case of high foraging yields, the implied probabilities of external threat seem quite low. This indicates that even quite small levels of threat would make it worthwhile to store maize in granaries on the cliff walls.

4. A Multi-Period Model of Granary Construction

Our analysis this far has abstracted from the cost of constructing granaries. This analysis would be correct if the granaries already existed. However, costs would be higher if it were necessary to construct the granary as well as transport maize to it.

The cost of constructing a granary, like that of transporting maize, consists of two types of costs: the direct additional caloric cost of moving the materials to the granary site, and the opportunity cost of the time spent building the granary. In the case of the granary, however, these costs are fixed costs. The construction costs are fixed regardless of the amount of grain actually stored, though clearly larger granaries would be both more expensive to construct and would hold more maize. The cost of maintaining a granary, mainly yearly repairs, would depend on how frequently it was used, but would still be independent of the amount of maize stored in any given year. Let the caloric cost of transporting the raw materials to the granary site plus the opportunity cost of time spent building the granary be G. Similarly, let the transport cost of materials and the opportunity cost of time spent for repair each year be H. Finally, let the useful lifetime of the granary be denoted T.

The most common granaries in Range Creek are groups of cylindrical or D-shaped, top-loading granaries built into cliffs. These are usually found in groups of two to five with accretional construction incorporating the adjacent walls of earlier granary chambers, rather than all granaries being built at the same time[17]. Construction materials that must be hauled up the cliff to build them, per typical granary chamber, include 8 to 15 courses of large, unshaped sandstone slabs, one to three basketloads of wet adobe or mud to act as mortar for the slabs to seal the circular, shaped sandstone granary lid, and four to eight structural timbers approximately 1.6 to 1.8 meters long by 4 to 8 cm in diameter. As an initial parameterization we assume this cost is 1750. That is, the caloric cost of preparing and transporting the materials is the same as the cost of transporting five burden baskets of maize to the site[18].

The opportunity cost of building the granary is assumed to be 3000 Kcal per hour as before. We assume as an initial guess that constructing a granary took two people two 12-hour days of work. This gives 48 hours of effort at 3000 Kcal for a total opportunity cost of 144,000. The total construction cost is thus, G=145,750.

Maintenance costs would have been a fraction of this figure, probably requiring only a few hours each year. We assume 4 hours or one-twelfth cost for H = 12,146.

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[17] Barlow et al. (2008)

[18] We are assuming roughly 2 loads of clay carried from the valley floor and 2 loads of stone carried from closer locations, but weighing more, plus the equivalent of 1 load of maize for transport of wood of various types.

Construction and maintenance costs will vary with the size of the granary. We have assumed a capacity of 20 bushels (a typical value for Range Creek) when choosing our numbers above. We denote the caloric storage capacity of the granary as Q.

In order for construction of a granary to make economic sense the discounted expected benefit over the life of the granary must exceed the sum of the construction cost plus the discounted expected maintenance and transportation costs. Keeping the assumption of only two locations from section 3, denote the cost and benefit of building a granary as K & B, respectively.

$$K = G + E\left\{\sum_{i=1}^{T} \beta^{i-1}(H + QV(i)t)\right\}$$

$$B = E\left\{\sum_{i=1}^{T} \beta^{i-1}\pi(i)dQV(i)\right\}$$

Here β is the subjective rate of time preference, $V(i)$ is the percent of granary capacity used in period i, and $\pi(i)$ is the probability of an incursion in period i.

Our risk-averse farmers lose $-s - \pi d - \frac{\gamma}{2}[\sigma_1^2 + \pi(1-\pi)d^2]$ in utility for every unit of maize transferred from the valley floor. They gain $-s - t - \pi d - \frac{\gamma}{2}\sigma_2^2$ in utility for every unit transferred to the new granary. Since $\sigma_1^2 = \sigma_2^2 = 0$, they have the following additional utility if the granary is built:

$$\Delta = \sum_{i=1}^{T} \beta^{i-1} QV(i)\{\pi(i)d - t + \frac{\gamma}{2}\pi(i)[1-\pi(i)]d^2\} - G + H \tag{17}$$

If we assume the incursion probability is constant over time, that the granary is filled to capacity every year, and that the effective life of the granary (with proper maintenance) is infinite[19], then (17) becomes:

$$\Delta = \frac{Q[\pi d - t + \frac{\gamma}{2}\pi(1-\pi)d^2]}{1-\beta} - G + H \tag{18}$$

We need to find the value of π that makes Δ greater than zero. That is, find the threat probability for which it makes sense to build and maintain a granary. This is found by solving the following quadratic equation.

$$\left[-\frac{\gamma}{2}Qd^2\right]\pi^2 + \left[Qd + \frac{\gamma}{2}Qd^2\right]\pi - [(H-G)(1-\beta) - Qt] > 0 \tag{19}$$

At the other extreme, if the planned life of the granary is only one year, then (17) becomes:

[19] Given these granaries are still standing today, this assumption is not as unreasonable as it may first seem.

$$\Delta = Q\{\pi d - t + \tfrac{\gamma}{2}\pi(1-\pi)d^2\} - G + H \tag{20}$$

And the critical values of π are found by solving (21).

$$\left[-\tfrac{\gamma}{2}Qd^2\right]\pi^2 + \left[Qd + \tfrac{\gamma}{2}Qd^2\right]\pi - [H - G - Qt] > 0 \tag{21}$$

The solutions to (19) and (21) are given in Table 2 for a variety of parameter values. As before, we consider values of γ equal to .5, 1.0 and 2.0. We also consider values of β equal to .9 and .7. The first two columns correspond to our baseline case discussed above. The second pair of columns corresponds to a higher transport cost. The third pair corresponds to the case of greater loss should an incursion occur. The fourth and fifth pairs correspond to low and high foraging yields, respectively. The sixth set of columns assumes a construction cost twice as high as the baseline case, and the final pair of columns assumes this cost is half the baseline.

As before, the perceived threat levels are all fairly low. For the high incursion loss case, the probabilities need only be 3% or less in order to make building a granary worthwhile. Even the most pessimistic cases, where risk aversion is low and 1) the cost of construction is high, 2) the transport cost is high, or 3) the opportunity costs of foraging are high, have perceived threats of only one in three or four.

5. Conclusion

This paper has used a simple one-period financial model and a simple multi-period economic production model to assess the probability of external raid on the stored maize of Fremont Indian farmers living in eastern Utah between AD 300 and 1300. We have roughly calibrated the model based on observed granaries in Range Creek, but similar granaries are found throughout the Fremont culture region, and some of the areas to the south occupied by farmers in the Pueblo region.

We find fairly low levels of threat can rationalize the building of granaries high on the side of canyons. In the case of existing granaries a threat in the range of 2% to 10% could have been enough to make transporting maize from the valley floor to higher elevations worth the cost. If the granaries did not exist and needed to be constructed first, we find threats in the range of 5% to 20% could have been sufficient.

Our model is appropriate if the only advantage of a granary is reducing losses due to external theft or robbery. However, the model could easily be modified to include two other explanations: construction/storage advantages of cliff granaries, and the need to control maize distribution within the farming group.

A construction or storage advantage on cliff granaries due to pest-impenetrable walls, for example, would only lower the threat necessary for higher elevation granary storage to make economic sense. It is possible that if this advantage were large enough maize would have been stored here even if the external threat were zero. Unfortunately, we have little in the way of data to indicate how large this construction/storage advantage might actually have been, making it impossible to incorporate these numbers into our calculations.

Our model could be easily modified to include internal as well as external threats. If the main objective of granaries was to deter filching of maize by members of the community or family that stored the maize, we could still model the losses as we have in this paper. The challenge would be to quantify the losses to the decision-maker(s). If grain was taken by members of the farming group it need not have been the same as the complete loss when grain was stolen by outsiders.

What does seem likely for the Range Creek Fremont is that storage of maize in cliff granaries made sense economically, even at low levels of expected confrontation or raids from people outside the small farming communities or families that stored the maize. The cliff granaries aided in the defense of the maize from theft by "outsiders," perhaps from Fremont foragers and horticulturists living in mobile, roving bands in nearby Desolation Canyon or the Tavaputs Plateau, perhaps from other, less successful Fremont farmers who lived in nearby communities, or , though less likely, perhaps even contemporary people from the Great Basin or nearby Puebloan cultures.

Table 1

Sensitivity Analysis for Implied Threat Probabilities from a One-Period Model

	baseline	higher transport cost	higher incursion loss	both	lower foraging yield	higher foraging yield
				γ=0.5		
transport hrs	1.5	3.0	1.5	3.0	1.5	1.5
incursion loss	50%	50%	95%	95%	50%	50%
foraging yield	3000	3000	3000	3000	1500	6000
lower	6.3%	13.1%	0.8%	1.6%	3.4%	12.4%
upper	8.3%	17.1%	1.9%	4.0%	4.5%	16.2%
				γ=1		
transport hrs	1.5	3.0	1.5	3.0	1.5	1.5
incursion loss	50%	50%	95%	95%	50%	50%
foraging yield	3000	3000	3000	3000	1500	6000
lower	5.0%	10.6%	0.5%	1.0%	2.7%	9.97%
upper	8.3%	17.1%	1.9%	4.0%	4.5%	16.2%
				γ=2		
transport hrs	1.5	3.0	1.5	3.0	1.5	1.5
incursion loss	50%	50%	95%	95%	50%	50%
foraging yield	3000	3000	3000	3000	1500	6000
lower	3.6%	7.5%	0.3%	0.6%	1.9%	7.1%
upper	8.3%	17.1%	1.9%	4.0%	4.5%	16.2%

foraging yields are in Kcal / hr.

Table 2

Sensitivity Analysis for Implied Threat Probabilities from a Multi-Period Model

	baseline		higher transport cost		higher incursion loss		lower foraging yield		higher foraging yield		higher construction cost		lower construction cost	
							γ=0.5							
discount factor	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7
incursion loss	50%	50%	50%	50%	95%	95%	50%	50%	50%	50%	50%	50%	50%	50%
transport hrs	1.5	1.5	3.0	3.0	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
days	2	2	2	2	2	2	2	2	2	2	4	4	1	1
foraging yield	3000	3000	3000	3000	3000	3000	1500	1500	6000	6000	3000	3000	3000	3000
T=infinity	8.1%	10.0%	15.9%	17.8%	1.2%	1.5%	4.3%	5.3%	16.0%	19.9%	9.0%	12.8%	7.6%	8.6%
T=1	16.7%	16.7%	24.8%	24.8%	2.6%	2.6%	8.6%	8.6%	33.7%	33.7%	26.5%	26.5%	11.9%	11.9%
							γ=1							
discount factor	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7
incursion loss	50%	50%	50%	50%	95%	95%	50%	50%	50%	50%	50%	50%	50%	50%
transport hrs	1.5	1.5	3.0	3.0	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
days	2	2	2	2	2	2	2	2	2	2	4	4	1	1
foraging yield	3000	3000	3000	3000	3000	3000	1500	1500	6000	6000	3000	3000	3000	3000
T=infinity	7.1%	8.8%	14.0%	15.8%	0.9%	1.1%	3.8%	4.6%	14.1%	17.6%	7.9%	11.3%	6.7%	7.5%
T=1	14.7%	14.7%	22.0%	22.0%	1.8%	1.8%	7.5%	7.5%	30.3%	30.3%	23.6%	23.6%	10.4%	10.4%
							γ=2							
discount factor	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7	0.9	0.7
incursion loss	50%	50%	50%	50%	95%	95%	50%	50%	50%	50%	50%	50%	50%	50%
transport hrs	1.5	1.5	3.0	3.0	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
days	2	2	2	2	2	2	2	2	2	2	4	4	1	1
foraging yield	3000	3000	3000	3000	3000	3000	1500	1500	6000	6000	3000	3000	3000	3000
T=infinity	5.7%	7.0%	11.3%	12.7%	0.5%	0.7%	3.0%	3.7%	11.4%	14.2%	6.3%	9.1%	5.3%	6.0%
T=1	11.8%	11.8%	17.8%	17.8%	1.1%	1.1%	6.0%	6.0%	24.7%	24.7%	19.1%	19.1%	8.4%	8.4%

foraging yields are in Kcal / hr.

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