Advance-Purchase Programs: When to Introduce and What to Inform Consumers

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Abstract. We study a two-stage model in which the information processed by consumers at the first stage (advance-purchase stage) is endogenously determined. In the model, the firm decides whether to introduce an advance-purchase program, chooses what attribute information to disclose and determines an advance-purchase price and a retail price. Forward-looking consumers strategically choose, based on the disclosed information, to buy in advance or to make a purchase decision at the second stage (retail stage) when all information is revealed. We characterize the firm's optimal choice on the advance-purchase program and the strategy of information disclosure. In particular, we show that the firm always prefers to introduce the advance-purchase program except when underlying consumer preferences are extremely homogenous. In addition, we find that fully revealing horizontal product information at the advance-purchase stage is never optimal to the firm, but revealing either partial or no product information can be optimal depending on the underlying consumer preferences. Our finding that partial information disclosure is sometimes optimal to the firm is in contrast to the result in the literature of horizontal information provision that a firm maximizes profit by revealing either no or full information to consumers.

Key words: advance purchase, information disclosure, dynamic pricing

JEL Classification: L12, M37.

1   Introduction

Advance-purchase programs (or pre-order programs) are widely used in the sale of future release products such as DVDs, books, albums, software, etc. An extensive literature has studied the condition necessary for a profit advantage from an advance-purchase program under the assumption that the information processed by consumers at the advance-purchase stage is taken to be exogenous (Gale and Holmes, 1992, 1993, Dana, 1998, 1999, Xie and Shugan, 2001, Möller and Watanabe, 2009, and Nocke and Peitz forthcoming). In reality, advance-purchase programs are, however, frequently accompanied by advertising and marketing activities that may contain product information. Thus, a firm has a substantial control over what the information to reveal to consumers at the advance-purchase stage. The revealed information influences consumers' expected valuations which in turn affect their purchase decisions. An important decision for the firm, then, is what to inform consumers. Examples of such a decision include, but not limited to, what contents in a trailer to be displayed for a future release DVD movies, what chapters in a coming-soon book to be excepted to show online to readers and which sample song to be played for a new album.

In this paper, we develop a simple dynamic model with an advance-purchase stage and a subsequent retail stage to study the firm's decision to reveal product information to consumers in the advance-purchase stage. In the model, consumers arrive (or become aware of the product) at each stage. The firm sells a product with two attributes: X and Y. We assume that Y attribute has a larger variance in consumer expected valuation which implies that the underlying consumer preferences are more diverse. At the advance-purchase stage, consumers initially have a same expected valuation though the ex-post realizations may differ. The firm decides whether to disclose the information on certain attribute(s). In particular, the firm has the options to reveal only X attribute, only Y attribute, both or neither of the attributes. The disclosed information facilitates consumers' learning of their individual valuations on the revealed attribute(s). In addition, the firm decides whether to introduce an advance-purchase program and determines an associated price. At the retail stage, the firm chooses a retail price as all product information becomes available to consumers. On the other side, consumers are forward-looking and strategically choose, based on the disclosed information, to buy in advance or to make purchase decisions at the retail stage.

We find that the firm's choice of advance-purchase program and the decision to reveal product

attribute importantly depend on the nature of the market. To facilitate analysis, we adopt a terminology, which is similar to Johnson and Myatt (2006), to categorize four types of markets, *Mass, Large, Small* and *Niche* market, based on the underlying consumer valuations. Generally speaking, if consumers' valuations for the product are relatively homogenous (heterogenous), the firm will choose to serve a large (small) fraction of consumers. Thus, we label the market according to the market size the firm chooses to serve when consumers are fully informed their valuations. We will give precise definitions for each type of market in section 3.

We first provide a simple condition under which it is optimal for the firm to introduce the advance-purchase program. In particular, we find that the firm always prefers to introduce the advance-purchase program except in the Mass market in which the underlying consumer preferences are extremely homogenous. We then show that fully revealing product information under the advance-purchase program is never optimal to the firm, but revealing either partial product information or no product information can be optimal depending on the underlying consumer preferences. Furthermore, we show that the equilibrium under the advance-purchase program can only possibly be two cases: a *Pooling Equilibrium* in which the firm discloses no information and all consumers at the advance-purchase stage purchase in advance and a *Separating Equilibrium* in which the firm reveals the information only on X attribute and consumers with a high realized valuation on X attribute buy at the advance-purchase stage while the remaining consumers wait and make purchase decisions at the retail stage. Finally, we show that the *Separating Equilibrium* arises in the Large market when the variance of Y attribute is sufficiently high while the *Pooling Equilibrium* emerges in the complementary situations. The firm's optimal choice of advance-purchase program and the decision on information disclosure is summarized in *Table 1*.

	Advance-purchase program	Attribute disclosed
Mass Market	No	None
Large Market with a high variance in Y	Yes	X only
Large Market with a low variance in Y	Yes	None
Small Market	Yes	None
Niche Market	Yes	None

Table 1

4

Our paper is related to the literature on advance purchase and informative advertising. While both are often used in practice, the existing papers only study advance-purchase program and informative advertising in separation. The literature on advance purchase has mainly focused the role of capacity constrain and aggregate demand uncertainty (Gale and Holmes, 1992, 1993, Dana, 1998, 1999, and Xie and Shugan, 2001). A couple of recent papers by Möller and Watanabe (2009) and Nocke and Peitz (forthcoming) show that advance purchase programs can be optimal when capacity is not a constrain or can be chosen ex-ante. In these papers, the information processed by consumers are assumed exogenous and thus, informative advertising plays no role. In contrast, our paper incorporates the firm's information disclosure decision and shows that how the firm may use informative advertising and advance purchase program complementarily to achieve profit maximization.

There is a small literature on horizontal information provision. Lewis and Sappington (1994) and Johnson and Myatt (2006), in a similar vein, examine models in which a monopoly firm chooses the degree of preciseness of a signal on horizontal product attributes. A more precise (or informative) signal allows consumers to have a better knowledge about how well the product matches their preferences. Focusing on static models, the literature draws a general conclusion that, as Bagwell (2007) notes, "the monopolist's expected profit often achieves its maximum at one of the extremes, thus a profit-maximizing monopolist either provides no or perfect information about product attributes"[1] Interestingly, by examining the incentive of information provision in a dynamic model with the possibility of advance-purchase, we find that partial information disclosure can be optimal to the firm.

The rest of the papers is organized as follows. In section 2, we develop a simple dynamic model with an advance-purchase stage and a retail stage. Section 3 analyzes the model and characterizes the firm's optimal strategy on the choice of the advance-purchase program and the decision of information disclosure. Section 4 concludes.

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[1]See Bagwell (2007), page 1784-1785.

2 The Model

A monopoly firm sells a new product which has two independent attributes, X and Y. Consumers are ex-ante identical and risk-neutral, each demands at most one unit of the product. A consumer's individual valuation is

$$V_i = \bar{w} + X_i + Y_i \tag{1}$$

where $\bar{w}$ is a fixed amount of expected utilities derived from the product and X_i (Y_i) is the consumer's expected utility on attribute X (Y). As shown below, the value of X_i can be either high (x_h) with probability $\alpha \in (0,1)$ or low (x_l) with probability $1 - \alpha$. Similarly, the value of Y_i can be either high (y_h) with probability $\beta \in (0,1)$ or low (y_l) with probability $1 - \beta$. The product can be produced at a constant marginal cost c. All information is common knowledge.

$$X_i = \begin{cases} x_h & prob = \alpha \\ x_l & prob = 1 - \alpha \end{cases} \quad \text{and} \quad Y_i = \begin{cases} y_h & prob = \beta \\ y_l & prob = 1 - \beta \end{cases}. \tag{2}$$

Note that a consumer derives at least x_l utility from X attribute and at least y_l from Y attribute. In other words, the only uncertainty for the consumer is $x_h - x_l$ in X attribute and $y_h - y_l$ in Y attribute. Hence, a consumer's utility function described in (1) can be normalized and written as

$$V_i = w + \tilde{X}_i + \tilde{Y}_i \tag{3}$$

where

$$\tilde{X}_i = \begin{cases} x & prob = \alpha \\ 0 & prob = 1 - \alpha \end{cases} \quad \text{and} \quad \tilde{Y}_i = \begin{cases} y & prob = \beta \\ 0 & prob = 1 - \beta \end{cases} \tag{4}$$

with $w = \bar{w} + x_l + y_l$, $x = x_h - x_l > 0$ and $y = y_h - y_l > 0$. Without loss of generality, we assume that $x \leq y$. Table 2 displays the possible realized consumer valuations and their associated probabilities.

Valuations	Probability
w	$(1 - \alpha)(1 - \beta)$
$w + x$	$\alpha(1 - \beta)$
$w + y$	$(1 - \alpha)\beta$
$w + x + y$	$\alpha\beta$

Table 2

In the following, we perform the analysis on the normalized framework as in (3) and (4). However, the results should be readily applied to the more general framework as in (1) and (2).

The model has two stages: an advance-purchase stage, $t = 0$ and a subsequent retail stage, $t = 1$. A positive measure of consumers arrive or become aware of the product at each stage. In particular, a unit measure of consumers enter the market at $t = 0$ while an additional m units of consumers arrive at $t = 1$. Thus, m captures the relative market size of consumers at the advance-purchase stage to the retail stage. At the advance-purchase stage, consumers initially do not know their valuations on neither X nor Y attribute. The firm decides what attribute(s) to reveal to consumers. In particular, the firm can choose to reveal only X, only Y, both or neither of the attributes. Consequently, consumers learn their individual valuations on the attributes that the firm chooses to reveal. Accompanied with the information disclosure, the firm also decides whether to introduce an advance-purchase program and determines the associated price p_0 for advance purchase. At the retail stage, the market is populated with the consumers who arrive at $t = 1$ and the consumers who arrived at $t = 0$ but did not make advance-purchases. All information on product attribute is assumed to be fully revealed to consumers.[2] The firm chooses a retail price p_1 and consumers make purchase decision accordingly.

Our focus is the firm's strategy on information disclosure. For simplicity, we make the following assumptions. (1) The information disclosure is costless. Thus, the firm strategically chooses what information to provide; (2) There is a positive but ε small cost associated with the advance-purchase program. By this assumption, we rule out the possibility of tie profits and avoid the complexity

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[2]This may be due to the availability of the product on market. For instance, consumers may visit a store to examine the product or obtain information through consumer review reports, forums, blogs or word of mouth.

arising from mixed strategies; (3) A consumer buys the product if both purchase and no purchase lead to a same expected payoff; (4) The marginal production cost is smaller than the fixed utility derived from the product, that is $c < w$. This ensures that the firm is able to make a positive sale even if consumers are not informed any attribute information; (5) The firm cannot commit to future prices; (6) The possibility of refund and product return is not considered.

The firm's strategy portfolio includes $Q = \{AP, NAP\}$, $S = \{\Phi, X, Y, XY\}$, $p_0 \in R^+$ and $p_1 \in R^+$ where (i) AP and NAP represent, respectively, introducing an advance-purchase program or not; (ii) S is the set of choices on information disclosure: revealing information on neither attribute, only X attribute, only Y attribute or both attributes; (iii) p_0 and p_1 are the advance-purchase price and retail price respectively. On the other side, consumers arriving at $t = 0$ decide whether to purchase in advance at the price p_0, buy in the retail stage at the price p_1 or exist the market without any purchase. Consumers arriving at $t = 1$ only have the options of purchasing at the retail stage or existing without a purchase.

3   Analysis

We use backward induction to solve the dynamic model. Our analysis will proceed as follows. We first provide a simple condition under which it is optimal for the firm to introduce the advance-purchase program. We then show that revealing both X and Y attribute or only Y attribute at the advance-purchase stage is never optimal to the firm. Thus, provided that the advance-purchase program is offered, the firm will only choose to reveal only X attribute or neither attribute. Finally, we identify the condition under which revealing only X attribute and neither attribute, respectively, is optimal.

Since consumers are fully informed about the product attributes at $t = 1$ it implies that the possible consumer valuations are w, $w + x$, $w + y$ and $w + x + y$. It follows that selling at a price that is not equal to any of the possible consumer valuations is not a profit-maximizing strategy.[3] Thus, we have the following lemma.

Lemma 1 *The optimal retail price can only be* w, $w + x$, $w + y$ *or* $w + x + y$.

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[3]For example, suppose that $p_1 \in (w, w + x)$. The firm then can sell to a same amount of consumers by charging a higher price $w + x$ and thus, earn a higher profit

We first consider the situation where the firm chooses not to introduce the advance-purchase program. Without the possibility of selling in advance, the firm only determines a price at the retail stage. By Lemma 1, the firm's optimal retail price p_1^{NAP} can be four possibilities. Accordingly, we define the type of a market as follows.

Definition 1 *(1) A market is defined, respectively, as a Mass, Large, Small and Niche market if it is optimal for the firm to charge a retail price p_1^{NAP} that is equal to w, $w + x$, $w + y$ and $w + x + y$ when consumers are fully informed about the product attributes.*

Table 3 summarizes the optimal price, sales volume and the profit in each type of market. From the Mass market to the Niche market, the price charged by the firm increases while the sales volume declines.

Market Type	Optimal Price	sales volume	Profit Π^{NAP}
Mass	$p_1^{NAP} = w$	$(1 + m)$	$(1 + m)(w - c)$
Large	$p_1^{NAP} = w + x$	$(1 + m)(\alpha + \beta - \alpha\beta)$	$(1 + m)(\alpha + \beta - \alpha\beta)(w + x - c)$
Small	$p_1^{NAP} = w + y$	$(1 + m)\beta$	$(1 + m)\beta(w + y - c)$
Niche	$p_1^{NAP} = w + x + y$	$(1 + m)\alpha\beta$	$(1 + m)\alpha\beta(w + x + y - c)$

Table 3

We first consider the situation where that the firm chooses to introduce the advance-purchase program. The firm has multiple options on what information to reveal to consumers. In particular, the firm can choose to reveal neither, both or one of the attributes. In any case, a positive sale has to be induced at the advance-purchase stage. Otherwise, the firm would be better off by not introducing the advance-purchase program due to a positive introduction cost. On the other hand, consumers will purchase in advance only if they expect gaining a higher expected utility

$$E\left(V_i|\Omega\right) - p_0 \geq E\left(V_i|XY\right) - p_1 \tag{5}$$

or, stating differently, if the firm offers a sufficiently low advance-purchase price

$$p_0 \leq p_1 - \left[E\left(V_i|XY\right) - E\left(V_i|\Omega\right)\right] \tag{6}$$

9

where $\Omega \in \{\Phi, X, Y, XY\}$ is the consumer information set. In the following, we discuss in turn the firm's choice on information disclosure.

Case 1 *Neither X nor Y attribute is revealed.*

Provided that no information has been disclosed, consumers arriving at $t = 0$ have a same expected valuation $w + \alpha x + \beta y$. To make a positive sale at $t = 0$ the firm has to charge a sufficiently low price p_0 that satisfies (6). The equilibrium is such that the firm introduces the advance-purchase program, sells to all the consumers arriving at $t = 0$ for the advance-purchase price p_0 and charges the consumers arriving at $t = 1$ for the retail price p_1. We show in the following lemma that the advance-purchase program is always preferred except in the Mass market.

Lemma 2 *Provided that neither attribute is revealed at the advance-purchase stage, the firm can always earn a higher profit by introducing the advance-purchase program except in the Mass market.*

Proof. If the firm chooses to introduce the advance-purchase program and discloses no information, she earns profit

$$(p_0 - c) D_0 + (p_1 - c) D_1 \tag{7}$$

where D_0 (D_1) is the number of consumers who purchase at $t = 0$ $(t = 1)$. Note that consumers arriving at $t = 0$ are willing to purchase in advance only if

$$
\begin{aligned}
w + \alpha x + \beta y - p_0 \quad > \quad & \alpha\beta \max\{w + x + y - p_1, 0\} + (1 - \alpha)\beta \max\{w + y - p_1, 0\} \\
& + \alpha(1 - \beta)\max\{w + x - p_1, 0\} + (1 - \alpha)(1 - \beta)\max\{w + x - p_1, 0\}
\end{aligned}
$$

where LHS is the expected utility from advance-purchase and RHS is the expected utility from waiting to make a purchase decision at $t = 1$. Thus, the firm will price optimally at

$$p_0 = w + \alpha x + \beta y - \left[\begin{array}{c} \alpha\beta \max\{w + x + y - p_1, 0\} + (1 - \alpha)\beta \max\{w + y - p_1, 0\} \\ + \alpha(1 - \beta)\max\{w + x - p_1, 0\} + (1 - \alpha)(1 - \beta)\max\{w + x - p_1, 0\} \end{array} \right] \tag{8}$$

We discuss the sub-cases for the Mass, Large, Small and Niche market respectively.

(i) **Mass market.** It follows that the optimal price at the retail stage is $p_1^{AP-\Phi} = w$. From (8),

10

$p_0 = w$. Thus, the profit is

$$\Pi^{AP-\Phi} = (1 + m)(w - c).$$

However, if the firm chooses not to introduce the advance-purchase program, she obtains, from *Table 3*, the same amount of profit. Hence, taking into account the positive introduction cost, the firm prefers to sell without the advance-purchase program.

(ii) **Large market.** It follows that $p_1^{AP-\Phi} = w + x$. From (8), $p_0 = w + (\alpha + \beta - \alpha\beta)x$. We have

$$
\begin{aligned}
\Pi^{AP-\Phi} &= w + (\alpha + \beta - \alpha\beta)x - c + m(\alpha + \beta - \alpha\beta)(w + x - c) \\
&= (\alpha + \beta - \alpha\beta)(w + x - c) + (1 - \alpha)(1 - \beta)(w - c) + m(\alpha + \beta - \alpha\beta)(w + x - c) \\
&> (\alpha + \beta - \alpha\beta)(w + x - c) + m(\alpha + \beta - \alpha\beta)(w + x - c) = \Pi^{NAP}
\end{aligned}
$$

where $w + (\alpha + \beta - \alpha\beta)x - c$ is the profit earned at the advance-purchase stage and $m(\alpha + \beta - \alpha\beta)(w + x - c)$ is the profit obtained at the retail stage.

(iii) **Small market.** It follows that $p_1^{AP-\Phi} = w + y$. From (8), $p_0 = w + \alpha(1 - \beta)x + \beta y$. We have

$$
\begin{aligned}
\Pi^{AP-\Phi} &= w + \alpha(1 - \beta)x + \beta y - c + m\beta(w + y - c) \\
&= \beta(w + y - c) + (1 - \beta)(w + \alpha x - c) + m\beta(w + y - c) \\
&> \beta(w + y - c) + m\beta(w + y - c) = \Pi^{NAP}.
\end{aligned}
$$

(iv) **Niche market.** It follows that $p_1^{AP-\Phi} = w + x + y$. From (8), $p_0 = w + \alpha x + \beta y$. We have

$$
\begin{aligned}
\Pi^{AP-\Phi} &= w + \alpha x + \beta y - c + m\alpha\beta(w + x + y - c) \\
&= \alpha\beta(w + x + y - c) + (1 - \alpha\beta)(w - c) + \alpha(1 - \beta)x + \beta(1 - \alpha)y + m\alpha\beta(w + x + y - c) \\
&> \alpha\beta(w + x + y - c) + m\alpha\beta(w + x + y - c) = \Pi^{NAP}.
\end{aligned}
$$

∎

The intuition behind the Lemma 2 is as follows. As no information is revealed at the advance-

purchase stage, the expected valuations of consumers are kept identical. This allows the firm to capture some additional consumer surplus. However, the ability of extracting consumer surplus depends on whether the firm is able to commit to a higher retail price if consumers would not purchase in advance. In the Mass market, the firm can only commit to a retail price w if no purchase occurs at $t = 0$. Consequently, the firm is not able to extract any additional consumer surplus through the advance-purchase program.

For future reference, we list the profit (net of the cost associated with the advance-purchase program) if the firm chooses not to reveal any attribute information under the advance-purchase program.[4]

$$
\pi^{AP-\Phi} = \begin{cases}
w + mw & if \quad p_1^{AP-\Phi} = w \\
w + (\alpha + \beta - \alpha\beta) x - c + m (\alpha + \beta - \alpha\beta) (w + x - c) & if \quad p_1^{AP-\Phi} = w + x \\
w + \alpha (1 - \beta) x + \beta y - c + m\beta (w + y - c) & if \quad p_1^{AP-\Phi} = w + y \\
w + \alpha x + \beta y - c + m\alpha\beta (w + x + y - c) & if \quad p_1^{AP-\Phi} = w + x + y
\end{cases}
\tag{9}
$$

Case 2 *Both X and Y attributes are revealed.*

Lemma 3 *Revealing both X and Y attributes at the advance-purchase stage cannot be an optimal strategy.*

Proof. We first note that since all information is revealed at $t = 0$, it has to be true that $p_0 = p_1$ in the equilibrium. We show it by contradiction. Suppose that $p_0 > p_1$. No consumers will purchase at $t = 0$ since buying at $t = 1$ will yield a higher utility. Suppose that $p_0 < p_1$. Consumers arriving at $t = 0$ with valuation below p_0 will not purchase neither at $t = 0$ nor at $t = 1$. In other words, pricing at p_1 is solely based on the consumer demand at $t = 1$. It follows that pricing at p_1 (instead of p_0) at $t = 0$ will lead to a higher profit. Therefore, we have $p_0 = p_1$ in the equilibrium. However, the firm can gain the same amount of profit and save the set-up cost by not introducing the advance-purchase program but just selling at the retail stage for a price p_1. Therefore, the strategy of revealing all information at the advance-purchase stage cannot be optimal. ∎

Intuitively, once consumers are informed all product information at the advance-purchase stage, the firm has to charge a same advance-purchase price as the retail price p_1. However, the firm can

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[4]The profits are computed in the proof of Lemma 2.

achieve the same amount of profit by selling at the same retail price p_1 without the advance-purchase program. This implies that the firm gains no extra benefit from disclosing all information but incurring the introduction cost of the advance-purchase program.

Case 3 *Only X attribute is revealed.*

In this case, consumers arriving at $t = 0$ are separated into two groups: consumers informed with $X = x$ and with $X = 0$. This provides an opportunity for the firm to practice price discrimination. In particular, there may exist an equilibrium such that the consumers informed with $X = x$ at $t = 0$ purchase in advance at p_0 while the consumers who are informed with $X = 0$ at $t = 0$ and who arrive at $t = 1$ make purchase decisions under a retail price p_1. In such an equilibrium, consumers informed with $X = x$ choose to purchase in advance only if, from (5),

$$w + x + \beta y - p_0 > \beta Max\left\{w + x + y - p_1, 0\right\} + (1 - \beta) Max\left\{w + x - p_1, 0\right\}$$

or if the firm charges the advance-purchase price

$$p_0 < w + x + \beta y - \beta Max\left\{w + x + y - p_1, 0\right\} - (1 - \beta) Max\left\{w + x - p_1, 0\right\}. \tag{10}$$

Clearly, the optimal advance-purchase price is constrained by the retail price. The following lemma helps identify the possible retail price in equilibrium.

Lemma 4 *If revealing only X attribute arises as an optimal strategy in equilibrium, the optimal retail price charged by the firm cannot be w, $w + x$ nor $w + x + y$.*

Proof. Suppose that there exists an equilibrium such that it is optimal for the firm to charge $p_1 = w + x$. By (10), it implies that $p_0 \le w + x$. However, if the firm charges $p_0 \le w + x$, she earns at most profit (not including a positive introduction cost of the advance-purchase program)

$$(1 + m)(\alpha + \beta - \alpha\beta)(w + x - c).$$

The firm can obtain the same amount of profit by revealing neither attribute and charging a price $w + x$ at the retail stage. Therefore, the $w + x$ cannot be the optimal retail price if the firm reveals

13

only X attribute. By a similar argument, we can rule out the possibility of w as the optimal retail price in the equilibrium in which the firm chooses to reveal only X attribute.

Suppose that the equilibrium is such that the firm charges $p_1 = w + x + y$. This implies that, from (10), $p_0 = w + x + \beta y$. Thus, the firm earns profit

$$\alpha (w + x + \beta y - c) + m\alpha\beta (w + x + y - c).$$

However, if the firm introduces the advance-purchase program but reveals neither attribute, she will optimally charge a retail price $w + x + y$ and enjoy profit, from (9)[5],

$$\begin{aligned} & w + \alpha x + \beta y - c + m\alpha\beta (w + x + y - c) \\ > \ & \alpha (w + \beta y - c) + \alpha x + m\alpha\beta (w + x + y - c) \\ = \ & \alpha (w + x + \beta y - c) + m\alpha\beta (w + x + y - c).\end{aligned}$$

Therefore, $w + x + y$ cannot be the optimal retail price if the firm reveals only X attribute. ∎

Two arguments help understand the intuition behind Lemma 4. First, if the retail price is low ($p_1 = w$ or $p_1 = w + x$), the firm has to charge a low price ($p_0 \leq p_1$) at $t = 0$ to induce consumers to purchase in advance. In either case, consumers who purchase in advance are those whose ex-post realized valuations are above p_1.[6] However, the firm can manage to sell to the same consumer group at p_1 and thus, obtain at least the same profit by offering a sale only at $t = 1$ without the advance-purchase program. It follows that the strategy of revealing only X attribute and charging a retail price at w or $w + x$ is a dominated strategy. Second, if the optimal retail price is $p_1 = w + x + y$ it implies that the market is the Niche market. The strategy of disclosing no information and charging the advance-purchase price as $w + \alpha x + \beta y$ results the maximum possible profit that the firm can possibly achieve. In fact, the firm earns a lower profit if she chooses to reveal only X attribute which makes the expected consumer valuations heterogenous at the advance-purchase stage.

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[5] The reason why the firm will charge a retail price $w + x + y$ when revealing no information is as follows. In the case under which the firm chooses to reveal only X attribute, the market demand at the retail stage consists of the consumers who are informed with $X = 0$ at $t = 0$ and the consumers who arrive at $t = 1$. Thus, if it is optimal for the firm to charge a retail price as $w + x + y$, it should be also optimal for the firm to charge the same price if the market demand consists of only the consumers who arrive at $t = 1$.

[6] In particular, if $p_1 = w$ the ex-post valuations of all consumers are higher than p_1. If $p_1 = w + x$ the firm is not able to induce the consumers whose valuation is w to purchase either in advance or at the retail stage.

Lemma 4 implies that, if only X attribute is revealed, the possible equilibrium is that the firm charges a retail price $w+y$. In such an equilibrium, from (10), the firm charges the advance-purchase price $w + (1 - \beta) x + \beta y$. Consumers who arrive at $t = 0$ and are informed with $X = x$ purchase in advance and the remaining consumers (who arrive at $t = 0$ and are informed with $X = 0$ and who arrive at $t = 1$) with valuation above $w + y$ purchase at $t = 1$. The firm earns profit

$$\Pi^{AP-X} = \alpha \left(w + (1 - \beta) x + \beta y - c\right) + (1 - \alpha) \beta \left(w + y - c\right) + m\beta \left(w + y - c\right). \qquad (11)$$

Case 4 *Only Y attribute is revealed.*

Lemma 5 *Revealing only Y attribute cannot arise as an optimal strategy in equilibrium.*

Proof. We show by contradiction. Suppose there exist a situation under which it is optimal for the firm to reveal only Y attribute. We can first rule out the possibility of $p_1 = w$, $p_1 = w + x$ or $p_1 = w + y$ by similar arguments as in Lemma 4.

Now, suppose that there is an equilibrium such that the firm charges $p_1 = w + x + y$. This implies that, by (6), $p_0 = w + y + \alpha x$. Thus, the firm earns profit

$$\beta \left(w + y + \alpha x - c\right) + m\alpha\beta \left(w + x + y - c\right)$$
$$= \beta \left(w + \alpha x - c\right) + \beta y + m\alpha\beta \left(w + x + y - c\right).$$

However, if the firm introduces the advance-purchase program but reveals neither attribute, she enjoys profit, from (9),

$$w + \alpha x + \beta y - c + m\alpha\beta \left(w + x + y - c\right)$$

A straightforward comparison shows that the latter strategy yields a higher profit. ∎

The arguments here are very similar to the case of revealing only X attribute. The reason why revealing only X attribute can be a profit-maximizing strategy in equilibrium while revealing only Y attribute cannot is as follows. By revealing X attribute (with a smaller variance), the firm is able to commit to a higher future price $(w + y)$ because a larger uncertainty in the high-variance attribute will be revealed at $t = 1$. However, if only Y attribute is revealed, the only higher future price that the firm can commit to is $w + x + y$. As we argue, the firm would rather choose to reveal

no information which enables her to extract the maximum possible profit.

We pause to summarize the results on the possible strategy and equilibrium under the advance-purchase program.

Summary 1 *Provided that the firm chooses to introduce an advance-purchase program, a possible equilibrium can be one of the two types: (1) A separating equilibrium: the firm reveals only X attribute and charges a low advance-purchase price and a high retail price; Consumers informed with $X = x$ purchase in advance while the remaining consumers make purchase decisions at the retail stage when all information is fully revealed. (2) A pooling equilibrium: the firm provides no information at the advance-purchase stage and offers a discounted price such that all consumers arriving at $t = 0$ purchase in advance. The firm charges a higher retail price for the consumers arriving at $t = 1$ who make purchase decisions accordingly.*

We now are in the position to fully characterize the firm's strategy on the choice of advance-purchase program and the decision of information disclosure. Define

$$y_1 = Max \left\{ \frac{1-\beta}{\beta} k, \left[1 + \frac{m\alpha(1-\beta)}{\beta(1-\alpha+m)} \right] x + \frac{m\alpha(1-\beta)}{\beta(1-\alpha+m)} k, \frac{m\alpha}{(1+m)(1-\alpha)} x - k \right\} \quad (12)$$

and

$$y_2 = \left[1 + \frac{m\alpha(1-\beta)}{\beta(1+m)} \right] x + \frac{(1-\alpha+m\alpha)(1-\beta)}{\beta(1+m)} k. \quad (13)$$

where $k = w - c$.

Proposition 1 *(i) It is optimal for the firm to sell at the retail stage without introducing an advance-purchase program in the Mass market; (ii) It is optimal for the firm to introduce an advance-purchase program and reveal information only on X attribute in the Large market with $y > \bar{y}$ where $\bar{y} = Max\{y_1, y_2\}$; (iii) It is optimal for the firm to introduce an advance-purchase program and reveal no information on product attribute in the Large market with $y < \bar{y}$, the Small market or the Niche market.*

Proof. See Appendix. ■

In the Mass market, consumer valuations are extremely homogenous. The firm is lack of ability to commit to a high retail price. In fact, it is always optimal for the firm to charge w as the retail

price. This in turn limits the possible advance-purchase price to w. Hence, the firm optimally chooses to sell only at the retail stage and save the introduction cost of the advance-purchase program.

In the Small or Niche market, the firm has no difficulty of committing to a high retail price $(w + y$ or $w + x + y)$. Thus, the firm optimally chooses to introduce the advance-purchase program. Moreover, by revealing no information on the product attributes, the firm is able to maintain consumer expected valuations at the same level. This allows the firm to extract the maximum possible consumer surplus.

In the Large market, the firm can only commit to a medium price $(w + x)$ if she chooses not to reveal any information. For a small increase in y (as long as in the Large market), it remains optimal for the firm to charge the same retail price and thus, from (6), the same advance-purchase price. This implies that the firm does not gain any extra profit from the increase in y. However, if the firm chooses to reveal only X attribute, the separating equilibrium will arise in which the profit earned by the firm DOES increase as y increases. This is because the separation of consumers in the advance-purchase stage allows the firm to commit to a higher retail price that is equal to $w + y$. In such a situation, as y increases, the optimal retail price charged by the firm increases. In addition, as the retail price becomes higher, the firm charges, from (10), a higher advance-purchase price. It follows that, for a sufficiently large y, the profit from revealing only X attribute (which leads to a separating equilibrium) is higher than the profit from revealing neither attribute (which leads to a pooling equilibrium).

By Definition 1, the Mass market arises if and only if

$$w - c > Max \left\{ (\alpha + \beta - \alpha\beta)(w + x - c), \beta(w + y - c), \alpha\beta(w + x + y - c) \right\}$$

which is equivalent to $x < x^o$ and $y < y^o$ where

$$x^o = \frac{(1 - \alpha)(1 - \beta)}{\alpha + \beta - \alpha\beta}(w - c) \tag{14}$$

and

$$y^o = Min \left\{ \frac{1 - \beta}{\beta}(w - c), \frac{1 - \alpha\beta}{\alpha\beta}(w - c) - x \right\}. \tag{15}$$

17

Moreover, the Large market emerges if and only if

$$(\alpha + \beta - \alpha\beta)(w + x - c) > Max\{w - c,\ \beta(w + y - c),\ \alpha\beta(w + x + y - c)\}$$

which is equivalent to $x > \bar{x}$ and $y \in (\bar{y}, \hat{y})$ where

$$\bar{x} = \frac{(1 - \alpha)(1 - \beta)}{\alpha + \beta - \alpha\beta}(w - c), \qquad \bar{y} = Max\{y_1,\ y_2\} \tag{16}$$

and

$$\hat{y} = Min\left\{x + \frac{\alpha - \alpha\beta}{\beta}(w + x - c),\ \frac{\alpha + \beta - 2\alpha\beta}{\alpha\beta}(w + x - c)\right\}. \tag{17}$$

These lead to the following corollary.

Corollary 1 *(1) It is optimal for the firm to sell at the retail stage without introducing an advance-purchase program if $(x, y) \in \Theta$ where $\Theta = (x, y)\,|\,x < x^o,\ y < y^o;$. (2) It is optimal for the firm to introduce an advance-purchase program and reveal information only on X attribute if $(x, y) \in \Gamma$ where $\Gamma = \{(x, y)\,|\,x > \bar{x},\ \bar{y} < y < \hat{y}\}$ (3) It is optimal for the firm to introduce an advance-purchase program but reveal no information on product attribute in other cases.*
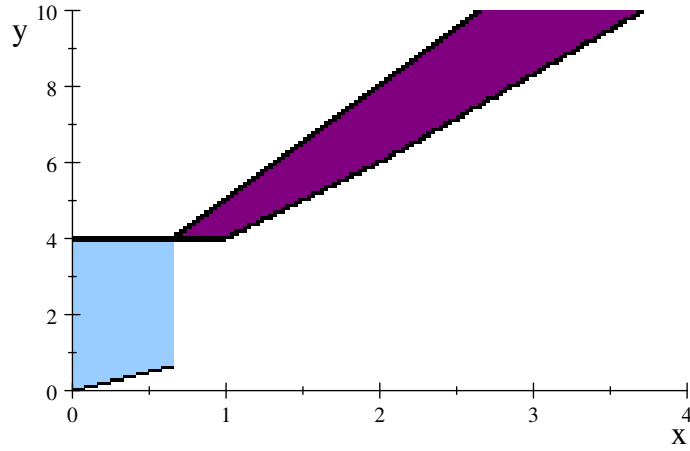
$$m = 1, \alpha = 0.5, \beta = 0.5, w = 4, c = 3$$

Figure 1

18

Figure 1 illustrates the result in Corollary 1. The blue (light) area represents the (x, y) region in which the firm chooses to sell at the retail stage without introducing the advance-purchase program; The purple (dark) area is the region in which the firm introduces the advance-purchase program and reveal only X attribute; The white area (the rest of the area) stands for the region in which the firm introduces the advance-purchase program and reveals neither attribute.

We next discuss how the changes in parameters affect the likelihood of the advance-purchase program as an optimal strategy. From (14),

$$\frac{\partial x^o}{\partial \alpha} = -\frac{1-\beta}{(\alpha + \beta - \alpha\beta)^2}(w - c) < 0,$$

$$\frac{\partial x^o}{\partial \beta} = -\frac{1-\alpha}{(\alpha + \beta - \alpha\beta)^2}(w - c) < 0,$$

$$\frac{\partial x^o}{\partial w} = \frac{(1-\alpha)(1-\beta)}{\alpha + \beta - \alpha\beta} > 0,$$

$$\frac{\partial x^o}{\partial c} = -\frac{(1-\alpha)(1-\beta)}{\alpha + \beta - \alpha\beta} < 0$$

and

$$\frac{\partial x^o}{\partial m} = 0.$$

In addition, from (15), we can show that y^o decreases as α increases, β increases, w decreases or c increases. Therefore, we have the following corollary.

Corollary 2 *Holding other things constant, the likelihood of the advance-purchase program as an optimal strategy increases as (i) x increases; (ii) y increases; (iii) α increases, (iv) β increases; (v) w decreases; (vi) c increases (vii) but is independent with m.*

We finally consider the effects of changes in parameters on the likelihood of revealing X attribute as an optimal strategy. From (12), (13) and Proposition 1, the firm is more likely to reveal only X attribute for a smaller x or a larger y. Moreover, we can show that y_1 decreases as β increases and

$$\frac{\partial y_2}{\partial \beta} = -\frac{m\alpha}{\beta^2(1+m)}x - \frac{1-\alpha+m\alpha}{\beta^2(1+m)}(w - c) < 0.$$

Thus, $\bar{y}$ decreases as β increases. This leads to the following corollary.

Corollary 3 *In the Large market, the likelihood of revealing partial information (X attribute) as an optimal strategy increases as (i) x decreases; (ii) y increases; or (iii) β increases;*

4   Conclusion

Departing from the existing literature, we have developed a two-stage dynamic model in which the firm is able to influence the information processed by consumers at the advance-purchase stage. We characterize the optimal strategy of the firm on whether to introduce an advance-purchase program and what information to disclose to consumers. We contribute to the literature on advance purchase by incorporating the firm's information disclosure decision. Our finding that partial information disclosure is sometimes optimal to the firm is in contrast to the well-known result in the literature of horizontal information provision that a firm maximizes profit by revealing either no or full information to consumers.

5   References

Anderson, S. and Renault R. "Advertising Content" American Economic Review, Vol. 96, pp 93-113, 2006

Bagwell, K., "The Economic Analysis of Advertising," in Mark Armstrong and Rob Porter eds., Handbook of Industrial Organization, Vol. 3, North-Holland: Amsterdam, , 1701-1844, 2007

Dana, J., "Advanced-Purchase Discounts and Price Discrimination in Competitive Markets" Journal of Political Economy, Vol. 106, pp. 395-422. 1998

Dana, J. "Using Yield Management to Shift Demand When the Peak Time is Unknown" RAND Journal of Economics, 30 (3) 45-474. 1999

Gale, I. and Holmes,T, "The Efficiency of Advance-Purchase Discounts in the Presence of Aggregate Demand Uncertainty" International Journal of Industrial Organization, 10, 413-437, 1992

Gale, I., and Holmes, T. "Advance-purchase discounts and monopoly allocation of capacity". American Economic Review, 83(1), March, 135-46, 1993

Hotz, V. and Xiao, M., "Strategic Information Disclosure: The case of Multi-Attribute Products with Heterogeneous Consumers" Economic Inquiry (Forthcoming)

Johnson, J., and Myatt, D., "On the Simple Economics of Advertising, Marketing, and Product Design" American Economic Review, 96 3 , 756-784. 2006

Lancaster, J. "A new Approach to Consumer Theory" Journal of Political Economy, Vol. 74, pp 132-157, 1966

Lewis, T. and Sappington, D., "Supplying Information to Facilitate Price Discrimination" International Economic Review, 35 2 , pp. 309-327. 1994

Möller, M, and Watanabe, M "Advance Purchase Discounts versus Clearance Sales". Economic Journal, forthcoming, 2009

Nelson, P. "Advertising as Information" Journal of Political Economy, Vol.82, pp 729-54, 1974

Nocke, V. Peitz, M and Rosar, F. "Advance-purchase discounts as a price discrimination device". Journal of Economic Theory (Forthcoming)

Sun, M, "Disclosing Multiple Product Attributes" Journal of Economics and Management Strategy (Forthcoming)

Xie, J., and Shugan, S. M. "Electronic Tickets, smart cards, and online prepayments: When and how to advance sell" Marketing Science, 20(3), Summer, 219-243, 2001

6   Appendix

6.1   Proof of Proposition 1

Proof. The proof consists of three parts.

Proof of (i). We have shown that, if the advance-purchase program is introduced, the firm either reveals only X attribute or no information in equilibrium. In addition, by Lemma 4, revealing only X attribute in the Mass market is a dominated strategy and thus, cannot be an optimal strategy.[7] Furthermore, by Lemma 2, the firm earns a higher profit in the Mass market by selling only at the retail stage than by introducing the advance-purchase program but with no information disclosure.

Proof of (ii).   The proof contains three steps.

---

[7] In the separating equilibrium, the demand consists of two groups of consumers: consumers who arrive at $t = 0$ and are informed $X = 0$ and who arrive at $t = 1$. We can show that the Mass market implies that it is optimal for the firm to charge w for each groups of consumers separately, thus the total population of the two groups.

Step 1. We show that in the Large market with $y > \bar{y}$, the following separating equilibrium exists: firm reveals information only on X attribute and charges an advance-purchase price $p_0^{AP-X} = w + (1 - \beta) x + \beta y$ and a retail price $p_1^{AP-X} = w + y$; Consumers who arrive at $t = 0$ and are informed with $X = x$ purchase in advance while the remaining consumers whose realized valuation is above p_1 will purchase at the retail stage. The separating equilibrium exists because (1) given the firm's strategy, consumers behave rationally; (2) given the consumers' purchase behavior, the prices charged by the firm are optimal in the separating equilibrium.

To see (1), we note that consumers arriving at $t = 0$ and informed $X = x$ will buy in advance because the expected utility from advance-purchase, $w + x + \beta y - p_0^{AP-X}$, is at least as high as the expected utility from waiting to make purchase decision at the retail stage which is $\beta(w + x + y - p_1^{AP-X})$. In addition, consumers arriving at $t = 0$ and informed with $X = 0$ will prefer to purchase at the retail stage because the expected utility from purchasing in advance, $w + \beta y - p_0^{AP-X}$ (a negative utility), is lower than the expected utility from waiting to make purchase decision at the retail stage, $\beta(w + y - p_1^{AP-X})$.

To see (2), note that the consumer demand at $t = 1$ consists the consumers who arrive at $t = 0$ and are informed with $X = 0$ and the consumers who arrive at $t = 1$. It is optimal for the firm to charge a price $w + y$ since, provided that $y > y_1$,

$$\beta (1 - \alpha + m) (w + y - c)$$
$$> \ Max \{(1 - \alpha + m) (w - c), ((1 - \alpha) \beta + m (\alpha + \beta - \alpha\beta)) (w + x - c), m\alpha\beta (w + x + y - c)\}$$

where the elements in the brackets on the RHS represent the profits for the firm to charge a retail price that is equal to $w, w + y$ and $w + x + y$ respectively.

Step 2. We show that the firm earns a higher profit by revealing X attribute than by disclosing neither attribute in the Large market with $y > \bar{y}$. This statement holds because, provided $y > y_2$

and from (11),

$$
\begin{aligned}
\Pi^{AP-X} &= \alpha\left(w+(1-\beta)x+\beta y-c\right)+(1-\alpha)\beta\left(w+y-c\right)+m\beta\left(w+y-c\right) \\
&= \left(\alpha+(1-\alpha)\beta+m\beta\right)\left(w-c\right)+\alpha\left(1-\beta\right)x+\beta\left(1+m\right)y \\
&> \left(\alpha+(1-\alpha)\beta+m\beta\right)\left(w-c\right)+\alpha\left(1-\beta\right)x \\
&\quad +\beta\left(1+m\right)\left[\left(1+\frac{m\alpha\left(1-\beta\right)}{\beta\left(1+m\right)}\right)x+\frac{\left(1-\alpha+m\alpha\right)\left(1-\beta\right)}{\beta\left(1+m\right)}\left(w-c\right)\right] \\
&= \left(1+m\left(\alpha+\beta-\alpha\beta\right)\right)\left(w-c\right)+\left(1+m\right)\left(\alpha+\beta-\alpha\beta\right)x \\
&= w+\left(\alpha+\beta-\alpha\beta\right)x-c+m\left(\alpha+\beta-\alpha\beta\right)\left(w+x-c\right)=\Pi^{AP-\Phi}
\end{aligned}
$$

Step 3. In the Large market, by Lemma 2, the firm earns a higher profit by introducing the advance-purchase program and revealing neither attribute than by selling only at the retail stage without introducing the advance-purchase program. From step 1-3, it follows that the optimal strategy for the firm in the Large market with $y>\bar{y}$ is to introduce the advance-purchase program and reveal only X attribute.

Proof of (iii) We proceed as follows.

Step 1. From Lemma 2, selling only at the retail stage without the advance-purchase program is a dominated strategy in the Large market, Small or Niche Market.

Step 2. We can show that the firm can achieve a higher profit by disclosing neither attribute than by revealing only X attribute in the Large market with $y<\bar{y}$. This directly follows from the result in the proof of (ii).

Step 3. The firm can achieve a higher profit by disclosing neither attribute than by revealing only X attribute in the Small market. Suppose that the optimal retail price is $p_1^{AP-\Phi}=w+y$ in the Small market. The profit from the pooling equilibrium is always higher than in the separating equilibrium. This is because, from (9)

$$
\begin{aligned}
\Pi^{AP-\Phi} &= w+\alpha\left(1-\beta\right)x+\beta y-c+m\beta\left(w+y-c\right) \\
&= \alpha\left(w+(1-\beta)x+\beta y-c\right)+(1-\alpha)\left(w+\beta y-c\right)+m\beta\left(w+y-c\right) \\
&> \alpha\left(w+(1-\beta)x+\beta y-c\right)+(1-\alpha)\beta\left(w+y-c\right)+m\beta\left(w+y-c\right)=\Pi^{AP-X}.
\end{aligned}
$$

Step 4. The firm can earn a higher profit by disclosing neither attribute than by revealing only X in the Niche market. Suppose that the optimal retail price is $p_1^{AP-\Phi} = w + x + y$ in the Niche market. The profit from the pooling equilibrium is always higher than in the separating equilibrium since, by (9)

$$
\begin{aligned}
\Pi^{AP-\Phi} &= w + \alpha x + \beta y - c + m\alpha\beta \left(w + x + y - c\right) \\
&> w + \alpha \left(1 - \beta\right) x + \beta y - c + m\alpha\beta \left(w + x + y - c\right) \\
&= \alpha \left(w + \left(1 - \beta\right) x + \beta y - c\right) + \left(1 - \alpha\right) \left(w + \beta y - c\right) + m\beta \left(w + y - c\right) \\
&> \alpha \left(w + \left(1 - \beta\right) x + \beta y - c\right) + \left(1 - \alpha\right) \beta \left(w + y - c\right) + m\beta \left(w + y - c\right) = \Pi^{AP-X}.
\end{aligned}
$$

This completes the proof. ∎