Pasinetti's Structural Change and Economic Growth: a conceptual excursus

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Pasinetti's *Structural Change and Economic Growth*: a conceptual *excursus*[*]

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Abstract A clear and organic exposition of Pasinetti's theoretical framework of *Structural Change and Economic Growth* is often complicated by misunderstandings and ambiguities concerning the basic categories and terminology.

The pre-institutional character of the approach, the nature of its equilibrium paths and the significance of the 'natural' economic system — together with its normative character — are some of the most controversial issues.

In particular, there seems to be a need for a clearcut distinction between the general dynamic analysis of the price and quantity systems and the specific dy-

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namics they follow when the sectoral proportions and levels of production exactly satisfy *dynamic equilibrium* conditions, and a particular closure of the price system is adopted, providing for specific *functional* income distribution and theory of value.

The aim of the present paper is therefore that of attempting at a conceptual *excursus* of the model, in order to establish a solid ground on the basis of which discussions with other *Classical* approaches can be fruitfully held.

Keywords Vertically (hyper-)integrated sectors, functional income distribution, 'natural' economic rates of profit, 'natural' economic system, pure labour theory of value.

JEL classification B51,O41

1 Introduction

Pasinetti's *Structural Change and Economic Growth* has been, since its publication in 1981, the object of many reviews and comments, and it is one of the most cited works as regards the topic of structural change (see for example Silva & Teixeira 2008).

However, many aspects of the book, both conceptual and analytical, have not been grasped, or have been grasped only partially, thus preventing from a complete understanding of the implications, and potentialities, of Pasinetti's approach.

The first stumbling block has usually been the pre-institutional character of the model, sometimes misinterpreted as a pre-industrial one.

A second problem is the often missed distinction between the general dynamic analysis of the price and quantity systems, the dynamic equilibrium paths — one for each possible *exogenous* combination of distributive variables — and the 'natural' economic system, resulting from a particular closure of the price system.

A third issue of importance is the vertically hyper-integrated character of the framework, on which we particularly insist in the paper, in order for the model — and some of its most far reaching insights — to be fully understood.

The present paper consists of a conceptual *excursus* of the model, and is organised as follows. Section 2 deals with the pre-institutional character of Pasinetti's (1981) model. Section 3 presents a synthetic exposition of the model, *before* the introduction of the 'natural' economic system. Section 4, then, gives the rationale for the particular closure of the price system adopted by Pasinetti (1981), highlighting some of the main insights. Section 5 is a methodological note on the notion of equilibrium and its role throughout the analysis. Finally, section 6 goes through the stages of development of the concept and analytical device of vertical hyper-integration.

2 The pre-institutional analysis of an industrial system

Before going into the details of Pasinetti's (1981) analytical formulation, it is worth putting forward a brief methodological introduction, in order for "this theoretical framework [...] [to be] appropriately understood and correctly used. It is a basic framework, a skeleton, so to speak, which is meant to remain at a *pre-institutional level of investigation*" (Pasinetti 1985, p. 274, italics added).

The above excerpt comes from the reply Pasinetti gave to a review, by Nina Shapiro (1984), of *Structural Change and Economic Growth*. The point he raised is crucial: the analytical framework he developed can be understood and correctly used only if its pre-institutional character is constantly and clearly kept in mind.

Therefore, even if the deepest implications of Pasinetti's (1981) methodology will be drawn later on, in section 5 below, a general hint must be given here, before going into the analytical description of the model, in order for the latter to be properly understood.

The first thing that should be made clear is the meaning Pasinetti attaches to the word 'capitalistic', as opposed to 'capitalist'. While the latter refers to the set of *social relations* of production typical of a capitalist economic system, in contrast to those typical, for example, of a centrally planned one, the former term describes the very physical-technological nature of the production process in any *industrial* system, to be intended as the *production of commodities by means of commodities*, i.e. the employment of capital goods as intermediate commodities, to be used together with labour, and accumulated, for the production process to take place.

Pasinetti's focus has *always* been, in all his works, on "industrial societies, with their tendency towards change and towards an evolving structure, as against the more static conditions of pre-industrial societies" (Pasinetti 2005, p. 247). Nonetheless, the pre-institutional analysis he puts forward has sometimes been (mis)interpreted as a pre-capitalist, or pre-capitalistic, one, in spite of the fact that he has never used such expressions, and that he has always made explicit, and repeated, reference to 'pure production systems'.

Anyway, Pasinetti's aim is that of analysing the working of a *capitalistic*, and not of a capitalist, economic system. As will be further discussed in section 5 below, the framework he develops is by no means an attempt at describing the functioning of an actual capitalist system. Nor it is an attempt at describing the functioning of a centrally planned economy, as someone could have been induced to conclude.

What does it mean, therefore, that Pasinetti's (1981) analysis has been carried out at the *pre-institutional* level?

The issue is not a trivial one. Quite apparently, many commentators did not succeed in grasping the meaning of such a statement, thereby failing to grasp the very nature of Pasinetti's framework. It is our contention that many — actual or pretended — ambiguities in Pasinetti's (1981) expositions are due to this misunderstanding.

As Pasinetti states in the Introduction of *Structural Change and Economic Growth*, his approach to economic theory starts from a very precise standpoint:

> It is my purpose [...] to develop first of all a theory which remains neutral with respect to the institutional organisation of society. My preoccupation will be that of singling out, to resume Ricardo's terminology, the 'primary and natural' features of a pure production system.
>
> (Pasinetti 1981, p. 25)

A 'separation' — as Pasinetti called it later, in his most recent book[1] — is therefore needed between two stages of analysis, each concerning a very specific kind of economic investigation. The rationale of this separation emerges very clearly from the Preface to the 1981 book:

> There is [...] a sharp discrimination between those economic problems that have to be solved on the ground of logic alone — for which economic theory is entirely autonomous — and those economic problems that arise in connection with particular institutions, or with particular groups' or individuals' behaviour — for which economic theory is no longer autonomous and

> needs to be integrated with further hypotheses, which may well come from other social sciences. It is with the first type of problems that the present work is basically concerned.
>
> (Pasinetti 1981, p. xiii)

Of course, Pasinetti's (1981) claim for the *logical* priority of the first stage — the *pre-institutional* one — with respect to the second stage — the *institutional* one — by no means implies that he is disregarding the role of institutions. On the contrary: their role is regarded as one of primary importance, as institutions are the *means* through which it is possible to shape the real world:

> All these considerations only come to confirm how important is to keep the logical problems concerning the 'natural' economic system quite separate from those concerning the institutions, and to consider the institutions they really are — means, and not ends in themselves. Once their instrumental role is properly understood and recognised, it becomes much easier also to operate on them in as detached a way as is possible; to treat them as instruments susceptible to be continually improved and changed, in relation to their suitability (or unsuitability) to ensure tendencies, or near-tendencies, towards agreed ends.
>
> (Pasinetti 1981, p. 155)

Institutions are means, not ends, but in order for them to be used to drive society 'towards agreed ends' it is first of all necessary to know the fundamental mechanisms they are called upon to counteract, or to favour, or simply to take advantage of. Without this knowledge, institutions cannot pursue any instrumental role.

Pasinetti's *vision* is that the 'primary and natural' features of an economic system have to be studied independently of a particular institutional set-up. Nonetheless, the task of describing an economic system without reference to a particular institutional set-up is not a trivial one. It is very difficult to realise how an economic system can be thought of without strong reference to the institutions which shape it, since no actual economic system could have been brought into existence without them.

This task can be accomplished by looking for those *physical* requirements necessary for an industrial system to carry out its production process, and *grow*. The way in which Pasinetti puts this idea into practice shall become clear by reading sections 3 and 4, below.

3 General dynamic analysis and equilibrium dynamics

Pasinetti's (1981) *Structural Change and Economic Growth* provides us with a model of economic growth starting from a complete description of an economic system in a single-period *equilibrium*, defined as "a situation in which there is full employment of the labour force and full utilisation of the existing productive capacity" (Pasinetti 1981, pp. 48-49). This situation can be thought of as the *initial condition* of a general multi-sector dynamic model, which "has been developed for the purpose of detecting the 'permanent' causes moving an economic system, irrespective of any accidental or transitory deviation which may temporarily occur" (Pasinetti 1981, p. 127).

We will now introduce a synthetic exposition of the model.[2] For basic notation see Table 1.

[Table 1 here]

3.1 Formulation of quantity and price systems

This single-period description consists of a physical quantity system and a commodity price system, each composed by $2m+1$ equations, where m is the number of final consumption commodities produced in the system. The production of each consumption commodity i requires a specific capital good k_i. As to the physical quantity system, this means that the equation concerning consumption commodity i and the equation concerning the corresponding capital good k_i together describe the total quantities produced by the vertically hyper-integrated sector[3] i ($x_i(t)$ and $x_{k_i}(t)$, $i = 1, 2, \ldots, m$). The last equation establishes the condition for the full employment of the total labour available in the system:

$$\begin{cases} x_i(t) - a_{in}(t)x_n(t) = 0 & \text{for} \quad i = 1, 2, \ldots, m \\ T_i^{-1}x_i(t) - x_{k_i}(t) - a_{k_in}(t)x_n(t) = 0 & \text{for} \quad i = 1, 2, \ldots, m \\ \sum_i a_{ni}(t)x_i(t) + \sum_i a_{nk_i}(t)x_{k_i}(t) - x_n(t) = 0 \end{cases} \tag{3.1}$$

As to the price system, there will be a price for each final consumption commodity i and a price for each capital good k_i associated to it ($p_i(t)$ and $p_{k_i}(t)$, $i = 1, 2, \ldots, m$). The last equation establishes the condition for the full expenditure of (full employment) national income:

$$\begin{cases} -p_i(t) + \left(\pi_i(t)\frac{k_i(t)}{x_n(t)} + \frac{1}{T_i}\right) p_{k_i}(t) + a_{ni}w = 0 & \text{for} \quad i = 1, 2, \ldots, m \\ -p_{k_i}(t) + a_{nk_i}(t)w(t) = 0 & \text{for} \quad i = 1, 2, \ldots, m \\ \sum_i a_{in}(t)p_i(t) + \sum_i \left(a_{k_in}(t) - \pi_i(t)\frac{k_i(t)}{x_n(t)}\right) p_{k_i}(t) - w(t) = 0 \end{cases} \tag{3.2}$$

3.2 Vertically hyper-integrated productive capacity

In general, the means of production required to obtain one unit of a final consumption good are a sector-specific composite commodity in which the same intermediate inputs enter in — technically given — proportions. This motivates the definition of a particular unit of measurement — one for each vertically hyper-integrated sector — for this physical composite commodity, a *unit of vertically* *hyper-integrated productive capacity.* Each of these units is the sum of three components: direct requirements for the production of one unit of final consumption commodity i; direct requirements for the replacement of worn-out direct and indirect capital goods needed for the production of one unit of final consumption commodity i; and direct requirements for the production of all intermediate commodities directly and indirectly needed for the expansion of productive capacity in line with the growth of final demand for consumption commodity i.

In Pasinetti's (1981) theoretical scheme, the units of productive capacity used as units of measurement for capital goods are actually units of *direct* productive capacity for the production of final consumption commodities. This becomes clear when looking at the most complex case, in which capital goods are produced by means of labour *and* capital goods (see Pasinetti 1981, pp. 43-45). However, it is our contention not only that units of vertically hyper-integrated productive capacity are the most appropriate units of measurement for capital goods, but also that Pasinetti himself, in 1981, had already begun to argue in terms of vertically hyper-integration, even if the complete analytical implications were still to be reached (many of them finally reached their rigorous formulation with the publication of Pasinetti (1988)).[4]

Anyway, in the present, simpler, case, no analytical difference can be found between direct, vertically integrated and vertically hyper-integrated productive capacity, since final consumption commodities are the only ones produced by means of capital goods. Therefore we can interpret units of productive capacity as being vertically hyper-integrated, without having to reformulate the analytical model in Pasinetti (1981). For the sake of simplicity, from now on, we will simply say 'sectors' instead of 'vertically hyper-integrated sectors'; and 'productive capacity' instead of 'vertically hyper-integrated productive capacity', except where the complete expressions shall be considered more appropriate.

A unit of productive capacity will refer to the specific final commodity that requires it, and therefore to the specific sector in which it is produced. In this way, the analysis opens up for the possibility of separating the pace of accumulation of the means of production (the number of units of productive capacity) from its physical composition.

In order to simplify exposition, Pasinetti (1981) regards these composite commodities as particular capital goods, specific to each consumption good. Therefore, as hinted at above, in the present context, a vertically hyper-integrated sector is made up by two industries: one producing the final consumption good, and the other one producing the corresponding capital good. These two industries play an asymmetric role, since "the physical quantities of the means of production appear as playing a sort of ancillary role with respect to the physical quantities of final demand [for consumption goods]; the former being, so to speak, 'at the service' of the latter" (Pasinetti 1988, pp. 125-126).

3.3 Conditions for flow and stock-equilibrium solutions

Both physical quantity and commodity price systems are formulated as sets of $2m + 1$ linear and homogeneous equations which have non-trivial solutions if the coefficient matrix is singular, i.e. if its determinat is zero. The condition for this to be true is the same for both equation systems, and can be written as:

$$\sum_i a_{in}(t)a_{ni}(t) + \sum_i T_i^{-1} a_{in}(t)a_{nk_i}(t) + \sum_i a_{k_in}(t)a_{nk_i}(t) = 1 \qquad (3.3)$$

If this condition is not satisfied, the systems are contradictory, i.e. each set of $2m + 1$ equations cannot simultaneously hold. However, because of the particular mathematical structure of the problem,[5] we can still get meaningful solutions for quantities and prices, but the last equation in each system will not be satisfied, i.e. we shall not be in a situation of full employment of the labour force and full expenditure of total income. On the contrary, if this single condition is satisfied, the solutions will correspond to a situation of full employment and full expenditure of income. To this situation we shall refer as a *flow-equilibrium* situation.

Assuming that condition (3.3) holds, we get two indeterminate linear homogeneous systems, which means that we have solutions for *relative* quantities and *relative* prices corresponding to a situation of flow-equilibrium, but we have to choose a scale factor for each system. For the quantity system this scale factor is total labour available, since this is an exogenous variable. Therefore, Pasinetti sets $x_n(t) = \overline{x}_n(t)$. For the price system, the choice of a scale factor is in fact an arbitrary one. Following Pasinetti (1981, pp. 92-93), we choose, for convenience, the wage rate, and therefore we take it as given both at a specific point in time and through time, i.e. we set $w(t) = \overline{w}$.

Therefore, for a given period t, the solutions for physical quantities and commodity prices — in a *flow* equilibrium — are, respectively:[6]

$$\begin{cases} x_i(t) = a_{in}(t)\overline{x}_n(t) \\ x_{k_i}(t) = T_i^{-1}a_{in}(t)\overline{x}_n(t) + a_{k_in}(t)\overline{x}_n(t) \end{cases} \tag{3.4}$$

and

$$\begin{cases} p_i(t) = \left(a_{ni}(t) + a_{nk_i}(t)\dfrac{k_i(t)}{x_i(t)}(T_i^{-1} + \pi_i(t))\right)\overline{w} \\ p_{k_i}(t) = a_{nk_i}(t)\overline{w} \end{cases} \tag{3.5}$$

for $\quad i = 1, 2, \ldots, m$

Expressions (3.4) and (3.5) are made up by m pairs of equations each. The first equation of each pair concerns consumption commodity i ($i = 1, 2, \ldots, m$); the second concerns the corresponding unit of vertically hyper-integrated productive capacity k_i $(i = 1, 2, \ldots, m)$.

Expressions (3.4) represent solutions for total physical quantities produced in the vertically hyper-integrated sectors $i = 1, 2, \ldots, m$. First, x_i is determined by the average per capita demand for final consumption commodity i multiplied by total population. Second, x_{k_i} is the sum of two components: $T_i^{-1} a_{in}(t)\overline{x}_n(t) = x'_{k_i}(t)$, i.e. the number of units of productive capacity for consumption good i necessary for the replacement of worn out productive capacity, and $a_{k_i n}(t)\overline{x}_n(t) = x''_{k_i}(t)$, i.e. the number of units of productive capacity demanded as new investment. The capital-producing industry has to produce not only those units of productive capacity necessary for keeping the initial stock of (units of) productive capacity intact, but also those units required to expand it.[7]

Expressions (3.5) are the solutions for commodity *production prices*. Since we are assuming that capital goods are produced by labour alone, each price p_{k_i} is determined by its direct labour requirements multiplied by the wage rate, while in the expressions for prices p_i the wage rate also multiplies a gross profit mark-up proportional to the direct labour required to produce a unit of the corresponding productive capacity.

In principle, there is no difference between production prices obtained when the price system is formulated in terms of industries and when it is formulated in terms of sectors. The technique in use and the distributive variables do not change as a consequence of adopting the procedure of vertical hyper-integration, which is simply a way of re-classifying and partitioning activities in order to explicitly acknowledge for the relationship between each activity producing a final consumption commodity and those activities producing the means of production for self-replacement and expansion of the corresponding productive capacity.[8]

The difference is introduced as a consequence of the specific adoption of the units of productive capacity for final consumption commodities as the units of measurement of capital goods. In this way, each price p_{k_i} does not stand for the price of one 'ordinary unit' of commodity k_i, but for the price of one unit of productive capacity for consumption good i.

As stated above (section 3, page 8), an equilibrium position entails full employment of the total labour available — which implies a *single* condition concerning *flows* — and full utilisation of the existing productive capacity in each vertically hyper integrated sector i ($i = 1, 2, \dots, m$) — a series of sectoral conditions concerning *stocks*.

The condition concerning the flows of the economic system has already emerged as the condition for non-trivial solutions to the quantity and price systems, i.e. expression (3.3), which is a *macroeconomic* condition, since it refers to the economic system as a whole, no matter how many sectors there are. Moreover, "it emerges from a model which has been developed on a *multi-sector* basis, thereby revealing its truly macro-economic nature" (Pasinetti 1981, p. 35).

As concerns stocks, we have a *series* of sectoral conditions, saying that in each sector, the number of units of productive capacity available at the beginning of the period as the capital stock endowment must be exactly equal to the number of units of final consumption good to be produced during the same time period, i.e.:

$$k_i(t) = x_i(t), \qquad i = 1, 2, \ldots, m \tag{3.6}$$

Expressions (3.4) and (3.5) together with conditions (3.3) and (3.6) exhaust the description of single-period equilibrium which is the starting point for the development of the general multi-sector dynamic model of growth.

For the analysis we are going to perform, we shall assume, from now on, that the economic system starts from a situation of both flow and stock equilibrium, i.e. we shall assume that, for time $t = 0$, both condition (3.3) and the series of conditions (3.6) hold true.

3.4 General dynamic analysis

The dynamic method Pasinetti (1981) adopts is that of specifying exponential laws of movement for the coefficients in (3.1) and (3.2) concerning total available labour (x_n), average per capita demand (a_{in}), and labour input requirements (a_{ni} and a_{nk_i}), according to:[9]

$$\begin{cases} x_n(t) = x_n(0)e^{gt} \\ a_{in}(t) = a_{in}(0)e^{r_i t} \\ a_{ni}(t) = a_{ni}(0)e^{-\varrho_i t} \\ a_{nk_i}(t) = a_{nk_i}(0)e^{-\varrho_{k_i} t} \end{cases} \qquad i = 1, 2, \dots, m \tag{3.7}$$

Solutions (3.4) and (3.5) are therefore linear structures whose components follow exponential dynamics. Taking expressions (3.4) and (3.5) evaluated at time period $t = 0$, and inserting the dynamics described in (3.7) we obtain the following solutions for physical quantities and commodity prices:

$$\begin{cases} x_i(t) = a_{in}(0)\overline{x}_n(0)e^{(g+r_i)t} \\ x_{k_i}(t) = T_i^{-1}a_{in}(0)\overline{x}_n(0)e^{(g+r_i)t} + a_{k_i n}(t)\overline{x}_n(0)e^{gt} \end{cases} \tag{3.8}$$

and

$$\begin{cases} p_i(t) = \left(a_{ni}(0)e^{-\varrho_i t} + a_{nk_i}(0)\frac{k_i(t)}{x_i(t)}(T_i^{-1} + \pi_i(t))e^{-\varrho_{k_i} t}\right)\overline{w} \\ p_{k_i}(t) = a_{nk_i}(0)e^{-\varrho_{k_i} t}\overline{w} \end{cases} \tag{3.9}$$

for $\quad i = 1, 2, \dots, m$

The dynamic movements in (3.8) and (3.9) do not imply full employment and full utilisation of productive capacity *after* time period $t = 0$. In particular, full utilisation of productive capacity depends on a series of *stock* conditions. In the present model, the stocks of the economic system change according to the flow of demand for new investment, linking one period to the following one. The accounting identity that describes this process of capital accumulation in each sector is:

$$\dot{k}_i(t) \equiv x''_{k_i}(t), \qquad i = 1, 2, \ldots, m \tag{3.10}$$

Given that $x''_{k_i}(t) = a_{k_i n}(t)x_n(t)$, we obtain $\dot{k}_i(t) = a_{k_i n}(t)x_n(t)$. Therefore, the series of coefficients $a_{k_i n}(t)$ "is the only one that affects the stocks of the economic system, i.e. productive capacity in each sector; hence it cannot be taken as given from outside" (Pasinetti 1981, p. 85).

This opens up for the possibility to perform a general dynamic analysis by specifying a law of movement for the level of per capita new investment demand ($a_{k_i n}$), allowing for the discrepancy between productive capacity available at the beginning of period t (k_i) and the units of productive capacity actually used up during period t (x_i). The specification of the dynamics of investment is a degree of freedom that, once closed, allows for performing an *institutional* analysis of different theories of capital accumulation.

Another degree of freedom can be opened by changing the last equation of both the physical quantity and the commodity price systems, in order to explicitly allow for the possibility of flow-disequilibrium, e.g. by writing:

$$\sum_i a_{ni}(t)x_i(t) + \sum_i a_{nk_i}(t)x_{k_i}(t) - \alpha x_n(t) = 0 \tag{3.11}$$

and

$$\sum_i a_{in}(t)p_i(t) + \sum_i \left(a_{k_i n}(t) - \pi_i(t)\frac{k_i(t)}{x_n(t)} \right) p_{k_i}(t) - \alpha w(t) = 0 \tag{3.12}$$

where $\alpha \gtreqless 1$. This will accordingly modify the condition for non-trivial solutions to exist, which will become:

$$\sum_i a_{in}(t)a_{ni}(t) + \sum_i T_i^{-1} a_{in}(t)a_{nk_i}(t) + \sum_i a_{k_i n}(t)a_{nk_i}(t) = \alpha \gtreqless 1 \tag{3.13}$$

meaning that macroeconomic condition (3.3) is not satisfied if $\alpha \neq 1$.[10]

3.5 Dynamic equilibrium conditions and vertical hyper-integration

Since Pasinetti's (1981) theoretical scheme aims at describing "the 'primary and natural' features of a pure production system [...] [which] will simply emerge as necessary requirements for equilibrium growth" (Pasinetti 1981, p. 25), he is concerned, on the one hand, with the condition for keeping full-employment through time (*flow-equilibrium*), and on the other hand, with the condition for maintaining full utilisation of productive capacity through time (*stock-equilibrium*).

As regards the flow-equilibrium, by inserting (3.7) into (3.3) we get:

$$\sum_i a_{in}(0)a_{ni}(0)e^{(r_i-\varrho_i)t}+\sum_i \frac{1}{T_i}a_{in}(0)a_{nk_i}(0)e^{(r_i-\varrho_{k_i})t}+\sum_i a_{k_in}(t)a_{nk_i}(0)e^{-\varrho_{k_i}t}=1 \tag{3.14}$$

It can be noticed that the demand coefficients for new investment are still taken as exogenously given, while their specification will be the subject of the following few paragraphs.

As regards the stock-equilibrium, the laws of motion of average per capita sectoral demands for new investment "must be such as to be compatible with the process of economic growth and will therefore themselves be determined as part of the equilibrium conditions" (Pasinetti 1981, p. 85).

Therefore, as $a_{k_in}(t)x_n(t) = \dot{k}_i(t)$ and, in stock equilibrium, $k_i(t) = x_i(t)$, demand for new investment must exactly satisfy the growth requirements of productive capacity in each sector, as determined by the growth of demand for each final consumption commodity ($\dot{x}_i(t) = (g + r_i)a_{in}(t)x_n(t)$) in all periods beyond $t = 0$, i.e. the following set of sectoral *capital accumulation conditions* must be satisfied:

$$a_{k_in}(t) = (g + r_i)a_{in}(t), \quad \forall i = 1, 2, \ldots, m; \quad t \geq 0 \tag{3.15}$$

which are the dynamic counterpart of stock-equilibrium conditions (3.6), stating the sectoral equilibrium *rates of new investment* — $(g+r_i)$ — defined as the number of units of productive capacity, per unit of final demand for each consumption commodity i, necessary as new investment for the expansion of the corresponding productive capacity.

The series of conditions (3.15) can also be expressed as ratios of sectoral new investment to production, at current prices:

$$\frac{p_{k_i}(t)x''_{k_i}(t)}{p_i(t)x_i(t)} = (g+r_i)\frac{p_{k_i}(t)k_i(t)}{p_i(t)x_i(t)} \equiv (g+r_i)\chi_i(t) \qquad (3.16)$$

When written in this way, sectoral conditions (3.15) tell us that, in stock-equilibrium, "the ratio of new investments to the level of production must be equal, in each sector, to the technologically determined capital/output ratio [$\chi_i(t)$] multiplied by [the sum of] the rate of population growth [and the rate of growth of per capita demand]" (Pasinetti 1981, pp. 54-55).

In order to fully acknowledge for the importance of conditions (3.16), we have first to notice the *vertically hyper-integrated character* of capital/output ratio $\chi_i(t)$. In a traditional inter-industry scheme, the net output of the economy is the set of commodities produced for final consumption and new investment. However, in a vertically hyper-integrated framework, the net output of the system is made up only of the set of commodities for final consumption, as new investment demand is part of the means of production required to expand productive capacity. Therefore, when thinking of the capital intensity of a sector i, its denominator (the net output) will be the value of final consumption commodity i produced in the system, while its numerator (the value of capital) will be the value of the units of productive capacity specific to each final consumption commodity i required to self-replace and expand the productive capacity during period t.

In the light of this, the specification of an equilibrium schedule of capital accumulation in vertically hyper-integrated terms reflects, on the one side, the interdependent nature of the production process, as — in the most general case — a single industry producing a basic commodity utilised as a capital good would participate in different sectors with a different capital intensity in each of them; and on the other side, it highlights the potential of working with vertically hyper-integrated sectors, as "the notion of a physical unit of productive capacity, by being defined with reference to the commodity that is produced, continues to make sense, as a physical unit, whatever complications technical change may cause to its composition in terms of ordinary commodities" (Pasinetti 1973, p. 24).[11]

This is the most remarkable property of the chosen unit of measurement: whatever the time period, whatever the stage of technical progress, whatever the technique actually in use, capital goods can always be measured in units of productive capacity, and the accumulation of capital can always be studied by evaluating the *number* of units of productive capacity that have to be produced during time period t to maintain stock-equilibrium at the beginning of time period $t+1$. In this way, we can link the stocks of different time periods through the simple capital accumulation (equilibrium) conditions (3.15) — or, equivalently, (3.16). Complementarily, the problem of the change in the *physical composition* of these units can be studied separately by exploiting the one-to-one correspondence between vertically hyper-integrated and inter-industry relations as "the production coefficients of a vertically [hyper-]integrated model turn out to be a linear combination of the production coefficients of the corresponding input-output model"(Pasinetti 1981, p. 111).

By substituting capital accumulation conditions (3.15) into the macroeconomic condition (3.14) and writing it as follows:

$$\sum_i a_{ni}(0)a_{in}(0)e^{(r_i-\varrho_i)t} + \sum_i \left(g + r_i + T_i^{-1}\right) a_{nk_i}(0)a_{in}(0)e^{(r_i-\varrho_{k_i})t} = 1 \quad (3.17)$$

we can notice that the two addenda distribute total labour of the system between the labour requirements of final consumption commodities and the labour requirements of equilibrium *gross* investments.

By looking at expression (3.17), it is immediately clear that, for any specific *composition* of final demand *for consumption*, the equilibrium amount of gross investment is univocally determined by the technique in use and by the dynamics of population and of final consumption demand itself. Hence, the left-hand side of (3.17) stands for the size of per-capita total effective demand in time period t. Therefore, (3.17) "may be called the *effective demand condition* for keeping full employment" (Pasinetti 1981, p. 54), since it establishes whether a given composition of final demand for consumption is compatible with flow-equilibrium, i.e with full-employment of the labour force.

It therefore follows that "the difficulty of increasing total effective demand is one of finding out, and achieving, at a sufficient speed, its appropriate structural composition, and not one of reaching any absolute level" (Pasinetti 1981, p. 242), highlighting the multisectoral foundation of an effective demand theory of output.

3.6 Vertically hyper-integrated labour

The units of productive capacity are one of the two constituent components of the technique of a vertically hyper-integrated sector, the other one being the *vertically hyper-integrated labour coefficients*. In order to define them, we shall start from the full-employment macroeconomic condition for flow-equilibrium. By inserting

(3.15) into (3.14) and rearranging, we get:

$$\sum_i a_{in}(0)e^{r_i t}\left(a_{ni}(0)e^{-\varrho_i t} + \frac{1}{T_i}a_{nk_i}(0)e^{-\varrho_{k_i} t} + (g+r_i)a_{nk_i}(0)e^{-\varrho_{k_i} t}\right) = 1 \quad (3.18)$$

which, by defining

$$\ell_i(t) \equiv a_{ni}(t) + \frac{1}{T_i}a_{nk_i}(t) + (g+r_i)a_{nk_i}(t) \quad (3.19)$$

can be written as:

$$\sum_i a_{in}(t)\ell_i(t) = 1 \quad (3.20)$$

$\ell_i(t)$ — the vertically hyper-integrated labour coefficient for sector i — is the sum of three components: $a_{ni}(t)$, i.e. direct labour for the production of one unit of final consumption commodity i (direct labour); $T_i^{-1}a_{nk_i}(t)$, i.e. direct labour for the replacement of worn-out units of productive capacity for vertically hyper-integrated sector i (indirect labour); and $(g+r_i)a_{nk_i}(t)$, i.e. direct labour required for the expansion of productive capacity of sector i according to the growth of final demand for consumption good i (hyper-indirect labour).[12]

We can now immediately take advantage of the just given definition in order to express prices in terms of vertically hyper-integrated labour. When conditions (3.15) hold, the prices of final consumption commodities in (3.5) can be written as:

$$p_i(t) = \left(a_{ni}(t) + \frac{1}{T_i} a_{nk_i}(t) + (g + r_i) a_{nk_i}(t) \right) \overline{w} + (\pi_i(t) - (g + r_i)) p_{k_i}(t)$$

or

$$p_i(t) = \ell_i(t)\overline{w} + (\pi_i(t) - (g + r_i)) p_{k_i}(t) \tag{3.21}$$

for $\quad i = 1, 2, \ldots, m$

It is interesting to notice that expression (3.21) establishes the production price of each final consumption commodity i as the sum of two components: the cost of vertically hyper-integrated labour embodied in it — $\ell_i(t)\overline{w}$ — and a profit-differential with respect to the sectoral equilibrium rate of new investment — $(\pi_i(t) - (g + r_i))$ — computed on the value of equilibrium productive capacity at current production prices — $p_{k_i}(t)$. This second component is not the (dual) value counterpart of necessary physical quantity requirements of (re-)production and expansion, but emerges as an amount of purchasing power created in excess to these requirements, that goes into the owners of the means of production through the process of income distribution. The amount of this magnitude is a direct consequence of the theory of the income distribution that shall be adopted to close the price system, and it will influence the whole process of structural dynamics, via its effect on the pattern of expenditure of real income.

Another important magnitude which we shall introduce into the analysis is the level of equilibrium employment in each sector i, given by the product of the corresponding vertically hyper-integrated labour coefficient and the physical quantity of final consumption commodity i produced during the time period:

$$L_i(t) = \ell_i(t)x_i(t), \qquad i = 1, 2, \ldots, m \tag{3.22}$$

It is relevant to stress the vertically hyper-integrated character of $L_i(t)$: in the most general specification of technology, a fraction of the total labour employed by a single industry producing a *basic* commodity would enter into the employment of *all* vertically hyper-integrated sectors, either directly and/or (hyper-)indirectly.

The comparison with the vertically hyper-integrated nature of the sectoral capital/output ratios — $\chi_i(t)$ in (3.16) — is straightforward. The composition of sectoral employment reflects not only the change in labour requirements of the industry producing the final consumption commodity concerned, but also the changing physical composition of the corresponding unit of productive capacity, and therefore the changes in labour requirements of *all* the industries composing the sector. It is for this reason that evaluating only the change in direct labour requirements cannot account for the interdependent and systemic nature of productivity changes. This opens up the possibility of performing empirical investigations on the dynamics of productivity taking vertically hyper-integrated sectors as the unit of analysis.[13]

We can now specify the dynamics of $\ell_i(t)$. By defining, for any variable $y(t)$ in the system, $\dot{y}(t) \equiv dy(t)/dt$, we can write:

$$-\frac{\dot{\ell}_i(t)}{\ell_i(t)} \equiv \varrho_i'(t) = \varrho_i \frac{a_{ni}(t)}{\ell_i(t)} + \varrho_{k_i} \frac{T_i^{-1} a_{nk_i}(t)}{\ell_i(t)} + \varrho_{k_i} \frac{(g + r_i) a_{nk_i}(t)}{\ell_i(t)} \tag{3.23}$$

$\varrho_i'(t)$ is the rate of growth of vertically hyper-integrated labour productivity of sector i, given by the weighted average of the rates of growth of direct, indirect and hyper-indirect labour productivity, the weights being the proportions of the three kinds of labour to total labour employed in vertically hyper-integrated sector i, respectively.

3.7 Dynamic equilibrium path for relative quantities, sectoral employment and relative prices

At this point, it is possible to describe the equilibrium path of relative quantities, sectoral employment and relative prices.

Let us start from the general dynamic movements for relative quantities and prices — given by expressions (3.8) and (3.9). As we have assumed that the capital accumulation equilibrium conditions (3.15), as well as the effective demand condition (3.20), hold, we can now specify the equilibrium path of relative quantities and prices, and the evolution of sectoral employment — given by expressions (3.22).

If "we choose to reckon prices in terms of Classical 'labour commanded' " (Pasinetti 1981, p. 99), the wage rate still being the basis for the price system, we set $\overline{w} = 1$. Hence, the equilibrium dynamic path of relative physical quantities, sectoral employment and commodity prices is given by:[14]

$$\begin{cases} x_i(t) = a_{in}(0)\overline{x}_n(0)e^{(g+r_i)t} \\ x_{k_i}(t) = \left(\frac{1}{T_i} + g + r_i\right) a_{in}(0)\overline{x}_n(0)e^{(g+r_i)t} \end{cases} \tag{3.24}$$

$$\begin{cases} L_i(t) = \ell_i(t)a_{in}(0)\overline{x}_n(0)e^{(g+r_i)t} \end{cases} \tag{3.25}$$

and

$$\begin{cases} p_i^{(w)}(t) = \ell_i(t) + \left(\pi_i(t) - (g + r_i)\right) a_{nk_i}(0)e^{-\varrho_{k_i} t} \\ p_{k_i}^{(w)}(t) = a_{nk_i}(0)e^{-\varrho_{k_i} t} \end{cases} \tag{3.26}$$

for $\quad i = 1, 2, \ldots, m$

where the expression for $p_i^{(w)}(t)$ is already written in the same form as in (3.21).[15]

The equilibrium solutions for physical quantities, given by expressions (3.24), together with equilibrium sectoral employment, given by expressions (3.25), represent a set of growing subsystems, one for each final consumption commodity i. Each growing subsystem or, equivalently, hyper-subsystem consists of three components: $x_i(t)$, $x_{k_i}(t)$ and $L_i(t)$. The first one represents the "production of one single consumption good i, expanding through time at its particular rate of growth $(g + r_i)$" (Pasinetti 1988, p. 127). The second one, represents the physical quantities for "the maintenance of a circular production process that *both* reproduces all the means of production which are absorbed by the production process for [each] consumption good [...] *and* also produces those means of production that are strictly necessary to expand such a circular process at a rate of growth $(g + r_i)$" (Pasinetti 1988, p. 127). Finally, the third one represents the "absorption of a physical quantity of labour $L_i(t)$" (Pasinetti 1988, p. 127) required to produce physical quantities $x_i(t)$ and $x_{k_i}(t)$.

To see the implied structural dynamics, it is worth computing the rates of change of relative quantities, sectoral employment and relative prices:

$$\frac{\dot{x}_i(t)}{x_i(t)} = \frac{\dot{x}_{k_i}(t)}{x_{k_i}(t)} = g + r_i \tag{3.27}$$

$$\frac{\dot{L}_i(t)}{L_i(t)} = g + r_i - \varrho_i'(t) \tag{3.28}$$

$$\frac{\dot{p}_i^{(w)}(t)}{p_i^{(w)}(t)} \equiv \sigma_i^{(w)}(t) = -\varrho_i'(t)\frac{\ell_i(t)}{p_i^{(w)}(t)} + \left(\frac{\dot{\pi}_i(t)}{\pi_i(t) - (g + r_i)} - \varrho_{k_i}\right)\frac{(\pi_i(t) - (g + r_i))p_{k_i}^{(w)}(t)}{p_i^{(w)}(t)} \tag{3.29}$$

$$\frac{\dot{p}_{k_i}^{(w)}(t)}{p_{k_i}^{(w)}(t)} \equiv \sigma_{k_i}^{(w)}(t) = -\varrho_{k_i} \tag{3.30}$$

where $\sigma_i^{(w)}(t)$ is the rate of change of the relative price of commodity i when the *numéraire* of the price system is the wage rate.[16]

As regards sectoral physical quantities, their equilibrium evolution, given by expressions (3.27), is completely determined by the evolution of effective demand for the corresponding final consumption commodity i on which each growing subsystem is built. This holds true for both x_i and x_{k_i}, due to the adoption of the units of productive capacity as the particular units of measurement for capital goods. Hence, the rate of change of physical quantities is given by the sum of two components: the rate of growth of population, g, common to all sectors; and the rate of change of sectoral per-capita demands for final consumption commodities, r_i, specific to each sector. Since these are different from sector to sector, the whole structure of relative quantities is changing through time.

As regards sectoral employment, its equilibrium evolution, given by expressions (3.28), is determined both by the equilibrium evolution of relative quantities, and by the dynamics of vertically hyper-integrated labour productivities. Since r_i is different from $\varrho_i'(t)$, and both are different from sector to sector, the whole structure of employment, i.e. the division of labour within the economic system, is continuously changing through time. This makes clear how a *sectoral reallocation* of employment is an essential requirement for the system to follow a full-employment path. It is worth noticing that, at the most general description of technology, a change in labour productivity in a single *industry* producing a basic commodity would be enough to change the *whole* structure of *sectoral* employment.

As regards relative prices, let us first notice that the equilibrium dynamics for the price of a unit of productive capacity for final consumption commodity i, given by expressions (3.30), is a particularly simple one, due to the simplifying assumption made, i.e. capital goods are produced by means of labour alone. As a consequence, prices are completely determined by labour costs, and therefore their equilibrium evolution only depends on the changes of labour productivity in the industry producing the specific capital good for sector i.

The equilibrium dynamics of commodity prices, given by expressions (3.29), reveals the process of change of a production price brought about by the interaction of technical progress and changes in the distribution of income. The rates of change of commodity prices are given by the weighted average of the rates of change of their two components. The first addendum shows how an increase in vertically hyper-integrated labour productivity exerts a univocally negative effect on production prices. The second addendum quantifies the effect of a change in income distribution — through a variation in the sectoral profit rate — on the 'labour commanded' production prices.

As we have already said, the first component reflects a necessary, physical self-replacement and expansion requirement; accordingly, its rate of change is completely determined by technology and equilibrium new investment, i.e. by the rate of change of vertically hyper-integrated labour $\varrho_i'(t)$. On the contrary, the second component of production price also reflects income distribution. Accordingly, its rate of change depends not only on labour productiviy in the capital goods producing industry — i.e. the rate of change of the price of the unit of productive capacity on which profits are computed — but also on the variation through time of the sectoral rate of profit — i.e. on the rate of change of the profit differential with respect to the sectoral equilibrium rate of new investment.

This analysis completes the description of the structural equilibrium dynamics of a growing economic system. In fact, we have described *a set* of equilibrium paths, one for each possible realisation of the *sequence* of sectoral rates of profit, so far considered as exogenous magnitudes.

4 The ‘natural’ economic system

We are now in a position to introduce what Pasinetti calls the ‘natural’ economic system, i.e. that particular equilibrium path associated to one specific sequence of sectoral rates of profit, that, “without recourse any longer to any exogenously given economic magnitude, now come to complete and close the whole relative price system of our theoretical scheme” (Pasinetti 1981, p. 131), due to the adoption of a particular *theory of the rate of profit*.

4.1 The ‘natural rates of profit’

As we have explained in section 2, the aim of Pasinetti’s (1981) book is that of developing a framework explaining the ‘primary and natural’ features of a growing economic system, independently of a particular institutional set-up. A reasoning in these terms, when coming to the issue of the distribution of income, would seem, at first sight, counterintuitive, since the way in which income is distributed crucially depends on the character of the *social relations of production*, no less than on cultural, ethic, legal considerations, that is to say, precisely on the institutional set-up of society. In fact, those analyses taking income distribution as exogenous, are clearly embedded in a specific institutional set-up.

Then, how can a theory of the rate of profit be conceived that is independent of it?

As Pasinetti states, the 'natural' economic system deals with *logical* relations, based on magnitudes given from outside economic analysis (and therefore taken as exogenous), and emerging from the physical growth requirements of the system itself. The problem must be therefore faced from this perspective: is there "a natural rate of profit (...) already *logically* implied in the previous theoretical framework *because the economic system* considered *is a growing one*"? (Pasinetti 1981, p. 128, italics added)

The answer to this question is: yes.

The crucial point is that at a pre-institutional stage of the analysis, a theory of the rate of profit is *not* a theory of income distribution among income recipients, i.e. individuals or groups of individuals. This is because the very definition of the categories among which the purchasing power generated in the process of production is to be distributed essentially depends on the social relations of production of a particular institutional set-up.

However, the very nature of an industrial system requires to perform a separation between the means of production that enter a circular process, and the set of commodities that are left out from the circular flow, once they are produced.

Moreover, when the system is a growing one, the new investment requirements become a *necessary* expansion of the means of production.

Now, hence, prices of production must, on the one hand, be precisely those exchange ratios that satisfy the conditions of re-production and growth — i.e. that *include* the growth of the means of production at the equilibrium rate of accumulation. But given that the equilibrium requirement to expand productive capacity differs among vertically hyper-integrated sectors, the surplus factor in the price of production of each consumption commodity must reflect this difference.

On the other hand, prices of production provide for the purchasing power both to self-replace and expand productive capacity and to consume those commodities not re-entering the circular flow. Consider that profits and wages just establish the amount of purchasing power that must be channeled to demand for means of production to expand productive capacity and to demand for final consumption commodities, respectively. In this sense, profits and wages would establish a truly *functional* distribution of income, as they stand for categories that channel purchasing power to two different economic *functions*. These functions arise from the conditions of production of physical quantities, in particular, from the need to separate what enters the circular flow (and is used as means of production) from what it does not (and is consumed).

As a consequence, from the reasoning stated above, it follows that profits must correspond to the purchasing power necessary for the equilibrium expansion of productive capacity in each vertically hyper-integrated sector to take place. In formal terms:

$$\pi_i^*(t)p_{k_i}(t)k_i(t) = (g + r_i)p_{k_i}(t)x_i(t), \quad \forall i = 1, 2, \ldots, m; \quad t \geq 0 \tag{4.1}$$

and therefore, since in equilibrium $x_i(t) = k_i(t)$:

$$\pi_i^*(t) = \pi_i^* = g + r_i, \quad i = 1, 2, \ldots, m \tag{4.2}$$

The equilibrium configuration corresponding to the just given structure of rates of profit is the only one that keeps the analysis at a strictly pre-institutional level.

4.2 A pure labour theory of value

The rates of profit in (4.2) are the 'natural' rates of profit. When inserted into the equilibrium solutions for consumption commodity prices in (3.26), these become:

$$p_i^{(w)*}(t) = \ell_i(t), \quad i = 1, 2, \ldots, m \tag{4.3}$$

their rate of change through time being:

$$\frac{\dot{p}_i^{(w)*}(t)}{p_i^{(w)*}(t)} = -\varrho_i'(t), \quad i = 1, 2, \ldots, m \tag{4.4}$$

Expressions (4.3) highlight the main result of the present formulation: when labour is the *numéraire* of the price system, and the rates of profit are the natural ones, prices — i.e. 'labour commanded' prices — come to be exactly equal to 'labour embodied'. Therefore, this theoretical scheme implies a generalisation of a *pure labour theory of value*, where the equality of 'labour commanded' and 'labour embodied' is achieved thanks to a "re-definition of the concept of 'labour embodied', which must be intended as the quantity of labour required directly, indirectly and hyper-indirectly to obtain the corresponding commodity as a consumption good" (Pasinetti 1988, pp. 131-132).

With the introduction of the 'natural' rates of profit, both the value of productive capacity for self-replacement and the profits computed on the value of existing productive capacity have the same function of "computing amounts of labour indirectly required elsewhere in the economic system for the equilibrium production of consumption good *i*" (Pasinetti 1981, p. 132).

Not less importantly, this result holds not at the level of the economic system as a whole, but in each single sector. Each growing subsystem following an equilibrium path of accumulation has a 'dual' value side that ascribes to natural prices a straightforward foundation, based on the "basic principle of equal rewards for equal amounts of homogeneous labour." (Pasinetti 1981, p. 133).

Equally interesting, at the most general specification of technology, the labour embodied in the basic commodities produced by a single industry participate in profits of all vertically hyper-integrated sectors. In this way, changes in the productiviy of *labour* in an industry alter the value of *profits* on capital of all sectors. Thus, it becomes clear that "it is not the 'productivity of capital', or of any commodity, that turns out to be the *raison d'être* of the rate of profit. It is the growth, and the increasing productivity, of labour!" (Pasinetti 1981, p. 133).

4.3 Natural profits, wages, new investments and consumption

There is a very clear asymmetrical relation between total natural profits and wages. The total national income produced in a specific time period, i.e. the *value* of total production, net of replacements, at current prices, is distributed among total (natural) profits and total wages. While the former emerge from the *physical* conditions for equilibrium growth as a necessity, if full employment and full capacity utilisation are to be maintained through time, the latter can be seen as a 'surplus', absorbing all the remaining national income. "To produce, and to continually increase this 'surplus', through technical progress, is precisely the purpose of the whole production process" (Pasinetti 1981, p. 144).

In the same way, there is an asymmetric relation between total new investments and consumption. The total quantities produced in a specific time period, net of replacements, must be devoted in part to new investments and in part to final consumption. While the former are determined — by the structure of final demand for consumption and its evolution through time — as a *physical* requirement for equilibrium growth, the aggregate level of the latter can be seen as a 'surplus', absorbing all the remaining purchasing power. Again, "to produce this surplus, and to continually increase it through technical progress, is the whole purpose of the production process" (Pasinetti 1981, p. 146).

As can be seen by the very definition of natural profits, emerging from conditions (4.1), in the 'natural' economic system, total profits will be equal to the value, at current prices, of total new investments, and correspondingly total wages will be equal to the value of total final consumption. But what is even more interesting is that this holds not as "a mere over-all averanging-out result, but [as] the consequence of a whole series of equalities realised at each single sectoral stage" (Pasinetti 1981, p. 147). In fact, condition (4.1) establishes that:

$$p^*_{k_i}(t)(g + r_i)a_{in}(t)x_n(t) = p^*_{k_i}(t)\pi^*_i x_i(t), \quad i = 1, 2, \ldots, m \tag{4.5}$$

and from the expression for natural prices of consumption commodities (4.3) it follows that:

$$p_i^*(t)a_{in}(t)x_n(t) = w(t)L_i(t), \quad i = 1, 2, \ldots, m \tag{4.6}$$

As a consequence, the value, at current prices, of total quantities, net of replacements, produced in each vertically hyper-integrated sector equals the total income it generates, i.e.:

$$p_i^*(t)a_{in}(t)x_n(t) + p_{k_i}^*(t)(g + r_i)a_{in}(t)x_n(t) = w(t)L_i(t) + p_{k_i}^*(t)\pi_i^* x_i(t) \tag{4.7}$$

for $\quad i = 1, 2, \ldots, m$

4.4 Changes in productivity and distributive variables

A straightforward consequence of expressions (4.4) is that any price reduction due to increases in labour productivity immediately translates into a corresponding increase in the real purchasing power of wages. This can be seen even more clearly by changing the *numéraire*.

Any commodity or composite commodity can be chosen as the *numéraire* of the price system; analytically, this amounts to setting its price equal to unity, and keeping it constant through time. For example, if commodity h is chosen as the *numéraire*, we set:

$$\begin{cases} p_h^{(h)}(0) = 1 \\ \sigma_h^{(h)}(t) = \sigma_h^{(h)} = 0 \end{cases} \tag{4.8}$$

Once the *numéraire* is specified, the wage rate has to be expressed in terms of it; this again means closing *two* degrees of freedom, i.e. we have to set both the wage rate at time zero and its rate of change in terms of the chosen *numéraire*. Within the 'natural' economic system, again taking commodity h as the *numéraire*, this means setting:

$$p_h^{(h)*}(t) = w(t)\ell_h(t) = 1 \tag{4.9}$$

from where we obtain:

$$w^{(h)}(t) = (\ell_h(t))^{-1} \tag{4.10}$$

and, therefore, we set:

$$\begin{cases} w^{(h)}(0) = (\ell_h(0))^{-1} \\ \frac{\dot{w}^{(h)}(t)}{w^{(h)}(t)} \equiv \sigma_w^{(h)}(t) = \varrho_h'(t) \end{cases} \tag{4.11}$$

and the rate of change of the price of any consumption commodity i is given by:

$$\frac{\dot{p}_i^{(h)*}(t)}{p_i^{(h)*}(t)} = \varrho'_h(t) - \varrho'_i(t) \tag{4.12}$$

Hence, the rate of change of the wage rate in terms of the chosen *numéraire* — the *real* wage rate — is given by the rate of increase of labour productivity in the corresponding vertically hyper-integrated sector, and the rate of change of the price of commodity i in terms of the chosen *numéraire* is given by the difference of the rate of change of labour productivity in the corresponding sector with respect to the rate of change in vertically hyper-integrated labour productivity in the sector producing the *numéraire* commodity.

As a consequence, within the 'natural' economic system, the dynamics of the wage rate and the sectoral rates of profit have two different orders or magnitude (see Pasinetti 1981, p. 143). The *level* of each π_i^* in (4.2) is given by two constant rates of change,[17] while the *rate of change* of the *real* wage rate is given by the rate of change of labour productivity in the vertically hyper-integrated sector producing the commodity chosen as *numéraire*. "In the long run, therefore, while the real wage rate will persistently grow, the rate(s) of profit cannot but roughly remain at the same level" (Pasinetti 1981, p. 143).

4.5 Natural structural dynamics

By closing the relative price system with the specific structure of sectoral rates of profit given by expressions (4.2) — the 'natural' rates of profit — we are actually closing the last degree of freedom left open at the end of section 3, focusing on the particular dynamic equilibrium path of:

(i) relative physical quantities of each growing subsystem — expressions (3.24) and (3.27);

(ii) sectoral employment — expressions (3.25) and (3.28);

(iii) final consumption commodity (relative) prices — expressions (4.3) and (4.4);

(iv) (relative) prices of the units of vertically hyper-integrated productive capacity — the second series of expressions in (3.26) and expressions (3.30).

which constitute the complete description of the 'natural' economic system.

It is worth concluding our description of the 'natural' economic system by noticing that the only explicitly different analytical formulations, with respect to the general case, are given by expressions (4.3) and (4.4), concerning final consumption commodity prices. The relative physical quantity system and sectoral employment would apparently be the same, irrespective of the particular rates of profit chosen.

However, this result is a consequence of the fact that, in this framework, demand coefficients are taken as given. But the structure of demand is strongly dependent on consumers' real income, which in turn is determined by the structure of relative prices and therefore also by the ruling rate(s) of profit. To be more precise, therefore, we are not considering the $a_{in}(t)$'s as exogenous, but we are considering as exogenous the *mechanism* by which changes in income distribution modify the structure of final consumption demand (and therefore of relative physical quantities). Such mechanism is constantly at work.[18] Demand coefficients in expressions (3.24) and (3.25) may therefore be different according to the particular configuration of the rate(s) of profit.

5 A methodological note

It might be useful to open this brief note on Pasinetti's (1981) method by recalling the definition of equilibrium, already given at the beginning of section 3, adopted all throughout the book:

> A *situation of equilibrium* will simply be taken to mean a situation in which there is full employment of the labour force and full utilisation of the existing productive capacity.
>
> (Pasinetti 1981, pp. 48-49)

It is not trivial that Pasinetti is referring to a '*situation* of equilibrium'; the choice of the term highlights the transitory character of any equilibrium position eventually reached at a certain point in time. In fact, "no connotation of automaticity and no association with any particular adjustment mechanism is intended to be implied by such an expression" (Pasinetti 1981, p. 48).

The 'natural' economic system is by no means an attempt at describing the functioning of an actual capitalist system; it is an attempt at singling out the 'primary and natural' features of an *industrial* system intended, i.e. "necessary requirements for equilibrium growth" (Pasinetti 1981, p. 25). Equilibrium growth, however, entails neither the identification of a 'normal position' towards which the system tends in the long run — since the very structural dynamics of the economic system makes it impossible to identify a 'normal position' *persistent* enough to the continuous changes in the system's proportions — nor a logical *succession* of temporary equilibria spontaneously realised.

The equilibrium dynamics defining the 'natural' economic system specifies the reproportioning of productive capacity, relative quantities — and therefore sectoral employment — and relative production prices *necessary to comply with* the ever-changing structure of final demand for consumption goods and with the pace of technical progress. It is important to stress that this process of re-proportioning is not *spontaneous*, but must be actively pursued if the new situation of equilibrium is to be reached period after period.

Furthermore, the empirical point of departure of the analysis must be explictly mentioned: “The coefficients that appear [...] in the present (vertically [hyper-]integrated) analysis must [...] be interpreted as representing those physical quantities which can actually be observed” (Pasinetti 1981, p. 110), to which there corresponds — for *each* time period — a specific equilibrium situation. These equilibrium situations, together with the *necessary* dynamic conditions connecting them through time, establish a ‘normative configuration’. In this sense, therefore, the ‘natural’ economic system is a “norm; and the norm is always there — even if it is not so much apparent — in the *short* no less than in the long run” (Pasinetti 1981, p. 127n).

Since the whole structure of physical quantities and technical production requirements are continuously changing through time, the problem arises of how to perform a truly dynamic analysis, connecting equilibrium situations with completely different characteristics. Pasinetti solves the problem by developing the analytical device of vertical hyper-integration: “By resolving all varieties of products into the same constituent elements — a flow of labour and a stock of capital goods both expressed in physical terms — the vertically [hyper-]integrated approach leads to relations whose permanence over time is independent of specific technical possibilities” (Pasinetti 1981, p. 116).

It worth stressing, however, that vertical hyper-integration is not a mere analytical device for making dynamic analysis possible; it also has a very impotant conceptual role within the development of the present theoretical framework — consider, for example, its role in the redefinition of the concept of 'labour embodied' in the theory of value implied by the 'natural' economic system.

To conclude, the 'natural' economic system is strongly rooted on the notion of vertically hyper-integration. The stages of development of this concept and its connection to Pasinetti's (1981) model of *Structural Change and Economic Growth* are explored in the next section.

6 Stages of development of the concept of vertical hyper-integration

The concept of vertical hyper-integration, together with its analytical formulation, is one of the cornerstones of the approach to structural dynamics put forward by Pasinetti. However, its development has not been immediate, as it clearly went through different stages. Pasinetti's (1981) book has been the final result of a process that began with his Doctoral dissertation at the University of Cambridge (Pasinetti 1962) — partially published in Pasinetti (1965). However, the book has itself been an intermediate outcome as regards the analytical elaboration of the device of vertical hyper-integration, which came to final accomplishment in Pasinetti (1988).

In his first general treatment of vertical integration (Pasinetti, 1973), the author explicitly recognised that he has “always been faced with questions” (Pasinetti 1973, p. 2) on “how to construct the vertically integrated sectors in the general case” (Pasinetti 1973, p. 2n), i.e. beyond the particular case where capital goods are made by labour alone. In fact, indications for proceeding towards a generalisation, in Pasinetti (1965), had been given “in a brief and incomplete way” (Pasinetti 1973, p. 2).

In Pasinetti (1973, p. 2) “The economic system is supposed to be viable, in the sense that is capable of producing larger quantities of commodities than those required to replace used-up capital goods”. From this basic assumption on the character of the circular process, the analysis proceeded towards obtaining a compact way to represent sub-systems in the sense of Sraffa (1960). In this case, the net output consisted of both final consumption commodities and new investment goods. However, the crucial difference between vertical integration and vertical hyper-integration departs from the consideration of new investments as belonging either to the net output — and therefore not entering the circular process — or to the means of production which re-enter the circular flow. As stated clearly in Pasinetti (1988):

> We now proceed in a way perfectly analogous to the one used in defining the earlier subsystems in Pasinetti (1973), but with the essential difference here of including in each hyper-subsystem *all* gross investments (both replacements *and* net investments).
>
> (Pasinetti 1988, p. 127)

In Pasinetti (1973), the analytical focus was mainly on the role of vertically integrated sectors in the theory of value and income distribution, while the analysis of economic growth with technical progress has been done by recalling the particular case of capital goods produced by labour alone. In fact, it has been only within the 'dual' exercise of an economy growing at a steady rate g in absence of technical progress that new investments were explicitly treated as part of the means of production expanding at the uniform growth rate. And the exercise was performed exclusively "in the search for an equilibrium growth solution" (Pasinetti 1973, p. 20).

In this sense, the analysis of dynamic models of equilibrium growth has proven to be an important intermediate analytical step in the discovery of vertical hyper-integration. It has been precisely within the discussion of the full-employment condition of the Dynamic Input-Output model in Pasinetti (1977, Chapter VII) that the first explicit reflection on hyper-indirect requirements came about:

> For every given exponential evolution of consumption [...] the solution

> [...] gives the evolution of the total physical quantities $\mathbf{Q}(t)$ which are required — as direct, indirect, and, we might add, hyper-indirect requirements (meaning by the last the requirements for new investment) — to keep the economic system in dynamic equilibrium.
>
> (Pasinetti 1977, p. 196)

It is interesting to note that the concept is formulated within the discussion of the necessary conditions for dynamic solutions to comply with full capacity utilisation and full employment of the labour force. The author reaches the conclusion that if the initial situation of the system is an appropriate one, both full employment and full capacity utilisation would follow through time.

The analysis of the Dynamic Input-Output model in Pasinetti (1977) marked a sharp difference with respect to Pasinetti (1973). The focus was not on the analytical treatment of subsystems but on the derivation of dynamic equilibrium solutions, on the one hand highlighting the method of analysis — by which the level (and structure) of per capita consumption was the only component of net-output considered as given (Pasinetti 1977, pp. 194-195) — and, on the other hand, showing the restrictions on the choice of the consumption structure imposed by the maximum rate of growth (Pasinetti 1977, p. 209). This second aspect would be taken up again in the Appendix to Chapter VI of Pasinetti (1981), where important insights have been presented on the difficulties of relying on von

Neumann proportional dynamics to analyse systems undergoing technical change.

But then, how come the analysis of the Dynamic Input-Output model had already been carried out with an explicit identification of vertically hyper-integrated magnitudes, while Pasinetti (1981) made only partial treatment of vertical hyper-integration? Hints at the stages of development of Pasinetti (1981) are found in the preface of the book. At this juncture — as has been pointed out in section 3.2 — Pasinetti himself had already begun to think in terms of vertically hyper-integrated sectors, though this is only reflected in some parts of the book. In fact, it can be seen that (almost) all the entries in the index concerning vertical hyper-integration belong to the chapters of the book which "have been almost entirely re-written" (Pasinetti 1981, p. xiv) since the time of his PhD Thesis, while in the remaining parts of the book expressions referring to vertically integrated magnitudes are still present.

It is clear from the analysis carried out so far that the model in Pasinetti (1981) adopts a method of singling out conditions for dynamic equilibrium based on necessary physical requirements for self-replacement and expansion — therefore considering new investments as part of the means of production re-entering the circular flow — but instead of dealing with a system growing at a uniform rate, develops the analysis within the framework of growing subsystems, even though a very simplified description of the technique is adopted — the one already present in Pasinetti (1965).

Therefore, it is our contention that Pasinetti's (1981) *Structural Change and Economic Growth* presents a vertically hyper-integrated model, though within a very simplified description of the technique. Many insights are further enriched when the model is seen through these lens. For example, the role of the vertically hyper-integrated units of productive capacity in the analysis of accumulation (presented in section 3.2) and that of vertically hyper-integrated labour coefficients in the analysis of value (presented in section 3.6 and in section 4.2 within the 'natural' economic system). Of course, a full generalisation of vertical hyper-integration in the context of non-proportional growth would only arrive with Pasinetti (1988), where growing subsystems would acquire their most general formulation.

Notes

[1]Pasinetti (2007), where he stresses in a much sharper way such a discrimination.

[2]We will expose here the specification of the model that considers capital goods produced by means of labour alone, for it is the main case Pasinetti (1981) deals with.

[3]The concept of vertically hyper-integration is already present in Pasinetti (1981), even though not always explicitly. For a rigorous statement and development of this concept, and of its analytical properties, see Pasinetti (1988).

[4]This is particularly clear when matching the chapters of the book which "have been almost entirely re-written" (Pasinetti 1981, p. xiv) since the time of his PhD Thesis with the entries in the index concerning vertical *hyper*-integration. We shall come back to this point later on, in section 6.

[5]For details, see Pasinetti (1981, pp. 33-34).

[6]In the solution for $p_i(t)$ given in Pasinetti (1981, p. 41) it is implicitly assumed that $x_i(t) = k_i(t)$:

$$p_i(t) = \left(a_{ni}(t) + a_{nk_i}(t)(T_i^{-1} + \pi_i(t))\right)\overline{w}$$

This amounts to stating that the productive capacity available at the beginning of time period t is exactly used up. In order to make the formulation as general as possible, we have decided not to make this assumption at this stage of the analysis.

[7]This is a crucial difference between the notion of vertically integrated sectors and that of vertically *hyper*-integrated sectors (see Pasinetti 1973, Pasinetti 1988).

[8]See Pasinetti (1973, p. 7, section 5) and Pasinetti (1988, p. 130, section 4).

[9]For the sake of simplicity, we are here assuming steady rates of change of the relevant variables, though this is not the procedure adopted by Pasinetti (1981), at least for the rate of change of final demand for consumption commodities (See Pasinetti 1981, p. 82). This is a crude simplification, though it is not possible — according to the authors — to take full advantage of the increasing realism of working with non-steady rates of change if the model is specified in continuous time. For the scope of the present work, moreover, the simplification adopted does not compromise the conclusions to be reached.

[10]For a hint at different cases that can occur as a consequence of flow and stock disequilibria, see Pasinetti (1981, pp. 47-48).

[11]In Pasinetti (1981), as each capital goods-producing industry is specific to each consumption goods-producing one, it is the *second* aspect that is emphasised, though the framework allows for further generalisation to reflect also the first one. See Pasinetti (1988).

[12]For details, see Pasinetti (1981, p. 102).

[13]For an empirical study taking this direction, see Garbellini & Wirkierman (2010).

[14]For a complete analysis of the equilibrium structural dynamics of a growing economic system, see Pasinetti (1981, pp. 91-99).

[15]In what follows, whenever a *nominal* magnitude has a letter in brackets as a superscript, that letter will indicate the *numéraire* commodity adopted. Therefore, $p_i^{(w)}(t)$ indicates the price of commodity i when the *numéraire* of the price system is the wage rate.

[16]It must be carefully noticed that expressions (3.29) have a finite value for $\pi_i(t) \neq$

$g + r_i, \quad i = 1, 2, \ldots, m.$

[17]When the hypothesis of steady rate of change of per capita demand for consumption commodity i ($i = 1, 2, \ldots, m$) is removed, the natural rates of profit are no more exactly constant through time, but shall exhibit a roughly constant trend.

[18]See Pasinetti (1981, pp. 71-77).

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Table 1: Basic notation in Pasinetti's *Structural Change and Economic Growth*

$x_i(t)$	number of units of final consumption commodity i produced during time period t in the vertically hyper-integrated sector i;
$x_{k_i}(t)$	gross investment in the vertically hyper-integrated sector i, i.e. number of units of productive capacity for final consumption commodity i produced during time period t;
$x_n(t)$	total units of labour available at the beginning of time period t;
$p_i(t)$	price of a unit of final consumption commodity i during time period t;
$p_{k_i}(t)$	price of a unit of productive capacity for final consumption commodity i during time period t;
$w(t)$	wage rate during time period t;
$\pi_i(t)$	profit rate of the industry producing final consumption good i during time period t;
$a_{in}(t)$	average per capita demand for final consumption commodity i during time period t;
$a_{k_in}(t)$	average per capita demand for units of productive capacity for final consumption commodity i during time period t;
$a_{ni}(t)$	direct labour requirements for the production of one unit of final consumption commodity i during time period t;
$a_{nk_i}(t)$	direct labour requirements for the production of one unit of productive capacity for final consumption commodity i during time period t;
T_i	reciprocal of the coefficient of wear and tear of one unit of productive capacity for final consumption commodity i;
$x'_{k_i}(t)$	demand for units of productive capacity for final consumption commodity i for replacement of worn out capacity during time period t;
$x''_{k_i}(t)$	net investment in the vertically hyper-integrated sector i, i.e. new investment demand for units of productive capacity for final consumption commodity i during time period t;
$k_i(t)$	stock of units of productive capacity for the vertically hyper-integrated sector i available at the beginning of time period t;
$\chi_i(t)$	capital/output ratio at current prices in time period t for vertically hyper-integrated sector i;