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Abstract

In this paper we provide a closed-form expression for one of the most popular index in Technical Analysis: the Relative Strength Index (RSI). Given that we show how the standard binomial model for the stock price can be used to predict RSI. The algorithm is as simple as to code a standard European option. In an empirical application to the Chilean exchange rate we show how the method works having a better out of sample performance than an ARMA(1,1) model.

JEL Codes: E43, G12

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1 Introduction

It is well-known that traders do not only rely in fundamental analysis to invest in their portfolios. Indeed, one of the most common tool across them is the use of technical analysis. This is particular important for developing countries, for example Abarca et al (2007) report that Chilean traders use technical analysis for investing in the Chilean peso.

In statistical sense this analysis is intended to extract the cyclical component of stock prices. One simple approach is to compare several moving average measures of the stock price. Comparing these the analyst is able to “predict” the most probable trend of the stock price.

In this paper we concentrate our effort in the Relative Strength Index (hereafter RSI), which is defined as the ratio between average-gains over average-gains plus average-losses (Wilder, 1978). To the best of our knowledge this index is very popular among traders, in particular the ones which portfolios are biased to currencies. We should note that the RSI is available in most the trader applications and for that the index is given for the traders.

Our first contribution is to derive a closed-form expression for the RSI which acts like a highly non-linear filter of the stock price. We should note that by definition the index is between 0 and 100 and for that the analysts say that an asset with RSI outside of the interval $[30, 70]$ is under or overpriced. This rule is based on experience and it will not be discussed in this paper, but it could be analyzed in the same framework that we work here.

The second and more attractive contribution is to provide a “financial forecast” of the index. For that purpose we use the Binomial Tree Model (BTM) as the distributional assumption for the stock price. Also the standard Geometric Brownian Motion (GBM) can be used for this purpose but it requires to use simulations. In order to avoid those computation burden we provide the asymptotic expansion for the first moment of the RSI which agrees with the BTM. Given that we believe that the binomial model is a good motivation to show that forecasting the RSI is similar as pricing a European option.

Finally, we provide an empirical application to the Chilean exchange rate. There we show how the parameters of the BTM could be calibrated minimizing Mean Squared Error (MSE) of the one period out-of-sample forecast. Based on daily data we show that the minimum MSE is obtained by using the last two months of data. This model has a better out-of-sample performance than the best ARMA model that we can estimate for the RSI.

2 Modeling the Relative Strength Index

In this section we provide a closed-form expression for the RSI and using standard distribution for the stock price we show how the expected value of RSI can be computed. For simplicity we rely on the binomial model, but complex distributions can be analyzed using simulations.

2.1 Working on the definition

For simplicity we consider the unscaled RSI, which implies that the range of this index will be $[0, 1]$. In particular, let Z_t be the RSI for the stock price S_t, and it is defined as

$$Z_t = \frac{A_t}{A_t + B_t} \tag{1}$$

where A_t and B_t are the average gains and losses up to the the time t. Both are computed as weighted average, where weights are inversely proportional to the size of the window selected (K). Usually, $K = 13$ (RSI-14) but this does not affect the main results of this paper as long as K^3 is big enough.

Component A_t accumulates average gains, but actually it accumulates non-negative changes, its definition is based on previous value as:

$$A_t = \left(\frac{K}{K+1}\right) A_{t-1} + \frac{1}{K+1} \left(S_t - S_{t-1}\right)^+ , \tag{2}$$

where $()^+$ is truncated at zero function, which means that $(x)^+ = x$ if $x > 0$ and zero otherwise. It is clear that A_t will decrease if stock price is going down. By the same idea, B_t is defined as follows

$$B_t = \left(\frac{K}{K+1}\right) B_{t-1} + \frac{1}{K+1} \left(S_{t-1} - S_t\right)^+ . \tag{3}$$

Following the previous example, B_t will increase if stock price is going down, as result of this RSI will decreases. Traders will see this fact as a signal to sell the asset.

Replacing (2) and (3) in (1), and defining $\phi = 1/K$, we have:

$$Z_t = \frac{A_{t-1} + \phi \left(S_t - S_{t-1}\right)^+}{A_{t-1} + \phi \left(S_t - S_{t-1}\right)^+ + B_{t-1} + \phi \left(S_{t-1} - S_t\right)^+}.$$

Note that $(S_t - S_{t-1})^+ + (S_{t-1} - S_t)^+ = |S_t - S_{t-1}|$, where $|\cdot|$ stands for absolute value. Finally, consider $(1 + R_t) = S_t/S_{t-1}$, and $X_t = S_t/(A_t + B_t)$, then we have

$$Z_t = \frac{Z_{t-1} + \phi X_{t-1} R_t^+}{1 + \phi X_{t-1}|R_t|}. \tag{4}$$

2.2 RSI meets Binomial Trees and Asymptotic Expansions

With information at time $t-1$, Z_t is a random variable. Note that randomness is generated by S_t. Standard textbooks introduce option valuation using binomial trees (Wilmott, 2007). This valuation uses a specific distribution for the stock price. That distribution was introduced in Cox, Ross, and Rubinstein (1979) as an alternative to the Geometric Brownian Motion proposed in Black and Scholes (1973).

Under this distributional assumption the stock price increases by u if the realized state is H and decreases by d if the state is L. Following the standard calibration, $u > 1$ and $d = 1/u$. In addition, the probability of state H is p, and distribution of the stock price is generated by the combination of these binomial trees. Each combination is defined as step (N). For example, a binomial tree with two steps ($N = 2$) has the following set of possible paths for the stock price $\Omega = \{HH, HL, LH, LL\}$. The total number of trajectories are 2^N which implies that for a reasonable size of N we have enough possible paths to describe any stochastic process. Finally, the probability of event HH can be computed by p^2.

In terms of RSI, we have four possible values: Z_t^{HH}, Z_t^{HL}, Z_t^{LH}, and Z_t^{LL}. Note that Z_t^{HH} is based on $S_t^{HH} = u^2 S_{t-1}$, then

$$Z_t^{HH} = \frac{Z_{t-1} + \phi X_{t-1}(u^2 - 1)}{1 + \phi X_{t-1}(u^2 - 1)}.$$

Z_t^{HL} and Z_t^{LH} are based on S_t^{HL} and S_t^{LH}, which are both equal to S_{t-1}, then $Z_t^{HL} = Z_t^{LH} = Z_{t-1}$. Finally, Z_t^{LL} is based on $S_t^{LL} = d^2 S_{t-1}$, then

$$Z_t^{LL} = \frac{Z_{t-1}}{1 + \phi X_{t-1}(1 - d^2)}.$$

Collecting the previous results we note that the expected value of Z_t with information at time $t-1$ can be computed as follows

$$
\begin{aligned}
E_{t-1}(Z_t) &= p^2 Z_t^{HH} + p(1-p)Z_t^{HL} + (1-p)pZ_t^{LH} + (1-p)^2 Z_t^{LL} \\
&= p^2\left[\frac{Z_{t-1}+\phi X_{t-1}(u^2-1)}{1+\phi X_{t-1}(u^2-1)}\right] + 2p(1-p)Z_{t-1} + (1-p)^2\left[\frac{Z_{t-1}}{1+\phi X_{t-1}(1-d^2)}\right].
\end{aligned}
$$

In order to understand the previous formula consider an Taylor approximation of d^2 around u_0^2, this means $d^2 = (1/u_0^2) - (1/u_0^4)(u^2 - u_0^2) + O(u^4)$. Evaluating at $u_0 = 1$ we have $d^2 \approx 2 - u^2$. Using this approximation, we have $\lambda_{t-1} \equiv \phi X_{t-1}(u^2-1) = \phi X_{t-1}(1-d^2)$, taking this and having states H and L with equal probabilities ($p = 1/2$) we have

$$
\begin{aligned}
E_{t-1}(Z_t) &\approx \frac{1}{4}\left(\frac{Z_{t-1}+\lambda_{t-1}}{1+\lambda_{t-1}} + 2Z_{t-1} + \frac{Z_{t-1}}{1+\lambda_{t-1}}\right) \\
&= \frac{1}{2}\left(\frac{Z_{t-1}}{1+\lambda_{t-1}}\right) + \frac{1}{2}Z_{t-1} + \frac{1}{4}\left(\frac{\lambda_{t-1}}{1+\lambda_{t-1}}\right) \\
&= Z_{t-1} + \frac{1}{2}\left(\frac{\lambda_{t-1}}{1+\lambda_{t-1}}\right)\left(\frac{1}{2} - Z_{t-1}\right).
\end{aligned}
$$

Note that $\lambda_{t-1} > 0$ by definition, then Z_{t-1} returns to one half. This makes the index stationary around that level. However, the stock price is a martingale.

$$
\begin{aligned}
E_{t-1}(S_t) &= p^2 S_t^{HH} + 2p(1-p)S_t^{HL} + (1-p)^2 S_t^{LL} \\
&= \left(\frac{u^2+2+d^2}{4}\right) S_{t-1} \approx S_{t-1}
\end{aligned}
$$

Note that if S_t is a martingale then simple averages preserve that property. In statistical terms, the stock price will have a unit root meanwhile Z_t is expected to be stationary. In fact, Z_t will be a martingale when $u = 1$ which implies that $\sigma = 0$. In practical terms, Z_t is suitable to be modeled by a standard ARMA process, as we do in the following section.

Noting that expected value of RSI is similar as the price of a European option, we have a closed form solution (van der Hoek and Elliot, 2000):

$$
E_{t-1}(Z_t) = \sum_{i=0}^{N} \frac{N!}{i!(N-i)!} p^i (1-p)^{N-i} \left[\frac{Z_{t-1} + \phi X_{t-1}\left(u^{2i-N}-1\right)^+}{1+\phi X_{t-1}|u^{2i-N}-1|}\right] \tag{5}
$$

where the estimators for probability p and factors u and d can be obtained by matching the moments of the binomial distribution with the moments of the stock returns. Following the standard approach (Wilmott, 2007) we could consider a binomial tree with N steps, and let μ and σ be the mean and standard deviation of logarithm returns, then $u = \exp(\sigma/\sqrt{N})$,

$d = 1/u$ and $p = (a - d)/(u - d)$ where $a = \exp(\mu/N)$.

Note that (5) provides a "financial forecast" based on the distributional assumption of stock price. Given that binomial distribution is a standard assumption in option pricing it seems fair to use it for this purpose. However, some traders may disagree about this distribution. For that reason, we introduce a asymptotic expansion of the variable Z_t in terms of the size of the window (K).

Asymptotic expansions are useful tools in econometrics and can be used to derived approximation of the moments of unknown distributions. In this context, there is three methods to approximate the moments of a unknown distribution: Laplace, Nagar, and Kadane (Ullah, 2004). Here, we use Nagar expansion which is based on terms of the sample size, here the size of the window. The asymptotic expansion relies in a big K (or small ϕ). Note that

$$\frac{1}{1 + \phi X_{t-1} |R_t|} = 1 - \phi X_{t-1} |R_t| + \phi^2 X_{t-1}^2 R_t^2 + O_p(\phi^3).$$

Replacing this into the formula of RSI, we have

$$\begin{aligned} Z_t &\approx \left(Z_{t-1} + \phi X_{t-1} R_t^+\right)\left(1 - \phi X_{t-1} |R_t| + \phi^2 X_{t-1}^2 R_t^2\right) \\ &= Z_{t-1} + \phi X_{t-1}\left(R_t^+ - Z_{t-1}|R_t|\right) + \phi^2 X_{t-1}^2\left(Z_{t-1} R_t^2 - R_t^+|R_t|\right) \end{aligned}$$

This equation can be used for any distributional assumption of the stock price. For example, consider the expansion up to $O_p(\phi^2)$ and the standard Geometric Brownian Motion as distributional assumption. Wilmott (2007) provides a simple approximation for the value of an option when it is at the money ($K = S_{t-1}e^r$) which is $E_{t-1}[(S_t - K)^+] \approx 0.39 S_{t-1}\sigma$. With that we have: $E_{t-1}(R_t^+) = (1/S_{t-1})E_{t-1}[(S_t - S_{t-1})^+] \approx 0.39\sigma$, and by the same argument $E_{t-1}(|R_t|) = 0.78\sigma$, then the expected value of RSI is

$$E_{t-1}(Z_t) = Z_{t-1} + 0.78\phi\sigma X_{t-1}\left(\frac{1}{2} - Z_{t-1}\right) + O\left(\phi^2\right). \tag{6}$$

We can compare this result with the one obtained by the binomial model using $N = 2$. In both cases RSI returns to its stationary value one half, and the size of the perturbation is proportional to the variance of the underlying process.

3 Empirical Application: the case of CLP

Abarca et al (2007) report than the RSI is widely used by Chilean analyst that trades in dollars. In this section we provide the empirical results of applying the financial forecast proposal using the Binomial Tree Model (BTM) for the underlying process. For the whole sample an ARMA(1,1) fits well the data, however, the BTM provides smaller errors in terms of the signals.

3.1 Sample Descriptive and ARMA model

For the application of BTM to the CLP we use daily data for the sample June 2000 and April 2009. The daily return and the RSI-14 are summarized in Table 1 in which is possible to see the increment in volatility after the bankrupcy of Lehman Brothers bank. Indeed, the standard deviation of daily return in the last part of the sample is about 2.3 times the one before that date. For the case of RSI-14 we note that unconditional mean is around 1/2 which was expected from the analysis of the previous section. Also, it is interesting to note that considering the first period, only 20% of sample is in the overpriced/oversold area.

Table 1: Descriptive Statistics

	Return		RSI-14	
	Jun'00-Aug'08	Sep'08-Apr'09	Jun'00-Aug'08	Sep'08-Apr'09
Mean	0.0000	0.0009	0.4933	0.5214
Std Dev.	0.0056	0.0130	0.1470	0.1363
Skewness	0.2775	0.4658	-0.0774	0.4649
Kurtosis	5.5017	4.6185	2.4921	2.3275
P10	-0.0064	-0.0131	0.2983	0.3647
P90	0.0066	0.0155	0.6760	0.7187

In Table 2 estimations of ARMA models are presented. Using the BIC criteria the ARMA(1,1) model is chosen. For that model we have an unconditional mean of $[0.0283/(1 - 0.9429)] = 0.4956$ which is consistent with the descriptive statistics.

Table 2: ARMA Models of RSI-14

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
C	.0222	.0249	.4957	.4957	.0283	.0330	.0260	.0290
	(.0036)	(.0034)	(.0082)	(.0083)	(.0040)	(.0055)	(.0040)	(.0134)
AR(1)	.9552	1.0724			.9429	.7095	.9475	.8392
	(.0068)	(.0181)			(.0076)	(.1310)	(.0077)	(.4458)
AR(2)		-.1227				.2238		.1021
		(.0180)				(.1257)		(.4202)
MA(1)			.8414	1.1848	.1429	.3720	.1332	.2411
			(.0151)	(.0227)	(.0195)	(.1274)	(.0203)	(.4456)
MA(2)				.6291			-.0401	-.0244
				(.0234)			(.0209)	(.0672)
AIC	-3.4436	-3.4578	-2.0425	-2.6119	-3.4596	-3.4602	-3.4602	-3.4594
BIC	-3.4384	-3.4501	-2.0373	-2.6042	-3.4519	-3.4499	-3.4500	-3.4465

Standard Errors in parentheses. AIC: Akaike Criteria, and BIC: Schwarz Criteria.

3.2 Implementing the Binomial Tree Model

For simplicity we calibrate the parameters of the binomial model (u and p) using the last M observations, then we need to minimize a loss function in two dimensions: the size of the window used for the calibration (M) and the number of steps of the tree (N). In this paper we consider 2 loss fuctions: the standard Mean Squared Error (MSE) and a the Mean Change in sign Error (MCE).

Let $\hat{Z}_t$ be the forecast of RSI, $\Delta\hat{Z}_t = \hat{Z}_t - \hat{Z}_{t-1}$ its change, and T the sample size, then

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(Z_t - \hat{Z}_t)^2 \text{ and } MCE = \frac{1}{T-1}\sum_{t=2}^{T} g(\Delta\hat{Z}_t, \Delta Z_t),$$

where $g(\Delta\hat{Z}_t, \Delta Z_t)$ is one if $|\Delta\hat{Z}_t|/\Delta\hat{Z}_t \neq |\Delta Z_t|/\Delta Z_t$, and zero otherwise.

It is clear that MSE will provide a model with more accurate forecasts, meanwhile MCE will choose a model with less noise in the buy/sell decision of the asset.

In Table 3 we have the results for RSI-14 using (5). Note that for each M we have 2 level of MSE: one for odd N's and another for even N's. This is consistent with the usual practice in option valuation that is taking the average price two models: one with N steps and another with $N+1$ steps in order to reduce the error. Given that it seems possible to reduce the MSE by this practice, but we leave this for a future research. Also, we note that MSE is decresing

Table 3: Grid Search for RSI-14

	M								
N	5	10	15	20	25	30	35	40	45
10	.2131	.2008	.1968	.1943	.1932	.1929	.1921	.1914	.1925
	(.4788)	(.4903)	(.4950)	(.5039)	(.5137)	(.5249)	(.5229)	(.5154)	(.5244)
11	.2125	.2001	.1962	.1937	.1928	.1925	.1918	.1912	.1923
	(.4797)	(.4962)	(.4946)	(.5030)	(.5155)	(.5285)	(.5210)	(.5186)	(.5276)
12	.2130	.2007	.1967	.1942	.1932	.1929	.1921	.1914	.1924
	(.4801)	(.4903)	(.4959)	(.5043)	(.5137)	(.5221)	(.5215)	(.5177)	(.5257)
13	.2125	.2001	.1962	.1937	.1928	.1925	.1918	.1912	.1923
	(.4792)	(.4975)	(.4950)	(.5025)	(.5141)	(.5276)	(.5201)	(.5186)	(.5280)
14	.2129	.2006	.1967	.1941	.1931	.1928	.1921	.1914	.1924
	(.4801)	(.4907)	(.4964)	(.5039)	(.5150)	(.5253)	(.5220)	(.5177)	(.5257)
15	.2125	.2001	.1962	.1937	.1928	.1925	.1918	.1912	.1923
	(.4792)	(.4975)	(.4941)	(.5020)	(.5141)	(.5276)	(.5192)	(.5186)	(.5280)
16	.2129	.2006	.1966	.1941	.1931	.1928	.1921	.1914	.1924
	(.4801)	(.4921)	(.4968)	(.5039)	(.5155)	(.5258)	(.5215)	(.5181)	(.5244)
17	.2125	.2002	.1962	.1937	.1928	.1925	.1918	.1912	.1923
	(.4788)	(.4971)	(.4946)	(.5020)	(.5141)	(.5281)	(.5197)	(.5177)	(.5280)
18	.2128	.2006	.1966	.1941	.1931	.1928	.1920	.1914	.1924
	(.4801)	(.4930)	(.4964)	(.5020)	(.5155)	(.5258)	(.5215)	(.5177)	(.5248)
19	.2125	.2002	.1962	.1937	.1928	.1926	.1919	.1912	.1923
	(.4792)	(.4962)	(.4950)	(.5020)	(.5141)	(.5281)	(.5197)	(.5167)	(.5280)
20	.2128	.2005	.1966	.1940	.1931	.1928	.1920	.1913	.1924
	(.4806)	(.4930)	(.4968)	(.5020)	(.5155)	(.5253)	(.5206)	(.5154)	(.5248)

MSE times 10^{-3} and MCE (in parentheses).

with M up to $M = 40$ which provides the minimum level, then we have a a minimum MSE at $M = 40$ and an odd N. Since we want to have a parsimonius model we pick $N = 11$.

In terms of MCE we do not observe a decresing or increasing function in with M neither with N. Indeed, the minimum levels of MCE are obtained under $M = 5$ which means that for this loss function a calibration of the binomial using the lastly data is better than considering a long period. Again for simplicity we choose $N = 10$.

3.3 Out of sample performance

Diebold and Mariano (1995) propose a test to compare the forecasts of two models. This test is based on the forecast errors and in our case we will compare the BTM model with the

Table 4: Diebold-Mariano tests for RSI-14

	z-value	p-value
d^K_{EC}	0.210	0.833
d^K_{ES}	-1.719	0.086

ARMA(1,1) model using two loss functions: MSE and MCE. The BTM model uses $N = 10$ and $M = 5$ meanwhile the ARMA(1,1) is estimated using windows of 300 observations. For the estimation of the standard errors of the Diebold-Mariano test we use the information of the partial autocorrelogram of the difference between forecast errors. Empirically, we note that MCE requires one lag for the estimation of a robust variance, meanwhile MSE does not need to include additional lags.

The results (non reported) show that using the MSE measure both BTM and ARMA(1,1) have the same forecast error, since there is not a significant difference between them. However, using the MCE measure the BTM model has a lower error than the ARMA model.

4 Conclusions

In this paper, we provide a closed-form expression for the Relative Strength Index (RSI), which is a popular index in Technical Analysis. Using that definition we show that a forecast of the Relative Strength Index (RSI) can be obtained by using the standard option valuation tools. Indeed, under the binomial model is easy to prove that RSI is stationary around its unconditional mean (one half).

Given that we use the valuation model for forecasting we call our approach "financial forecast". It is important to stress that underlying model is indeed consistent with many financial contracts, and for that the use in forecasting opens an opportunity. Using the Chilean exchange rate as empirical application we show that our financial forecast, based on the Binomial Tree Model (BTM) provides a similar Mean Squared Error than modelling the RSI-14 using the standard time-series approach (ARMA processes). However, in terms of the signals the BTM is superior to an ARMA(1,1) process.

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