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Deep Habits, Nominal Rigidities and Interest Rate Rules

Sarah Zubairy*

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Abstract

This paper explores how the introduction of deep habits in a standard new-Keynesian model affects the properties of widely used interest rate rules. In particular, an interest rate rule satisfying the Taylor principle is no longer a sufficient condition to guarantee determinacy. Including interest rate smoothing and a response to output deviations from steady state significantly improve the regions of determinacy. However, under all the simple interest rate rules considered here with contemporaneous variables, determinacy is not guaranteed for very high degree of deep habits. The intuition behind these findings is tied to how deep habits give rise to counter-cyclical markups, a property that makes it an appealing feature in the study of demand shocks.

Keywords: Taylor principle, interest rate rules, sticky prices, deep habits

JEL Classification: E31, E52

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1 Introduction

Simple interest rate rules have been shown to approximate the actual monetary policy rather successfully, and are widely used in the literature.¹ An interest rate rule where the nominal interest rate adjusts by more than one-for-one in response to inflation, is often argued to be a necessary and sufficient requirement to guarantee a locally unique rational expectations equilibrium. This rule is said to satisfy the Taylor principle, following Woodford (2001).

However, a number of papers have pointed out the limitation of the Taylor principle in avoiding indeterminacy and fluctuations driven by self-fulfilling fluctuations, when departing from standard modeling assumptions. This includes among others Benhabib et. al (2001) and Carlstrom and Fuerst (2001) who consider different modeling choices for money, Gali et. al (2004) who consider a model with rule-of-thumb consumers and Sveen and Weinke (2005) who model firm-specific capital. The conditions for determinacy of a unique equilibrium thus seems to be model-dependent, and so the robustness of proposed rules to model specification is a concern.

In this paper, I show how introducing deep habits into a model affect the performance of simple interest rate rules, and assess whether the Taylor principle is a sufficient condition for determinacy.² I analyze a standard new Keynesian model economy with capital accumulation, and in this framework allow for households to exhibit deep habits, which is essentially external habit formation (or keeping up with the Joneses) on a good-by-good basis. Habit formation is a desirable feature in macroeconomic models since it helps account for the hump-shaped and persistent response of consumption to various shocks in the economy. Studying a model with deep habits, as introduced in Ravn et. al (2006), is of special interest since it is a more generalized version of habit formation, as agents form habits over consumption of individual goods that form the composite consumption good. Deep habits give rise to the same consumption Euler equation, but unlike the more widely used habit formation at the level of a single aggregate good, they have additional consequences for the supply side of the economy. They render the firm's pricing problem dynamic and give rise to time-varying markups of price over marginal cost. The implied countercyclical markups are consistent with the findings of the empirical literature (e.g. Rotemberg and Woodford (1999)), and additionally act as a transmission mechanism for the observed effects of demand shocks, such as government spending shocks (Ravn et. al (2007) and Zubairy (2009)).

The main findings of this paper can be summarized as follows. In a model with deep habits, if the monetary authority follows a rule where the nominal interest rate strictly responds to current inflation, then the Taylor principle is too weak a condition to render stability. Specifically, the response to inflation required in order to guarantee a determinate equilibrium is increasing in the degree of deep habit. I also show that including interest rate smoothing and a response to output deviations from steady state into the monetary policy rule significantly improve the regions

¹This is shown in the seminal paper, Taylor (1993) and is also one of the main conclusion of the contributions to the Taylor (1999) volume.

²In this paper only *operational* rules, as described in McCallum and Nelson(1999), are considered, where the nominal interest rate responds to past inflation or expectations of current inflation and output gap.

of determinacy. However, under all the simple interest rate rule considered here, when nominal interest rate responds to contemporaneous variables, determinacy is not guaranteed for very high degrees of deep habits. Lastly, a backward looking rule is considered, where the nominal interest rate responds to past inflation. This rule is shown, in general, to perform better than interest rate rules that respond to current inflation, for high value of deep habits.

The remainder of the paper is as follows: Section 2 describes the model with deep habits and derives the optimality conditions of the households and firms. Section 3 analyzes the conditions required for the determinacy of a local unique equilibrium under various simple interest rate rules. Section 4 examines the robustness of these results under a backward-looking interest rate rule and habit formation at the level of a single aggregate good. And finally, Section 5 concludes.

2 Theoretical Model

I am considering a model economy that features optimizing households and a continuum of profit maximizing firms producing intermediate goods. This is a canonical new-Keynesian model with investment and the only departure is the presence of deep habit formation, or habit formation at the level of intermediate goods, for public and private consumption goods, as first introduced in Ravn et. al (2006). As will become apparent in the following section, the existence of deep habits gives rise to demand functions with pro-cyclical price-elasticity, and therefore time-varying counter-cyclical markups, even in the absence of any nominal rigidities.

2.1 Households

The economy is populated by a continuum of identical households of measure one indexed by $j \in [0, 1]$. Each household $j \in [0, 1]$ derives utility from consumption, x_t^c and disutility from labor supply, h_t and seeks to maximize lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t^c, h_t), \quad (1)$$

where the utility function has the following functional form, $U(x^c, h) = \frac{[(x^c)^{1-\nu}(1-h)^\nu]^{1-\sigma} - 1}{1-\sigma}$. The introduction of deep habits means that the agents do not form habits at the level of the aggregate consumption basket, given here by x_t^c , but at the level of individualized goods. This is then habit formation for a narrower category of goods. Thus, the variable x_t^c is a composite of habit adjusted consumption of a continuum of differentiated goods indexed by $i \in [0, 1]$.

$$x_t^c = \left[\int_0^1 (c_{it} - b^c s_{it-1}^C)^{1-\frac{1}{\eta}} di \right]^{1/(1-\frac{1}{\eta})}, \quad (2)$$

where s_{it-1} denotes the stock of habit in consuming good i in period t . In principle, households could exhibit a different degree of habit formation across the different individualized goods but for

the sake of tractability, I assume it to be the same across the differentiated goods. The parameter $b^c \in [0, 1)$ measures the degree of external habit formation, and when b^c is zero, the households do not exhibit deep habit formation. The stock of external habit is assumed to depend on a weighted average of consumption in all past periods. Habits evolve over time according to the following law of motion,

$$s_{it}^C = \rho^c s_{it-1}^C + (1 - \rho^c) c_{it}. \quad (3)$$

The parameter $\rho^c \in [0, 1)$ measures the speed of adjustment of the stock of external habit to variations in the cross-sectional average level of consumption of variety i . When ρ^c takes the value zero, habit is measured by past consumption. For any given level of consumption of x_t^c , purchases of each individual variety of goods $i \in [0, 1]$ in period t must solve the dual problem of minimizing total expenditure, $\int_0^1 P_{it} c_{it} di$, subject to the aggregation constraint (2), where P_{it} denotes the nominal price of a good of variety i at time t . The optimal level of demand, c_{it} for $i \in [0, 1]$ is then given by

$$c_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} x_t^c + b^c s_{it-1}^C, \quad (4)$$

where P_t is a nominal price index defined as $P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}$.

Note that consumption of each variety is decreasing in its relative price, P_{it}/P_t and increasing in the level of habit adjusted consumption x_t^c . At the same time, the demand function has a second price-inelastic component given by $b^c s_{it-1}^C$. When there is an increase in aggregate demand, the price-elastic part gets a higher weight, which implies that price-elasticity is pro-cyclical and since markup is given by the inverse of the price-elasticity, it is counter-cyclical. Additionally, firms are forward-looking and internalize that the demand function has a backward looking term. When they expect high future demand, they have an additional incentive to lower their markups in order to appeal to a broader customer base and carry it over to the following period.

The household is assumed to own physical capital, k_t , which accumulates according to the following law of motion,

$$k_{t+1} = (1 - \delta)k_t + i_t, \quad (5)$$

where i_t denotes investment by the household and δ denotes the rate of depreciation of physical capital.

Households are assumed to have access to a complete set of nominal state-contingent assets. Specifically, each period $t \geq 0$, consumers can purchase any desired state-contingent nominal payment A_{t+1}^h in period $t + 1$ at the dollar cost $E_t r_{t,t+1} A_{t+1}^h$. The variable $r_{t,t+1}$ denotes a stochastic nominal discount factor between periods t and $t + 1$. Households pay real lump-sum taxes in the amount τ_t per period, and each period receive a labor income $w_t h_t$ and income from renting out capital given by $r_t^k k_t$.

The household's period-by-period budget constraint is then given by,

$$E_t r_{t,t+1} a_{t+1}^h + \left(\int_0^1 \frac{P_{it}}{P_t} c_{it} di \right) + \omega_t + i_t + \tau_t = \frac{a_t^h}{\pi_t} + r_t^k k_t + w_t h_t + \phi_t, \quad (6)$$

where $\omega_t = b^c \int_0^1 P_{it} s_{it-1}^C / P_t di$. The variable a_t^h / π_t denotes the real payoff in period t of nominal state-contingent assets purchased in period $t - 1$. The variable ϕ_t denotes dividends received from the ownership of firms and $\pi_t \equiv P_t / P_{t-1}$ denotes the gross rate of consumer-price inflation.

The household chooses sequences for x_t^c , h_t , a_{t+1}^h , k_{t+1} , i_t so as to maximize the utility function (1) subject to (5) and (6), and a no-Ponzi game constraint.

The first-order conditions from the optimizing household's problem with respect to x_t^c , a_{t+1}^h , h_t and k_{t+1} in that order, are given by

$$U_x(x_t^c, h_t) = \lambda_t, \quad (7)$$

$$\lambda_t r_{t,t+1} = \beta \lambda_{t+1} \frac{P_t}{P_{t+1}}, \quad (8)$$

$$-U_h(x_t^c, h_t) = \lambda_t w_t, \quad (9)$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1} \left[r_{t+1}^k + q_{t+1}(1 - \delta) \right]. \quad (10)$$

2.2 Government

Like households, the government is also assumed to form habits over its consumption of individual varieties of goods. This can be thought of as households deriving utility from public goods that is separable from private consumption and leisure, and they exhibit good-by-good habit formation for these particular goods also. For instance, households care about the provision of individual public goods, such as trash removal or street lighting, in their own constituency versus others. The government allocates spending over individual varieties of goods, g_{it} , so as to maximize the quantity of composite good produced with the differentiated varieties of goods according to the relation,

$$x_t^g = \left[\int_0^1 (g_{it} - b^g s_{it-1}^G)^{1-1/\eta} \right]^{1/(1-1/\eta)}.$$

The variable s_{it}^G denotes the government's stock of habit in good i and is assumed to evolve as follows,

$$s_{it}^G = \rho^g s_{it-1}^G + (1 - \rho^g) g_{it}. \quad (11)$$

The government's problem consists of choosing g_{it} , $i \in [0, 1]$, so as to maximize x_t^g subject to the budget constraint $\int_0^1 P_{it} g_{it} di \leq P_t g_t$. The resulting demand function for each differentiated good $i \in [0, 1]$ by the public sector is,

$$g_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} x_t^g + b^g s_{it-1}^G. \quad (12)$$

Lump sum taxes are assumed to balance out government spending expenditures each period. Real government expenditures, denoted by g_t are assumed to be exogenous, stochastic and follow the following univariate first-order autoregressive process,

$$\hat{g}_t = \tilde{\rho}_g \hat{g}_{t-1} + \epsilon_t^g, \quad (13)$$

where \hat{g}_t is the log deviation of spending from its steady state.

2.3 Firms

Each variety of final goods is produced by a single firm in a monopolistically competitive environment. Each firm $i \in [0, 1]$ produces output using capital services, k_{it} , and labor services, h_{it} as factor inputs. The production technology is given by $F(k_{it}, h_{it})$, where the function F is assumed to be homogenous of degree one, concave, and strictly increasing in both arguments and has the following constant returns to scale functional form, $F(k, h) = k^\theta h^{1-\theta}$.

The firm is assumed to satisfy demand at the posted price. Formally,

$$F(k_{it}, h_{it}) \geq c_{it} + i_{it} + g_{it}. \quad (14)$$

The objective of the firm is to choose contingent plans for P_{it} , h_{it} and k_{it} in order to maximize the present discounted value of dividend payments, given by $E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \phi_{it+s}$ where,

$$\phi_{it} = \frac{P_{it}}{P_t} (c_{it} + i_{it} + g_{it}) - r_t^k k_{it} - w_t h_{it} - \frac{\alpha}{2} \left(\frac{P_{it}}{P_{it-1}} - 1 \right)^2, \quad (15)$$

under constraints given by (3), (4), (11), and (12) and $i_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} i_t$. Note that sluggish price adjustment is introduced following Rotemberg (1982), by assuming that the firms incur a quadratic price adjustment cost for the good it produces. This modeling choice of price stickiness produces qualitatively similar aggregate dynamics as pricing mechanism based on Calvo (1983).³

The first order conditions with respect to h_{it} , k_{it} , c_{it} , s_{it}^C , g_{it} , s_{it}^G , i_{it} and p_{it} are,

$$mc_{it} F_2(k_{it}, h_{it}) = w_t, \quad (16)$$

$$mc_{it} F_1(k_{it}, h_{it}) = r_t^k, \quad (17)$$

$$\left(\frac{P_{it}}{P_t} \right) - mc_{it} - \tilde{v}_t^c + \tilde{\lambda}_t^c (1 - \rho^c) = 0, \quad (18)$$

$$\left(\frac{P_{it}}{P_t} \right) - mc_{it} - \tilde{v}_t^g + \tilde{\lambda}_t^g (1 - \rho^g) = 0, \quad (19)$$

$$q_t P_t \tilde{\lambda}_t^c = E_t q_{t+1} P_{t+1} (b^c \tilde{v}_{t+1}^c + \rho^c \tilde{\lambda}_{t+1}^c), \quad (20)$$

³The presence of deep habits makes the pricing problem dynamic and so additionally accounting for dynamics due to Calvo-style pricing makes aggregation non-trivial.

$$q_t P_t \tilde{\lambda}_t^g = E_t q_{t+1} P_{t+1} (b^g \tilde{\nu}_{t+1}^g + \rho^g \tilde{\lambda}_{t+1}^g), \quad (21)$$

$$\left(\frac{P_{it}}{P_t} \right) - mc_{it} - \tilde{\nu}_t^i = 0, \quad (22)$$

$$\eta \left(\frac{P_{it}}{P_t} \right)^{-\eta-1} (\tilde{\nu}_t^c x_t^c + \tilde{\nu}_t^g x_t^g + \tilde{\nu}_t^i i_t) + \alpha \frac{P_t}{P_{it-1}} \left(\frac{P_{it}}{P_{it-1}} - \tilde{\pi}_t \right) - (c_{it} + g_{it} + i_{it}) = \alpha E_t \left[\frac{q_{t+1}}{q_t} \frac{P_{t+1}}{P_t} \frac{P_{it+1}}{P_t} \frac{P_t}{P_{it}} \left(\frac{P_{it+1}}{P_{it}} - 1 \right) \right]. \quad (23)$$

Note that because all firms face the same factor prices and since they all have access to the same production technology with the function F being linearly homogeneous, marginal costs, mc_{it} , are identical across firms. Also note that combining the first order conditions with respect to c_{it} and s_{it}^C to eliminate $\tilde{\lambda}_t^c$ yields,

$$\frac{\left(\frac{P_{it}}{P_t} \right) - mc_{it} - \tilde{\nu}_t^c}{\rho^c - 1} = E_t r_{t,t+1} \pi_{t+1} \left[b^c \tilde{\nu}_{t+1}^c + \frac{\rho^c}{\rho^c - 1} \left\{ \left(\frac{P_{it+1}}{P_{t+1}} \right) - mc_{it+1} - \tilde{\nu}_{t+1}^c \right\} \right], \quad (24)$$

and similarly for g_{it} and s_{it}^G .

2.4 Market Clearing

The market clearing conditions yield $\int_0^1 h_{it} di = h_t$, $\int_0^1 k_{it} di = k_t$, and in the final goods market,

$$F(k_t, h_t) = c_t + g_t + i_t + \frac{\alpha}{2} (\pi_t - 1)^2. \quad (25)$$

Also, all firms set identical prices in a symmetric equilibrium.

2.5 Monetary policy rule

The log-linearized monetary policy rule is assumed to have the following form,

$$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1 - \alpha_R) (\alpha_\pi \hat{\pi}_t + \alpha_Y \hat{y}_t), \quad (26)$$

where $\alpha_R \geq 0$, $\alpha_\pi \geq 0$ and $\alpha_Y \geq 0$, and \hat{R}_t , $\hat{\pi}_t$ and \hat{y}_t represent nominal interest rate, inflation and output log deviations from respective steady states.

2.6 Baseline Calibration

The model is calibrated to quarterly frequency. The preference parameter, σ , which is the reciprocal of intertemporal elasticity of substitution, is set equal to 1, which implies that the utility function is separable and logarithmic. The discount factor β , is set at $1.03^{-1/4}$, which implies a steady-state annualized real interest rate of 3 percent. The depreciation rate, δ , is set at 0.025, which implies an annual rate of depreciation on capital equal to 10 percent. θ is set at 0.3, which corresponds to

a steady state share of capital income roughly equal to 30%. Goods elasticity of substitution, η is set at 6, which implies a steady state markup of 20 percent in the absence of deep habits. Also, the steady state labor is set at 0.3, and the share of government spending in aggregate output is taken at 0.2.

In the baseline calibration, the price stickiness parameter, α , following Schmitt-Grohé and Uribe (2004), where they model price stickiness with a Rotemberg price adjustment cost, is set to be 17.5.⁴ Also, it is assumed that the speed of adjustment of the stock of habit for all kinds of goods is equal, so $\rho = \rho^c = \rho^g$, and this parameter is calibrated to be 0.9, based on estimates in Ravn et. al (2006) and Zubairy (2009).⁵ However, I will be conducting sensitivity analysis for varying degrees of price stickiness (α) and the rate at which the habit stock accumulates (ρ). For sake of simplicity, deep habits parameters for household consumption and public consumption goods are restricted to be the same, so $b = b^c = b^g$.

3 Equilibrium Dynamics

This section explores the conditions required for the existence of a local unique equilibrium. Of special interest is the role played by the monetary policy rule given by Equation (26), the degree of price stickiness and their interaction with the deep habit parameters.

3.1 Local Determinacy

The system of log-linearized equations describing the model can be written as follows,

$$E_t y_{t+1} = A y_t + B z_t,$$

$$z_t = R z_{t-1} + \epsilon_t,$$

where y_t is a vector of endogenous variables, and consists of both predetermined and non-predetermined variables.⁶ z_t is a vector of exogenous variables and ϵ_t is a vector of zero mean, serially uncorrelated shocks. Both A and R are assumed to be non-singular matrices and all eigenvalues of R lie inside the unit circle, so that z_t is covariance stationary.⁷ Note that the entries of the coefficient matrices are typically functions of the underlying deep parameters of the model. The goal here is to derive the parameter restrictions that render a unique equilibrium for the model.

The existence and uniqueness of a local equilibrium in this case depends on the number of

⁴This value of the price stickiness parameter implies that firms change their price on average every 3 quarters in a Calvo-Yun staggered-price setting model, based on estimates of a linear new-Keynesian Phillips curve by Sbordone (2002). Refer to a more detailed discussion in Schmitt-Grohé and Uribe (2004).

⁵Ravn et. al(2006) assume $\rho = \rho^c = \rho^g$, and estimate it be 0.85. Zubairy (2009) allows different values for ρ^c and ρ^g and estimates them to be 0.89 and 0.98, respectively.

⁶The perfect foresight version of this model will consist of only the first equation, and this model has the additional second equation because of the exogenous process given for government spending in Equation (13).

⁷The eigenvalue of R here corresponds to the autocorrelation parameter for the exogenous process for public spending in the model, $\tilde{\rho}_g$, which is calibrated to be 0.85, based on estimates in the literature.

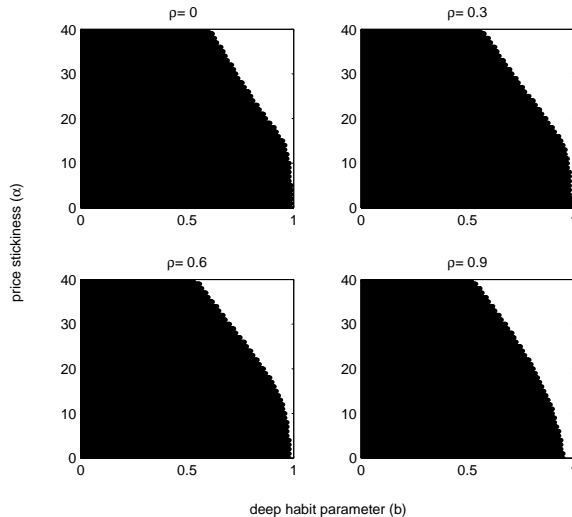


Figure 1: Region of determinacy in the baseline model for various values of the speed of adjustment of the habit stock (ρ), and a monetary policy rule with strict inflation targeting with $\alpha_\pi = 1.5$ and $\alpha_R = \alpha_Y = 0$.

eigenvalues of matrix A which lie within the unit circle. For determinacy, it is required that number of stable (i.e. lying within the unit circle) eigenvalues of A equal the number of predetermined variables. Alternatively, if the number of stable eigenvalues exceeds the number of predetermined variables then for different initial conditions on the predetermined variables, there exist a continuum of equilibrium paths that converge to steady state, and a possibility of sunspot fluctuations arise. On the other hand if the number of stable eigenvalues is less than the number of predetermined variables, then no local equilibrium exists.

In the analysis that follows, I characterize regions of determinacy by searching over the parameter space and numerically computing the eigenvalues of A and checking for the number of eigenvalues that are within the unit circle.

3.2 Deep Habits and Indeterminacy

The main objective of this paper is to understand how the presence of deep habits affects the determinacy property of the model. As shown in Woodford (2001), in the case of a simple new-Keynesian model, if we restrict attention to monetary policy rules with $\alpha_\pi, \alpha_Y > 0$, the necessary and sufficient condition to guarantee the existence of a local unique equilibrium is given by,

$$\alpha_\pi + \frac{1 + \beta}{\kappa} \alpha_Y > 1,$$

and is said to satisfy the Taylor principle.⁸ When there is a zero coefficient on output deviations from steady state, namely $\alpha_Y = 0$, then $\alpha_\pi > 1$, satisfies the Taylor principle. Such a rule guarantees that in response to a change in inflation, the nominal interest rate adjusts more than one for one.

⁸Here κ is the coefficient on the output gap in the Phillips curve in the standard new-Keynesian model.

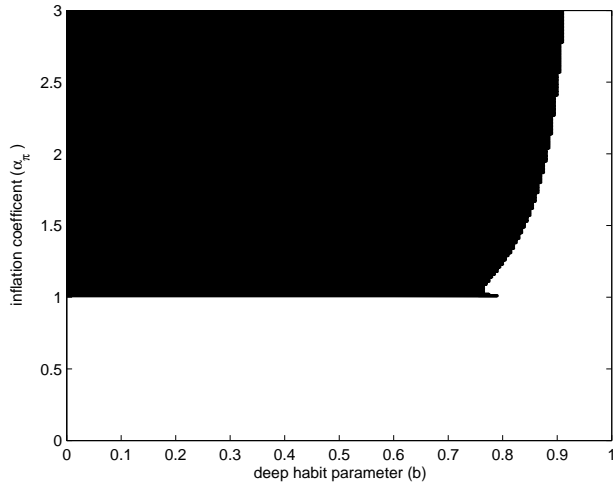


Figure 2: Region of determinacy under baseline calibration and a strict inflation targeting monetary rule with $\alpha_R = \alpha_Y = 0$.

In order to isolate the effects of deep habits, I first consider the conditions for a determinate equilibrium as a function of the degree of price stickiness, given by α , and the degree of deep habit formation, given by the parameter b , under a monetary policy rule with strict inflation targeting.⁹ Figure 1 shows what the determinacy region looks like in our baseline model with the price stickiness parameter, α increasing along the y-axis, the deep habit parameter, b along the x-axis and under the assumption $\alpha_\pi = 1.5$ and $\alpha_R = \alpha_Y = 0$. Four different cases are shown with varying values of ρ , the speed of adjustment of the habit stock. Looking at the figure it is clear that when there are no deep habits in the model, i.e. $b = 0$, a unique local equilibrium exists. However, the degree of deep habits plays a crucial role and for a combination of high values of the deep habit parameter and high degree of price stickiness, the economy runs into a region of indeterminacy even when nominal interest rate is adjusting more than one-for-one with inflation. Note in the right most lower panel, that even in the case of no price rigidities, for the case of $\rho = 0.9$, multiple equilibria exist in the case of high degree of deep habits.

Thus, it is clear that the Taylor principle is no longer a sufficient condition to ensure the existence of a local unique equilibrium in the case of strong nominal rigidities and high degree of deep habit formation.

3.3 Monetary Policy Rules and Indeterminacy

In order to characterize conditions on monetary policy rule coefficients, required to ensure the existence of a unique equilibrium, I follow by formally analyzing variation in these coefficients.

In Figure 1, the existence of a determinate equilibrium for the baseline model was considered under the case of $\alpha_\pi = 1.5$. Now, I expand the analysis to consider a broader range of values for

⁹This implies a monetary policy rule of the form $\hat{R}_t = \alpha_\pi \hat{\pi}_t$, and so $\alpha_R = \alpha_Y = 0$.

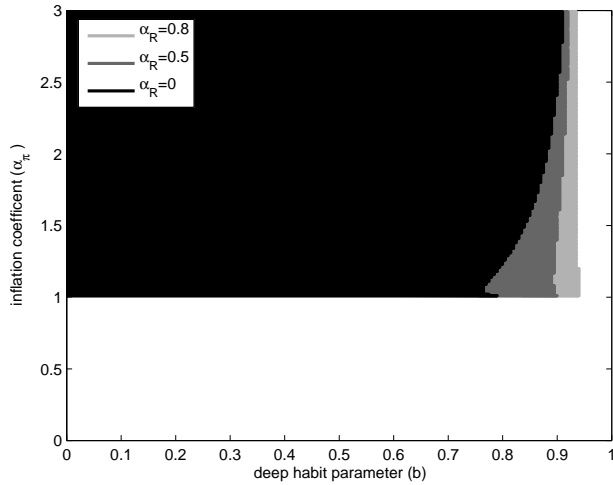


Figure 3: Regions of determinacy under baseline calibration, and $\alpha_Y = 0$.

the inflation coefficient in the monetary policy rule. Figure 2 shows the determinacy region as I vary the deep habit parameter along with the inflation coefficient under a strict inflation targeting rule in our baseline model. All other parameters are calibrated at their baseline values.

It is apparent in Figure 2 that for the case of no deep habit formation, $b = 0$, or low values of the deep habit parameter, a unique equilibrium is guaranteed for $\alpha_\pi > 1$, but this is not the case for high degree of deep habit formation. Thus the next question that arises is if the region of determinacy can be improved upon by modifying the monetary policy rule. So far, only the case of strict inflation targeting has been considered, where $\alpha_R = \alpha_Y = 0$ in the monetary policy rule. Next, different values for these parameters are considered.

First the role of interest rate smoothing is studied. The size of the region of indeterminacy shrinks gradually as α_R is increased, as shown in Figure 3. This suggests that inertial rules are more desirable in order to render macroeconomic stability.

Next, nominal interest rate is allowed to respond to deviations of output from steady state, and once again increasing α_Y improves the region of determinacy. While there is a significant improvement between the case of no response to output deviations ($\alpha_Y = 0$) and the case of $\alpha_Y = 0.5$, the region of determinate equilibria are not affected much by considering $\alpha_Y = 1$ relative to $\alpha_Y = 0.5$. So, a response of nominal interest rate to economic activity is also a desirable feature for an interest rate rule to lead to determinacy.

Here the values considered for α_R and α_Y are based on estimates by Clarida et. al (2000) and Orphanides (2001). The finding that combining active monetary policy with interest rate smoothing and responsiveness of nominal interest rate to economic activity improves the determinacy properties of the model is a common across significantly different models.¹⁰ Note, however, that

¹⁰Among others, Gali et. al (2004) and Sveen and Weinke (2005). Sveen and Weinke (2005) have output gap in the policy rule, which is the difference between output and its natural level (level of output absent any nominal rigidities).

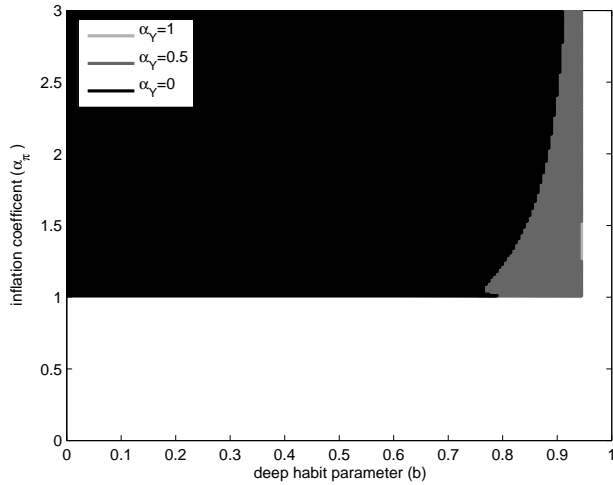


Figure 4: Regions of determinacy under baseline calibration, and $\alpha_R = 0$.

allowing for interest rate smoothing and/or response to economic activity still gives us indeterminacy for very large degrees of deep habits. The estimates for deep habit parameter in the context of medium-scale dynamic general equilibrium models as well as simpler frameworks similar to the one considered here, are usually between 0.6 and 0.9.¹¹ The equilibrium is determinate for these values of the deep habit parameter under some of the calibrations for α_R and α_Y considered here.

3.4 Indeterminacy and Impulse Response Analysis

The analysis above clearly suggests that for very high degree of deep habits, a unique equilibrium converging to a steady state does not exist for the different interest rate rules studied here. To get further insight into this finding, let us consider the baseline model and assume that in the monetary policy rule the nominal interest rate only responds to current inflation. I consider the case where there are no adjustment costs for prices, so $\alpha = 0$, since in this case, the dimension of indeterminacy is 1.¹² Once price stickiness is incorporated in the model, the region of indeterminacy where the Taylor principle is satisfied, has a dimension of indeterminacy being 2. Thus, it is not possible to get the impulse responses of the endogenous variables to a sunspot shock, as shown in Gali(1997).

Suppose households anticipate an increase in aggregate demand, without any shocks to fundamentals to justify it. This increase in demand would be accompanied by an increase in hours worked, lower markups due to deep habits, and high inflation as the firms adjust prices to get to their wanted markups. But an interest rate rule that has $\alpha_\pi > 1$, will generate high real interest rate along the adjustment path and imply lower consumption and investment relative to steady state. Thus it would not be possible to sustain a boom in demand, and so it is not consistent with rational expectations.

¹¹See Ravn et. al (2006) and Zubairy (2009)

¹²Dimension of indeterminacy is 1 if there is one more stable eigenvalue than the number of predetermined variables.

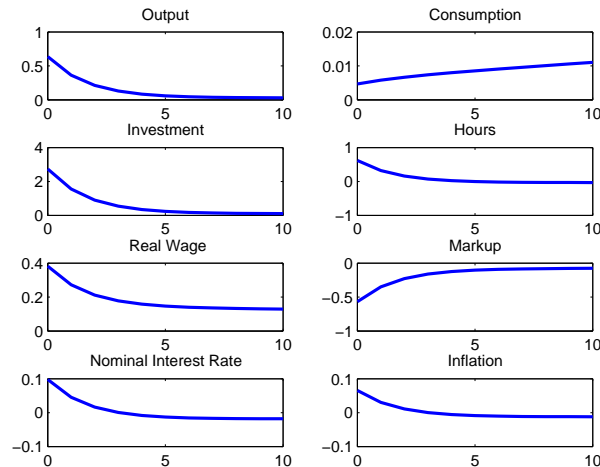


Figure 5: Response to a sunspot shock, where $b = 0.96$ and the monetary policy rule is given by $\hat{R}_t = 1.5\hat{\pi}_t$.

On the other hand, consider the case where the degree of deep habits is sufficiently high to allow multiple equilibria. The impulse response functions for such an expansionary sunspot shock are shown in Figure 5. Here the model is calibrated so the Taylor principle is satisfied, $\alpha_\pi = 1.5$, and the deep habit parameter, $b = 0.96$. Now even if the interest rate rule follows the Taylor principle, the higher degree of deep habits will drive the markups countercyclical to a greater extent. This high deep habit formation helps in driving the markup sufficiently down so it ultimately leads to a rise in real wages. The increase in wages causes the households to substitute away from leisure to consumption, and so consumption of households rises. In other words, in such a case the degree of deep habit formation leads to intra-temporal substitution effects which are larger than the intertemporal substitution effects. This in turn will lead to a realization of an increase in demand, as anticipated by agents in the economy, and thus give rise to self-fulfilling expectations.

4 Robustness Analysis

4.1 Backward Looking Interest Rate Rule

In this section, a backward looking monetary rule is considered, where the nominal interest rate responds to past inflation. The motivation behind adopting this rule is related to the information available to the monetary authority at any given time. Backward looking rules have also been recommended in the literature by many others, for instance, Carlstrom and Fuerst (2000) and Benhabib et. al (2001).¹³ The rule considered has the following form,

$$\hat{R}_t = \alpha_\pi \hat{\pi}_{t-1}.$$

¹³Benhabib, Schmitt-Grohé and Uribe (2003) show however, that to guarantee global stability, the interest rate responding to past inflation is not sufficient, and the interest rate should also be set as a function of past interest rate.

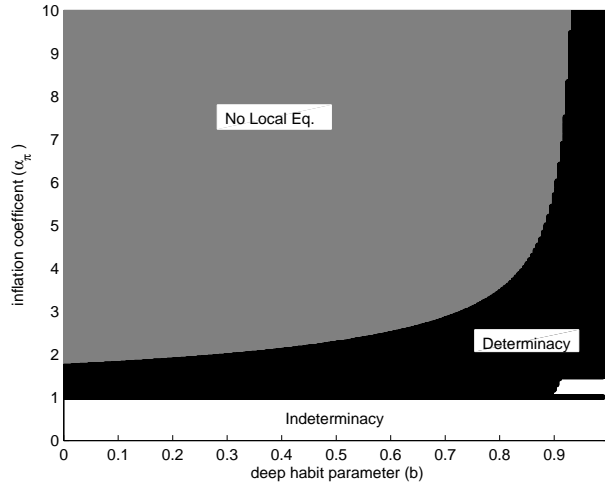


Figure 6: Region of determinacy under baseline calibration and a backward looking monetary policy rule of the form $\hat{R}_t = \alpha_\pi \hat{\pi}_{t-1}$.

Figure 6 shows the region of determinacy when the deep habit parameter, b is varied along with the inflation coefficient, α_π , in the backward looking rule, in a model where all other parameters are set at their baseline values. In this case, the equilibrium is unique for higher degree of deep habits, however for lower values of b , there is determinacy in the case where the monetary policy actively responds to past inflation so $\alpha_\pi > 1$, but not too actively, so that α_π that guarantees a unique local equilibrium is bounded above. Notice, that this exercise is analogous to Figure 2 except that in Figure 6 the interest rate responds to past inflation instead of current inflation. Just by comparison of the two figures, under a backward looking rule, the equilibrium is determinate for higher degrees of deep habits, and unlike the case where nominal interest rate responds to current inflation, it is not always determinate for lower values of the deep habit parameter, b . This happens because when nominal interest rate responds to past inflation, it is predetermined in any given period. Therefore, now in the region where there was determinacy with a contemporaneous rule, the number of predetermined variables exceeds the number of eigenvalues within the unit circle by one, and thus there is no local equilibrium in that parameter space. There are still, however, multiple equilibria for very high degree of deep habit when α_π is greater than but close to 1.

4.2 Habit Formation Over a Single Aggregate Good

The main question addressed in this paper is how introducing deep habits into a new-Keynesian model affects the determinacy properties of the model. This section shows that the results shown in Figure 2 are driven by deep habit formation and how it affects the supply side of the economy, and similar results do not hold for habit formation at the level of the a single aggregate good, which only affects the demand side. In particular, when habits are formed over the composite consumption

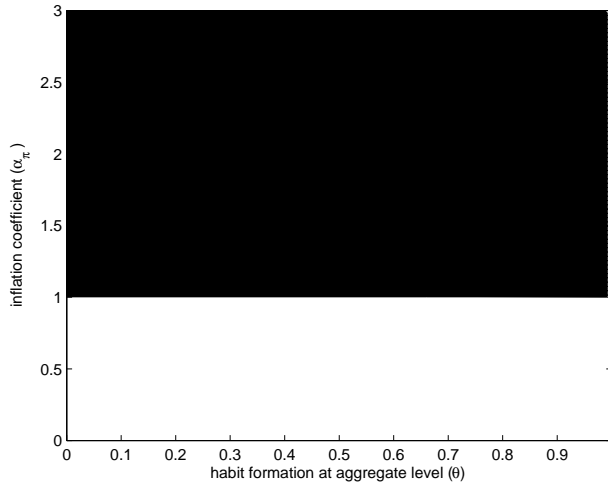


Figure 7: Region of determinacy in a model with nominal rigidities and superficial habits, and a strict inflation targeting monetary rule with $\alpha_R = \alpha_Y = 0$.

good, each household maximizes its utility function,

$$U(c_t, h_t) = \frac{\left[(c_t - \theta \tilde{c}_{t-1})^{1-\nu} (1-h)^\nu \right]^{1-\sigma} - 1}{1-\sigma},$$

where $c_t = \left[\int_0^1 c_{it}^{1-\frac{1}{\eta}} \right]^{\frac{1}{1-\frac{1}{\eta}}}$. The parameter $\theta \in [0, 1)$ measures the degree of external habit formation, and \tilde{c}_{t-1} is the average consumption last period. The demand function for good i for a household in this case is given by,

$$c_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} c_t.$$

This specification of external habit formation is the same as commonly found in the literature, e.g. in Smets and Wouters (2007), in order to match the persistence in the consumption response to macroeconomic shocks.

Figure 7 shows the region of determinacy for superficial habits in a model with monopolistically competitive product markets, nominal rigidities and a strict inflation targeting monetary policy rule. All parameters are calibrated at their baseline values. Note that the Taylor principle is a necessary and sufficient condition to guarantee determinacy in this framework. The indeterminacy region in Figure 2, is therefore precisely due to how deep habits affect the firm's problem and give rise to countercyclical markups.

5 Conclusion

This paper shows how introducing deep habits into a model affects the performance of simple interest rate rules, where the nominal interest rate responds to inflation, output or is subject to

interest-rate smoothing. The results suggest that the Taylor principle is too weak a condition to guarantee stability. In this standard new-Keynesian model with deep habits, including interest rate smoothing and a response to output deviations from steady state into the interest rate rule significantly improve the regions of determinacy. But, under all these rules, the equilibrium is not uniquely determined for very high degree of deep habits.

The main intuition behind this finding is that at very high degrees of deep habits, the markups generated are extremely large and countercyclical with resulting effects on wages that counter otherwise stabilizing effects of changes in real interest rate due to the monetary policy rule.

It is also shown that a backward looking rule, where nominal interest rate responds to past inflation, in general, performs better and results in a unique local equilibrium for high values of the deep habits parameter b , where the monetary rules responding to contemporaneous variables fail to render determinacy.

To put these findings in perspective, inflation targeting alone does not give us determinacy in a model with deep habits. The interest rate rules are not evaluated based on welfare, but if the objective is to avoid indeterminacy, then the interest rate rules that improve the region of determinacy include interest rate smoothing and/or a response to output. This paper adds to the literature that suggests that the recommendations for monetary rules that render unique equilibrium are model dependent. It is then important to be more careful and aware of these problems of indeterminacy when augmenting models with new features.

The introduction of deep habits is an increasingly important new feature. It generalizes the concept of habit persistence so that habits are formed over consumption of individual goods instead of at the level of aggregate consumption. While deep habits affect the demand side exactly like the standard habit formation, they have additional important consequences for the supply side of the economy, giving rise to counter-cyclical markups, in line with the existing empirical literature. Deep habits with their implied strong counter-cyclical movements of markups have been shown to successfully explain the rise in wages and consumption in response to a government spending shock, an empirical observation that most standard model fail to predict (see Ravn et. al (2007) or Zubairy (2009)).

In this paper, monetary policy rules are evaluated on the basis of guaranteeing uniqueness of equilibrium when deep habits are introduced. Therefore, no recommendations for a policy rule are made from a welfare point of view.¹⁴ An interesting next step would be to use second order approximation to find optimal monetary policy in a model with deep habits.

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¹⁴There is new work in this direction by Leith et. al (2009).

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