Fake switch points

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Abstract

Based on C.Bidard's and E.Klimovsky's "Switches and Fake switches in methods of production", an attempt will be made to show if fake switch points (as named) are in fact, and opposite of what Bidard and Klimovsky claim, real switch points.

JEL codes: C610, C670, O330

Key Words: Fake switch points, Choice of Techniques, Input-Output Models

1. Assumptions-Preliminaries

The following assumptions are similar with Bidard's και Klimovsky's:

Let n commodities be produced with m available production processes (m>n).

The systems of production use linear techniques of joint production. As a consequence accrue m+1 different square techniques and m+1 w-r relations.

The m+1 w-r relations, have been accrued of, the same for all alternative subsystems, price normalizing.

Based on the above assumptions they try to prove the existence of fake switch points[2].

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[2] Before going further to the existence or not of the fake switch points, a short reference on how Bidard and Klimovsky prescribe switch points, would be useful:
Firstly on single production systems:

"Let there k+1 commodities and k+1 methods of production. A switch point is a level r^* of the rate of profits such that the k+1 methods are equally profitable for some price-and-wage vector." Bidard Ch. and Ed. Klimovsky (2004). This definition of Bidard-Klimovsky about Switch points, seems to be valid in the case of single production.

But in the case of joint production :

"In a multiple-product system, let r^* be a switch point. The price-and-wage vector is the same for all of the k+1 systems. Therefore, whatever the numeraire is, the k+1 wage-profits curves have a common point (r^*, w^*)."

Simplifying things, likewise C. Bidard και E. Klimovsky, we examine the special case of 2 commodities (X,Y), produced by 3 different methods of production let,1,2,3. Consequently there will be three square techniques made up of the methods (1,2) (1,3), (2,3). Consequently there will be formed the input and output matrices, $A_{ij}, B_{ij}, i \neq j = 1,2,3$, respectably . Let w, r be the nominal wage rate and the profit rate respectably.

Let the production prices be normalized with any standard commodity common to the 3 techniques. As usual, the w-r relations are being exported for each technique and the w-r space. In this case, for a given r, two of tree w-r curves intersect in a given point. Therefore for Bidard-Klimovsky:

The intersection points of these techniques are not real but a fake switch point, as it contradicts to the definition of footnote 2.

If the "fake switch point" is in the outer envelope of the w-r curves, then a transition occurs to a point, that no switch of techniques is occurred.

Furthermore the real switch points, according to Bidard and Klimovsky do not appear/disappear with price normalization.

In other words, for Bidard-Klimovsky, the w-r criterion is not a criterion of univocal ranking of techniques, as it implies a transition to techniques, which, according to them, nothing can be said about being the most profitable.

Bidard and Klimovsky, try to prove the existence of fake switch points. They move in the following analytical framework:

Prices are been normalized with a typical commodity, u, u≥0

For a given r, $r_0 > 0$, such that the direction of net product is the same of the typical commodities

Let r,

$r_0 > 0 : b_i - a_i(1 + r_0) = w_0 u, m = 1,2,3, \ldots, m$

, such an r exists if the typical commodity is found in the ankle, that is formed by :

$b_i - a_i$ and $b_i - a_i(1 + R)$.

In bibliography has been referred, that the point where all w-r curves intersect, is called a switch point. This point has the following properties:

- In switch point(s) the profit rate and, consequently, the nominal wage of all alternative systems are in common
- In switch point(s) all the typical subsystems, normalized with the same way, have the same vector of production prices for the given profit rate.
- In switch point(s) all the typical subsystems have the same capital intensity in (price terms)

The same properties stand for the reswitch points.

In ccontroversy in fake switch points two or more (but not all) w-r curves, corresponding to the typical subsystems, intersect. This implies that the above properties hold not for all typical subsystems, in general, but for some of them.

A fake switch point can be found, under, above or over the upper envelope (of w-r curves). When a fake switch is found under the upper envelope there will be not a significant problem, as it is found on the intersection of two sublime techniques, which by definition are not being chosen. The problem arises when the fake switch point is found on the upper envelope. In this case, according to Bidard, the w-r criterion implies a switch to a point, that according to Bidard and Klimovsky no switch occurs.

Bidard and Klimovsky claim that r_0 is a fake switch point. The reason is because for r_0, every system, that contains the above process I, can produce w_0 units of typical commodity.

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2. Real and "Fake" switch points

The main purpose of this paper is to prove that the «fake switch points» are real switch points. This will be proved, not only in terms of the numerical example of Bidard and Klimovsky, but in the general case as well, using a non-decomposable system of joint production.

2.1. The numerical Example

The facts of Bidard's and Klimovsky example are being reminded:

Let A≥0, and B≥0, with A+B>0, be the nxm matrices of inputs and outputs respectively. And let ℓ, ℓ>0 be the 1xm vector of direct labor of the system

$$A = \begin{bmatrix} 20 & 20 & 30 \\ 20 & 20 & 30 \end{bmatrix}, B = \begin{bmatrix} 21 & 23 & 36 \\ 27 & 25 & 34 \end{bmatrix}, \ell = [1,1,1]$$

The produced relation for production prices are:

$p_{ij} = p_{ij} A_{ij} (1+r) + w\ell_{ij}$, i≠j, i,j=1,2,3

Therefore for the relative prices and the w-r relation holds:

$p_{12} = [a, a]$

$w_{12} = a(8 - 40r)$

$p_{13} = [b(3+10r), b(5-10r)]$

$w_{13} = b(38 - 220r)$

$p_{23} = [c(1+10r), c(3-10r)]$

$w_{23} = c(18 - 100r)$

The prices of each technique ij are being normalized as follow:

$p_{1,ij} = 1$

So each technique's w-r holds:

$w_{12} = (8 - 40r)$

$$w_{13} = \frac{(38-220r)}{3+10r}$$

$$w_{23} = \frac{(18-100r)}{1+10r}$$

The results for the production prices for the first price normalization are:

p(x)=1

r	w12	w13	w23
0,005	7,800	12,098	16,667
0,01	7,600	11,548	15,455
0,015	7,400	11,016	14,348
0,02	7,200	10,500	13,333
0,025	7,000	10,000	12,400
0,03	6,800	9,515	11,538
0,035	6,600	9,045	10,741
0,04	6,400	8,588	10,000
0,045	6,200	8,145	9,310
0,05	6,000	7,714	8,667
0,055	5,800	7,296	8,065
0,06	5,600	6,889	7,500
0,065	5,400	6,493	6,970
0,07	5,200	6,108	6,471
0,075	5,000	5,733	6,000
0,08	4,800	5,368	5,556
0,085	4,600	5,013	5,135
0,09	4,400	4,667	4,737
0,095	4,200	4,329	4,359
0,1	4,000	4,000	4,000
0,105	3,800	3,679	3,659
0,11	3,600	3,366	3,333
0,115	3,400	3,060	3,023
0,12	3,200	2,762	2,727
0,125	3,000	2,471	2,444
0,13	2,800	2,186	2,174
0,135	2,600	1,908	1,915
0,14	2,400	1,636	1,667
0,145	2,200	1,371	1,429
0,15	2,000	1,111	1,200
0,155	1,800	0,857	0,980
0,16	1,600	0,609	0,769
0,165	1,400	0,366	0,566
0,17	1,200	0,128	0,370
0,175	1,000	-0,105	0,182
0,18	0,800	-0,333	0,000
0,185	0,600	-0,557	-0,175
0,19	0,400	-0,776	-0,345
0,195	0,200	-0,990	-0,508
0,2	0,000	-1,200	-0,667
0,205	-0,200	-1,406	-0,820
0,21	-0,400	-1,608	-0,968
0,215	-0,600	-1,806	-1,111

Table 1

And also the w-r relation:

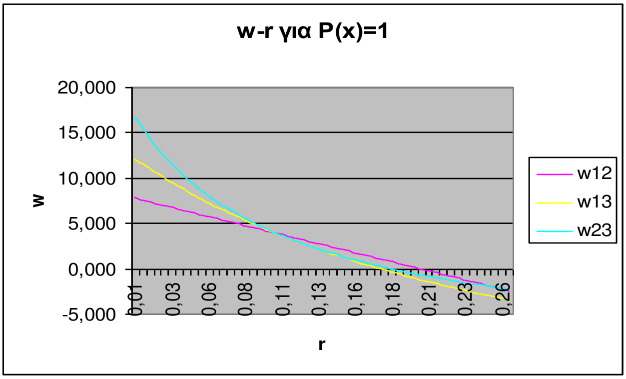

Figure 1.

About the capital intensity (in price terms) stand the following[3]:
From the production prices system stands:

$$w = K_q(R-r) \Rightarrow K_q = \frac{w}{R-r}$$

Kq is the capital intensity in the typical subsystem q.
Therefore the capital intensities Kij , i≠j, i,j=1,2,3 are:

$$K_{12} = \frac{8-40r}{0,2-r}$$

$$K_{13} = \frac{\frac{38-220r}{3+10r}}{\frac{19}{110}-r}$$

$$K_{23} = \frac{\frac{18-100r}{1+10r}}{\frac{9}{50}-r}$$

For the capital intensities the following results occur:

r	k12	k13	k23
0,005	40,000	73,631	95,238
0,01	40,000	73,948	90,909
0,015	40,000	74,284	86,957
0,02	40,000	74,643	83,333
0,025	40,000	75,026	80,000

[3] G.Stamatis (1997a)

0,03	40,000	75,435	76,923
0,035	40,000	75,875	74,074
0,04	40,000	76,347	71,429
0,045	40,000	76,856	68,966
0,05	40,000	77,407	66,667
0,055	40,000	78,005	64,516
0,06	40,000	78,656	62,500
0,065	40,000	79,367	60,606
0,07	40,000	80,147	58,824
0,075	40,000	81,008	57,143
0,08	40,000	81,961	55,556
0,085	40,000	83,022	54,054
0,09	40,000	84,212	52,632
0,095	40,000	85,556	51,282
0,1	40,000	87,083	50,000
0,105	40,000	88,837	48,780
0,11	40,000	90,870	47,619
0,115	40,000	93,255	46,512
0,12	40,000	96,092	45,455
0,125	40,000	99,524	44,444
0,13	40,000	103,759	43,478
0,135	40,000	109,116	42,553
0,14	40,000	116,111	41,667
0,145	40,000	125,628	40,816
0,15	40,000	139,333	40,000
0,155	40,000	160,769	39,216
0,16	40,000	199,048	38,462
0,165	40,000	286,863	37,736
0,17	40,000	696,667	37,037
0,175	40,000	-696,667	36,364
0,18	40,000	-174,167	n/a
0,185	40,000	-77,407	35,088
0,19	40,000	-36,667	34,483
0,195	40,000	-14,218	33,898
0,2	0,000	0,000	33,333
0,205	40,000	9,812	32,787
0,21	40,000	16,992	32,258
0,215	40,000	22,473	31,746

Table 2

And the relation:

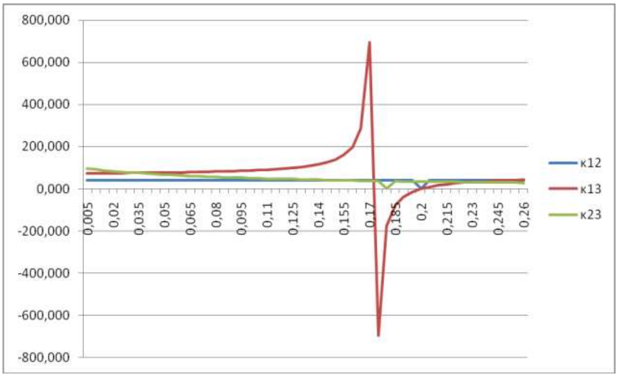

Figure 2

$p_{2,ij} = 1$

For each technique's w-r stands:

$w_{12} = (8 - 40r)$

$$w_{13} = \frac{(38 - 220r)}{5 - 10r}$$

$$w_{23} = \frac{(18 - 100r)}{3 - 10r}$$

The results of the production prices for the first price normalization are:

p(y)=1

r	w12	w13	w23
0,005	7,800	7,455	5,932
0,01	7,600	7,306	5,862
0,015	7,400	7,155	5,789
0,02	7,200	7,000	5,714
0,025	7,000	6,842	5,636
0,03	6,800	6,681	5,556
0,035	6,600	6,516	5,472
0,04	6,400	6,348	5,385
0,045	6,200	6,176	5,294
0,05	6,000	6,000	5,200
0,055	5,800	5,820	5,102
0,06	5,600	5,636	5,000
0,065	5,400	5,448	4,894
0,07	5,200	5,256	4,783
0,075	5,000	5,059	4,667
0,08	4,800	4,857	4,545
0,085	4,600	4,651	4,419
0,09	4,400	4,439	4,286
0,095	4,200	4,222	4,146
0,1	4,000	4,000	4,000
0,105	3,800	3,772	3,846
0,11	3,600	3,538	3,684
0,115	3,400	3,299	3,514

0,12	3,200	3,053	3,333
0,125	3,000	2,800	3,143
0,13	2,800	2,541	2,941
0,135	2,600	2,274	2,727
0,14	2,400	2,000	2,500
0,145	2,200	1,718	2,258
0,15	2,000	1,429	2,000
0,155	1,800	1,130	1,724
0,16	1,600	0,824	1,429
0,165	1,400	0,507	1,111
0,17	1,200	0,182	0,769
0,175	1,000	-0,154	0,400
0,18	0,800	-0,500	0,000
0,185	0,600	-0,857	-0,435
0,19	0,400	-1,226	-0,909
0,195	0,200	-1,607	-1,429
0,2	0,000	-2,000	-2,000
0,205	-0,200	-2,407	-2,632
0,21	-0,400	-2,828	-3,333
0,215	-0,600	-3,263	-4,118

Table 3.

And w-r relation:

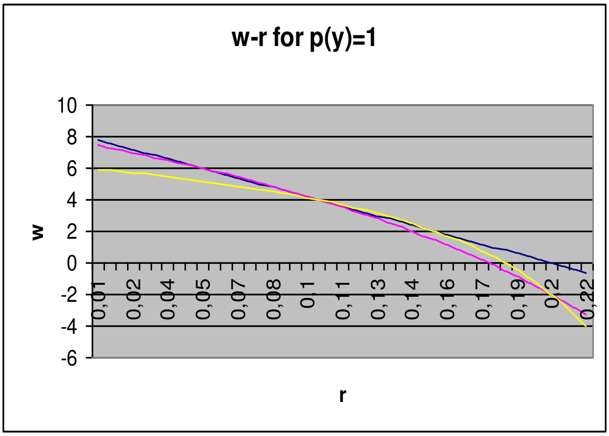

Figure 3

Similar, based on the price system, occurs for the capital intensity:

$$w = K_q(R-r) \Rightarrow K_q = \frac{w}{R-r}$$

K_q is the capital intensity for the typical subsystem q.

The capital intensities Kij , i≠j, i,j=1,2,3 are the following:

$$K_{12} = \frac{8 - 40r}{0,2 - r}$$

$$K_{13} = \frac{\frac{38 - 220r}{5 - 10r}}{\frac{33}{210} - r}$$

$$K_{23} = \frac{\frac{18 - 100r}{3 - 10r}}{\frac{15}{90} - r}$$

For the capital intensities the following hold:

r	k12	k13	k32
0,005	40	48,99701	36,69404
0,01	40	49,65326	37,41746
0,015	40	50,33414	38,17235
0,02	40	51,04167	38,96104
0,025	40	51,77809	39,7861
0,03	40	52,54602	40,65041
0,035	40	53,34842	41,5572
0,04	40	54,18876	42,51012
0,045	40	55,07104	43,5133
0,05	40	56	44,57143
0,055	40	56,98122	45,68992
0,06	40	58,02139	46,875
0,065	40	59,12858	48,13394
0,07	40	60,31262	49,47526
0,075	40	61,58568	50,90909
0,08	40	62,96296	52,44755
0,085	40	64,4638	54,10536
0,09	40	66,11313	55,90062
0,095	40	67,94381	57,85593
0,1	40	70	60
0,105	40	72,34264	62,37006
0,11	40	75,05828	65,01548
0,115	40	78,27427	68,00349
0,12	40	82,18623	71,42857
0,125	40	87,11111	75,42857
0,13	40	93,59886	80,2139
0,135	40	102,6955	86,1244
0,14	40	116,6667	93,75
0,145	40	141,5079	104,2184

0,15	40	200	120
0,155	40	527,5362	147,7833
0,16	40	-288,235	214,2857
0,165	40	-64,5862	666,6667
0,17	40	-14,1414	-230,769
0,175	40	8,615385	-48
0,18	40	21,875	0
0,185	40	30,76923	23,71542
0,19	40	37,30715	38,96104
0,195	40	42,43736	50,42017
0,2	Δ.Ο.	46,66667	60
0,205	40	50,29092	68,64989
0,21	40	53,49487	76,92308

Table 4

The above can be described in a figure:

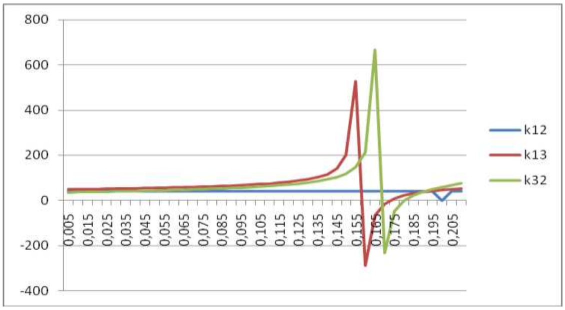

Figure 4

It is obvious that in the case of price normalization with p(x)=1 there is only one switch point. In ccontroversy with price normalizing with p(Y)=1 there are tree switch points (it will be shown later if they are real or fake switch points) for r=0.05,r=0.1, r=0.15.

Normalized with p(x)=1, there is only one switch point4 in r=0.1. But things change with price normalized with p(Y)=1. Once more the "real" switch point in r=0,1 and the "fake" switch points in r=0.05 and r=0.15 appear.

It has been shown that the switch point r=0.1 does not change with price normalization, in controversy with switch points r=0.05 ,r=0.15, which appear only when the normalization is p(Y)=1.

Based on w-r criterion, there is no significant reason, why the "fake" switch point, should not occur a transaction from a less profitable technique, to a more profitable5. In other words for a given nominal wage, if r (which corresponds to the "fake" switch

[4] A switch point can be found in the inner envelope, but it has not a significant economic meaning

[5] By the so far analysis, it is evident that there is no relation between choosing a technique, according to the profit rate, with the capital intensity of each technique. In other words there is no monotonic relation between the capital intensity and the profit rate. Also in switch points, the two techniques have not the sane capital intensity.

points) had not been chosen, then there would exist a technique that does not bring the maximum profit rate for the capitalists.

It is known from the bibliography6, that in special cases, is possible for the switch points (no matter if they are real or fake) to appear or disappear for a given price normalization even in the special case of the indecomposable single production techniques.

Bidard and Klimovsky, claim that the fake switch points can appear or disappear only in special the case of joint production. From Bharadwaj's paper7 is known that, it is possible for two techniques to bring a different price vector in switch points, in the case the two techniques are not neighboring.

In this case not only the price vectors differ in switch points, but also the (dis)appearance of switch points, is affected by the changes in price normalization.

H.Kurz και N.Salvadori[8] give a first definition, according to a technique is cost minimizing if there is no technique that brings extra profits[9]:

$p_I \leq (1+r)p_I A_{II} + wl_{II}$

$p_I = p_I A_I (1+r) + wl_I$

$p_I d = 1$

In other words we import the prices that occur for $R_{I\max}$, in the profit maximization criterion for technique $[A_{II}, l_{II}]$. If term (3) is satisfied, no extra profits occur, and technique $[A_I, l_I]$ is chosen as the cost minimizing one.

In the case of joint production the above terms will become:

$p_I B_{II} \leq (1+r)p_I A_{II} + wl_{II} (1)$

$p_I B_I = p_I (1+r) p_I A_I + wl_I (2)$

$p_I d = 1 (3)$, where $r = R_{I_{\max}}$

For the following price normalizations:

$p_{1,ij} = 1$

For each technique's w-r stand:

$w_{12} = (8-40r)$ (4)

$w_{13} = \dfrac{(38-220r)}{3+10r}$ (5)

$w_{23} = \dfrac{(18-100r)}{1+10r}$ (6)

First for technique (12) and for $r = R_{\max(12)} = 0,2$ the following will stand

The price vector for technique (12) is $p_{12} = [1,1]$.

Implying term (1) for techniques (13) and (23):

$p_{12} B_{13} \leq (1+r) p_{12} A_{13} + wl_{13} (9)$ και

[6] Th Mariolis, (1994)
[7] Kr. Bharadway (1970)
[8] H.D.Kurz and Neri Salvadori, (1995)
[9] Ch. Bidard (1990)

$p_{12}B_{23} \le (1+r)p_{12}A_{23} + wl_{23}(10)$

$p_{12}B_{23} \le (1+r)p_{12}A_{23} + wl_{23}(10)$

First we check term's (9) direction in r=0.1:

$48 \;\; 70 \;=\; 48 \;\; 70$

In the same way we check the direction of term (10) in switch point r=0.1:

$48 \;\; 70 \;=\; 48 \;\; 70$

We conclude that in switch point r=0.1 production techniques (12) (13) and (23) are equivalent for prices of technique (12) and for technique p1=1.

$p_{2,ij} = 1$

For each technique's w-r:

$w_{12} = (8-40r)$

$w_{13} = \dfrac{(38-220r)}{5-10r}$

$w_{23} = \dfrac{(18-100r)}{3-10r}$

First for technique (12) we have for $r = R_{\max(12)} = 0,2$

$p_{12} = [1,1]$ [10]

Initially we check out what stands in switch point r=0.1. According to the cost minimization criterion:

$p_{12}B_{13} = p_{12}A_{13}(1+r) + w_{13}\ell_{13} \rightarrow [40,70] = [48,70]$

Therefore technique (13) minimizes cost as no extra profits occur from using this technique.

In the same way for technique (23):

$p_{12}B_{23} = p_{12}A_{23}(1+r) + w_{23}\ell_{23} \rightarrow [48,70] = [48,70]$

Technique (13) minimizes cost as no extra profits occur. In other words all the three techniques are cost minimizing.

It is evident that in "real" switch point r=0.1 all three techniques are equivalent. In the numerical example of Bidard and Klimovsky stands:

$p_{12} = p_{13} = p_{23} = [1,1]$

Furthermore we check what in "fake" switch point r=0.05 stands. According to the cost minimization criterion:

$p_{12}B_{13} \underset{\le}{>} p_{12}A_{13}(1+r) + w_{13}\ell_{13} \rightarrow [40,70] \underset{\le}{>} [48,69]$

Therefore nothing can be said whether technique (23) minimizes cost or not. In other words the cost minimization criterion can not lead the system to cost minimizing technique.

In the same way for technique (23):

$p_{12}B_{23} \underset{\le}{>} p_{12}A_{23}(1+r) + w_{23}\ell_{23} \rightarrow [40,70] \underset{\le}{>} [47.2,68.2]$

Therefore nothing can be said whether technique (23) minimizes cost or not. In other words the cost minimization criterion can not lead the system to cost minimizing technique.

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[10] The vector [1,1] emerge for every r, $0 \le r \le 0.2$

The price vectors for these techniques are:

$p_{12} = [1,1]$

$p_{13} = [0.778,1]$

$p_{23} = [0.6,1]$

In "fake" switch point r=0.15, also according to cost minimization criterion stand:

$p_{12}B_{13} < p_{12}A_{13}(1+r) + w_{13}\ell_{13} \rightarrow [40,70] < [47.429,70.428]$

Therefore technique (13) does not bring extra profits.

In the same way for technique (23):

$p_{12}B_{23} \leq p_{12}A_{23}(1+r) + w_{23}\ell_{23} \rightarrow [48,70] \leq [48,71]$

Therefore technique (23) minimizes cost as occurs extra profits. In other words the cost minimizing technique is (23).

Last for the price vectors of the above techniques stand:

$p_{12} = [1,1]$

$p_{13} = [1.285,1]$

$p_{23} = [1.667,1]$

It is implied that in "fake switch" points, the intersected techniques have not the same price vectors[11]. The last does not imply, nevertheless, that in fake switch points, no change in choice of techniques is occurred.

2.1. The General Case

In terms of the w-r criterion a non-decomposable productive[12] joint production technique, let (a) is chosen instead of (b) when it stands:

$w^a > w^b$, that implies:

$\ell^a[B^a - A^a(1+r)]^{-1}y < \ell^b[B^b - A^b(1+r)]^{-1}y$, for normalization with y, $y \geq 0$[13].

In the same way, in case we have more than two techniques:

$w^a > w^b > w^c$

$\ell^a[B^a - A^a(1+r)]^{-1}y < \ell^b[B^b - A^b(1+r)]^{-1}y < \ell^c[B^c - A^c(1+r)]^{-1}y$

$p^i y = 1$,i=a,b,c.

First in the case of a switch point:

$w^a = w^b = w^c$

$\ell^a[B^a - A^a(1+r)]^{-1}y = \ell^b[B^b - A^b(1+r)]^{-1}y = \ell^c[B^c - A^c(1+r)]^{-1}y$

In order for a switch point to be independent of price normalization it is necessary:

$\ell^a[B^a - A^a(1+r)]^{-1} = \ell^b[B^b - A^b(1+r)]^{-1} = \ell^c[B^c - A^c(1+r)]^{-1}$,

In other words, it is necessary all vectors $\ell^i[B^i - A^i(1+r)]^{-1}$,for i=1,2,3 to be equivalent[14].

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[11] In this point, it is necessary to refer, that not only in the case of joint production but also in the case of non neighboring single production techniques as well, the w-r criterion does not match with the cost minimization criterion.

[12]In other words holds: $[B - A(1+r)]^{-1} \geq 0$

[13] Y can be any nx1 vector of standard commodity

But in the case of fake switch points stands:

$w^a = w^b \neq w^c$

$\ell^a[B^a - A^a(1+r)]^{-1}y = \ell^b[B^b - A^b(1+r)]^{-1}y \neq \ell^c[B^c - A^c(1+r)]^{-1}y$

Let the fake switch point –according to Bidard- to be found on the upper envelope of w-r curves, then:

$w^a = w^b > w^c$

$\ell^a[B^a - A^a(1+r)]^{-1}y = \ell^b[B^b - A^b(1+r)]^{-1}y < \ell^c[B^c - A^c(1+r)]^{-1}y$

It is evident that in this case, according to the w-r criterion, the systems operates with either technique a or b. If the system operated with technique (c) then it operates below it's production potential[15].

Nevertheless the fake switch points, according to the w-r criterion are real switch points – although Bidard and Klimovsky claim the opposite. The fact that prices are different in these points doesn't seem to affect the final choice of techniques. The fact that «fake switch points» appear and disappear[16], is a phenomenon that can be found, even in indecomposable single production systems[17], and that because price normalizing affects the relative position of w-r curves[18]. But according to the cost minimization criterion we may decide which technique will be operated. The last is not something new, as it is know from bibliography[19] that, in the case of joint production the w-r criterion and the cost minimization criterion do not come up to the same technological change decision.

3. Conclusions

In this analysis so far, an effort was made, to be shown in a numerical example of Bidard and Klimovsky, that the existence of "Fake" switch points does not change the aspect of choice of techniques.

The fact that in these switch point do not have the same price vectors, does not affect the choice of techniques. According not only to the w-r but to the cost minimization criterion as well, it is evident that a technique is chosen after all[20].

An other characteristic, according to which the switch points called fake, was the fact that the switch points were appearing or disappearing with a change in price normalization. But in bibliography it is known that even the "real" switch points can appear or disappear with a change in price normalization

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[14] In the same way, in the case that two techniques i=a,b compete each other , the change of technique, in only then unchanged, when the vectors can be compared $\ell^i[B^i - A^i(1+r)]^{-1}$. In other words it is necessary to be a order relation between them.

[15] For the w-r relation: $w = \pi_\varepsilon - K_n r$, where π_ε is the labor productivity and K_n the capital intensity in price terms. In this case in order the relation $w^a = w^b > w^c$ to stand, the labor productivity should ceteris paribus should be reduced (as for given r the capital intensity is constant). The last does not seem to be an orthological decision.

[16] The existence of real switch point in r=0.1, does not related with it' s real switch point nature, but mostly related of it's property as the ratio of the compared typical subsystems

[17] Th. Mariolis (1994)

[18] This exalts choice of technique to a choice of typical subsystems instead.

[19] G.Stamatis (1997)

[20] The last was shown in the numerical example

The fact that in the above numerical example, the switch point in r=0.1 is not affected by a change in the standard commodity, is related with the fact that:

- r=0.1 is the standard ration of surplus product to the used means of production/ used commodities.

In economic theory the ratio of surplus product to the used means of production/ used commodities is the same for all typical subsystems[21].

In other words in mathematical terms, the rows and columns of the input matrices are linear dependent.

Consequently the case of Bidard's and Klimovsky's example is a special case, that can not stand in general.

In the general case, the choice of technique depends on the price. The (re)switch points can appear or disappear with a change in the standard commodity.

Eventually when we refer to choice of techniques, we refer to choice of typical subsystems.

Finally not only the w-r criterion, but the cost minimization criterion as well, does not stand in general, because they are affected by the price normalization.

The only case that the choice of techniques is univocal is the charassofian systems of production and the corn economies. John von Neumann's criterion[22] can be counted as an application of the charassofian systems[23].

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