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Abstract We examine the effects of child policies on both the transitional dynamics and long-run demo-economic outcomes in the conventional overlapping generations model of neoclassical growth extended with endogenous longevity and endogenous fertility. The government invests in public health (Chakraborty, 2004) and the individual survival probability at the end of youth depends on health expenditure through an S-shaped longevity function. This may give rise to four steady states and, hence, development traps are possible. However, poverty or prosperity may not depend on initial conditions, while being the result of a child policy design. In particular, a child tax can be used to effectively allow those economies that were entrapped into poverty to prosper irrespective of where they start from.

Keywords Child policy; Endogenous fertility; Health; Life expectancy; OLG model

JEL Classification I1; J13; O4

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1. Introduction

Over the past century and, in particular, in the recent decades, citizens in most countries in the world have experienced dramatic increases in life expectancy (e.g., Livi-Bacci, 1997). The importance of longevity in determining the macroeconomic and demographic performances of an economy over the very long run is the object of a growing body of economic literature, see, e.g., the influential papers by Blackburn and Cipriani (2002) and Chakraborty (2004).

In particular, Blackburn and Cipriani (2002) considered a general equilibrium overlapping generations (OLG) economy, with endogenous fertility and endogenous longevity and three-period lived individuals that accumulate human capital through education – which is the main determinant of adult survival probability. Individuals produce and consume output, invest in education and spend a fraction of their time endowment to take care of their descendants. A rise in the individual life span creates a virtuous cycle of events on the process of development: it increases, in fact, the productivity of labour by raising the returns to human capital accumulation; this causes a reduction in the child bearing time as well as a rise in the time that agents devote to education; this chain of events promotes human capital accumulation that causes, in turn, a reduction in both adult mortality and population growth. In this context, they found that the existence of low and high development regimes, the former being characterised by low income, high fertility and a relatively low length of life, the latter by high income, low fertility and a relatively high length of life. Depending on initial conditions, therefore, an economy may be entrapped into poverty or prosper. Their model accords with the empirical evidence of the demographic transition.[1]

Chakraborty (2004), instead, introduced endogenous lifetime in the basic Diamond's (1965) style OLG model of neoclassical growth with exogenous fertility. The probability of surviving from work

[1] In a model with educational investments and endogenous fertility, Chen (2010) developed an OLG model showing that (*i*) with exogenous lifetime, multiple development regimes with club convergence can exist when mortality is high enough, and (*ii*) with endogenous lifetime, a unique stable steady state exist if mortality is low enough.

time to retirement time is a non-decreasing concave function of the individual health status which is determined by public investments. A rise in the health tax to finance health expenditure may lead individuals to live longer and this, in turn, provides an impetus to capital accumulation and a higher life span as well. His main finding is that, when the output elasticity of capital in the Cobb-Douglas production function is relatively high, endogenous mortality may cause development traps (represented by the stable zero equilibrium), so that low-income high mortality and high-income low-morality societies can exist. Chakraborty (2004) considered public health investments as a prerequisite for sustained economic growth and found that improving the health status of people may be beneficial for growth and development because it directly reduces the risk of adult mortality and this causes, in turn, an impetus to higher capital accumulation and lower adult mortality as well.

As regards population, it is well established that in recent decades several developed countries have experienced dramatic drops in fertility that have produced, in turn, a reduction in the number of children even well below the replacement rate (e.g., Germany, Italy, Spain and Japan), while also causing a declining ratio of economically active to retired people. The response of many governments has been the implementation of child support programmes mainly based on the public provision of child subsidies (e.g., a direct monetary transfer to families with children) in order to incentive child care activities of households and favour the fertility recovery, even if other policies such as child care facilities (e.g., investments in infrastructure for day-care centres, schools and so on) and child tax credits have been adopted, especially in North European countries. However, even the opposite problem of excessive population growth may represent a serious concern for economic growth and sustainable development in some countries in the world. In these countries, therefore, a child-tax[2] policy may be implemented with anti-natalist purposes to alleviate possible

[2] We note that while child support programmes have been extensively examined in economic literature (see, e.g., Momota, 2000; van Groezen et al., 2003; Apps and Rees, 2004; van Groezen and Meijdam, 2008), the theoretical analysis of the effects of child taxes on long-run demo-economic performances is, to the best of our knowledge, relatively scarce. For instance, in the literature with endogenous fertility, Bental (1989) represents one of the first

environmental problems and social conflicts that an excessive population may cause. This is the well known case of the one-child policy (or, alternatively, family planning policy, see Coale, 1981) enforced by the Chinese government since the 1979, which probably represents the sole example of application of tax penalties for couples with more than one child all over the world. As regards some of the technical rules of implementation, the one-child policy restricts the number of children that couples decide to have to one, although several exceptions exist, for instance, for couples living in certain rural areas of China or for people belonging to ethnic minorities. In particular, Chinese families subject to the restrictions of the family planning policy must pay fines based on their income if they choose to have more than one child. The monetary penalties, however, raises more than proportionally for any additional newborn. Of course, the enforcement of this policy is controversial because it had several unpleasant effects (especially as regards the moral feasibility to pursue the goal of restricting the freedom of people to have children as well as some of the methods adopted to realise it) that, however, we do not want to discuss in this paper since they would require a more careful and thorough analysis.

Moreover, as is well known at least starting from the seminal paper by Becker (1960), economic literature has argued that the choice of the number of children should be the result of a rational choice of individuals, especially in developed economies.[3] In particular, the theoretical literature on endogenous fertility is of greater importance in the theory of economic growth (e.g., Becker and

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attempt to discuss, amongst other things, the effects of the child tax instrument in a model where children are considered as a capital good that furnish services to retired parents (i.e., the old-age security hypothesis), and concluded that a tax on children can achieve the optimal capital-labour ratio but fails, however, to realise the optimal population growth rate. Recently, Fanti and Gori (2009) have shown that a child tax can be used to actually raise population growth in the long run, while also raising income per worker.

[3] For instance, van Groezen et al. (2003, p. 237) argued that "The rate of fertility should therefore be treated as an endogenous variable, that is, as the result of a rational choice which is influenced by economic constraints and incentives. Economic theory can thus help in explaining why the observed decline in the (desired) number of children would occur."

Barro, 1988; Barro and Becker, 1989), even as an explanation of multiple regimes of developments when human capital is considered (e.g., Becker et al., 1990; Blackburn and Cipriani, 2002).

This paper represents a first attempt to introduce endogenous fertility (with children being considered as a desirable good) in the basic OLG model with endogenous lifetime à la Chakraborty.[4] Different from Chakraborty (2004), however, we consider a more general (S-shaped) longevity function of health capital and find that development traps with a positive stock of capital – and, hence, positive levels of production and consumption – are possible.[5] This certainly represents an interesting point, especially as regards the empirical relevance of the results.

Moreover, in this paper attention essentially is focused on the crucial role that family policies, consisting in either a tax or subsidy on children, can play on both the transitional dynamics and demo-economic outcomes over the very long run. The main finding is that poverty or prosperity may not depend on initial conditions, while being the result of a child policy design. In particular, a large enough increase in the child tax reduces both fertility and adult mortality. This stimulates capital accumulation and eliminate the low equilibrium so that an economy that were entrapped into poverty due to unfavourable initial conditions will converge towards the unique high development regime, where income per worker is high, life expectancy is high and fertility is low.

The present paper is differentiated from each of the above mentioned contributions in terms its specific objectives, analyses and results. From a broader perspective, our paper belongs to the demo-economic literature that treats the key demographic variables – i.e. fertility and longevity – as endogenously determined in the model rather than exogenously given, and links them into the process of economic growth in the simple and intuitive context of the standard OLG model (e.g.

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[4] Blackburn and Cipriani (2002), in fact, assumed human capital accumulation through education, rather than public health care investments, as the main determinant of the surviving probability of people.

[5] Blackrun and Issa (2002) noted the importance of the existence of a low equilibrium with positive levels of production and consumption, and introduced bequests in an OLG growth model with exogenous fertility and uncertain lifetimes to avoid degeneracy in a low development equilibrium.

Zhang et al., 2001). This paper may also be viewed as a contribution to the wider literature on multiple equilibria, poverty traps and demographic changes over the very long run (e.g. Azariadis and Drazen, 1990).

The remainder of the paper is organised as follows. In Section 2 we set up the model. In Section 3 we study the dynamic path of capital accumulation and provide necessary conditions for the existence of multiple (four) steady states. In Section 4 we analyse the effects of child policies on economic growth and the stages of development. Section 5 concludes.

2. The model

Consider a general equilibrium overlapping generations (OLG) closed economy populated by identical individuals, identical firms and a government that runs both health and child polices at a balanced budget.

The lifetime of the typical agent is divided into childhood and adulthood, the latter being, in turn, divided into working time (youth) and retirement time (old age). As a child she does not make economic decisions. When adult she draws utility from consumption over the life cycle and the number of children.[6]

Young individuals of generation t (N_t) are endowed with one unit of time supplied inelastically on the labour market, while receiving wage income at the competitive rate w_t.

It is assumed that the probability of surviving from youth to old age is endogenous and determined by the individual health level, that is, in turn, augmented by the public provision of health investments such as, for instance, hospitals, vaccination programmes and so on (see Chakraborty, 2004). The survival probability at the end of youth of an individual entered the

[6] See Eckstein and Wolpin (1985) and Galor and Weil (1996). Note that the way of modelling children as a desirable good that directly enters the parents' utility has been called by Zhang and Zhang (1998) a weak form of altruism towards children.

working period at t, π_t, depends upon her health capital, h_t, and is given by a non-decreasing – though bounded – function $\pi_t = \pi(h_t)$. Following Blackburn and Cipriani (2002) and Blackburn and Issa (2002), we model this relationship with the following rather general function of health capital:[7]

$$\pi_t = \pi(h_t) = \frac{\pi_0 + \pi_1 \Delta h_t^{\delta}}{1 + \Delta h_t^{\delta}}, \tag{1}$$

where $\delta, \Delta > 0$, $0 < \pi_1 \leq 1$, $0 < \pi_0 < \pi_1$, $\pi(0) = \pi_0 > 0$, $\pi'_h(h) = \dfrac{\delta \Delta h^{\delta-1}(\pi_1 - \pi_0)}{(1 + \Delta h^{\delta})^2} > 0$,

$\lim_{h \to \infty} \pi(h) = \pi_1 \leq 1$, $\pi''_{hh}(h) < 0$ if $\delta \leq 1$ and $\pi''_{hh}(h) \gtrless 0$ for any $h \lessgtr h_T := \left[\dfrac{\delta - 1}{(1+\delta)\Delta}\right]^{\frac{1}{\delta}}$ if $\delta > 1$.

Eq. (1) allows us to capture various aspects of the evolution of the length of life of the typical agent as a function of the individual health measure h: it encompasses, in fact, the "saturating" function used in the numerical examples by Chakraborty (2004) when $\delta = \Delta = 1$ and $\pi_0 = 0$ as well as the S-shaped function when $\delta > 1$, while also preserving (different from Chakraborty, 2004) a positive constant rate of longevity regardless of public health spending. Some clarifications on Eq. (1) are now useful. First, we define π_0 as being an exogenous measure of the "natural" rate of longevity of people in a country (see, e.g., Ehrlich, 2000; Leung and Wang, 2010): it represents the "biological" individual survival probability at the end of the working period irrespective of whether the government invests in public health or not. This measure of individual mortality of the adult

[7] Although the independent variable of the longevity function in the analysis by Blackburn and Cipriani (2002) is human capital instead of public health capital, the line of reasoning to justify this formulation may be the same. To this purpose, in fact, and to capture the idea that life expectancy is positively correlated with the level of development, Blackburn and Issa (2002) argued, in the first part of their paper, that the probability of surviving at the end of youth is non-decreasing bounded function of the stock capital installed in the economy (and used a stepwise function of the stock of capital to show the possibility of multiple development regimes), while introducing, in the second part of the paper, an S-shaped function to justify the relationship between longevity and public health spending.

may be thought to be affected by both economic and non-economic factors, e.g. the lifestyle of people, education, economic growth and the standards of living, the degree of culture and civilisation, weather and climate changes, ethnical and civil wars, endemic diseases and so on (which are, however, left exogenous in the present context[8]). Thus, we may expect π_0 to be higher in developed rather than developing or under-developed nations, and the more individuals naturally live longer, the smaller the reduction in adult mortality when the public health expenditure raises. Second, the parameter π_1 captures the intensity of the efficiency of health capital on the rate of longevity. A rise in π_1 may be interpreted as exogenous medical advances due, for instance, to scientific research, vaccination programmes and so on. Third, we may think, realistically, that health investments have a more intense effect in reducing adult mortality when a certain threshold level of health capital is achieved, while becoming scarcely effective when longevity is close to its saturating value (e.g., the functional relationship between public health expenditure and longevity may be S-shaped). The parameters δ and Δ allow to capture such an idea and determine both the turning point of $\pi'_h(h)$ and speed of convergence from the natural length of life π_0 to the saturating value π_1. In particular, given the value of Δ,[9] the parameter δ may represent the degree of effectiveness of public health investments as an inducement to higher longevity, other things being unchanged. In other words, it measures how an additional unit of health capital is transformed into higher longevity through the health technology. If $\delta \leq 1$, $\pi(h)$ is concave for any h and, hence, no threshold effects exist so that longevity increases less than proportionally from the starting point π_0 to the saturating value π_1 as h rises, and the more δ is close to zero the more efficiently and rapidly an additional unit of health capital is transformed into higher longevity when h is relatively

[8] The endogenous determination of π_0 in the longevity function Eq. (1) may give rise to interesting findings as regards the dynamical features of the model that are, however, left for future research.

[9] In the numerical example presented throughout the paper, we used $\Delta = 1$ without loss of generality, given the purely technical (and not economically interpretable) nature of such a parameter.

low, while reaching the saturating value π_1 more slowly as h becomes larger. Figure 1 illustrates in a stylised way the function $\pi(h)$ when $\delta \le 1$: the solid (dashed) [dotted] line refers to the case $\delta = 1$ ($\delta < 1$) [$\delta \to 0$]. As can readily be seen from Figure 1, the lower (higher) δ is the more (less) efficiently an additional unit of health capital is transformed into higher longevity until (once) a certain level of h is reached.

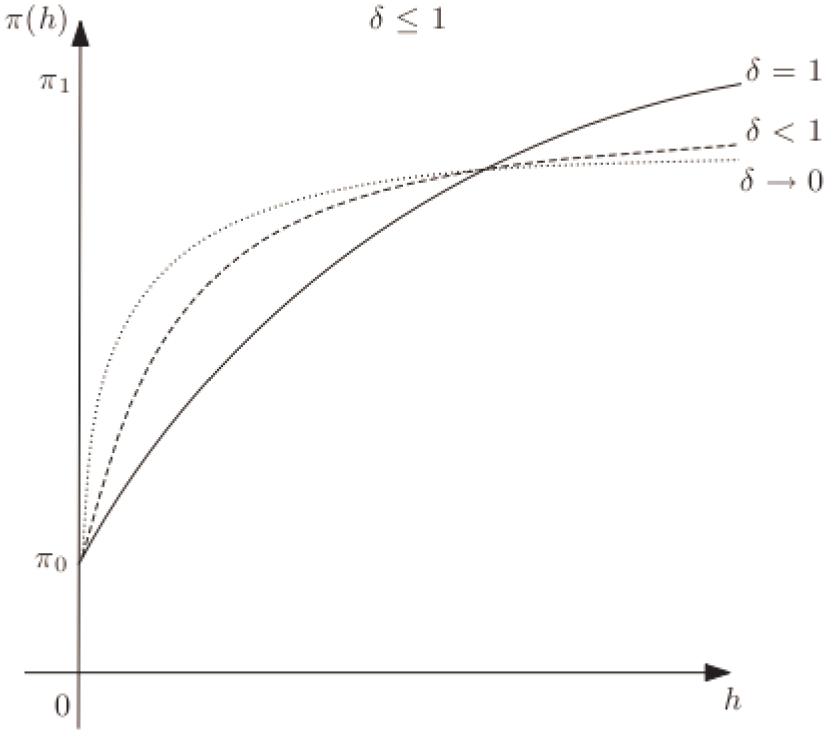

Figure 1. Longevity and public health capital when $\delta \le 1$.

In contrast, when $\delta > 1$ the longevity function is S-shaped and threshold effects exist. In particular, longevity increases more (less) than proportionally until (once) the turning point h_T is achieved. However, a rise in δ shifts the longevity function to the right while also increasing the speed of convergence from π_0 to π_1, as clearly shown in Figure 2, where the solid (dashed) [dotted] line refers to δ_{low} (δ_{high}) [$\delta \to +\infty$]. This means that the more threshold effects are intense (high values of δ), the slower an additional unit of health capital is transformed into a higher life span when h

is relatively low, while reaching the saturating value π_1 more efficiently and rapidly as h becomes larger.

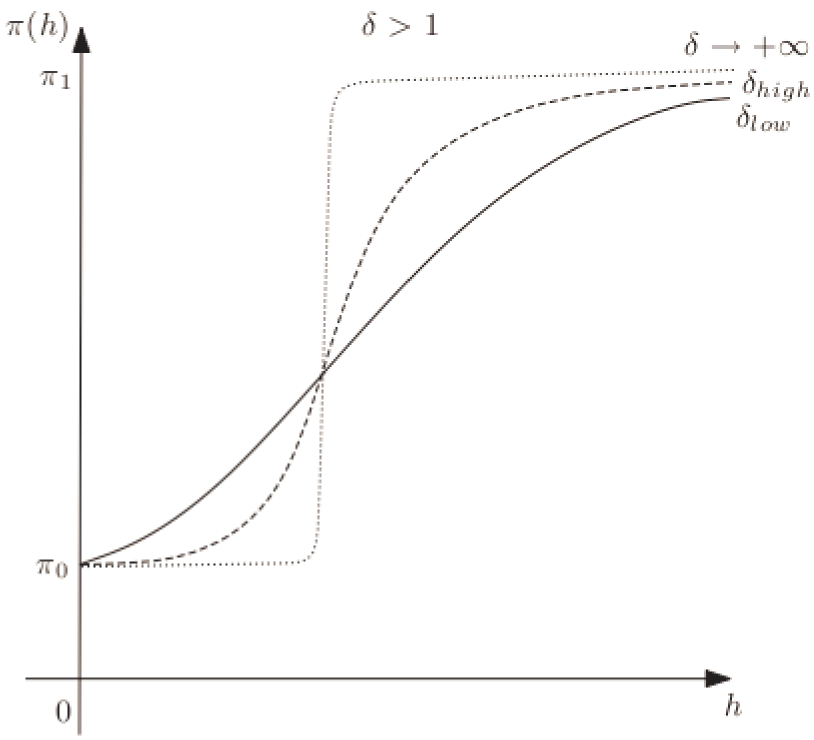

Figure 2. Longevity and public health capital when $\delta > 1$.

In other words, increasing public health investments is not effective until a threshold value of health capital is achieved (and this value is larger the larger is δ), because, for instance, a certain degree of knowledge to enable such investments to be effectively transformed into higher longevity has not yet been reached.[10]

[10] As an example, we may think about the existence of threshold effects in the accumulation of knowledge required for new medical advances and discoveries in the treatment of diseases (e.g. vaccines) to be effective: the public health expenditure to finance new research projects may be high and apparently useless until a certain degree of knowledge is achieved. Beyond such a threshold, however, a "sudden" effect exists that allows to trigger and bring to light the beneficial effects of the new discoveries, to make them efficient, usable and operative across population and eventually transformed into higher longevity.

As in Chakraborty (2004), we assume that health capital per worker at t is augmented through public investments financed at a balanced budget with a (constant) proportional wage income tax $0 < \tau < 1$, that is:

$$h_t = \tau w_t . \tag{2}$$

As regards child care activities, we assume that raising children is costly and, in particular, the amount of resources needed to take care of them is given by a monetary cost $q w_t$ per child, with $0 < q < 1$ being the percentage of child-rearing cost on the parents' working income.[11] Moreover, we assume that in every period the government finances a wage income subsidy be levying a fixed child tax for each newborn at a balanced budget.[12] Therefore, at time t the per worker child policy budget of the government reads as

$$\theta_t w_t = b w_t n_t , \tag{3}$$

the left-hand side being the wage subsidy expenditure and the right-hand side the child tax receipt, where $b > 0$ is the fixed percentage of wage income collected by the government as a tax for an additional newborn, $\theta_t > 0$ is the wage subsidy rate adjusted over time to balance out the budget and n_t the average number of children at time t.

Therefore, in period t the budget constraint faced by an individual of the younger working-age (child-bearing) generation reads as:

$$c_{1,t} + s_t + (q + b) w_t n_t = w_t (1 - \tau + \theta_t), \tag{4.1}$$

[11] This child cost structure is similar to that adopted, amongst many others, by Wigger (1999) and Boldrin and Jones (2002) and may be considered as a proxy of the time cost of children.

[12] For instance, the tax penalties imposed by the Chinese birth planning programme on parents with multiple descendants are computed as a fraction of either the disposable income of people living in urban areas or the cash income, estimated by the local authorities, of people living in rural area, and are also proportional to the number of children that exceeds the quota planned by the government.

i.e. wage income – net of the contribution paid to finance health expenditure plus the wage subsidy financed with the fixed child tax – is divided into material consumption when young, $c_{1,t}$, savings, s_t, and the (net) cost of raising n children.

Old individuals are retired and live uniquely with the amount of resources saved when young plus the interest accrued from time t to time $t+1$ at the rate r_{t+1}. The existence of a perfect annuity market (where savings are intermediated through mutual funds) implies that old survivors will benefit not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased. Hence, the budget constraint of an old retired individual started working at t can be expressed as

$$c_{2,t+1} = \frac{1+r_{t+1}}{\pi_t} s_t, \tag{4.2}$$

where $c_{2,t+1}$ is old-aged consumption.

The representative individual entering the working period at time t must choose (*i*) how much to save out of her disposable income and (*ii*) how many children to raise in order to maximise the lifetime utility function

$$U_t = \ln(c_{1,t}) + \pi_t \ln(c_{2,t+1}) + \gamma \ln(n_t), \tag{5}$$

subject to Eqs. (4), where $\gamma > 0$ captures the parents' relative taste for children.

The constrained maximisation of Eq. (5) gives the demand for children and the saving rate, respectively:

$$n_t = \frac{\gamma(1-\tau+\theta_t)}{(1+\pi_t+\gamma)(q+b)}, \tag{6.1}$$

$$s_t = \frac{\pi_t w_t (1-\tau+\theta_t)}{1+\pi_t+\gamma}. \tag{6.2}$$

Now, using Eq. (3) to eliminate θ_t and rearranging terms, Eqs. (6) can definitively be written as:

$$n_t = \frac{\gamma(1-\tau)}{(1+\pi_t)(q+b)+\gamma q}, \tag{7.1}$$

$$s_t = \frac{\pi_t w_t (1-\tau)(q+b)}{(1+\pi_t)(q+b)+\gamma q}, \tag{7.2}$$

with π_t being jointly determined by Eqs. (1) and (2).[13]

Since one of the objective of this paper is the study of the effects of the child policy variable b on both the transitional dynamics and demo-economic outcomes over the very long run, it is first interesting to briefly notice the role played by b in a partial equilibrium context. In particular, a rise in the child tax increases the marginal cost of bearing an extra child and then makes more convenient to substitute children with consumption. From Eqs. (7.1) and (7.2), in fact, we observe that as a direct partial equilibrium effect a higher child tax reduces the demand for children and increases savings.

In the following section we introduce production and characterise the general equilibrium features of the model.

2.1. Production and equilibrium

Firms are identical and act competitively on the market. The (aggregate) constant returns to scale Cobb-Douglas technology is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t and $L_t = N_t$ are output, capital and the labour input at time t respectively, $A > 0$ represents a scale parameter and $0 < \alpha < 1$ is the output elasticity of capital. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per worker, the intensive form production function can be written as $y_t = Ak_t^{\alpha}$. Standard profit maximisation implies that factor inputs are paid their marginal products, that is[14]

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{8}$$

[13] Note that the individual lifetime budget constraint is always verified irrespective of whether the child tax is fixed either at high or low levels.

[14] The price of final output is normalised to unity and capital totally depreciates at the end of each period.

$$w_t = (1-\alpha)Ak_t^{\alpha}. \tag{9}$$

Knowing that $N_{t+1} = n_t N_t$, market-clearing in goods and capital market gives the usual equilibrium condition (in per worker terms) $n_t k_{t+1} = s_t$, that is combined with Eqs. (7) to obtain:

$$k_{t+1} = \frac{\pi_t}{\gamma} w_t (q+b). \tag{10.1}$$

Now, using Eqs. (1), (2), (9) and (10.1), the dynamic path of capital accumulation is described by the following first order non-linear difference equation:

$$k_{t+1} = \frac{Dk_t^{\alpha}\left(\pi_0 + \pi_1 Bk_t^{\alpha\delta}\right)}{\gamma\left(1 + Bk_t^{\alpha\delta}\right)}, \tag{10.2}$$

where $B := \Delta[\tau(1-\alpha)A]^{\delta}$ and $D := (q+b)(1-\alpha)A$ are two positive constant used to simplify notation. Despite Eq. (10.2) is a simple first order non-linear difference equation, the dynamical features originated by it are interesting to be investigated and discussed.

3. Dynamics

In this section we analyse both the transitional dynamics and steady states of the economy. From Eq. (10.2) it is easy to observe that when $\tau = 0$ a unique stable steady (as in Diamond, 1965) exists because $B = 0$ in that case. In contrast, when $0 < \tau < 1$, that is $B > 0$, the following proposition shows that development traps are possible.

Proposition 1. *The dynamic system described by Eq. (10.2) admits either two steady states* $\{0, \bar{k}\}$, *with* $\bar{k} > 0$ *(only the positive one being asymptotically stable) or four steady states* $\{0, \bar{k}_1, \bar{k}_2, \bar{k}_3\}$, *with* $\bar{k}_3 > \bar{k}_2 > \bar{k}_1 > 0$ *(only the second and the forth being asymptotically stable). Moreover, (1) a sufficient condition to avoid development traps is* $\Lambda_2 > 0$ *and* $\Lambda_3 > 0$, *and (2) a necessary condition for the existence of multiple steady states is that at least either* $\Lambda_2 < 0$ *or* $\Lambda_3 < 0$ *holds,* *where* $\Lambda_2 := BE[1-\alpha(1-\delta)] + F[1-\alpha(1+2\delta)]$, $\Lambda_3 := \pi_0 B[1-\alpha(1-2\delta)] + E[1-\alpha(1+\delta)]$, $E := [\pi_0 + \pi_1 + \delta(\pi_1 - \pi_0)]B > 0$ *and* $F := \pi_1 B^2 > 0$.

Proof. Let first the following lemma be established.

Lemma 1. *Define the right-hand side of (10.2) as* $G(k)$. *Then, we have: (1.i)* $G(0)=0$, *(1.ii)* $G'_k(k)>0$ *for any* $k>0$, *(1.iii)* $\lim_{k\to 0^+} G'_k(k) = +\infty$, *(1.iv)* $\lim_{k\to+\infty} G'_k(k) = 0$, *(1.v)* $G''_{kk}(k)$ *admits at most three roots and* $G''_{kk}(0) \neq 0$.

From Eq. (10.2), property (1.*i*) is straightforward. Differentiating the right-hand side of Eq. (10.2) with respect to k gives

$$G'_k(k) = \frac{\alpha D\left(Fk^{2\alpha\delta} + Ek^{\alpha\delta} + \pi_0\right)}{\gamma k^{1-\alpha}\left(1+Bk^{\alpha\delta}\right)^2}. \tag{11.1}$$

Defining $k^{\alpha\delta} := x$ as a new supporting variable, (11.1) can be transformed to:

$$g(k,x) = \frac{\alpha D\left(Fx^2 + Ex + \pi_0\right)}{\gamma k^{1-\alpha}(1+Bx)^2}. \tag{11.2}$$

Since no positive real roots of (11.2) can exist, then (11.1) implies that $G'_k(k)>0$ for any $k>0$. This proves (1.*ii*).

Moreover,

$$\lim_{k\to 0^+} G'_k(k) = \frac{\alpha D}{\gamma}\lim_{k\to 0^+} \frac{Fk^{2\alpha\delta} + Ek^{\alpha\delta} + \pi_0}{k^{1-\alpha}\left(1+Bk^{\alpha\delta}\right)^2} = +\infty.$$

and

$$\lim_{k\to+\infty} G'_k(k) = \lim_{k\to+\infty} \frac{\alpha D\left(Fk^{2\alpha\delta} + Ek^{\alpha\delta} + \pi_0\right)}{\gamma k^{1-\alpha}\left(1+Bk^{\alpha\delta}\right)^2} = \frac{\alpha D}{\gamma}\lim_{k\to+\infty} \frac{\dfrac{\pi_0}{k^{2\alpha\delta}} + \dfrac{E}{k^{\alpha\delta}} + F}{k^{1-\alpha}\left(\dfrac{1}{k^{2\alpha\delta}} + \dfrac{2B}{k^{\alpha\delta}} + B^2\right)} = 0,$$

which prove (1.*iii*) and (1.*iv*), respectively. Now, differentiating (11.1) with respect to k gives

$$G''_{kk}(k) = \frac{-\alpha D\left(\Lambda_1 k^{3\alpha\delta} + \Lambda_2 k^{2\alpha\delta} + \Lambda_3 k^{\alpha\delta} + \Lambda_4\right)}{\gamma k^{2-\alpha}\left(1 + Bk^{\alpha\delta}\right)^3}, \tag{12.1}$$

where $\Lambda_1 := (1-\alpha)BF > 0$ and $\Lambda_4 := \pi_0(1-\alpha) > 0$. Knowing that $k^{\alpha\delta} := x$, Eq. (12.1) can be rewritten as

$$f(k,x) = \frac{-\alpha D\left(\Lambda_1 x^3 + \Lambda_2 x^2 + \Lambda_3 x + \Lambda_4\right)}{\gamma k^{2-\alpha}(1 + Bx)^3}. \tag{12.2}$$

From (12.2), it is straightforward to see that $f(k,x)$ admits at most three roots for x and $f(k,0) \neq 0$. Hence, from (12.1), $G''_{kk}(k)$ admits at most three roots for k and $G''_{kk}(0) \neq 0$. This proves (1.*v*).

Proposition 1 therefore follows. In fact, by properties (1.*i*) and (1.*iii*), zero is always an unstable steady state of Eq. (10.2). By (1.*ii*)-(1.*iv*), $G(k)$ is a monotonic increasing function of k and eventually falls below the 45° line, so that at least one positive stable steady state exists for any $k > 0$.

Now, assume *ad absurdum* the existence of an odd number of equilibria. By (1.*ii*)-(1.*iv*), the inflection points cannot be odd-numbered for any $k > 0$. By property (1.*v*), therefore, the number of inflection points of $G(k)$ is either zero or two for any $k > 0$. Since at least one positive stable steady state exists, then for any $k > 0$ the phase map $G(k)$ may intersect the 45° line from below *at most* once before falling below it. Hence, an even number of equilibria must necessarily exist. In particular, the steady states are either two, with the positive one being the unique asymptotically stable equilibrium, or *at most* one positive steady state separates the lowest asymptotically stable steady state from the highest asymptotically stable one, and, thus, the number of equilibria is four.

Moreover, from Eqs. (12.1) and (12.2) we observe that if $\Lambda_2 > 0$ and $\Lambda_3 > 0$ then no inflection points of $G(k)$ exist for any $k > 0$, $G''_{kk}(k) < 0$ and two steady states exist in that case. In contrast,

$G(k)$ has two inflection points for any $k > 0$ if at least either $\Lambda_2 < 0$ or $\Lambda_3 < 0$ is fulfilled and, hence, four steady states *can* exist in that case. **Q.E.D.**

Proposition 1 says that multiple development regimes are possible when longevity is endogenous and determined by an individual health measure augmented by public investments through the S-shaped longevity function Eq. (1).[15]

Note that the scenario $\pi_0 = 0$ and $\delta = 1$ resumes the case studied by Chakraborty (2004) in a model with exogenous fertility. The assumption of a positive natural rate of longevity, however, exposes the economy to a dramatic change: the zero equilibrium – which is a catching point when $\pi_0 = 0$ and the output elasticity of capital is high enough – becomes always unstable when $0 < \pi_0 < \pi_1$, and the number of steady states passes from three to four, thus making certainly more plausible the comparison of demo-economic performances between low and high income countries. In fact, although the existence of a stable zero equilibrium where the economy is collapsed to may be a useful abstraction to represent poorer economies, it certainly suffers from a sort of lack of realism, especially as regards the empirical relevance of the results.

Proposition 1 provides sufficient conditions to avoid development traps and necessary but not sufficient conditions to have multiple steady states, which depend – as can be ascertained from Eq. (12.2) – on the key parameters of the problem and the policy variable τ. However, as extensive numerical simulations revealed, the existence of multiple regimes of development crucially depends

[15] It is worth noting that Proposition 1 still holds if longevity depended on health capital according to $\pi(h_t) = \pi_0 + \frac{\pi_1 \Delta h_t^{\delta}}{1 + \Delta h_t^{\delta}}$, where $0 < \pi_0 < 1$, $0 < \pi_1 < 1$, $0 < \pi_0 + \pi_1 \le 1$, $\pi(0) = \pi_0 > 0$, $\lim_{h \to \infty} \pi(h) = \pi_0 + \pi_1 \le 1$ and $\pi'_h(h) = \frac{\pi_1 \delta \Delta h^{\delta - 1}}{\left(1 + \Delta h^{\delta}\right)^2} > 0$, which is another realistic way of modelling a positive rate of longevity regardless of public health spending. Moreover, note that values of b for which we have a unique steady state or multiplicity of steady states are indeed consistent with positive consumption.

on the mutual relationship between the output elasticity of capital (α) and the degree of effectiveness of public health investments on longevity (δ). In particular, for any given value of δ development traps are more likely to occur when production is relatively capital-oriented (high values of α). In particular, when $\delta = 1$ multiple steady states appear when α exceeds $1/2$,[16] and this threshold monotonically shrinks as δ raises. Therefore, when threshold effects of health capital on longevity exist (i.e. $\delta > 1$), a wider range of economies is prone to be characterised by development traps, since the output elasticity of capital that discriminates between a unique regime and multiple regimes of development is empirically more plausible and smaller than one half (as shown in the numerical example below).

Now, assume that economies exclusively differ as regards the initial condition k_0. Figure 3 depicts in a stylised way all the possible outcomes of an economy with endogenous longevity and endogenous fertility. The figure clearly shows that an economy that starts below the unstable equilibrium $\bar{k}_2$ is entrapped into the low regime ($\bar{k}_1$) where income per worker is low, fertility is high and mortality is high. In contrast, an economy that starts beyond the threshold $\bar{k}_2$ converges towards the high regime ($\bar{k}_3$) where income per worker is high, fertility is low and mortality is low. Therefore, as expected, an exogenous shift in k_0 may cause a change in the development regime.

[16] This result is in accord with Chakraborty (2004, Proposition 1, (i), p. 126). By passing we note that Bunzel and Qiao (2005) have shown that the second part of Proposition 1 (i) by Chakraborty "is incorrectly stated" (Bunzel and Qiao, 2005, p. 4), because $\alpha > 1/2$ represents a necessary but not sufficient condition for the existence of multiple steady states (a high value of the scale parameter A of course is required to avoid the existence of a unique, degenerate equilibrium in that case). However, it still remains true that when $\alpha > 1/2$ three steady states *can* exist, so that the central result by Chakraborty (2004) is kept unaltered even after the critique by Bunzel and Qiao (2005). It is important to note, however, that since $k = 0$ is a catching point in such a context, high values A can drastically reduce the basin of attraction towards the low stable equilibrium, but the possibility to eliminate the poverty trap does not exist whatever the level of technological development.

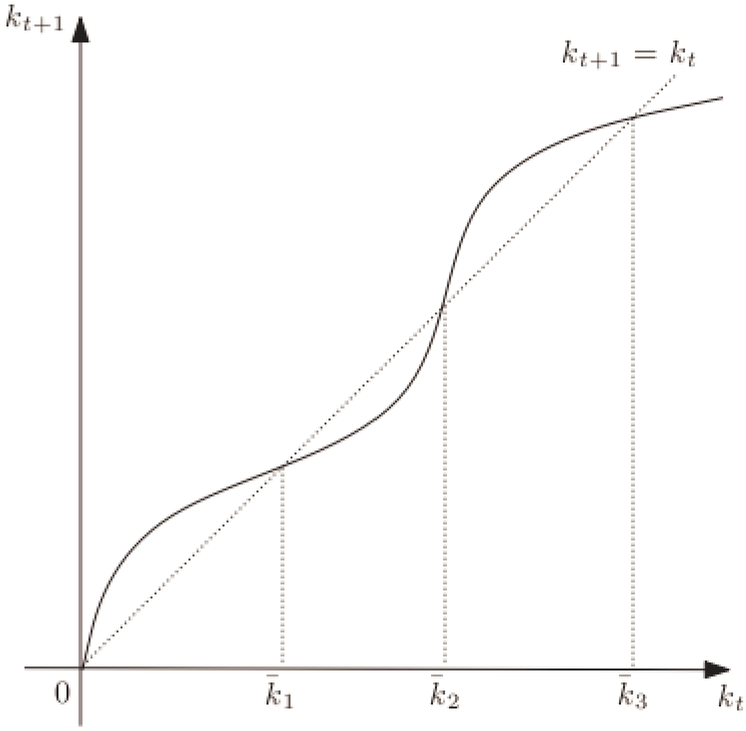

Figure 3. Multiple steady states.

A numerical experiment to illustrate Proposition 1 now follows. We take the parameter values (chosen only for illustrative purposes) $A = 10.5$, $\alpha = 0.33$,[17] $\gamma = 0.30$, $\pi_0 = 0.30$, $\pi_1 = 0.95$, $\delta = 10$, $\Delta = 1$, $q = 0.14$, $\tau = 0.10$ and $b = 0$. Therefore, the low regime is characterised by the equilibrium values $\bar{k}_1 = 1.105$, $\pi(\bar{k}_1) = 0.325$ and $n(\bar{k}_1) = 1.186$, and the high regime instead by $\bar{k}_3 = 4.324$, $\pi(\bar{k}_3) = 0.812$ and $n(\bar{k}_3) = 0.912$. The unstable equilibrium that discriminates between poor and rich countries is $\bar{k}_2 = 2.834$. Therefore, an economy that for some exogenous reasons starts with a stock of capital below (beyond) such a threshold level of development will end up in the low (high) regime, where income per worker is low (high) and mortality and fertility rates are high (low), confirming some of the most striking aspects of the so-called demographic transition.

But this is not the end of the story, however, since poverty or prosperity may not depend on initial conditions. A value added of this paper, in fact, grounds on the importance of determining the

[17] We used $\alpha = 0.33$ as it is usually assumed in literature (for estimates of α in several countries see, e.g., Jones, 2003).

role of the child policy variable b on both the transitional dynamics and long-run demo-economic outcomes of the economy. The next section deals with this argument and the main finding is that a large enough increase in the child tax is sufficient to eliminate the vicious cycle and ill-health of poverty, thus allowing those economies that were entrapped into poverty due to unfavourable initial conditions to end up in the high development regime.

4. Child policy

Child allowances in the form of direct monetary transfers entitled to families with children have often been suggested both by politicians and economists as a remedy against below-replacement fertility, and have been extensively used in several European countries, which are among the countries most plagued by sharp reductions in population growth rates in the last decades.[18] In contrast, the Chinese one-child per family programme was enforced, among other things, as a stimulus to economic growth because of the opposite problem of over-population, and maybe it represents the most interesting case of application of tax penalties on children in the world.[19]

It may be interesting, therefore, to study the effects of child taxes on demo-economic outcomes in a context of multiple steady states. Analysis of b from Eq. (10.2) gives the following proposition:

Proposition 2. *When the government invests in public health and development traps exist, a large enough increase in the child tax produces the loss of the lowest stable steady state,* $\bar{k}_1$, *thus*

[18] Policies consisting in cash subsidies for children are largely adopted in several countries. As an example, consider that in Italy a 1,000 euro child grant for each new born was introduced in the year 2005, while in Poland every woman will benefit from a one-off 258 euro payment for each child, and women from poorer families will receive double the previous amount.

[19] For instance, the one-child policy had the effect of reducing the Total Fertility Rate in China from more than five births per woman in the 1970s to slightly less than two births per woman in recent years.

allowing poorer economies to permanently escape from poverty and converge towards the highest equilibrium, $\bar{k}_3$.

Proof. Differentiation of Eq. (10.2) with respect to b gives:

$$G'_b(k) = \frac{k^{\alpha}\left(\pi_0 + \pi_1 B k^{\alpha\delta}\right)}{\gamma\left(1 + B k^{\alpha\delta}\right)} \cdot \frac{\partial D}{\partial b}, \tag{13}$$

where $\partial D / \partial b = (1-\alpha)A > 0$ for any $b \in [0,+\infty)$. Since $G'_b(k,b) > 0$ for any $k > 0$ and $b \in [0,+\infty)$, then for any $k^* > 0$ such that $G(k^*, b_1) < k^*$ with $b_1 \in [0,+\infty)$, there exists $b^* > b_1$ such that $G(k^*, b^*) = k^*$. Therefore, for any $b > b^*$, $G(k^*, b) > k^*$ holds. **Q.E.D.**

Proposition 2 shows that in a context of multiple steady states, a rise in the child tax increases the extreme stable steady states, reduces the intermediate unstable steady state, while shrinking the size of the basin of attraction of the poverty trap. Moreover, a large enough increase in the child tax shifts upward the phase map $G(k)$ in such a way to cause the loss of the lowest stable equilibrium, thus allowing those economies that were entrapped into poverty due to unfavourable initial conditions to end up in the high regime of development, where fertility is low and income per worker and longevity are high.

Given Proposition 2 the following result holds:

Result 1. *For any given value of the health tax rate* τ*, poverty or prosperity might not depend on initial conditions, while being the result of a child policy design.*

As discussed in Section 2, in fact, a rise in the child tax reduces fertility and increases savings due to a partial equilibrium effect. This causes a direct impetus to capital accumulation that shifts the phase map $G(k)$ upward for any $k > 0$, while also raising the stable steady states and thus the equilibrium wage rate earned by the young workers in both low and high income countries. Higher wages, however, translate into a higher health expenditure per worker – *ceteris paribus* as regards the value of the health tax τ – that produces a reduction in adult mortality at the steady state. The increased survival probability causes an indirect general equilibrium feedback effect that acts negatively on fertility and positively on savings, and thus properly works as a stimulus to accumulate capital further on. As a consequence, the equilibrium output per worker increases, while the steady-state adult mortality and fertility rates shrink in both regimes. A large enough increase in the child tax, however, is sufficient to create a stimulus to capital accumulation to dramatically eliminate the low equilibrium and allow those economies that were entrapped into poverty to prosper, irrespective of initial conditions.

We now illustrate Proposition 2 and Result 1 with the following numerical examples. We take the same parameter values as that used in Section 3 and look at the effects of the child policy variable b on both macroeconomic and demographic performances over the very long run. Starting from the case $b = 0$, where the world is divided into poor and rich countries, Table 1 shows how the main steady-state variables react following a rise in the child tax. As can be seen from Table 1, a slight increase in the child tax (from $b = 0$ to $b = 0.01$) represents an engine for capital accumulation, while also causing a relatively large reduction in adult mortality and fertility in both regimes of development. If we assume that childhood is 20 years of life and adulthood is divided, in turn, into a 30-year working time and a 30-year retirement time, a rise in the child tax from 0 to 0.01 raises the individual survival probability in poor countries that passes from 32.5 per cent of the whole time after the end of youth (i.e. individuals live about 9.75 years beyond the working time) to 34.8 per cent of the whole time after the end of youth (i.e. individuals live about 10.5 years beyond the working time), with an increase of almost 0.75 years of life. Adult mortality shrinks of course even in rich countries but its reduction in more sensible because the percentage increase in capital accumulation (and, hence, in the wage rate) is higher in that case. In particular, raising the child tax from 0 to 0.01 causes a sharp increase in the lifetime of people in rich countries that moves from

81.2 per cent of the whole time after the end of youth (i.e. individuals live about 24.3 years beyond the working time) to 87.3 per cent of the whole time after the end of youth (i.e. individuals live about 26.1 years beyond the working time), with a gain of almost 1.8 years of life.

Increasing further the child tax, however, sharply stimulates capital accumulation and produces the loss of lowest stable steady state. Therefore, the low regime of development vanishes and the economies that were entrapped into poverty due to unfavourable initial conditions will converge towards the unique high stable steady state (i.e. the phase map $G(k)$ lies everywhere above the 45° line and falls below it only once the high equilibrium $\bar{k}_3$ is achieved), with dramatic consequences for both macroeconomic and demographic outcomes. In fact, capital accumulation monotonically increases with the child tax and this raises, in turn, the equilibrium wage rate. As a consequence, for any given value of the health tax τ, the public health expenditure raises because of the increased child tax and the length of life of people increases approaching its saturating value π_1, while the fertility rate considerably shrinks.

Table 1. Child taxes and multiple development regimes.

Low regime

b	0	0.01
$\bar{k}_1$	1.105	1.358
$w(\bar{k}_1)$	7.272	7.782
$h(\bar{k}_1)$	0.727	0.778
$\pi(\bar{k}_1)$	0.325	0.348
$n(\bar{k}_1)$	1.186	1.104

High regime

b	0	0.01	0.05	0.10	0.15
$\bar{k}_3$	4.324	5.338	8.355	12.098	16.136
$w(\bar{k}_3)$	11.405	12.226	14.174	16.016	17.613
$h(\bar{k}_3)$	1.140	1.222	1.417	1.601	1.761
$\pi(\bar{k}_3)$	0.812	0.873	0.930	0.944	0.947
$n(\bar{k}_3)$	0.912	0.839	0.660	0.530	0.444

At this point, it is useful studying the effects of the opposite – and widely adopted – child allowance policy in a context of multiple steady states. We define the child allowance (financed by a wage income tax $0<|\theta_t|<1$) as $0<|b|<q$, i.e. the net cost of children should remain positive to guarantee the existence of a finite positive solution for n_t. As a consequence of the introduction of the child allowance scheme, a partial equilibrium effect exists that increases fertility and reduces savings. Capital accumulation, therefore, will be lower. This causes a reduction in the steady state stock of capital and, hence, in the wage rate. Therefore, for any given value of the health tax τ, the health expenditure per worker shrinks and then adult mortality raises in equilibrium. The reduced life span induces agents to have more children and this, in turn, decreases capital accumulation further on. Moreover, a large enough increase in the child subsidy can produce the loss of the high equilibrium (i.e. the phase map $G(k)$ lies everywhere below the 45° line once the low equilibrium $\bar{k}_1$ is achieved), and this in turn implies that, irrespective of initial conditions, all economies will end up in the low regime of development, where income per worker is low, adult mortality is high and fertility is high.

Table 2 shows the effects of child allowances on the main steady states variables in both low and high income countries. As regards the high regime of development, from Table 2 it can be seen that a slight increase in the child allowance (from 0 to 0.003) increases fertility and causes a sharp reduction in both capital accumulation and adult mortality (with a loss of almost 2 years of life). The high equilibrium vanishes as a consequence of a further increase in the child allowance and, thus, the economies converge towards the low regime where capital accumulation is low, fertility is high and the lifetime of people tends to the natural rate π_0.

Table 2. Child allowances and multiple development regimes.

High regime

$\lvert b \rvert < q$	0	-0.003
$\bar{k}_1$	4.324	3.752
$w(\bar{k}_1)$	11.405	10.884
$h(\bar{k}_1)$	1.140	1.088
$\pi(\bar{k}_1)$	0.812	0.754
$n(\bar{k}_1)$	0.912	0.955

Low regime

$\lvert b \rvert < q$	0	-0.003	-0.01	-0.05	-0.10
$\bar{k}_3$	1.105	1.052	0.943	0.510	0.150
$w(\bar{k}_3)$	7.272	7.153	6.902	5.636	3.767
$h(\bar{k}_3)$	0.727	0.715	0.690	0.563	0.376
$\pi(\bar{k}_3)$	0.325	0.322	0.315	0.302	0.300
$n(\bar{k}_3)$	1.186	1.210	1.267	1.696	2.872

5. Conclusions

We studied the effect of child policies in the basic overlapping generations model with endogenous fertility and endogenous longevity determined by public health investments (see Chakraborty, 2004). We assumed a sufficiently general form of the relationship between longevity and public health spending to include rather realistic features of the evolution of the length of life of agents.

We found that four (instead of the usual setup with three) steady states can exist, and this enriched set of long-run outcomes is possible under more plausible economic conditions with respect to those described by Chakraborty (2004). In particular, when threshold effects of health capital on longevity exist, development traps appear under less stringent (and more empirically relevant) conditions on the output elasticity of capital. Thus, depending on initial conditions an economy may be either entrapped into a low development regime, where income and life expectancy are low are fertility is high, or converge towards a high development regime where income and life expectancy are high and fertility is low.

However, the main message of this paper is that the limiting outcomes of the economy may not depend on initial conditions, while being the result of child policy design. In particular, regardless of whether an economy starts out with either a low or high stock of capital, a child tax programme can be adopted to escape permanently the vicious cycle of poverty and ill-health, since it properly works as a stimulus to accumulate capital, increase income per worker and life expectancy and reduce population growth.

This paper suggests that the child tax policy may have favoured the economic growth performances of an economy such as the Chinese one in the recent years. Conversely, our results may also constitute a warning as regards the effects of the more traditional child-subsidy policy: the price to be paid for stimulating the fertility recovery in developed countries may be not only the expected (in a neoclassical growth model) reduction in per capita income, but also the formation of development traps which may be attractive also for developing countries in the case of negative economic shocks.

Finally, other lines along which the present model can be extended are the following: (*i*) a private provision of health capital can be introduced and compared with the public health system; (*ii*) both young and old people can be entitled to public healthcare services; (*iii*) the effects on macroeconomic and demographic variables of other public policies such as pension and public education programmes can be evaluated in a context of multiple steady states; (*iv*) improvements in health care can have important economic effects beyond those engendered by greater life expectancy: for instance, lower morbidity and better functionality can raise both the productivity and wage of individuals (e.g. Strauss and Thomas, 1998) and thus may affect economic growth.

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