Optimal Monetary and Fiscal Policies In a Search-theoretic Model of Money and Unemployment

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Optimal Monetary and Fiscal Policies In a Search-theoretic Model of Money and Unemployment*

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Abstract

In this paper we study the optimal monetary and fiscal policies of a general equilibrium model of unemployment and money with search frictions both in labor and goods markets as in Berentsen, Menzio and Wright (2010). We abstract from revenue-raising motives to focus on the welfare-enhancing properties of optimal policies. We show that some of the inefficiencies in the Berentsen, Menzio and Wright (2010) framework can be restored with appropriate fiscal policies. In particular, when lump sum monetary transfers are possible, a production subsidy financed by money printing can increase output in the decentralized market and a vacancy subsidy financed by a dividend tax even when the Hosios' rule does not hold.

JEL Classification: E52, E63.
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1 Introduction

The relationship between inflation and unemployment has been extensively analyzed over the past fifty years. One of the most robust monetary features of post-war U.S. data is its positive

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correlation. Many economists view this as an important tool for the conduct of monetary policy. Understanding the origins of the correlations associated with the Phillips curve is a crucial first step to understand the implications of that relationship when designing optimal policies. One thus needs to specify the economic environment where agents make choices regarding their labor supply and savings decisions; linking labor and money markets.

Given that agents interacting in frictionless Walrasian markets do not require money for their transactions and are always employed, models with frictions are key in understanding the relationships between unemployment and inflation.[1] Despite the growing use of search theoretic models of unemployment to study the Phillips curve, little is known about the nature of optimal fiscal and monetary policies in these class of models. An exception is that of Cooley and Quadrini (2004) who study optimal monetary policy in a model that integrates the modern theory of unemployment with firms facing cash-in-advance constraints to purchase the intermediate input. These authors show that when the economy is subject to productivity shocks, the optimal policy is pro-cyclical and with commitment the optimal inflation rate is inversely related to the bargaining power of workers. Within the search theoretic models models of money, Lehmann's (2006) shows that the optimal monetary growth rate decreases with the workers' bargaining power, the level of unemployment benefits and the payroll tax rate. Finally, Berentsen, Rocheteau and Shi (2007) studies optimal monetary policy in an economy with endogenous search decisions. They show that the same frictions that give fiat money a positive value generate an inefficient quantity of goods in each trade and an inefficient number of trades. The Friedman rule eliminates the first inefficiency and the Hosios rule the second. A monetary equilibrium attains the social optimum if and only if both rules are satisfied. When they cannot be satisfied simultaneously, optimal monetary policy achieves only the second best.

In this paper we follow the tradition of search theoretic models of money and unemployment and study the design of optimal monetary and fiscal policies. The underlying environment is based on search frictions in labor and goods markets as in Berentsen, Menzio and Wright (2010). We abstract from revenue-raising motives to focus on the welfare-enhancing properties of optimal policy. Our analysis considers alternative fiscal instruments to study the following issues. First, can fiscal and monetary policy restore efficiency of equilibria? Is the

[1] For instance, within a fully flexible cash in advance framework, Cooley and Quadrini (1999) integrate a model of equilibrium unemployment. When price rigidities are considered, Walsh (2003), among others, studies the relationship between inflation and unemployment. When search theoretic models of money are considered, Lehmann (2006), Kumar (2008) and Berentsen, Menzio and Wright (2010), among others explore the relationship between inflation and unemployment. None of these papers examine the corresponding optimal fiscal and monetary policies.

Friedman rule an optimal policy once unemployment exists and fiscal policy is available?

The environment we consider has three sources of inefficiency. First, in monetary exchanges agents pay a cost today (production) to receive a future benefit (money that can be used to purchase goods in future trades); i.e., the intertemporal distortion present in most monetary models. The second one is a direct consequence of the properties of Nash's solution to the bargaining problem when agents trade goods for money. Finally, the third source of inefficiency derives from the matching frictions in the labor market.[2] The existence of these inefficiencies motivates our work in designing fiscal and monetary policies that can attenuate them. The Friedman rule eliminates the intertemporal one and attenuates the impact of bargaining inefficiencies. However, equilibrium allocations are not efficient without active fiscal policies. Hence, we propose different fiscal and monetary policies that can restore efficiency of monetary equilibrium.

In this paper, by establishing a search-theoretical model of money and unemployment, we find the following policy implications:

- Production subsidies, paid in money, can be used to increase production in the decentralized market but they may be inflationary. If costless lump-sum monetary transfers are available, these can be used to extract the money introduced through the subsidy and thus inflation can be easily contained. In this environment, we find multiple combinations of taxes, subsidies, and (sometimes strictly positive) inflation rates such that efficiency is attained.

- The optimal fiscal and monetary policy are used to resort efficiency in both labor market and goods market. A simple combination of production subsidy s_q^* and inflation rate π^* can help to achieve efficiency in good market. When both Hosios' rule and a similar "contribution-compensating" rule are hold, this simple policy combination implements efficiency in both markets. However, s_q^* can deteriorate the inefficiency in labor market. In this case, other fiscal policies are required.

- If government wants a larger vacancy-unemployment ratio $\theta = \frac{v}{u}$, the government can either raise firms' incentive to recruit, or lower unemployment workers' welfare, or do both. When the vacancy-unemployment ratio is higher than efficient level, government may want to tax firms or subsidize the unemployed.

The findings of this paper confirm the observation by Kocherlakota (2005) and Wright (2005) that fiscal and monetary policies may have important interactions, particularly in frameworks

[2]As shown in Hosios (1990), if the worker's share of the matching surplus is too small, there will be an excessive creation of vacancies due to the high profitability of a match for the firm.

with microeconomic foundations for the existence of fiat money, and should always be jointly considered in the design of optimal government policy.

2 Model

Time is discrete and continues forever. There are two types of agents, firms and households, indexed by f and h. The set of households denoted by h belong to the [0,1] interval, the set of firms denoted by f is arbitrarily large, but not all are active at any point in time. Households work, consume, and enjoy utility while firms maximize profits and pay dividends.

In each period, there are three distinct markets where economic activity takes place: a labor market in the spirit of Mortensen-Pissarides, a specialized goods market in the spirit of Kiyotaki-Wright, and a general market in the spirit of Arrow-Debreu. We refer these submarkets as LM, DM and CM, respectively.

The timing of the environment is such that LM convenes first, then DM, and finally CM. In the first subperiod agents enter the labor market with different employment status. Note that in the LM market, h and f can match bilaterally to create a job. To denote the outcome of that bilateral meeting we have an index, e, that represents the employment status: $e = 1$ if an agent is matched and $e = 0$ otherwise. Employed workers are matched with firms to produce output y, while unemployed workers do not produce. After the labor market takes place, both types of workers (employed and unemployed) enter into the next submarket, DM, where they purchase the goods that have been just produced. In the final subperiod, employed (unemployed) workers receive their wage payments (unemployment benefits), as well as dividend income from firms. Through exchanges in a Walrasian market, workers acquire consumption of a general good and rebalance their currency to carry into the next period.

We define a value function for LM, DM and CM submarkets as $U_e^j(z)$, $V_e^j(z)$, $W_e^j(z)$ respectively, which depend on type $j \in \{h, f\}$, employment status $e \in \{0, 1\}$ and real money balances $z \in [0, \infty)$, where $z = m/p$ and p is the current price level. As in Berentsen, Menzio and Wright (2010) we adopt the following convention for measuring real balances. When an agent brings in m dollars to the CM, the agent then takes $\hat{z} = \hat{m}/p$ out of that market and into the next period. In the next CM market the price level is $\hat{p}$, so the real value of the money is $\hat{z}\hat{\rho}$, where $\hat{\rho} = p/\hat{p}$ converts $\hat{z}$ into units of the *numeraire* general consumption good X in that market.

In addition to firms and households, there exists a government who conducts monetary and fiscal policies. In the CM, the government collects labor income taxes and dividend taxes to finance vacancy subsidies and unemployment benefits. The government also conducts mone-

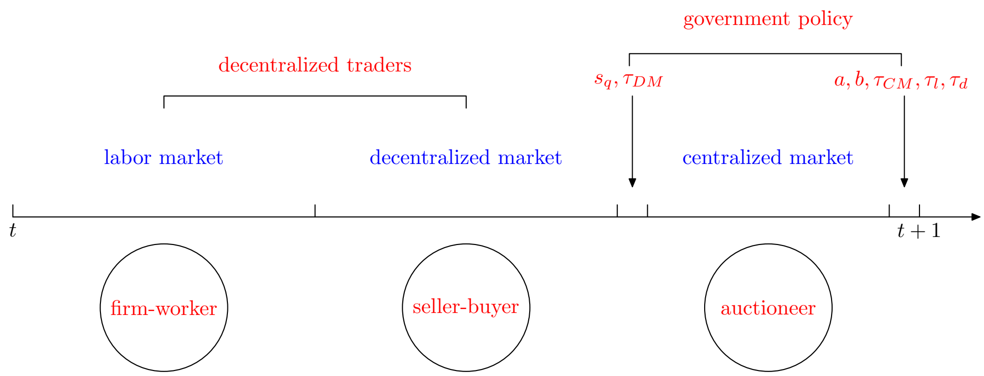

Figure 1: Timing.

tary policy by printing money at different times of the day. In particular, at the end of the DM, the government provides a monetary production subsidy. On the other hand, at the end of CM, new money is injected through a lump-sum transfer (or tax). See Figure 1 for the exact timing.

2.1 Household's Problem

We now examine households' value functions in each of the subperiods, starting with the final one. A household that enters the CM with employment status, e, and real money holdings, z, chooses consumption and real money holdings for next period, $\hat{z}$, to solve the following problem:

$$W_e^h(z) = \max_{X,\hat{z}}\{X + (1-e)l + \beta U_e^h(\hat{z})\}$$
$$\text{s.t.} \quad X = ew(1-\tau_l) + (1-e)b + \triangle(1-\tau_d) + z - \hat{z} + \tau_z;$$

where X is the general CM consumption good, l represents leisure, $\triangle$ denotes dividend income, b is the unemployment benefit, τ_l denotes the labor income tax rate, τ_d represents the dividend income tax rate, and τ_z is the real lump-sum real monetary transfers.

Substituting X into the objective function we obtain the following simplified problem:

$$W_e^h(z) = \max_{\hat{z}}\{ew(1-\tau_l) + (1-e)(l+b) + \triangle(1-\tau_d) + z - \hat{z} + \tau_z + \beta U_e^h(\hat{z})\}. \tag{1}$$

As in Lagos and Wright (2005), with quasi-linear preferences the optimal choice of future real balances, $\hat{z}$, is independent of the current ones, z. It depends, however, on the workers' employment status e. To determine this link let us now explore the problem of a representative household in the DM which is given by:

$$V_e^h(z) = \alpha_h v(q) + \alpha_h W_e^h(\rho(z-d)) + (1-\alpha_h) W_e^h(\rho z) \tag{2}$$

where $v(q)$ denotes the utility that a buyer obtains when consumes q units of the specialized DM good, $\rho = p/\hat{p}$ and α_h is the probability of trading. The probability of trade is given by a matching function $\alpha_h = \mathscr{M}(B,S)/B$, where B and S are the measures of buyers and sellers in the market. Assuming $\mathscr{M}(B,S)$ satisfies the constant returns scale property, $\alpha_h = \mathscr{M}(Q,1)/Q$, where $Q = B/S$ is the queue length or market tightness. All households participate in the DM market, so $B = 1$; only firms with $e = 1$ participate, so $S = 1 - u$ where u is unemployment. Thus, $\alpha_h = \mathscr{M}(1, 1-u)$.

Now exploiting the linearity of $W_e^h(z)$, we can rewrite the DM value function as follows:

$$V_e^h(z) = \alpha_h[v(q) - \rho d] + W_e^h(0) + \rho z. \tag{3}$$

To close the household problem we need to specify the choices of the representative household in the LM which are characterized by:

$$U_{e=1}^h(z) = V_1^h(z) + \delta[V_0^h(z) - V_1^h(z)] \tag{4}$$

and

$$U_{e=0}^h(z) = V_0^h(z) + \lambda_h[V_1^h(z) - V_0^h(z)];$$

where δ denotes the exogenous rate at which matches are destroyed and λ_h is the endogenous rate at which they are created. The latter is determined by another standard matching function, $\lambda_h = \mathscr{N}(u,v)/u$, where u is unemployment and v is the number of vacancies posted by firms.

Implicit in this set up is that wages are determined when f and h meet in the LM, although they are paid in the next CM market. Moreover, wages can be renegotiated each period.

Now that all the value functions for the different submarkets have been specified, the problem of the representative household can be rewritten as follows:

$$W_e^h(z) = I_e + z + \max_{\hat{z}}\{-\hat{z} + \beta\alpha_h[v(q) - \rho d] + \beta\rho\hat{z}\} + \beta\mathbb{E}W_e^h(0) \tag{5}$$

where $I_e = ew(1-\tau_l) + (1-e)(l+b) + \triangle(1-\tau_d) + \tau_z$ and $\mathbb{E}W_e^h(0)$ is the expectation with respect to next period's employment status.

2.2 Firm's Problem

Firms carry no real balance out of CM. The problem of a firm in the LM is given by

$$U_{e=1}^f(z) = V_1^f(z) + \delta[V_0^f(z) - V_1^f(z)] \tag{6}$$

and

$$U_{e=0}^f(z) = V_0^f(z) + \lambda_f[V_1^f(z) - V_0^f(z)];$$

where λ_f is the endogenous rate at which matches are created which is given by $\lambda_f = \mathcal{N}(u,v)/v$. By constant returns, $\lambda_f = \mathcal{N}(u/v,1)$, where v/u is labor market tightness.

In the first subperiod, firms with $e = 0$ have no production activities. If a firm enters the LM with a job match, it can produce y units of output in the LM which can then be sold in the DM, with a probability $\alpha_f = \alpha_h Q$ with $Q = B/S$. The problem of the firm in the DM is given by:

$$V_1^f(z) = \alpha_f W_1^f(y - c(q), \rho d(1+s_q)) + (1-\alpha_f) W_1^f(y,0) \tag{7}$$

where s_q is the monetary subsidy to production in DM and $c(q)$ is opportunity cost of a sale. As in Berentsen, Menzio and Wright (2010) we assume that $c(q)$ units of CM good are transferred into q units of DM good so that $y - c(q)$ units of general good will be carried to the next CM market. Thus one could interpret x and q as the same physical good that a firm can store across markets, bearing in mind that consumers generally value it differently in the two markets.

The CM value of a firm with inventory ,x, real money balances, z, and wage commitment, w, is given by:

$$W_1^f(x,z) = x + z - w + \beta U_1^f. \tag{8}$$

Thus the DM value of a firm is given by:

$$V_1^f(z) = R - w + \beta[\delta V_0^f + (1-\delta) V_1^f] \tag{9}$$

where $R = y + \alpha_f[(1+s_q)\rho d - c(q)]$ is the expected revenue.

To model entry, any firm with $e = 0$ can pay k, in units of X, in the CM to enter the next labor market. Thus we have that:

$$W_0^f = \max\{0, -k + a + \beta\lambda_f V_1^f + \beta(1-\lambda_f) V_0^f\} \tag{10}$$

where a denotes the vacancy subsidy that the government gives to firms. The free entry condition then implies that $k - a = \beta\lambda_f V_1^f$. Now taking into account equation (9), the entry condition becomes:

$$k - a = \frac{\beta\lambda_f(R-w)}{1-\beta(1-\delta)}.$$

The resulting profits over all firms are then given by:

$$\widetilde{\Pi} = (1-u)(R-w) - vk + va.$$

Since households own shares of all firms, profits define the dividend payment: $\widetilde{\Pi} = \triangle$.

2.3 The Government

In this environment, the government's available monetary and fiscal policy tools are used to attenuate the frictions in the economic environment. The government pays unemployment benefits, b, and vacancy subsidies, a, which are financed through labor taxes, τ_l, and dividend taxes, τ_d, in the CM market. Thus the fiscal budget constraint is given by:

$$va + ub = \tau_d \,\Delta + \alpha_f \tau_l w$$

which holds at every date.

Following Gomis-Porqueras and Peralta-Alva (2010), the monetary authority injects money at different times within a period to finance a production subsidy, s_q, in the DM as well as to affect the money supply at the end of CM. Thus the corresponding monetary budget constrains is given by

$$\alpha_f s_q q = \tau_{DM} z$$

$$(1+\tau_{DM})(1+\tau_{CM}) = 1+\pi$$

where τ_{DM} (τ_{DM}) correspond to the DM (CM) money growth rate and π equals the inflation rate in steady state.

The monetary production subsidy we consider can be mapped in real-life economies to policies that prescribe paying interest to some money holders, in our case firms. This type of policy prescription has a long tradition in monetary economics and has been advocated by Tolley (1957) and Friedman (1959), among others. Regarding the feasibility and implementability of this policy, Feinman (1993) notes that the Federal Reserve has explicitly supported legislation authorizing the payment of interest on reserves since the 1970s. Our paper then re-examines an established monetary policy prescription in the new search models of money.

3 Pricing Mechanisms

To characterize the terms of trade, we need to specify the trading protocols in each of the three markets. Following the standard specifications in the literature, we consider price taking in the CM and Nash bilateral bargaining in LM and DM submarkets. Although one can easily modify the model to allow other pricing options. The reason for this choice is that we can use seller's bargaining power as a proxy for the degree of competition in the goods market, and examine how different market structures affect the implied inflation-unemployment relation.

3.1 Nash Bargaining in DM

Suppose that when a worker and firm meet in the DM, they bargain over the terms of trade (q,d), subject to the worker's cash constraint $d \leq z$, where d is the amount of real money balances the buyer gives to the seller in exchange for q. Let ϕ denote the seller's bargaining power. The trading protocol in DM is the generalized Nash bargaining which is defined as follows:

$$\max_{q,d}[v(q)-\rho d]^{\phi}[\rho d(1+s_q)-c(q)]^{1-\phi} \text{ s.t. } \quad d \leq z \quad \text{and} \quad c(q) \leq y.$$

The Nash bargaining solution is given a pair (q,d) that satisfies the following conditions:

$$d = z \quad \text{and} \quad q = g^{-1}(\rho z)$$

where $g(\cdot)$ is defined as follows:

$$g(q) = \frac{\phi c(q) v'(q) + (1-\phi) v(q) c'(q)}{\phi(1+s_q) v'(q) + (1-\phi) c'(q)}.$$

Given this bargaining outcome, we can rewrite the choice of $\hat{z}$ by a household in CM as follows:

$$\max_{\hat{z}\geq 0}\{-\hat{z} + \beta\alpha_h v(g^{-1}(\rho\hat{z})) + \beta(1-\alpha_h)\rho\hat{z}\}$$

and the solution to this latter problem is given by:

$$\frac{1}{\beta\rho} = \alpha_h \frac{v'(q)}{g'(q)} + (1-\alpha_h)$$

which can be rewritten using Fisher equation, $1/\beta = (1+i)\rho$, as follows:

$$\frac{i}{\mathscr{M}(1,1-u)} = \frac{v'(q)}{g'(q)} - 1; \tag{11}$$

where i is the nominal interest rate. Equation (11) will be referred to as the LW curve. This curve represents the (q,u) pairs that are consistent with the household optimization problem. This curve determines the DM output, q, as in Lagos and Wight(2005), except in their model α_h was fixed and now $\alpha_h = \mathscr{M}(1,1-u)$. Moreover, in this environment the production subsidy, s_q also alters the incentives to produce in DM. The properties of the LW curve follow the well-known result that simple conditions guarantee $\frac{v'(q)}{g'(q)}$ is monotone, so there is a unique q solve the previous condition with $\frac{\partial q}{\partial u} < 0$.

Proposition 1 *Let q^* solve $v'(q^*) = c'(q^*)$. For all $i > 0$ the LW curve slopes downward in (u,q) space, with $u = 0$ and $s_q = 0$ implying $q \in (0,q^*)$ and $u = 1$ implying $q = 0$. As $i \to 0$, $q \to q_0$ for all $u < 1$, where q_0 is independent of u.*

We refer the reader to the appendix for the proofs of all our results.

An increase in u affects q because higher unemployment makes it less attractive to be a buyer by adversely affecting the probability and/or terms of trade. The different subsidies considered in this paper change the incentives to produce more and possibly increase welfare.

According to Fisher equation $1/\beta = (1+i)\rho$, LW curve moves upward when inflation rate $\pi = 1/\rho - 1$ decreases. This effect is intuitive: a high i increases the cost of holding money, and thus lowers the trading quantity which is constrained by money holding of buyers. When the subsidy to production s_q increases, firm has larger incentive to recruit in the labor market and this helps to reduce the unemployment.

Lemma 1 *LW curve shifts upward, i.e., q is increasing for any given u, when monetary subsidy to production s_q increases or inflation rate π decreases.*

3.2 Nash Bargaining in LM

In the labor market, when a firm with a vacancy meets with an unemployed worker they bargain over wage, w, with threat points given by their continuation values. Recall that the difference in value functions between different employment status for a household are given by:

$$\begin{aligned} U^h_{e=1} - U^h_{e=0} &= (1-\delta-\lambda_h)(V^h_1(z) - V^h_0(z)) \\ &= (1-\delta-\lambda_h)(W^h_1(z) - W^h_0(z)) \\ &= (1-\delta-\lambda_h)(w(1-\tau_l) - b - l + \beta(1-\delta-\lambda_h)\hat{S}^h); \end{aligned}$$

where $\hat{S}^h = \frac{w-(b+l)}{1-\beta(1-\delta-\lambda_h)}$ represents the steady state difference of the household's continuation values. From equation (6), we have that the difference in value functions between different employment status for the firm is given by:

$$\begin{aligned} U^f_{e=1} - U^f_{e=0} &= (1-\delta-\lambda_f)(V^f_1 - V^f_0) \\ &= (1-\delta-\lambda_f)(R - w + \beta(1-\delta)\hat{S}^f); \end{aligned}$$

where $\hat{S}^f = \frac{R-w}{1-\beta(1-\delta)}$ is the steady state difference of the firm's continuation value and the firm's revenues, R, which are given by:

$$R = y + \alpha_f[(1+s_q)\rho z - c(q)].$$

Having specified the continuation values for the firm and the household, the Nash bargaining problem with firm's bargaining power η can be written as follows:

$$\max_w \left[w(1-\tau_l) - b - l + \beta(1-\delta-\lambda_h)\hat{S}^h\right]^{1-\eta} \left[R - w + \beta(1-\delta)\hat{S}^f\right]^{\eta}.$$

The solution to this problem is given by:

$$w = \frac{\eta[1-\beta(1-\delta)](\frac{b+l}{1-\tau_l}) + (1-\eta)[1-\beta(1-\delta-\lambda_h)]R}{1-\beta(1-\delta)+(1-\eta)\beta\lambda_h}.$$

Knowing the household wage in the LM, w, as well as the firm's revenues, R, we can substitute these expressions and determine the corresponding free entry condition, which is given by:

$$k-a = \frac{\lambda_f \eta \left[y - (\frac{b+l}{1-\tau_l}) + \alpha_f[(1+s_q)\rho d - c(q)]\right]}{(r+\delta)+(1-\eta)\lambda_h};$$

where r is the real interest rate.

To simplify things, let us use the Beveridge curve which describes the relationship between unemployment and the job vacancy rate. In steady state we have that $(1-u)\delta = \mathcal{N}(u,v)$ which implicitly defines $v = v(u)$ so we can rewrite $\alpha_f = \frac{\mathcal{M}(1,1-u)}{1-u}$ $\lambda_f = \frac{\mathcal{N}(u,v(u))}{v(u)}$, $\lambda_h = \frac{\mathcal{N}(u,v(u))}{u}$. Recalling that $\rho d = g(q)$, the free entry condition can be written as follows

$$k-a = \frac{\frac{\mathcal{N}(u,v(u))}{v(u)}\eta\{y - (\frac{b+l}{1-\tau_l}) + \frac{\mathcal{M}(1,1-u)}{1-u}[(1+s_q)g(q) - c(q)]\}}{(r+\delta)+(1-\eta)\frac{\mathcal{N}(u,v(u))}{u}}; \quad (12)$$

which we refer from now on as the MP curve. The MP curve, given by equation (12), relates the firm's optimal (u,q) which depends on different tax instruments, namely, the production and vacancy subsidies and the tax rate on labor income.

Proposition 2 *The MP curve slopes downward in (u,q) space. It passes through $(1,\underline{q})$, where $\underline{q} > 0$, if $(k-a)(r+\delta) > \eta(y - \frac{b+l}{1-\tau_l})$, and it passes through $(\bar{u},0)$, where $\bar{u} > 0$, if $(k-a)(r+\delta) \leq \eta(y - \frac{b+l}{1-\tau_l})$.*

MP curve determines unemployment rate u as in Mortensen and Pissarides (1994), except the total surplus (the term in braces) includes not just $y - \frac{b+l}{1-\tau_l}$ but also the expected gain from trade in decentralized market. Different from Berentsen, Menzio and Wright(2010), the labor market equilibrium can be driven by multiple fiscal policies. The following lemma summarizes the effects of fiscal policy instruments on MP curve.

Lemma 2 *MP curve moves upward when labor income tax rate τ_l or unemployment benefit b increases. MP curve moves downward when monetary subsidy to production s_q or vacancy subsidy a increases.*

Intuitively, there are three effects from an increase in u all of which encourage entry: (i) it is easier for firms to hire; (ii) it is harder for households to get hired, which lowers w; (iii) it is easier for firms to compete in the DM market.

In contrast to the LW curve, labor income taxes and vacancy subsidy affect the firms' optimal (u,q) which translate in shifts in the MP curve.

4 Monetary Equilibrium

We break the analysis of equilibrium into three parts. First, following Lagos-Wright (2005), we determine the value of DM production measured by q, taking unemployment u as given, the LW curve. We then determine u, taking q as given, as in Mortensen-Pissarides (1996) which yields the MP curve previously derived. It is convenient to depict these two relationships graphically in (u,q) space with the LW and MP curves, see Figure 2. Their intersection determines the equilibrium unemployment rate and value of money (u,q), from which all of the other endogenous variables easily follow.

Proposition 3 *Steady state equilibrium exists. If $(k-a)(r+\delta) \geq \eta(y - \frac{b+l}{1-\tau_l})$, there is a non-monetary steady state at (1,0) and monetary steady state may also exist. If $(k-a)(r+\delta) < \eta(y - \frac{b+l}{1-\tau_l})$, there is a non-monetary steady state at $(\bar{u},0)$, where $\hat{u} \in (0,1)$, is the intersection between MP curve and horizontal axis, and at least one monetary steady state. If the monetary steady state is unique, a rise in i or b decreases q and increases u in equilibrium, while a rise in τ_l or a increases q and decreases u in equilibrium.*

As in Berentsen, Menzio and Wright (2010), there may be multiple equilibria. The LW and MP curves both slope downward in the (u,q) space. LW curve enters (u,q) space from the left at $(0,q^0)$ and exits from the right at $(1,0)$. When the expected cost of creating a vacancy is greater than its benefit, i.e., $(k-a)(r+\delta) \geq \eta(y - \frac{b+l}{1-\tau_l})$, MP curve enters (u,q) space from the top at $(\underline{u},q^0)$ and exits from the right at $(1,q_1)$. In this case, there exists a non-monetary equilibrium at $(1,0)$ and, depending on parameter values, monetary equilibria may also exist(see the curves labeled MP_2 and MP_3). If $(k-a)(r+\delta) < \eta(y - \frac{b+l}{1-\tau_l})$, MP curve enters from the top at $(\underline{u},q^0)$ and exits from the bottom at $(\bar{u},0)$(see the curve labeled MP_1). In this case, a non-monetary equilibrium exists at $(\bar{u},0)$[3] as well as at least one monetary equilibrium.

Aruoba, Rocheteau and Waller (2007) emphasize that in the Lagos and Wright (2005) frameworks, Nash bargaining generates non-monotonic surpluses for buyers which lead to inefficient

[3]In the first type of non-monetary equilibrium, as $(1,0)$, both DM and LM markets shut down, the equilibrium degenerates to a standard Walrasian equilibrium with home production. In the second type of non-monetary equilibrium, as $(\bar{u},0)$, only DM market shuts down and some workers are still employed in the LM market to produce general goods. This is similar to an equilibrium in the standard Mortensen-Pissarides model.

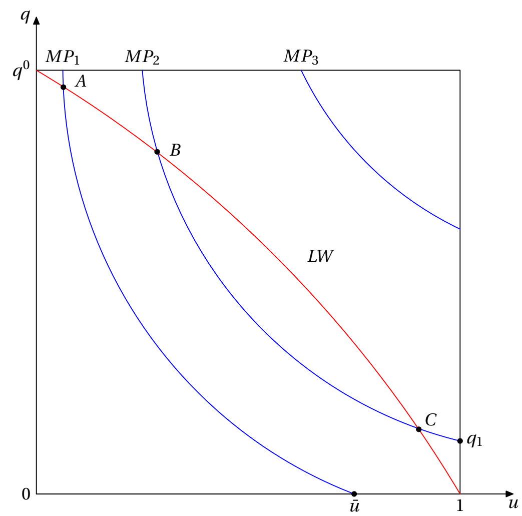

Figure 2: Monetary equilibrium.

DM production. Moreover, from Mortensen and Pissarides (1994) if the Hoisos condition is not satisfied, the economic environment will have inefficient outcomes. These two properties are inherited by our environment, thus leaving a role for monetary and fiscal policies. To determine the set of policies capable of implementing efficiency, we first characterize the efficient allocation in the next section.

5 Efficient Allocation

The social planner's problem can be summarized as follows:

$$J(u) = \max_{q,v}\{(1-u)y + \alpha_h[v(q) - c(q)] + u\,l - v\,k + \beta J(\hat{u})\}$$
$$\text{s.t. } \hat{u} = u + (1-u)\delta - \mathcal{N}(u,v)$$

Notice that the choice of q does not depend on state variable u. The first-order condition with respect to q is simply $v'(q) = c'(q)$. The first-order condition with respect to v is $-k - \beta J'(\hat{u})\mathcal{N}_2(u,v)$ since current vacancies will affect unemployment rate in the next period. This fact together with the law of motion $\hat{u} = u + (1-u)\delta - \mathcal{N}(u,v)$ generates a decision rule for v as a function of u, determining the optimal combination of (u,v).

Using the envelope theorem, we have the following:

$$J'(u) = l - y - \mathcal{M}_2(1, 1-u) + \beta J'(\hat{u})[1 - \delta - \mathcal{N}_1(u,v)].$$

Substituting this last expression into the first-order condition with respect to v, we have the

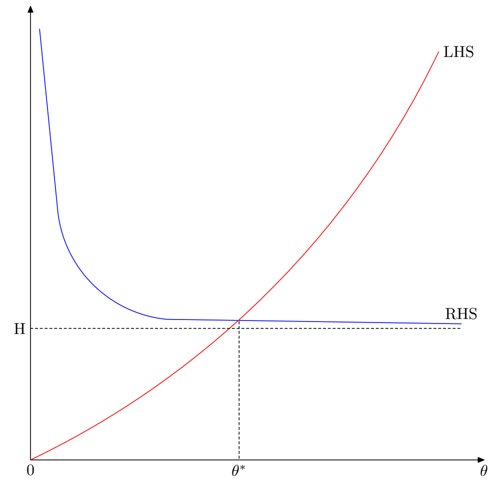

Figure 3: Uniqueness of efficient allocation.

following social planner's Euler equation:

$$\frac{k}{\beta\mathcal{N}_2(u,v)} = y - l + \mathcal{M}_2(1, 1-\hat{u})[v(\hat{q}) - c(\hat{q})] + \frac{k[1-\delta-\mathcal{N}_2(\hat{u},\hat{v})]}{\mathcal{N}_1(\hat{u},\hat{v})}.$$

In steady state, the planner's solution generates an optimal unemployment u^* that satisfies the following relationship:

$$k = \frac{\mathcal{N}_2(u^*, v(u^*))\left[y - l + \mathcal{M}_2(1, 1-u^*)[v(q^*) - c(q^*)]\right]}{r + \delta + \mathcal{N}_1(u^*, v(u^*))}$$

Proposition 4 *When the matching technologies in the LW and DM are given by $\mathcal{N}(u,v) = u^\sigma v^{1-\sigma}$ and $\mathcal{M}(B,S) = B^\gamma S^{1-\gamma}$, respectively, then there exists a unique social optimal plan given by $\theta^* \equiv \frac{v^*}{u^*}$ and q^*.*

Figure 3 provides a graphical summary of Proposition 4. Now that the first best is characterized we can determine if fiscal and monetary instruments can implement such an allocation.

6 Optimal Monetary and Fiscal Policies

The government's problem consists of choosing inflation, unemployment benefits, tax and subsidy rates that maximize social welfare subject to the constraint that production and consumption in all markets are stationary monetary equilibria. Thus, we are contemplating an environment in which the government sets an inflationary and fiscal plan that will not change over time. In the next subsection we explore if there exists a set of policies that can replicate the social planer's allocation (u^*, v^*, q^*).

6.1 Benchmark

As a baseline situation, let us consider an economy where the government does not provide unemployment insurance, b, nor vacancy subsidy, a, then there is no need to have taxes on labor, $\tau_l = 0$, nor taxes on dividend income $\tau_d = 0$. Let us further assume that the government sets production subsidy $s_q = s_q^*$, the subsidy rate that can achieve efficient trade quantity q^* in DM for any given matching probability α_h. The next lemma ensures that it is possible to do so.

Lemma 3 *Consider any given value of the buyer's bargaining power,* $0 < \phi \leq 1$*, and any given inflation rate,* $\pi \geq \beta - 1$*. Then, there exists value* s_q^* *that achieves the first-best allocation in DM market.*

Lemma 3 indicates that, by using proper production subsidy rate s_q^*, efficiency in decentralized market can always be achieved for any unemployment level u. The extra revenue from subsidies to trade encourages sellers to be more generous in the DM bargaining and produce up to the efficient level. However, subsidies in DM may also create problem in both high inflation and inefficiency in LM. On the one hand, given that the government has access to lump-sum monetary taxes, government can always undo the inflation that results from these subsidies. On the other hand, the efficiency in LM creates the role for other fiscal policy instruments.

Following Gomis-Porqueras and Peralta-Alva (2010), the monetary authority injects money at different times within a period as to pay a production subsidy in the DM as well as to affect the money supply at the end of CM. Thus the corresponding budgets constrains are then:

$$\alpha_f^* s_q^* q^* = \tau_{DM} z^*$$

$$(1+\tau_{DM})(1+\tau_{CM}) = 1+\pi$$

Observe that the availability of lump sum monetary transfers at the end of the centralized market can neutralize any increase of the money supply from the payment of monetary subsidies at the centralized market (where we measure inflation). Thus we can deviate from the Friedman rule.

The resulting labor market equilibrium, MP curve, can be written as follows:

$$[(r+\delta)\theta^\sigma + (1-\eta)\theta]k = \eta\{y - l + (1+\delta\theta^{\sigma-1})^\gamma[(1+s_q^*)g(q^*) - c(q^*)]\}.$$

Rearranging the social planner's solution and the equilibrium MP curve, respectively; we have the following relationships:

$$[(r+\delta)\theta^\sigma + \sigma\theta]\frac{k}{(1-\sigma)} = y - l + (1+\delta\theta^{\sigma-1})^\gamma(1-\gamma)[v(q^*) - c(q^*)] \tag{EF}$$

$$[(r+\delta)\theta^\sigma + (1-\eta)\theta]\frac{k}{\eta} = y - l + (1+\delta\theta^{\sigma-1})^\gamma[(1+s_q^*)g(q^*) - c(q^*)]. \tag{MP}$$

In this search environment, there exist efficiency issues in both LM and DM markets. While the government uses subsidy s_q^* to correct the trade inefficiency in DM, the efficient unemployment rate θ^* is not guaranteed if there is no further policy interference. We use θ^{MP} to denote the equilibrium market tightness when $s_q = s_q^*$ has been chosen. We summarize the relative value of θ^{MP} when compared to θ^* in the following proposition.

Proposition 5 *When the trade efficiency is attained by setting $q^{MP} = q^*$, we have following properties regarding market tightness θ^{MP} in LM:*

- *when $\sigma = 1-\eta$ (Hosios' rule) and $(1-\gamma)[v(q^*)-q^*] = (1+s_q^*)g(q^*)-c(q^*)$, $\theta^{MP} = \theta^*$;*
- *when $\sigma > 1-\eta$ and $(1-\gamma)[v(q^*)-q^*] \leq (1+s_q^*)g(q^*)-c(q^*)$, $\theta^{MP} > \theta^*$;*
- *when $\sigma < 1-\eta$ and $(1-\gamma)[v(q^*)-q^*] \geq (1+s_q^*)g(q^*)-c(q^*)$, $\theta^{MP} < \theta^*$.*

Proposition 5 is intuitive. The efficiency in LM depends on whether firms and workers are properly compensated in the matching and bargaining process of both LM and DM. When Hosios' rule holds, the conventional inefficiency in LM is eliminated. However, on top of Hosios' rule, firms and workers need to be compensated by their contribution to DM matching so that the efficiency in LM can be virtually implemented. This happens when $(1-\gamma)[v(q^*)-q^*] = (1+s_q^*)g(q^*)-c(q^*)$, where the LHS is firm' contribution to generating surplus in DM and the RHS is firm's real payoff in DM as an equilibrium outcome. When $\sigma \geq 1-\eta$ and $(1-\gamma)[v(q^*)-q^*] < (1+s_q^*)g(q^*)-c(q^*)$, firms are favored in the bargaining of both LM and DM and have larger incentive to post vacancies. Therefore, $\theta^{MP} > \theta^*$. The $\theta^{MP} < \theta^*$ result can be seen through a similar reasoning. However, when bargaining in LM is firm-biased(worker biased) and bargaining in DM is worker-biased(firm-biased), the value of market tightness is not clear since these opposite effects may counterbalance each other to various degree.

An immediate implication of our previous results is that, when we implement efficiency in DM by choosing s_q^*, efficiency in labor market may not be achieved. This is the case as it depends on the underlying parameters governing the matching processes in the labor and goods market as well as the bargaining power of firms and workers. In the next subsection we explore if active fiscal policies can help us achieve efficiency.

6.2 Active Fiscal Instruments and Efficiency

An important insight of the previous section was the importance of the DM production subsidy in helping achieve the efficient production in the DM. Thus, after setting s_q^*, the government needs to choose a vacancy subsidy, a, unemployment insurance (or tax), b, and labor income tax rate ,τ_l, to move MP curve such that efficiency in LM and DM markets can be achieved. In other words, we need to specify policies such that $\theta^{MP} = \theta^*$ and $q = q^*$. In order to determine the right set of policies we need to determine the efficient labor market gap, i.e., $\theta^{MP} - \theta^*$.

Recall that the MP curve was given by:

$$\left[(r+\delta)\theta^{\sigma} + (1-\eta)\theta\right] \frac{k-a}{\eta} = y - \frac{b+l}{1-\tau_l} + (1+\delta\theta^{\sigma-1})^{\gamma}[(1+s_q^*)g(q^*) - c(q^*)].$$

From Lemma 2, we know that the MP curve shifts downward(i.e., u decreases for any fixed q) in (u,q) space when the vacancy subsidy, a, increases or unemployment benefits, b, decreases. Intuitively, if government wants a larger vacancy-unemployment ratio $\theta = \frac{v}{u}$, the government can either raise firms' incentive to recruit, or lower unemployment workers' welfare, or do both.

For an initial set of fiscal and monetary policies, if the vacancy-unemployment ratio is higher than efficient level, $\theta^{MP} > \theta^*$, government may want to tax firms or subsidize the unemployed. Thus, in principle (u^*, q^*) is an allocation on the equilibrium MP curve.

Before we determine the appropriate policy instruments, we need to consider the different possibilities when $s_q = s_q^*$. First, if the MP curve is to the right of (u^*, q^*), the government can change the MP allocations by increasing τ_l, increasing a or decreasing b until the MP curve crosses (u^*, q^*). Then using the appropriate combination of policies we can construct a pair of LW and MP curves that crosses at (u^*, q^*). In order to be an efficient market equilibrium, the policy instruments τ_l, τ_d, a and b need to satisfy the government budget constraint which is given by:

$$v^*a + u^*b = \tau_d \,\Delta^* + \alpha_f^* \tau_l^* w$$

which holds at every date. The fact that τ_d has no effect on the LW and MP curves, is going to be important as this instrument can help the government balance its budget. After the government sets a, b and τ_l to shift the MP curve so that LW and MP curves cross at (u^*, q^*), then the fiscal authority choosesτ_d to balance the budget.

However, if the MP curve is to the left of (u^*, q^*) when $s_q = s_q^*$, $\theta^{MP} < \theta^*$, it is still possible to achieve efficiency by setting $a < 0$, thus moving MP curve to the right. But since we can at most set $a = -k$, efficiency in DM market can not be resorted if MP curve is too far away from (u^*, q^*).

Define ζ as follows:

$$\zeta = \frac{(1-\gamma)[v(q^*) - q^*]}{(1+s_q^*)g(q^*, s_q^*) - c(q^*)}$$

Generally, if $\zeta > 1$, the government can always achieve labor market efficiency by setting positive a, b and τ_l as stated in the following proposition.

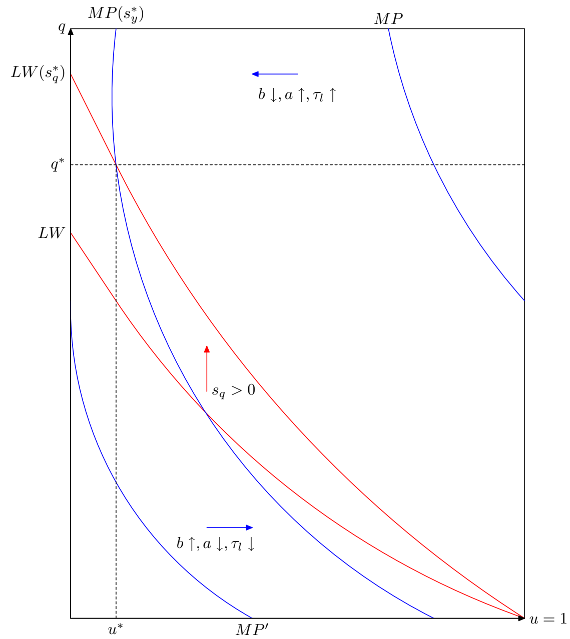

Figure 4: Existence of optimal policies.

Proposition 6 *When Hosios' rule holds. Labor market efficiency can be achieved through appropriate fiscal policies* $\{a^*, b^*, \tau_l^*, \tau_d^*\}$ *such that they satisfy the following conditions:*

$$\frac{k}{k-a} = \frac{y}{y - \frac{b+l}{1-\tau_l}} = \zeta$$

$$v^* a^* + u^* b^* = \tau_d^* \Delta^* + \alpha_f^* \tau_l^* w.$$

- *when firms are favored in both LM and DM, i.e.,*

$$\sigma > 1-\eta \quad and \quad (1-\gamma)[v(q^*) - q^*] \leq (1+s_q^*)g(q^*) - c(q^*)$$

to achieve labor market efficiency, the government needs to implement a small positive or even negative a *and increase* τ_l *and* b.

- *when workers are favored in both LM and DM, i.e.,*

$$\sigma < 1-\eta \quad and \quad (1-\gamma)[v(q^*) - q^*] \geq (1+s_q^*)g(q^*) - c(q^*)$$

to achieve labor market efficiency, government needs to implement small positive or even negative τ_l*, increase* a *and decreases* b.

6.3 Numerical Example

For a given parameterization, the government's problem can be easily solved with standard numerical methods. For ease of comparison with existing analysis, we follow closely the parameterization of Berentsen, Menzio and Wright (2010). These authors consider the following preferences

$$v(q) = \frac{A * q^{(1-\alpha)}}{1-\alpha};$$

and the values for the underlying parameters are $\beta = 0.992$, l=0.1, $y = 3$, A=1.86, δ=0.05 k=8, $\sigma = 0.28$, γ=0.28, η=0.28, ϕ=0.275 and α=0.17. Given this structure we analyze the quantitative implications for optimal fiscal and monetary policies.

For this particular parametrization we find that there exist a unique monetary equilibrium. Nevertheless, there are multiple fiscal and monetary policies that yield the efficient allocation. The optimal fiscal and monetary policies are reported in Table 1 where $\pi = \beta - 1$, $\tau_l = b$=0 with optimal unemployment, vacancy, DM production 0.076, 0.037 and 12.863, respectively.

Table 1: Welfare Maximizing Policies

Policies	Values
s_q	0.356
τ_d	0.547
a	0.282

With this particular parametrization, we obtain a unique monetary equilibrium that exhibits an endogenous long run Phillips curve. Moreover, the environment is such that a labor tax as well as unemployment benefits are redundant instruments. This is the case as the the government is able to implement a vacancy subsidy that shifts the MP curve enough so that (u^*, q^*) belongs to both the MP and LW curves in equilibrium.

7 Conclusion

The objective of this paper is to provide a better understanding of the interactions between monetary and fiscal policies in an economic environment with microeconomic foundations for fiat money and unemployment. The underlying framework is tractable where search-and-bargaining frictions are considered as to create a role for money and to deliver unemployment as equilibrium outcomes.

One of our main results is to show that, even when terms of trade in the decentralized market are given by Nash's bargaining solution, some of the inefficiencies in the Berentsen, Menzio and Wright (2010) framework can be restored with appropriate fiscal policies. In particular, when lump sum monetary transfers are possible, a production subsidy financed by money printing can increase output in the decentralized market and a vacancy subsidy financed by a dividend tax even when the Hosios' rule does not hold.

We also showed there exist multiple subsidies and (sometimes strictly positive) inflation rates that yield the efficient allocation. The Friedman rule is one of the possible policy options that yields efficiency and is always an optimal policy regardless of the bargaining power of the buyer.

Finally, under any of the operating procedures for monetary and fiscal policy considered in this paper, with or without lump sum taxes, large welfare gains are achieved by having fiscal and monetary policies in place. Thus, ignoring active fiscal policies can be quite costly.

The findings of this paper confirm the conjecture of Kocherlakota (2005) and Wright (2005) that fiscal and monetary policies have important interactions in frameworks with micro foundations for the existence of fiat money; thus, they should always be jointly considered in the design of optimal government policy.

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8 Appendix

Proof of Proposition 1

Proof. The LW curve , determines q for a given u exactly as in Lagos and Wright(2005), except for the production subsidy q_s in $g'(q)$. We implicitly impose mild assumption to guarantee $\frac{v'(q)}{g'(q)}$ is globally decreasing in q. Examples of such conditions can be: (i) v' is log-concave; or (ii) $\phi \approx 1$. Thus, when u increases, the LHS of equation (11) increases, q will decrease as $\frac{v'(q)}{g'(q)}$ is a decreasing function of q and therefore LW curve is downward slopping.

If $u = 0$ and $s_q = 0$, $q < q^*$ as in Lagos and Wright(2005). If $u = 1$, it implies that the LHS of (11) goes to ∞ and therefore $q \to 0$ as $v'(0) = \infty$. When workers own full bargaining power, i.e. $\phi = 1$, and Friedman rule $i \to 0$ is implemented, we have $\frac{v'(q)}{c'(q)} \to 1$, i.e., $q \to q^*$. Furthermore, when $i \to 0$, the RHS of (11) goes to 0 so that q is independent of u. ■

Proof of Lemma 1

Proof. It is straightforward to see when u is fixed, q decreases as i increases, given that $\frac{v'(a)}{g'(q)}$ is decreasing in q. From the representation of $g(q, s_q)$ and by applying implicit function theorem, we have that

$$\frac{\partial q}{\partial s_q} = -\frac{\phi z v'(q)}{\phi[-v' + v''(z(1+s_q) - q)] - (1-\phi)v'}.$$

Thus, given $v' \geq 0$, $v'' < 0$ and $z(1+s_q) - q \geq 0$, we have $\frac{\partial q}{\partial s_q} \geq 0$. Therefore, a higher monetary subsidy to production shifts the LW curve upward. ■

Proof of Proposition 2

Proof. First, notice that $\frac{\mathcal{N}(u,v)}{v}$ and $\frac{\mathcal{M}(1,1-u)}{1-u}$ are increasing in u, while $\frac{\mathcal{N}(u,v)}{u}$ is decreasing in u. Then, when other parameters are unchanged, a higher u will lead to a lower q when $q \leq q^*$ given that $g'(q) > 0$. MP curve is thus downward slopping in (u, q) space. Second, time both sides of equation (12) by $(r+\delta) + (1-\eta)\frac{\mathcal{N}(u,v)}{u}$, we have

$$(k-a)\left[(r+\delta) + (1-\eta)\frac{\mathcal{N}(u,v(u))}{u}\right] = \frac{\mathcal{N}(u,v(u))}{v(u)}\eta\{y - (\frac{b+l}{1-\tau_l}) + \frac{\mathcal{M}(1,1-u)}{1-u}[(1+s_q)g(q) - c(q)]\}.$$

The LHS reaches its minimum when $\frac{\mathcal{N}(u,v)}{u} \to 0$. And the RHS reaches its maximum when $\frac{\mathcal{N}(u,v)}{v} \to 1$ and $\frac{\mathcal{M}(1,1-u)}{1-u} \to 1$. Thus, if

$$(k-a)(r+\delta) \geq \eta\left\{y - \frac{b+l}{1-\tau_l} + \left[(1+s_q)g(q^*) - c(q^*)\right]\right\},$$

the market will shut down. Finally, suppose $(k-a)(r+\delta) \geq \eta\left\{y - \left[\frac{b+l}{1-\tau_l}\right]\right\}$, q must be positive at $u = 1$, otherwise the above equation can not hold. ■

Proof of Lemma 2

Proof. The effect of policy instruments a, b and τ_l on u and q are immediate from the equation (12). Also notice that

$$\frac{d(1+s_q)g(q)}{ds_q} = g(q) + (1+s_q)\frac{dg(q)}{dq}\frac{dq}{ds_q} \geq 0.$$

This implies that when u and other parameters are fixed, q decreases as s_q increases. ■

Proof of Proposition 3

Proof. We only focus on steady state equilibrium. When $(k-a)(r+\delta) \geq \eta(y - \frac{b+l}{1-\tau_l})$, MP curve crosses $(1, \underline{q})$ by lemma 2. It is clear from figure 2, the MP curve may(e.g. MP_2) or may not(e.g. MP_3) intersect with LW curve. However, since LW curve crosses $(1,0)$, there always exists a non-monetary steady state where there is no production and trade at all. When $(k-a)(r+\delta) < \eta(y - \frac{b+l}{1-\tau_l})$, MP curve enters (u,q) space from $(\bar{u}, 0)$(e.g. MP_1). Therefore, there exists a non-monetary steady state $(\bar{u}, 0)$ and at least one monetary steady state(e.g. point A in figure 2). There may exist multiple monetary steady state(e.g. point B and C in figure 2).

Suppose monetary steady state exists. Because both LW and MP curves are downward sloping, any shift of one curve creates a new monetary equilibrium(if it exists) with u and q changing in opposite directions. By lemma 1, a higher i makes LW curve move downward and thus decrease q and increases u. By lemma 2, a rise in a or τ_l, or a decrease in b makes MP curve shift upward and thus leads to a steady state with higher q and lower u. ■

Proof of Proposition 4

Proof. By choosing Cobb-Douglas matching technologies and using Beveridge curve $u\mathcal{N}(1,\theta) = \mathcal{N}(u,v) = (1-u)\delta$, we have $\mathcal{N}_1(u,v) = \sigma\theta^{1-\sigma}$, $\mathcal{N}_2(u,v) = (1-\sigma)\theta^{-\sigma}$ and $\mathcal{M}_2(1,1-u) = (1-\gamma)(1+\sigma\theta^{\sigma-1})^{\gamma}$. The social planner's optimal choice can be written as

$$[(r+\delta)\theta^{\sigma}+\sigma\theta]k = (1-\sigma)\left[y-l+(1-\gamma)(1+\delta\theta^{\sigma-1})^{\gamma}[v(q^*)-c(q^*)]\right]$$

The left hand-side(LHS) tends to 0 as $\theta\to 0$ while tends to ∞ as $\theta\to\infty$. And

$$\frac{\partial LHS}{\partial\theta} = [(r+\delta)\theta^{\sigma-1}+1]\sigma k > 0$$

The right hand-side(RHS) tends to ∞ as $\theta\to 0$ while tends to $H = (1-\sigma)\left[y-l+(1-\gamma)[v(q^*)-c(q^*)]\right]$. And

$$\frac{\partial RHS}{\partial\theta} = -(1-\sigma)^2(1-\gamma)\gamma\sigma[v(q^*)-c(q^*)](1+\delta\theta^{\sigma-1})^{\gamma-1} < 0$$

Thus, there exists an unique θ^* which satisfies the social optimality. Given q^* is also unique, there exist an unique social optimal plan given by θ^* and q^*. ■

Proof of Lemma 3

Proof. Given the L-W curve $\frac{i}{\mathcal{M}(1,1-u)} = \frac{v'(q)}{g_q(q,s_q)} - 1$ and assume $c(q)=q$, where

$$g_q(q,s) = \frac{\phi(1+s_q)v'(q)^2+(1-\phi)v'(q)-(1-\phi)\phi v'(q)[(1+s_q)v(q)-q]}{[\phi(1+s_q)v'(q)+1-\phi]^2}$$

The optimal policy requires the government to pick s_q given q such that $g_q(q,s)=1$ which is the true marginal cost. Such a policy eliminates any bargaining inefficiency that arises. Under such a subsidy policy we have then

$$\frac{i}{\mathcal{M}(1,1-u)} = v'(q)-1$$

which means q only depends on i. As a result setting $i=0$(i.e. $\pi=\beta-1$) eliminates the time cost of holding money yields q^*. Given this policy we can now solve for the optimal subsidy. We have

$$1 = \frac{\phi(1+s_q)v'(q)^2+(1-\phi)v'(q)-(1-\phi)\phi v''(q)[(1+s_q)v(q)-q]}{[\phi(1+s_q)v'(q)+1-\phi]^2}$$

which is a quadratic equation in s_q. Imposing $v'(q^*) = 1$ we can solve for a positive s_q^*. Also notice that, given $i = 0$ is chosen, s_q^* does not depend on u. ■

Proof of Proposition 5

Proof. Comparing the social planer's allocation (EF) and the market equilibrium allocation (MP), we can consider three different cases.

Case 1: $\sigma = 1 - \eta$. The matching elasticity with respect to u, $\frac{u\mathcal{N}_2(u,v)}{\mathcal{N}(u,v)}$, is σ under our Cobb-Douglas specification. Hosios(1990) states that the labor market efficiency is going to be achieved whenever the bargaining power reflects party's contribution to matching, i.e. $\sigma = 1 - \eta$. Thus this scenario satisfies the so-called Hosios' rule. In this case, the LHS of equation (EF), and the LHS of equation (MP) are equal for same market tightness θ. Thus, if the RHS of equations (EF) and (MP) satisfy

$$(1-\gamma)[v(q^*) - c(q^*)] = (1 + s_q^*)g(q^*) - c(q^*)$$

we must have that the equilibrium labor market tightness θ^{MP} equals to the efficient tightness θ^*. This in turn implies that the optimal policy scheme (s_q^*, π^*) implements the efficient outcome in both markets.

Now suppose that $(1-\gamma)[v(q^*)-q^*] > (1+s_q^*)g(q^*)-q^*$. Then, for the same market tightness θ, the RHS of equation (EF) is larger than the RHS of equation (MP). As a result, we have that $\theta^{MP} < \theta^*$ when $q^{MP} = q^*$. Similarly, if $(1-\gamma)[v(q^*) - q^*] < (1 + s_q^*)g(q^*) - q^*$, we have $\theta^{MP} > \theta^*$ given that $q^{MP} = q^*$.

Case 2: $\sigma > 1 - \eta$. The Hosios' rule is no longer holding due to the excessive bargaining power of firms. The LHS of equation (EF) is larger than the LHS of equation (MP) for same θ. If $(1 - \gamma)[v(q^*) - q^*] \leq (1 + s_q^*)g(q^*) - q^*$, we must have that $\theta^{MP} \geq \theta^*$ when $q^{MP} = q^*$. If $(1-\gamma)[v(q^*) - q^*] > (1 + s_q^*)g(q^*) - q^*$, the relative value of θ^{MP} when compared to θ^* is ambiguous.

Case 3: $\sigma < 1 - \eta$. The Hosios' rule is not satisfied due to the excessive bargaining power of workers. The LHS of equation of (EF) is smaller than the LHS of equation (MP) for same θ. If $(1-\gamma)[v(q^*)-q^*] \geq (1+s_q^*)g(q^*)-q^*$, we must have $\theta^{MP} \leq \theta^*$ when $q^{MP} = q^*$. If $(1-\gamma)[v(q^*)-q^*] < (1 + s_q^*)g(q^*) - q^*$, the relative value of θ^{MP} when compared to θ^* is ambiguous. ■

Proof of Proposition 6

Proof. First, suppose that Hosios' rule holds. From equations (EF) and (12), it is easy to see θ^* can be achieved in labor market if and only if $\frac{k}{k-a}$, $\frac{y}{y-\frac{b+l}{1-\tau_l}}$ and $\frac{(1-\gamma)[v(q^*)-c(q^*)]}{(1+s_q^*)g(q^*)-c(q^*)}$ are of the same value. The first two fractions can be adjusted by choosing policy instruments a, b and τ_l. Whether the efficiency can be resorted depends on the value of the last fraction.

Suppose that $\frac{(1-\gamma)[v(q^*)-c(q^*)]}{(1+s_q^*)g(q^*)-c(q^*)} = \zeta > 1$. All we need is to choose a, b and τ_l such that

$$\frac{k}{k-a} = \frac{y}{y - \frac{b+l}{1-\tau_l}} = \zeta$$

This can be easily done by choosing nonnegative policy instruments. Suppose that $\zeta < 1$. Now we need to choose negative a and τ_l. But since the entry tax a can not be less than $-k$ and τ_l is also restricted by budget constraint, the feasible set of policies is restricted. Thus, only when ζ is not too small, efficiency can be achieved.

When Hosios' rule does not hold, choice of policy becomes complicated as it depends on the detailed parameters of matching and bargaining process. However, the direction which heads to efficiency is clear. When

$$\sigma > 1-\eta \quad \text{and} \quad (1-\gamma)[v(q^*)-q^*] \le (1+s_q^*)g(q^*)-c(q^*),$$

the MP curve determined by (MP) is lower than (u^*, q^*) by proposition 5. Thus, large b and τ_l, and small(or even negative) a should be chosen. Similar argument can be formed for the opposite case.

Of course, all policy combinations above should satisfy government's budget constraint $v^*a^* + u^*b^* = \tau_d^* \triangle^* + \alpha_f^* \tau_l^* w$. ■