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Abstract

Model risk in the estimation of value-at-risk is a challenging threat for the success of any financial investments. The degree of the model risk increases when the estimation process is constructed with a portfolio in the emerging markets. The proper model should both provide flexible joint distributions by splitting the marginality from the dependencies among the financial assets within the portfolio and also capture the non-linear behaviours and extremes in the returns arising from the special features of the emerging markets. In this paper, we use time-varying copula to estimate the value-at-risk of the portfolio comprised of the Bovespa and the IPC Mexico in equal and constant weights. The performance comparison of the copula model to the EWMA portfolio model made by the Christoffersen back-test shows that the copula model captures the extremes most successfully. The copula model, by estimating the portfolio value-at-risk with the least violation number in the back-tests, provides the investors to allocate the minimum regulatory capital requirement in accordance with the Basel II Accord.

Key Words: *Time-varying Copula, portfolio value-at-risk, Latin American equity markets, portfolio GARCH.*

JEL classification: C14, C51, G1

1. Introduction

The total losses of any portfolios should be estimated correctly in order to allocate economic capital for the investments in a proper way. The correlation among the price or volatility behaviours of the financial assets within a portfolio is a crucial dimension for the proper estimation of the value-at-risk amount. However, restrictions on the joint distributions of the financial assets within the portfolio might decrease the performance of the value-at-risk estimation. The joint distribution of the portfolio should be free from any normality assumptions especially if the portfolio is composed of assets from emerging markets where there exists high volatility, regime switches, and non-linearities in the returns.

Emerging markets are distinguished features that create complicated financial behaviours to be captured in the estimation process of the value-at-risk. Risk-return relationship might not be linear due to turmoil in the investment environment, asymmetric perceptions of the investors and non-stable political decisions and regulatory structures.

Parametric models like GARCH and its derivatives have certain assumptions, whether normal or asymmetric, on the distributional characteristics of the individual financial time series or portfolio returns. On the other hand, financial markets in the emerging economies might show abnormal behaviours due to their chaotic and dynamic climates. For that reason, modeling value-at-risk in the emerging markets requires flexible and non-parametric approaches to reach a successful estimation performance.

This paper chooses copula methodology to estimate the value-at-risk of a portfolio constructed with the Brazilian stock exchange market and Mexican stock exchange market in equal proportion. The research aims at examining the performance of copula methodology with respect to those of alternative parametric model EWMA estimation for the time period between 02.01.2001 and 15.02.2007. In that framework, this paper is the first research on the estimation of the value-at-risk of the equity portfolio consists of Latin American stock markets, as much as we know. The empirical results of the Christoffersen back-tests show that the copula method captures the extremes in the equity markets of the Latin America. The success in the performance of the model is crucial for the regulatory capital, as well. Due to relatively lower fails in the back-tests, the model is adequate for the minimum regulatory capital within the Basel II norms.

The paper is constructed as follows. In the next section, copula methodology for the estimation of the portfolio value-at-risk is introduced in detail. In the third part of the article, the performance of the copula method in value-at-risk estimation is discussed with the empirical results of the back-tests. The paper ends with suggestions for the future research on risk estimation with nonparametric methodologies in the emerging markets.

2. Literature Review

The copula method is based on the Sklar (1959) theorem which describes the copula as an indicator of the dependencies among the variables. According to Dowd (2005), the strenght of the copula comes from its feature that it does not have any assumptions on the joint distributions among the financial assets in a portfolio. It creates N marginal distribution for the joint distribution with N dynamics. In fact, the normality is rarely an adequate assumption in finance. For example, Longin and Solnick (2001) and Ang and Chen (2002) empirically show that asset returns are more highly correlated during volatile markets and during market downturns. For that reason, the deviation from normality might lead to inadequate value-at-risk estimations.

The copula as an risk measurement technique has been started to use in the financial risk estimation in recent years. Frey and McNeil (2003), Hamerle and Raosch (2005), Goorbergh, Genest and Werker (2005) use the copula in the option valuation. The method is employed by Junker, Szimayer and Wagner (2006) in the analysis of term structures of the interest rates; Giesecke (2004) and Meneguzzo and Vecchiato (2004) in the credit risk analysis and Neslehova, Embrechts and Demoulin (2006) for the calculation of the operational risk at the banks.

The researches using copula for the estimation of value-at-risk go back from the last five years. The copula methodology used in the first researches does not include conditionality, in other words, it does not have time-varying feature. Empirical and methodological discussions for the constant copula can be found in Cherubini, Luciano and Vecchiato (2004), Rockinger and Jondeau (2001), Fortin and Kuzmics (2002) Embrechts, McNeil and Straumann (2002), Embrechts, Hoing and Juri (2003), Chen and Fan (2002), Cherubini and Luciano (2001), Fermanian and Scaillet (2003), Li (2000) and Rosenberg (2003).

Patton (2002) constructs the conditional copula by allowing the first and second conditional moments to vary on time. After the methodological expansion of Patton (2002), the conditional copula has been started to use in the estimation of valu-at-risk. Jondeau and Rockinger (2006) use normal GARCH based copula for the value-at-risk estimation of the portfolio composed of international equity indices. Junker, Szimayer and Wagner (2006) employ copula to model the yield curve on the US interest rates between 1982-2001. Chen and Fan (2006) use copula method to construct a semi-parametric model based on the Markov chain GARCH.

Poon et al. (2004) use Gaussian and Gumbel copulas for estimation of loss of the portfolio with linear assets. Diversification breakdown in a portfolio is examined by Loretan and English (2000), Campbell et al. (2002) and Ang and Chen (2002). Rockinger and Jondeau (2001) use a dependence measure to check if the linear dependence varies with the time by applying the Plackett's copula with the returns of European stock indices, the S&P500 and the Nikkei 225. They underline that the disadvantage of the Plackett's copula is that it cannot be easily used with portfolios composed of financial assets more than two. Cherubini and Luciano (2001) estimate the value-at-risk using the copula and the historical empirical distribution in the estimation of marginal distributions.

The copula can be seen as an alternative for the multivariate GARCH models. Lee and Long (2005), on the other hand, combine copula with multivariate GARCH model, which allows very flexible joint distributions. They propose copula-multivariate GARCH model with uncorrelated dependent errors to compare with three MGARCH models and empirically show that the mixed model outperforms multivarate GARCH in terms of in- sample model selection criteria and out-of-sample multivariate density forecast. Palaro and Hotta (2006) use a mixed model with the conditional copula and multivariate GARCH to estimate the value-at-risk of a portfolio composed of Nasdaq and S&P500 indices.

The copula method is used with expected short fall to estimate the fat tails. Embrechts at all (2005) apply the copula method to create value-at-risk scenarios for the worst cases. Juri and Wuthrichts (2002) combine the copula with extreme value theory, Mendes and Souza (2004) do with the stress scenarios to calculate the value-at-risk amount.

We write Matlab codes to estimate the models used in this research. The Matlab codes written by Patton (2002, 2006a, 2006b) and Sheppard (2006) for the conditional copula are the references for us to create the software for the models employed. Quantile copula multivariate GARCH is calculated with two steps maximum probability in parallel to the method used by Bauwens at all (2006).

3. Methodology

Conditional copula was constructed based on the Sklar's (1959) dependency theory. Under the assumption that for t = 1,........T, we assume that equation 6 represents the historical data till time t. Under this assumption, Sklar (1959) theorem can be expressed equation 7.

$$\xi_t = \sigma \{ X_{1t-1}, X_{2t-1}, \ldots\ldots, X_{nt-1}, X_{t-2}, X_{2t-2}, \ldots\ldots, X_{nt-2}, \ldots\ldots \} \tag{6}$$

$$F_1(X_{1t}, X_{2t} \ldots\ldots\ldots\ldots X_{nt} \mid \xi_t) = C_t(F_{1t}(X_{1t} \mid \xi_t), F_{2t}(X_{2t} \mid \xi_t), \ldots\ldots, F_{nt}(X_{nt} \mid \xi_t \mid \xi_t) \tag{7}$$

In the equation, C_t should have copula function for each t. Patton (2002, 2006a) assumes that conditional mean is created on autoregressive process, while conditional variance is done based on GARCH (1,1) process.

Symmetric Joe-Clayton (SJC) can be expressed with equation 8 (Patton, 2006a). In the equation, τ^U and τ^L represent the tail distributions.

$$C_{SJC}(u,v \mid \tau^U, \tau^L) = 0.5.(C_{SJC}(u,v \mid \tau^U, \tau^L) + (C_{SJC}(1-u,1-v \mid \tau^U, \tau^L) + u + v - 1) \tag{8}$$

The equation of $\tau^U = \tau^L$ makes the model symmetric. On the other hand, Patton (2001,2006a,2006b) uses copula to model the conditional dependency varying in time, as well. The minimum and maximum dependency values of conditional symmetric Joe-Clayton copula are expressed on equation 9 and equation 10, respectively (Patton,2006a).

$$\tau_t^U = \Lambda\left(\omega_U + \beta_U \tau_{t-1}^U + \alpha_U . \frac{1}{10} \sum_{j=1}^{10} \left| u_{t-i} - v_{t-i} \right| \right) \tag{9}$$

$$\tau_t^L = \Lambda\left(\omega_L + \beta_L \tau_{t-1}^L + \alpha_L . \frac{1}{10} \sum_{j=1}^{10} \left| u_{t-i} - v_{t-i} \right| \right) \tag{10}$$

The equation of $\Lambda(x) \equiv (1 + e^{-x})$ is the logistic transformation that fixes τ^U and τ^L parameters to take $(0,1)$ values.

For reliable empirical evidence, we should create marginal distribution for each stock index and a conditional copula function for the whole portfolio.
Marginal distribution is calculated with normal GARCH (1,1) expressed in equation 11.

$$X_t = \varepsilon_t \tag{11}$$

$$h_t^x = \omega_x + \beta_x h^x_{t-1} + \alpha_x \varepsilon^2_{t-1}$$
$$\varepsilon_{t/} h_t^x \sim N(0,1) \ \ (x)$$

X_t represents the logarithmic difference of the financial asset. After estimating marginal distributions, the joint distribution of two financial assets is reached. The correlation parameter for conditional symmetric Joe-Clayton copula, ρ, is expressed with equation 12.

$$\rho_t = \Lambda\,(\omega_\rho + \beta_\rho\,\rho_{t-1} + \alpha_\rho\,1/\rho \sum_{j=1}^{p} \text{Ø}^{-1}(u_{t-j})\text{Ø}^{-1}\,(v_{t-j})) \tag{12}$$

$\Lambda(x)$ is the hyperbolic tangent function fixing ρ_t between (-1,1). The equation 13 is, on the other hand, dependency parameter that enables to capture the changes in the dependency.

$$1/\rho \sum_{j=1}^{p} \text{Ø}^{-1}(u_{t-j})\text{Ø}^{-1}\,(v_{t-j})) \tag{13}$$

Christoffersen test (Christoffersen, 1998) focuses on the probability of failure rate. The importance of testing conditional coverage arises with volatility clustering in financial time series. Christoffersen test might be more proper to detect fat-tail in the returns as compared to the alternative models like Kupiec test.

To apply the test, we firstly define $p^{\alpha} = \Pr(y_t < VaR_t(\alpha))$ and test $H_0 : p^{\alpha} = \alpha$ against $H_1 : p^{\alpha} \neq \alpha$.

We consider $\{1(y_t < VaR(\alpha)\}$ which has a binomial likelihood $L(p^{\alpha}) = (1-p^{\alpha})^{n_0}(p^{\alpha})^{n_1}$ (Saltoglu, 2003)

where $n_0 = \sum_{t=R}^{T} 1(y_t > VaR_t(\alpha))$ and $n_1 = \sum_{t=R}^{T} 1(y_t < VaR_t(\alpha))$.

Under the null hypothesis, it becomes $L(\alpha) = (1-\alpha)^{n_0}\alpha^{n_1}$. Thus the likelihood ratio test statistics is in equation below.

$$LR = -2In(L(\alpha))/L(\hat{p})) \xrightarrow{d} \chi(1) \tag{25}$$

We estimate VaR with α =0.01 confidence interval and backtest VaR models with Kupiec Christoffersen out-of-sample forecasting test. We chose %99 confidence level in accordance to Basel II requirement.

4. Data and Empirical Results

Data

The research aims at examining the performance of copula methodology with respect to those of alternative parametric model portfolio EWMA estimations for the time period between 02.01.2001 and 28.02.2007 with 1498 daily observations. We use time-varying copula and EWMA portfolio to estimate the value-at-risk of the portfolio comprised of the Bovespa and the IPC Mexico in equal and constant weights. Data that is used in this paper Bovespa and IPS Mexico index are from Bloomberg. Figure 1 shows estimated stock indexes in log-differenced series and Figure 2 shows scatter plot of two stock indexes. Although there is positive correlation between two stock indexes correlation is not constant and different in tails. Thus requires complex methods like copula to estimate portfolio value-at-risk with marginal distributions effects.

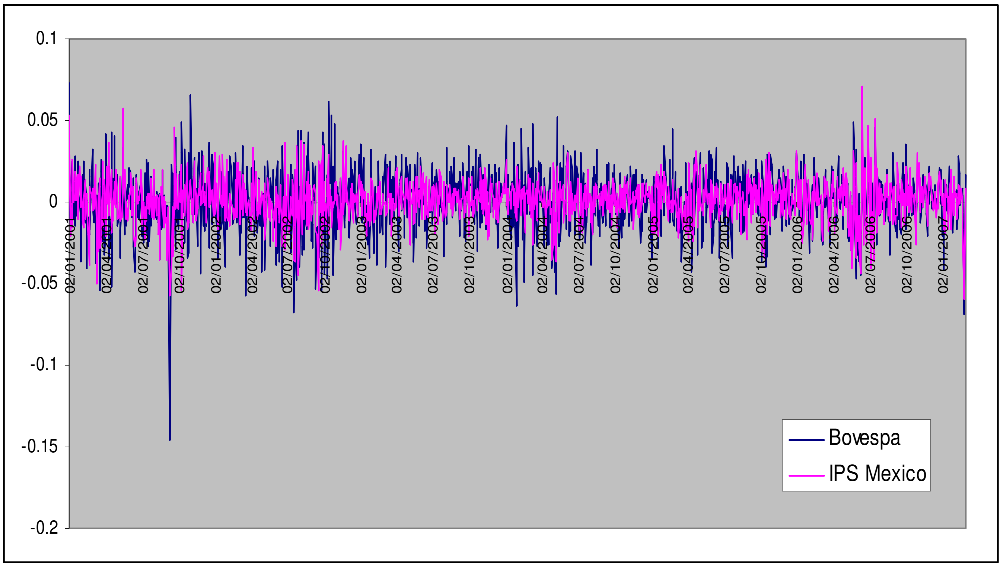

Fig. 1 Log-differenced series (Bovespa and IPS Mexico)

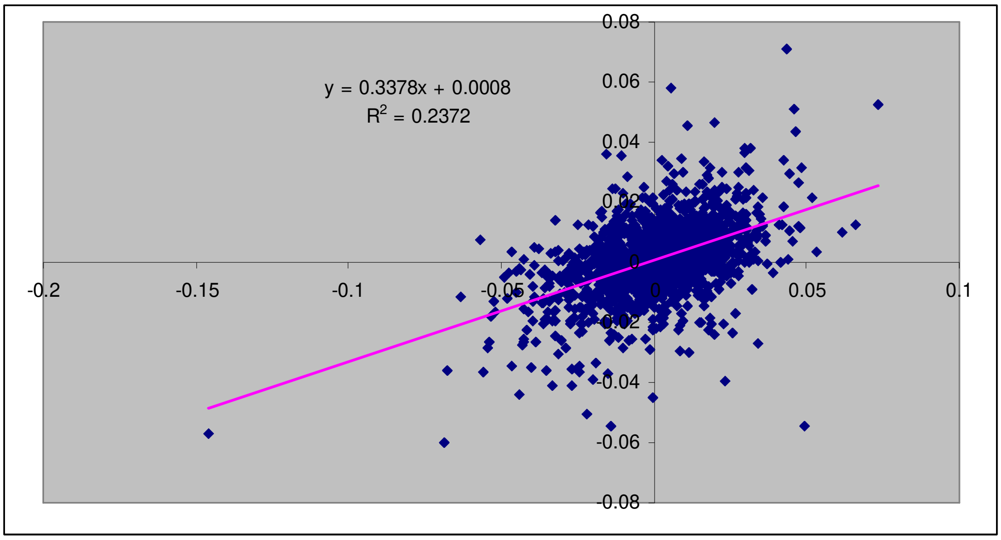

Fig. 2 Scatter plot of Bovespa and IPS Mexico

Empirical Results

In Table 1, there are stability values of stock returns at the level I(0) and I(1) based on Phillips-Peron test (Phillips and Peron, 1988) and Augmented Dickey Fuller test (Dickey and Fuller, 1981). Series are not stationary at I(0) where stationary at I~(1). I(1) represents log-difference level based on both Phillips-Peron(Phillips and Peron, 1988) and Augmented Dickey Fuller test (Dickey and Fuller, 1981) unit root tests.

Table 1. Unit Root Test Results

	Phillips- Peron test I(1)	Augmented D-F test I(1)
Bovespa	0.689875	0.698002
IPS Mexico	1.98546	1.85796
LLBovespa	-36.9297*	-36.9475*
LLIPS Mexico	-34.7981*	-34.8686*

* Stationary in %1 C.I.

Main Statistical Properties of Log Differenced Series are shown in Table 2. Both of the series' kurtosis and skewness values are not close to normal distribution based on jargue-bera stats and series are left tailed as shown in Figure 3. Density plot of the series' distributions also shows that both of the series are not normally distributed. Linear correlation between series is %48,7 and thus indicate that copula can be applied to improve forecasting with marginal distribution affects.

Table 2. Main Statistical Properties of Log Differenced Series*

	LLBovespa	LLIPS Mexico
Mean	0.000746561	0.00108077
Standard Dev.	0.01817	0.0126084
Kurtosis	6.42424	6.02487
Skewness	-0.492586	-0.141769
Min	-0.145659	-0.0597751
Max	0.0733529	0.0711869
Jarque-Bera statistic	792.968	576.503
Linear Correlation	***0,487***	

* LL represents log-differencing.

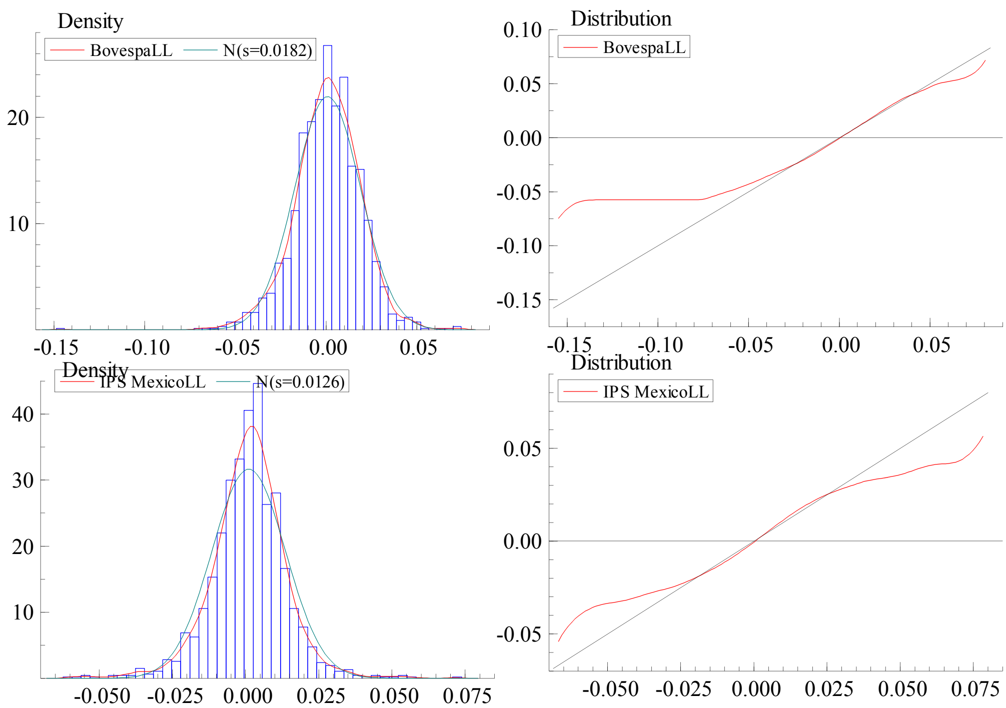

Fig. 3 Density and Distribution Plots of Bovespa and IPS Mexico

In Table 2, six copula and three conditional copula models' akaike values are reported for Bovespa and IPS Mexico equally weighted portfolio. Student-t copula is the best copula model in nonconditional copula models where symmetrised joe-clayton copula is the best model in conditional and all copula models. This indicate that symmetrised joe-clayton copula should be applied for Bovespa and IPS Mexico equally weighted portfolio. In Table 3, symmetrised joe-clayton copula and conditional symmetrised joe-clayton copula's parameters are reported. For standard symmetrised joe-clayton copula τ^U and τ^L parameters are 0.45 and 0.20(Patton, 2002) respectively and as reported in Table 3, our portfolio's symmetrised joe-clayton copula τ^U ve τ^L paremeters are 0,34191 and 0,23436 respectively. Copula likelihood also shows that conditional symmetrised joe-clayton copula is better than unconditional symmetrised joe-clayton copula.

Table 2. Comparison of Copula Models

Model	**Akaike Value**
1. Normal Copula	-382,75
2. Clayton Copula	-346,13
3. Frank Copula	Inf
4. Gumbel Copula	-346,15
5. Student-t Copula	-412,52
6. Symmetrised Joe-Clayton Copula	-404,15
7. Conditional Normal Copula	-383,88
8. Conditional Gumbel Copula	**-426,65**
9. Conditional Symmetric Joe-Clayton Copula	**-441,77**

Table 3. Copula Models

Symmetrised Joe-Clayton Copula	
Parameters	**Value**
τ^U	0,34191 [0,1247]
τ^L	0,23436 [0,0787]
Copula Likelihood	202,08
Conditional Symmetrised Joe-Clayton Copula	
ω^U	-1,1943 [0,054]
α^U	-3,3279 [0,1401]
β^U	3,2087 [0,0367]
ω^L	-1,5913 [0,1283]
α^L	-1,1755 [1,6746]
β^L	3,4716 [0,0917]
Copula Likelihood	220,89

[ ] Standard errors

Contour plot of Symmetrised Joe-Clayton Copula for Bovespa and IPS Mexico portfolio shown in Figure 5. As it can be seen Figure 5 simulates Figure 2 with marginal distributions.

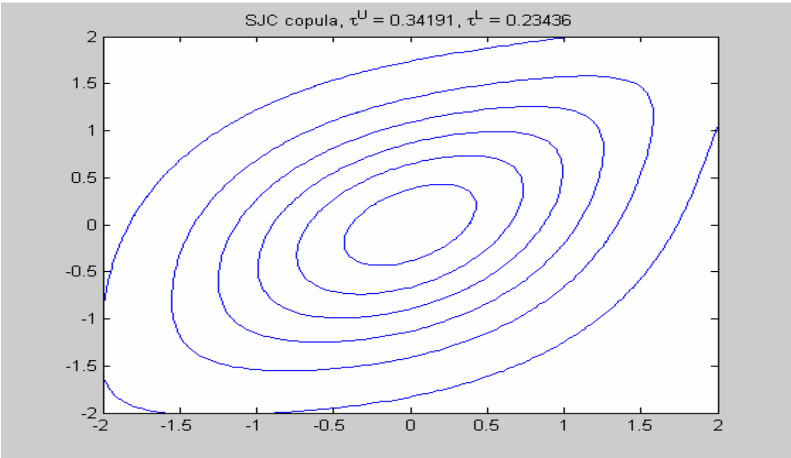

Fig. 5 SJC Copula Contour plot for Bovespa and IPS Mexico

Time varying correlation based on symmetrised Joe-Clayton copula is shown in Figure 6 for Bovespa and IPS Mexico portfolio. Lower tail depencende is more volatilite than upper tail dependence and correlation is not constant over time.

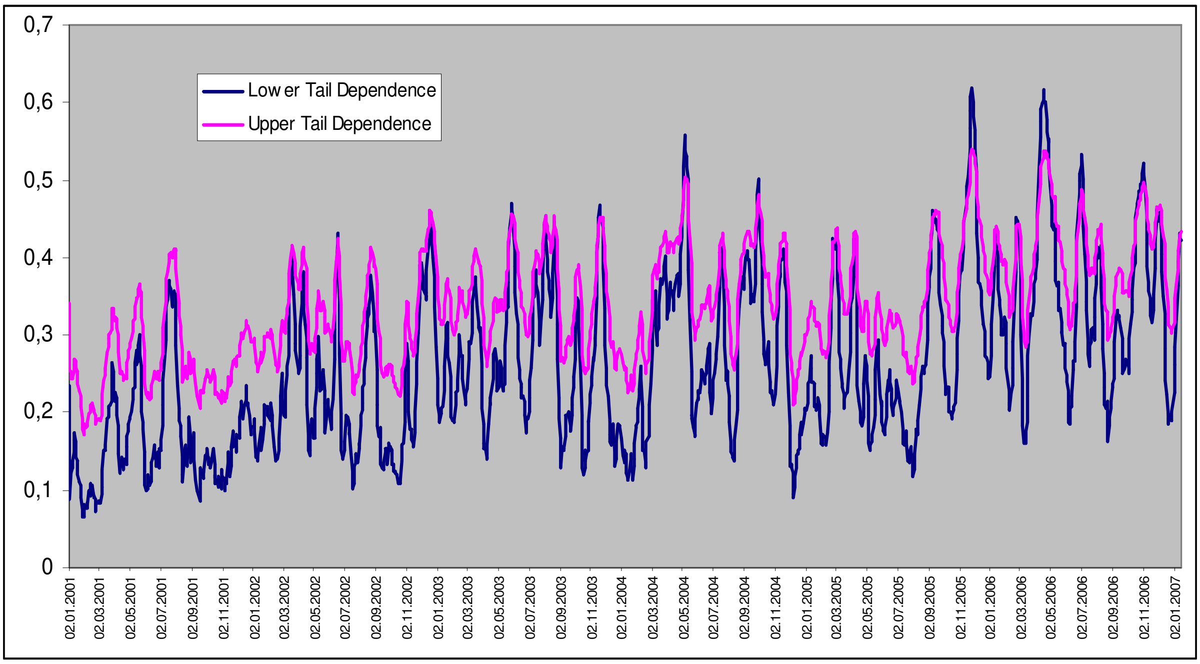

Fig. 6 Time Varying Correlation based on Conditional SJC Copula

Figure 8 shows Conditinal Symmetrised Joe-Clayton Copula and EWMA portfolio value-at-risk for Bovespa and IPS Mexico portfolio. Symmetrised Joe-Clayton Copula captures the extremes most successfully compare to EWMA portfolio.

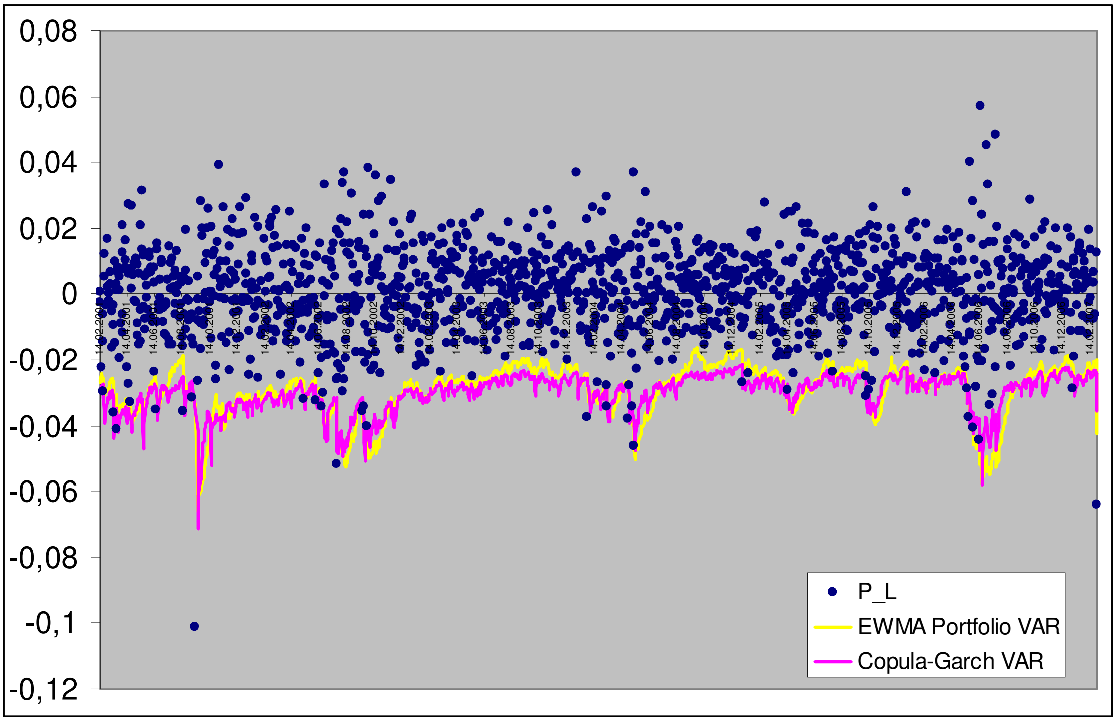

Fig. 8 Portfolio Return and EWMA Portfolio VAR

Table 5 shows christoffersen backtesting results. Conditinal Symmetrised Joe-Clayton Copula is reasonably weel according to christoffersen test stat. Number of exceptions is 43 for EWMA Portfolio where 36 for Conditinal Symmetrised Joe-Clayton Copula. This shows that Conditinal Symmetrised Joe-Clayton Copula also satisfies regulatory capital requirement as Basel II Accord.

Table 5. Christoffersen Backtest Results*

Models	**Test Value***	**No. of Exceptions**
EWMA Portfolio VAR	0,855296	43
Conditional SJC Copula VAR	0,19348	36

* %99 C.I.

5. Conclusion

Model risk in the estimation of value-at-risk is a challenging threat for the success of any financial investments. The degree of the model risk increases when the estimation process is constructed with a portfolio in the emerging markets. The proper model should both provide flexible joint distributions by splitting the marginality from the dependencies among the financial assets within the portfolio and also capture the non-linear behaviours and extremes in the returns arising from the special features of the emerging markets. In this paper, we use time-varying copula to estimate the value-at-risk of the portfolio comprised of the Bovespa and the IPC Mexico in equal and constant weights. The performance comparison of the copula model to the EWMA portfolio model made by the Christoffersen back-test shows that the copula model captures the extremes most successfully. The copula model, by estimating the portfolio value-at-risk with the least violation number in the back-tests, provides the investors to allocate the minimum regulatory capital requirement in accordance with the Basel II Accord.

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