A real-time trading rule

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Abstract

If prices of assets are not aligned to their net present value, a trading strategy may be implemented when actual prices revert to fundamentals. A real-time trading strategy is introduced based on the assumption that reversion occurs in later periods. The fundamental price is constructed in real time using the net present value approach which requires the series for expected dividends, expected returns and expected dividend growth rate. These series, typically unobservable, are derived from a structural state space model. A battery of tests comparing the rule to the passive Buy and Hold Strategy illustrates that the rule is marginally better for shorter horizons.

Key Words: Trading Rule,Asset Pricing, State Space Modeling
JEL References: G12, G14, G17

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1 Introduction

If an asset is overpriced or underpriced relative to its true valuation, mean reversion towards its true value should be anticipated in subsequent periods. This statement emanates from a property of the Efficient Markets where rational agents will arbitrage this riskless gain so that the price is aligned with the value of the asset. In the context of an equity market, if it is possible to identify whether the market is underpriced or overpriced relative to the fundamental value, a simple strategy would be to go long on equity or bonds (depending on the direction of the mispricing). The trading rule is developed in line with real time varying expected returns and expected dividend growth. As mentioned in previous literature, the stock market is excessively too volatile to be justified by the rational expectations model (**Shiller and Beltratti (1993), Poterba and Summers (1988)**). The rule that we develop attempts to take advantage of the volatile nature of stock markets, while resting on the assumption of Efficient Markets (as defined in **Timmermann and Granger (2004)**) and Rational Expectations. We construct the theoretical price (net present value) of the market index using values of dividends and expected returns and dividend growth optimized from a net present value state space model. The theoretical price is then compared to the actual price. Such comparison will identify buying the index or the risk free asset before the mean reversion takes place.

Since the net present value is calculated in real time, the expected returns and expected dividend growth are also derived by a time varying procedure. I make use of **Koijen and Van Birsbergen (2010)** model to derive the expected returns and expected dividends component in a state space, where the Kalman Filter is used to derive the log likelihood function. The intuition behind this methodology is that expected returns and expected dividend growth are unobservable to the econometrician. However we do observe the realized values The Expected Returns and Expected Dividend Growth series are filtered from realized observations based on the Kalman procedure, where expectations are updated as a new observation of the realized value. The most common way to derive the series for these variables is to use a latent variables approach. I derive the law of motion for the price dividend ratio from the Campbell and Shiller(1988) identity, assuming that the expected returns and the dividend growth rate follow an autoregressive process. The state space model is derived from the net present value relationship between Price Dividend ratio, expected returns and expected dividend growth. The Kalman Filter is applied to the model parameters which are optimized using the conditional Maximum Likelihood procedure.

Consistent with rational expectations and the Efficient Markets Hypothesis, the structural decomposition of the expected returns and expected dividend growth rate rely on the Price -Dividend Ratio as being the key variable governing the law of motion and having theoretical links with the two components we want to derive. As such, the build of the state space model involves the Price Dividend Ratio and realized dividend growths as being the measurement equation with the expected returns and expected dividend growth processes forming the state equation. The interesting aspect of such a model, just as in the recursive and rolling model, is that it is robust to structural breaks and does not require the estimation of a large number of parameters **(Rytchkov 2007)**.

Having derived the expected returns and expected dividend growth series, the rational present value (fundamental price) of the index may be easily computed. The fundamental price is compared to the realised price to decide whether to go long on equity or treasury bills. If the theoretical price is higher than the actual price, implying the market is underpriced, the proper strategy would be to go long on equity, assuming that actual price revert to the fundamental price. On the other hand if the market price is high relative to the theoretical price, this will lead to a capital loss and hence the proper strategy would be to shift the assets from equity to bonds.

2 Literature review

In this section we look at related studies on the trading rule, the net present value model as applied in the financial markets, and the state space models application in the derivation of the expected returns and expected dividend growth.

2.1 Trading Rule

The theoretical underpinning of the rule involves the comparison of the Actual Price with the Efficient Markets Price (fundamental value) in order to define whether it is profitable to go long or short on it. Theoretically, the EMH price is determined by the expectations of agents and the type of process they used to model the data generating process of dividends **(Timmermann 1993)**. Any difference between the EMH and the actual price, will lead agents to revise their expectations and their own behavior will move the market back to equilibrium. Hence a profitable opportunity might arise during the adjustment of the market price towards the fundamental value. For instance if the stock market price is higher than the one postulated by EMH, implying that the market is overpriced, reversion to the EMH market price implies that the price will move downwards, hence leading to a capital loss. On the other hand if the stock market price is underpriced relative to the market, we should expect that the price will rise then reaping a capital gain.

The model was put to use in **Bulkley and Tonks (1989)** where the expost rational price was compared with the net present value. The rule involves choosing to invest in either the market index or in the risk free asset. If the market is overpriced, the trading rule posits going long on the risk free asset and invest zero weight on the equity market. When the stock market is underpriced, the investor should shift his assets from bonds to equity. The rule makes out the best of the movement of assets. They also showed how the revision in the parameters in an estimated model of dividends is likely to explain the excess volatility in the UK stock market, as an overall objective of testing whether the stock market can be efficient in the weak form. Another variant of the paper **(Bulkley & Tonks, 1992)** was used to compare the Buy and Hold Strategy and Trading Rules in the S&P 500 market with the same outcome as in the UK market. The study extends on further implications on the rule's returns when risks and transaction costs are included are accounted for.**Taylor and Bulkley (1996)** who use the same REPV formulae in a Price conditional VAR model to derive the theoretical price. The objective however was to test whether underpriced portfolios tend to generate higher returns than overpriced portfolios over several years horizon going until 10 years. **Rambaccussing (2009)** tests the rule in a real time context by assuming that agents have an econometric model from which they can forecast dividends.The expected returns and dividend growth rate are generated in the same way. He finds that the rule works better for longer holding horizons.

2.2 Net Present Value Approach for Deriving EMH price

The Net Present Value approach to deriving the theoretical price involves the assumption of a general equilibrium economy where any riskless return may be arbitraged away. An interesting derivation of the model may be found in Cochrane (2002). The basic premise of the net present value is that the theoretical price is derived by the infinitely discounted payoffs from the asset. The theoretical price (or NPV) is given by :

$$P_t^* = E_t \sum_{i=1}^{\infty} [r_{t,t+j} D_{t+j}] \tag{1}$$

where E_t is the expectations operator at time t, $r_{t,t+j}$ is the return from time t to t+j and D_{t+j} relates to the dividend at time t+j.

If dividends are growing at a rate g_{t+1} and $E_t\left[r_{t+1}\right]$ is greater than g_{t+1}, equation **14** can be written as:

$$P_t^* = \frac{1}{E_t[r_{t+1} - g_{t+1}]} E_t[D_{t+1}] \tag{2}$$

Equation 16 is used to compute the REPV price. The rational valuation formulae has been extensively to derive the Efficient Markets Hypothesis price.For applications, see **(Cuthbertson & Hyde, 2002)**, **(Kanas, 2005)**, **(Rangvid, 2006)**, **(Shiller & Beltratti, 1993)**, **(Allen & Yang, 2004)** , **(Caporale & Gil-Alana, 2004) (Strauss & Yigit, 2001) (Bohl & Siklos, 2004) (Mills, 1993)**

2.3 Time Varying Expected returns and Dividend Growth

The asset pricing model applies state space modeling to the derived present value of Price Dividend ratio. We review papers that apply the The objective of applying the state space model is to derive expected returns and expected dividend growth which are both latent variables but which are linked through the Price Dividend ratio identity. Decomposing the Price Dividend Ratio between expected growth and returns hence is theoretically sound since it is in line with the net present value approach and is formed with rational expectations foundations. According to theory, if prices and dividends are cointegrated, then all the variation in the dividend -price ratio must come from the variation of expected returns and dividend growth. Past empiricism has found the latter to be unpredictable, hence in this sense, all the variation in the price dividend ratio comes from the changes in expected returns.

The methods for deriving the expected returns and expected dividend growth rate can include various classifications such as the simple trend, predictive Ordinary Least Squares,. Bayesian models and State Space models. State space models of expected returns provide more robust estimates with respect to structural breaks in the data (**Rytchkov 2007**). Since both expected returns and expected dividend growth rate are unobservable, the state space model can be used to provide most efficient estimates of these two variables given observed data. The filtering technique used to uncover expected returns has been used widely in the literature. **Conrad and Kaul** (1988) apply the Kalman Filter to extract expected returns from the history of realized returns. **Brandt and Kang** (2004) model conditional mean and volatility as unobservable variables which follow a latent VAR model and filter them from observed returns. In the same line of thinking, Cochrane (2008) shows that the VAR model can be represented in state space form. **Pastor and Stambaugh** (2006) make use of the fact that imperfect exogenous predictors imply that there is no errors in variables. and hence may be used to uncover the unobservable expected returns from realized returns. **Koijen and Van Binsbergen (2010)** use the state space model to model cash invested and market invested dividends.

3 Methodology

3.1 Net Present Value Model

In this section, we derive the net present value relationship between the Price Dividend Ratio and expected returns and expected dividend growth. Interestingly the series is developed from a theoretical assumption that both expected returns and dividend growth rate follows an autoregressive process of order 1. We start by defining some standard equations in the literature and then we derive the Campbell and Shiller (1988) log linearized model:

The rate of return is defined as

$$r_t = \log(\frac{P_{t+1} + D_{t+1}}{P_t}) \tag{3}$$

The Price Dividend ratio is defined as

$$PD_t = \frac{P_t}{D_t} \tag{4}$$

The Dividend Growth rate is defined as

$$\Delta d_{t+1} = \log(\frac{D_{t+1}}{D_t}) \tag{5}$$

One of the important assumptions that we put forward for the process of expected returns and dividend growth concerns the order of the process. The intuitive idea concerning the functional form of the process is that there should be in theory near to the data generating process. However, this endeavour of finding a best model is hectic and involves a lot of data mining. We shall assume our own functional form of the model. The mean adjusted conditional expected returns and dividend growth rate are modelled as an autoregressive process as in equations 6 and 7 respectively :

$$\mu_{t+1} - \delta_0 = \delta_1(\mu_t - \delta_0) + \varepsilon^{\mu}_{t+1} \tag{6}$$

$$g_{t+1} - \gamma_0 = \gamma_1(g_t - \gamma_0) + \varepsilon^{g}_{t+1} \tag{7}$$

where $\mu_t = E_t(r_{t+1})$ and $g_t = E_t(g_{t+1})$

Equation 6 and 7 relates to the mean deviation of the expected returns and expected dividend growth rate where δ_0 and γ_0 represents the unconditional mean of the expected returns and dividend growth respectively. δ_1 and γ_1 represents the autoregressive parameters. ε^{μ}_{t+1} and ε^{g}_{t+1} represents the shocks to the expected returns and the dividend growth rate processes. $\varepsilon^{\mu}_{t+1} \sim N(0,\sigma^2_{\mu})$ and $\varepsilon^{g}_{t+1} \sim N(0,\sigma^2_{g})$. However, we do not implement any restrictions between the covariance of $\varepsilon^{\mu}_{t+1}$and ε^{g}_{t+1}because a shock to the expected return process might actually affect the dividend growth process as well.

The realized dividend growth rate are defined as the expected dividend growth rate and expected returns and the unobserved shock ε^{d}_{t+1}, where by :

$$\Delta d_{t+1} = g_t + \varepsilon^{d}_{t+1} \tag{8}$$

ε^{d}_{t+1} and g_t are assumed to be orthogonal to each other. $\mathrm{E}(\varepsilon^{d}_{t+1}, g_t) = 0$.

The Campbell and Shiller (1988) log linearized return equation (derived in appendix 1) may be written as :

$$r_{t+1} = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t \tag{9}$$

where $pd_t = E[\log(PD_t)], \kappa$ is an arbitrary constant defined as $log(1 + exp(\overline{pd})) - \rho\overline{pd}$ and $\rho = \frac{\exp(\overline{pd})}{1+\exp(\overline{pd})}$

The equation can be further be reduced to :

$$r_{t+1} = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t \tag{10}$$

To study the dynamics of the price dividend ratio, the process may be written with pd_t being the subject of the formula:

$$pd_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - r_{t+1}$$

The full derivation of the of the price dividend ratio model is explained in appendix 1.

By replacing lagged iterated values of pd_{t+1}in the equation, the process may be written as :

$$pd_t = \sum_{i=0}^{\infty} \rho^i \kappa + \rho^{\infty} pd_{\infty} + \sum_{i=1}^{\infty} \rho^{i-1}(\Delta d_{t+i} - r_{t+i})$$

$$pd_t = \frac{\kappa}{1-\rho} + \rho^{\infty} pd_{\infty} + \sum_{i=1}^{\infty} \rho^{i-1}(\Delta d_{t+i} - r_{t+i})$$

3.2 State Space Model

The state space model makes use of a transition equation and a measurement equation. The Kalman Filter best illustrates the dynamics of the estimates of $\mu_t$and g_t.The model parameters are estimated before making the forecasts. The maximum likelihood estimator is used to obtain the parameters of the Kalman filter. The Maximum likelihood is optimized using the MaxBFGS procedure.

There are two transition equations, one governing the dividend growth rate and the other one governing the mean return:

$$\widehat{g}_{t+1} = \gamma_1 \widehat{g}_t + \varepsilon^g_{t+1} \tag{11}$$

$$\widehat{\mu}_{t+1} = \delta_1 \widehat{\mu}_t + \varepsilon^\mu_{t+1} \tag{12}$$

the two measurement equations are given by :

$$\Delta d_{t+1} = \gamma_0 + \widehat{g}_t + \varepsilon^d_{t+1} \tag{13}$$

$$pd_t = A - B\widehat{\mu}_t + B\widehat{g}_t \tag{14}$$

Equation 12 can be rearranged into 14 such that there are only two measurement equations and only one state space model.

$$\widehat{g}_{t+1} = \gamma_1 \widehat{g}_t + \varepsilon^g_{t+1} \tag{15}$$

$$\Delta d_{t+1} = \gamma_0 + \widehat{g}_t + \varepsilon^d_{t+1} \tag{16}$$

$$pd_{t+1} = (1-\delta_1)A - B_2(\gamma_1 - \delta_1)\widehat{g}_t + \delta_1 pd_t - B_1\varepsilon^\mu_{t+1} + B_2\varepsilon^g_{t+1} \tag{17}$$

Equation 15 defines the transition (state) equation. The measurement equation relates the obsevable variable to theunobserved variables. In our case this is given by equation 16 and 17. A is equal to $\frac{\kappa}{1-\rho} + \frac{\gamma_0 - \delta_0}{1-\rho}$, $B_1 = \frac{1}{1-\rho\delta_1}$, $B_2 = \frac{1}{1-\rho\gamma_1}$.

Equation 16 and 17 relate to the measurement equation. This can be put into a state space form as shown in appendix. Since all the equations are linear, we can implement the Kalman Filter and obtain the likelihood which is maximized over the following vector of parameters.

$$\Theta = (\gamma_0, \delta_0, \gamma_1, \delta_1, \sigma_g, \sigma_\mu, \sigma_D, \rho_{g\mu}, \rho_{gD}, \rho_{\mu D})$$

The individual elements of the state and measurement vectors are given in appendix 2.

The filtered series for the expected dividend growth is just taken to be the first element for the state vector X_t (Refer to appendix 2 for a more detailed explanation). The state vector is derived according to the following update :

$$X_{t|t-1} = FX_{t-1|t-1} \tag{18}$$

In the case of the demeaned expected returns, the expected returns is defined as :

$$\widehat{\mu}_{t-1|t-1} = B_1^{-1}(pd_t - A - B_2\widehat{g}_{t-1|t-1}) \tag{19}$$

3.3 Expected Future Dividends

In this section, we look at the performance of a trading rule using the expected returns and expected dividend growth rate we derived in the earlier section. The theoretical price or net present value of the market equity is derived by using the net present value approach to returns. This is given by equation 20

$$P_t^* = \frac{1}{\mu_t - g_t} E_t[D_{t+1}] \tag{20}$$

where $E_t[r_{t+1}]$ and $E_t[g_{t+1}]$ are the expected returns and growth series that we derive using the state space model.

$E_t[D_{t+1}]$ is a real time valuation of expected dividends, which is computed as :

$$E_t[D_{t+1}] = D_t(1 + g_t)$$

The expected future dividend based on expectations at time t is made up of the realized dividend compounded with the expected growth rate. It should be noted that at time t, the realized value of the dividends is known.

The trading rule option can be summarized as follows:

1) Go long on the equity index if

$$\frac{P_t^*}{P_t} > 1$$

2) Go long on the risk free asset if :

$$\frac{P_t^*}{P_t} < 1$$

where P_t^* is the theoretical price or the net present at time t, whose computation will be set out in the next section. P_t is the actual price

4 Data and Results

The monthly series of treasury bill rates, realized market returns, dividend and price is retrieved from Shiller's website for the period 1899 to 2008. The indicators and returns variables are controlled for inflationary tendencies. The state space model is optimized using data from December 1899 to December 2008. .

Parameter	Coefficient	Std error
$\boldsymbol{\gamma}_0$	0.0012	0.001
$\boldsymbol{\delta}_0$	0.0405	0.0503
$\boldsymbol{\gamma}_1$	0.8056	0.0246
$\boldsymbol{\delta}_1$	0.9988	0.0032
$\boldsymbol{\sigma}_g$	0.0066	0.0005
$\boldsymbol{\sigma}_d$	0.0023	0.0002
$\boldsymbol{\sigma}_\mu$	0.0071	0.0003
$\boldsymbol{\rho}_{g\mu}$	0.6006	0.0326
$\boldsymbol{\rho}_{\mu D}$	0.1013	0.0321

Table 1:
Optimization of State Space Model.The optimal values of the various parameters are given in the second colomn. The associated standard errors are computed analytically from the Hessian Matrix.

The interesting features represented from the table is that unconditional mean of expected dividend growth is vert low. The unconditional expected returns is low at 4 %, which is approximately the same actual returns. The standard errors are quite low in both cases. There is high persistence in both expected returns and dividend growth. Expected Returns tend to be exhibit near unit root behavior. There is also a higher level of variability in the expected returns than in the case of dividend growth. There is also a high positive correlation between the expected returns and dividend growth rate. However expected dividend returns and expected returns tend to exhibit a low correlation.

4.1 Properties of Expected Returns and Dividend Growth

We plot the realized and expected returns and dividend growth series in figures 1 and 2 and report the summary statistics in table 2.

	$\mathbf{R}_t$	$\boldsymbol{\mu}_t$	$\boldsymbol{\Delta}\mathbf{d}_t$	$\mathbf{g}_t$
Mean	0.0036	0.0035	9.4 x 10^{-4}	0.001
Std. Deviation	0.044	0.017	0.013	0.008
Skewness	0.202	-0.812	-0.926	-0.859
Kurtosis	14.41	3.563	9.97	0.11
Jarque-Bera	7105	161	2788	2918

Table 2:
Summary Statistics. The mean, standard deviation,and other moments are reported for the realized and expected values for returns and dividend growth.

Both the mean relaized and expected returns tend to be the same. The volatility tends to be roughly the same. Dividend growth tend to be highly negatively skewed. The kurtosis and Jarque Bera statistic rejects the fact that the distribution could be normal. Stationarity tests and correlation across time for the different series are presented in tables 7,8 and 9. With respect to the stationarity tests, The null hypothesis of stationarity is not rejected by all tests in the realized and expected dividend growth series. However, in the case of expected returns, the results are mixed. The possibility of a non stationary expected returns series is further reinforced by the non stationarity tests. All of the tests do not reject the possibility of a unit root in the series. However, all the other processes are found to be stationary. In terms of correlatedness, both the expected divvidend growth and realised returns have diminishing correlation with their past lags. There is a high contemporaneous relationship between expected and realized dividend growth, which tends to disappear as time increases.

4.2 Results from the trading Rule

In this section, we report the results from applying the trading rule. Before probing deeper, we would like the reader to understand the important role played by the horizon involved. The cumulated returns from the rule may perform worse or as good as the equity market for some periods of time but still may overtake the equity market return for the next period. A good switch in assets when the equity market return is going down or the treasury bill returns is going up may increase the cumulated returns at the terminal horizon. However, in shorter horizons, for instance one month, less number of switches may be made because of market trends. However, over longer horizons, there are more possibilities for the market to be undervalued or overvalued, hence giving rise to arbitrage opportunities.

Figure 4.2 shows the cumulated returns over time.

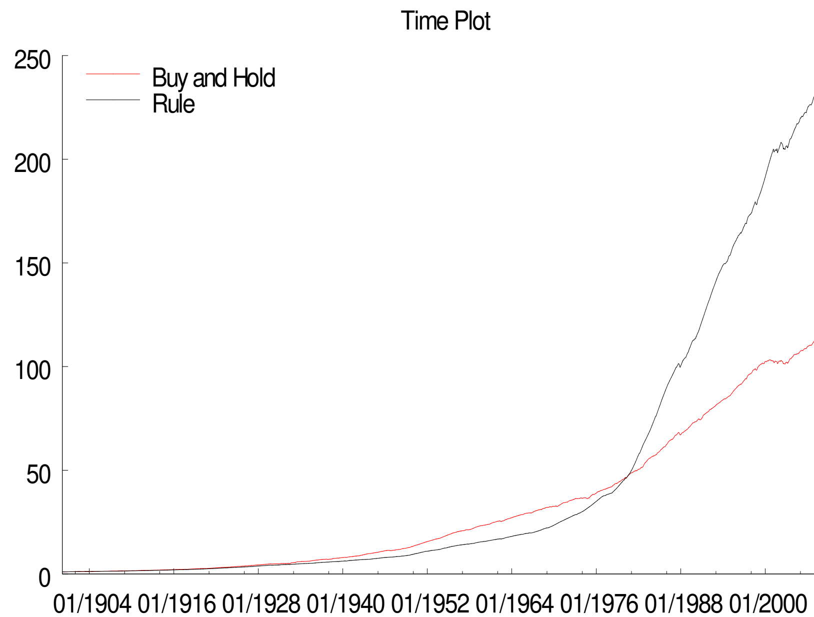

Figure 4.2. **Cumulated Returns** The graphical plot illustrates the cumulative returns over the horizon 1900-2008 from the Buy and Hold Strategy and the Rule.

The rule tends to beat the equity index at the end date of the horizon. However for a long time, the equity index turns out to perform better. Passive investment in the equity index. The plot shows that the market tends to perform better than the trading rule index. However the rule tends to perform better only during the period 1975 to 1990's. Interestingly, the success of the rule might come from the fact that there might have been some shocks during those periods that might have reduced the share prices but with a high net present value of and hence according to the rule required the investor to invest in the market in subsequent periods.

Figure 3 in the appendix D supports the earlier claims. The Trading rule return is plotted against the best return, which is defined as the maximum of the equity and risk free return rate in a particular date. The rule tends to perform badly back in the 1940's, witnessing drops in the return. However the rule makes the most out of the 60's, 70's and 80's. However for the 2006 -2008 crises, we find that the rule does not tend to perform well. The rule would work reasonably well if only mean reversion started happening within the actual date of shifting the asset to the next period when the investor computes the $\frac{P_t^*}{P_t}$ again This is interesting since in this case, we find a situation where mean reversion does not happen instantly. In fact the actual price goes further from its rational value.

The rule makes only 58 % of the times the good decision of selecting the higher return. This is quite low comparatively. Throughout the sample, the number of switches posited by the rule is 208 times, while the best accumulated returns would be reached by making 344 switches. The accumulated wealth is 2.05 times higher than the Buy and Hold strategy. The reason for a higher cumulated return is simply because the switches were made later rather than earlier in the sample. Both strategies have relatively the same standard deviation. The model illustrates that the market is highly underpriced especially during once off events. Figure 4 shows the probability distribution of $\frac{P_t^*}{P_t}$. The figure shows that the distribution is right skewed and has a mean slightly higher than the efficient markets, implying that most of the time the market is slightly underpriced. Hence the rule posits going long for most of the time (54 %) while the best outcome is to stay in the market 54.7 % of the time.

4.3 Tests on the trading rule

Application of the rule is interesting if it can yield higher returns over the short run. We also report the mean return and the respective standard error for the horizon of 12, 24, 36, 48 and 60 years in table 3.

Period	R^{BH}	R^{TR}	S.E^{BH}	S.E^{TR}
1 year	0.044	0.052	0.0034	0.0027
2 year	0.092	0.107	0.0035	0.0028
3 year	0.142	0.166	0.0036	0.0028
4 year	0.194	0.230	0.0036	0.0029
5 year	0.249	0.296	0.0036	0.0029

Table 3:
Cummulated Returns over Horizons. The table provides the cummulated returns over horizons of 12, 24, 36, 48 months for both the model and the Buy and Hold strategy. The standard errors are also reported to illustrate the riskiness of the returns.The Buy and Hold and Trading Rule returns are computed as $R^{BH}(k) = \frac{1}{T-k}\sum_{h=1}^{T-k}\prod_{i=h-k}^{K}(1+R_{m,i})$ and $R^{TR}(k) = \frac{1}{T-k}\sum_{h=1}^{T-k}\prod_{i=h-k}^{K}(1+R_{tr,i})$ respectively.

Interestingly, contrary to the graphical depiction, we find that marginally the trading rule seems to have a higher average return with a lower standard error as well. As the horizon increases, return margin between the rule and the equity index tend to increase as well, average roughly 4.7 % more than the equity index over five years of horizon. The standard error tends to remain more or less stagnant. The increasing margin shows that the rule would be better suited for long horizons since for shorter horizons, the marginal return may be eroded by transaction costs. Hence the rule commands both higher returns and lower risk as illustrated by the figures. The standardized return is obviously higher when the rule is put to use for all horizons involved.

4.4 Robustness of the rule

In this section, we check the robustness of the earlier results. We test for whether there is statistical evidence of the rule beating the market. We apply three test, namely a simple test of paired correlation, the Sweeney's X statistic, and a sampling method.

4.4.1 Test of Paired Correlation

We also test whether the returns from the Buy and Hold strategy are significantly outperformed by the trading rule returns by performing a test of pairwise correlated means. The t-statistic is given by:

$$t(k) = \frac{\overline{R_{BH}}(k) - \overline{R_{TR}}(k)}{S^2_{\overline{R_{BH}}} + S^2_{\overline{R_{TR}}} - 2rS_{\overline{R_{BH}}}S_{\overline{R_{TR}}}}$$

where $t(k)$ refers to the t-statistic for a horizon of k months. $\overline{R_{BH}}$ refers to the mean return on the market (Buy and Hold Strategy) and $\overline{R_{TR}}$ refers to the mean return under the trading rule. $S_{\overline{R_{BH}}}$ refers to the standard deviation on the market return and $S_{\overline{R_{TR}}}$ is the market return under the rule with r being the correlation coefficient. The results are reported in table 4

Period	Paired Correlation	Mean Differences	Std error	t-statistic
1 year	0.368	-0.010	0.0007	-13.99
2 year	0.329	-0.021	0.0014	-14.88
3 year	0.297	-0.035	0.0022	-15.40
4 year	0.268	-0.049	0.0031	-15.69
5 year	0.247	-0.066	0.0041	-15.88

Table 4:
Test of Correlated Means. The right hand side column illustrates the holding period (k). The correlation between the two return series are also reported. the $\overline{R_{BH}}(k) - \overline{R_{TR}}(k)$ refers to the mean difference. The denominator in the test is given by the std error. The degrees of freedom for the 1,2,3, 4, 5 years of horizons were 1295, 1283, 1271, 1259 and 1247 respectively.

We find that the rule significantly outperforms the Buy and Hold strategy. The null hypothesis that the mean difference is zero is rejected in all cases. As the horizon increases, the t-statistic becomes larger. Hence over the long horizon, there is a higher mean reversion towards the equilibrium. The correlation between the rule and thhe Buy and Hold strategy tends to decrease over time which is normal as there is a higher probability of assets being held in bonds for one or more months.

4.4.2 Test of Riskiness

As a further test for the whole horizon, we perform the Sweeney's X statistic designed to take into account the number of periods that the asset is stored in the equity or in the bond market. It is highly probable that when the asset is held in the stock market, it possesses higher risk than when the asset is kept in the bond market. Sweeney's test procedure (1986) is used in an attempt to account for the riskiness of shifting assets. The test is given as follows

$$X = R^{tr} - (1-f)R^{BH}$$

$$\sigma_x = \sigma[f(1-f)/N]^{\frac{1}{2}}$$

where R_{TR} relates to the returns under the trading rule over the period, R_{BH} refers to the returns under the Buy and Hold Strategy, (1-f) is the proportion of months in which the investor's wealth is placed in the equity market, N is the number months the rule is put to the test and σ is the standard error of the monthly returns under the Buy and Hold strategy. The X statistic turns out to be 0.0022 with a standard error of 5.4 x 10^{-5}. The resulting t-statistic rejects the null hypothesis of equal returns for both the rule and the buy and hold strategy after accounting for the riskiness of the equity index.

4.4.3 Sampling with Replacement

The earlier results for the different horizons are based on the whole sample of returns. Hence the results may be biased due to periods of randomly high returns the rule posited. The results may be biased if we have some high values of returns for particular horizons, which may produce higher returns under the rule than under the Buy and Hold Strategy. In the following test, random dates are picked from the larger sample, and the success of the rule vis- a-vis the Buy and Hold strategy are investigated.

The selection of the random dates is derived from a uniform probability distribution. A vector of dates is generated using a random number generator where an equally size vector between zero and one is randomly chosen from the uniform probability distribution. This vector is then multiplied with n-k where n is the end date of the sample and k is the length of horizon we are looking at, for example k = 12, 24.....60. Subtraction of k ensures that returns under the passive Buy and Hold can be calculated for horizon k, especially if the draw is near the end of the sample. The number of dates (initial point) picked out are 20,40, 80 and 160. Adopting a large sample may add power to the results. However, we note that having a large sample may not exclude the presence of dependence in the results. For example, just by luck , there is a higher probability of two dates being drawn near to each other such that some element of double counting may not be excluded. The results are reported in table 5

Period	**20**	**40**	**80**	**120**
1 year	40	40	46.25	43.13
2 year	35	47.5	43.75	43.75
3 year	35	47.5	40	40.63
4 year	40	47.5	42.5	41.88
5 year	45	52.5	41.25	41.88

Table 5:
Performance of Model versus Buy and Hold. The column period relates to the number of months returns have been cummulated. The first row illustrate the sample size of the number of dates randomly picked up. The figures illustrate the percentage number of times the trading rule strictly beats the Buy and Hold strategy.

The table illustrates that the Buy and Hold strategy marginally beats the rule. This is not unexpected. In the cumulated returns plot **??**, the Buy and Hold strategy seems to exceed the rule for approximately 68 % of the whole sample. Hence it seems appropriate that a random date was picked up in time where the rule performed badly. We also report the percentage number of times where the rule beats or equals the Buy and Hold strategy in table 6

Period	**20**	**40**	**80**	**120**
1 year	55	60	61.25	60
2 year	40	52.5	53.75	53.12
3 year	40	47.5	41.25	41.87
4 year	45	50	40	41.25
5 year	45	52.5	42.5	43.12

Table 6:
Performance of Model versus Buy and Hold. This table illustrates the number of times that the rule beats or equals the Buy and Hold Strategy.

The reason for reporting table 5 and 6 separately lies in the subjective preferences of investors. For example an investor mind not mind getting the same return as the Buy and Hold Strategy. As expected the percentage of times, the trading rule beats or equals the Buy and Hold strategy tends to increase. However. In this case, the rule seems to work only for 1 and 2 years of accumulated returns. However for longer periods, the rule does not seem to beat the market. A potential reason that can be highlighted for this phenomenon is that markets tend to suffer from behavioral biases that may hinder the adjustment from the actual price to the theoretical price. Or within a rational expectations context, over the long periods, even if the market is efficient, which implies that $P_t^* = P_t$, marginal changes in the P_t^* through measurement errors in either μ_t and g_t will lead to changes in the theoretical price. Marginal measurement errors in forming the theoretical price P_t^* can lead to an improper selection of the asset.

5 Conclusion

This paper draws attention to a relatively passive trading rule that can be implemented by identifying whether equity indices are overpriced or underpriced. If the equity market is underpriced, the asset should be held long in the equity market and vice versa. A conceptual issue addressed in this paper is the valuation of the asset, which is dependent on the future dividends, expected returns and expected dividend growth rates. All these three series are unidentifiable in real time. The series for the three variables were derived using a state space framework.

The interesting finding of this paper is that the rule does work for long periods of time. The cumulative returns over a long monthly sample was derived and found to be higher for the rule. Even when risk of shifting the asset was accounted for, the rule produced higher returns. However, when a sample of dates was randomly selected, the rule tends to perform worse over longer periods that it was put to use. It seems to beat or equal the buy and hold strategy marginally for short periods of time. The success of the rule, a priori, relies on whether markets are dynamically efficient. However, if markets are not efficient, the rule fails since the actual price will not adjust to the theoretical price set by rational agents. The behavioral story behind it is that there seems to be periods where there is no convergence of actual price to the theoretical price and that would explain the current model However, small measurement errors in the neighborhood of the $\frac{P_t^*}{P_t} = 1$,may actually make lead to an incorrect timing of the market. Moreover, market sentiments might actually perpetuate any disequilibrium from the efficient market price. In other words, it may take more than one month, which is our period of measurement, for the process of mean reversion to take place.

This simple study raises further research questions. For instance, the frequency of the data is quite important since it is highly possible in lower frequency data that any disequilibrium will not take time to be updated in the information set of agents and hence smooth the path of the adjustment of the actual price to the efficient market's price. Moreover, it is worth examining the reliability of using earlier data. Moreover, transaction costs may be included in order to see whether the return from the rule is not arbitraged away.

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A The Net Present Value Model.

Equations 3 , 4 and 5 are represented

$$r_t = \log(\frac{P_{t+1} + D_{t+1}}{P_t}) \tag{21}$$

$$PD_t = \frac{P_t}{D_t} \tag{22}$$

$$\Delta d_{t+1} = \log(\frac{D_{t+1}}{D_t}) \tag{23}$$

The return process can be written as

$$\begin{aligned}
r_t \quad &= \quad \log((\frac{P_{t+1} + D_{t+1}}{P_t}).\frac{D_t}{D_t}.\frac{D_{t+1}}{D_{t+1}}) \\
&\log((\frac{P_{t+1}D_t + D_{t+1}D_t}{P_tD_{t+1}}).\frac{D_{t+1}}{D_t}) && (24) \\
&\log((\frac{D_t}{P_t}.\frac{P_{t+1}}{D_{t+1}} + \frac{D_t}{P_t}).\frac{D_{t+1}}{D_t}) && (25) \\
&\log((\frac{P_{t+1}}{D_{t+1}} + 1)\frac{D_{t+1}}{D_t}.\frac{D_t}{P_t}) && (26) \\
&\log(1 + e^{pd_{t+1})}) + \Delta d_{t+1} - pd_t && (27)
\end{aligned}$$

Assuming the log linearization of Campbell and Shiller (1988) the returns can be written as

$$r_t \simeq \log((1 + e^{pd_{t+1})}) + \frac{\exp(\overline{pd})}{1 + \exp(\overline{pd})} + \Delta d_{t+1} - pd_t$$

$$r_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t$$

Hence,

$$pd_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - r_{t+1}$$

B State Space Model.

In this section, we describe the Kalman filter procedure. The model has been coded using Ox 5. From the paper, there are two measurement equation and one transition equation. Equations 15, 16 and 17 can be written in this form:

$$X_t = FX_{t-1} + R\varepsilon_t$$

$$Y_t = M_0 + M_1 Y_{t-1} + M_2 X_t$$

where $Y_t = \begin{bmatrix} \nabla d_t \\ pd_t \end{bmatrix}$

The variables of the transition equation are $X_t$and $\varepsilon^x_{t+1}$and are made up of the following elements:

$$X_t = \begin{bmatrix} \widehat{g}_{t-1} \\ \varepsilon^D_t \\ \varepsilon^g_t \\ \varepsilon^\mu_t \end{bmatrix} \qquad F = \begin{bmatrix} \gamma_1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \varepsilon^x_{t+1} = \begin{bmatrix} \varepsilon^D_{t+1} \\ \varepsilon^g_{t+1} \\ \varepsilon^\mu_{t+1} \end{bmatrix}$$

The parameters of the measurement equation include parameters of the net present value model to be estimated. These are defined as :

$$M_0 = \begin{bmatrix} \gamma_0 \\ (1-\delta_1) * A \end{bmatrix} \qquad M_1 = \begin{bmatrix} 0 & 0 \\ 0 & \delta_1 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ B_2(\gamma_1 - \delta_1) & 0 & B_2 & -B_1 \end{bmatrix}$$

The variance covariance matrix from the state space model is given by :

$$\Sigma = var \begin{bmatrix} \varepsilon^g_{t+1} \\ \varepsilon^\mu_{t+1} \\ \varepsilon^d_{t+1} \end{bmatrix} = \begin{bmatrix} \sigma^2_g & \sigma_{g\mu} & \sigma_{gd} \\ \sigma_{g\mu} & \sigma^2_\mu & \sigma_{D\mu} \\ \sigma_{gd} & \sigma_{D\mu} & \sigma^2_D \end{bmatrix}$$

The Kalman Filter procedure is given by the following equations :

$$\begin{aligned}
X_{0|0} &= E[X_0] \\
P_{0|0} &= E[X_t X_t'] \\
X_{t|t-1} &= FX_{t-1|t-1} \\
P_{t|t-1} &= FP_{t-1|t-1}F' + R\Sigma R' \\
\eta_t &= Y_t - M_0 - M_1 Y_{t-1} - M_2 X_{t|t-1} \\
S_t &= M_2 P_{t|t-1} M_2' \\
K_t &= P_{t|t-1} M_2' S_t^{-1} \\
X_{t|t} &= X_{t|t-1} + K_t \eta_t \\
P_{t|t} &= (I - K_t M_2) P_{t|t-1}
\end{aligned}$$

The parameters to be optimized are :

$$\Theta = (\gamma_0, \delta_0, \gamma_1, \delta_1, \sigma_g, \sigma_\mu, \sigma_D, \rho_{g\mu}, \rho_{gD}, \rho_{\mu D})$$

C Other Statistical results

	$\mathbf{R}_t$	$\boldsymbol{\mu}_t$	$\boldsymbol{\Delta}\mathbf{d}_t$	$\mathbf{g}_t$
Stationarity test of I(0)				
Robinson-Lobato	9.54 (0)	5.327 (0)	1.263 (0.103)	-0.676 (0.751)
KPSS test	0.574 (<0.02)	0.878 (<0.01)	0.054 (<1)	0.049 {<1)
Lo's RS	1.316 (<0.4)	1.386 (0.3)	0.971 (<0.9)	0.962 (<0.9)
Stationarity test of I(1)				
Augmented Dickey-Fuller	-25.34 (<0.01)	-2.199 (<0.9)	-17.07 (<0.01)	-11.85 (<0.01)
Phillips-Perron	-25.34 (<0.01)	-2.601 (<0.1)	-16.99 (<0.01)	-11.75 (<0.01)
DF-GLS	-25.34 (<0.01)	-2.201 (<1)	-16.74 (<0.01)	-11.76 (<0.01)
P	0.549 (<0.01)	1.819 (<0.01)	0.151 (<0.01)	0.160 (<0.01)

Table 7:
Tests of Stationarity. The table reports the computed statistics for the tests of stationarity. The first three tests have for null hypothesis that the series is stationary, while the rest have the null hypothesis of a nonstationary series. The figures in brackets are the p-values.

Lag	Δd_t**vs** Δd_{t-j}	g_t**vs** g_{t-j}	Δd_t**vs** g_{t-j}	g_t**vs** Δd_{t-j}
0.	1	1	0.649	0.649
1.	0.636	0.807	0.519	0.962
2.	0.506	0.643	0.420	0.781
3.	0.386	0.520	0.402	0.626
4.	0.387	0.470	0.349	0.489
5.	0.332	0.416	0.319	0.448
6.	0.317	0.374	0.261	0.395
7.	0.258	0.317	0.215	0.366
8.	0.224	0.264	0.160	0.314
9.	0.165	0.208	0.125	0.268
10.	0.128	0.165	0.112	0.212
11.	0.117	0.141	0.092	0.167
12.	0.103	0.116	0.089	0.144

Table 8:
Correlation of Realized and Expected Dividend Growth. The table illustrates the correlation of realized and expected dividend growth rates at specific lags.

Lag	R_t**vs** R_{t-j}	μ_t**vs** μ_{t-j}	R_t**vs** μ_{t-j}	μ_t**vs** R_{t-j}
0.	1	1	0.255	0.255
1.	0.340	0.992	0.353	0.220
2.	0.097	0.981	0.372	0.211
3.	0.038	0.969	0.365	0.215
4.	0.102	0.960	0.353	0.211
5.	0.155	0.951	0.351	0.205
6.	0.145	0.942	0.356	0.198
7.	0.118	0.931	0.362	0.193
8.	0.113	0.921	0.361	0.189
9.	0.103	0.911	0.357	0.186
10.	0.115	0.901	0.357	0.179
11.	0.097	0.890	0.360	0.176
12.	0.071	0.88	0.359	0.177

Table 9:
Correlation of Realized and Expected Returns. The table illustrates the correlation of realized and expected returns at specific lags.

D Graphical Plots

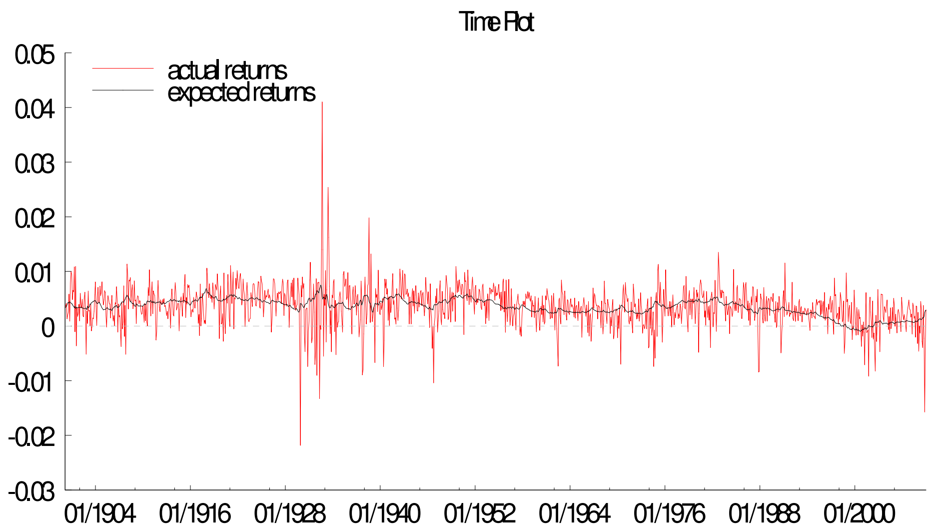

Figure 1: Plot of Expected and Realized Returns

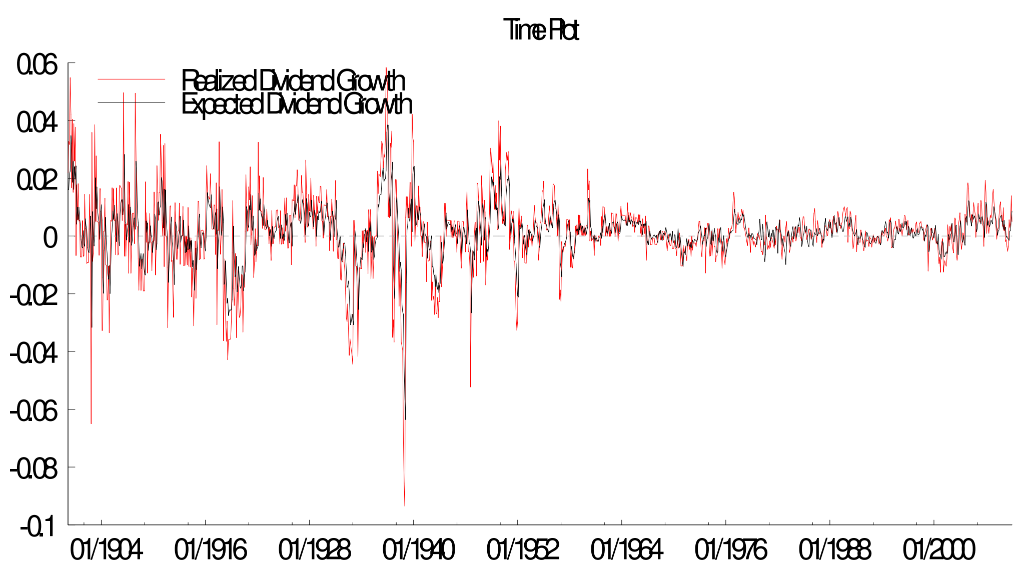

Figure 2: Plot of Expected and Realized Dividend Growth Rate

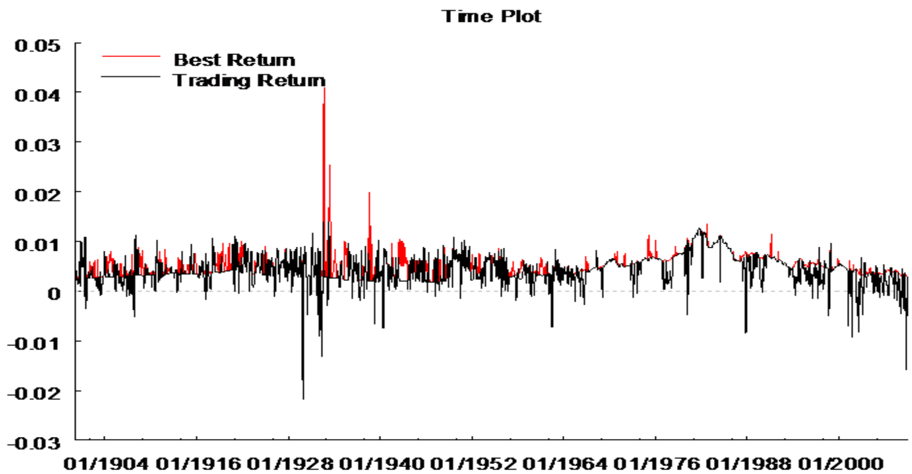

Figure 3: Plot of Trading Rule Return and Best Return.

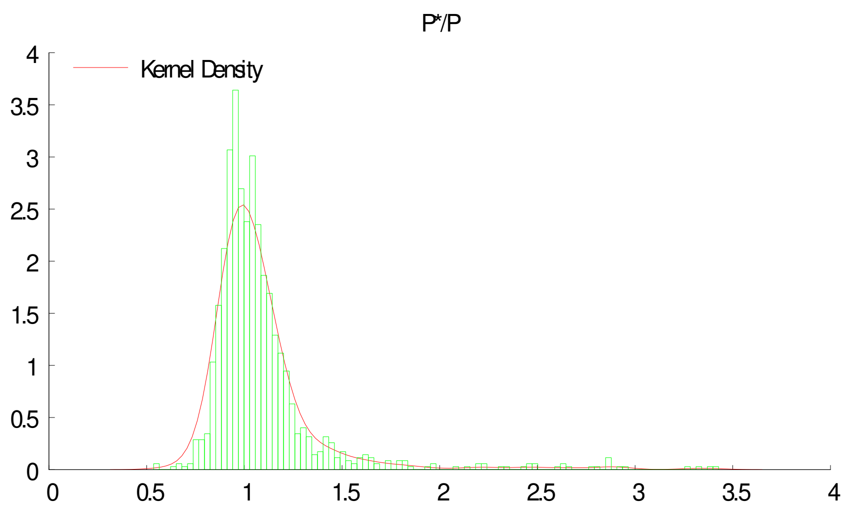

Figure 4: Probabibility Distribution of P_t^*/P_t