Revisiting Indicators of Public Debt Sustainability: Capital Expenditure, Growth and Public Debt in India

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Abstract

The paper tests whether productive expenditures share a long run relationship with debt to GDP ratio by using a multivariate time series framework. The theoretical model is based on dynamic optimization of utility and productive expenditure with respect to capital and debt. Literature on growth theory has suggested that all less productive expenditures can have a negative effect on the growth rate of real GDP per capita until the optimal level of productive expenditure is reached. This would indeed lead to higher level of debt as growth rate will be reduced. Aggregate yearly data for India covering the period 1980-2009 have been used. The CAPRATIO and Debt to GDP ratio are cointegrated. VAR modeling with error correction reveals that the model can be used for forecasts. The regression coefficient between the two variables is negative, signifying the inverse relationship. Having proved the hypothesis of an inverse long run relationship between the two variables, a new indicator based on the Government Inter-temporal budget constraint is suggested, revolving around capital expenditure.

JEL Classification: 3 C, 30 P

Keywords: Public Debt sustainability indicators, Capital Expenditure, Growth.

1. Introduction

Public debt sustainability is vital for both developing and developed nations. While recent literature has laid great emphasis on analyzing this issue, the approach has always been biased towards the revenue account of the government.

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In some countries, especially India, this problem has actually been reduced to a puzzle, wherein, the reason for why the country has not landed into a crisis is not clear given its non stationary public debt. This paper aims at validating if debt sustainability should be measured in terms of a new parameters more suited for developing countries like India namely the change in the composition of public expenditure from current towards capital. Public expenditure is generally classified as consumption and investment expenditure. A positive shift should be inversely proportional to public debt and deficits.

The expenditure side of the government in sustainability debates has always been neglected based on the premise of fiscal policy serving as a stabilization process. Instead, fiscal policy can actually be used to promote growth and long run welfare of a country. It is not that the issue has been forgotten, in fact, it was never stressed upon thinking taxation is much easily controlled (Blanchard 1991). Paul. A. Samuelson in his paper on aspects of public expenditure theories[24] said "Economic theorists have done work of high quality and great quantity in the field of taxation. Public expenditure seems to have been relatively neglected." Even the famous treasure of *A study in public finances* by Pigou devoted most of its attention to taxes with only half a dozen pages on public expenditure and very little on pure public expenditure. However, recent revival of interest in growth theory has also revived interest among researchers in understanding the role of elements of public expenditure that bear significant association with economic growth. And this had led to deep analysis of the role of current and capital expenditure and evaluation of their relationship with economic growth. Traditional theories of macroeconomics which talk of a robust fiscal policy to combat the problem of public debt accumulation [14] stress on the fact that government must redirect expenditure towards sectors where they will see an improvement in the long run, in other words, the composition should be changed towards capital expenditure. A recent analysis by (Bose, Osborne, Haque 2003) suggest some very relevant empirical findings related to the topic in discussion. Their analysis is spread over a belt of developed and developing nations, keeping in mind the peculiarities of the developing world block. They find that the share of capital expenditure in GDP, is positively and significantly correlated with economic growth, while the growth effect of current expenditure is insignificant for most of the countries. This makes capital expenditure and it's growth rate a very important factor in determining fiscal sustainability. Another recent study by (Gupta, Clements, Granados 2005) finds that composition of public outlays matters. Countries where spending is concentrated on wages tend to have lower growth, while those that allocate higher shares to capital and now wage goods and services enjoy faster output expansion[1, 2]. A caveat is that once the optimal level of capital expenditure has been reached, the composition of public expenditure needs to be revisited(Devarajan, Swaroop, Zou 1996). Hence, a comprehensive analysis would also imply deriving an expression for 'optimal capital expenditure'.

More specifically a number of economists have studied the debt sustainability issue for India. Buiter and Patel(2004) used the traditional stationarity tests Phillips and Perron(1998) and KPSS and others(1992). The paper argues that while deficits in India are large , at least in the short run the risk of a deficit-induced crisis is minimal. Their analysis was among one of the first contributions to time series based empirical studies on public debt sustainability for India. Jha and Sharma(2004)[18] performed a more extensive analysis on this issue by testing for cointegration between public expenditure and revenue. They argue that if the two series are stationary or stationary in first differences but cointegrated, Indian public debt is sustainable. Their empirical analysis is based on data covering both the pre and post independence period 1871-1997. Since their analysis suggests that the revenue and expenditure series are I(1) and cointegrated with regime shifts, Indian public debt may not be unsustainable. While the above two studies dealt with the issue of debt sustainability only for the Central Government, Goyal, Kundarapakam et.al(2005)[12] analyzed the same issue for all levels of government. They test for conventional stationarity tested as was done by Buiter and Patel(2004) but employ the Gregory and Hansen tests of cointegration with structural breaks. By addressing the issue of regime shift, their paper finds that while the fiscal stance of the Central and the State Government at the individual level is unsustainable, it is weakly sustainable for the combined finances as it nets out inter-governmental financial flows. Thus, claims about sustainability of India's public finance, made on the basis of the assessment of individual finances and neglecting inter-governmental flows and the possibility of regime shifts seem exaggerated.

We make two contributions to the theoretical literature on public debt sustainability in developing countries and one contribution to empirical literature on debt sustainability in India. First, we re frame the dynamic optimization problem(Devarajan, Swaroop, Zou 1996) of maximizing consumption with respect to private capital by introducing productive expenditure as a control variable and the law of motion of debt as another state variable. Second, even though it is acknowledged that capital expenditure does share a positive correlation with economic growth and composition of expenditure tilted towards investment can help in enjoying faster output expansion, very less literature exists on examining the long run relation between capital expenditure ratio in total expenditure and public debt in a co-integrating framework. We bridge this gap in the literature by testing for cointegration and then representing the relationship in a VECM framework[1], and evaluating its suitability to forecast debt. India with its federal structure, increasing growth rate, yet exceedingly high public debt levels seems to be a good case in the point.

Third, having established the long run relationship we reformulate the Government Inter-temporal budget constraint proposed by Blanchard(1991) to derive the 'capital expenditure gap' indicator. While this is just a slight variant of the 'tax gap indicator' suggested by Blanchard it would be useful in analyzing the gap between the current capital expenditure and the optimal level, which marks the threshold for policy makers suggesting fiscal consolidation related to attempted increase in capital expenditure. A more important suggestion of the

[1] Owing to statistically significant cointegration results discussed in detail in Section 4 of the paper.

paper is the 'capitalexpenditure ratio ' indicator which clearly on basis of our empirical results could be much more useful for policy makers to forecast debt . Modeling debt taking advantage of the long-run relationship between capital expenditure and debt could improve the precision of forecasting debt for developing nations. Thus, 'capitalexpenditure ratio' can be used as a predictor of debt dynamics. At the same time, fiscal consolidation aimed at restructuring expenditure can also help in reducing debt levels.

2. Public Sector: Expenditure, Budget Constraint and Growth

2.1 Public Expenditure and Growth

Since the 1960s, researchers have been looking at the relationship between fiscal policy and the economy's growth rate. Among some very important contributions, in 1970, Arrow and Kurz, developed a model where consumers derive utility from private consumption as well as public capital stock. The literature on endogenous growth theories has further generated models linking public spending with economy's long-term growth rate. One of the economists who analyzed this issue both theoretically to an extent and empirically was Robert Barro in his paper on '*Government spending in a simple model of endogenous growth*'[3]. In this paper Barro introduces the government in the utility function to be maximized along with the private sector and classifies the expenditure as consumption and investment expenditure. His empirical findings[4] suggest that all non productive expenditures[2] can have a negative effect on the growth rate of real GDP per capita in the long term. This would indeed lead to higher level of debt as growth rate will be reduced. However, a caveat in both these models is that public spending only affects the economy's transitional growth rate, while the steady-state growth rate remains unaltered. Hence, these models cannot be used until the effect of public spending components on growth and debt respectively is accounted for as endogenous. Devarajan et.al(1996) is an improvement on the earlier models as they relax the assumption of the exogenous public spending. They build an optimization problem with two types of expenditure, namely productive and unproductive. These are optimized with respect to capital stock in the economy to de termine the relation between these expenditures and the growth rate of consumption.

Among more extensive empirical studies, Bose et.al(2003) examine the growth effects of government expenditure for a panel of 30 developing economies with a focus on sectoral expenditures during the 1970s and 80s. This study is important

[2]While Barro calls this element non productive expenditure Landau(1983) calls these consumption expenditure and they have a close bearing to the definition of current expenditure used in this model. Expenditure that provides long term stimulus to growth and thus helps in reducing public debt

in assessing the empirical analysis on the said topic for developing countries, as previous literature was more inclined towards analysis on developed countries or a mixed sample of developing and developed countries[3] . Their main empirical finding is that the share of government capital expenditure in GDP is positively and significantly correlated with economic growth, while the growth effect of current expenditure is insignificant for our group of countries. The study classifies the variables into three distinct sets: I, M and Z. The set I consists of variables that commonly appear as conditioning variables in growth regressions. The set Z includes variables that often have been included in previous studies as indicators for monetary policies, trade policies, and market distortion. Finally, the set M consists of variables that are of particular interest for the present study, namely Central Government expenditures and their major components at aggregate and sectoral levels. The first base regression is done between M and I. Out of the twenty categories of public expenditure examined, they report the results only for the six categories (total investment, investment in education, investment in transport and communication, total expenditure on education, total expenditure on transport and communication and total expenditure on defense) that they find to display a significant association with growth, using a 10 percent significance level. While this confirms the hypothesis of capital expenditure being vital for economic growth, the second part of the paper moves ahead to also consider variables in the inter-temporal budget constraint explained in more detail in part III. Fearing the possibility of an omitted variables bias, in the above results, they integrate the regression performed with the inter-temporal budget constraint.[4] The component left behind from the budget constraint is non-tax revenue. This omission is based on the theoretical prediction by Barro(1990) that variation in non-distortionary tax revenue is likely to create insignificant growth effects. This results in only three expenditures of the 6 mentioned above to be significant namely capital expenditure, outlay in education sector and investment expenditures in the education sector.

Gupta et.al (2005) test the effects of fiscal consolidation and expenditure composition on economic growth in a sample of 39 low-income countries during the 1990s. The most important contribution of the paper is its extensive data coverage and an in-depth econometric evaluation. The results of the study confirm that there is a strong link between public expenditure reform and growth , as fiscal consolidations achieved through curtailing current expenditures are, in general, more conducive to growth. Even they mention that for the developed economies, more choice can be exercised over expenditure priorities and higher public spending even if of current expenditure form could not necessarily contract economic activity. Their simple correlation analysis[5] shows that higher capital outlays are associated with more buoyant growth , while higher current expenditures and domestic financing of deficit are associated with less favorable economic performance. Further, three main regression models are used, the

[3]Exceptions include Landau(1986) and Devarajan et al(1996)

[4]They include the expenditure side of the budget constraint, and we explicitly include tax revenue (TX) and the budget surplus/deficit (GD), both as percentages of GDP

[5]Correlation coefficients are significant for capital outlays

first two being the most relevant ones. Model A specifies a relationship between expenditure items and deficit, while Model B analyzes the relationship between expenditure composition and growth. The models are estimated in both levels and first differences to differentiate between short-term and long-run effects. In Model A, a one percentage point of GDP increase in spending on wages and salaries reduces growth by half a percentage point, while a one percentage point increase in the ratio of capital outlays to GDP increases growth by more than half a percentage point. Expenditures on other goods and services are also found to raise the growth rate, but only in the short term. Interest payments have a statistically insignificant impact on growth. Finally, in the models that assess the impact of expenditure composition directly (Models B and C), the coefficients for spending on wages are significant, but only in the long run. The share of capital expenditures in total outlays is positively related to growth under both A and B model specifications. Empirical literature with similar results include Landau(1983) and Summers and Heston(1984). They used 115 countries[21] in their analysis , using data on government consumption. The data used and analysis done was a pooled crossection , time series analysis, using data averaged over 5 year intervals.

2.2 Government Inter-temporal Budget Constraint

After having discussed the literature on relationship between public expenditure, growth and debt in the previous section, we discuss some important existing indicators of debt sustainability. Blanchard(1990) proposed two indicators of fiscal sustainability, the primary gap indicator (PGI) and the Tax Gap Indicator (TGI). PGI calculates the adjustment in the primary balance needed to stabilize the outstanding of public debt ratio. $PGI = ps = ps* = -(d + (r - g)b$ where ps is the current primary balance, $ps*$ is the constant primary balance that stabilizes the debt ratio at its current level, whereas r and g are the real rate of interest and the growth rate. The value of r and g are in constant value over the last 10 years or so. However, this gap does not capture the change of economic fundamental and policies. Therefore, Blanchard proposed another indicator.

This other indicator knows as the *'Tax gap indicator'*[5] is derived from the inter temporal budget constraint. The original tax gap indicator answers a fundamental question which asks about a desired tax rate to ensure sustainability. In other words, it is the difference between existing and desired tax rate. Using a similar framework we can derive the desired productive expenditure to ensure sustainability and compare it with existing categories of public expenditure. This would help further in understanding by how much should productive expenditure be changed and henceforth, the effect on the composition of public expenditure. Thus we start once again with the basic equation of the inter temporal budget constraint for the government, The law of motion of the dynamic budget constraint is mathematically represented by (1). Here $\frac{dB}{ds}$ represents the law of motion of public debt, G_1 represents public productive expenditure, G_2

represents less productive expenditure, H represents the total transfers, T the total taxes and r is the real interest rate. D represents the deficit as a whole :

$$\frac{dB}{ds} = G_1 + G_2 + H - T + rB = D + rB \tag{1}$$

Taking ratios to GDP for all variables in the equation we obtain (2). The mathematical passage to arrive at (2) is as follows.
$B^{\cdot} = D + rB$, $b^{\cdot} = {}^{d(B/Y)}/_{dt} = (B^{\cdot}Y - BY^{\cdot})/Y^2$,
$b^{\cdot} = {}^{B^{\cdot}}/Y - \frac{B}{Y} \cdot \frac{Y^{\cdot}}{Y}$, $b^{\cdot} = \frac{(D+rB}{Y} - b\theta = d + rb - b\theta = d + (r-\theta)b$.
θrepresents the rate of growth of the economy.

$$\frac{db}{ds} = g_1 + g_2 + h - t + (r-\theta)b \tag{2}$$

Integrating this equation forward we get the final inter temporal budget constraint

$$\int dexp - (r-\theta)sds = -b_0 \tag{3}$$

Now we can derive the '*expenditure gap*' indicator as follows by substituting $d = g_1 + g_2 + h - t$:
$\int (g_1 + g_2 + h - t)exp - (r-\theta)sds = -b_0$
$\int (g_2 + h - t)exp - (r-\theta)sds + \int g_1 exp - (r-\theta)sds = -b_0$
$\int (g_2 + h - t)exp - (r-\theta)sds + b_0 = -\int g_1 exp - (r-\theta)sds$
$\int (g_2 + h - t)exp - (r-\theta)sds + b_0 = g_1 \frac{exp-(r-\theta)s}{(r-\theta)}$
$g_1^* = (r-\theta)(\int (g_2 + h - t)exp - (r-\theta)sds + b_0)$

$$g_1^* = (r-\theta)[(\int (g_2 + h - t + (r-\theta)b_0)(exp - (r-\theta)sds)] \tag{4}$$

The above expression (4) defines the threshold of the level of productive expenditure in the economy. Revisiting the composition of expenditure cannot directly decrease debt, however, focusing on productive spending can help in handling the debt situation in a better way. Nevertheless, current expenditure cannot always be ignored owing to current pressing needs of the economy. Hence, g_1^*defines the optimal level of productive expenditure in the economy. In the part on model and method we suggest an indicator based on the expenditure aspect of public debt.

3. Model and Method

3.1 Optimization Model on Productive Expenditure and Growth

In this section we develop a mathematical and economic relationship model between composition of public expenditure, growth and public debt. The model expresses the difference between productive and unproductive expenditures by how a shift in the mix between the two alters the economy's long-term growth rate and public debt. The aggregate production function has capital stock, k, and two types of government spending , g_1[6] and g_2[7]. g_1 represents expenditure that contributes to future productivity of output and hence is part of capital accumulation by the government while $g_2$contributes only to current output. g_2 enters the capital stock equation indirectly. Considering the functional form to be CES(constant elasticity of substitution) then the relationship can be expressed as

$$y = f(k, g_{1,}g_2) = \left[\alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi}\right]^{\frac{-1}{\xi}} \tag{5}$$

where $\alpha > 0$, $\beta \geq 0$, $0 \leq \gamma < \beta$, $\alpha + \beta + \gamma = 1$, $\xi \geq -1$

The share , $\lambda(0 \leq \lambda \leq 1)$, of total government expenditure which goes toward g_1 is given by

$g_1 = \lambda g$ and $g_2 = (1 - \lambda)g$

Utility in this model is assumed to be in the iso-elastic form (7) and the representative agent maximizes his welfare by choosing consumption, c based on the utility function and the constraints discussed further on .

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{6}$$

We consider an optimal control problem with (8) as the function to be maximized with two state variables, k and b , namely the capital stock and public debt with their equations of motion as represented by (9) and (10) and two control variables, c and g_1, consumption and productive expenditure respectively. (5), (6) and (7) are also constraints in the optimization problem. n this model we refrain from discussion on transfers done by the government, consumer preferences are iso elastic, government productive expenditure contributes to future productive capacity while less productive expenditure contributes only to current output.

Maximize

$$U = \int_0^\infty u(c)e^{-\rho t}dt \tag{7}$$

[6]Capital expenditure, which has been empirically found to give high stimulus to growth up til a certain level

[7]Current expenditure, which is considered to give less stimulus to growth

subject to

$$k^{\cdot} = (1-\tau)y - c + g_1 \tag{8}$$

$$b^{\cdot} = g_1 + g_2 - t + (r-\theta)b \tag{9}$$

Instead of assuming that the government only obtains taxes and does spending, we introduce government debt as well in the form of the dynamic inter temporal budget constraint (2) where t represents the taxes collected by the government and r and θ are the interest rate on debt and growth rate of output in the economy respectively.

Then we have the current value Hamiltonian

$$H(c,h) = \frac{c^{1-\sigma}-1}{1-\sigma} + \mu_k \left[(1-\tau)\left\{\alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi}\right\}^{-1/\xi} - c + g_1\right]$$

$$+\mu_b \left[g_1 + g_2 - t + (r-\theta)b\right]$$

where μ_k and μ_b represent the shadow prices of k and b respectively. From this we get the conditions

$$c^{-\sigma} = \mu_k \tag{10}$$

$$\mu_k \left[(1-\tau)\beta g_1^{-\xi-1}\left\{\alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi}\right\}^{-(1+\xi)/\xi} + 1\right] + \mu_b = 0 \tag{11}$$

$$(1-\tau)\beta g_1^{-(\xi+1)}\left\{\alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi}\right\}^{-(1+\xi)/\xi} + 1 = -\frac{\mu_b}{\mu_k} \tag{12}$$

(13) represents the co state equation with productive expenditure g_1 and the shadow price of debt μ_b. Since the relationship between the two is inverse we can conclude that a positive movement towards productive expenditure helps in repayment of debt much more smoothly over time.

In addition to the $k^{\cdot}$ and $b^{\cdot}$ equations given in the problem statement, the maximum principle requires the following equations of motion for the co state variables:

$$\mu_k^{\cdot} = \mu_k \left[(1-\tau)\alpha k^{-\xi-1}\left\{\alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi}\right\}^{-(1+\xi)/\xi}\right] + \rho\mu_k \tag{13}$$

$$\mu_b^{\cdot} = \mu_b(r-\theta) + \rho\mu_b \tag{14}$$

Since there are four differential equations, the system cannot be analyzed with a phase diagram. But our main question of interest is to see the relationship between productive expenditure, growth rate and public debt. So how does the productive expenditure affect the shadow price of debt? What is the relation between the proportion of productive expenditure λ with respect to growth rate of the economy and growth rate of consumption?

The basic features of such a steady state is that all the state and control variables grow at the same rate. We thus have in steady state

$$\dot{\mu_k} = -\sigma c^{-\sigma-1} \dot{c} \tag{15}$$

Using (11) we get

$$\frac{\dot{\mu_k}}{\mu_k} = -\sigma \frac{\dot{c}}{c} \tag{16}$$

Now we substitute (14) into (17) and obtain

$$\frac{\dot{c}}{c} = -\frac{\left[(1-\tau)\alpha k^{-\xi-1}\left\{\alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi}\right\}^{-(1+\xi)/\xi}\right] + \rho}{\sigma} \tag{17}$$

Now we use the other constraints and substitute them in (18) and get (19) $g_1 = \lambda g$ and $g_2 = (1-\lambda)g$, hence $c_\theta = \frac{\dot{c}}{c}$, the growth rate of consumption

$$c_\theta = -\frac{\left[(1-\tau)\alpha\left[(\alpha + (g/k)^{-\xi}(\beta\lambda^{-\xi} + \gamma(1-\lambda)^{-\xi}))\right]^{-(1+\xi)/\xi}\right] + \rho}{\sigma}$$

$$\frac{dc_\theta}{d\lambda} = \frac{\alpha(1-\tau)(1+\xi)(g/k)^{-\xi}\left[\beta\lambda^{-(1+\xi)} - \gamma(1-\lambda)^{-(1+\xi)}\right]}{\sigma\left\{\left[(\alpha + (g/k)^{-\xi}(\beta\lambda^{-\xi} + \gamma(1-\lambda)^{-\xi}))\right]^{-(1+2\xi)/\xi}\right\}} \tag{18}$$

Expression(19) should be positive if CAPRATIO should have a positive effect on growth. The right hand side this equation will be positive if $(1+\xi)\left[\beta\lambda^{-(1+\xi)} - (1-\lambda)^{-(1+\xi)}\right] > 0$. It follows that $\xi \geq -1$, hence $\frac{d\theta}{d\lambda} > 0$ if $\left(\frac{\beta}{\gamma}\right)^{\eta} > \frac{\lambda}{1-\lambda}$ where $\eta = 1/(1+\xi)$ is the elasticity of substitution. Since λ is an increasing proportion until optimality is reached, the left side will always be bigger than the right hand side. A caveat, is that the increase in growth on account of capital expenditure apart from depending on β and γ, depends also on λ, which is the initial share of productive expenditure. [8]Thus if initial $\lambda$is very high, $\beta > \gamma$ may not necessarily raise the growth rate. However, such debates would be more relevant for developed countries where the productive threshold of capital/productive expenditure has already been reached.

Since $\frac{\dot{\mu_k}}{\mu_k} = \frac{\dot{\mu_b}}{\mu_b}$ characterizes steady state, we can equate

$$\theta = (r - \left[\alpha(1-\tau)\left\{\alpha + (g/k)\right)^{-\xi}(\beta(\lambda)^{-\xi} + \gamma((1-\lambda))^{-\xi}\right\}^{-(1+\xi)/\xi}\right]) \tag{19}$$

[8]This result is in consistence with that of Devarajan et.al. In (20) we obtain such an analytical condition also for growth rate of the economy

This equation shows the growth of output in the economy and its relationship with composition of public expenditure. The analytical condition obtained here is the same as that of the growth of consumption(18).

$$\frac{d\theta}{d\lambda} = (1-\tau)(1+\xi)\alpha\left\{\alpha + (g/k)^{-\xi}(\beta(\lambda)^{-\xi} + \gamma((1-\lambda))^{-\xi}\right\} *^{-(1+2\xi)/\xi} \tag{20}$$

$$(g/k)^{-\xi}\left[\beta\lambda^{-(1+\xi)} + \gamma(1-\lambda)^{-(1+\xi)}\right]$$

Summing up, the two main results are given by (13) and (20). The first is that the relationship between productive expenditure and growth rate of the economy is governed by initial shares of expenditures as well as the current proportions. The ratio of initial shares would always be more than that of current proportion of productive expenditure. Thus there is a constraint on the amount up til which the investment expenditure can be increased in the economy. Additionally, an increase in productive expenditure decreases the utility denominated value of debt which means that the welfare cost of an increase in government debt falls. Further, part of government expenditure considered to be highly productive is used to generate productive capacity in the future reducing the burden of debt. This helps the government in choosing a consumption path which helps in reabsorbing the value of debt slowly and extends the repayment time period. Thus it would be worthwhile to develop indicators of debt sustainability based on λ, the productive expenditure component. In the next section we derive an indicator based on λ, which has empirically been seen as the capital expenditure ratio[9] in total expenditure(CAPRATIO).

3.2 Proposed Indicator : CAPRATIO

We propose an indicator that can be called as CAPRATIO defined as Capital Expenditure Ratio to aggregate expenditure (λ) and mathematically can defined as

$$\lambda = \frac{g_{ca}}{(g_{ca} + g_c)} \tag{21}$$

In line with the growth theories outlined above this indicator measures the share of capital expenditure in total public expenditure. As λ increases, it is expected that the public debt levels will react inversely for developing countries. This unique feature makes it more feasible to be used for debt sustainability simulations in comparison to the existing indicators. It would be noteworthy to understand how this indicators maps with the Government inter-temporal budget constraint for policy and simulation purposes.

Having understood that CAPRATIO affects growth positively, and growth and debt share a negative relationship, we can infer a negative relationship

[9]See Table 1

between CAPRATIO and debt intuitively. However, it is imperative to test this empirically, done in part IV. Since the results confirm this intuition we can represent this relationship in the form of an indicator that can be a predictor of debt dynamics.

(2) provides us with a representation of the inter-temporal constraint with all variables in a ratio to GDP form. (22) can be used to substitute for $(g_{ca}+g_c)$ back in (2). The resulting equation is as follows:

$$\int \frac{db}{ds} = \int \frac{g_{ca}}{\lambda} + h - t + (r-\theta)bds \tag{22}$$

$\int \frac{g_{ca}}{\lambda} + h - t)exp - (r-\theta)sds = -b_0$
$\int (h-t)exp - (r-\theta)sds + \int \frac{g_{ca}}{\lambda} exp - (r-\theta)sds = -b_0$
$\int (h-t)exp - (r-\theta)sds + b_0 = -\int \frac{g_{ca}}{\lambda} exp - (r-\theta)sds$
$\int (h-t)exp - (r-\theta)sds + b_0 = \frac{g_{ca}}{\lambda}\frac{exp-(r-\theta)s}{(r-\theta)}$
$\frac{g_{ca}}{\lambda} = (r-\theta)(\left[\int (h-t)exp - (r-\theta)sds\right] + b_0)$

$$\lambda = \frac{g_{ca}}{(r-\theta)(\left[\int (h-t)exp - (r-\theta)sds\right] + b_0)} \tag{23}$$

(24) represents the capital expenditure ratio indicator. While dynamic future simulations are not in the focus of the paper, the empirical analysis in the next part will aim to test the theoretical hypothesis of inverse relationship between $\lambda$and b_t with the use of multivariate time series analysis followed by static VECM based simulations. This is mandatory for understanding the precision of this indicator for forecasting purposes.

3.3 Empirical Test

The long-run relation between debt to GDP ratio(b)and Capital expenditure ratio(λ) can be expressed as in (24):

$$b_t = \frac{\beta}{\lambda_t} - \gamma\alpha \tag{24}$$

[10], where $\gamma = 1$ if $r = \theta$ which means that the interest rate on debt equals the growth rate of the economy.

A unit coefficient ($\beta = 1$) would imply that capital expenditure ratio does affect the debt to GDP level in a perfect market. However, in reality since we are in an imperfect market this parameter should exceed one.

We apply Johansen's(1992,1995) multivariate method to estimate the long-run relation[11] between debt to GDP ratio(b_t)and Capital expenditure ratio(λ_t). Under this approach, a system of n endogenous variables can be parametrized into a vector error correction model:

[10] Detailed Derivation in appendix in the section of VECM Properties

[11] The parameters of equation(25)

$$\triangle X_t = \mu + \Gamma_1 \triangle X_{t-1} + \Gamma_2 \triangle X_{t-2} + ... + \Gamma_{k-1} \triangle X_{t-k+1} + \Pi X_{t-k} + \varphi D_t + u_t \quad (25)$$

where $X_t$is an (n ,1) vector ; $\Gamma_i$and $\Pi$are (n , n) coefficient matrices ; $D_t$are deterministic components, such as seasonal and impulse dummies ; $\mu$is a constant term ; k is the lag length ; and $u_t$is a vector of normally and independently distributed error terms. In our system, $X_t = [b_t, \lambda_t]$is a 2 * 1 vector, and $\Gamma_i$and $\Pi$are (2 * 2) coefficient matrices. A cointegrated system implies that $\Pi = \alpha\beta^{'}$is reduced rank, r, for $r < n$.

To understand this in more detail, we can take a deeper look at a multivariate generalization of single equation dynamic models, known as VAR

$$\gamma_t = \begin{bmatrix} b_t \\ \lambda_t \end{bmatrix} = A_1 \begin{bmatrix} b_{t-1} \\ \lambda_{t-1} \end{bmatrix} + A_2 \begin{bmatrix} b_{t-2} \\ \lambda_{t-2} \end{bmatrix} + ... + A_n \begin{bmatrix} b_{t-n} \\ \lambda_{t-n} \end{bmatrix} + u_t \quad (26)$$

Now representing this model in levels and first differences, we subtract $\gamma_{t-1}$from both sides of the VAR ; we obtain

$$\triangle \gamma_t = (A_1 - 1)\gamma_{t-1} + A_2\gamma_{t-2} + ... + A_n\gamma_{t-n} + u_t$$

Further we subtract $(A_1 - 1)\gamma_{t-2}$from both sides until $n - 1$

$$\triangle \gamma_t = \Pi_1 \triangle \gamma_{t-1} + \Pi_1 \triangle \gamma_{t-2} + ... + \Pi \triangle \gamma_{t-n} + u_t \quad (27)$$

$$= \sum_{i=1}^{n-1} \Pi_i \triangle \gamma_{t-i} + \Pi \triangle \gamma_{t-n} + u_t \quad (28)$$

where

$$\Pi_i = (I - \sum_{j=1}^{i} A_j)$$

$$\Pi = (I - \sum_{j=1}^{n} A_i)$$

The above equation (28) is the parametrization of the VAR model as a VECM[12]. The next section throws more light on how this econometric framework can be tested with data, results observed and their interpretation.

4. Results

The empirical analysis has been done on Indian data covering the time period 1980-2009 for all three levels of government namely Central, State and Consolidated General Government. There are two main reasons as to why India

[12]Refer to appendix for details on long run properties of the VECM and links with cointegration methodology of Johansen(1995)

has been selected for the analysis. Firstly, India is a developing nation, with huge level of public debt and deficits at all levels of Government. The puzzle, however, is that even then it has escaped a debt crisis so far. Partly, this can be explained by its burgeoning growth rate but other factors contributing to this still remain a mystery. Secondly, owing to a strong federal structure, there is a clear demarcation on expenditure prerogatives.

Before embarking on the empirical analysis it is imperative to take a look at the federal structure of the government. This is because India's federal structures are an important aspect of its political and economic system. The Indian Constitution, in its Seventh Schedule, assigns the powers and functions of the Center and the States. The schedule specifies the exclusive powers of the Center in the Union list; exclusive powers of the States in the State list; and those falling under the joint jurisdiction are placed in the Concurrent list. All residuary powers are assigned to the Center. The nature of the assignments is fairly typical of federal nations. The functions of the central government are those required to maintain macroeconomic stability, international trade and relations and those having implications for more than one state. The major subjects assigned to the states comprise public order, public health, agriculture, irrigation, land rights, fisheries and industries and minor minerals. The States also assume a significant role for subjects in the concurrent list like education and transportation, social security and social insurance. According to the Indian constitution, capital disbursements are the responsibility of the Central Government, while the State Government is assigned current and social disbursements[26]. The proposed theoretical model in section 3 of the paper considers the government as a body that governs the country in entirety. However, if this model has to be fitted in case of India, each level of Government must be separately analyzed .Thus, the following empirical analysis bridges the gap between the theoretical model and existing federal structure. Essentially this means that empirically the effect of capital expenditure on debt should be more pronounced for the Central and Consolidated General Government, than for the State Governments.

As a precursor to the cointegration tests, we regress the CAPRATIO and CURATIO (defined as current expenditure to aggregate expenditure) on DEBT. This is a common approach when short run relationships between two variables need to be established. The results would mainly aim at checking the sign of the coefficients, and not necessarily on their statistical significance. Additionally, they can also help explain which of the categories of expenditure are more productive, in consistence with equation (13). Broadly consistent with the earlier discussion on the inverse relationship between the two time series in question, we do obtain a negative coefficient for CAPRATIO and a positive one with CURATIO respectively for all levels of Government . Additionally, we observe that the regression is significant for Centre, consolidated Gen Government and insignificant for the States. Table 1 summarizes the key results of this regression.

4.1 Unit Root Tests

Before testing for co-integrating relations, uni variate time-series properties of capital expenditure ratio and Public debt are examined using two unit root tests: the KPSS(Kwiatkowski et al.1992) and the augmented Dickey Fuller(ADF; Dickey and Fuller 1979). The KPSS tests the null of stationarity, whereas the ADF tests the null of the unit root. If the KPSS test rejects the null but the ADF test does not, both tests support the same conclusions; that is, the series in question is a unit root process. Results of the ADF and KPSS tests are reported in Table 2. [13]

In case of Consolidated General Government and the Centre, the ADF tests cannot reject the unit root null in any of the indexes(ratio/log level) , whereas the KPSS tests reject the null of stationarity for all indexes. At the first differences, the ADF reject the unit root for CAPRATIO but for DEBT the significance can be weakly seen at 10% levels. It has already been seen in the past literature that debt series for India suffer from structural breaks(explain and citation). A caveat of the unit root tests is that stationary series with structural breaks may appear non stationary. Failure to allow for such shifts could bias the unit root tests in favour of non stationarity. Since work on structural breaks does show such considerations for India, and KPSS, which is a much powerful unit root tests shows DEBT to be I(1) we can proceed with the cointegration analysis for further conclusive answers. The KPSS test however, clears this doubt by not rejecting the null for stationarity. Additionally, correlogram analysis(represented in Table 3) show the DEBT and CAPRATIO to be non stationary ratios and stationary in first differences respectively. Correlograms show the Auto correlation and Partial Auto correlations for a particular time series. For a non stationary time series, the auto correlations are extremely high and p-values are low. Additionally, the AC coefficient starts at a very high value and then declines slowly. They also test the null of stationarity, hence when p-values are low we reject the null. The correlograms for Centre and State are in the Table 3 of the appendix and. Thus, ADF and KPSS tests and correlogram analysis, confirm that both DEBT and CAPRATIO are unit root processes and seem to be I(1) for the Central and Consolidated General Government.

In case of the state level analysis as was perceived, while the log levels of the variables are I(0) , the first differences seem to be I(2). To endorse this further, we check for cointegration between DEBT(debt to GDP ratio) and CAPRATIO(proportion of capital expenditure in total public expenditure) for each levels of government. In case of the Centre and Consolidated General Government, the two variables share a common I(1) trend, and DEBT is unlikely to be second-order cointegrated. However, in case of the State level analysis, we do not find any cointegration between CAPRATIO and DEBT which shows that the CAPRATIO and debt relation is less pronounced for the States than for the Centre and Consolidated General government. Thus, reemphasizing the fact that capital expenditure is the responsibility of the Central Government.

[13]No mention of * indicates that the variable is significant at multiple levels

Further analysis and discussions on forecasting emit the State level Analysis.

4.2 Cointegration and VAR/VECM

To estimate the Vector autoregressive (VAR) models, identifying the order of the VAR is important. We identify the lag lengths . The reason for using a VAR model here is that all time series variables are endogenous and there is cointegration between the two. We have 30 observations for each time series variables and including too many lagged terms will consume degrees of freedom, and also a probability of multicollinearity could arise. On the other hand, too few lags call for specification errors. We identify the lag lengths following Sim's(1980) likelihood (LR) tests and multivariate Akaike information criterion(AIC). Under the LR tests, we begin with a maximum lag lenght(k-max) of 7 and sequentially test down, deleting one VAR lag at a time until the deleted lags are jointly significant. Information criteria normally choose a shorter lag lenght, which is not always sufficient to flush serial correlation from the VAR residuals. However, it is important to render VAR residuals uncorrelated(Johansen 1992). To circumvent this, we restrict the AIC search between k-max=7 and k-min=1. The VAR lengths specified by both the methods are reported in Table 4. In our case the VAR length selection is uniform since both LR and AIC identify the same lengths. Hence, we adopt VAR length of 3, as represented in the last column of Table 4. [14]

The next table reports the results of the Johansen cointegration test. Table 5 shows the trace tests for the cointegration rank r, for DEBT and CAPRATIO. The trace test equation indicates 1 cointegrating equation at 0.05 level. This means that DEBT and CAPRATIO are cointegrated, which suggests a long-run relation between these two variables. On grounds that debt and capital expenditure bear a long run inverse relationship, an error correction representation of them can be used to assess whether an indicator like the one suggested in (6) would be useful for forecasting and fiscal consolidation policies. If the VECM model does indicate significant coefficients in the cointegration equation, this could be useful for policy makers because they can refer policies suited towards redesigning of expenditure in developing countries.

The VAR[15] specification for the analysis is as represented in equation(7) and (8) below and DEBT is represented by b_t while CAPRATIO by λ_t. However, in this case, since we have a cointegration between the two variables, we reparameterized the VAR into a vector error correction model[16]. Table 6 below shows the results of the VECM representations. The cointegration equation coefficient for DEBT is statistically significant, and so is the constant. In addition, the second lag coefficient is also significant. The lag coefficients for CAPRATIO for the first lag is significant too. The coefficient in the cointegration equation

[14]The VAR length specification is particularly important for CAPRATIO since its dependency has to be checked with that of DEBT.

[15]Refer to Table 7 in the appendix

[16]Already defined in section 3 of the paper

for CAPRATIO is highly significant for the consolidated General government whereas for the Centre the level of significance is not so high. .[17]. Thus, the error correction lends further support to the hypothesis that CAPRATIO affects DEBT. The not so significant coefficient of the CAPRATIO does not affect further analysis, because the forecast has to be done for DEBT only. The overall $R^2$is 0.46 and 0.39 respectively for the Consolidated and Central Governments respectively which makes the regression non-spurious statistically.

4.3 VECM Forecasting Simulations

Having obtained significant coefficients in the VECM, we proceed in evaluating the model for forecasting purposes for the Consolidated General Government and the Centre. The representations below define the VECM model, and can be used for static and dynamic forecasts. While dynamic future forecasting is out of the scope of the paper, we can use the VECM model to estimate/forecast for the period between 1980-2010 and compare the forecast with the baseline. A converging pattern would suggest high precision of the indicator. Figure (1) in the appendix shows the graphical comparison of forecasts with actuals. It can clearly be observed, that the forecasts do converge around the observed values for both types of government. The baseline trajectory represents the simulations while Actual refers to the values observed historically.

5. Summary and Conclusion

We propose a theoretical framework for devising optimal productive public expenditure in an economy in relation with the capital stock in the economy and public debt of the government. Co state equation and steady state conditions help in deriving two analytical results. The first being the fact that an inverse relationship is seen between productive expenditure and shadow price of debt, allowing the government to smooth the path for repayment of its debt. Additionally the ratio of this expenditure in total expenditure should always be lower than the ratio of the initial shares of productive and less productive expenditures.

Further, we examine whether capital expenditure has a long run relationship with public debt. We use capital expenditure and public debt annual time series spanning 30 years from India for all levels of government. . We find capital expenditure, capital expenditure ratio and Public debt are cointegrated, which implies a long-run relation between them. While the VECM might not be the best procedure for testing future simulations it does empirically help in adding weight to use the indicator in (24) for forecasting and policy purposes.

Our investigation of annual data at all levels of government, extends the

[17]Significant at 12%, could be due to the small size of the sample, as all data tested for is annual

strands of empirical literature. We provide a robust empirical analysis by formally testing for stationarity and cointegration of debt to GDP ratio and capital expenditure ratio. In both analysis we identify the two variables to be integrated of first order and bear a cointegration relationship. The application of error correction representation, improves the results of the VAR model. Furthermore, dynamic simulations increase the confidence in using the suggested indicator. Overall, our empirical findings suggest that for developing countries like India, the percentage of capital expenditure in total public expenditure bears an inverse long-run relationship with debt, and the suggested 'capital expenditure ratio gap' indicator could be used for forecasting purposes.

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6. Appendix

6.1 Data

The data used in this study are obtained from *Handbook of Statistics on the Indian economy(2009) , National Accounts Statistics(CSO)* and *Indian Public Finance Statistics*-various issues. All variables used in analysis are ratio to GDP. CAPRATIO specifically has been calculated as capital expenditure divided by total public expenditure. The sample covers 1980-2010, yielding 30 observations for each variables at each level of Government. The other variables used in the analysis are total public expenditure, public debt, GDP at Factor cost and current expenditure.

The figure (2) represents some graphs that show the trajectories of CAPRATIO, CURATIO and debt for each level of government. The graphs are in sequence Consolidated, Centre and State Governments respectively. It is evident that for India the debt trajectory is an increasing one with CURATIO higher than CAPRATIO. However, from 2006 the CAPRATIO has been increasing at the Consolidated and State levels, while there is a decrease in the value of the same for the Centre. This increase and decrease has been stagnant ever since. Policymakers could aim at increasing the CAPRATIO since it lies even below that of CURATIO for better public debt management.

6.2 VECM(Long Run Properties) and Econometric Relationship derivation between λ_t and b_t

$\lambda = \frac{g_{ca}}{(r-\theta)([\int(h-t)exp-(r-\theta)sds]+b_0)}$represents the CAPRATIO indicator. Normally a long run linear relationship between two variables, is defined by $b_t = \alpha + \beta\lambda_t$. However in this case since the relationship does not seem linear, we would need to deduce the relationship by rearranging the equation as follows.

$$\lambda - \frac{g_{ca}}{(r-\theta)(\left[\int(h-t)exp-(r-\theta)sds\right]+b_0)} = 0$$

$$\frac{\lambda\left\{(r-\theta)(\left[\int(h-t)exp-(r-\theta)sds\right]+b_0)\right\}-g_{ca}}{(r-\theta)(\left[\int(h-t)exp-(r-\theta)sds\right]+b_0)}=0$$

For this expression to hold good, we would need the numerator to be zero. Equating the numerator as 0 and assuming $\alpha = \left[\int(h-t)exp-(r-\theta)sds\right]$, $\beta = g_{ca}$, $(r-\theta)=\gamma$, we get

$$g_{ca} = \lambda_t\left[(r-\theta)\alpha + b_t\right]$$

Final rearrangement and substitution yields

$$b_t = \frac{\beta}{\lambda_t} - \gamma\alpha \tag{29}$$

Π's properties explain the long run properties of the VECM model.

rank(Π) = 0,non stationary with no cointegration

rank(Π) = 2,full rank, which means that the system is stationary as a whole even if individual series are not

rank(Π) = 1,non stationary with 1 cointegrating relationships.

For the Johansen test, the rank of the matrix= no. of characteristic roots that differ from zero. In case of no cointegration rank of $\Pi$is 0 and all characteristic roots equal zero. $1-\lambda_i = 0$.

If rank(Π) = 1, which is the case in point here, $0 < \lambda_1 < 1$, we have the following model which can be represented as a VAR.

$\begin{bmatrix} b_t \\ \lambda_t \end{bmatrix} = A_0 + A_1 \begin{bmatrix} b_{t-1} \\ \lambda_{t-1} \end{bmatrix} + A_2 \begin{bmatrix} b_{t-2} \\ \lambda_{t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u2t \end{bmatrix}$

The VECM form hence would be

$\begin{bmatrix} \triangle b_t \\ \triangle\lambda_t \end{bmatrix} = \Pi_0 + \Pi_1 \begin{bmatrix} b_{t-1} \\ \lambda_{t-1} \end{bmatrix} + \Pi_2 \begin{bmatrix} \triangle b_{t-1} \\ \triangle\lambda_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u2t \end{bmatrix}$

$\Pi = \alpha\beta^{'}$and $\Pi = -\left(I - \sum_{i=1}^{2} A_i\right)$

6.3 Appendix Tables

Table 1

Table1. OLS Regression of CURATIO and CAPRATIO on L(Debt)						
	Con Gen Govt.		**Centre**		**State**	
	CAPRATIO	**CURATIO**	**CAPRATIO**	**CURATIO**	**CAPRATIO**	**CURATIO**
Coefficient	**-16.5** (0.65)	16.6 (0.65)	**-13.8** (0.45)	11.1 (0.42)	-11.0 (1.11)	11.07 (1.11)
R^2	0.71	0.71	0.86	0.87	0.13	0.16
P-value	0.000	0.00	0.000	0.000	0.033	0.033
DW	0.29	0.29	1.04	2.13	0.06	0.06
Note: The L(debt) is used to avoid problems of autocorrelation and spurious regression. The value of R^2 is very high for the Consolidated and Central Government. For the States the value is very low which means that he relationship between CAPRATIO and DEBT is not very well explained on the basis of the data.						

Table 2

Table 2. Augmented Dickey-Fuller and Kwiatkowski, Phillips, Schmidt, and Shin Tests						
Log Levels					First Differences	
Con Gen Govt.	**ADF Const**	**ADFTrend**	**KPSSConst**	**KPSSTrend**	**ADFConst**	**KPSSConst**
CAPRATIO	-2.3***	-1.62***	0.53**	0.17**	-7.51***	0.49***
DEBT	2.43***	2.55***	0.61**	0.18**	3.16**	0.10***
Centre	**ADF Const**	**ADFTrend**	**KPSSConst**	**KPSSTrend**	**ADFConst**	**KPSSConst**
CAPRATIO	-1.15***	-3.80***	0.66**	0.19**	-4.63***	0.50**
DEBT	3.00***	-0.79***	0.63***	0.18**	3.80***	0.55**
State	**ADF Const**	**ADFTrend**	**KPSSConst**	**KPSSTrend**	**ADFConst**	**KPSSConst**
CAPRATIO	-2.04***	-1.74***	0.25	0.25	-3.95***	0.18
DEBT	2.22***	1.97***	0.61**	0.61	-2.39	0.75

Note: ADF= augmented Dickey-Fuller(1979) ; KPSS= Kwiatkowski, Phillips, Schmidt, and Shin(1992) ; CAPRATIO= Capital expenditure component in total public expenditure ; DEBT R = Public Debt to GDP ratio. The ADF tests are conducted by setting a lag length (k) of 7 as explained in the test. The KPSS tests are reported on the automatic (k) selection of 4 since the sample is small. The ADF tests , ADF Const denotes the only constant term in the estimating equation, whereas Trend denotes both the constant term and linear time trend. For ADF Trend log values of variables have been used. Same notations are used for constant and trend in the KPSS model.

Critical Values:

	ADFConst	**ADFTrend**	**KPSSConst**	**KPSSTrend**
1%	-3.73	-4.33	0.739	0.216
5%	-2.99	-3.58	0.463	0.146

*** Significant at the 1% level
** Significant at the 5% level
* Significant at the 10% level

Table 3A Con Gen Govt.

Correlogram CAPRATIO

Lag	AC	PAC	Q-Stat	Prob
1	0.840	0.840	21.975	0.000
2	0.738	0.108	39.572	0.000
3	0.613	-0.106	52.212	0.000
4	0.556	0.140	63.045	0.000
5	0.394	-0.346	68.718	0.000
6	0.279	-0.038	71.688	0.000
7	0.156	-0.037	72.657	0.000
8	0.051	-0.166	72.764	0.000
9	-0.043	0.088	72.846	0.000
10	-0.120	-0.074	73.521	0.000
11	-0.181	-0.027	75.134	0.000
12	-0.252	-0.067	78.472	0.000

Correlogram DEBT

Lag	AC	PAC	Q-Stat	Prob
1	0.893	0.893	24.817	0.000
2	0.784	-0.066	44.695	0.000
3	0.678	-0.051	60.120	0.000
4	0.574	-0.049	71.636	0.000
5	0.473	-0.052	79.791	0.000
6	0.373	-0.060	85.106	0.000
7	0.277	-0.054	88.182	0.000
8	0.186	-0.053	89.636	0.000
9	0.100	-0.051	90.078	0.000
10	0.018	-0.058	90.093	0.000
11	-0.059	-0.059	90.267	0.000
12	-0.128	-0.040	91.126	0.000

Correlogram firstdiff CAPRATIO

Lag	AC	PAC	Q-Stat	Prob
1	-0.402	-0.402	4.8607	0.027
2	0.265	0.124	7.0605	0.029
3	-0.040	0.124	7.1133	0.068
4	0.246	0.291	9.1691	0.057
5	-0.128	0.042	9.7567	0.082
6	0.116	-0.027	10.254	0.114
7	0.057	0.076	10.381	0.168
8	0.009	0.005	10.384	0.239
9	-0.128	-0.192	11.101	0.269
10	0.031	-0.174	11.145	0.346
11	0.060	0.052	11.322	0.417
12	-0.184	-0.101	13.090	0.363

Correlogram firstdiff DEBT

Lag	AC	PAC	Q-Stat	Prob
1	0.599	0.599	10.815	0.001
2	0.416	0.089	16.231	0.000
3	0.232	-0.076	17.981	0.000
4	0.225	0.141	19.710	0.001
5	0.034	-0.225	19.750	0.001
6	-0.119	-0.174	20.274	0.002
7	-0.213	-0.047	22.043	0.002
8	-0.071	0.209	22.251	0.004
9	0.026	0.143	22.280	0.008
10	-0.111	-0.304	22.844	0.011
11	0.044	0.364	22.939	0.018
12	0.066	-0.143	23.164	0.026

Table 3B Centre

Correlogram CAPRATIO

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.834	0.834	22.328	0.000
		2	0.659	-0.119	36.799	0.000
		3	0.570	0.180	48.033	0.000
		4	0.489	-0.056	56.628	0.000
		5	0.376	-0.106	61.909	0.000
		6	0.298	0.060	65.381	0.000
		7	0.244	-0.031	67.812	0.000
		8	0.193	0.008	69.414	0.000
		9	0.115	-0.118	70.012	0.000
		10	0.031	-0.083	70.056	0.000
		11	-0.062	-0.131	70.245	0.000
		12	-0.119	0.026	70.990	0.000

Correlogram DEBT

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.865	0.865	24.010	0.000
		2	0.740	-0.032	42.230	0.000
		3	0.622	-0.042	55.602	0.000
		4	0.511	-0.044	64.987	0.000
		5	0.407	-0.043	71.191	0.000
		6	0.310	-0.043	74.955	0.000
		7	0.219	-0.048	76.923	0.000
		8	0.137	-0.039	77.727	0.000
		9	0.060	-0.048	77.891	0.000
		10	-0.008	-0.040	77.895	0.000
		11	-0.068	-0.035	78.125	0.000
		12	-0.121	-0.042	78.901	0.000

Correlogram firstdiff CAPRATIO

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.011	0.011	0.0037	0.952
		2	-0.561	-0.561	10.182	0.006
		3	0.031	0.068	10.214	0.017
		4	0.010	-0.452	10.218	0.037
		5	-0.118	-0.068	10.729	0.057
		6	0.209	-0.052	12.389	0.054
		7	0.004	-0.236	12.390	0.088
		8	-0.085	0.150	12.695	0.123
		9	0.147	-0.044	13.648	0.135
		10	0.012	0.154	13.654	0.189
		11	-0.178	-0.045	15.213	0.173
		12	-0.065	-0.109	15.433	0.219

Correlogram firstdiff DEBT

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.547	0.547	9.3066	0.002
		2	0.330	0.045	12.834	0.002
		3	0.240	0.061	14.761	0.002
		4	0.269	0.153	17.286	0.002
		5	0.077	-0.205	17.506	0.004
		6	-0.132	-0.222	18.171	0.006
		7	-0.082	0.116	18.437	0.010
		8	0.031	0.111	18.477	0.018
		9	0.026	0.013	18.507	0.030
		10	-0.197	-0.259	20.311	0.026
		11	0.003	0.295	20.311	0.041
		12	0.044	-0.117	20.415	0.060

Table 3C State

Correlogram CAPRATIO

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.798	0.798	19.818	0.000
		2	0.551	-0.238	29.617	0.000
		3	0.376	0.065	34.377	0.000
		4	0.247	-0.048	36.510	0.000
		5	0.095	-0.172	36.839	0.000
		6	-0.040	-0.046	36.900	0.000
		7	-0.114	0.009	37.415	0.000
		8	-0.163	-0.077	38.527	0.000
		9	-0.192	-0.001	40.150	0.000
		10	-0.253	-0.180	43.128	0.000
		11	-0.310	-0.080	47.887	0.000
		12	-0.331	-0.030	53.640	0.000

Correlogram DEBT

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.874	0.874	23.778	0.000
		2	0.758	-0.027	42.331	0.000
		3	0.646	-0.045	56.361	0.000
		4	0.532	-0.077	66.267	0.000
		5	0.417	-0.078	72.619	0.000
		6	0.310	-0.046	76.295	0.000
		7	0.209	-0.054	78.050	0.000
		8	0.118	-0.042	78.632	0.000
		9	0.033	-0.050	78.681	0.000
		10	-0.043	-0.046	78.767	0.000
		11	-0.111	-0.049	79.378	0.000
		12	-0.165	-0.021	80.816	0.000

Correlogram firstdiff CAPRATIO

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.196	0.196	1.1577	0.282
		2	-0.271	-0.322	3.4542	0.178
		3	-0.065	0.079	3.5930	0.309
		4	0.084	-0.007	3.8329	0.429
		5	0.083	0.070	4.0802	0.538
		6	-0.070	-0.092	4.2619	0.641
		7	-0.133	-0.062	4.9542	0.666
		8	-0.098	-0.109	5.3497	0.720
		9	0.151	0.171	6.3384	0.706
		10	0.025	-0.139	6.3667	0.784
		11	-0.103	0.043	6.8852	0.808
		12	-0.044	-0.062	6.9865	0.859

Correlogram firstdiff DEBT

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.589	0.589	10.444	0.001
		2	0.350	0.004	14.274	0.001
		3	0.103	-0.160	14.620	0.002
		4	0.052	0.081	14.713	0.005
		5	-0.076	-0.145	14.918	0.011
		6	-0.231	-0.228	16.908	0.010
		7	-0.298	-0.046	20.386	0.005
		8	-0.419	-0.267	27.634	0.001
		9	-0.223	0.235	29.800	0.000
		10	-0.103	0.055	30.293	0.001
		11	0.013	-0.056	30.300	0.001
		12	0.040	0.072	30.385	0.002

Table 4

Centre Lag selection criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	16.79994	NA	0.000893	-1.345449	-1.246263	-1.322084
1	104.9028	152.1777*	4.28e-07	-8.991166	-8.693609*	-8.921070
2	108.3395	5.311225	4.57e-07	-8.939954	-8.444026	-8.823129
3	109.9656	2.217454	5.84e-07	-8.724148	-8.029849	-8.560592
4	113.2885	3.927067	6.56e-07	-8.662594	-7.769923	-8.452308
5	120.2371	6.948584	5.52e-07	-8.930647	-7.839605	-8.673630
6	129.8713	7.882521	3.87e-07*	-9.442846	-8.153433	-9.139099*
7	134.2044	2.757428	4.82e-07	-9.473128*	-7.985343	-9.122651

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Con Gen Govt. Lag selection criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-307.3591	NA	1.65e+09	26.90079	26.99953	26.92562
1	-184.8020	213.1427*	55199.82*	16.59148*	16.88770*	16.66598*
2	-182.5467	3.530061	64960.68	16.74319	17.23688	16.86735
3	-180.7113	2.553595	80457.93	16.93142	17.62259	17.10525
4	-178.8482	2.268125	101683.5	17.11723	18.00588	17.34073
5	-175.9710	3.002326	121659.0	17.21487	18.30099	17.48802

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Table 5

Table 5. Cointegration Tests.			
Trace (Eigen Values) Statistics H_0 = Rank=r (Con Gen Govt			
r= 0	r ≤ 1	Coefficient of Cointegration	α
15.61 [0.048] (15.12)	0.49** [0.48] (0.49)	0.97	0.001
Trace (Eigen Value) Statistics H_0 = Rank=r (Centre)			
r= 0	r ≤ 2	Coefficient of Cointegration	α
32.5 [0.0001] (17.23)	15.32* [0.0001] (15.32)	0.69	-0.015

Note: P-values are reported in brackets for this test. The 5% critical values of the trace statistics for $H_0 = 0$ are 15.49 and for $H_0 \leq 1$ are 3.84 respectively. In case of Central Government 2 cointegrating vectors are observed. The lag lengths used are as per the last column of Table 4.

*** Significant at the 1% level

** Significant at the 5% level

* Significant at the 10% level

Table 6

Table 6. Error –Correction models		
Con Gen Govt.	**Lag 1**	**Lag 2**
Change in DEBT	0.39(0.21)	-0.08(0.17)
Change in CAPRATIO	-0.72(0.20)	-0.32(0.21)
Constant	10.389	
Centre	**Lag 1**	**Lag 2**
Change in DEBT	0.30(0.21)	-0.11(0.18)
Change in CAPRATIO	0.12(0.17)	-0.49(0.17)
Constant	-0.33	

Note: The cointegration equation for DEBTR is statistically significant with a value of 78% with a t-stat of -3.7 for the consolidate general government.
The 10% critical value for the t-stat is 1.31 for n= 24, where n is the number of degrees of freedom.
The constant term is also significant.
*** Significant at the 1% level
** Significant at the 5% level
* Significant at the 10% level

Table 7

Con Gen Govt.

	CAPRATIO	LDEBT
CAPRATIO(-1)	0.397849	-0.199682
	(0.18933)	(0.17746)
	[2.10136]	[-1.12522]
CAPRATIO(-2)	0.489493	0.218534
	(0.19494)	(0.18272)
	[2.51094]	[1.19599]
LDEBT(-1)	0.013787	1.191236
	(0.19346)	(0.18133)
	[0.07126]	[6.56927]
LDEBT(-2)	-0.009544	-0.204854
	(0.18915)	(0.17729)
	[-0.05046]	[-1.15545]
C	-0.046573	0.295017
	(0.14359)	(0.13459)
	[-0.32434]	[2.19200]
R-squared	0.889123	0.999784
Adj. R-squared	0.868003	0.999743
Sum sq. resids	0.007458	0.006552
S.E. equation	0.018845	0.017664
F-statistic	42.09973	24328.68
Log likelihood	69.14261	70.82598
Akaike AIC	-4.934047	-5.063537
Schwarz SC	-4.692105	-4.821595
Mean dependent	0.181930	13.61147
S.D. dependent	0.051871	1.102186
Determinant resid covariance (dof adj.)		1.10E-07
Determinant resid covariance		7.21E-08
Log likelihood		140.0075
Akaike information criterion		-10.00058
Schwarz criterion		-9.516695

Centre_

	CAPRATIO	LDEBT
CAPRATIO(-1)	0.564912	0.011120
	(0.19856)	(0.14139)
	[2.84508]	[0.07865]
CAPRATIO(-2)	-0.425072	-0.131578
	(0.19500)	(0.13886)
	[-2.17981]	[-0.94755]
LDEBT(-1)	0.058772	1.068888
	(0.24731)	(0.17611)
	[0.23764]	[6.06949]
LDEBT(-2)	-0.109376	-0.095621
	(0.24416)	(0.17387)
	[-0.44796]	[-0.54997]
C	0.851117	0.514015
	(0.24250)	(0.17268)
	[3.50975]	[2.97664]
R-squared	0.886971	0.999735
Adj. R-squared	0.866420	0.999687
Sum sq. resids	0.016546	0.008390
S.E. equation	0.027424	0.019528
F-statistic	43.16003	20782.22
Log likelihood	61.55447	70.72223
Akaike AIC	-4.189220	-4.868314
Schwarz SC	-3.949250	-4.628344
Mean dependent	0.220448	13.46404
S.D. dependent	0.075034	1.104369
Determinant resid covariance (dof adj.)		2.80E-07
Determinant resid covariance		1.86E-07
Log likelihood		132.5971
Akaike information criterion		-9.081268
Schwarz criterion		-8.601329

Figure 1a-Con Gen Govt. Simulations

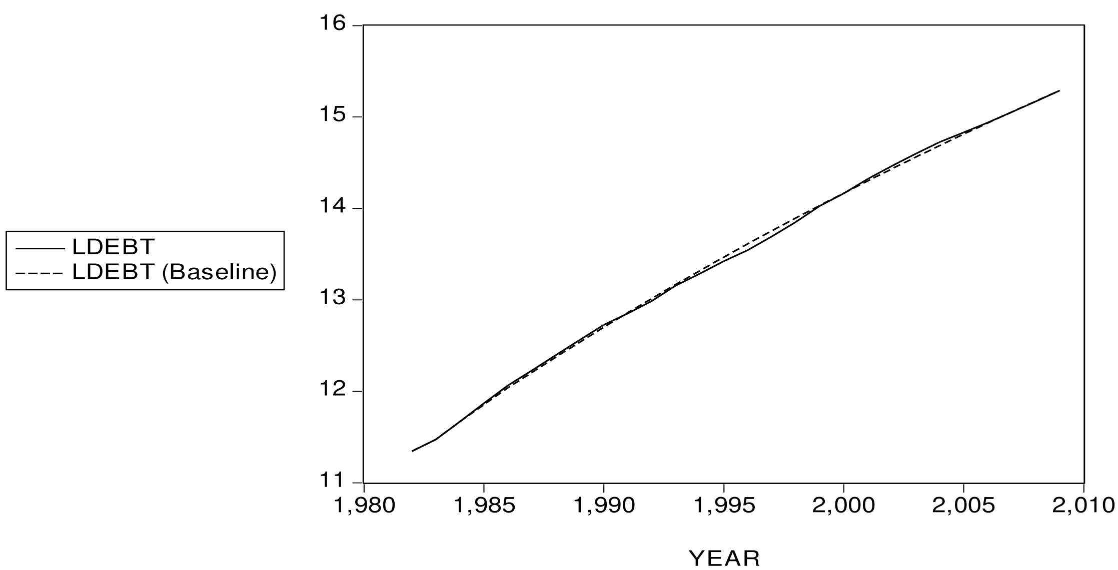


Figure 1b-Centre Simulations

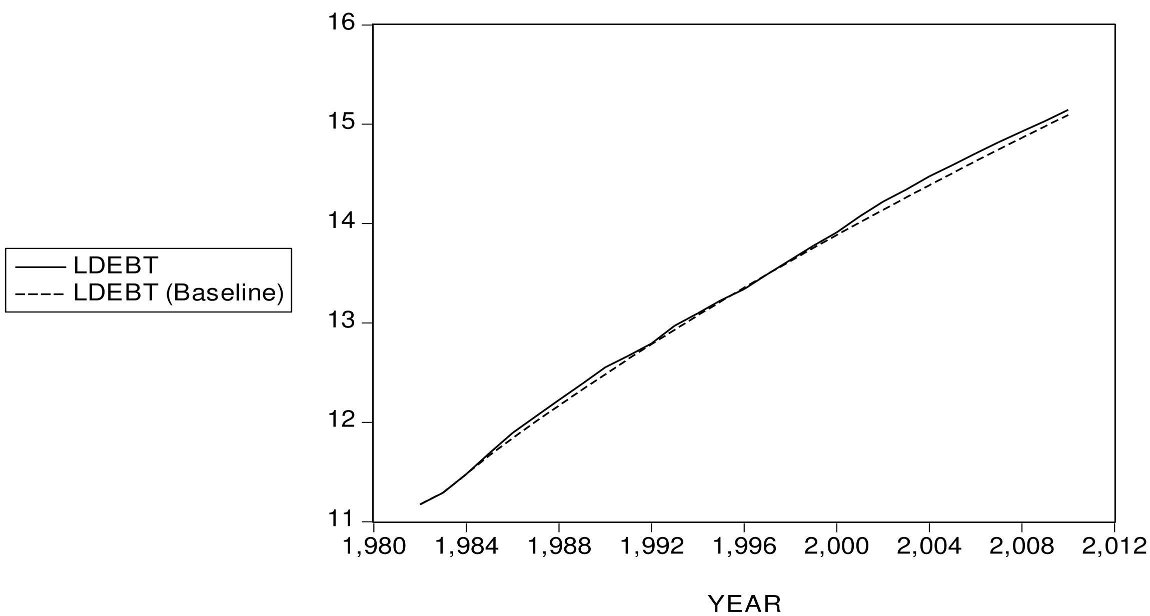
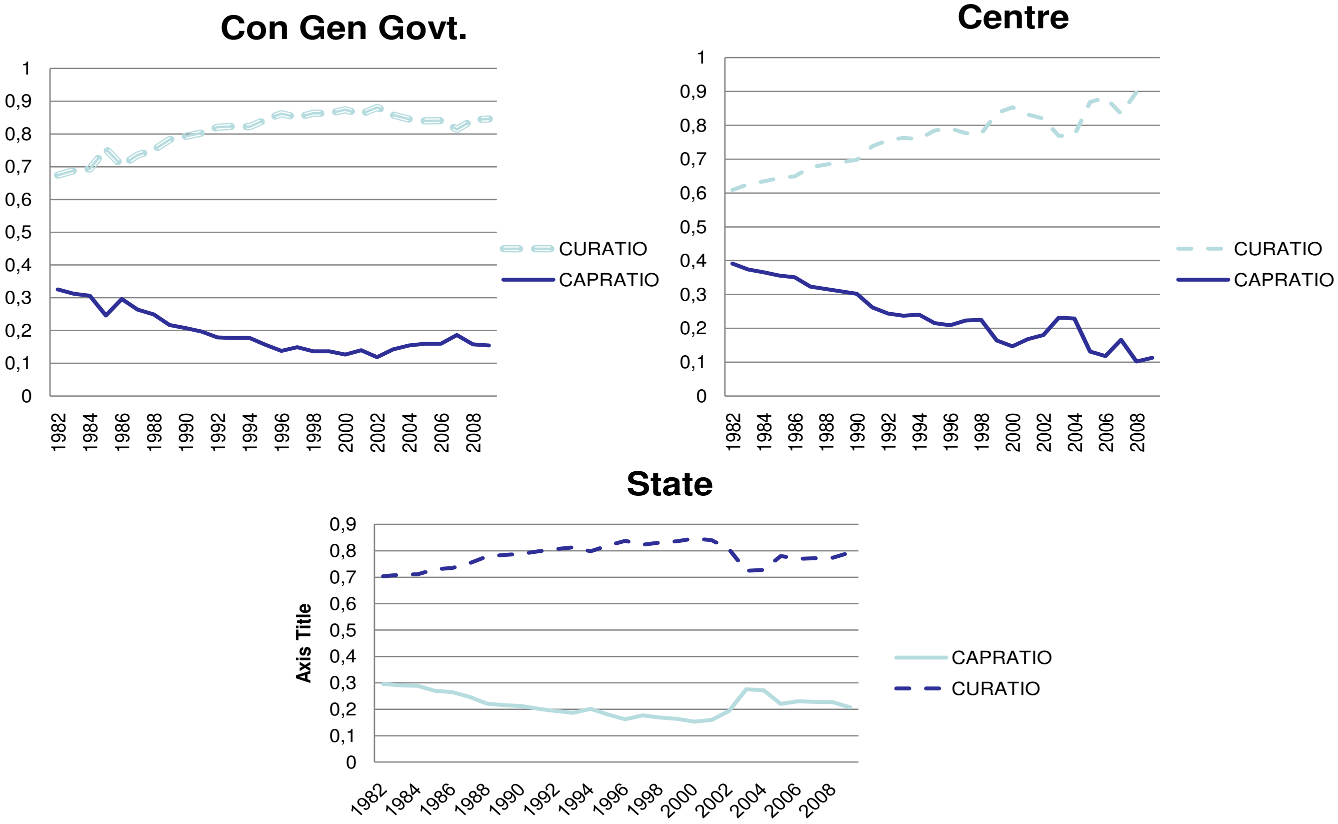


Figure 2



Con Gen Govt.
1
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0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
CURATIO
CAPRATIO
Centre
CURATIO
CAPRATIO
State
Axis Title
CAPRATIO
CURATIO