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# Linking Decision and Time Utilities

Krzysztof Kontek<sup>1</sup>

## Abstract

This paper presents the functional relationship between two areas of interest in contemporary behavioral economics: one concerning choices under conditions of risk, the other concerning choices in time. The paper first presents the general formula of the relationship between decision utility, the survival function, and the discounting function, where decision utility is an alternative to Cumulative Prospect Theory in describing choices under risk (Kontek, 2010). The stretched exponential function appears to be a simple functional form of the resulting discounting function. Solutions obtained using more complex forms of decision utility and survival functions are also considered. These likewise lead to the stretched exponential discounting function. The paper shows that the relationship may also have other forms, including the hyperbolic functions typically used to describe the intertemporal experimental results. This solution has however several descriptive disadvantages, which restricts its common use in the description of lottery and intertemporal choices, and in financial asset valuations.

**JEL classification:** C91, D03, D81, D90, E43, G12

**Keywords:** Discounted Utility, Hyperbolic Discounting, Decision Utility, Prospect Theory, Asset Valuation

## 1 Introduction

The Discounted Utility Model was introduced by Paul Samuelson in 1937. This model specifies a decision maker's intertemporal preferences over consumption profiles. It assumes that an individual's intertemporal utility function can be described by the following functional form:

$$U^t(c_t, \dots, c_T) = \sum_{k=0}^{T-t} d(k) u(c_{t+k}) \quad (1.1)$$

where

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$$d(k) = \left( \frac{1}{1 + \rho} \right)^k \quad (1.2)$$

The function  $u(c_{t+k})$  is often interpreted as the individual's cardinal instantaneous utility function, and  $d(k)$  as the individual's discount function (Frederick et al., 2002). The model assumes a constant discount rate of  $\rho$ , which implies that an individual's intertemporal preferences are time-consistent. Empirical research has documented various inadequacies of the DU model as a descriptive model of behavior. First, experimentally observed discount rates are not constant but decrease over time. Furthermore, discount rates vary across different types of intertemporal choices: gains are discounted more than losses, small amounts more than large amounts, and explicit sequences of multiple outcomes differently than outcomes considered singly (Frederick et al., 2002).

Several types of functions have been proposed to describe the observed phenomenon commonly referred to as hyperbolic discounting. A comprehensive review can be found in Doyle (2010) and Andersen et al. (2010). Only those most frequently used are mentioned below. The hyperbolic discounting function:

$$d(t) = \frac{1}{1 + \alpha t} \quad (1.3)$$

is due to Mazur (1987). The model proposed by Loewenstein and Prelec (1992) and Myerson and Green (1995) generalizes the hyperbolic function:

$$d(t) = \frac{1}{(1 + \alpha t)^\beta} \quad (1.4)$$

Another general form of the hyperbolic function:

$$d(t) = \frac{1}{1 + \alpha t^\beta} \quad (1.5)$$

was analyzed by Rachlin (2006). A quasi-hyperbolic model was proposed by Laibson (1997):

$$d(t) = \begin{cases} 1 & \text{if } t = 0 \\ \beta \delta^t & \text{if } t > 0 \end{cases} \quad (1.6)$$

where  $\beta$  and  $\delta$  are constants between 0 and 1. The constant sensitivity discounting function:

$$d(t) = e^{-(\alpha t)^\beta} \quad (1.7)$$

was proposed by Read (2001) and Ebert and Prelec (2007).

It was Prelec and Loewenstein (1991) who first noticed that risk taking and discounting behavior exhibit a number of parallels. For a long time, however, research on time discounting was conducted independently of research on lottery behavior. It is only recently that a common approach has attracted significant interest. Halevy (2008) showed that diminishing impatience may be a consequence of nonlinear probability weighting. Saito (2008, 2009) proposed a model in which a failure of expected utility captured by the Allais paradox is equivalent to a failure of exponential discounting captured by hyperbolic discounting. Walther (2008) stated that the S-shape of the probability weighting function induces imputed hyperbolic discount rates. Epper et al. (2009) showed that people’s risk behavior is a significant determinant of their time discounting behavior.

It would seem, however, that no simple direct functional link between these two areas of interest has so far been presented. This may be due to Prospect Theory, which consists of the value and the probability weighting functions; this two-equation form may create some problems in capturing the link to the time domain. It may also be a corollary of the functional form of the probability weighting proposed by Tversky and Kahneman (1992), which is unfriendly to transform. It appears, moreover, that hyperbolic functions make the functional link more difficult to establish; however, the term “hyperbolic discounting” has dominated the literature to such an extent that other types of descriptive function are considered far more rarely.

This paper presents such a link between the decision and time utilities.

## 2 Basic Model

The essence of time discounting is that an immediate reward is preferred over a future reward of the same magnitude. A plausible explanation of this behavior is the risk that the future reward will not be realized. This type of problem may be analyzed using survival functions<sup>2</sup>, a basic failure analysis tool. The survival function  $S(t)$  expresses the probability  $p$  that the time of failure (death)  $T$  is later than some specific time  $t$ :

$$S(t) = p[T > t] \tag{2.1}$$

The probability of survival is thus complementary to the probability of failure. It follows that the expectation of a future reward may be regarded as equivalent to taking part in lottery

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<sup>2</sup> The term “reliability function” is common in engineering.

whose probability of winning is expressed by the survival function. For  $t = 0$ , the probability of survival and the probability of winning the lottery assume a value of 1. The probability of winning the lottery decreases over time, as does the probability of survival. As  $t$  approaches infinity, the probability of survival, and consequently the probability of winning the lottery, approach 0.

There are several theories describing lottery behavior, of which the best known is Prospect Theory (Kahneman, Tversky, 1979, 1992). In this paper, however, Decision Utility Theory (Kontek, 2010) is used. This theory is an alternative to Prospect Theory, but has been proved to present several descriptive advantages over Cumulative Prospect Theory, especially in the case of multi-outcome lotteries. The theory makes no use of the probability weighting function concept, but rather distinguishes decision utility from perception utility. Decision utility describes how decisions are made under conditions of risk, whereas perception utility describes how people perceive different welfare or income levels. The basic model is expressed as:

$$p = D(r) \quad (2.2)$$

where  $p$  denotes probability,  $D$  denotes decision utility and  $r$  denotes the framed (relatively expressed, normalized) outcome defined as:

$$r = \frac{v(ce) - v(P_{min})}{v(P_{max}) - v(P_{min})} \quad (2.3)$$

where  $v$  denotes perception utility,  $ce$  denotes the certainty equivalent,  $P_{max} = \text{Max}(x)$  is the maximum lottery outcome, and  $P_{min} = \text{Min}(x)$  is the minimum lottery outcome. Decision Utility Theory postulates a double S-shaped decision utility curve similar to the one hypothesized by Markowitz (1952), and applies the expected *decision* utility value similarly to Expected Utility Theory.

Equation (2.2) directly expresses the probability of winning the lottery so it can be immediately compared with the probability of survival (2.1).

$$D(r) = S(t) \quad (2.4)$$

Equation (2.4), which states that decision utility equals the survival function, presents the connection between the two areas of interest, i.e. lottery and time decision making. Probability, which is the link between the two, disappears as a result from considerations. This solution can be inverted to give:

$$r(t) = D^{-1}[S(t)] \quad (2.5)$$

Equation (2.5) describes the relatively expressed outcome over the interval  $[0,1]$  as a function of time. This is the general form of the time discounting function  $r(t)$  expressed in terms of the decision utility  $D$  and survival function  $S$ .

### 3 Basic Functional Form

The functional form of the discounting function (2.5) depends on the forms used to describe the survival function (2.1) and decision utility (2.2). Their simple forms will be considered first, before discussing more complex ones in Point 5. Let us assume that the pace at which the survival function decreases depends on its actual value and the failure function<sup>3</sup>  $h(t)$ :

$$\frac{dS(t)}{dt} = -S(t)h(t) \quad (3.1)$$

Let us assume the simplest case, so that the failure rate is constant over time:

$$h(t) = \frac{1}{\tau} \quad (3.2)$$

Solving the differential equation (3.1) with the condition  $S(t=0) = 1$  results in the following exponential survival function, which holds for  $t \geq 0$ :

$$S(t) = e^{-\frac{t}{\tau}} \quad (3.3)$$

The constant  $\tau$  is the time after which the probability of survival (obtaining the reward) reduces to  $1/e$ .

Decision utility will be described using the form proposed by Prelec (1998, 2000). This was originally intended to describe the probability weighting function:

$$D(r) = e^{-(-\ln r)^{\frac{1}{\alpha}}} \quad (3.4)$$

Decision utility is linear for  $\alpha = 1$  (Figure 3.1, left). For  $\alpha < 1$ , the decision utility curve assumes an S-type shape. This reflects risk seeking below  $r = 1/e$  and risk aversion above this value<sup>4</sup>. For  $\alpha > 1$ , the decision utility curve assumes an inverse S-shape. This describes the opposite, quite unusual behavior.

Despite its limited descriptive capabilities, this single-parameter form appears to be ade-

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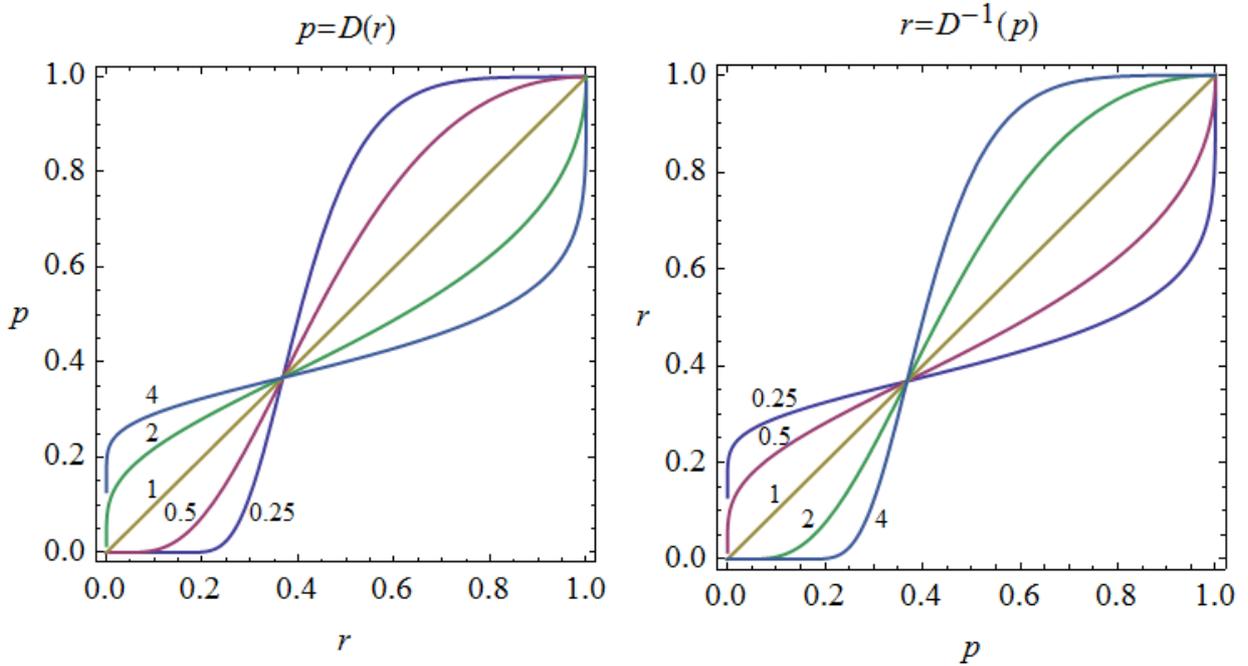
<sup>3</sup> The term “hazard function” is also used.

<sup>4</sup> Using the form (3.4), the point  $r = 1/e$  happens to be the aspiration level, according to Decision Utility Theory. This provides a plausible psychological explanation of risk attitudes.

quate to proceed further. The inversed decision utility, required in (2.5), has a symmetrical form:

$$D^{-1}(p) = e^{-(-\ln p)^\alpha} \quad (3.5)$$

Its shape is presented in Figure 3.1 (right)<sup>5</sup>.



**Figure 3.1** The shape of the decision utility curves (left) and the inversed decision utility curves (right) using Prelec’s functional form for different values of  $\alpha$ .

Substituting (3.3) and (3.5) in (2.5) results in:

$$r(t) = e^{-\left[-\ln\left(e^{-\frac{t}{\tau}}\right)\right]^\alpha} \quad (3.6)$$

which, after simplification, leads to the time discounting function:

$$r(t) = e^{-\left(\frac{t}{\tau}\right)^\alpha} \quad (3.7)$$

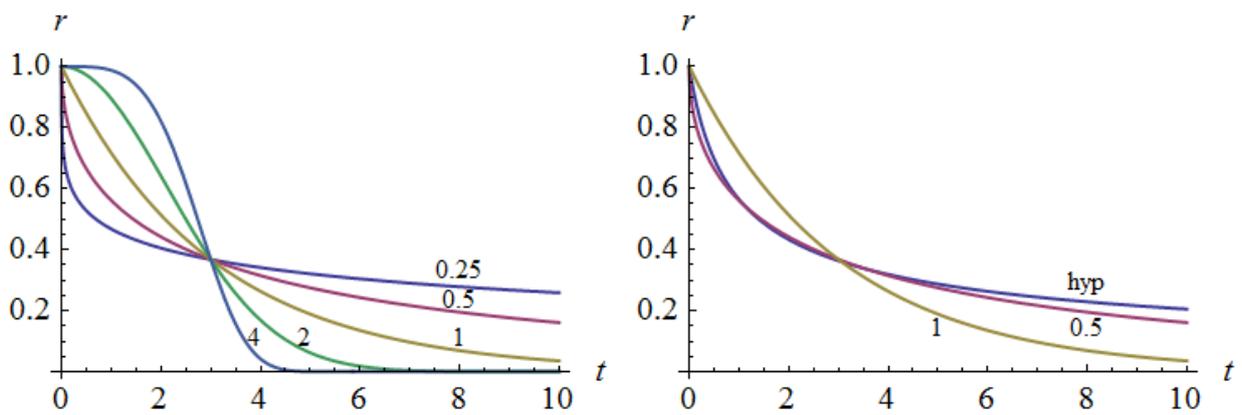
The function (3.7) is known as the stretched exponential function<sup>6</sup> and is essentially the same as (1.7). Its shape is presented in Figure 3.2 (left). For  $\alpha = 1$ , the shape is exponential. For  $\alpha < 1$ , the curve falls faster than exponentially for  $t < \tau$ , but slower for  $t > \tau$ . For  $\alpha > 1$ , it exhibits

<sup>5</sup> The shape of the inversed decision utility resembles the one of the probability weighting function. However both concepts lead to very much different model properties, especially in the case of multi-outcome lotteries (Kontek, 2010).

<sup>6</sup> This function has been used in physics since 1854. The name was coined to describe longer than exponential decays. In the present context, in which the initial part of the curve is of most interest, “condensed exponential” might have been a more appropriate name.

the opposite behavior. The curves for all values of  $\alpha$  intersect at  $t = \tau$ . At this point, the discounting function assumes a value of  $1/e$ .

The shape of the discounting function corresponds with that of the decision utility function. The case of  $\alpha < 1$  is considered first. For  $t < \tau$ , the discounting function assumes values greater than  $1/e$ , for which decision utility demonstrates an attitude of risk aversion. Therefore the discounting function falls faster than exponentially. For  $t > \tau$ , the discounting function assumes values less than  $1/e$ , for which decision utility demonstrates a risk seeking attitude. The discounting function therefore falls slower than exponentially. For  $\alpha > 1$ , this reasoning is reversed.



**Figure 3.2**The shape of the stretched exponential function for different values of  $\alpha$  (left). The shape of the stretched exponential function compared with the general hyperbolic function (1.4).  $\tau = 3$  is assumed.

The same  $\alpha$  parameter controls the shape of both curves. The greater the difference between  $\alpha$  and 1, the greater the departure from both the linear shape of decision utility and the exponential shape of the discounting function. The clear link between decision and time utility has been presented. The greater the curvature of decision utility, the greater the magnitude of the so called “hyperbolic discounting”.

The shape of the stretched exponential function for  $\alpha = 0.5$  is compared with the general hyperbolic function (1.4) (see Figure 3.2, right). The shapes are clearly similar. This may indicate that the hyperbolic function, which gave “hyperbolic discounting” its name, was mistakenly assumed to describe the experimental results, and that the phenomenon can be better explained using a more or less “stretched” exponential function. This observation will be further discussed in the next Points.

## 4 Hyperbolic Functions

Specific functional forms of decision utility and the survival functions have been assumed in the considerations presented so far in order to determine the time discounting function (the stretched exponential one). Let us now approach the problem from the opposite direction. Let us consider the hyperbolic discounting function (1.3):

$$r(t) = \frac{1}{1 + \frac{t}{\tau}} \quad (4.1)$$

and determine the functional form of the decision utility that corresponds with (4.1). According to (2.5), this should satisfy:

$$\frac{1}{1 + \frac{t}{\tau}} = D^{-1}\left(e^{-\frac{t}{\tau}}\right) \quad (4.2)$$

Although a general rule for determining  $D^{-1}$  seems unobtainable, the sought function can be deduced:

$$D^{-1}(p) = \frac{1}{1 - \ln p} \quad (4.3)$$

In order to verify the result, (4.3) and (3.3) are substituted into (2.5) leading to the desired form:

$$r(t) = \frac{1}{1 - \ln e^{-\frac{t}{\tau}}} = \frac{1}{1 + \frac{t}{\tau}} \quad (4.4)$$

The form (4.3) is unable to describe different lottery behaviors as it has no parameters. Therefore the general hyperbolic function (1.4) is next considered:

$$r(t) = \frac{1}{\left(1 + \frac{t}{\tau}\right)^\alpha} \quad (4.5)$$

for which the decision utility function  $D$  has to satisfy:

$$\frac{1}{\left(1 + \frac{t}{\tau}\right)^\alpha} = D^{-1}\left(e^{-\frac{t}{\tau}}\right) \quad (4.6)$$

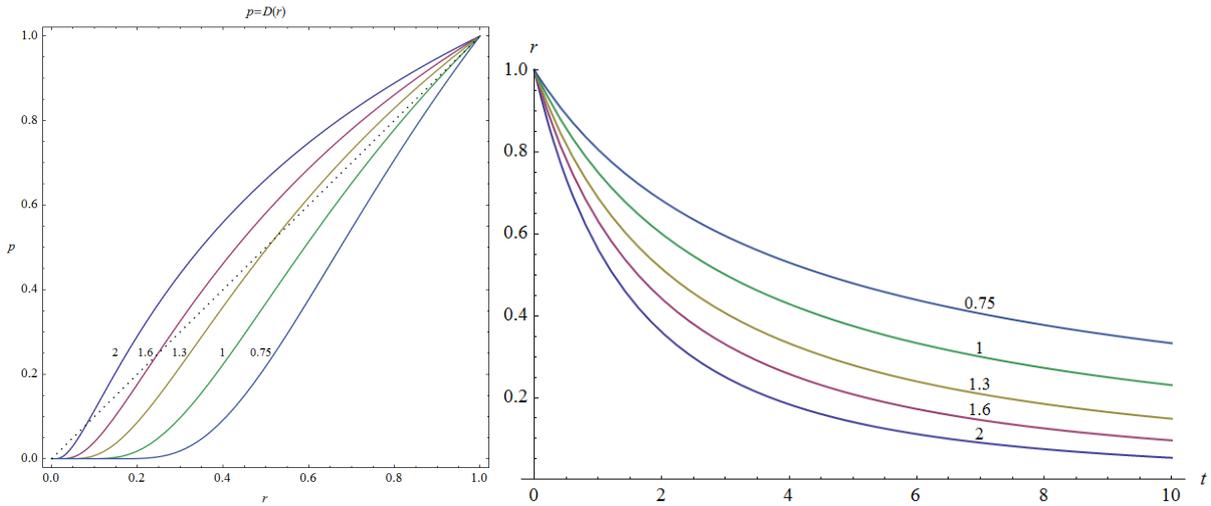
The desired function can be deduced analogously to (4.3):

$$D^{-1}(p) = \frac{1}{(1 - \ln p)^\alpha} \quad (4.7)$$

The functional form of decision utility can thus be determined:

$$D(r) = e^{1-r \frac{1}{\alpha}} \quad (4.8)$$

The shape of decision utility (4.8) is plotted for several values of  $\alpha$  (see Figure 4.1, left). The corresponding shape of the discounting function (4.5) is presented in Figure 4.1 (right).



**Figure 4.1** Decision utility (4.8) and the corresponding hyperbolic discounting function (4.5) for several values of  $\alpha$ .  $\tau = 3$  is assumed for the discounting function.

As in the case considered in Point 3, the same  $\alpha$  parameter controls the shape of the both decision utility and discounting functions. However, this model does not have such attractive properties as the one already presented. First, (4.8) has limited capabilities to model the shape of the function; varying the  $\alpha$  parameter merely shifts the decision utility curve, and a linear and an inverse S-shaped decision utility can not be described. Second, the exponential discounting function may be described by only using (4.5) in the limiting case when  $\alpha$  and  $\tau$  approach infinity. Third, the discounting function does not assume a constant value at  $t = \tau$  for different values of  $\alpha$ : this means that  $\tau$  cannot be unequivocally interpreted in this model. Further limitations of this solution are encountered in financial asset valuations (see Point 7).

Let us repeat the presented approach using another general form of the hyperbolic function (1.5):

$$r(t) = \frac{1}{1 + \left(\frac{t}{\tau}\right)^\alpha} \quad (4.9)$$

The decision utility function  $D$  needs to satisfy:

$$\frac{1}{1 + \left(\frac{t}{\tau}\right)^\alpha} = D^{-1}\left(e^{-\frac{t}{\tau}}\right) \quad (4.10)$$

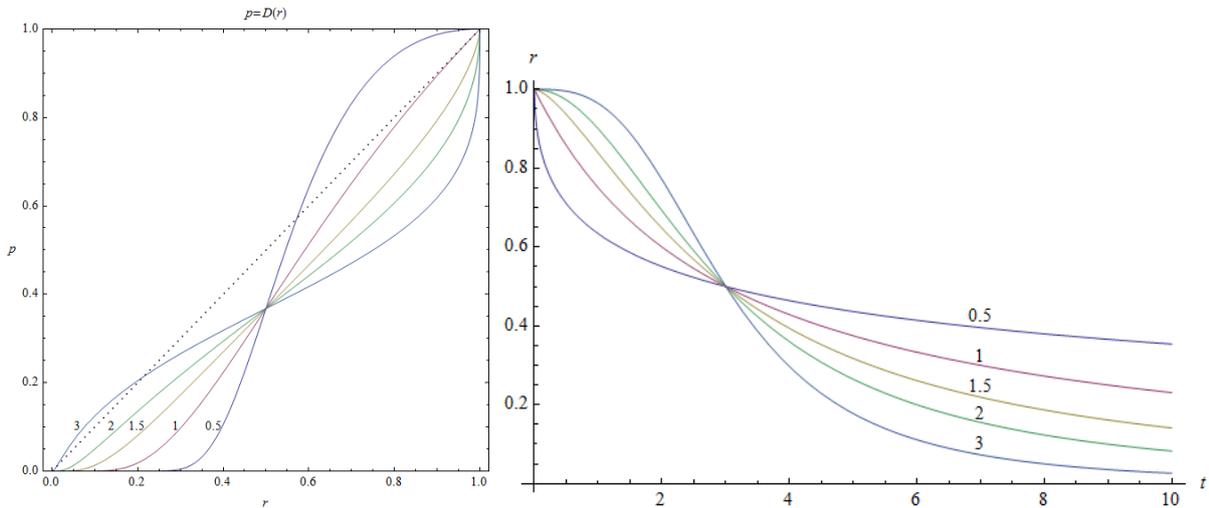
The inversed decision utility is deduced to be:

$$D^{-1}(p) = \frac{1}{1 + (-\ln p)^\alpha} \quad (4.11)$$

This leads to the decision utility functional form:

$$D(r) = e^{-\left(\frac{1}{r}-1\right)^\frac{1}{\alpha}} \quad (4.12)$$

The shape of the decision utility curve (4.12) for several values of  $\alpha$ , together with the corresponding shape of the discounting function (4.9) is presented in Figure 4.2.



**Figure 4.2 Decision utility (4.12) and the corresponding hyperbolic discounting function (4.9) for several values of  $\alpha$ .  $\tau = 3$  is assumed for the discounting function.**

In this case, the plots more closely resemble those considered in Point 3, but this result is not entirely satisfactory either. First, the decision utilities intersect at the point  $p = 1/e \approx 0.37$  and  $r = 0.5$ , which describes very unlikely behavior (as the probability of winning the lottery is typically *greater* than the relative certainty equivalent). Second, the linear decision utility cannot be described using (4.12). Third, (4.9) cannot describe the exponential discounting, even in limiting cases. Fourth, the “hyperbolic” shape only obtains for  $\alpha < 1$ , which is a very inconvenient attribute in financial applications (see Point 7).

## 5 More Advanced Forms

The stretched exponential discounting function (3.7) has been derived using the simple functional forms of decision utility and the survival function. It is interesting to check the result using their more advanced forms. First, the two-parameter function proposed by Prelec (2000) is used to describe decision utility:

$$D(r) = e^{-\left(\frac{\ln r}{\beta}\right)^{\frac{1}{\alpha}}} \quad (5.1)$$

The inversed decision utility is given by:

$$D^{-1}(p) = e^{-\beta(-\ln p)^{\alpha}} \quad (5.2)$$

Substituting (5.2) and the exponential survival function (3.3) into (2.5) results in:

$$r(t) = e^{-\beta\left(\frac{t}{\tau}\right)^{\alpha}} \quad (5.3)$$

As one of the parameters  $\beta$  or  $\tau$  appears to be redundant, (5.3) can be presented as:

$$r(t) = e^{-\left(\frac{t}{\tau'}\right)^{\alpha}} \quad (5.4)$$

where  $\tau' = \tau \beta^{\frac{1}{\alpha}}$ . Despite using a more complex functional form of decision utility, the resulting discounting function thus remains the stretched exponential function.

More complex forms of the survival function can also be used. Let us assume that it is described using the Weibull distribution, which has an additional  $\beta$  parameter to cater for a variable failure rate. The complementary cumulative Weibull distribution is a stretched exponential function, quite coincidentally to the considerations presented so far. The Weibull survival function is therefore expressed as:

$$S(t) = e^{-\left(\frac{t}{\tau}\right)^{\beta}} \quad (5.5)$$

For  $\beta = 1$ , the Weibull survival function reduces to the exponential, which has already been considered (see (3.3)). Substituting (5.5) and Prelec's basic form (3.5) into (2.5) leads to:

$$r(t) = e^{-\left(\frac{t}{\tau}\right)^{\alpha'}} \quad (5.6)$$

which is another stretched exponential function with  $\alpha' = \alpha \beta$ . The stretched exponential dis-

counting function thus remains a solution when more complex functional forms of decision utility and the survival function are used.

Obviously, this should not be treated as a general rule, as Prelec's functions are not the only means of describing decision utility, and the Weibull distribution is not the most advanced form of describing survival functions. Let us assume the cumulative beta distribution to describe the inversed decision utility function:

$$D^{-1}(p) = I(p, \alpha, \alpha) \quad (5.7)$$

where  $I$  denotes a regularized incomplete beta function (cumulative beta). Only a single parameter  $\alpha$  is used to model the curvature of the decision utility function with an intersection point of  $r = 0.5$  rather than  $r = 1/e$  as in the case of Prelec's function (3.4). Substituting (5.7) and (3.3) into (2.5) gives:

$$r(t) = I\left(e^{-\frac{t}{\tau}}, \alpha, \alpha\right) \quad (5.8)$$

which is a possible form of the time discounting function.

## 6 Time Utility

The present value of a future outcome will now be calculated. We refer to the definition of the relative outcome  $r$  (2.3). Please note that the minimum outcome assumes a value of 0 in the presented considerations. Therefore (2.3) simplifies to:

$$r = \frac{v(PV)}{v(P)} \quad (6.1)$$

where  $PV$  denotes the present value of a future outcome  $P$ , and  $v$  is the perception utility. Thus:

$$v(PV) = v(P)r \quad (6.2)$$

As  $r$  describes the discounting function, it follows that:

$$v(PV) = v(P)r(t) \quad (6.3)$$

Assuming that perception utility can be described using Steven's power law, and that the discounting function is described using the stretched exponential function, (6.3) can be presented as:

$$PV^\gamma = P^\gamma e^{-\left(\frac{t}{\tau}\right)^\alpha} \quad (6.4)$$

where  $\gamma$  denotes the power coefficient. This allows the present value to be determined in absolute terms:

$$PV = P e^{-\left(\frac{t}{\tau}\right)^{\alpha'}} \quad (6.5)$$

where  $\alpha' = \alpha / \gamma$ . Including the power perception utility thus leads to the stretched exponential discounting function once more. More complex outcome pattern will be analyzed in the Point 7.

## 7 Behavioral Present Value of Future Cash Flows

The discounting function is of great importance in financial valuations, in which the Discounted Utility model (1.1) is usually used. Let us verify this model using behavioral discounting functions. Let us assume a continuous, constant future cash flow of  $C$  whose behavioral present value can be calculated using the stretched exponential function:

$$PV = \int_0^{\infty} C e^{-\left(\frac{t}{\tau}\right)^{\alpha}} dt = C \tau \Gamma\left(1 + \frac{1}{\alpha}\right) \quad (7.1)$$

where  $\Gamma$  is the gamma function. For  $\alpha = 1$ , i.e. the exponential discounting, (7.1) reduces to  $C\tau$ . Using the hyperbolic function, the integral does not converge:

$$PV = \int_0^{\infty} C \frac{1}{1 + \frac{t}{\tau}} dt = \infty \quad (7.2)$$

This precludes this function from being used in financial applications, as the perceived price of perpetual bonds (like British Consols) would be infinite. Using the general hyperbolic discounting function (4.5) results in:

$$PV = \int_0^{\infty} C \frac{1}{\left(1 + \frac{t}{\tau}\right)^{\alpha}} dt = C \frac{\tau}{\alpha - 1} \quad (7.3)$$

which only has a finite value for  $\alpha > 1$ . This restricts the use of this function in asset valuations. It also limits the possible shapes of the decision utility curve. Using the other general hyperbolic function (4.9) results in:

$$PV = \int_0^{\infty} C \frac{1}{1 + \left(\frac{t}{\tau}\right)^{\alpha}} dt = C \frac{\pi \tau}{\alpha \sin \frac{\pi}{\alpha}} \quad (7.4)$$

which likewise only holds for  $\alpha > 1$ . However, the discounting function (4.9) presents the reversed “hyperbolic” shape for such parameter values (see Figure 4.2). To put it another way, the “hyperbolic” discounting described by (4.9) always results in an infinite integral value.

The verification of the functions used to describe “hyperbolic” discounting presented here have not as yet been presented in the behavioral literature on the subject. The finite integral for all values of  $\alpha$  demonstrates a major advantage of the stretched exponential function over hyperbolic functions. The latter appear to have limited applications outside the laboratory.

This remark does not exhaust the topic of behavioral discounting in financial applications. One of the most intriguing subjects to be found under this heading is the shape of yield curves, which present the relation between the interest rate and the time to maturity. These curves typically indicate an *increasing* interest rate for longer maturity periods. This means, a premium is paid for lending money for a longer time<sup>7</sup>. This starkly contradicts the *decreasing* discount rate established experimentally by behavioral economics. A detailed discussion of this reversed pattern (which is “normal” in the financial world) is beyond the scope of this paper. Nevertheless, it is worth noting that the hyperbolic functions (4.1) and (4.5) are unable to describe this phenomenon, as their discount (failure) rates only ever decrease over time.

## 8 Conclusions

This paper presented a simple functional link between the lottery and time decision making theories. This was done using Decision Utility Theory (Kontek, 2010). Decision utility was assumed to be equal the survival function, with probability being the link between the two areas of decision making. This general formula allowed the time discounting function to be determined. The stretched exponential function appeared to be its simple functional form. This function also resulted when more advanced forms of decision utility and survival function were used. However, other forms of the relationship, including hyperbolic functions, are possible. Unfortunately, these come up against many descriptive and practical hurdles. The stretched exponential function thus not only captures the relationship between lotteries and time decision making, but appears to be the most versatile in valuing financial assets.

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<sup>7</sup> Therefore this pattern is called the “normal yield curve”. Flat and decreasing interest rates are met less frequently. The latter pattern is called the “inverted yield curve”.

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