Estimation of the Business Cycles - Selected Methodological Problems of the Hodrick-Prescott Filter Application

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The aim of this article is to highlight a number of problems due to a rather „mechanical“ application of the Hodrick-Prescott Filter and its possible implication for the monetary policy decision-making process. The author concludes that HP filter is not able to estimate the potential output, however it could be used as the efficient tool in monetary policy decision making process and its stabilizing function.

Keywords

economic activity indicator, time series, BDS test, stochastic process, non-linearity

JEL Classification

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Introduction

For the purposes of this article the expression economic cycle means *„more or less regular pattern of expansion (recovery) and contraction (recession) in economic activity around the path of trend growth.“* (Dornbusch, 1984, pp.9) Various phases of the economic cycle and alternating peaks and troughs, when the economic activity is higher or lower than the long-term trend, are deeply wedded to the inflation and unemployment rates. For this reason the question of definition and identification of economic cycles is in the forefront of interest not only for economists but especially the policy makers, whose aim is the inflation stabilization and economic growth or unemployment maintaining in the long-run.

In historical reports one can find a number of logical arguments for the origin of economic activity fluctuation. Whether these are bad weather conditions causing poor harvest, discrepancy between organic and non-organic products or political influences, contemporary economists keep looking for causes of economic cycles. The first economic cycle indicators in 18th century England are described by Sherman and Kolk (1996). One can connect their existence with a certain level of economic maturity and understand them as part of the economic development. However the basic precondition for the existence of cyclical fluctuation are innovations. These have the character of structural slumps which trigger investment activities. Nevertheless, in the short run, investment activities exceed the real possibilities of the given economics, and then, in the long run, they return to a balanced state. The periodic occurrence of innovations causes economic cycle which, in real economics, is distinguished by irregularity both in time, meaning the frequency of alternating various stages, and in the fluctuation rate. (Schumpeter, 1939)

According to Keynes the main causes of cyclical fluctuation are aggregate demand and induced investement. These function as criteria of real economic activity which grows in case of nonzero induced investment. However the accelerated tempo of the economic activity growth slows downint the moment the economics reaches the realm of its production possibilities. Thereafter there is disinvestment followed by a necessary break during which the used physical capital needs to be replaced. On the contrary, monetarists accentuate the negative effect of demand-oriented economic state policy, in conjunction with the possibility of deepening economic oscillation. According to the monetarists the main source of cyclical economic activity fluctuation are external shocks whose consequences we can be regulated through monetary politics. New classical economics complements the given approaches with the element of rational expectations where the real output, and therefore the economic performance, are influenced by unexpected changes in economic policy. The influence of exogenous shocks on economic activity is further accentuated by a theory of real economic cycles. One can label it as a stochastic version of Walrasian principles of perfect market balance. The precondition is absolute awareness of the economic entities and identification of real (not the nominal) economic activity fluctuation. New Keynes Economy is concerned with the markets´ shortcomings such as the source of economic activity fluctuation, particularly price and emoluments rigidity (Holman, 2001).

In empirical analyses of economic cycles the main causes for the origin of economic activity fluctuation around the long-term trend are the structural slumps in market economy, institutional changes (initiated by the economic policy makers and market entities), discretionary economic policy, erroneous expectations of economic entities, external shocks, technological, information and knowledge changes and changes in the earnings and price flexibility (Zarnowitz, 1992, pp.77-124).

It is apparent that one can find a number of theoretical approaches in contemporary literature same as the explanations for the existence of cyclical economic activity fluctuation around the long-term trend. Their simple explanation of the impact of one or several factors or shocks on the economic activity under mutual interaction and ceteris paribus conditions is very attractive and illustrative but rather imperfect when using empiric analyses. Cyclical fluctuation of economic activity actually aggregates all the above mentioned and not mentioned phenomena, while the expectations of economic entities and information availability on the market play an equally important role.

Irregularity in cyclical recurrence of economic activity fluctuation is actually increased by different length of individual economic cycle types (waves). The economic theory most commonly distinguishes between the long Kondratiev waves (on average from 15 to 25 years long), the intermediate Juglar waves (on overage from 10 to 11 years) and the short Kitchin waves (40 months). However it is not possible to say that one economic wave causes the next one. The Kondratiev wave can be connected with changes in manufacturing technologies, monetary phenomena and political events, the Juglar wave with investments into machinery and equipment, the Kitchin wave with seasonal influences (fluctuation in the construction field, etc.). One can assume that these waves are interconnected.

The application of quantitative methods for the indentification of economic cycles, which uses deterministic approach of trend modelling, does not prove as convenient enough due to a great deal of irregularities in time series. If we consider cyclical economic activity fluctuation around long-term trend as random, there is suitable alternative in form of stochastic trend modelling. *„In the first model, known as the deterministic trend model, variables evolve as a function in time along a linear trend. In the second model (the stochastic trend model) variables evolve as a function of their foregoing values and a shock shifts the value of the variable from the lagged value. In this second case any shock does evidently affect the value of the variable at all leads and, therefore it has a persistent effect. Moreover the time series is entirely determined by the occurrence of all past shocks.“* Bevilacqua a Zon (2001, pp.2)

However random character of observation in time series, which monitor economic activity, is only illusive from the economic point of view. The reason being high degree of indicator complexity, such as GDP. The problem therefore lies primarily in the information which we are able to acquire from real data available, and also in the shortcomings of statistical methods which are able to test the importance of this information. The complexity of economic activity indicators which triggers the apparent impression of stochastic data character gave ground to the development of „mechanical“ filter techniques for the identification of cyclical fluctuation in macroeconomic time series. Among the techniques often used for estimating cyclical fluctuation in economic activity also belongs the Hodrick-Prescott Filter (HP Filter).

Experimental procedures

The precondition of the application of filter techniques for the identification of economic cycle is the schotastic character of time series. The criterium for accepting this assumption is the Brock-Decher-Scheinkman test, known as BDS test (Brock et al., 1991). However this test cannot identify the schotastic or deterministic character of the time series directly, only through nonlinearity. The assumptions that the underlying system is a non-linear process or deterministic system are based on the chaos theory. The null hypothesis is defined that individual observations forming the time series are independently and identically distributed (I.I.D.) against an unspecified alternative, where the time series is formed nonlinearly (Bevilacqua and Zon, 2001, pp. 13). The test is based on the calculation of correlation integral

$$c_{m,n}(\varepsilon) \equiv \frac{2}{(n-m+1)(n-m)} \sum_{s=m}^{n} \sum_{t=s+1}^{n} \prod_{j=0}^{m-1} I_{\varepsilon}\left(y_{s-j}, y_{t-j}\right) \tag{1}$$

and the test criterium

$$w_{m,n}(\varepsilon) = \sqrt{n-m+1}\,\frac{c_{m,n}(\varepsilon) - c_{1,n-m+1}(\varepsilon)^m}{\sigma_{m,n}(\varepsilon)} \tag{2}$$

where the variance estimation of $c_{m,n}(\varepsilon) - c_{1,n-m+1}(\varepsilon)^m$ is determined by the relation

$$\sigma_{m,n}^2(\varepsilon) = 4\left\{k^m + 2\sum_{j=1}^{m-1} k^{m-j}c^{2j} + (m-1)^2 c^{2m} - m^2 k c^{2m-2}\right\} \tag{3}$$

and where the proximity parameters ε and the space dimension m are selected arbitrarily.[1] Application of the BDS test is actually considerably limited by the duration of the time series. In case the data files are smaller than 400 observations, the null hypothesis may be rejected by mistake. For the purposes of this article both the parameters and the critical value of the BDS test for 50 observations, provided that the data distribution is normal[2], have been taken over from the tabulated data through Monte Carlo simulations (Kanzler, 1999).

Concerning the empirical analysis itself it is necessary to distinguish between a classic and growth approach to the identification of the economic cycle. The classic approach regards the individual phases of the cycle as peaks or troughs in economic activity. In the empirical model can be qualified as a long-term trend y^g:

$$y_t = y_t^g + \varepsilon_t\,. \tag{4}$$

By contrast the growth approach differs the cyclical fluctuation from the long-term trend:

$$y_t = y_t^g + y_t^c + \varepsilon_t\,. \tag{5}$$

In accordance with the original definition of the economic cycle as a cyclical fluctuation of economic activity around long-term trend, in relation (5) labeled y^c, the classic approach to the economic cycles estimation will not be considered in the rest of the article. Among others, the growth approach to the economic cycle modeling proves to be generally more suitable for transitive economies where higher growth of the economic activity is expected than in others.

The HP Filter, used in empirical analysis for the economic cycle estimation, is based on minimizing variability of cyclical component on second difference of the trend component:

$$\left\{y_t^g\right\}_{t=0}^{T+1} = \min \sum_{t=1}^{T}\left[(y_t - y_t^g)^2 + \lambda\left[(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)\right]^2\right] \tag{6}$$

[1] The ambiguity of statistical theory when choosing the proximity parameter ε and space dimension m is criticized by Kočenda (2001) who expands the issue by adding his own calculation of parameters through correlative integral.

[2] The normality of data distribution is tested by a Jarque-Bera test based on the coefficients of skew and kurtosis.

The second difference of the trend component is actually weighed by smoother parameter λ whose growth is accompanied by more effective smoothing of the trend component.
The zero value of the smoother parameter leads to a simple solution of minimizing procedure, where $y_t = y_t^g$ and the trend component equals the readings of the time series y_t. Parameter λ goes to infinity represents the trend component in the form of a linear trend.

It is appropriate to mention that the HP Filter is intended for application on logarithmic values of the time series in order for the first difference is a growth.

Results

If we base our conclusions on relation (4), we can say that when modeling the economic cycle Hodrick and Prescott assumed that white noise covers the cyclical component y^c and residuum ε_t. Sufficient smoothness of the trend component y^g, which is modeled by the HP filter, is guaranteed by indirect effect of shocks on the trend level. It is because shocks in economic activity influence the tempo of the trend component y^g growth.

According to Guay and St-Amant (1996, pp. 6), in comparison with other methods for time series modeling, it is not possible to evaluate the informative efficiency of the HP Filter from the perspective of the optimal time series smoothing with the objection to minimize mean squared error:

$$MSE = (1/T)\sum_{t=1}^{T}(\hat{y}_t^c - y_t^c)^2 \ . \qquad (7)$$

Among the main arguments of the above-mentioned authors belong: the non-existence of spurious regression between short-term and long-term time series component, second variance of the process y_t (macroeconomic time series are usually transformed by first difference or stationarity is expected), controversy of character presupposition of short-term error such as white noise and feasibility of the smoother parameter λ. Among other limitations, which are necessary to be taken into consideration when applying the HP Filter, is the boundary effect in case the values do not intercept a similar phase of the cycle. When applied to yearly or quarterly values, the shifting is averaged at three years (Baxter and King, 1995).

The essence of the problem of evaluating the informative efficiency of the HP Filter and above all setting the optimal smoother parameter λ, resides mainly in its own definition of cyclical component y_t^c. This should be defined through economic activity and should represent the real economic activity fluctuation around the long-term rising trend.

Nevertheless, the problematic issue is the repeatedly mentioned complexity of economic activity indicators and the definition of economic cycle wave length connected with the reason for fluctuation.

Economic theory uniquely defines component y^g as a long-term trend of economic growth determined by the amount and productivity of human and capital resources. Whereas the trend component of economic activity time series is influenced only by real factors, the cyclical fluctuation y^c is caused not only by short-term changes of real factors, but also by nominal factors.

As mentioned above, we can distinguish between cycles of different length: the long Kondratiev waves where structural long-term changes in economies belong to its main generators (for example the fall of the Iron Curtain in 1989, the change in exchange regime of CZK into managed floating in 1997, the Czech entry into the EU in 2004), whereas in the intermediate Juglar economic waves these are mainly investments, and the Kitchin waves is related to short-term, on average 40-month, fluctuation within individual major national industries and is caused by changes in supplies and production in process.

An important factor for the identification of economic cycles when using filter techniques is the expected length and the type of cycles. Burns and Mitchell (1946) defined economic cycles in connection with the application of Baxter-King Filter as six to twenty-three quarters long.

When designing the HP Filter, the theory of Hodrick and Prescott (1981) was based on empirical data of the U.S. economic activity in the years 1947-1973. In doing so, the authors suppose that cyclical economic activity fluctuation is closely connected with the number of working hours whereas the population size is non-relevant. Fluctuation in capital stock and productivity (economic performance per working hour) is, according to Hodrick and Prescott´s theory, irrelevantly connected with high-frequency fluctuation of economic activity.

If we proceed from the assumptions of neoclassic theories of growth, which are mentioned by both authors in their work, it is possible to determine the trend economic activity component y^{g} principally by the capital stock and its productivity. And further, the number of working hours should be based on the actual changes in stock and production in process so that we define the cyclical component y^{c} as Kitchin cycle, average length of 40 months.

In accordance with the assumptions of the Solow Neoclassical growth model related to the oscillation of the given economy around a stable equilibrium as the result of from long-term balanced growth, where the capital growth and potential product equals the growth of population, the authors of the HP Filter expect mild oscillation of component y^g in time t. The relationship between the trend y^g and cyclical component y^c is determined by smoother parameter λ.

The smoother parameter λ is therefore closely connected with the cyclical fluctuation type and its sources. If the cyclical component y^c and the second difference of the trend component y^g are independently and identically distributed and the entry data are subject to normal distribution with zero mean value, then:

$$\lambda = \sigma_1^2 / \sigma_2^2 , \tag{8}$$

where σ_1^2 is variation of cyclical component $(y_t - y_t^g)$ and σ_1^2 represents variation of trend component growth. Then the relation (6) can be described as:

$$\{y_t^g\}_{t=0}^{T+1} = \min \sum_{t=1}^{T} \left[\frac{1}{\sigma_1^2}(y_t - y_t^g)^2 + \frac{1}{\sigma_2^2}\left[(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)\right]^2 \right]. \tag{9}$$

With the parameter growing, higher significance is given to the second term of the relation (9). In their work, Hodrick and Prescott (1981) identified the values σ_1=5, σ_2=1/8 and the value of smoother parameter λ=1600 for the quarterly data. The authors assume that 5% of cyclical fluctuation of time series component y^c is formed at 1/8% by change in growth component y^g. Thus the economic cycle identified by the HP Filter is formed from 97,56% by component y^c determined by Kitchin Waves and from 2,44% by cyclical economic activity fluctuation caused by determinants of long-term economic growth such as a number of capital resources and their productivity or possibly other influences given by other kinds of cyclical fluctuation.

Discussion of results

Discussions concerning the application of the HP Filter when identifying economic cycles are connected mainly with neoclassical growth models as the base for the theory of real economic cycles. External economic shocks and their performance in economic activity of stochastic character are considered as the source of fluctuation. The precondition of cyclical economic activity fluctuation around the long-term trend is its irregularity in time and degree of fluctuation. However no uniform consensus concerning the economic cycle estimation methodology exists in contemporary literature. In spite of the claim that the methodology of economic growth modeling and economic cycles should be based on the same principles, the dynamic economic theory is not able to identify the trend component or the relation between long-term and short-term cyclical fluctuations. HP filter for quarterly observations connected with the smoother parameter $\lambda = 1600$ identifies cyclical fluctuation in the interval of approximately 4-6 years (Mechado, 2001). The compilation study by this author further points out various sources for fluctuation in connection with the length of the cycles and distinguishes between centennial, seasonal, cyclical and irregular components, which are contained in one time series. As mentioned above, it is not only the complexity of economic activity identifiers but also spurious relationship between the different types of waves creates problems when modeling the time series through deterministic trend.

Criticism of the HP Filter application as means of economic cycle identification does not concern only the choice of the smoother parameter but also the filter itself. Guay and St-Amant (1996) point out the biased estimation of the cycle through mechanical filters in those sections of the time series which do not contain a top or a bottom. However during practical application HP achieves very good results.

It suffices to mention the argument of Prescott himself: the result of the HP Filter is nothing less than a well-defined statistics which measures nothing that would not have a direct connection to economic theory (Kydland and Prescott, 1996, pp. 76-77).[3] It is impossible to fully accept the view that the HP Filter as a time series modeling method belongs to atheoric approaches. As implied by the derivation of the filter itself and the smoother parameter λ, the authors Hodrick and Prescott (1981) based their assumptions on the fact that short-term cyclical oscillation in economic activity is given by the variability of the number of working hours and long-term cyclical oscillation by the number of capital resources and their productivity. Provided that there is spurious relationship between the number of working hours and relevant change in stock and production in process, we can qualify the short-term oscillation in economic activity as Juglar waves with the average length of 40 months. The authors of the HP Filter further suppose that 5% of cyclical fluctuation of time series component y^c is formed at 1/8% by change in growth component y^g. These assumption were drawn from empirical data of economic activity in the U.S. in the years 1947 - 1973. The only thing we can label as atheoretic is the mechanical application of the HP Filter for other purposes than economic activity in the U.S. in the years 1947 – 1973 or the application the smoother parameter λ. The work of Hodrick and Prescott (1981) suggests that

[3] Further discussion concerning the application of the HP Filter in connection with economic theory can be found in Mechado (2001).

minor change of the parameter has no major influence on the results of empirical analyses using the HP Filter.

Conclusions

Although the mechanical application of HP Filter or the choice of smoother parameter λ have been highly discussed, smoothening of the economic activity time series through HP Filter is still efficient in order to identify cyclical fluctuation around long-term trend. The question is the applicability in the economic policy´s decision making process.

The most important measurement issue is how the monetary authorities determine whether the economy is operating above or below its maximum sustainable level. The answer is based on the potential output as the level of output consistent with the maximum sustainable level of employment, simultaneously the level of output at which aggregate demand and supply are balanced and inflation tends to its expected value in long-run. De Grauwe (2007, pp.168) contributes that the inflation bias disappears if the monetary authorities stop aiming to permanently lower the unemployment rate below the natural rate.

It is evident from the previous chapter that HP Filter is not able to identify the maximum sustainable level of economic growth or natural level of unemployment. Its application is not based on the real economic growth sources (technological changes, capital and labor from the neoclassical point of view) identification. However, if the economic activity fluctuates more or less regularly around the steady equilibrium (trend growth), the HP filter is very useful methodology in the monetary policy decision making process.

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