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Yield Curve Analysis: Choosing the optimal maturity date of investments and financing

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Yield Curve Analysis

Choosing the optimal maturity date of investments
and financing

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December 30, 2010

The shape of the yield curve determines the relationship between interest rate risk and return of investments. The analysis of the yield curve can help the investor or financier decide whether to take a short- or long term bond or loan. The management decision of choosing an optimal maturity depends on three form-giving factors of the yield curve: the general level of interest rates, the slope and the curvature of the curve. By using implicit forward rates the decision situation of investors and financiers is modeled and general decision rules for financial managers are derived.

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1 Introduction

Every finance or investment decision always induces a decision concerning the maturity of bonds or loans as well. This is how capital can be raised long-term with the acquisition of one equal duration loan or by a sequence of several short-term loans. When choosing the maturity of funding, as with all economical decisions, risk and return are to be assessed and weighed up against each other. At a 'normal' shaped yield curve, interest rates for short-term loans are lower than those for long-term loans. In the case of successive short-term loans, these interest savings, however, carry the risk of a future rise in market interest rates. So which maturity for funding should be chosen? The analysis of the yield curve respectively the term structure of interest rates indicates an optimal term allocation of investments and financing. A yield curve oriented term structure of debt or financial assets can either notably reduce a company's financing expenses on the liability side of the balance sheet or effectively increase the attainable return of investment on the asset side. The choice of the optimal term period is therefore a substantial factor for the success of corporate investment and financing decisions.

The yield curve is subject to research of an uncountable number of academic papers, where mostly economic issues dominate. The analysis of the yield curve with respect to its meaning for long-term oriented investment and financing decisions, however, is mostly ignored in finance textbooks or research papers. This is rather surprising, as empirical studies prove that numerous companies align the term structures of liabilities with the shape of the yield curve¹. Goal of this article is to fill this gap and to explain the coherence of the term structure of investments and the shape of the yield curve. The understanding of the relationship between return, risk, and duration which is determined by the shape of the yield curve, is fundamental for every financial decision and should, to that extent, be a substantial component of lectures in Finance.

2 The yield curve and market expectations

Investment, as well as financing, entail a future stream of cash flows with certain risks. Financing is different from the investment only by the sign in the cash flow stream. Future payments bare risks concerning inflation, credit risk, the market's change in interest rates, as well as the liquidity of tradable loans. In bond markets these future cash flow streams are tradable as securities, so that the market participants' expectations about the

¹see for instances Guedes, Opler: 1996; Scherr, Hulburt: 2001; Baker, Greenwood, Wurgler: 2002; Antoniou, Guney, Paudyal: 2002.

above mentioned risks are reflected by the term structure of interest rates². Through this the capital market - unlike the market of real investments or the non-tradable loan market - provides the market participant with an extensive transparency of the current assessment of the risks involved in future cash flow streams. The assessment differences can be read off the yield curve, which documents the relationship between market yields and the maturity of bonds. As bond are tradable loans the yield curve in the bond market indicates the term structure of interest rates for investments and for financing within an economy.

For calculation of the yield curve, spot rates or so-called zero-coupon yields are used. A spot rate is the yield of a zero bond, with an interest that is reflected in the difference between a bond's issuing and redemption price. Between the time of issue $t = 0$ and the time of redemption $t = n$, no interest is paid.

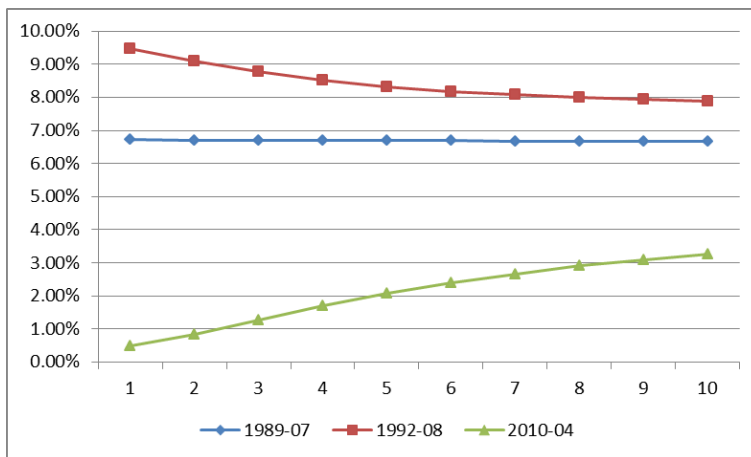


Figure 1 – Normal, flat and inverse yield curves of German government bonds, (data source: Deutsche Bundesbank.)

An interest rate term structure is called "normal" when it shows a positive slope (see fig. 1: 2010-04). In capital markets investors request a higher return (risk premium) for taking a higher risk, so risk and return are strongly correlated. If investors extend the maturity of investments the uncertainty concerning future inflation, the credit risk, the interest rate risk and the liquidity risk increases. As higher the uncertainty, as higher the risk premium agreed upon by creditors and debtors. Consequently spot rates rise with increasing maturity. Theoretically, the market traded spot rate $r_{0,t}$ with the duration 0, t (from today until maturity t) can be divided into the components of a risk free interest rate plus risk premiums for inflation,

²In the case of bonds, not the price, but the yield is traded at the market: The bond price (including accrued interest) represents the present value of the future cash flow stream discounted by the yield. Thus the bond price is a residual of the yield. The yield is the determining pricing factor.

credit risk, interest risk, and liquidity risk. In doing so, the risk free interest rate is usually approximated with the yield of a short-term loan of the most solvent debtor, for example a German government loan. Depending on the market expectations concerning the future economical development, the total risk premium rises more or less significantly with increasing maturity of financing or investment given a normal shaped curve. The flat or inverse term structures characterize extreme market situations mostly in economic boom or deep recession phases and are usually short-lived (see fig. 1: 1989-07 and 1992-08). The flat yield curve shows that spot rates stay the same, unaltered by the maturity. In the case of the inverse yield curve, short maturities are higher-yielding than long maturities.³

German Government bonds are triple-A rated by rating agencies which indicates highest credit quality respectively lowest probability of default. Therefore the German state pays so far the lowest interest rate for financing its state debt in European capital markets. Categorizing bond issuers in regard to their credit quality the bond market is segmented in AAA, AA, A, BBB (and so on) debtor classes and its corresponding yield curves. Debtors with higher credit risk have to pay a higher yield for financing. The difference in yield between debtors of different credit segments (at the same maturity) is called "credit spread". Usually credit spreads between different credit segments are rising with increasing maturity, which implies that debtors with lower credit quality have to pay higher yields with extending the maturity of finance. Hence the distances between credit curves expand with longer maturities (see figure 2).

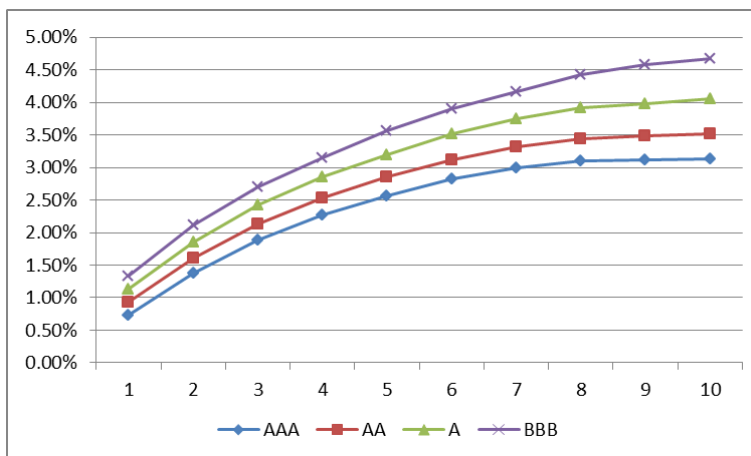


Figure 2 – yield curves of different credit segments (schematic diagram).

The actual term structure in interest rates, however, is not static; it changes constantly with the market participants' changing risk assessment concerning future economical development. The analysis of the yield curve,

³In most financial calculations, a totally flat yield curve is assumed - see for instance the present value formula.

to that extent, always requires a constant monitoring of daily changes in interest rates, since each term structure only represents a momentary depiction of the present assessment by market participants.

Market expectations concerning future interest rates can be read off the yield curve by calculating implicit forward rates. Implicit forward rates are expected future interest rates based on the given term structure. The following example explains the calculation of implicit forward rates: An investor with a two-year investment horizon can either acquire a two-year zero-coupon bond with a spot rate of $r_{0,2} = 2,5\%$ or a one-year zero-coupon bond with $r_{0,1} = 2,0\%$ and in one year again a one-year zero-coupon bond $r_{1,2} = x\%$. The bond's interest rate in one year with a duration of one year is still unknown to the investor at the time of acquisition, but he already has an expectation for the future interest rate. Assuming that the recorded expectations for the spot rates are accurate, the principle of an arbitrage free capital market induces that the investment alternatives must lead to equal returns. Such an ideal capital market requires information efficiency and further includes assumptions such as the lack of taxes and transaction fees, as well as the assumption, that every market participant can raise and invest funds at the same interest rate. The capital market is information efficient, when all information is used to obtain a profit and hence market prices reflect all available information. Assuming this is the case, the expected spot rate in one year time for the duration of one year (so called "implicit forward rate": $r_{1,2}$, from year 1 until year 2) can be calculated by the given one year and two year spot rates ($r_{0,1}, r_{0,2}$). This is the basic statement of the expectation theory of interest rates. If the market participant's expectation about future interest rates are correct and realize in the future than the return of an investment into the two-year loan must equal the return of the combination of both one-year loans.

$$(1 + r_{0,2})^2 = (1 + r_{0,1})(1 + r_{1,2})$$

$$r_{1,2} = \frac{(1 + r_{0,2})^2}{1 + r_{0,1}} - 1 = 0.03002$$

The two year interest rate equals the geometric mean value of the two one year interest rates. If this would not be the case, under the presumption of an ideal capital market, the investor can raise (invest) capital for the two-year interest rate, invest (raise) it for the two one-year periods, and so generate a risk free return. This contradicts the assumption of arbitrage free markets. Thus - assuming that the market participants' expectations for future interest rates are accurate - the investor cannot benefit from splitting the total term of an investment into a sequence of shorter maturities. In other words if the expectation theory holds market participants are indifferent concerning the choice of maturity.

Implicit forward rates can also analogously be estimated for longer maturities, where their number increases with extending the maturity. The three-year spot rate, for example, contains the implicit single period forward rates $r_{1,2}$ and $r_{2,3}$ as well as the implicit dual period forward rate $r_{1,3}$.

$$\begin{aligned}(1 + r_{0,3}) &= (1 + r_{0,1})(1 + r_{1,2})(1 + r_{2,3}) \\(1 + r_{0,3}) &= (1 + r_{0,2})^2(1 + r_{1,2}) \\(1 + r_{0,3}) &= (1 + r_{0,1})(1 + r_{1,3})^2\end{aligned}$$

The restrictive assumption, that the market participants' expectations for future risks are accurate, however, does not correspond with reality. A number of empirical studies point out, that the implicit forward rates only hold a limited prediction quality concerning future interest developments⁴. This can be exemplified by means of market data of any randomly chosen market day. End of August 2007 the state bonds' yields were as follows:

$$\begin{aligned}r_{0,2} &= r_{2007,2009} = 0.0407 \\r_{0,10} &= r_{2007,2017} = 0.0436\end{aligned}$$

In the year 2007 the implicit forward rate $r_{2,10}$ (from year 2009 until 2017) is at 4.43%

$$r_{2,10} = r_{2009,2017} = \sqrt[8]{\frac{1.0436^{10}}{1.0407^2}} - 1 = 4.4326\%$$

At the end of August 2009, the yield for an eight-year loan, however, only is at $r_{0,8} = r_{2009,2017} = 3.2\%$.

Therefore, in hindsight, a long-term investment was more profitable than the sequence of a two-year and a eight-year loan. The implicit forward rate at the end of August 2007 does not accurately predict the eight-year spot rate $r_{0,8}$ in August 2009. Additionally a historic comparison of a one-year spot rate $r_{0,1}$ and the implicit forward rate $r_{1,2}$, shifted by a time difference of one year, repeatedly shows significant deviations between forward rates and future spot rates.

⁴Concerning the prediction quality of implicit forward rates see: Fildes, Fitzgerald, 1980, Gerlach 1997, Cochrane, Piazzesi 2005 und Kalev, Inder 2006

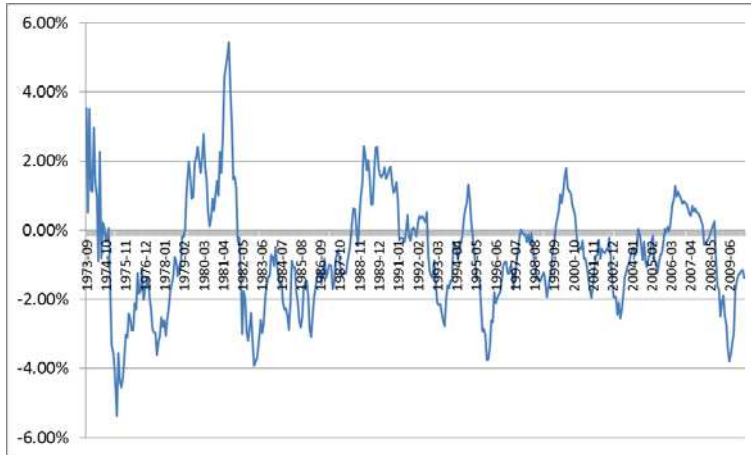


Figure 3 – Difference one-year spot rate and one-year implicit forward rate shifted by one year. (Calculations of implicit forward rates are based on data from the Deutsche Bundesbank for the time-span 09/1973 through 04/2010.)

Therefore, the investor is not indifferent in terms of the choice of maturity. On the contrary: the knowledge, that the implicit forward rates do not project future interest development, makes the choice concerning the maturity date of a loan a crucial determinant for the return. A conscious management of the term structure of investments and financing can substantially reduce a company’s financing costs on the liability side of the balance sheet or increase the attainable return of investment on the asset side.

3 The decision-making problem with the choice of maturity

Although current market expectations concerning future interest development may be read off from the shape of the yield curve, nonetheless implicit forward rates are unable to predict the future interest rates and so the interest rate change risk remains for market participants. For investors as well as for financiers this creates a decision-making problem in terms of choosing the optimal maturity date.

3.1 The investor’s perspective

The investor, as in every economical decision, only takes an additional risk, when it faces an accordingly higher return as a risk premium. When the yield structure is normal, an investment in long-term bond with a fixed nominal interest rate (“coupon bond”) carries the benefit of higher interest earnings in relation to short matured bonds. However, the interest rate change risk rises simultaneously as the investment’s duration increases, because on the one hand, the probability of an interest rate rise increases with extending the maturity and on the other hand the opportunity costs for an interest rate

rise increase with longer maturities. In financial math, this relationship can be illustrated by the changes of the present value. The present value formula shows, that the longer the investment's maturity is, the more strongly the present value reacts to changes in the discounting interest rate⁵. For long-term bonds, a rising market interest rate, hence, leads to a higher loss in bond prices than in the case of short-term bonds. Meaning as longer the maturity of an investment as higher is the risk of an interest rate change. Schematically the investor's decision-making problem can be illustrated as follows:

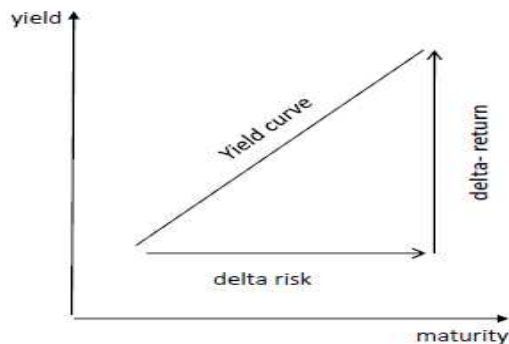


Figure 4 – The investor's decision-making problem

The position of a yield curve determines in how far the market compensates the risk of a maturity extension (delta-risk) by a risk premium in form of a higher yield (delta-return). In the case of a normal yield curve, the increasing risk of a maturity extension is compensated by a yield pick-up when attaining long-term bonds (see figure 4).

The decision the investor has to make now is to assess, whether the risk premiums of a maturity extension offered at the market over- or underestimate the risk of an increase in spot rates during maturity. The preceding historic comparison between the future spot rates and the implicit forward rates (figure 3) documents that the market risk premiums oftentimes estimate the risk of future interest changes to be either too high or too low. If the market's risk premiums would be accurate (the expectation theory holds), the implicit forward rates precisely reflect the future development of spot rates and so the choice of duration is insignificant to the return. But this is not the case.

If the investor chooses to buy a long-term bond he implicitly assumes, that the risk premium offered by the market (delta-return) overestimates the risk of an interest rate increase during the duration period. At a normal yield curve, this is the case, when the future spot rates are below the implicit

⁵A bond's interest rate change risk is described by the "Modified Duration" ratio. It measures the percentage change of the bond's present value if the market yield (discount rate) changes by one percentage point.

forward rates of the current yield structure. Vice versa, when choosing short maturities, the investor presumes that the market risk premium underestimates the rise of spot rates and the future spot rates, to that extent, are above the implicit forward rates.

The following example clarifies the investor's decision-making situation: With the given yield curve, an investor with a ten-year investment horizon can either take up a ten-year investment with a 3.49% spot rate (alternative A) or with the expectation of a future interest rate increase, take up one two-year investment with a 1.38% spot rate and in two years an eight-year investment (alternative B).

maturity	yield
2 years	$r_{0,2} = 1.38\%$
8 years	$r_{0,8} = 3.2\%$
10 years	$r_{0,10} = 3.49\%$

Table 1 – Given interest rate term structure

The risk premium offered by the market for a maturity extension of eight years (from two to ten years) amounts to 2.11 percentage points. With the implicit forward rate $r_{2,10}$ it can be estimated at which future spot rate the two alternatives A and B will provide the same return. Choosing a short maturity (alternative B) is only profitable, when in two years, the eight-year spot rate of currently 3.2% yields above the implicit forward rate $r_{2,10}$ of 4.02%.

$$r_{2,10} = \sqrt[8]{\frac{1.0349^{10}}{1.0138^2}} - 1 = 4.02\%$$

If the investor realistically estimates an increase in the eight-year spot rate by more than 0.82 percentage points within the next two years, he chooses alternative B. Otherwise he chooses alternative A. The difference of 0.82 percentage points between the implicit forward rate $r_{2,10} = 4.02\%$ and the present spot rate of an eight-year investment $r_{0,8} = 3,2\%$, so becomes the central decision-making variable for the investor.

The investor's decision-making problem can be formalized as follows: An investor prefers a long-term investment with the maturity n over a short-term investment with maturity j , when he expects the future spot-rate \dot{r}_{n-j} in $n-j$ years to yield below the implicit forward rate $r_{j,n-j}$. Then:

$$\dot{r}_{0,n-j} < r_{j,n-j} \text{ from this it follows that } [(1 + r_{0,j})^j \times (1 + \dot{r}_{0,n-j})^{n-j}] < r_{0,n}$$

Principally an investor will tend to extend (shorten) the maturity of an investment, the larger (smaller) the difference between the current spot rates and the implicit forward rates. Because, the larger the difference, the higher the existing risk buffer against a possible interest rate increase.

3.2 The financier's perspective

As mentioned earlier, investing and financing only differ in the sign of the future cash flow stream. This difference in sign, however, causes the relationship earlier established for investments (asset side) between risk, return and choice of maturity to be an exact inverted reflection for financing (liability side). Based on a risk assessment, the financier wants to pay the lowest possible interest rate for long-term capital funding. In the case of a normal shaped yield curve, the incentive is to shorten the loans' maturity and to facilitate a long-term funding through the sequence of several short-term loans⁶. The benefit of shortening the maturity is the lower yield in comparison to that of the long-term loan.

The risk of a long-term investment with a fixed interest rate consists in a decrease in market rates during the loans' term. Here again, as longer the chosen maturity is, as higher the risk. Such risk disclosure, however, is inconsistent with the previously formulated decision-making problem of the financier, who only takes an additional risk, if it brings along a higher return - a risk premium. This risk disclosure implies, that he attains a higher return with less risk involved when choosing shorter maturities. But, as risk and return always show a positive correlation, there is no "free lunch" on the financial market. Therefore the interest rate change risk in the case of financing, refers to the risk of refinancing at a higher interest rate after the short-term loan's termination, the so-called "roll over risk". The shorter the first loan's maturity the longer is the remaining time period of financing with possibly higher interest rates. Consequently the risk of an interest rate rise increases with shortening the maturity. The burden of an increase in interest rates rises with the duration of the remaining maturity after the roll over from the first to the second loan. In financing too, the position of the yield curve determines to what extent the market compensates the risk of a shortened maturity with a risk premium in form of interest savings. This relationship between risk and return can be illustrated as follows (see figure 5):

⁶The sequence of short-term loans could be realized by a long term loan with variable interest rates. The interest rate of such a loan is periodically (for instance every six-months) adjusted to current market conditions as it is linked to a money market reference rate like Euribor or Libor rates.

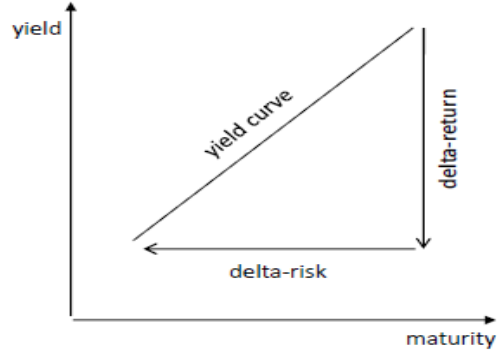


Figure 5 – The financier’s decision-making problem

Referring to the previous example, an investor with a long-term capital demand, on one hand has the option of taking up a ten-year loan with a 3.49% spot rate (alternative A) or on the other hand, expecting that the spot rates will linger on the current level or even drop within the next couple of years, he can take up a two-year loan at 1.39% today and in two years an eight-year loan (alternative B). The implicit forward rate $r_{2,10} = 4.02\%$ marks the return equality between alternative A and B. Alternative B is only profitable, when in two years, the eight-year spot rate is below the implicit forward rate $r_{2,10}$ of 4.02%. If the financier chooses the short maturity, he expects eight year spot rates to increase less than the market does, so one that is below 0.82 percent points within the next two years. In the reverse case, the financier chooses a long investment maturity (alternative A).

Formally, the financier’s decision-making problem can be described as follows: He prefers a long-term loan with the maturity n over a short-term credit with the run-time j , when he expects that the future spot rate $\dot{r}_{j,-j}$ in $n-j$ years is above the implicit forward rate. Then:

$$\dot{r}_{0,n-j} > r_{j,n-j} \text{ from this it follows that } [(1 + r_{0,j})^j \times (1 + \dot{r}_{0,n-j})^{n-j}] > r_{0,n}$$

Contrary to the investor’s decision-making problem, the financier more often tends towards a long-term (short-term) loan, the smaller (larger) the difference between the current spot rates and the implicit forward rates is.

4 Spot rates and implicit forward rates

As this analysis shows so far, the difference between the current spot rates and the implicit forward rates is an important determinant in the choice of maturity for both the investor as well as the financier. The difference between forward rates and spot rates depends on the slope and the curvature of the yield curve. The slope of the yield curve is reflected in the yield spread between short-term and long-term maturities. A yield curve is said to be

steep (flat), when the yield spread exceeds considerably (below) the historic average.

Based on the positive slope of a normal yield curve, the implicit forward rates are yielding above spot rates, while they are below spot rates in the case of an inverse curve. On an entirely flat yield curve, the spot rates and the implicit forward rates are identical. The steeper a yield curve, as higher is the difference between spot rates and implicit forward rates. In the case of a normal shape, the yield curve shows a slight or strong concave curvature. The concavity implies a monotone decreasing slope of tangents from left to right respectively a decreasing yield pick-up with the extension of maturity. This means, that the investor acquires a yield pick-up with the extension of maturity, but the marginal yield increase however decreases continuously. With the extension of maturity, the concavity consequently leads to a flattening of the yield curve at the long end. At the eight to ten year maturity segment the curve than either is very flat, or it may even show a slightly negative slope. The curvature can be measured as a deviation of a straight line connecting one- and ten-year yields. Furthermore a comparison of spreads between one- and five-year yields relative to the (overall) slope of the yield curve (yield spread 1-10 years) also indicates the intensity of the curvature. The more distinctive the yield curve's curvature is, the more the difference between spot rates and implicit forward rates decreases with the extension of maturity.

These facts can be illustrated with the yield curve for German government bonds at the end of August 2009 (figure 6). In historical comparison, the curve presents a high yield spread between one- and ten-year bonds of 2.76 percentage points, through which the implicit forward rates' curves are considerably above the spot rate' curve. The concave shape of the yield curve implies, that the difference between the curves becomes less as the maturity increases. Noticeable is the high difference between the forward rates and spot rates in the segment 1 through 4 years, because here a maturity extension leads to a decreasing, and yet very high yield increase. The yield curve's curvature is in so far very distinct.

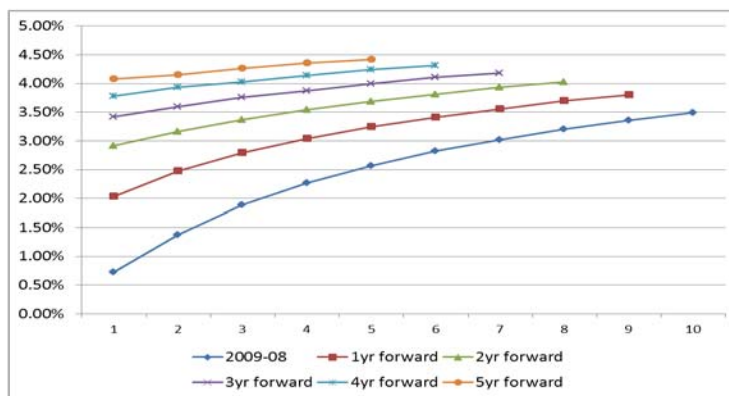


Figure 6 – Spot rates and implicit forward rates 08/2009.(Calculations are based on data from the Deutsche Bundesbank for German government bond yields at 08/2009.)

5 Decision-making rules for the choice of maturity

Aside from the two beforehand mentioned formative factors, slope and curvature, a third factor, the current level of interest rates, complements the description of the yield curve’s shape. In the historical comparison, the current interest level can be low, average, or high. This factor has no direct influence on the difference between spot rates and implicit forward rates; nevertheless, it is relevant to the deduction of decision-making rules.

In the following, each of the three formative factors, level, slope, and curvature are analyzed individually with respect to the choice of the optimal maturity. In its shape, every curve is a combination of the three formative factors. Consequently, the factors’ combination possibilities are to be identified in the next step and coherent decision-making rules are to be deduced for the choice of maturity.

5.1 Level, slope and curvature

A maturity recommendation with respect to the interest rate level is simple: In the case of historically low (high) interest rates, the financier should, when expecting a future market rate rise, take a long-term loan (short-term). Vice versa, it is recommendable, that the investor should in the hope of a future interest rate increase, take a short-term (long-term) investment. Both of these maturity recommendations are based on the implicit expectation, that the deviation from the historic mean is corrected in the not so far future. In this context, (CHUA ET AL., 2006, p. 20) speaks of so-called ”mean-reverting strategies”.

With respect to the slope of the yield curve, the investor takes a long

maturity if the yield curve is steep. A steep curve implies that the increasing interest rate risk by taking a long maturity is compensated with a respective higher yield. When the focus lies solely on the yield curve's slope, and the curve is, in a historical comparison, rather steep (flat), the investment maturity should be extended (shortened). Reversely, in financing, it is true, that in the case of a steep yield curve, the risk of shortening the maturity is compensated with considerably lower interest rates. Consequently, the steeper (the more flat) the yield curve is, the maturity of financing should occur at the shorter (longer) end of the curve. Here it is possible, to argue with the distance between spot rates and implicit forward rates. The steeper the yield curve is, the larger the difference between implicit forward rates and spot rates. As larger the difference as higher is the risk-buffer against a future rise in interest rates.

At a normal shape, yield curves show a concave curvature. This means that the difference between short- and long-term yields (yield spread), is not distributed proportionally throughout the varying maturities. Hence the yield spread between one- and five-year bonds is - although the difference in maturity is only 4 years - regularly higher than the yield spread 10-5 years. Considering solely the curvature of the yield curve, the following statements can be formulated: In the case of a strongly arched curve, the investor can hardly acquire a yield pick-up with the extension of maturity in the five to ten years segment, which compensates the disproportionately increase in risk. The stronger the concavity of the curve is, the more investors should prefer the short to medium maturity segment. For the financing, however, a strongly arched yield curve means, that the risk of a shortening in maturity is compensated through higher interest savings - a higher risk premium - than in the case of a less arched curve.

Summarizing the above analysis of the yield curve's shaping factors and their influence on the choice of maturity of investments and financing, the following table can be rendered:

factor	level		slope		concavity	
	low	high	flat	steep	slight	strong
financing	long	short	long	short	short (+)	short (++)
investment	short	long	short	long	short (+)	short (++)

Table 2 – Maturity recommendations

In table 2, a normal yield curve shape is presumed. A completely flat

as well as an inverse yield curve are extreme market situations and usually of short duration, so that these are not being taken into consideration here. Furthermore, only the characteristics, low/high, steep/flat, and slight/strong are being observed. Each decision for a choice of a short or long maturity recorded in table 2 is solely based on each factor's isolated observation. The curve's concavity always implies a recommendation for shortening the maturity. This recommendation's intensity, however, is much greater in the case of a strong distinction, than in the case of a slightly distinct curve. The usage of the plus signs indicates this.

5.2 Interdependencies of the shaping factors

Every yield curve's shape is always a combination of the three shaping factors. Three factors with two distinctions each lead to eight possible combinations. In relation to the choice of maturity, the investor's or financier's decision-making situation is comparatively simple, when the combination of factors leads to a unanimous recommendation. This, however, is only the case with two combinations. For the other six combinations, the recommendation for the choice of maturity is not definite. Therefore, the interdependencies between the factors must be taken into consideration as well.

Curvature - Slope

The economical relevance of the factor curvature depends on the yield curve's slope. This becomes obvious, when the yield structure is thought to be a straight line with a slight slope. The low yield spread between one- and ten-year investments on such a straight line is distributed evenly throughout the various maturities. The concave curvature, on the contrary, implies an uneven distribution of the yield spread according to size, decreasing from left to right. In the case of a nearly flat curve with a low yield spread between one and ten years, there is a lack in redistribution mass, whereby even in the case of a relatively strong curvature only a slight deviation from the straight line is possible. This is different for a steep yield curve. Here, the high yield spread between short and long ends enables a significant deviation from the linear distribution even in the case of a slight curvature. A strong curvature enhances this effect. Consequently for a flat yield curve the factor curvature can be disregarded as a decision making variable. In the reverse case of a steep yield curve, the curve's concavity becomes a significant influencing factor for the choice of maturity. A steep curve shows at the long end either a very flat, or a negative slope depending on the degree of concavity. For the long-term oriented investor, this effect implies that he cannot achieve a sufficient yield pick-up, which rectifies the increasing risk of

taking a long maturity in the 7 to 10yrs segment of the curve⁷. The previous considerations result in the following maturity recommendation (see table 3) for investments differentiated by the distinction of the criteria slope and curvature:

Investment									
Slope: steep					Curvature: slight				
1	2	3	4	5	6	7	8	9	10
Slope: steep					Curvature: strong				
1	2	3	4	5	6	7	8	9	10
Slope: flat					Curvature: slight				
1	2	3	4	5	6	7	8	9	10
Slope: flat					Curvature: strong				
1	2	3	4	5	6	7	8	9	10

Table 3 – Decision-making rules for investments

For the financing, the risk-return-assessment for a steep curve results in a clear recommendation for the short end and in the case of a flat curve for the long end of the curve. This polarization in short or long maturities as a recommendation results not least because the curve’s concavity leads to a strong flattening of the curve at the long end.

Financing									
Slope: steep					Curvature: strong				
1	2	3	4	5	6	7	8	9	10
Slope: steep					Curvature: slight				
1	2	3	4	5	6	7	8	9	10
Slope: flat					Curvature: strong				
1	2	3	4	5	6	7	8	9	10
Slope: flat					Curvature: slight				
1	2	3	4	5	6	7	8	9	10

Table 4 – Decision-making rules for financing

General level of interest rates versus curvature and slope

The decision-making rules based on the curvature and slope of the yield curve rest upon an assessment of risk and return. In comparison, a maturity decision with regard to a historically low or high interest rate level is based on a directional bet concerning a correction of this deviation from a normal state. This different approach leads to a divergence in maturity recommendation for some combinations of shaping factors. So, for example, in financing, when the interest rate level is very low, but the curve is very steep. If the interest rate level orients the financier, he should make use of low interest rate for long-term funding. However, if he regards the high

⁷For the short-term investment strategies, investments in long-term bonds with a steeper slope and high interest level may, despite concavity, be profitable, because here capital gains have priority in the case of a decrease in market interest rates.

yield spread between short- and long-term loans combined with the concavity, instead, it is advisable to take up short-term loans. Logically, there is no compromising solution, in the sense that the mean maturity period could be recommended. Rather, in this case as in all of the previously discussed cases, the decision-maker's individual risk-preference is pivotal, as well as question whether a dynamic adjustment of risk positions to changing market conditions is possible within the lapse of time. In the previous case, the retail investor most likely tends to the solution with the least risk. Given a low interest rate level, he takes up a long-term loan. This creates the benefit of a fixed basis for calculation of financing costs. Aside from the lower risk, what needs to be taken into consideration is that retail investors cannot easily change the once taken credit position, due to high transaction costs at accelerated termination of the loan agreement. In addition, the uses of interest derivatives, like interest rate swaps, require a certain minimum transaction volume, so that this market is not open to most retail investors or financiers. Therefore, the private investor's risk position is static. On the other hand, institutional investors like companies could use interest rate swaps for changing the maturity anytime without high transaction costs. Therefore institutional investors are doing a risk-return analysis and follow the maturity recommendations based on the slope and the curvature of the curve. For these type of investors or financiers the overall level of interest rates is of minor importance. In the case of a steep curve a company finances on the short end of the curve. In the case of a significant increase of short-term interest rates, an interest rate swap agreement enables the financier to shift the maturity easily from the short to the long end of the curve. Due to higher financing volume of the company and the application of interest rate swaps, a dynamic adjustment of the risk position to market conditions is always possible. However the usage of such financial derivatives presumes expert knowledge in finance, as well as a continuous monitoring of capital market conditions.

6 Final remark

The analysis of the yield curve provides important information for the maturity allocation for investments and financing. Naturally the uncertainty and the risk concerning the future market development of spot rates remain, because an interest rate forecast for a time range of several years is hardly possible. The understanding and the analysis of the yield curve with respect to risk and return, however, enables the finance manager to make a rational decision.

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