# Growth and Welfare Effects of Patentability and Knowledge Spillovers of Basic RD

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# Growth and Welfare Effects of Patentability and Knowledge Spillovers of Basic R&D

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**Abstract**

This study develops an R&D-based growth model with basic and applied research to analyze the growth and welfare effects of two patent instruments (a) the patentability of basic R&D and (b) the division of profit between basic and applied researchers. We find that for the purpose of stimulating basic R&D and economic growth simultaneously, increasing the share of profit assigned to basic researchers is more effective than raising the patentability of basic R&D, which has either a negative effect or an inverted-U effect on technological progress. Nonetheless, a benevolent patent authority requires both patent instruments to achieve the socially optimal allocation in the decentralized economy.

*JEL classification*: O31, O34, O40
*Keywords*: economic growth, innovation, intellectual property rights

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"Our ambition to build a knowledge-based society and a European Research Area requires a strong science base and high quality human capital. Basic research is the answer to both demands. Today's fundamental research will turn into tomorrow's growth, competitiveness and better quality of life. The US has understood this. The EU is still lagging behind. Ours is a wake-up call: we need to act now to reverse this situation and fill the gap." European Research Commissioner Philippe Busquin.[1]

# 1 Introduction

Basic (or fundamental) research is an important part of the innovation process by expanding the frontier of human knowledge. However, unlike applied research, it may not lead to immediate marketable applications. Therefore, basic research is often underprovided and has to be funded by the public sector. Given the importance of basic R&D, the European Research Council was officially launched in 2007 as the first European funding body to support and promote fundamental research. An important policy lever for incentivizing basic research is the patentability of basic R&D. For example, the European Union directive on biotechnological patents (passed in 1998 and implemented by all of the 27 EU member states by the end of 2006) has increased the patentability of biotechnological inventions in Europe. This directive provides that "inventions ... shall be patentable even if they concern a product consisting of or containing biological material or a process by means of which biological material is produced, processed or used" and that "biological material which is isolated from its natural environment or produced by means of a technical process may be the subject of an invention even if it previously occurred in nature." In other words, biological material could be patentable in Europe as a result of this directive. As for the US, in the court case of Diamond vs. Chakrabarty of 1980, the Supreme Court ruled that genetically modified organisms could be patentable.

Another important example of patentability of basic R&D is the Bayh-Dole Act or University and Small Business Patent Procedures Act of 1980.

[1]European Research Commission's press release "Commission calls for a boost in basic research." January 15, 2004.

As a result of this Act, universities in the US are granted the right to patent and license the results of federal government funded research. In a comprehensive review of the Bayh-Dole Act, Mowery *et al.* (2004) argue that although this Act is one of the several key factors that contributed to the significant increase in patenting and licensing of university inventions, many of the historical contributions from universities to industrial innovation took place without patenting. Furthermore, when universities patent their inventions, other researchers are restricted from using these basic research outputs until the patents expire. Therefore, Mowery *et al.* (2004) conclude that it is important for universities to recommit to the free flow of knowledge that has historically enhanced industrial innovation. However, most OECD countries currently give universities the right to patent their government-funded inventions (OECD 2002).

In this study, we develop an R&D-based growth model with basic and applied research to analyze how the patentability of basic R&D affects economic growth and social welfare. On the one hand, we find that increasing the patentability of basic R&D increases patented basic inventions that contribute to economic growth as the conventional argument suggests. On the other hand, it reduces knowledge spillovers because more basic research outputs are patented and hence not accessible by other researchers. Given these opposing effects, we find that patentability of basic R&D may have an inverted-U effect on technological progress.[2] The intuition behind this non-monotonic effect is based on the tradeoff between patent protection and knowledge spillovers. Putting it simply, increasing the patentability of basic R&D increases patented basic inventions that contribute to economic growth; however, this policy change also decreases the accumulation of what we call "pure knowledge," which is freely available to all researchers. Our analysis reveals that these two opposite forces interact with each other to potentially generate a non-monotonic relationship between patent protection for basic inventions and technological progress. Furthermore, we find that patentability of basic R&D has a monotonically negative effect on economic growth when knowledge spillovers depend only on pure knowledge (but not on patented basic inventions). Intuitively, an increase in the patentability of

[2]Recently, Qian (2007) and Lerner (2009) reported novel empirical evidence that enhancing intellectual property rights protection reduces innovation activities when the protection is already strong. This suggests that the relationship between patent protection and innovation follows an inverted-U shape. See Furukawa (2010) for a review on theoretical models.

basic R&D generates a negative effect on the accumulation of pure knowledge as expected and an additional surprising negative crowding-out effect on basic R&D. This crowding-out effect on basic R&D occurs because raising the patentability of basic R&D increases the stock of industrially applicable basic inventions, which in turn improves the incentives for applied R&D by so much that basic R&D falls. Therefore, patentability of basic R&D may not be an ideal policy instrument for stimulating basic research.

In addition to the patentability of basic R&D, we also consider a second related patent instrument that is the division of profit between basic and applied researchers.[3] We show that unlike the patentability of basic R&D, the share of profit assigned to basic researchers has a monotonically positive effect on the equilibrium growth rate. Intuitively, strengthening the bargaining power of basic researchers stimulates basic R&D without stifling the spillover effects of pure knowledge while increasing the patentability of basic R&D reduces spillovers from pure knowledge. Therefore, strengthening the bargaining power of basic researchers relative to applied researchers may be a superior policy instrument in achieving the dual objectives of stimulating basic R&D and economic growth. However, characterizing the optimal coordination of the two patent instruments, we find that a benevolent patent authority requires both patent instruments to achieve the socially optimal allocation in the decentralized economy.

Our study relates to the R&D-based growth literature.[4] This literature emphasizes two fundamental factors for endogenous technological progress. First, the patent institution matters. Without sufficient patent protection, investors would have insufficient incentives to invest resources in R&D activities due to the public nature of knowledge. Another essence is the wide-spread spillover of knowledge, which plays the critical role as a source of long-run economic growth. An inevitable tradeoff emerges between patent protection and knowledge spillovers. Patent protection encourages private incentives for innovation but also limits the wide-spread use of patented inventions. This latter effect weakens knowledge spillovers from pure knowl-

[3] This profit-division rule can be interpreted as the outcome of a bargaining game, in which the relative bargaining power of the basic and applied researchers is influenced by the relative strength of patent protection on the basic and applied inventions. Therefore, it is not unrealistic to treat the profit-division rule as a patent policy lever.

[4] See Romer (1990), Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) for seminal studies and also Jones (2005) for a comprehensive review of this literature.

edge. Although patent protection and knowledge spillovers are fundamental for long-run technological progress, there hasn't been much analysis on their tradeoff in the context of an R&D-based growth model. We fill this gap in the literature by explicitly analyzing this tradeoff in an endogenous growth model with basic and applied R&D.

Our study also relates to the literature on optimal patent design for which Nordhaus (1969) provides the seminal analysis on optimal patent length. Scotchmer (2004) provides a comprehensive review on the subsequent developments in this patent-design literature. While studies in this literature are mostly based on a partial-equilibrium framework, our study follows more closely a related macroeconomic literature by providing a dynamic general-equilibrium (DGE) analysis on patent policy. In the macroeconomic literature on patent policy, the seminal DGE analysis on optimal patent length is Judd (1985), who shows that the optimal patent length can be infinite. Subsequent studies by Horowitz and Lai (1996), Kwan and Lai (2003), Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007) show that the optimal patent length is finite in the Romer model. While these studies focus on patent length, other studies analyze the growth and welfare effects of other patent instruments in R&D-based growth models. See, for example, Cozzi (2001) and Cozzi and Spinesi (2006) on intellectual appropriability, Li (2001) on patent breadth, O'Donoghue and Zweimuller (2004) on forward patent protection and patentability requirement, Furukawa (2007, 2010) and Horii and Iwaisako (2007) on patent protection against imitation, and Chu (2009) on blocking patents. The present paper complements these studies by analyzing the optimal coordination of multiple patent instruments, which is often neglected in this literature, in an R&D-based growth model with multiple research sectors.[5]

The rest of this study is organized as follows. Section 2 sets up the model. Section 3 analyzes the effects of patent policies on economic growth and social welfare. The final section concludes.

[5] See also Chu (2010) for a quantitative analysis on uniform versus sector-specific optimal patent breadth in a two-sector quality-ladder growth model.

# 2 An R&D-based growth model with basic and applied R&D

In this section, we extend the seminal R&D-based growth model in Romer (1990) and Rivera-Batiz and Romer (1991) by introducing two types of innovative activities, basic R&D and applied R&D.[6] To model the patentability of basic R&D, we assume that some basic research outputs are patentable while others are *not*. The probability that a basic research output is patentable captures the degree of patentability of basic R&D. A basic invention that is not patented becomes pure knowledge, which is freely available to all researchers. A patented basic invention is eventually matched with an applied invention subject to a stochastic process. After this match occurs, the matched inventions generate monopolistic profits, and the profits are shared between the basic and applied researchers subject to a profit-division rule. For simplicity, we assume that patent length is infinite as in the seminal Romer model.

## 2.1 The basic setup

Consider a continuous-time model, in which there is an infinitely lived representative household. As in Aghion and Howitt (1996), the household inelastically supplies $L$ units of unskilled labor and $H$ units of skilled labor at each date $t$. Unskilled and skilled labors are used for manufacturing and R&D respectively. The household is endowed with a standard log utility function $U = \int_0^\infty e^{-\rho t} \ln C_t dt$, where $\rho > 0$ is the discount rate and $C_t$ is the consumption of final goods at date $t$. Final goods are used for consumption only. Given $C_t$ as the numeraire, standard dynamic optimization yields the familiar Euler equation given by $\dot{C}_t/C_t = r_t - \rho$, where $r_t$ is the interest rate. Consumption goods are produced by a standard CES (constant elasticity of substitution) aggregator over a continuum of patented intermediate goods

[6] See Aghion and Howitt (1996) for a seminal growth model with basic and applied R&D. Our model is also related to Michelacci (2003), who explicitly distinguishes between R&D and entrepreneurship as two types of innovative activities in a quality-ladder setting. See also Akiyama (2009) and Cozzi and Galli (2009a). However, these studies do not consider the patentability of basic R&D or the knowledge spillovers of pure knowledge.

$X_t(i)$ distributed on $[0, N_t]$.

$$C_t = \left( \int_0^{N_t} X_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $N_t$ is the number of intermediate goods (or industries) and $\varepsilon > 1$ is the elasticity of substitution. Consumption goods firms are perfectly competitive.

Each variety of intermediate goods is monopolistically manufactured by a monopolistic producer who holds the patents on the manufacturing technology for intermediate goods $i$. Each unit of intermediate goods is produced with one unit of unskilled labor; therefore, the marginal cost is equal to the wage rate of unskilled labor $w_t^u$. The monopolistic price for patented intermediate goods $i$ is equal to $[\varepsilon/(\varepsilon-1)]w_t^u$. From profit maximization of consumption goods firms, the conditional demand and profit functions of intermediate goods are as follows.

$$X_t(i) = \left( \frac{\varepsilon - 1}{\varepsilon} \right) \frac{C_t}{w_t^u N_t} \equiv X_t, \quad \Pi_t(i) = \frac{C_t}{\varepsilon N_t} \equiv \Pi_t. \tag{2}$$

An industrial monopolist $i$ holds the patents on the manufacturing technology of product $i$ and produces $X_t(i) = X_t$ units of intermediate goods earning profit $\Pi_t(i) = \Pi_t$ at each date $t$. The value of being the monopolist in industry $i$ is $V_t(i) = \int_t^\infty e^{-(R_\tau - R_t)} \Pi_\tau d\tau \equiv V_t$, where $R_\tau = \int_0^\tau r_s ds$ is the cumulative interest rates up to date $\tau$.

## 2.2 Patented basic inventions and pure knowledge

The economy grows endogenously due to two forms of technological progress. The first growth engine is the accumulation of patented inventions. As mentioned above, an industrial monopolist $i$ holds the two patents of well-matched basic and applied inventions. A basic invention is a preliminary idea, which cannot bear any profit *by itself* but may establish the basis for a future invention that generates profits by introducing a new variety of intermediate goods. We call such a profitable invention an applied invention. When an applied invention is developed on a patented basic invention that previously has not been matched, a new industry is introduced into the intermediate goods market by the industrial monopolist holding the patents of basic and applied inventions. This process of patented inventions continuously occurs and contributes towards the variety $N_t$ of intermediate goods in (1). The productivity of consumption goods firms thus increases over time.

The second engine of technological progress is the accumulation of "pure knowledge." We introduce pure knowledge into the model as a by-product of basic research activity in the following manner. With our consideration on the patentability of basic R&D, some basic inventions are randomly chosen to be patentable by the patent institution while others are not patentable. Formally, each newly developed basic invention is patentable with probability $\phi$. An unpatented basic invention contributes to pure knowledge that is freely available to all researchers, consequently having a knowledge spillover effect on R&D activity as in the knowledge-driven growth model of Romer (1990) and Rivera-Batiz and Romer (1991). Here we assume that applied inventions must be derived from patented basic inventions because any applied invention that is derived from public pure knowledge is not patentable.

Current research productivity depends on both patented basic inventions and pure knowledge. Denote by $K_t$ the cumulative number of unpatented basic inventions that have been accumulated as of date $t$. This $K_t$ represents the stock of pure knowledge in the economy at date $t$. In this setting, we intend to differentiate between pure knowledge and patented inventions in their external spillover effects. To do so, we assume that the knowledge spillover effect is a function of the stocks of pure knowledge and patented inventions, but these two inputs are not perfectly substitutable.[7] Furthermore, we consider two special cases in which the spillover effect depends only on either pure knowledge or patented basic inventions.

[7] It is useful to consider an alternative but related interpretation on $\phi$. If we think of $\phi$ as a technological parameter instead of a policy instrument, $\phi$ could be interpreted as the share of basic inventions that are industrially applicable. These basic inventions are naturally granted a patent. The remaining share $1 - \phi$ of basic inventions is pure research output, such as a mathematical theorem. Under this interpretation, it is natural that applied inventions are developed only on patented (i.e., industrially applicable) basic inventions and that the stocks of pure knowledge and patented inventions are not perfectly substitutable. Also, we can relate this interpretation to the patentability of basic R&D. Increasing the patentability of basic R&D significantly encourages patenting of university inventions (Mowery *et al.* 2004), resulting into an increase in the share of industrially applicable basic inventions. Therefore, increasing the patentability of basic R&D can also be captured by an increase in $\phi$ under this alternative interpretation.

## 2.3 Basic and applied R&D

As mentioned above, the economy grows as the two factors of technological progress, $N_t$ and $K_t$, accumulate. The two factors increase as basic inventions are developed and then industrially applied. In this section, we explain how inventions are made and applied to industries and how inventions increase pure knowledge of the economy. There are infinitely many potential researchers. Potential researchers can make both basic and applied inventions. Denote by $B_t$ the cumulative number (stock) of total basic inventions. The stock of basic inventions $B_t$ increases due to basic research activity by researchers. The accumulation of $B_t$ gradually increases the two growth factors, $N_t$ and $K_t$, through the three stages of research to be described below.

### 2.3.1 Stage I (Patented basic inventions and pure knowledge)

When a potential researcher invests $b/S\left(K_t, B_t - K_t\right)$ units of skilled labor in basic research, she can develop a basic invention $j$ without any risk of failure. In this setting, the spillover function $S(.)$ captures the extent to which the stocks of pure knowledge $K_t$ and patented basic inventions $B_t - K_t$ affect research productivity through knowledge spillovers. We adopt a simple Cobb-Douglas functional form for the spillover function $S\left(K_t, B_t - K_t\right) = K_t^{\psi}\left(B_t - K_t\right)^{1-\psi}$,[8] where $\psi \in [0,1]$ is a factor share parameter that controls how pure knowledge $K_t$ affects the magnitude of knowledge spillovers. $\psi \in \{0,1\}$ captures the special cases in which the spillover effect depends only on either patented basic inventions $B_t - K_t$ or pure knowledge $K_t$.

Each newly developed basic invention is judged as patentable with probability $\phi \in (0,1)$, and basic inventions that are not patented become pure knowledge. Thus, at date $t$, $\phi \dot{B}_t$ units of patentable basic inventions and $(1-\phi)\dot{B}_t$ units of pure knowledge are introduced. We can describe the evolution of pure knowledge as

$$\dot{K}_t = (1-\phi)\,\dot{B}_t. \tag{3}$$

Denote $Z_t$ as the market value of a basic invention that is patentable. Free

[8] It is useful to note that our results are robust to generalizing the spillover function to a CES form. For simplicity, we focus on the Cobb-Douglas spillover function in this study.

entry guarantees the following no-arbitrage condition in equilibrium.

$$\phi Z_t = \frac{b w_t^s}{S\left(K_t, B_t - K_t\right)}, \tag{4}$$

where $w_t^s$ denotes the wage rate of skilled labor at date $t$.

#### 2.3.2 Stage II (Basic inventions waiting for applied inventions)

Basic inventions that are patentable at date $t$, with size $\phi \dot{B}_t$, immediately go to a waiting pool of patented inventions, where patented basic inventions (that have not been industrially applied) await for applied inventions. Denote $W_t$ as the pool of patented basic inventions waiting for industrial applications. The inflow to the waiting pool is $\phi \dot{B}_t$, and the outflow is the number of applied inventions $\dot{N}_t$ that are recently matched with basic inventions in the waiting pool. Then, we have

$$\dot{W}_t = \phi \dot{B}_t - \dot{N}_t. \tag{5}$$

#### 2.3.3 Stage III (Basic inventions becoming industrially applied)

We now turn to applied research activities. Applied researchers make applied inventions that can be matched with basic inventions in the waiting pool. As a result of a successful match, a new industry is introduced into the intermediate goods sector increasing $N_t$. When an applied researcher invests $\iota_t a / S\left(K_t, B_t - K_t\right)$ units of skilled labor in matching with basic invention $j$, she can develop an applied invention that is well matched with the basic invention with probability $\iota_t$. At the aggregate level, it holds that

$$\dot{N}_t = \iota_t W_t. \tag{6}$$

In other words, a fraction $\iota_t$ of the waiting basic inventions $W_t$ becomes industrially applied at date $t$.

Once basic and applied inventions are matched, the patent holders of these inventions can earn and share the final market value $V_t$. This value $V_t$ is shared between the patent holders of basic and applied inventions according to a profit-division rule $s \in (0, 1)$. The basic invention takes $sV_t$ and the applied invention takes $(1-s)V_t$. Then, we can describe the free-entry condition to applied R&D as

$$(1-s)V_t = \frac{aw_t^s}{S(K_t, B_t - K_t)}. \tag{7}$$

Recall that the patent holder of a basic invention takes $sV_t$, earning a net value $sV_t - Z_t$. Therefore, the market value $Z_t$ of a patented basic invention satisfies the following no-arbitrage condition.

$$r_t Z_t = \dot{Z}_t + \iota_t (sV_t - Z_t). \tag{8}$$

The final value of an invention $V_t$ (i.e., the value of a pair of matched basic and applied inventions) follows the familiar Bellman equation.

$$r_t V_t = \Pi_t + \dot{V}_t. \tag{9}$$

Through the above three stages, basic and applied R&D governs technological progress and economic growth. There are two roles of basic inventions in technological progress. One is the role to increase the stock of pure knowledge $K_t$ in the first stage. The other is the role to increase the number of industrially applicable basic inventions $W_t$ and eventually the variety of intermediate goods $N_t$. These two roles interact with each other to drive technological progress and economic growth.

## 2.4 Market equilibrium

The stock of total basic inventions $B_t$ can be divided into three parts as follows.

$$B_t = K_t + W_t + N_t. \tag{10}$$

This equation states that the stock of basic inventions $B_t$ is divided into pure knowledge $K_t$, patented basic inventions $W_t$ that have not been industrially applied, and patented basic inventions $N_t$ that have been industrially applied. We now close the model by considering the labor market equilibrium conditions.

$$L = N_t X_t \tag{11}$$

for unskilled labor, and

$$1 = H_{A,t} + H_{B,t} = \frac{a\dot{N}_t}{S\left(K_t, B_t - K_t\right)} + \frac{b\dot{B}_t}{S\left(K_t, B_t - K_t\right)} \tag{12}$$

for skilled labor. Here we normalize the total supply of skilled labor to unity. The left-hand side of (12) is the total supply of skilled labor and the right-hand side is the total demand for skilled labor from applied research and basic research, respectively.

From (2)–(12), we can completely characterize the equilibrium dynamics of our model. Before proceeding, we define a balanced growth path in our model. On the balanced growth path, variables for cumulative inventions $B_t$, $K_t$, $N_t$, and $W_t$ grow at a constant rate $g^*$, which we call the equilibrium growth rate of technology, and the balanced growth rate of consumption is equal to $\dot{C}_t/C_t = g_c^* = g^*/(\varepsilon - 1)$. Taking into account the laws of motion (3), (5) and (6), we have the following steady-state ratios $K/B = 1 - \phi$, $N/B = \phi\iota^*/(\iota^* + g^*)$ and $W/B = \phi g^*/(\iota^* + g^*)$ on the balanced-growth path. Using these ratios and (12), the equilibrium growth rate of technology is

$$g^* = \frac{\dot{B}_t}{B_t} = (1-\phi)^{\psi}(\phi)^{1-\psi}\left(\frac{H_B^*}{b}\right), \tag{13}$$

where $H_B^* \in (0,1)$ is the equilibrium amount of high-skill labor allocated to basic R&D. Similarly, the equilibrium arrival rate $\iota^*$ of applied inventions is

$$\iota^* = \frac{\dot{N}_t}{W_t} = \frac{S\left(K_t, B_t - K_t\right)/B_t}{W_t/B_t}\left(\frac{H_A^*}{a}\right) = \left(\frac{1-\phi}{\phi}\right)^{\psi}\left(\frac{\iota^* + g^*}{g^*}\right)\left(\frac{H_A^*}{a}\right), \tag{14}$$

where $H_A^*$ is the equilibrium amount of high-skill labor allocated to applied R&D.

Equating (4) and (7) yields a no-arbitrage condition between basic and applied R&D given by $(1-s)\,V_t/a = \phi Z_t/b$. Imposing the balanced-growth condition on (8) yields $Z_t = sV_t\iota^*/(\iota^* + r^* - g_z^*)$, where $r^* = \rho + g_c^*$ from the Euler equation and $g_z^* = g_c^* - g^*$ from (2). Combining these two conditions yields

$$\frac{b}{a}\left(\frac{1-s}{s}\right)\frac{1}{\phi} = \frac{\iota^*}{\iota^* + \rho + g^*}. \tag{15}$$

This condition along with $1 = H_A^* + H_B^*$, (13) and (14) solves the model. In the following analysis, we restrict attention to the feasible region of $\phi$ given by $\phi \in (\phi_L, 1)$, where $\phi_L \equiv (b/a)(1-s)/s$. Given $\iota^* > 0$ and $g^* > 0$, the right hand side of (15) is less than one. Therefore, $\phi > \phi_L$ must hold in order for the left-hand side of (15) to be also less than one. Then we have the following theorem. A formal proof is relegated to Appendix A.

**Theorem 1** *If $\rho$ is sufficiently small, the economy converges to a unique balanced growth path that is locally stable. On the balanced growth path, the equilibrium growth rate $g^*$ of technology is positive and determined by the following condition.*

$$\frac{(1-\phi)^{\psi}(\phi)^{1-\psi}/b - g^*}{\rho + g^*} = \frac{\phi(1-s)g^*}{\phi s g^* + \rho(1-s)b/a}. \tag{16}$$

Equation (16) is a quadratic equation for which the solution is quite complicated.[9] Therefore, we will analyze (16) in the next section. However, it is useful to first consider a limiting case of $g^*$ given by $\rho$ approaching zero.

$$\lim_{\rho \to 0} g^* = (1-\phi)^{\psi}(\phi)^{1-\psi}\left(\frac{s}{b}\right). \tag{17}$$

This special case previews our results that the growth rate $g^*$ is a strictly increasing function in the share $s$ of profit assigned to basic R&D and a potentially inverted-U function in patentability $\phi$.[10] From the above expression, it may seem that the non-monotonic effect of $\phi$ on $g^*$ is entirely built-in through the spillover function because $\lim_{\rho \to 0} H_B^* = s$ is independent of $\phi$. However, this is not true in the general case of $\rho > 0$. In the case of $\rho > 0$, patentability $\phi$ has both positive and negative effects on the equilibrium allocation of $H_B^*(s, \phi)$. The positive effect of $\phi$ on $H_B^*$ arises from the direct effect of raising the patentability of basic inventions that increases the incentives for basic R&D. The negative effect of $\phi$ on $H_B^*$ arises from an indirect effect of patentability $\phi$ that increases applied R&D by so much that it crowds out basic R&D through the resource constraint on skilled labor. Intuitively, there is a positive effect of $\phi$ on applied R&D $H_A^*$ because a larger $\phi$ increases the

[9] See Appendix A for an explicit solution of $g^*$.

[10] To be more precise, $\lim_{\rho \to 0} g^*$ is an inverted-U function within the feasible range of $\phi$ if and only if $0 < \psi < 1 - \phi_L$. If $\psi \geq 1 - \phi_L$, then $\lim_{\rho \to 0} g^*$ would be a monotonically decreasing function in $\phi \in (\phi_L, 1)$. If $\psi = 0$, then $\lim_{\rho \to 0} g^*$ would be a monotonically increasing function in $\phi$.

waiting pool of industrially applicable basic inventions. For a given arrival rate of applied inventions, a larger waiting pool requires more applied R&D, which in turn crowds out basic R&D. When the discount rate $\rho$ approaches zero, this negative crowding-out effect and the positive incentive effect cancel each other. When $\rho$ is strictly positive, it is the interaction between these general-equilibrium effects and the spillover function that drives our results in the next section. Finally, $H_B^*(s, \phi)$ continues to be a monotonically increasing function in $s$ when $\rho > 0$.

# 3 Effects of patent instruments

In the previous section, we have developed an R&D-based growth model with basic and applied research as well as characterizing the dynamic behavior of the model (Theorem 1). In this section, we investigate how the patentability of basic R&D and the division of profit between basic and applied researchers affect economic growth.

Patentability $\phi$ of basic R&D has four effects on technological progress. First, it increases patented basic inventions by raising the probability that a basic invention is patentable. Second, increasing patentability $\phi$ reduces knowledge spillovers from pure knowledge. Finally, increasing patentability $\phi$ has the positive incentive effect and the negative crowding-out effect on basic R&D $H_B^*$ as described in the previous section. The following proposition is our first key result, which demonstrates that the effect of raising the patentability $\phi$ of basic R&D on the equilibrium growth rate $g^*$ is generally non-monotonic except when $\psi$ is either sufficiently large or equal to zero. When $\psi$ is sufficiently large, $g^*$ is monotonically decreasing in $\phi$. In the unlikely case that the spillover function is independent of pure knowledge (i.e., $\psi = 0$), $g^*$ is monotonically increasing in $\phi$.

**Proposition 1** *If $\psi \geq 1 - \phi_L$, then $g^*$ is monotonically decreasing in $\phi$ for $\phi \in (\phi_L, 1)$. If $0 < \psi \leq s - (1-s)b/a$,*[11] *then $g^*$ is firstly increasing and eventually decreasing in the patentability $\phi$ of basic R&D; therefore, the relationship between $g^*$ and $\phi$ is non-monotonic. If $\psi = 0$, then $g^*$ is monotonically increasing in $\phi$.*

[11] It is useful to note that $\psi \leq s - (1-s)b/a$ can be re-expressed as $\psi \leq s(1 - \phi_L)$, which is a sufficient condition for $\psi < 1 - \phi_L$ given $s \in (0, 1)$.

**Proof.** In (16), the left-hand side ($LHS$) is decreasing in $g$ while the right-hand side ($RHS$) is increasing in $g$. Furthermore, $RHS$ is increasing in $\phi$. Therefore, if $LHS$ is weakly decreasing in $\phi$, then the equilibrium growth rate $g^*$ must be decreasing in $\phi$. The necessary and sufficient condition for $RHS$ to be weakly decreasing in $\phi$ is $\psi \geq 1-\phi$. In other words, if $\psi \geq 1-\phi_L$, then $g^*$ is monotonically decreasing in $\phi$ for $\phi \in (\phi_L, 1)$. Taking the total differentials with respect to $g^*$ and $\phi$ in (16) yields

$$\frac{dg^*}{d\phi} = \frac{\left[g^* - \psi[\phi/(1-\phi)]^{1-\psi}/b\right]/[\phi^2(g^*+\rho)] + s(1-s)(g^*)^2/[s\phi g^* + \rho(1-s)b/a]^2}{\rho(1-s)^2 b/[a(s\phi g^* + \rho(1-s)b/a)^2] + \left[\rho/\phi + (1/\phi - 1)^{\psi}/b\right]/(g^*+\rho)^2}.$$

As $\phi \to \phi_L = (b/a)(1-s)/s$, $g^* \to (1-s)\left[(a/b)s/(1-s)-1\right]^{\psi}/a$. Therefore,

$$\lim_{\phi \to \phi_L} \frac{dg^*}{d\phi} = \frac{[s-(1-s)b/a-\psi][\phi/(1-\phi)]^{1-\psi}/[b\phi^2(g^*+\rho)] + s(1-s)(g^*)^2/[s\phi g^* + \rho(1-s)b/a]^2}{\rho(1-s)^2 b/[a(s\phi g^* + \rho(1-s)b/a)^2] + \left[\rho/\phi + (1/\phi - 1)^{\psi}/b\right]/(g^*+\rho)^2} > 0,$$

in which the inequality holds if $\psi \leq s - (1-s)b/a$. Furthermore, we know that so long as $\psi > 0$, $\lim_{\phi \to 1} dg^*/d\phi < 0$. Therefore, $g^*$ must be a non-monotonic function in $\phi$ if $0 < \psi \leq s - (1-s)b/a$. Finally, if $\psi = 0$, then $dg^*/d\phi > 0$. ■

In addition to the analytical result in Proposition 1, we have also conducted a large number of numerical simulations, and the only non-linear configuration that we find is an inverted U. These results explain the *pros and cons* of increasing the patentability of basic R&D. As mentioned above, increasing the patentability of basic research has the following effects on $g^*$. First, it increases the probability that a basic invention is patentable. As a result, the size of the waiting pool $W_t$ increases leading to a larger number of matched invention pairs $N_t$. It can be shown that $\frac{\partial}{\partial \phi}\left(\frac{W_t}{B_t}\right) > 0$ and $\frac{\partial}{\partial \phi}\left(\frac{N_t}{B_t}\right) > 0$ hold on the balanced growth path.[12] This positive effect of patentability $\phi$ on technological progress is reflected in the upward sloping part of the inverted U unless $\psi$ is sufficiently large. If $\psi \geq 1 - \phi_L$, then the equilibrium growth rate is monotonically decreasing in $\phi$ because the remaining positive incentive effect of $\phi$ is dominated by its negative effects on pure knowledge spillovers and the crowding out of basic R&D.

A negative effect of $\phi$ on $g^*$ emerges from the externality of pure knowledge. Raising the patentability of basic inventions reduces the accumulation

[12] See Appendix A for the derivations.

of pure knowledge. Recall that the cumulative stock of pure knowledge $K_t$ has a spillover effect on research productivity. Therefore, the degree of knowledge spillovers from pure knowledge decreases with $\phi$. It can be shown that $\frac{\partial}{\partial \phi}\left(\frac{K_t}{B_t}\right) < 0$ holds on the balanced growth path. Because the knowledge spillover effect is a fundamental source of long-run growth (see for example Romer (1990) and Rivera-Batiz and Romer (1991)), the decrease in knowledge spillovers slows down economic growth. This negative effect of $\phi$ on economic growth is reflected in the downward sloping part of the inverted U unless $\psi$ is equal to zero. When $\psi = 0$, the equilibrium growth rate becomes monotonically increasing in $\phi$ because the remaining negative crowding-out effect of $\phi$ is dominated by its positive effects on patented basic inventions and the incentives for basic R&D.

Next we consider the profit-division rule $s$ as an alternative patent instrument. The profit-division rule $s$ controls the capital gain that is received by basic researchers, $sV_t - Z_t$, whereas the patentability parameter $\phi$ affects the initial expected return on basic research, $\phi Z_t$. We may interpret an increase in $s$ as a strengthening of patent protection for basic R&D relative to that of applied R&D.[13]

**Proposition 2** *The relationship between the profit share $s$ of basic R&D and the technology growth rate $g^*$ is monotonically positive.*

**Proof.** Recall that the $LHS$ of (16) is decreasing in $g$ while the $RHS$ is increasing in $g$. Furthermore, $RHS$ is decreasing in $s$ and $LHS$ is independent of $s$. Therefore, the equilibrium growth rate $g^*$ must be increasing in $s$. ■

Proposition 2 shows that the growth rate is an increasing function in the profit share $s$ of basic research. Intuitively, the growth rate of $B_t$ is determined by basic R&D $H_B^*$, which in turn is strictly increasing in $s$. Furthermore, on the balanced-growth path, the growth rate of $N_t$ is equal to the growth rate of $B_t$. Although increasing $s$ reduces the equilibrium arrival rate $\iota^*$ of applied inventions, this reduction in $\iota^*$ does not affect the growth rate but only the steady-state ratio of $N/B$, which has a level effect on social welfare as shown in the next section. Therefore, strengthening patent

[13] See also Cozzi and Galli (2009b) for an interesting analysis of profit division between basic and applied R&D on wage inequality and human-capital accumulation in a quality-ladder model.

protection for basic R&D relative to applied R&D results in a faster growth rate. Because $s < 1$, there does not exist an interior growth-maximizing profit-division rule.

The drastically different growth effects of the two patent instruments arise for the following reasons. Strengthening patent protection for basic R&D via the profit-division rule stimulates basic R&D without stifling the spillover effects of pure knowledge. However, increasing the patentability of basic R&D eventually decreases basic research due to a crowding-out effect on skilled labor and also reduces the spillover effect from pure knowledge.

## 3.1 Optimal coordination of patent instruments

Propositions 1 and 2 reveal that the two patent instruments, the patentability $\phi$ of basic R&D and the profit-division rule $s$, are useful policy levers for controlling the equilibrium growth rate in the decentralized economy. In this section, we demonstrate that the steady-state welfare-maximizing allocation can be achieved in the decentralized economy only if both patent instruments are present. Therefore, although the profit-division rule may be more effective than the patentability of basic R&D in stimulating basic research and technological progress simultaneously, a benevolent patent authority requires both patent instruments in achieving the socially optimal allocation.

Consider a social planner's problem as follows.[14]

$$\max_{\phi, H_A, H_B} U = \int_0^\infty e^{-\rho t} \ln C_t dt$$

subject to the resource constraint $H_A + H_B = 1$, the laws of motion (3), (5), and (6) along with $\dot{N} = aH_A/S\left(K_t, B_t - K_t\right)$ and $\dot{B} = bH_B/S\left(K_t, B_t - K_t\right)$. Imposing symmetry of $X_t(i)$ on (1) yields

$$C_t = \left(\int_0^{N_t} X_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} = (N_t)^{\frac{\varepsilon}{\varepsilon-1}} X_t. \tag{18}$$

The resource constraint for unskilled labor implies $X_t = L/N_t$, where $L$ is the total supply of unskilled labor. Therefore, (18) becomes $C_t = (N_t)^{\frac{1}{\varepsilon-1}} L$.

[14] In the case of a social planner, it is more appropriate to view $1 - \phi$ as the fraction of basic inventions $B$ that the planner chooses as pure knowledge $K$.

Taking log yields $\ln C_t = (\varepsilon - 1)^{-1} \ln N_t + \ln L$, and the utility of households on the balanced-growth path becomes

$$U = \frac{1}{\rho(\varepsilon - 1)} \left( \ln N_0 + \frac{g}{\rho} \right), \tag{19}$$

where the exogenous $L$ has been dropped. The equilibrium growth rate of technology in (13) and the equilibrium arrival rate of applied inventions in (14) hold along the balanced growth path for both the decentralized and centralized economies. Using (13) and (14), we can re-express $N/B$ as[15]

$$\frac{N}{B} = \frac{(\phi)^{1-\psi}(1-\phi)^{\psi}}{g} \left( \frac{H_A}{a} \right) = \frac{b}{a} \left( \frac{H_A}{H_B} \right) \leq \phi.$$

Substituting $N/B$ and $g$ into the utility of households (19) yields

$$U = \frac{1}{\rho(\varepsilon - 1)} \left[ \ln \left( \frac{b}{a} B_0 \right) + \ln \left( \frac{H_A}{H_B} \right) + \frac{(1-\phi)^{\psi}(\phi)^{1-\psi}}{\rho} \left( \frac{H_B}{b} \right) \right], \tag{20}$$

where $B_0$ is the initial number of basic inventions. The resource constraint for high-skill labor implies $H_A = 1 - H_B$, and $N/B \leq \phi$ implies $H_B \geq b/(b + a\phi)$.[16]

Simple optimization yields the socially optimal $\phi^{**}$ given by

$$\phi^{**} = 1 - \psi, \tag{21}$$

where $1 - \psi > \phi_L$ is assumed. The socially optimal $H_B^{**}$ is characterized by the following equation.

$$H_B(1 - H_B) = \frac{b\rho}{(\psi)^{\psi}(1-\psi)^{1-\psi}}.$$

This quadratic equation has two solutions. Deriving the second-order condition, one can easily show that the larger solution is the locally optimal $H_B^{**}$ given by

$$H_B^{**} = \frac{1}{2} \left( 1 + \sqrt{1 - \frac{4b\rho}{(\psi)^{\psi}(1-\psi)^{1-\psi}}} \right). \tag{22}$$

[15] It is useful to recall the following steady state ratios (a) $K/B = 1 - \phi$, (b) $N/B = \phi\iota^*/(\iota^* + g^*)$, and (c) $W/B = \phi g^*/(\iota^* + g^*)$.

[16] When this constraint is violated, the arrival rate $\iota$ of applied inventions becomes negative.

To ensure that this interior optimum is achievable, we naturally assume that

$$H_B^{**} > b/(b + a\phi^{**}),$$

which must hold given a sufficiently small $\rho$. This parameter restriction simply implies that at $H_B = H_B^{**}$, $N/B < \phi^{**}$. To ensure that the locally optimal $H_B^{**}$ is also the global optimum, we impose the following condition.[17]

$$U|_{H_B = H_B^{**}} > U|_{H_B = b/(b + a\phi^{**})}.$$

Finally, by (13), the market equilibrium $H_B^*$ in the decentralized economy can be expressed as

$$H_B^*(s, \phi) = \left( \frac{b}{(1-\phi)^{\psi} (\phi)^{1-\psi}} \right) g^*(s, \phi), \tag{23}$$

where the equilibrium growth rate $g^*$ satisfies (16). Comparing the optimal $H_B^{**}$ and the equilibrium $H_B^*$, we can show the following proposition.

**Proposition 3** *If $\psi < 1 - \phi_L$, then the patent authority can achieve the steady-state welfare-maximizing allocation in the decentralized economy by using the two patent instruments $\{\phi, s\}$. To do so, the patentability parameter $\phi$ is set to its optimal level $\phi^{**} = 1 - \psi$, and the profit-division rule $s$ is set to an intermediate value $s^{**}$ within the feasible range of $s$.*

**Proof.** First, set $\phi$ to the optimal level $\phi^{**} = 1 - \psi$ as in (21). Then, we show that there exists a feasible value of $s = s^{**}$ that equates the equilibrium $H_B^*$ to the optimal $H_B^{**}$. By (22), the optimal $H_B^{**} \in (0, 1)$ is independent of $s$ while the equilibrium $H_B^*$ is strictly increasing in $s$ by Proposition 2. As $s \to 1$, it can be shown by using (16) and (23) that $H_B^* \to 1$. Therefore, it suffices to show that as $s$ approaches its lower bound given by $b/\left(b + a\phi^{**}\right)$ (from $\phi^{**} > \phi_L$), $H_B^*$ approaches a value that is below the optimal $H_B^{**}$. As $s \to b/(b + a\phi^{**})$, it can be shown by using (16) and (23) that $H_B^* \to b/(b + a\phi^{**})$, which is less than $H_B^{**}$. ■

[17] It can be shown that this equality must hold given a sufficiently small $a$.

# 4 Conclusion

In this study, we have developed an R&D-based growth model with basic and applied research to analyze the growth and welfare effects of two patent instruments (a) the patentability of basic R&D and (b) the division of profit between basic and applied researchers. We find that the patentability of basic R&D has either a monotonically negative effect or an inverted-U effect on technological progress. Therefore, although increasing the patentability of basic R&D may have contributed to economic growth since the 1980's with continual technological progress on biotechnology and information technology, the inverted-U relationship suggests that further increasing the patentability of basic R&D might eventually depress economic growth. As Mowery *et al.* (2004) argue, increasing the patentability of basic R&D makes basic research discoveries less available to researchers resulting into a reduction in knowledge spillovers. Furthermore, we find that the equilibrium growth rate is monotonically increasing in the share of profit assigned to basic researchers. Therefore, strengthening the bargaining power of basic researchers relative to applied researchers may be a superior policy lever for achieving the dual objectives of stimulating basic R&D and economic growth. Nonetheless, for a benevolent patent authority, both patent instruments are needed for achieving the socially optimal allocation in the decentralized economy.

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## Appendix A: Proof of Theorem 1

By using the profit function in (2) and the free-entry and non-arbitrage conditions in (4), (7), (8), and (9), and by defining $c_t \equiv C_t/(\varepsilon N_t V_t)$, $k_t \equiv K_t/B_t$, $n_t \equiv N_t/B_t$ and $u_t \equiv W_t/B_t$, we obtain

$$\iota_t \left( \frac{as\phi}{b(1-s)} - 1 \right) = c_t. \tag{24}$$

**Claim 1** *Note that $\phi > (b/a)(1-s)/s$ must hold in order for $\iota_t > 0$, which gives rise to the lower bound of $\phi$ given by $\phi_L \equiv (b/a)(1-s)/s$.*

Since $\iota_t = \dot{N}_t/W_t$ by (5),

$$\frac{\dot{N}_t}{N_t} = \frac{b(1-s)}{as\phi - b(1-s)} \frac{u_t c_t}{n_t}. \tag{25}$$

By using the Euler equation,

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{V}_t}{V_t} = c_t - \rho \tag{26}$$

is also derived. From (3),

$$\frac{\dot{K}_t}{K_t} = (1-\phi) \frac{1}{k_t} \frac{\dot{B}_t}{B_t}. \tag{27}$$

From (6),

$$\frac{\dot{W}_t}{W_t} = \frac{\phi}{u_t} \frac{\dot{B}_t}{B_t} - \frac{n_t}{u_t} \frac{\dot{N}_t}{N_t}. \tag{28}$$

From (2), (7), (12), and (25), noting the definition of the spillover function $S(.)$,

$$\frac{\dot{B}_t}{B_t} = \frac{k_t^{\psi} (1-k_t)^{1-\psi}}{b} - \frac{a(1-s) c_t u_t}{as\phi - b(1-s)}. \tag{29}$$

From (2), (7), and (11), the skill premium is given by

$$\frac{w_t^s}{w_t^u} = \frac{(1-s) L}{a(\varepsilon - 1)} k_t^{\psi} (1-k_t)^{1-\psi} \frac{1}{c_t n_t}. \tag{30}$$

From (24)–(29), the equilibrium dynamical system of our model is given by

$$\frac{\dot{c}_t}{c_t} = c_t - \rho - \frac{\dot{N}_t}{N_t}, \tag{31}$$

$$\frac{\dot{k}_t}{k_t} = \left(\frac{1-\phi}{k_t} - 1\right)\frac{\dot{B}_t}{B_t}, \tag{32}$$

$$\frac{\dot{n}_t}{n_t} = \frac{b\,(1-s)}{as\phi - b\,(1-s)}\frac{u_t c_t}{n_t} - \frac{\dot{B}_t}{B_t}, \tag{33}$$

and

$$\frac{\dot{u}_t}{u_t} = \left(\frac{\phi}{u_t} - 1\right)\frac{\dot{B}_t}{B_t} - \frac{b\,(1-s)}{as\phi - b\,(1-s)}c_t, \tag{34}$$

in which $\dot{B}_t/B_t$ satisfies (29).

Firstly, steady states of the system are identified in what follows. In a balanced growth path (BGP), all variables grow at constant rates; specifically, in our model, $\dot{K}_t/K_t = \dot{B}_t/B_t = \dot{W}_t/W_t = \dot{N}_t/N_t = \dot{C}_t/C_t - \dot{V}_t/V_t$ holds. Denote by $g^*$ the steady-state growth rate of variables $B_t$, $K_t$, $N_t$, and $W_t$.[18] Denote the values of $(c_t, k_t, n_t, u_t)$ along a BGP by $(c^*, k^*, n^*, u^*)$.

The trivial BGP is excluded by assuming $g^* > 0$, which implies

$$k^* = 1 - \phi, \tag{35}$$

in which the use has been made of $\dot{K}_t/K_t = \dot{B}_t/B_t$ with (27). Noting $\dot{W}_t/W_t = \dot{B}_t/B_t$ with (28), $g^* > 0$ also implies

$$n^* = \phi - u^*. \tag{36}$$

By $\dot{C}_t/C_t - V_t/V_t = \dot{N}_t/N_t$, equations (25) and (26) imply

$$u^* = \frac{as\phi - b\,(1-s)}{b\,(1-s)}\left(1 - \frac{\rho}{c^*}\right)n^*. \tag{37}$$

Cancelling out $n^*$ from (36) and (37),

$$u^* = \phi\left[\frac{as\phi - b\,(1-s)}{b\,(1-s)}\left(1 - \frac{\rho}{c^*}\right)\right]\left[1 + \frac{as\phi - b\,(1-s)}{b\,(1-s)}\left(1 - \frac{\rho}{c^*}\right)\right]^{-1}, \tag{38}$$

[18] From (1) and (11), $\frac{\dot{C}_t}{C_t} = \frac{1}{\varepsilon - 1}g^*$, which implies $\frac{\dot{V}_t}{V_t} = \left(\frac{1}{\varepsilon-1} - 1\right)g^*$.

and

$$n^* = \phi \left[ 1 + \frac{as\phi - b(1-s)}{b(1-s)} \left( 1 - \frac{\rho}{c^*} \right) \right]^{-1}. \tag{39a}$$

By $\dot{N}_t/N_t = \dot{B}_t/B_t$ with (25) and (29),

$$\frac{(k^*)^{\psi} (1-k^*)^{1-\psi}}{b} = \frac{b(1-s) u^*}{as\phi - b(1-s)} \left( \frac{c^*}{n^*} + \frac{a}{b} c^* \right). \tag{40}$$

Cancelling out $c^*$, $k^*$, and $u^*$ from (40), with (35)–(39a) and $g^* = c^* - \rho$ by (26),

$$\frac{(1-\phi)^{\psi} (\phi)^{1-\psi}/b - g^*}{\rho + g^*} = \frac{\phi (1-s) g^*}{\phi s g^* + \rho (1-s) b/a}.$$

Because the left-hand side is a strictly decreasing function in $g^*$ and the right-hand side is a strictly increasing function in $g^*$, a unique positive BGP growth rate $g^*$ exists. Because $g^* > 0$ holds, $c^* > \rho$ holds $(1 - \rho/c^* > 0)$. This ensures the feasible values of the fractions $n^*$ and $u^*$: $n^* \in (0,1)$ and $u^* \in (0,1)$ hold by (38) and (39a). The fraction of pure knowledge $k^*$ also satisfies $k^* \in (0,1)$ by (35).

In what follows, the saddle-path stability of the dynamical system for $(c_t, k_t, n_t, u_t)$, (31)–(34), is proved. From (32) and (35), since $\frac{\dot{B}_t}{B_t} > 0$ by the assumption, $\dot{k}_t > 0$ holds for all $k_t < k^*$ and $\dot{k}_t < 0$ holds for all $k_t > k^*$. Thus, for any small $\delta > 0$, there exists a sufficiently large $T < \infty$ such that $k_t \in (k^* - \delta, k^* + \delta)$ holds for all $t \geq T$. Due to the continuity of the system, it would suffice to analyze the stability of the $3 \times 3$ dynamical system omitting $\dot{k}_t$ where $k_0 = k^*$. We consider this abbreviated system omitting $\dot{k}_t$. Using (35)–(40), the log-linearized version of this abbreviated system with $(\hat{c}_t, \hat{n}_t, \hat{u}_t) = \left( \ln \frac{c_t}{c^*}, \ln \frac{n_t}{n^*}, \ln \frac{u_t}{u^*} \right)$ has the following coefficient matrix, $M$:[19]

$$M = \begin{pmatrix} \rho & g^* & -g^* \\ g^* + \zeta & -g^* & g^* + \zeta \\ -\zeta \frac{b(1-s)}{as\phi - b(1-s)} \frac{g^*+\rho}{g^*} - \frac{b(1-s)(g^*+\rho)}{as\phi - b(1-s)} & 0 & -\frac{as\phi g^* + b(1-s)\rho}{as\phi - b(1-s)} - \zeta \frac{b(1-s)}{as\phi - b(1-s)} \frac{g^*+\rho}{g^*} \end{pmatrix}$$

[19] We can formally prove that the coefficient matrx for the original $4 \times 4$ linearized system (including $\dot{k}_t$) has the three same eigenvalues as those of the coefficient matrix for the abbreviated $3 \times 3$ linearized system (omitting $\dot{k}_t$ with $k_t = k^*$). We can also show that the remaining one eigenvalue is a multiple root that is equal to $-g^*$, which is negative. Therefore, since $k_t$ is a non-jumpable variable, it suffices to verify only the stability of the abbreviated $3 \times 3$ system in our model.

where $\zeta = \frac{a\phi}{b} \frac{g^*(g^*+\rho)}{\frac{as\phi}{b(1-s)} g^* + \rho}$. We can show that the determinant of $M$ is always positive.[20] The trace of $M$ is negative if $\rho - \frac{as\phi g^* + b(1-s)\rho}{as\phi - b(1-s)} < 0$ (a sufficient condition). This inequality can be re-expressed as $g^* > \rho(1 - 2\phi_L/\phi)$, where $\phi_L \equiv (b/a)(1-s)/s$. Solving the quadratic equation in (16) yields

$$g^* = \frac{1}{2}\left(-\Phi + \sqrt{\Phi^2 + 4\left(\frac{1-\phi}{\phi}\right)^{\psi}(1-s)\frac{\rho}{a}}\right),$$

where $\Phi \equiv \rho(1-s)[1 + b/(a\phi)] - (1-\phi)^{\psi}(\phi)^{1-\psi} s/b$ is a composite parameter. Using this expression, we find that $g^* > \rho(1-2\phi_L/\phi)$ can be re-expressed as

$$\frac{s(1-\phi)^{\psi}(\phi)^{1-\psi}}{(2-s)b} > \rho(1 - 2\phi_L/\phi).$$

When this inequality holds, the eigenvalues of $M$, denoted by $\lambda_1$, $\lambda_2$, and $\lambda_3$, satisfy $\lambda_1\lambda_2\lambda_3 > 0$ and $\lambda_1 + \lambda_2 + \lambda_3 < 0$. This implies that $M$ has two negative and one positive eigenvalues, which proves the saddle-path stability of the system.

**Claim 2** *If $\rho$ is sufficiently small (or $\phi < 2\phi_L$), then the dynamical system is locally saddle-path stable.*

[20] Note that the determinant of $M$ is equal to

$$b(1-s)\rho\left(\rho(g^* + \zeta) + (g^*)^2 + 2\zeta g^*\right) + as\phi(g^*)^2(g^* + \rho + \zeta) .$$