Efficient Committed Budget for Implementing Target Audit Probability for Many Inspectees

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EFFICIENT COMMITTED BUDGET FOR IMPLEMENTING TARGET AUDIT PROBABILITY FOR MANY INSPECTEES†

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KEYWORDS: Audit sampling plan, audit budget, tax audit, tax compliance, tax evasion, inspection game, appropriation and rescission.

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EFFICIENT COMMITTED BUDGET FOR IMPLEMENTING TARGET AUDIT PROBABILITY FOR MANY INSPECTEES

1. Introduction

Private-sector organizations often begin a period of operations with the allocation of a budget to each business unit. The budget committed to a unit limits its activities and thereby imposes a constraint on its operational scale. Under extraordinary circumstances, a unit might be able to obtain extra budget with special approval. But usually the committed budget is a "hard" constraint that has to be met. Similarly, public-sector and international organizations, such as US Internal Revenue Service (IRS) and International Atomic Energy Agency (IAEA), plan their activities according to the committed budget of a period.[1]

If the unit in concern or the organization itself is an auditor responsible for performing audits to many inspectees, the committed budget restrains the audit sampling plan that can be implemented to attain a target level of audit probability.[2] An overly-committed audit budget ties up resources that could have been allocated for better alternative uses.[3] When randomized audits are desirable, the

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[1] *Committed budget* refers to the budgeted amount of resources *reserved* for a unit or organization at the beginning of a period of operations. Owing to uncertain factors, it usually differs from the actual amount of resources used in the period, or the expected amount anticipated at the beginning of the period.

In US federal budget terms, the committed budget that sets aside the amount of money an agency is allowed to incur obligations and make payments is called *appropriation* (US GAO [2005], US OMB [2006], and Coven and Kogan [2006]). For example, US IRS was approved appropriations of $9,998 million and $10,461 million for tax administration and operations for fiscal years 2005 and 2006, respectively. These, however, were not fully used, with rescissions of unobligated balances amounting to $80.6 million and $104.6 million in those years, respectively (US Treasury [2005; 2006]).

[2] I use auditor-auditing-auditees or inspector-inspection-inspectees interchangeably to represent strategic relationships such as those between tax collector and taxpayers, insurer and policyholders, auditor and clients, regulator and banks, environmental protection agency and polluters, welfare agency and welfare recipients, cartel and member producers, multilateral treaty monitoring agency and member states, etc. See footnote 4 for citations of related studies.

[3] Obviously, there are opportunity costs associated with unnecessarily reserved resources. Tying up of unneeded funds is a serious concern of administrators, even though unobligated balances of previously committed budgets may later be rescinded for other uses. For example, US Government Accountability Office (GAO) routinely conducts reviews to identify unobligated balances that could be rescinded. In a review report, it concludes that "regularly tracking [information about highway projects] … to identify unneeded unobligated balances, and submitting it to Congress could result in more timely rescissions of unobligated balances that the states no longer need, freeing funds for other purposes. " (US GAO [2004]).

auditor can conserve the committed budget for implementing an audit probability by proper selection of an audit sampling plan.

Models of an inspection game with randomized audits usually are analyzed in terms of the one-to-one interaction between the auditor and an inspectee.[4] The optimal or equilibrium audit probability determined from the standard analysis renders an audit sampling plan, namely *Poisson sampling*, that obviously solves the many-inspectee extension of the one-to-one game.[5] But this obvious solution is neither a realistic description of the actual practice nor analytically an *efficient* one, in terms of the committed budget it demands. The audit probability determined from the standard analysis has also been interpreted as doing *proportional simple random sampling* (*SRS*).[6] While this reduces the committed budget required, it remains far from efficient.

A main objective of this paper is to study the minimum committed budget required to implement a target audit probability when (i) the audit sample can be contingent on "red flags" due to signals of inspectees' private information (e.g., from self-reporting) and (ii) the number of inspectees is large.[7] I propose an audit sampling plan called *bounded SRS*, which requires no more

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[4] Such models have been formulated to study financial audits, insurance claim investigations, environmental regulation / pollution control, multilateral treaty monitoring, deterrence theory / law enforcement, optimal tax policy / tax compliance, etc. For example, (a) *financial audits*: Chatterjee, Morton, and Mukherji [2006] and minor modifications of Newman, Patterson, and Smith [2001] and Patterson and Noel [2003]; (b) *insurance claim investigations*: Mookherjee and Png [1989] and Picard [1996]; (c) *environmental regulation / pollution control*: Bontems and Bourgeon [2005] and minor modification of Florens and Foucher [1999]; (d) *multilateral treaty monitoring*: minor modifications of Avenhaus, von Stengel, and Zamir [2002] and Hohzaki [2007]; (e) *deterrence theory / law enforcement*: Kaplow and Shavell [1994], Polinsky and Shavell [1998; 2000], and Polinsky [2007]; (f) *optimal tax policy / tax compliance*: Border and Sobel [1987], Sanchez and Sobel [1993], Cremer and Gahvari [1995], Sansing [1993], Rhoades [1997; 1999], Mills and Sansing [2000], and Feltham and Paquette [2002].

Related empirical studies that assume randomized audits include Dubin, Graetz, and Wilde [1987], Alm, Bahl, and Murray [1993], Chang, Steinbart, and Tuckman [1993], Mete [2002], and Alm, Blackwell, and McKee [2004].

[5] This audit sampling plan is generalized directly from the standard analysis by independently selecting each inspectee into a sample according to the optimal or equilibrium audit probability, as if the auditor is playing many independent one-to-one games. Poisson sampling, which generates samples with stochastic sizes, was first studied by Hajek (see Chapter 15 of his collected works [1998]). An application of (unequal-probability) Poisson sampling is called Sieve sampling, which is a selection method for obtaining monetary-unit samples for auditing (Wurst, Neter, and Godfrey [1989] and Horgan [1997; 2003]).

[6] Loosely speaking, proportional SRS means randomly selecting a constant proportion of the suspicious inspectees for audit.

[7] For example, some research suggests tax collectors pay attention to "red flags" such as book-tax differences in reported income (Mills [1998] and Cho, Wong, and Wong [2006]). Official documentations and anecdotal

committed budget to support than proportional SRS or Poisson sampling. Simply put, bounded SRS chooses an audit sample from the population of "red-flagged" inspectees, with the sample size bounded by a ceiling whenever the population size exceeds a threshold, or otherwise proportional to the population size. In contrast, proportional SRS always has the sample size proportional to the population size. Unlike proportional SRS or Poisson sampling, bounded SRS is asymptotically efficient. That means, when the number of inspectees is large enough, the sampling plan is nearly as good as any *efficient* sampling plan, which demands the *lowest* committed budget *necessary* to implement the target audit probability.

The multiple-inspectee formulation studied here differs from the one-to-one game in three interesting aspects:

(i) The auditor has more latitude to formulate an audit strategy, which becomes an audit sampling plan conditional on the number of "red flags" observed. Each plan requires possibly a different level of committed budget to support, leading to the interesting issue of identifying strategies that attain the minimum level. This issue does not exist in the one-to-one game.

(ii) Unlike the auditor, an inspectee cannot observe the total number of "red flags" that might trigger a different level of audit sampling. However, if he turns the "red flag" on by violating the regulation, he knows that the number of "red flags" observed by the auditor is at least one because he knows his type and action. This information is strategically important, so long as the population size is not as low as one, which would return to the one-to-one game.

(iii) The auditor needs not reserve an overly-committed budget to prepare for auditing all inspectees. She can wisely reduce the committed budget to the minimum level required to support an equilibrium. This is in stark contrast to the one-to-one game where the auditor must always be ready to do a full audit.

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observations also indicate that US IRS uses the Discriminant Index Function (DIF) score and the newly designed Unreported Income (UI) DIF score to "red-flag" tax returns for further audit selection decisions (Rogers [1999], US IRS [2004; 2006], IRS Audit [n.d.], and Gutkin [n.d.]). See Pentland and Carlile [1996] for additional information about US IRS's practice of tax auditing.

Using the multiple-inspectee formulation of this paper, I clarify the difference between committed budget and expected audit cost. The formulation is consistent with the commonly used budgeting practice in public- and private-sector organizations. By including the opportunity cost of reserving a committed budget in the model, the auditor's objective function changes to one that is consistent with the concept of *residual income*. With this formulation, I characterize in closed form a class of optimal audit strategies with a "ceiling" structure. Specifically, it means: Audit all "red-flagged" inspectees when the total is no more than the maximum number of audits allowed by the committed budget, and merely this maximum otherwise. I call this the "kernel" bounded SRS rule.

Owing to the discrete nature of conducting each extra audit, I am unable to provide a direct characterization of the optimal committed budget. Instead, I examine the class of bounded SRS strategies that generalizes the "kernel" rule. The budget-related characteristics of bounded SRS are compared to those of proportional SRS and Poisson sampling. The committed budget required to support bounded SRS is the lowest among the three and is also asymptotically efficient.

This study makes three contributions. First, it formulates a multiple-inspectee extension of the classic one-to-one game with three interesting differences as explained earlier. Second, by characterizing a class of optimal audit strategies and examining the properties of a generalization of this class, the paper addresses an important implementation issue long neglected in the literature. My results offer insights on how audit strategies may be formulated to reduce inefficiency and what budget usage ratios should be expected accordingly. These findings are novel in the literature. Third, the study provides a connection between the strategic auditing literature and the practitioner-oriented audit sampling literature. Results in the strategic auditing literature are predominantly based on one-to-one analyses that do not involve audit sampling, or they are about sampling for multiple observations of an agent's performance. Neither of these has drawn substantial attention in the practitioner-oriented audit sampling literature, which cares more about how to draw audit samples efficiently from a population.

For ease of exposition, the ideas of my study are explained using a multiple-taxpayer extension of the classic tax compliance game introduced by Graetz, Reinganum, and Wilde [1986].

Some recently formulated tax compliance models continue to build upon the basic structure of their model.[8] Though falling outside the scope of this paper, generalization of the results to other auditor-inspectees relationships should be straightforward.[9]

The rest of the paper is organized as follows. In the next section, I review the classic tax compliance game and the equilibrium derived with the one-to-one standard analysis. Directly replicating the one-to-one equilibrium audit probability to a multiple-taxpayer setting yields the Poisson sampling audit strategy. Alternatively, the equilibrium audit probability can be implemented with the proportional SRS audit strategy. The ideas of the "kernel" and generalized bounded SRS are also introduced at the end of the section. I include there a numerical example illustrating the key features of bounded SRS and how it conserves the committed budget and yet maintains the induced audit probability at the same level. Section 3 explains how the decisions of the tax collector and taxpayers differ in the multiple-taxpayer and one-to-one formulations. A characterization of optimal audit strategies and a discussion of the optimal committed budget are given in Section 4. In Section 5, I examine properties of bounded SRS, comparing its budget-related characteristics to those of proportional SRS and Poisson sampling. Section 6 contains concluding remarks that summarize the results, contributions, and limitation of my analysis. Technical proofs and derivations are relegated to the appendix.

2. Classic Tax Compliance Game

2.1. One-to-one Analysis

Consider a taxpayer population constituted of high-income and low-income taxpayers.[10] Both

[8] For example, Rhoades [1999], Mills and Sansing [2000], and Feltham and Paquette [2002]. A recent review of theoretical models on tax compliance is McCubbin [2003]. Related discussions can also be found in Andreoni, Erard, and Feinstein [1998], Franzoni [2000], Slemrod and Yitzhaki [2002], Cowell [2004], and Sandmo [2005].

[9] Such relationships are those mentioned in footnotes 2 and 4. The results, however, are not relevant to auditor-inspectee models with only non-randomized audits, exogenous or endogenous. Examples of such models are Baiman and Demski [1980a; 1980b], Beck and Jung [1989], and Fuente and Marin [1996].

[10] This is a simplified version of Graetz, Reinganum, and Wilde's model. In their original setup, the taxpayer population contains four types of individuals based on combinations of two aspects: (i) "strategic non-compliers" versus "habitual compliers;" (ii) "high income class" versus "low income class." The results here can be easily modified to allow for habitual compliers.

groups are strategic players driven purely by economic interests without any inherent preference for honesty or similar values. Each taxpayer privately knows his true taxable income and independently files a tax return to report whether he has "high income" or "low income." Based on the reported income, the tax collector decides whether to conduct an audit. Taxpayers are weakly risk-averse expected utility maximizers. The von-Neumann-Morgenstern utility function u of a taxpayer is increasing in his *disposable income*, which equals his after-tax income minus the fine for non-compliance, if any. The tax collector is an expected *net tax revenue* maximizer with the net tax revenue equal to the tax revenues and fines collected minus the audit costs incurred.

Below is a list of the notations used in the model:

q = the *ex ante* probability of a taxpayer having high income ($0 < q < 1$);

β = the *under-reporting probability* (of a high-income taxpayer), i.e., the probability of a high income taxpayer filing a "low-income" tax return ($0 \le \beta \le 1$);

$\nu(\beta)$ = the probability that a taxpayer reporting low income actually has high taxable income; by Bayes' rule, $\nu(\beta) = \beta/(\beta + q^{-1} - 1)$;

α = the *audit probability* (for a "low-income" tax return), i.e., the probability of auditing a taxpayer filing a "low-income" tax return ($0 \le \alpha \le 1$);

c = the cost per audit ($c \ge 0$);

I_H = the income of a high-income taxpayer;

I_L = the income of a low-income taxpayer ($0 < I_L < I_H$);

T_H = the tax owed by a high-income taxpayer;

T_L = the tax owed by a low-income taxpayer ($0 \le T_L < T_H$ and $T_L \le I_L$);

F = the fine for tax evasion ($0 < F \le I_H - T_H$).

It is assumed that if a high-income taxpayer always files a "low-income" tax return (i.e., $\beta = 1$ so that $\nu(\beta) = q$), the expected gain from conducting an audit will exceed the audit cost, i.e., $q(F + T_H - T_L) > c$. Otherwise, it is never gainful to do any audit, and the equilibrium is not interesting.

Since a low-income taxpayer can never gain by reporting untruthfully, compliance is his dominant strategy. So there is no point in auditing a "high-income" tax return, for it must have come from a high-income taxpayer reporting honestly. Consequently, no auditing is the tax collector's unique best response to a "high-income" tax return. Considering these points, we may simply use the under-reporting probability β (of a high-income taxpayer) and the audit probability α (for a questionable tax return, i.e., a "low-income" tax return) to characterize the interaction between the tax collector and *an individual taxpayer*.

Given a conjecture α on the tax collector's strategy, a high-income taxpayer's expected utility from choosing an under-reporting probability β is

$$\beta[\alpha u(I_H - T_H - F) + (1-\alpha)u(I_H - T_L)] + (1-\beta)u(I_H - T_H).$$

Similarly, given a conjecture β on a high-income taxpayer's strategy, the tax collector's expected net tax revenue from choosing an audit probability α is

$$\alpha[\nu(\beta)(T_H + F - c) + (1-\nu(\beta))(T_L - c)] + (1-\alpha)T_L.$$

The interaction between the tax collector and an individual taxpayer has a unique equilibrium characterized by the following under-reporting probability b and audit probability a:

$$b = (1-q)c/q(F + T_H - T_L - c) \quad \text{and}$$

$$a = [u(I_H - T_L) - u(I_H - T_H)]/[u(I_H - T_L) - u(I_H - T_H - F)].$$

The probability of the tax collector receiving a "low-income" return from a taxpayer thus equals $p = 1 - q(1-b)$ in equilibrium. The game tree in Figure 1 summarizes the one-to-one interaction discussed here.

Insert Figure 1 around here.

2.2. Multiple-taxpayer Setting: Poisson Sampling and Proportional SRS

Unless the taxpayer population has only one individual, (b,a) strictly speaking is not an equilibrium of the game between the tax collector and the whole taxpayer population. To generalize, assume that the size N of the taxpayer population is at least two.[11] An equilibrium audit strategy of the tax collector for the *whole* game (i.e., the game against the whole taxpayer population) can be obtained simply by independently replicating the one-to-one strategy, characterized by a. This strategy for the whole game is referred to as a *Poisson sampling* audit strategy. Because this is like

[11] For simplicity, N is assumed to be publicly known by all parties.

playing N independent one-to-one games against the taxpayers, the probability pair (b,a) now describes an equilibrium for the whole game that is essentially an N-time replication of the (b,a) equilibrium for the one-to-one game.

Though simple, the Poisson sampling audit strategy has an undesirable feature. To implement the strategy, the tax collector must prepare to audit possibly every tax return, even though the expected number of audits to be conducted is much smaller. Unless the tax collector has a committed audit budget of at least cN, the strategy is not implementable.

The standard analysis has focused on the one-to-one interaction between the tax collector and an individual taxpayer. Nonetheless, the taxpayer is often interpreted as representing the whole population of taxpayers in a multiple-taxpayer setting. With this interpretation, a may be viewed as a non-contingent sampling rate used by the tax collector to determine the audit sample size for any realization of the number L of "low-income" tax returns received. That means, she will always randomly audit a fraction a of the L "low-income" tax returns received.[12] This strategy, with its structure illustrated in Figure 2, is referred to as the *proportional SRS* audit strategy with sampling rate a. Formally, it may be expressed as follows:

$$s(L) = a \qquad\qquad \text{for } L = 1, 2, ..., N,$$

where $s(L)$ is referred to as the sampling rate contingent on L.

Insert Figure 2 around here.

Given this strategy, the chance of being audited remains a from a non-compliant high-income taxpayer's perspective.[13] So b is still a high-income taxpayer's equilibrium under-reporting strategy. Given b, the proportional SRS strategy is a best response of the tax collector.[14] Thus, the probability pair (b,a) continues to describe an equilibrium of the whole game, though with a now standing for

[12] If aL is not an integer, the number of "low-income" tax returns audited will be set to $\lceil aL \rceil$ with probability $aL-\lfloor aL \rfloor$, and $\lfloor aL \rfloor$ with probability $\lceil aL \rceil - aL$, where $\lceil aL \rceil$ and $\lfloor aL \rfloor$ are the integers rounded up and rounded down from aL, respectively.

[13] This will be clear shortly in Section 3.2 below.

[14] This is implied by Lemma 1 to be stated in Section 3.1 below.

the constant sampling rate of the equilibrium audit strategy.

Note that the audit sample size determined with the equilibrium proportional SRS strategy is at most aN even when $L = N$.[15] Therefore, the committed audit budget necessary to support the strategy is caN. This is merely a fraction a of the committed budget cN required by the equilibrium Poisson sampling strategy.

Although the two strategies differ in the committed budgets necessary to support them, they have identical expected audit costs incurred, which equal $cNpa$, where $p = 1-q(1-b)$. This follows from the simple fact that under either strategy, the mean audit sample size is given by $\mathcal{E}[s(L)L \mid b] = Npa$.[16] Intuitively, the amount $cNpa$ is simply a consequence of the following facts. Note that there are a total of N individuals in the taxpayer population, of which a fraction p is expected to file "low-income" tax returns. On average a fraction a of these questionable tax returns will be selected for audit, with each conducted at the cost of c. Therefore the expected audit cost is $cNpa$. Budget-related characteristics of the proportional SRS and Poisson sampling audit strategies are summarized as follows:

Audit Probability = a	Proportional SRS	Poisson Sampling
Committed Budget Necessary to Support	caN	cN
Expected Audit Costs Incurred	$cpaN$	$cpaN$
Expected Budget Usage Ratio	p	pa
Expected Unused Percentage	$1 - p$	$1 - pa$

[15] More precisely, it is at most $\lceil aN \rceil$, where $\lceil aN \rceil$ is the integer rounded up from aN. For notational simplicity, rounding is ignored in presenting/discussing results when this is not crucial to conveying the main insights.

[16] First of all, independent distribution of types and choices of action imply that L, the number of "low-income" tax returns received, follows the binomial distribution. To see this, first note that "low-income" tax returns may come from low-income taxpayers, who have no incentive to mis-report and always file truthfully with probability 1. Alternatively, a "low-income" tax return could come from a non-compliant high-income taxpayer. If each high-income taxpayer under-reports with probability β, the chance of seeing a "low-income" tax return filed by a taxpayer is $(1 - q) + q\beta$. There are N individuals in the population. Thus, L follows the binomial distribution $\text{Bin}(1-q(1-\beta),N)$.

Under proportional SRS, $s(L)$ is simply a, and thus $\mathcal{E}[s(L)L \mid b] = \mathcal{E}[aL \mid b] = Npa$. Under Poisson sampling, $s(L)L$ is equivalent to the expectation, when conditional on L, of the audit sample size A that follows the binomial distribution $\text{Bin}(a, L)$ with mean La. Hence, $\mathcal{E}[s(L)L \mid b] = \mathcal{E}[La \mid b] = Npa$ as well.

2.3. Bounded SRS: Numerical Example

The innovation of this paper is the *bounded SRS* audit rule that implements the same target audit probability but requires no more committed budget to support than the level necessary under the proportional SRS or Poisson sampling rule. Additionally, when the size of the taxpayer population is sufficiently large, this rule is nearly as good as the best available in attaining the efficient level of committed budget.

Insert Table 1 around here.

The key features of bounded SRS are illustrated with a numerical example given in Table 1.[17] Panel A of the table contains the parameter choices consistent with a target audit probability of a = 0.55.[18] This is the equilibrium audit probability suggested by the one-to-one analysis. It can be implement by Poisson sampling or proportional SRS. In the latter case, it means regardless of the total number L of "low-income" returns received, the tax collector audits 55% of them. Note that 55% of L in general is not a whole number. Thus, a sampling rate of 55% essentially means if L = 6, with a probability 0.3 a random sample of size 4 will be drawn and with a probability of 0.7 a sample of size 3 will be drawn. As a result, on average a total of $0.3 \times 4 + 0.7 \times 3 = 55\% \times 6$ of the "low-income" returns received are audited when $L = 6$. Changes are made analogously for other values of L. The committed budget required to support proportional SRS with a sampling rate of 55% must allow conducting at least 4 audits, which is the whole number rounded up from 55% × 6. Any committed budget less than this will force the tax collector to audit less than the constant sampling rate of 55% prescribed by the proportional SRS rule considered here.

In contrast, implementing $a = 0.55$ by bounded SRS requires no more than 2 audits. This is in stark contrast to 4 audits required by the proportional SRS rule. The difference represents a

[17] The example was constructed with a precision of eight decimal places; for brevity, only up to four significant digits are reported in the table.

[18] I assume a power-expo utility function suggested by recent research measuring risk preferences with laboratory experiments (Holt and Laury [2002]). This functional form subsumes the CARA and CRRA utility functions commonly seen in theoretical studies.

substantial reduction of at least 33% of resources set aside, even when taking the conservative view that the 4th audit required by proportional SRS is largely due to indivisibility. Panel B illustrates how a bounded SRS rule with a "committed budget" of $K = 2$ accomplishes the mission. The sampling rate prescribed by this rule can be formally expressed as

$$s^K(L) = \min\{ K/L, 1 \} \qquad \text{for } L = 0, 1, ..., N.$$

Intuitively, it says: Audit all L received when its value is no more than the tax collector's "audit capacity," namely $K = 2$, and merely 2 when L exceeds 2. The audit sample size, $s^K(L)L$, for different contingencies is given in the second row in Panel B, with the corresponding sampling rate given in the third row.

The bounded SRS rule in this example illustrates in general three distinct regions of L that may have strategic implications to a high-income taxpayer deciding to underreport:

a. If he files a "low-income" return while at most one of the others file "low-income" returns as well, his return will surely trigger an audit under this rule;
b. If he is among the only three taxpayers filing "low-income" returns, the chance of being selected for audit is not certainly 100% but still higher than 55%, the sampling rate under proportional SRS.
c. If he files a "low-income" return and including him there are a total of 4 to 6 taxpayers doing so, the chance of being selected for audit is lower than 55%.

Compared to proportional SRS, the sampling effort for the contingencies with larger values of L is reallocated to those with smaller values, yet leaving the overall deterrence power unchanged. So under the bounded SRS rule larger values of L do not trigger audits, allowing the committed budget to be smaller without affecting the audit probability induced.

To see exactly how bounded SRS works, consider first $q = 0.5$. Given the conjectured $b = 0.1818$ and accordingly $p = 1 - q(1 - b) = 0.59$, the audit probability to a non-compliant higher-income taxpayer is determined by averging the sampling rates, $s^K(L)$, under different contingencies of L, weighted by the chances $\Pr\{L^{N-1} = L - 1\}$. The number of "low-income" returns filed by other taxpayers, L^{N-1}, matters here because a non-compliant high-income taxpayer knows for sure that L

$= 1+L^{N-1}$. The audit probability induced by the bounded SRS rule is provided in the rightmost column of Panel B. As shown, bounded SRS with K = 2 indeed implements the same audit probability, just like proportional SRS with a sampling rate of 55%.

Because K is a whole number, bounded SRS of the form $s^K(L)$ in general may "over-audit." To avoid this, the following generalized form of bounded SRS is also considered:

$$s(L) = \begin{cases} \theta & \text{for } L = 1, 2, ..., K; \\ \theta K/L & \text{for } L = K+1, K+2, ..., N, \end{cases}$$

where $0 \leq \theta \leq 1$. Setting the parameter θ below 1 can tune down the deterrence power of the "kernel" bounded SRS rule $s^K(L)$. The bottom part of Panel B demonstrates that when q = 0.61 and the conjectured b = 0.1162, the "kernel" rule $s^K(L)$ induces an audit probability of 0.66, which cannot constitute an equilibrium. However, the generalized rule $s(L)$ with θ = 5/6 can.

In the next section, I introduce the audit strategy in a multiple-taxpayer setting as a randomized sampling plan. Then I analyze the decision problems facing the tax collector and an individual taxpayer. In Section 4, I show that a strategy characterized by the "kernel" rule $s^K(L)$ is optimal to the tax collector. The rest of the paper shows that the generalized rule $s(L)$ with $\theta \in [0,1]$ implements the same target audit probability as proportional SRS or Poisson sampling but requires no more committed budget to support. Moreover, bounded SRS is asymptotically efficient in terms of the committed budget required.

3. Decisions of the Tax Collector and Taxpayers in the Multiple-taxpayer Setting

3.1. Tax Collector's Decisions

With a population of taxpayers, the tax collector actually has greater latitude to formulate an audit strategy than simply conducting independent randomized audits. Different *sampling plans* may be used to draw an audit sample from the taxpayer population. Since the tax collector knows the set of taxpayers filing "low-income" tax returns before drawing the audit sample, her choice of an audit sampling plan can be contingent on the set. However, the taxpayers in the set all look the same to the tax collector. Therefore, what matters is only the number of "low-income" tax returns received, i.e., $L \in \{1, 2, .., N\}$, rather than the exact composition of the set. For the same reason, the size rather than the composition of the audit sample is what actually affects the level of deterrence induced by an audit strategy. Consequently, an audit strategy of the tax collector may be defined as follows:

DEFINITION 1: An *audit strategy*, denoted by $\boldsymbol{g} = [g(i \mid m)]_{i=0,1,\ldots,N;\, m=0,1,\ldots,N}$, is a (probabilistic) plan of determining the audit sample size A contingent on the number L of "low-income" tax returns received, where $g(i \mid m)$ is the probability of setting $A = i$ given $L = m$, with $0 \le g(i \mid m) \le 1$, $\sum_{i=0}^{m} g(i \mid m) = 1$, and hence $g(i \mid m) = 0$ for $i > m$.

An organization, whether in the public or private sector, typically requires its units to have their budget plans approved before they can spend money on activities of the following year. Resources set aside for a unit's activities have an opportunity cost to the organization that is not embedded in the "unit variable cost" c of conducting an audit. If the resources set aside were just enough to do a maximum of K audits but not a single audit was conducted during a year, the "operating cost" of the year would be zero. However, the opportunity cost on each dollar reserved, denoted by $r > 0$, would not be avoided, and the total amounts to $r(cK)$. Suppose, for example, a committed budget of only $c(K - 1)$ is enough to support an audit strategy in equilibrium. Setting a committed budget of any amount above that, e.g. cK, is thus inefficient.

Before moving on, below summarizes the sequence of events in this multiple-taxpayer setting:

1. The tax collector chooses a committed budget K to set aside resources that may be used to support audits done in the current period.
2. Each individual in the population of size N has a chance of q to be a high-income taxpayer, and the number of high-income taxpayers, N_H, is realized.
3. Each high-income taxpayer chooses to under-report with probability β, and the number of non-compliant high-income taxpayers, L_H, is realized. With no incentive to mis-report, low-income taxpayers always file "low-income" tax returns.
4. All tax returns are received. The number of "low-income" tax returns, L =

$N - N_H + L_H$, is realized. The tax collector observes L but not its components N_H and L_H separately.

5. Contingent on L, the tax collector chooses a probabilistic audit sampling plan $[g(i \mid L)]_{i=0,1,\ldots,N}$ to determine the audit sample size A.

6. An audit sample of size A is drawn from the "low-income" tax returns for audits. The number of "violators caught," V, is realized, and the tax collector and taxpayers receive their payoffs accordingly.

Usually units of an organization are competing for resources. Assume a unit only competes for resources that add value to the fulfillment of its functions. A shortcut way to model this incentive to conserve resources, even though the opportunity cost is not directly borne by the tax collector, is by assuming the following *"residual" net revenue* payoff function for her:

$$R(\boldsymbol{g}; \beta) - r(cK).$$

The tax collector sets a committed budget characterized by the maximum number K of audits allowed and chooses an audit strategy $\boldsymbol{g}$ to maximize the payoff subject to the constraint that $\boldsymbol{g}$ is implementable under K.[19] The first component $R(\boldsymbol{g}; \beta)$ of the payoff function is the expected net revenue collected with $\boldsymbol{g}$, given the conjecture β on the taxpayers' under-reporting probability. The second component $r(cK)$ is analogous to the capital charge in the *residual income* concept in accounting.[20] In other words, the reserving of a committed budget is viewed as an investment that ties up resources in this formulation of the tax compliance game with multiple taxpayers.

More specifically, the tax collector's decision problem can be represented by the following two-step optimization problem, given any conjecture β on the taxpayers' under-reporting probability:

$$\text{(CB-}\beta\text{)} \qquad \text{Max}_{K} \; R(\boldsymbol{g}^K; \beta) - r(cK),$$

where for any given K, $\boldsymbol{g}^K$ is an optimal audit strategy and $R(\boldsymbol{g}^K; \beta)$ is the maximized expected net

[19] It suffices to consider only committed budgets in the multiple of c, for any amount in between is obviously inefficient.

[20] In the one-to-one analysis of the classic tax compliance game, c is assumed to be sufficiently small so that the more interesting mixed-strategy equilibrium exists. Similarly, I assume the per-dollar opportunity cost r is sufficiently small so that $K > 0$ in equilibrium.

revenue from the program below:

(AS-β) Max $_{\boldsymbol{g}}$ $R(\boldsymbol{g}; \beta)$

subject to

P0: $\boldsymbol{g} \geq \mathbf{0}$,

P1: $\sum_{i=0}^{N} g(i \mid m) = 1$ for all m,

FS: $g(i \mid m) = 0$ for all $i > m$,

BG: $g(i \mid m) = 0$ for all i, m with $K+1 \leq i \leq m$,

where i, $m = 0, 1, ..., N$. Constraints P0 and P1 are standard requirements on $[g(i \mid m)]_{i=0,1,\ldots,N}$ as a valid vector of probabilities from a probability distribution. Constraints FS recognize that a sampling plan is not feasible if it ever requires drawing an audit sample with a size greater than the number of "low-income" tax returns received. Finally, constraints BG recognize that only sampling plans consistent with the committed budget characterized by K are implementable.

Given any number L of "low-income" tax returns received, suppose A of them are selected for audits, with V of them being "violators caught" (i.e., discovered to be non-compliant high-income taxpayers). Then the net revenue of the tax collector is as follows:

$$R^V(A, L) = V(F + T_H - T_L) - Ac + LT_L.$$

Accordingly, the expected net revenue collected with an implementable audit strategy $\boldsymbol{g}$ is

$$R(\boldsymbol{g}; \beta) = \sum_{m=0}^{N} \sum_{i=0}^{N} \mathcal{E}[R^V(A, L) \mid A = i, L = m, \beta]\, g(i \mid m) \Pr\{L = m \mid \beta\}.$$

To better understand how $\boldsymbol{g}$ affects the expected net revenue $R(\boldsymbol{g}; \beta)$, a discussion of the "constituents" of L and their probability distributions is useful. Let N_H denote the number of *high-income* taxpayers. Since q is the *ex ante* probability of a taxpayer having a high income, N_H follows the binomial distribution Bin(q,N). Denote by L_H the number of non-compliant high-income taxpayers (i.e., *high-income* taxpayers filing "*low-income*" tax returns). Given the conjecture on the taxpayers' under-reporting probability β, L_H follows the binomial distribution Bin(β,N_H). When conditional on L, it has a mean as provided in the lemma below:

LEMMA 1 (POSTERIOR EXPECTED NUMBER OF "VIOLATORS"): *Given the conjecture* β *on the* *taxpayers' under-reporting probability,*

$$\mathcal{E}[L_{\mathrm{H}} \mid L, \beta] = L\beta/(\beta + q^{-1} - 1).$$

Catching non-compliant high-income taxpayers from an audit sample of size A drawn from "low-income" tax returns is like trying to find red balls by drawing A balls out of an urn that contains L balls with L_{H} of them being red. Note that $N - L$ is the number of "high-income" tax returns received. Only compliant high-income taxpayers would have filed these tax returns. So the number of such taxpayers must be the same as the number of high-income taxpayers minus that of non-compliant high-income taxpayers, i.e., $N - L = N_{\mathrm{H}} - L_{\mathrm{H}}$. Conditional on L_{H} and N_{H} (and thus also on $L = N - N_{\mathrm{H}} + L_{\mathrm{H}}$), the number V of "violators caught" from an audit sample of size A follows the hypergeometric distribution $\mathrm{Hyp}(V; A, L_{\mathrm{H}}, L)$. The mean of this distribution is $\mathcal{E}(V \mid A, L_{\mathrm{H}}, L) = A(L_{\mathrm{H}}/L)$, provided $L > 0$. If $L = 0$, $L_{\mathrm{H}} = A = V = 0$ with certainty.

By Lemma 1 and other properties of the "constituents" of L, the expected net revenue collected with strategy $\boldsymbol{g}$ can be expressed as follows:[21]

$$R(\boldsymbol{g}; \beta) = \mathcal{E}[s^{\boldsymbol{g}}(L)L \mid \beta]\,[(F + T_{\mathrm{H}} - T_{\mathrm{L}})\beta/(\beta + q^{-1} - 1) - c] + \mathcal{E}[L \mid \beta]T_{\mathrm{L}},$$

where $s^{\boldsymbol{g}}(L) = \sum_{i=0}^{N} i\, g(i \mid L)/L$ for $L > 0$ and $s^{\boldsymbol{g}}(L) = 0$ for $L = 0$. In the first component of the expression, inside the expectation operator is

$$s^{\boldsymbol{g}}(L)L = \sum_{i=0}^{N} i\, g(i \mid L) = \mathcal{E}[A \mid L].$$

In other words, it is the (conditional) mean audit sample size induced by strategy $\boldsymbol{g}$; accordingly, for $L > 0$, $s^{\boldsymbol{g}}(L)$ is the mean *sampling rate* specified by $\boldsymbol{g}$.

The remaining part of the first component of $R(\boldsymbol{g}; \beta)$ is the following difference:

$$(F + T_{\mathrm{H}} - T_{\mathrm{L}})\beta/(\beta + q^{-1} - 1) - c.$$

Note that $F + T_{\mathrm{H}} - T_{\mathrm{L}}$ is the incremental "revenue" collected from a violator caught, and $\beta/(\beta + q^{-1} - 1)$ is the probability of catching a violator by auditing a "low-income" tax return. So the difference is simply the incremental expected net revenue from doing one extra audit. The last component of $R(\boldsymbol{g}; \beta)$ is the baseline tax revenue that would be expected from "low-income" return filers should

[21] The derivation is given in the appendix.

the tax collector do no audit at all.

With the conjecture $b = (1-q)c/q(F + T_H - T_L - c)$, the tax collector's expected net revenue, $R(\boldsymbol{g};\ b) = \mathcal{E}[L \mid b]T_L$, is independent of her choice of the audit strategy $\boldsymbol{g}$. Consequently, any implementable audit strategy is a best response to b. This finding is stated as Lemma 2 below.

LEMMA 2 (IMPLEMENTABLE AUDIT STRATEGIES AS OPTIMAL RESPONSES TO EQUILIRBIUM UNDER-REPORTING PROBABILITY): *Any audit strategy implementable by the tax collector's committed budget is a best response to the conjecture* $b = (1-q)c/q(F + T_H - T_L - c)$ *on the taxpayers' under-reporting probability.*

When the tax collector starts to deal with the whole taxpayer population rather than each taxpayer individually, the knowledge of L might convey information about the number of non-compliant high-income taxpayers, denoted by L_H (i.e., the number of high-income taxpayers filing "low-income" tax returns). As L changes, the conditional distribution of L_H given L might also change. Since different distributions of L_H stand for different expected net gains from conducting audits, the tax collector might have a preference for certain audit sample size depending on the L observed. However, given the conjecture b on the taxpayers' under-reporting probability, audit samples of different sizes turn out to be equally good to the tax collector, regardless of L. Consequently, any implementable audit strategy is an optimal response to b.

3.2. Taxpayer's Decision

While only the tax collector can directly observe the number of "low-income" tax returns received, a taxpayer may also know something about it. If a high-income taxpayer chooses to be non-compliant, he knows that the realized value of L observed by the tax collector must be $1 + L^{N-1}$, where L^{N-1} is the number of "low-income" tax returns filed by the remaining $N-1$ taxpayers. Independent distribution of types and choices of action imply that given any conjecture β on other taxpayers' under-reporting probability, L^{N-1} follows the binomial distribution $\mathrm{Bin}(1-q(1-\beta), N-1)$.

Viewing himself as a "white ball" in an urn with L balls, a non-compliant high-income taxpayer worries only about the audit probability he is facing, regardless of other details of the tax collector's audit strategy. If the tax collector draws an audit sample of size A from all "low-income" tax returns received, the chance of including the "white ball" in the sample is $A/(1+L^{N-1})$. So given any conjecture $\boldsymbol{g}$ on the audit strategy, the audit probability facing a non-compliant high-income taxpayer is

$$\sum_{l=0}^{N-1} \Pr\{\text{“selected for audit”} \mid L = 1+l\}\Pr\{L^{N-1} = l \mid \beta\}$$

$$= \sum_{l=0}^{N-1} \Big[\sum_{i=0}^{N} [i/(1+l)]\, g(i \mid 1+l) \Big] \Pr\{L^{N-1} = l \mid \beta\}$$

$$= \sum_{l=0}^{N-1} s^{\boldsymbol{g}}(1+l) \Pr\{L^{N-1} = l \mid \beta\}$$

$$= \mathcal{E}[s^{\boldsymbol{g}}(1+L^{N-1}) \mid \beta].$$

In other words, as long as two audit strategies have the same mean sampling rate $\mathcal{E}[s^{\boldsymbol{g}}(1+L^{N-1}) \mid \beta]$, their deterrence effects are identical. Hence, a high-income taxpayer will under-report with certainty if $\mathcal{E}[s^{\boldsymbol{g}}(1+L^{N-1}) \mid \beta] < a$, where a is the equilibrium audit probability defined in Section 2. Similarly, he will be compliant with certainty if $\mathcal{E}[s^{\boldsymbol{g}}(1+L^{N-1}) \mid \beta] > a$, with any under-reporting probability β being a best response to a conjecture $\boldsymbol{g}$ on the audit strategy that has $\mathcal{E}[s^{\boldsymbol{g}}(1+L^{N-1}) \mid \beta] = a$.

To conclude this section, Table 2 summarizes the distribution properties of variables affecting and/or affected by the decisions of the tax collector and taxpayers.

Insert Table 2 around here.

4. Optimal Audit Strategy and Committed Budget

Characterizing in closed form the optimal solution for the tax collector's two-step decision problem is not straightforward. The difficulty is partly due to the choice variable K that represents the committed budget. Instead of entering as a parameter of some given set of constraints, K affects the number of constraints to be included in the second-stage decision. In the following, I will characterize an optimal audit strategy for a given K, through the mean sampling rates the strategy induces under different realizations of L.

First of all, note that for any conjecture β with $(F + T_H - T_L)\beta/(\beta + q^{-1} - 1) < c$, the "no audit" strategy with $g(0 \mid m) = 1$ is obviously optimal. Secondly, if the conjecture β is b, where b is the equilibrium underreporting probability defined in Section 2, then $(F + T_H - T_L)\beta/(\beta + q^{-1} - 1) = c$, and any implementable audit strategy is optimal. Therefore, it suffices to consider below only a conjecture with $(F + T_H - T_L)\beta/(\beta + q^{-1} - 1) > c$. Under such circumstances, the tax collector's objective is simply to maximize $\mathcal{E}[s^{\boldsymbol{g}}(L)L \mid \beta]$, i.e., the mean audit sample size induced by an implementable strategy $\boldsymbol{g}$.

Clearly, what matters is only the conditional mean audit sample size for each L, namely

$$s^{\boldsymbol{g}}(L)L = \left[\, \textstyle\sum_{i=0}^{N} i\, g(i \mid L) \,\right],$$

which is characterized by the mean sampling rate $s^{\boldsymbol{g}}(L)$. Therefore, to identify an optimal audit strategy, it suffices to solve for an optimal sampling (rate) plan of the program below.

SR-β $\qquad$ $\text{Max}_{\,s} \ \sum_{l=0}^{N} [\, s(l)l \,] \Pr\{L = l \mid \beta\}$

subject to

S0: $\qquad$ $s(l) \geq 0,$ $\qquad$ for all l,

S1: $\qquad$ $s(l) \leq 1,$ $\qquad$ for all l,

BC: $\qquad$ $s(l)l \leq K,$ $\qquad$ for all l,

where $l = 0, 1, ..., N$. Constraints S0 and S1 ensure a sampling rate is between 0 and 1. Constraints BC rule out sampling rates that yield a mean sample size greater than K, which are obviously inconsistent with audit strategies implementable under K.

Because there is no tradeoff between sampling rates for different L, an optimal plan will have $s(L)$ set to the maximum feasible level, which is

$$s^K(L) = \min\{\, K/L, 1 \,\} \qquad \text{for } L = 0, 1, ..., N.$$

An audit strategy implementable under K that induces this optimal sampling plan is as follows:

$$g^K(\min\{K, L\} \mid L) = 1 \qquad \text{for } L = 0, 1, ..., N.$$

Consequently, this is an optimal audit strategy for the the committed budget characterized by K. This finding is the first main result of the paper.

PROPOSITION 1 ("KERNEL" BOUNDED SRS AS A CLASS OF OPTIMAL AUDIT STRATEGIES): *Given a conjecture on the taxpayers' under-reporting probability* β *with* $(F + T_H - T_L)\beta/(\beta + q^{-1} - 1) \geq c$, *and a committed budget characterized by K, an audit strategy that always draws an audit sample of size* $\min\{K, L\}$ *for any realization L of the number of "low-income" tax returns received is a best response to* β.

By the results above, for any conjecture β with $(F + T_H - T_L)\beta/(\beta + q^{-1} - 1) > c$, the tax collector's first-stage decision problem is equivalent to the following:

$$\text{Max}_{K} \quad A^K(\beta)[(F + T_H - T_L)\beta/(\beta + q^{-1} - 1) - c] - r(cK),$$

where

$$A^K(\beta) = \mathcal{E}[\min\{K, L\} \mid \beta] = \mathcal{E}[L \mid L \leq K] \Pr\{L \leq K \mid \beta\} + K \Pr\{L > K \mid \beta\}$$

is the expected audit sample size induced by the optimal audit strategy $\boldsymbol{g}^K$. Obviously, $A^K(\beta)$ is increasing in K, with $A^0(\beta) = 0$ and $A^N(\beta) = N[1-q(1-\beta)]$. Note that $A^N(\beta) - A^{N-1}(\beta) = \Pr\{L = N \mid \beta\}$.[22] Therefore, setting $K = N$ is suboptimal if

$$\Pr\{L = N \mid \beta\} \ < \ rc/[(F + T_H - T_L)\beta/(\beta + q^{-1} - 1) - c].$$

Intuitively, if the chance of seeing $L = N$ is sufficiently small, reserving a committed budget large enough to audit possibly all tax returns is likely to be a pure waste. This finding is stated as the following proposition:

PROPOSITION 2 (CONDITION FOR SUBOPTIMALITY OF FULL-AUDIT COMMITTED BUDGET): *Given a conjecture on the taxpayers' under-reporting probability* β *with* $(F + T_H - T_L)\beta/(\beta + q^{-1} - 1) \geq c$, *a committed budget with* $K = N$ *is suboptimal if*

$$\Pr\{L = N \mid \beta\} \ < \ rc/[(F + T_H - T_L)\beta/(\beta + q^{-1} - 1) - c].$$

Fully characterizing the optimal K is not easy owing to the discrete nature of $A^K(\beta)$. A full characterization, however, is unnecessary if the focus is to understand only the optimal K in

[22] To see this, note that $A^N(\beta) = \mathcal{E}[\min\{N, L\} \mid \beta] = \mathcal{E}[L \mid \beta] = \mathcal{E}[L \mid L \leq N-1] \Pr\{L \leq N-1 \mid \beta\} + N \Pr\{L = N \mid \beta\}$. Thus, $A^N(\beta) - A^{N-1}(\beta) = \Pr\{L = N \mid \beta\}$.

equilibrium. In particular, I will in the following focus on the equilibrium characterized by the probability pair (b,a), i.e., the pair of equilibrium underreporting and audit probabilities defined in Section 2. A committed budget, characterized by K, cannot constitute such an equilibrium unless, given the conjecture b, an optimal strategy $\boldsymbol{g}$ supported by the budget yields an audit probability equal to a, i.e.,.

$$\mathcal{E}[s^{\boldsymbol{g}}(1+L^{N-1}) \mid b] = a.$$

If I restrict only to "kernel" bounded SRS characterized by $\boldsymbol{s}^K$ discussed earlier, there is no guarantee that the equation below has an integer solution for K:

$$\mathcal{E}[\ \min\{\ K/(1+L^{N-1}),\ 1\ \} \mid b\ ] = a.$$

Therefore, considering the following generalized form of bounded SRS is useful:

$$s(L) = \begin{cases} \theta & \text{for } L = 1, 2, ..., K; \\ \theta K/L & \text{for } L = K{+}1, K{+}2, ..., N, \end{cases}$$

where $0 \le \theta \le 1$. Figure 3 illustrates the structure of this type of plans. In words, it entails randomly selecting a fraction θ of all the "low-income" tax returns to audit if $L \le K$ and a fraction $\theta K/L$ if $L > K$.[23]

Insert Figure 3 around here.

From the taxpayers' perspective, bounded SRS strategies with different θ's induce different audit probabilities. Depicted in Figure 4 is the strategy for different values of θ. With $\theta < a$, the strategy lies everywhere below the equilibrium proportional SRS strategy. As a result, the strategy provides less deterrence to taxpayers and cannot constitute an equilibrium. By increasing θ beyond a, the audit probability induced by the strategy may be raised to a level closer to a or even above it. If $\theta = 1$, the induced audit probability will reach the maximum level attainable by the K that characterizes the committed budget.

Insert Figure 4 around here.

[23] If θL or θK is not an integer, the exact size of the sample drawn with this strategy will be determined in the fashion described in footnote 12.

In the next section, I will discuss further some properties of bounded SRS, followed by a comparison of its budget-related characteristics with those of proportional SRS and Poisson sampling.

5. Equilibrium Bounded SRS

So long as the committed budget characterized by K is not too restrictive, an audit strategy consistent with the bounded SRS can constitute an equilibrium, if θ is set at the right level. This second main result of the paper is stated as the proposition below.

PROPOSITION 3 (SUFFICIENCY FOR BOUNDED SRS AUDIT STRATEGY TO CONSTITUTE AN EQUILIBRIUM): *Suppose the number of audits K that characterizes the tax collector's committed budget satisfies the condition* $\lambda(K) \geq a$*, where*

$$\lambda(K) = \frac{\mathcal{E}[\, \min\{K, L\} \mid b \,]}{Np}$$

and $p = 1 - q(1-b)$*. Then there exists an equilibrium such that bounded SRS with parameter* $\theta = a/\lambda(K)$ *is an equilibrium audit strategy.*

The ratio $\lambda(K)$ defined above may be interpreted as the audit probability induced by a bounded SRS strategy with parameter $\theta = 1$ under a committed budget characterized by K. For $K = 0$, $\lambda(K) = 0$, meaning that the chance of being audited has to be zero when the committed budget is insufficient to conduct even a single audit. By contrast, $\lambda(N) = \mathcal{E}(L)/Np = 1$. When the committed budget is big enough to audit even the whole taxpayer population, a bounded SRS strategy with $\theta = 1$ means a constant 100% sampling rate, regardless of the L observed. Suppose $K > 0$ but $\lambda(K) < a$. In words, the committed budget allows some audits to be done but is not large enough to ensure $\lambda(K) \geq a$. If so, even setting the sampling rate parameter to its maximum value (i.e., $\theta = 1$) will not be high enough to implement the equilibrium audit probability a.

Proposition 3 says the parameter θ for implementing an equilibrium bounded SRS strategy should be set to $a/\lambda(K)$ in order to fix the induced audit probability at a. What is the intuition behind this? Note that $\lambda(K)$ is the audit probability induced by the "most deterrent" bounded SRS strategy (i.e., with θ set to 1) allowed by the committed budget characterized by K. When this budget is not too small (i.e., $\lambda(K) \geq a$), such a strategy will audit too often and fail to constitute an equilibrium. To form an equilibrium, θ must be reduced from 1 to some appropriate level so that the induced audit probability equals a exactly. The simple structure of the model ensures that the adjustment can be achieved proportionally; that is, the correct choice of θ is given by the equation: $\theta/1 = a/\lambda(K)$. Of course, this equation will have no solution if $\lambda(K) < a$, i.e., when the condition in Proposition 3 is violated. Can the condition ever be met? The following corollary gives an instance where it is met.

COROLLARY 1 (COMMITTED BUDGET SUFFICIENT TO IMPLEMENT BOUNDED SRS AUDIT STRATEGY IN EQUILIBRIUM): *If the tax collector has a committed budget characterized by* $K > aN$, *then* $\lambda(K) \geq a$ *and hence bounded SRS with parameter* $\theta = a/\lambda(K)$ *is an equilibrium audit strategy.*

This corollary implies that if a committed budget is big enough to support the equilibrium proportional SRS strategy with sampling rate a, it will also be sufficient to implement the equilibrium bounded SRS strategy with parameter $\theta = a/\lambda(K)$. The reverse, however, is not true.[24] Similarly, the committed budget cN necessary to support a Poisson sampling strategy will also support the equilibrium proportional SRS strategy. However, a committed budget with $K \in \{\lceil aN \rceil, \lceil aN \rceil + 1, \ldots, N-1\}$ that can support proportional SRS in equilibrium will not support Poisson sampling. The efficiency ranking of the three audit strategies, based on the committed budgets necessary to support them, is stated formally as the proposition below:

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[24] There may exist a committed budget large enough to support an equilibrium bounded SRS strategy but not an equilibrium proportional SRS strategy. The following is an example. Consider a setting with $N = 2$, $q = 1/2$, $b = 1/3$, $a = 9/16$, and a committed budget with $K = 1$. Then $p = 2/3$, $K < \lceil aN \rceil = 2$, and

$$\begin{aligned}\lambda(K) &= [\mathcal{E}(L \mid L \leq K)\Pr\{L \leq K\} + K\Pr\{L > K\}]/Np \\ &= [\mathcal{E}(L \mid L \leq 1)\Pr\{L \leq 1\} + \Pr\{L = 2\}]/2p \\ &= [\Pr\{L = 1\} + \Pr\{L = 2\}]/2p \\ &= [2p(1-p) + p^2]/2p \\ &= 2/3 \\ &> a.\end{aligned}$$

So the committed budget can support an equilibrium bounded SRS strategy with parameter $\theta = 27/32$ but not the proportional SRS strategy with sampling rate $a = 9/16$.

PROPOSITION 4 (RELATIVE EFFICIENCY OF BOUNDED SRS, PROPORTIONAL SRS, AND POISSION SAMPLING AUDIT STRATEGIES): *In terms of the committed budget necessary to support an equilibrium audit strategy, bounded SRS is weakly more efficient than proportional SRS, which in turn is weakly more efficient than Poisson sampling. In other words, a bounded SRS strategy will constitute an equilibrium whenever the committed budget can implement proportional SRS in equilibrium, but not vice versa; a proportional SRS strategy will constitute an equilibrium whenever the committed budget can implement Poisson sampling in equilibrium, but not vice versa.*

Some preparation is needed before providing the last main result of the paper. First of all, note that the K of any committed budget that supports an equilibrium audit strategy must be at least Npa, which is the expected number of audits done under proportional SRS and Poisson sampling. Intuitively, if this is not satisfied, there is no way to audit sufficiently often so that the audit probability facing each non-compliant high-income taxpayer is maintained at a, the equilibrium level.[25]

Let $\underline{K}$ denote the number of audits that characterizes the committed budget necessary to support an equilibrium bounded SRS strategy, i.e., $\underline{K} = \min\{ K \mid \lambda(K) \geq a \}$, or equivalently,

$$\underline{K} = \min\Big\{ K \;\Big|\; \mathcal{E}[\min\{K, L\} \mid b] \geq Npa \Big\}.$$

With this K, the parameter of the equilibrium bounded SRS strategy would be set to $\theta = a/\lambda(\underline{K})$. Since the minimum committed budget must exceed the expected audit cost, implying that $\underline{K} \geq Npa$, pa sets a lower bound on the ratio $\underline{K}/N$. If any equilibrium audit strategy, including bounded SRS, manages to get arbitrarily close to this lower bound, it must be nearly as good as an *efficient* equilibrium audit strategy.

[25] Consider any committed budget with $K < Npa$. By definition, any equilibrium audit strategy it supports must have a sampling plan $\boldsymbol{s}$ such that $s(L)L \leq K$ for all $L = 0, 1, \ldots, N$. This implies $s(1+l) \leq K/(1+l)$ for all $l = 0, 1, \ldots, N-1$. Sum up both sides with probability weights of the binomial distribution $\mathrm{Bin}(p, N-1)$. It follows that the audit probability is less than a:

$$\mathcal{E}[s(1+L^{N-1}) \mid b] = K \textstyle\sum_{l=0}^{N-1} (1+l)^{-1}\binom{N-1}{l} p^l (1-p)^{N-1-l}] = K/Np < a,$$

which means the strategy with $\boldsymbol{s}$ cannot constitute a (b,a) equilibrium. Therefore, the K of any committed budget that supports an equilibrium audit strategy must be at least Npa.

To determine the limiting value of $\underline{K}/N$, an upper bound is needed. Corollary 1 says a committed budget with $K = \lceil aN \rceil$ can implement an equilibrium bounded SRS strategy. Therefore, $\underline{K}$ must be weakly below $\lceil aN \rceil < 1 + aN$, or equivalently, $\underline{K}/N < 1/N + a$. The lemma below helps to derive a tighter upper bound on the ratio $\underline{K}/N$, which is used to establish Proposition 5 that follows.

LEMMA 3 (UPPER BOUND ON $\underline{K}$): *Let $\underline{K} = \min\{ K \mid \lambda(K) \geq a \}$, i.e., the number of audits characterizing the committed budget necessary to support the bounded SRS audit strategy with parameter $\theta = a/\lambda(\underline{K})$ in equilibrium. There exists an N^* such that for any $N > N^*$, $\underline{K} \leq (N-1)p$.*

With this lemma, the ratio $\underline{K}/N$ can be shown to reach nearly its lower bound pa, which may only be attained by an efficient equilibrium audit strategy, if ever. In other words, if the taxpayer population is large enough, the committed budget necessary to support an equilibrium bounded SRS audit strategy will be nearly as low as $cNpa$.

By definition, even an efficient equilibrium audit strategy requires at least this level of committed budget to support. Moreover, the strategy must require no more committed budget to support than the level required by an equilibrium bounded SRS strategy. Therefore, *$cNpa$ is asymptotically the efficient committed budget for implementing the audit probability a* with some equilibrium audit strategy, including a bounded SRS strategy. The asymptotic efficiency of bounded SRS with parameter $\theta = a/\lambda(\underline{K})$ is stated in the proposition below, in terms of expected budget usage ratio, i.e., expected audit cost incurred as a percentage of the committed budget.

PROPOSITION 5 (ASYMPTOTIC EFFICIENCY OF BOUNDED SRS AUDIT STRATEGY): *Let $\underline{K} = \min\{ K \mid \lambda(K) \geq a \}$. The equilibrium bounded SRS audit strategy with parameter $\theta = a/\lambda(\underline{K})$ is asymptotically efficient. In contrast, the equilibrium proportional SRS audit strategy is asymptotically less efficient, whereas the equilibrium Poisson sampling audit strategy is asymptotically least efficient among the three. Specifically, the limiting values of the expected budget usage ratios of the three equilibrium audit strategies are as follows:*

	Expected Budget Usage Ratio at the Limit
Bounded SRS	$\lim_{N\uparrow\infty} \mathcal{E}[s(L)L \mid b] / \underline{K} = 1$
Proportional SRS	$\lim_{N\uparrow\infty} \mathcal{E}[s(L)L \mid b] / aN = p$
Poisson Sampling	$\lim_{N\uparrow\infty} \mathcal{E}[s(L)L \mid b] / N = pa$

This proposition tells us that when the taxpayer population is large, on average nearly all of the committed budget $c\underline{K}$ necessary to support the equilibrium bounded SRS strategy with parameter $\theta = a/\lambda(\underline{K})$ will actually be incurred for conducting audits. To see the intuition, note that when the population size is sufficiently large, the number L of "low-income" tax returns received is highly likely to be somewhere around Np. This is an implication of the law of large numbers. It also means nearly all of the time the audit sample size $s(L)L = \min\{\theta K, \theta L\}$ is around $\theta\min\{K, Np\}$. Consequently, the audit probability $\mathcal{E}[s(1+L^{N-1}) \mid b]$ is around $\theta\min\{K, Np\}/Np$. For the bounded SRS to constitute an equilibrium, this audit probability needs to be set at a, or equivalently, it requires that $\theta\min\{K, Np\} = Npa$. The most efficient way to do this is by setting θ to 1 and K to Npa. Doing this is possible when N is sufficiently large so that Npa is very close to the integer $\lceil Npa \rceil$, which allows θ to be set to almost 1. As a result, the committed budget characterized by the K is at a level almost equal to the expected audit cost $c\mathcal{E}[s(L)L \mid b] \approx cNpa$.

6. Concluding Remarks

The virtue of the bounded SRS audit rule stems from its simplicity and the substantial reduction in the committed budget required, compared to rules suggested by the traditional analysis. Research in this direction has practical relevance to issues like audit staff planning and compliance enforcement efficiency. Despite the importance, there has been no discussion in this direction in the literature.

Using a multiple-taxpayer extension of the classic tax compliance game, I study the important but neglected implementation issue. Audit sampling plans inducing the same target audit probability can differ significantly in the committed budgets necessary to support the strategies. Because there is an opportunity cost associated with the committed budget, how it can be conserved to free unneeded resources for better uses is often a serious concern of administrators (see, e.g., US GAO [2004]). In this paper, I discuss how an audit sampling plan can be formulated *efficiently* to conserve the committed budget necessary for implementing the target audit probability.

Proportional SRS and Poisson sampling are audit rules generalized naturally from the traditional, one-to-one analysis. They can implement the equilibrium audit probability in the multiple-taxpayer setting but demand inefficient committed budgets to support. By contrast, the bounded SRS rule never needs more committed budget to support than the levels required by the other rules. Moreover, the committed budget it requires can be nearly as low as the *efficient* committed budget, i.e., the lowest attainable by any equilibrium audit strategies.

Thirty-five years after Allingham and Sandmo [1972] published the first model of tax evasion, tax compliance issues continue to draw researchers' attention.[26] One of the growing interests in this area concerns tax audit selection decisions and assessments. As Andreoni, Erard, and Feinstein [1998] have pointed out, "[i]mproved understanding of the audit process is likely to provide guidance in a number of policy areas, including the comparison and evaluation of alternative tax administration systems and the development of better audit selection methods." I contribute to this literature by examining different audit sampling plans and their efficiencies in terms of the committed budgets necessary to support them. The results offer novel insights on how audit strategies may be formulated to reduce inefficiency and what budget usage ratios should be expected accordingly.

This paper also contributes to a new perspective on audit sampling issues. Traditional audit sampling research takes the perspective of the acceptance sampling literature (see, e.g., Kinney and Warren [1979], Adcock [1988], and Guy, Carmichael, and Whittington [2002]). It concerns how sampling plans should be formulated so as to optimally accept or reject reported balances, with the

[26] Examples of recent studies on tax compliance include Alm and McKee [2004], Fisman and Wei [2004], Besim and Jenkins [2005], and Feldman and Slemrod [2007].

false-positive and false-negative risks properly controlled.[27] By contrast, the concern in strategic settings can be quite different.[28] In the stylized tax compliance setting examined here, controlling false-positive and false-negative risks is not an issue. Instead, the concern is about how a target level of audit probability can be implemented with a sampling plan that needs only the minimum committed budget to support.

In other contexts like financial audits, the analysis of this paper may or may not apply, depending on the nature of the issue in concern. For instance, from an audit firm's point of view, my analysis about determining the efficient "audit capacity" has obvious relevance to audit staff planning issues. The analysis may also be relevant to an audit client's strategic decision to choose between a Big4 auditor and a smaller one, in addition to considerations like deep-pocket insurance. For example, the committed-budget constraint of a Big4 auditor might be "softer" than a smaller auditor's. As a result, the audit quality might also be different.

My analysis can be adapted to study independent employee thefts and embezzlements that may exist in different segments of a large corporation. An analogy between auditor-employees and tax collector-taxpayers can be easily drawn. However, the analysis is not applicable to orchestrated financial frauds by an executive or a gang of managers. In such circumstances, whether a transaction, out of the many related to a fraud, would ultimately become a red flag is not an independent event. An important assumption of my analysis is thus violated.

Mainly because of Lemma 2, proving the dominance of bounded SRS over proportional SRS or Poisson sampling is not a formidable task in my analysis. The lemma says any audit sampling plan implementable by the committed budget is optimal. Therefore I can reallocate the sampling rates for different contingencies on the number of "low-income" returns received, without making the strategy sub-optimal. I only need to ensure that the induced audit probability remains the same. To guarantee this, I consider the generalized bounded SRS with an additional parameter, rather than only the

[27] Such risks are also called Type-I and Type-II error risks or alpha and beta risks.

[28] For example, in the context of statistical sampling for process control, Bushman and Kanodia [1996] point out that "in strategic settings, where the stochastic process is significantly impacted by the actions of human agents, *deterrence* rather than ex post detection is the main issue." (emphasis added).

"kernel" bounded SRS. By tuning the parameter appropriately, the induced audit probability can be fixed at the right level. The equilibrium suggested by the one-to-one analysis can thus be implemented using bounded SRS with a lower committed budget.

Lemma 2 relies heavily on the additive separability of the tax collector's linear payoff structure.[29] While such a structure appears reasonable in tax compliance situations or when audits are conducted by larger firms, it is an open question as to how the results would generalize to other settings. I conjecture that a multiplicatively separable payoff function would still yield a result analogous to that of Lemma 2, although it may not be possible to establish that every implementable audit strategy is optimal. Additional explorations along these lines would go a long way to expanding the applicability of the results in this paper.

I have used the notion of opportunity cost to model the waste of an overly-committed budget. This actually understates the severity of the problem. For example, in the context of audit staff planning for financial audits, audit firms must hire enough college graduates during the summer to prepare for the peak demand near the end of the year. In the context of tax compliance, the IRS must have enough trained tax auditors in order to support the work in the upcoming year. In other words, professionals qualified for the job typically are in short supply. So the issue is not simply about budget money left unused. "Overly-committed budget" discussed here may actually mean outlay costs spent to build up excessive capacity because qualified people cannot be hired instantaneously. This is a waste substantially higher than merely the opportunity cost of unused budget money, making the implementation issue discussed in this paper even more important.

[29] See the derivation of the tax collector's expected net revenue function in the appendix.

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Appendix: Proofs and Derivations

PROOF OF LEMMA 1: Let $H_H = N_H - L_H$, which is the number of compliant high-income taxpayers. Since $N - L = N_H - L_H = H_H$, conditioning on L is equivalent to conditioning on H_H. Therefore, $\mathcal{E}[L_H \mid L, \beta] = \mathcal{E}[L_H \mid H_H, \beta] = \mathcal{E}[\mathcal{E}(L_H \mid N_H, H_H, \beta) \mid H_H, \beta]$. For notational simplicity, I will in the following omit β from the information set indicated in conditional means and probabilities.

First of all, let me consider the conditional mean $\mathcal{E}(L_H \mid N_H, H_H)$ and the conditional probability distribution $\Pr\{N_H = j \mid H_H = h\}$. By definition, $\mathcal{E}(L_H \mid N_H, H_H) = N_H - H_H$ and $\Pr\{N_H = j \mid H_H = h\} = \Pr\{N_H = j, H_H = h\} / \Pr\{H_H = h\}$. Recall that N_H is distributed with the binomial distribution Bin(q,N), i.e., $\Pr\{N_H = j\} = \binom{N}{j} q^j (1-q)^{N-j}$. Moreover, given the conjecture β on the taxpayers' under-reporting probability, L_H is distributed with the binomial distribution Bin(β,N_H), i.e., $\Pr\{H_H = h \mid N_H = j\} = \binom{j}{h} \beta^{j-h}(1-\beta)^h$.

Therefore,
$$
\begin{aligned}
&\Pr\{N_H = j, H_H = h\} \\
&= \Pr\{H_H = h \mid N_H = j\} \Pr\{N_H = j\} \\
&= \tbinom{j}{h} \beta^{j-h}(1-\beta)^h \tbinom{N}{j} q^j (1-q)^{N-j} \\
&= [(\beta^{-1}-1)^h/h!][N!/(N-j)!(j-h)!](1-q)^{N-j}(q\beta)^j
\end{aligned}
$$
for $j \geq h$ and $\Pr\{N_H = j, H_H = h\} = 0$ otherwise. As a result,
$$
\begin{aligned}
&\Pr\{H_H = h\} \\
&= \textstyle\sum_{j=h}^{N} \Pr\{N_H = j, H_H = h\} \\
&= [(\beta^{-1}-1)^h/h!] \textstyle\sum_{j=h}^{N} [N!/(N-j)!(j-h)!](1-q)^{N-j}(q\beta)^j \\
&= [N!/(N-h)!h!](q\beta)^h(\beta^{-1}-1)^h \textstyle\sum_{i=0}^{N-h} [(N-h)!/(N-h-i)!i!](1-q)^{N-h-i}(q\beta)^i \\
&= [N!/(N-h)!h!](q\beta)^h(\beta^{-1}-1)^h(1-q+q\beta)^{N-h}.
\end{aligned}
$$

Similarly, $\sum_{j=h}^{N} \mathcal{E}(L_H | j, h) \Pr\{N_H = j, H_H = h\}$

$$= \sum_{j=h}^{N} (j-h) \Pr\{N_H = j, H_H = h\}$$
$$= [(\beta^{-1}-1)^h/h!] \sum_{j=h+1}^{N} [N!/(N-j)!(j-h-1)!](1-q)^{N-j}(q\beta)^j$$
$$= [N!/(N-h-1)!h!](q\beta)^{h+1}(\beta^{-1}-1)^h$$
$$\times \sum_{i=0}^{N-h-1} [(N-h-1)!/(N-h-1-i)!i!](1-q)^{N-h-1-i}(q\beta)^i$$
$$= [N!/(N-h-1)!h!](q\beta)^{h+1}(\beta^{-1}-1)^h(1-q+q\beta)^{N-h-1}.$$

Thus, $\mathcal{E}(L_H|H_H)$

$$= \mathcal{E}[\mathcal{E}(L_H \mid N_H, H_H) \mid H_H]$$
$$= [\textstyle\sum_{j=h}^{N} \mathcal{E}(L_H| j, h) \Pr\{N_H = j, H_H = h\}]/\Pr\{H_H = h\}$$
$$= [N!/(N-h-1)!h!](q\beta)^{h+1}(\beta^{-1}-1)^h(1-q+q\beta)^{N-h-1}$$
$$/ [N!/(N-h)!h!](q\beta)^h(\beta^{-1}-1)^h(1-q+q\beta)^{N-h}$$
$$= (N-h)(q\beta)/(1-q+q\beta)$$
$$= (N - H_H)\beta/(\beta + q^{-1} - 1)$$
$$= L\beta/(\beta + q^{-1} - 1).$$ ■

DERIVATION OF TAX COLLECTOR'S EXPECTED NET REVENUE FUNCTION: Conditional on (i) the audit sample size A, (ii) the number of "low-income" tax returns received L, with $L > 0$, and (iii) the conjecture on the taxpayers' under-reporting probability β, the expected net revenue of the tax collector can be expressed as follows:

$\mathcal{E}[R^V(A, L) \mid A, L, \beta]$

$$= \mathcal{E}[V \mid A, L, \beta](F + T_H - T_L) - Ac + LT_L$$
$$= \mathcal{E}[L_H/L \mid A, L, \beta]A(F + T_H - T_L) - Ac + LT_L$$
$$= \mathcal{E}[L_H/L \mid L, \beta]A(F + T_H - T_L) - Ac + LT_L.$$

The last equality follows from the fact that L_H and L are realized before A is determined. If $L = 0$, obviously $\mathcal{E}[R^V(A, L) \mid A, L, \beta] = 0$. Thus,

$R(\boldsymbol{g}; \beta)$

$$= \mathcal{E}[\ \mathcal{E}[R^V(A, L) \mid A, L, \beta] \mid \beta\ ]$$
$$= \mathcal{E}[\ \mathcal{E}[R^V(A, L) \mid A, L, \beta] \mid L > 0, \beta\ ] \Pr\{L > 0 \mid \beta\}$$
$$= \mathcal{E}[\ \mathcal{E}[L_H/L \mid L, \beta]A(F + T_H - T_L) - Ac + LT_L \mid L > 0, \beta\ ] \Pr\{L > 0 \mid \beta\}$$
$$= \mathcal{E}[\ \mathcal{E}[L_H/L \mid L, \beta]\mathcal{E}[A \mid L, \beta] \mid L > 0, \beta\ ] \Pr\{L > 0 \mid \beta\}(F + T_H - T_L) - \mathcal{E}[\ \mathcal{E}[A \mid L, \beta] \mid \beta\ ]c$$
$$+ \mathcal{E}[L \mid \beta]T_L$$
$$= \mathcal{E}[\ \mathcal{E}[L_H/L \mid L, \beta]\mathcal{E}[A \mid L] \mid L > 0, \beta\ ] \Pr\{L > 0 \mid \beta\}(F + T_H - T_L) - \mathcal{E}[\ \mathcal{E}[A \mid L] \mid \beta\ ]c + \mathcal{E}[L \mid \beta]T_L$$

The fourth inequality follows from the fact that $\mathcal{E}[-Ac + LT_{\mathrm{L}} \mid L = 0, \beta] = 0$. The last equality follows from the fact that although β affects the probability distribution of L, when conditional on L the sampling plan $[g(i \mid L)]_{i=0,1,\ldots,N}$ completely determines the audit sample size A.

By Lemma 1, $\mathcal{E}[L_{\mathrm{H}}/L \mid L, \beta] = \beta/(\beta + q^{-1} - 1)$ for $L > 0$. Therefore,

$$R(\boldsymbol{g}; \beta) = \mathcal{E}[\,\mathcal{E}[A \mid L] \mid \beta\,]\,[(F + T_{\mathrm{H}} - T_{\mathrm{L}})\beta/(\beta + q^{-1} - 1) - c] + \mathcal{E}[L \mid \beta]T_{\mathrm{L}}.$$

Let $s^{\boldsymbol{g}}(L) = \sum_{i=0}^{N} i\, g(i \mid L)/L$ for $L > 0$ and $s^{\boldsymbol{g}}(L) = 0$ for $L = 0$. Since $s^{\boldsymbol{g}}(L)L = \sum_{i=0}^{N} i\, g(i \mid L) = \mathcal{E}[A \mid L]$, the expected net revenue can be expressed as

$$R(\boldsymbol{g}; \beta) = \mathcal{E}[s^{\boldsymbol{g}}(L)L \mid \beta]\,[(F + T_{\mathrm{H}} - T_{\mathrm{L}})\beta/(\beta + q^{-1} - 1) - c] + \mathcal{E}[L \mid \beta]T_{\mathrm{L}}.$$

■

PROOF OF PROPOSITION 3: Since $0 < a \le \lambda(K)$, $\theta = a/\lambda(K) \in [0,1]$. Therefore, the bounded SRS strategy with parameter θ is an implementable audit strategy. When a high-income taxpayer chooses to be non-compliant, he knows that the realized value of L observed by the tax collector must be $1 + L^{N-1}$, where L^{N-1} is the number of "low-income" tax returns filed by the remaining $N-1$ taxpayers. Independent distribution of types and choices of action imply that L^{N-1} follows the binomial distribution Bin(p, $N-1$), where $p = 1-q(1-b)$. So given θ, the audit probability facing a non-compliant high-income taxpayer is

$$\begin{aligned}
&\textstyle\sum_{l=0}^{N-1} \Pr\{\text{"selected for audit"} \mid L = 1+l\}\Pr\{L^{N-1} = l\} \\
&\quad= \textstyle\sum_{l=0}^{K-1} \theta \Pr\{L^{N-1} = l\} + \sum_{l=K}^{N-1} [\theta K/(1+l)] \Pr\{L^{N-1} = l\} \\
&\quad= \theta\, [\textstyle\sum_{l=0}^{K-1} \Pr\{L^{N-1} = l\} + K \sum_{l=K}^{N-1} (1+l)^{-1} \Pr\{L^{N-1} = l\}] \\
&\quad= \theta\, [\textstyle\sum_{l=0}^{K-1} \binom{N-1}{l} p^l(1-p)^{N-1-l} + K \sum_{l=K}^{N-1} (1+l)^{-1}\binom{N-1}{l} p^l(1-p)^{N-1-l}] \\
&\quad= (\theta/Np)[\textstyle\sum_{l=1}^{K} l \Pr\{L = l\} + K \sum_{l=K+1}^{N} \Pr\{L = l\}] \\
&\quad= (\theta/Np)[\textstyle\sum_{l=0}^{K} l \Pr\{L = l\} + K \Pr\{L > K\}] \\
&\quad= (\theta/Np)[\mathcal{E}(L \mid L \le K)\Pr\{L \le K\} + K \Pr\{L > K\}] \\
&\quad= a.
\end{aligned}$$

This audit probability makes him indifferent between being non-compliant and compliant. Thus, b is a best response to θ. By Lemma 2, any $\theta \in [0,1]$ is a best response to b. Hence, $(b, a/\lambda(K))$ is an equilibrium with the induced audit probability equal to a. ■

PROOF OF COROLLARY 1: It suffices to prove that $K \ge aN$ implies $\lambda(K) \ge a$. The rest then follows immediately from Proposition 3. If $K \ge aN$,

$$\begin{aligned}
\lambda(K) &= [\mathcal{E}(L \mid L \le K)\Pr\{L \le K\} + K \Pr\{L > K\}]/Np \\
&= \textstyle\sum_{l=0}^{K-1} \Pr\{L^{N-1} = l\} + \sum_{l=K}^{N-1} [K/(1+l)] \Pr\{L^{N-1} = l\} \\
&\ge \textstyle\sum_{l=0}^{K-1} \Pr\{L^{N-1} = l\} + a\sum_{l=K}^{N-1} [N/(1+l)] \Pr\{L^{N-1} = l\}
\end{aligned}$$

$$\geq \sum_{l=0}^{K-1} \Pr\{L^{N-1} = l\} + a\sum_{l=K}^{N-1} \Pr\{L^{N-1} = l\}$$

$$\geq a. \qquad \blacksquare$$

PROOF OF LEMMA 3: Suppose to the contrary that there does not exist such an N^*. This means for any N^*, there exists $\underline{K} > (N-1)p$ for some $N > N^*$. Let $\underline{K}^N$ denote the $\underline{K}$ satisfying this condition for any such an N. For any sufficiently small $\varepsilon > 0$, $\underline{K}^N > (N-1)(p+\varepsilon)$ as well.

Now for any N with the $\underline{K}^N$, consider $K = \underline{K}^N$, which has

$$\begin{aligned}
\lambda(K) &= [\mathcal{E}(L \mid L \leq K)\Pr\{L \leq K\} + K \Pr\{L > K\}]/Np \\
&= \sum_{l=0}^{K-1} \Pr\{L^{N-1} = l\} + \sum_{l=K}^{N-1} [K/(1+l)] \Pr\{L^{N-1} = l\} \\
&\geq \sum_{l=0}^{K-1} \Pr\{L^{N-1} = l\} + (p+\varepsilon)\sum_{l=K}^{N-1} [N/(1+l)] \Pr\{L^{N-1} = l\} \\
&\geq \sum_{l=0}^{K-1} \Pr\{L^{N-1} = l\} + (p+\varepsilon)\sum_{l=K}^{N-1} \Pr\{L^{N-1} = l\} \\
&= 1 - [1-(p+\varepsilon)]\Pr\{L^{N-1} \geq K\} \\
&\geq 1 - [1-(p+\varepsilon)]\Pr\{L^{N-1} \geq (N-1)(p+\varepsilon)\}.
\end{aligned}$$

Following the reasoning of the law of large numbers (e.g., see Feller [1968], p. 152), $\Pr\{L^{N-1} \geq (N-1)(p+\varepsilon)\}$ goes to zero as N goes to infinity. Therefore, $\lambda(\underline{K}^N)$ must be arbitrarily close to 1 for some $N > N^*$. This, however, implies that for some $N > N^*$, $\underline{K}^N$ is arbitrarily close to N and hence $\underline{K}^N - 1 > aN$. Analogous to the Proof of Corollary 1, this in turn implies $\lambda(\underline{K}^N - 1) \geq a$, a contradiction to the definition that $\underline{K}^N$ is the minimum K satisfying $\lambda(K) \geq a$ for the given N.

Consequently, there must exist an N^* such that for any $N > N^*$, $\underline{K} \leq (N-1)p$. $\blacksquare$

PROOF OF PROPOSITION 5: By definition, $\underline{K}$ is the smallest number in $\{1, 2, ..., N\}$ that allows the existence of an equilibrium $(b, a/\lambda(\underline{K}))$ with the induced audit probability $\theta\lambda(\underline{K})$ equal to a. This implies $0 \leq \theta = a/\lambda(\underline{K}) \leq 1 < a/\lambda(\underline{K}-1)$. Because $\lambda(K) = A^K(b)/Np$, where

$$A^K(b) = \mathcal{E}[\min\{K, L\} \mid b] = \mathcal{E}[L \mid L \leq K] \Pr\{L \leq K \mid b\} + K \Pr\{L > K \mid b\},$$

it follows that

$$A^{\underline{K}}(b) = \mathcal{E}(L \mid L \leq \underline{K})\Pr\{L \leq \underline{K}\} + \underline{K} \Pr\{L > \underline{K}\} \geq Npa$$

$$A^{\underline{K}-1}(b) = \mathcal{E}(L \mid L \leq \underline{K}-1)\Pr\{L \leq \underline{K}-1\} + (\underline{K}-1)\Pr\{L > \underline{K}-1\} < Npa.$$

Note that the difference

$$\begin{aligned}
&A^{\underline{K}}(b) - A^{\underline{K}-1}(b) \\
&\quad = \mathcal{E}(L \mid L \leq \underline{K})\Pr\{L \leq \underline{K}\} + \underline{K} \Pr\{L > \underline{K}\} \\
&\qquad\quad - \mathcal{E}(L \mid L \leq \underline{K}-1)\Pr\{L \leq \underline{K}-1\} - (\underline{K}-1)\Pr\{L > \underline{K}-1\} \\
&\quad = \underline{K}\tbinom{N}{\underline{K}}p^{\underline{K}}(1-p)^{N-\underline{K}} - \underline{K}\,[\Pr\{L > \underline{K}-1\} - \Pr\{L > \underline{K}\}] + \Pr\{L > \underline{K}-1\} \\
&\quad = \Pr\{L > \underline{K}-1\}.
\end{aligned}$$

Thus, $\lim_{N\uparrow\infty} [A^{\underline{K}}(b) - A^{\underline{K}-1}(b)]/N = 0$. Consequently, it must be that $\lim_{N\uparrow\infty} A^{\underline{K}}(b)/N = \lim_{N\uparrow\infty} A^{\underline{K}-1}(b)/N = pa$.

In the following, I will show that $\lim_{N\uparrow\infty} \underline{K}/N = pa$. By definition

$$A^{\underline{K}}(b) = \mathcal{E}(L \mid L \le \underline{K})\Pr\{L \le \underline{K}\} + \underline{K}\Pr\{L > \underline{K}\}$$
$$= \mathcal{E}(L) - \mathcal{E}(L - \underline{K} \mid L > \underline{K})\Pr\{L > \underline{K}\}.$$

For any given $\varepsilon > 0$, I can break the second term on the right into two for L above or below $N(p+\varepsilon)$ and then divide all terms on both sides by N to obtain

$$A^{\underline{K}}(b)/N = p - \mathcal{E}[L - \underline{K} \mid \underline{K} < L \le N(p+\varepsilon)] \Pr\{\underline{K} < L \le N(p+\varepsilon)\}/N$$
$$- \mathcal{E}[L - \underline{K} \mid L > N(p+\varepsilon)] \Pr\{L > N(p+\varepsilon)\}/N.$$

By Lemma 3, there exists an N^* such that for any $N > N^*$, $\underline{K} \le (N-1)p$. Hence, for any $N > N^*$,

$$0 \le \mathcal{E}[L - \underline{K} \mid L > N(p+\varepsilon)] \Pr\{L > N(p+\varepsilon)\}/N \le \Pr\{L > N(p+\varepsilon)\}.$$

Following the reasoning of the law of large numbers (e.g., see Feller [1968], p. 152), $\Pr\{L > N(p+\varepsilon)\}$ goes to zero as N goes to infinity. Therefore, $\lim_{N\uparrow\infty} \mathcal{E}[L - \underline{K} \mid L > N(p+\varepsilon)] \Pr\{L > N(p+\varepsilon)\}/N = 0$. Consequently,

$$\lim_{N\uparrow\infty} \mathcal{E}[L - \underline{K} \mid \underline{K} < L \le N(p+\varepsilon)] \Pr\{\underline{K} < L \le N(p+\varepsilon)\}/N$$
$$= p - \lim_{N\uparrow\infty} A^{\underline{K}}(b)/N$$
$$= p(1-a).$$

Since for any $N > N^*$, $0 \le \mathcal{E}[L - \underline{K} \mid \underline{K} < L \le N(p+\varepsilon)] \le N(p+\varepsilon) - \underline{K}$ for any given $\varepsilon > 0$, it follows that

$$p(1-a) = \lim_{N\uparrow\infty} \mathcal{E}[L - \underline{K} \mid \underline{K} < L \le N(p+\varepsilon)] \Pr\{\underline{K} < L \le N(p+\varepsilon)\}/N$$
$$\le [\lim_{N\uparrow\infty} N(p+\varepsilon)/N - \lim_{N\uparrow\infty} \underline{K}/N] \lim_{N\uparrow\infty} \Pr\{\underline{K} < L \le N(p+\varepsilon)\}$$
$$\le [p + \varepsilon - \lim_{N\uparrow\infty} \underline{K}/N].$$

Because this holds for any $\varepsilon > 0$, $\lim_{N\uparrow\infty} \underline{K}/N \le pa$. However, $\underline{K} \ge Npa$. I conclude that $\lim_{N\uparrow\infty} \underline{K}/N = pa$.

By definition, $s(L)L \le \underline{K}$ for all L. Additionally, note that

$$\mathcal{E}[s(L)L \mid b] \ge \mathcal{E}[\min\{\theta\underline{K}, \theta L\} \mid b] = \theta\mathcal{E}[\min\{\underline{K}, L\} \mid b] = [a/\lambda(\underline{K})]\mathcal{E}[\min\{\underline{K}, L\} \mid b] = Npa.$$

Therefore,

$$pa \le \lim_{N\uparrow\infty} \mathcal{E}[s(L)L \mid b]/N \le \lim_{N\uparrow\infty} \underline{K}/N = pa,$$

which implies the limiting expected budget usage ratio for the bounded SRS is $\lim_{N\uparrow\infty} \mathcal{E}[s(L)L \mid b]/\underline{K} = 1$.

Recall that for the proportional SRS and Poisson sampling strategies, the minimum K required is $\lceil aN \rceil$ and N, respectively. According to footnote 16, both have $\mathcal{E}[s(L)L \mid b] = Npa$. Therefore, their expected budget usage ratios for a sufficiently large N are close to p and pa, respectively.

■

Table 1 Numerical Example Illustrating a Bounded SRS Rule

Panel A: Parameter Choices

Taxpayer population size	N	=	6
Committed budget for bounded SRS rule	K	=	2
Cost per audit	c	=	2.12
Income of H taxpayer	I_{H}	=	23.17
Income of L taxpayer	I_{L}	=	9.27
Tax for an (honest) H taxpayer	T_{H}	=	11.59
Tax for an (honest) L taxpayer	T_{L}	=	3.24
Fine for a dishonest H taxpayer caught	F	=	5.42
Power-expo utility function	$u(w)=\dfrac{1-\exp[-\rho_{\mathrm{a}} w^{1-\rho_{\mathrm{r}}}]}{\rho_{\mathrm{a}}}$		
Constant absolute risk aversion (when ρ_{r} = 0)	ρ_{a}	=	0.029
Constant relative risk aversion (when ρ_{a} = 0)	ρ_{r}	=	0.269
Target audit probability	$\boldsymbol{a}$	**=**	**55%**

Panel B: Audit Probability Induced by Bounded SRS Rule

L =	0	1	2	3	4	5	6	Audit prob. induced
$s^K(L)L$	0	1	2	2	2	2	2	
$s^K(L)$	100%	100%	100%	66.7%	50%	40%	33.3%	
***q* = 0.5**								
With **conjectured *b* = 18.18%** and 1–q(1–b) = 0.59,								
Pr{$L^{N\text{-}1}$=L–1}		0.011	0.083	0.239	0.345	0.249	0.072	
$\boldsymbol{s^K(L)}$Pr{$L^{N\text{-}1}$=L–1}		0.011	0.083	0.159	0.173	0.100	0.024	**55%**
***q* = 0.61**								
With **conjectured *b* = 11.62%** and 1–q(1–b) = 0.46,								
Pr{$L^{N\text{-}1}$=L–1}		0.046	0.195	0.333	0.284	0.122	0.021	
$s^K(L)$Pr{$L^{N\text{-}1}$=L–1}		0.046	0.195	0.222	0.142	0.049	0.007	66%
For θ = 5/6,								
$\boldsymbol{\theta s^K(L)}$Pr{$L^{N\text{-}1}$=$L$–1}		0.038	0.162	0.185	0.119	0.041	0.006	**55%**

Table 2 Distributional Properties of Selected Variables

$N_H \sim \text{Bin}(q,N)$	Number of high-income taxpayers.
$L_H \sim \text{Bin}(\beta,N_H)$	Number of non-compliant high-income taxpayers, given the conjectured underreporting probability β.
$L = N - N_H + L_H$ $\sim \text{Bin}(1-q(1-\beta),N)$	Number of "low-income" tax returns received by the tax collector, given the conjectured β.
$A \sim g(A \mid L)$	Audit sample size, given the probabilistic audit sampling plan $[g(i \mid L)]_{i=0,1,\ldots,N}$ chosen by the tax collector contingent on L.
$V \sim \text{Hyp}(A, L_H, L)$	Number of "violators caught"
$L^{N-1} \sim \text{Bin}(1-q(1-\beta), N-1)$	Number of "low-income" tax returns filed by the remaining $N-1$ taxpayers, given any conjecture β on other taxpayers' under-reporting probability
$1 + L^{N-1}$	Value of L to be observed by the tax collector if a high-income taxpayer chooses to be non-compliant.
$\mathcal{E}[s^{\boldsymbol{g}}(1+L^{N-1}) \mid \beta]$	Audit probability facing a non-compliant high-income taxpayer, given any conjecture $\boldsymbol{g}$ on the tax collector's audit strategy and the conjectured β.

Note. Bin = Binomial; Hyp = Hypergeometric

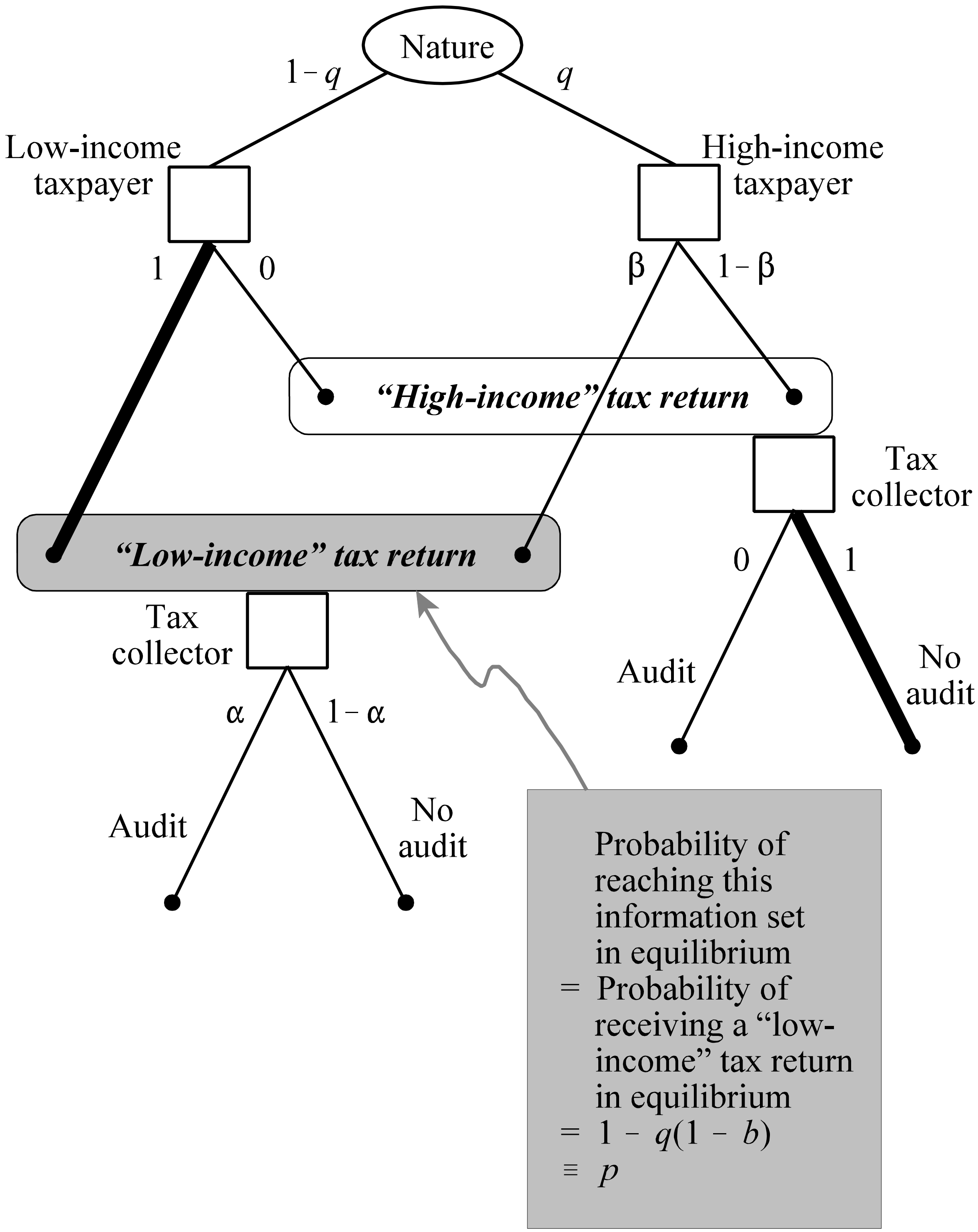

Figure 1. Moves in the Classic Tax Compliance Game

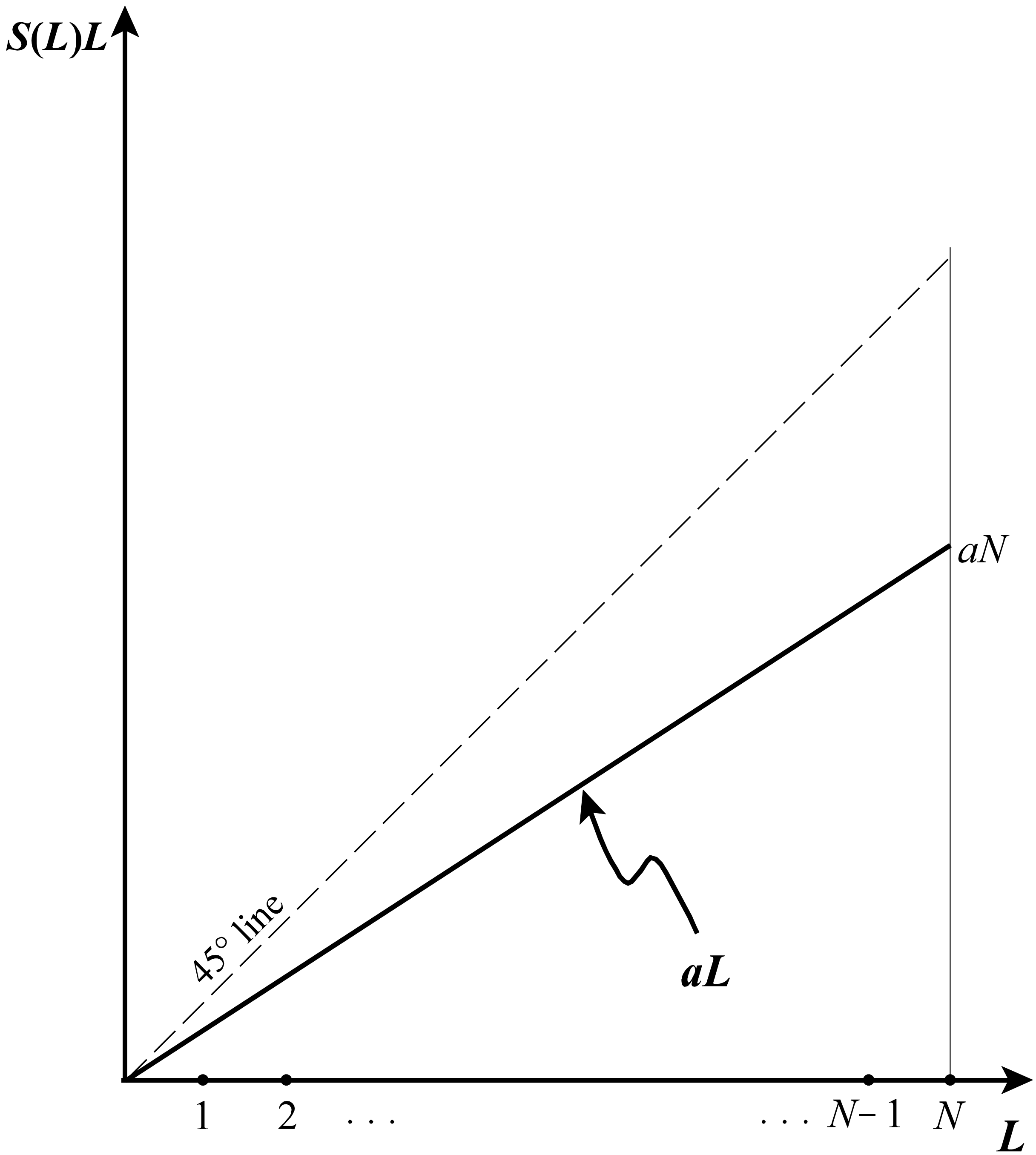

Figure 2. Proportional SRS Audit Strategy

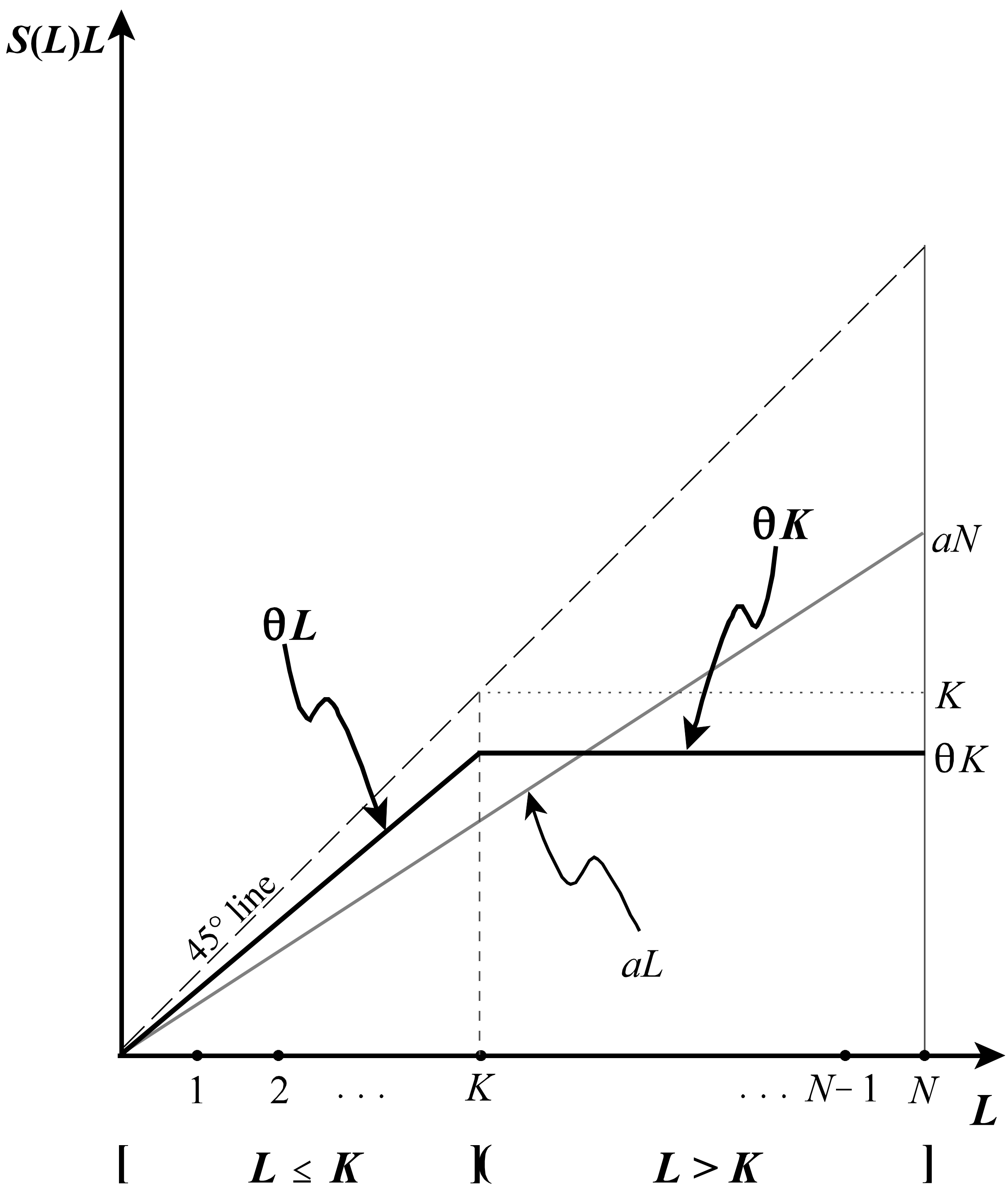

Figure 3. Bounded SRS Audit Strategy

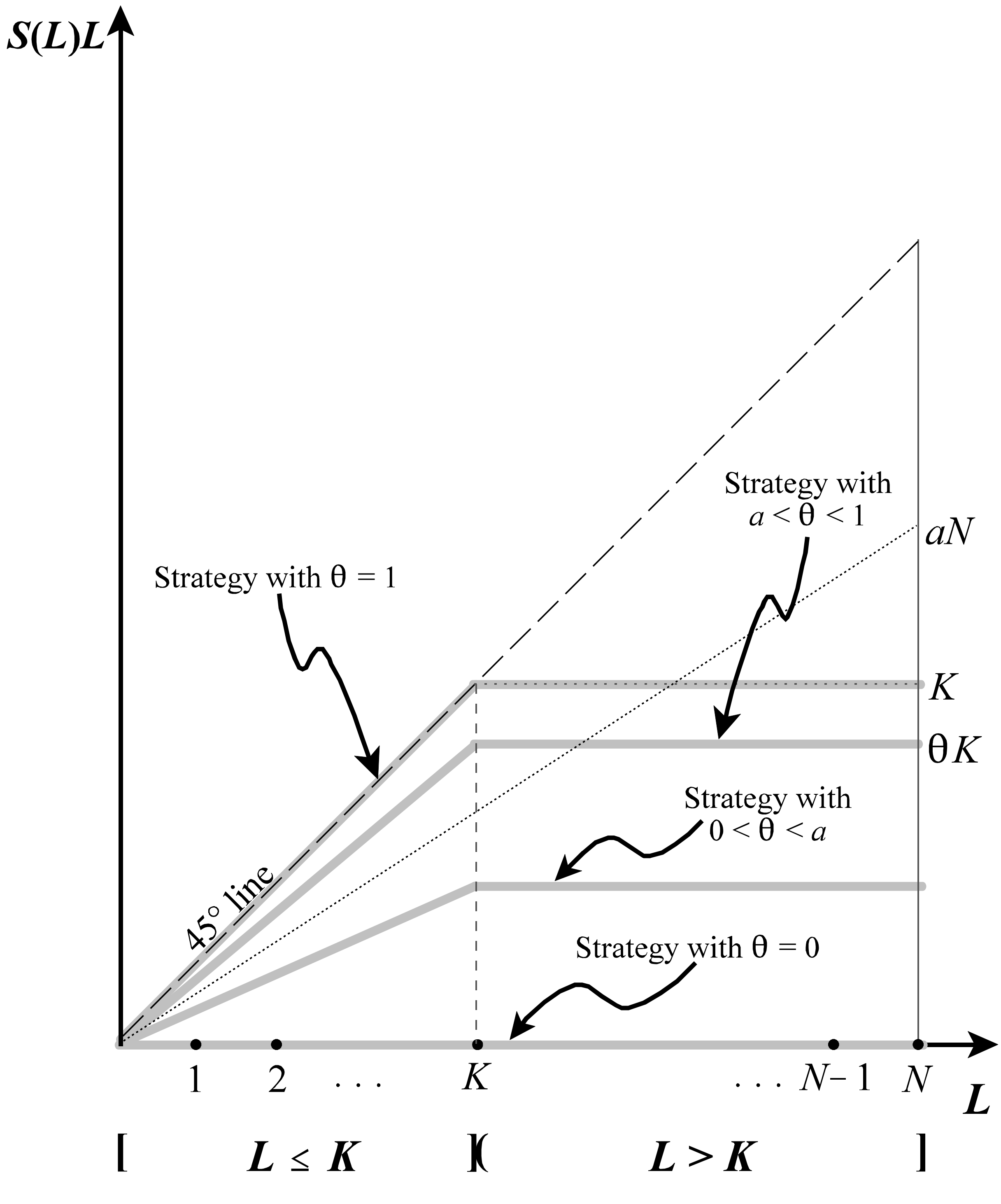

Figure 4. Bounded SRS Audit Strategies with θ Ranging from 0 to 1