Quality Cost and Failure Risk in the Choice of Single versus Multiple Sourcing

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QUALITY COST AND FAILURE RISK IN THE CHOICE OF SINGLE VERSUS MULTIPLE SOURCING

1. Introduction

This paper examines the supply base composition problem, in particular the choice of single versus multiple sourcing, from a quality cost control perspective. There have been over twenty years of interest in understanding the relative advantages of single versus multiple sourcing, beginning with Deming (1986) and Porter (1985). Many issues have been considered in the literature regarding selecting suppliers to form a supply base. For example, Weber, Current, and Benton (1991) have identified more than twenty criteria for supplier selection decisions. Among the many factors considered, three of them, namely price, delivery, and quality, are often regarded as the core to consider (Lemke et al. 2000). The emphasis of this paper is on the last, but not the least, of these major factors: *quality*.

A strand of the literature on multiple sourcing has focused on the advantage of price reduction due to more suppliers competing with each other (the *competition* advantage).[1] A second, more recent strand examines the advantage arising from the protection against supplier failures (the *protection* advantage).[2] The supplier failures that have been studied are a variety of *unreliable supply* such as late/insufficient/no delivery due to reasons like machine breakdowns, labor strikes, natural disasters, and financial defaults (e.g., Babich, Burnetas, and Ritchken 2007, Burke, Carrillo, and Vakharia 2007, Dada, Petruzzi, and Schwarz 2007, Federgruen and Yang 2008, 2009). Such failures adversely affect the buying company's production and may result in the loss of sale in the end product market. In contrast, the supplier failure studied here is about *undependable* (or even unsafe) *products* sold to the customers of the buying company, as a result of using product parts with latent defects provided by its suppliers.

I emphasize the difference between unreliable supply and undependable products. The former is caused by random yields, which have been studied extensively (see reviews by Yano and Lee 1995 and Grosfeld-Nir and Gerchak 2004). The latter is due to latent defects, which is receiving growing attention

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[1] Studies in this strand often use modeling techniques of the auction literature (e.g., Tunca and Wu 2009). See Mishra and Tadikamalla (2006) for a concise review of major results of the models in this strand.

[2] Examples in this strand are Berger, Gerstenfeld, and Zeng (2004), Berger and Zeng (2006), and Ruiz-Torres and Mahmoodi (2006, 2007). These studies rely mainly on numerical simulation analyses to obtain results, in contrast to the analytical modeling method used in this paper.

(e.g., Chao, Iravani, and Savaskan 2009). *Latent defects* are flaws or weaknesses in product items that could not be discovered by reasonable inspection prior to the sale. They later manifest as field failures in the hands of end customers. Consequences to the buying company (i.e., the manufacturer/retailer to which the supplier has sold the faulty items) are all sorts of quality costs. They include costs due to warranty repairs, product recalls, defect liability claims, reputation damage, loss of sale, and customer confidence restoration efforts (Nagar and Rajan 2001). Such costs originating from latent defects can be huge. For instance, in 2002, Ford announced that it would spend $3 billion to replace millions of Firestone tires causing customer safety concerns.

If latent defects are due to occasional independent mistakes, the impact probably is limited. Latent defects, however, can result from design flaws, systematic manufacturing faults, or the like. In that case, defects can occur in a correlated manner in product items from the same supplier.[3] As a result, voluminous amounts of product items may be affected. Some of these items with latent defects later are identified when field failures occur. Depending on the nature of the product, the consequences to end customers can be substantial or even fatal, e.g., the 34 deaths in accidents allegedly caused by runaway Toyota vehicles (Shepardson 2010). It is thus crucial to understand how the risk of such failures can be diversified and the costs of doing so. I add to the understanding of this issue by analyzing a supply base composition model for the *LUX* (*L*atent defect-*U*ndependable product-e*X*ternal failure) setting described above.

The question asked in this study is similar to Benjaafar, Elahi, and Donohue's (2007). They examine the outsourcing of a fixed demand for a service at a fixed price to a set of potential suppliers. The two competition mechanisms they compare, namely the *supplier-selection* (SS) versus the *supplier-allocation* (SA) approach, are equivalent to single versus multiple sourcing. Their analysis focuses on how the relative advantage of SS versus SA is affected by the presence/magnitude of demand-independent versus demand-dependent service costs. By contrast, the emphasis here is on the convexity of the external

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[3] An example of products with correlated latent defects is hard disk drive. It affects many aspects of modern living, from home electronic appliances like digital video recorders to large automation systems controlled by computers that have hard disk drives as massive storage devices. According to Paris and Long (2006), "[b]atch-correlated failures result from the manifestation of a common defect in most, if not all, disk drives belonging to the same production batch. They are much less frequent than

failure cost rather than the service/production cost structure.

The buyer in my model can rank suppliers by a composite score,[4] which for simplicity is represented by a supplier's quality level (i.e., a parameter affecting the defect rate). Multiple sourcing cannot be more advantageous than single sourcing unless the expected profit maximizing buyer has a "risk-averse" preference. In the model, this is due to a quadratic external failure cost representing the reputation damage suffered by the buyer, as a result of field failures of undependable products.

When ignoring any desire for risk diversification, the buyer would source only from the supplier of the highest quality because this minimizes the quality cost. However, if risk diversification is desirable, the buyer might want to admit another supplier into the supply base and thereby diversify the failure risk somewhat. Whether the buyer would continue to admit more depends on how difficult it is to maintain the overall quality level of the supply base. With more suppliers selected into the supply base, the buyer had to choose from suppliers of further lower qualities. By weighing the incremental quality cost against the incremental benefit from further risk diversification, the optimal supply base is determined.

I show that the optimal supply base has the following properties. If a supplier is selected to constitute the supply base, any supplier of higher quality must also be selected. Moreover, larger production quotas should be assigned to higher-quality suppliers. These results hold under a mild condition on the differences among suppliers' production costs. This includes equal costs as a special case but also allows unequal costs. As long as the condition is fulfilled, the suppliers' production costs do not need to have any particular order (e.g., higher-quality suppliers may or may not have higher production costs).

I also derive a necessary and sufficient condition for determining the size of the optimal supply base, provided the mild condition on suppliers' production costs holds. If the question is not about determining the exact size but merely about whether single or multiple sourcing is optimal, alternative necessary and sufficient condition can be derived without requiring any precondition to hold.

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random disk failures but can cause catastrophic data losses." Paris and Long advise that redundant copies of the same data should be stored on disks from *different batches* and, possibly, *different manufacturer*s to reduce impact of batch- correlated failures.

[4] Firms like Magna Closures require interested companies to provide detailed information about their business operations in order to become certified suppliers of the firms. Only after then would the suppliers receive invitations to submit tenders for the firm's procurement programs. (See, e.g., Magna Closures' *Supplier Quality Manual & Requirements* at http://iweb01ds.

This paper contributes to the literature on multiple sourcing by analyzing its protection against external failure risks arising from undependable products due to latent defects. The model highlights the sourcing decision as one about selecting the right *combination* of suppliers to balance between risk diversification and quality cost reduction. Prior studies instead concern mainly the *number* of suppliers that determines the level of competition.

The paper adds a new view point to the debate on single versus multiple sourcing. Deming (1986) argues that a buying company should take quality costs into consideration and choose single sourcing by selecting a high-quality supplier that minimizes *total cost* of sourcing. The result of this paper suggests that in minimizing total cost of sourcing, one might need to use multiple sourcing and include suppliers of lower quality. This can happen when the total cost of sourcing counts in the benefit from supplier failure risk diversification. The breakup of the long-term buyer-supplier relationship between Ford and Firestone demonstrates that sometimes it needs more than loyalty and trust to diversify the risk of latent defects.

This paper's closed-form characterization of the optimal supply base with "quality-driven" properties substantially reduces the computational complexity of finding the set of selected suppliers. The tractability of the model provides promising potential for using it as a building block to integrate with another model (e.g., Chao et al. 2009 or Arya and Mittendorf 2007) to study interesting questions. An example is the joint use of multiple sourcing and product recall cost sharing to reduce external failure risks. Another is the interplay between internal transfers and external procurement in controlling quality costs in a LUX setting.

In the next section, I formulate a quality-cost model of supply base composition. The analysis of the model is provided in Section 3. Further discussion on the importance of analyzing such a model for the LUX setting is given in Section 4, where related recent studies are also reviewed. Section 5 contains concluding remarks, with technical proofs and derivations relegated to the appendix. Table 1 summarizes the notations used in the paper.

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2. A Quality-cost Model of Supply Base Composition

Consider a setting with a single buyer and $n \geq 2$ available suppliers, indexed by $i \in N = \{1, 2, \ldots, n\}$. All of them are expected profit maximizers. The buyer designs a product, owns the brand, and operates as a make-to-order manufacturer. Except for a component part (e.g., the accelerator pedal of an automobile), the rest of the product is manufactured by the buyer and the related designs have been used in previous generations of the product for a long time. It is common knowledge to all parties in this model that except for the component part in concern, the designs of the rest of the product are foolproof, and the production quality of the buyer is virtually perfect. In other words, if a product failure occurs, it must be due to the failure of the component part.

The buyer plans to outsource the manufacturing of Q units of the component part to some/all of the suppliers. I suppose that the quantity $Q > 0$ is either the optimal quantity it has already figured out, or otherwise what follows is part of the thought process to figure out the optimal total procurement quantity by first finding out the minimized cost associated with a given quantity. For simplicity, I assume Q is also the quantity of the finished products supplied to end customers. Inventory holding issues are suppressed to focus on other economic forces driving the sourcing decision.

The buyer selects some suppliers to form the supply base by offering each of the n suppliers a take-it-or-leave-it procurement contract. The contract for supplier i specifies a production quota $Q_i \geq 0$ for the supplier and a procurement payment $T_i \geq 0$ for the units supplied according to the quota. Feasibility requires $\sum_{i=1}^{n} Q_i = Q$. For analytical convenience, I will treat Q_i like a real variable although it is assumed to take an integer value for ease of interpretation. The approximate solution so obtained would not be too far from the exact solution because in practice the production target Q is likely to be quite large and the number of selected suppliers tends to be small. The indivisibility of Q_i should be negligible.

Because the buyer will never offer a positive payment for nothing, contracts assigning zero quotas must also specify zero payments. Such contracts are called *null contracts*. It is analytically more convenient to have the buyer offering contracts to all suppliers, with some being null contracts, than offering contracts only to the selected suppliers. Suppliers getting null contracts are losers in the supplier competition. For convenience, I suppose suppliers will always reject null contracts. In reality, the buyer may simply ignore the non-selected suppliers. Suppliers getting "genuine" contracts will accept them if the resulting expected profits are no worse than the profits that could have been earned from their best alternative opportunities. For simplicity, these reservation profits are assumed to be zero.

Related economic studies suggest that scale economies in production inherently bias against multiple sourcing, whereas scale diseconomies bias towards it. (More precisely, the multiple sourcing arrangement analyzed in this paper is of the *split* sense. See the review of related economic studies provided in the appendix for a clarification of the different senses the term "multiple sourcing" might mean.) To eliminate such inherent biases, I assume the suppliers have constant marginal costs of production, c_i's, and no fixed costs.[5] The production cost incurred by supplier i to meet its assigned quota thus equals c_iQ_i. For simplicity, I normalize the buyer's fixed and marginal costs of production to zero.

The design of the component part is newly introduced by the buyer to replace an expensive (but infallible) design used in the past. The new design is much more affordable than the old one and believed to be very robust under a wide range of operating conditions. Still, it is not infallible. Whether a field failure of the component part will occur depends on whether it has a latent defect. By definition, such a defect cannot be discovered by inspection prior to sale. In this model, it is assumed that if a latent defect exists in a component part, it will surely reveal itself through a field failure. The chance of having a defective part depends on the buyer's design quality level, as well as the production quality of the supplier that manufactures the part.

Specifically, let D_i denote the amount of defective parts in the Q_i units of parts produced by supplier i. I assume defects occur in a manner following stochastically proportional yield loss, i.e., $D_i = R_i\delta_iQ_i$. The random variables R_i's are independently and identically distributed with mean $E(R_i) = \mu$, where $0 < \mu \leq \bar{\mu} << 1$, and variance $\mathrm{var}(R_i) = \sigma^2 > 0$. Intuitively, $R_i\delta_i$ may be referred to as the *random yield loss*, although for convenience I sometimes use this to refer to R_i alone.

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[5] In accounting, constant marginal cost is arguably a more widely accepted assumption on cost behavior than increasing or decreasing marginal cost. For example, in product costing, the variable cost per unit is typically assumed to be constant within the relevant range of production. Zero fixed cost, though uncommon, is a justifiable assumption if accepting the procurement

Besides the variable μ that characterizes the buyer's design quality level, the random yield loss also depends on the parameter δ_i, where $0 < \delta_i \leq 1$. I refer to $1-\delta_i$ as the *quality-based scoring index* of supplier i, or simply its *quality* level. For analytical convenience, assume δ_i's are distinct. Without loss of generality, I suppose $\delta_1 < \delta_2 < \ldots < \delta_n$ so that supplier 1 has the highest quality level, and other suppliers are ordered accordingly.

Suppose the latent defect described above is the only sort of defects that could occur in the buyer's or a supplier's production process. Thus, the total quantity ordered by the buyer is always the same as the total quantity delivered by the suppliers. The buyer sells all Q units of finished product to end customers. Denote by $D = \sum_{i=1}^{n} D_i$ the total amount of defective products sold. I assume that each customer with a defective product will experience a field failure and take the product back for warranty repair. Knowing that the product has a defective part, the buyer finds it in its best interest to replace the part by another based on the old design, which is expensive but infallible.

Suppose that all warranty related costs, net of the salvage values of the replaced parts, are proportional to the amount of products returned for repair, i.e., ωD, where $\omega > 0$. Moreover, the defective products will lead to customer dissatisfaction and eventually some additional external failure costs borne by the buyer. In particular, I suppose that such costs are primarily due to reputation damage. I argue that it is increasingly more likely to catch the attention of mass media as D grows, and the resulting reputation damage is increasingly more detrimental. This feature is captured by the analytically tractable assumption of a quadratic *other external failure cost* function: $C_E(D) = c_E D^2$, where $c_E > 0$.

Summing up this and the warranty related costs gives the total external failure cost $\omega D + c_E D^2$. Because the increasing marginal cost of C_E is with respect to D, not its individual constituents D_i's, the quadratic functional form does not by itself favors multiple sourcing.[6] To keep the model simple, the suppliers are assumed to have no external failure cost besides (possibly) sharing the buyer's through a

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contract offer is an incremental production decision falling within the relevant range of production of a supplier.

[6] To see this, imagine the hypothetical case where the proportions of D_i's in D could be directly controlled by the buyer. Then it is clear that as long as the total, D, remained constant, there could not be any gain or loss from shifting among the D_i's. Even though D_i's actually are random variables, multiple sourcing cannot create any benefit unless the randomness of D_i's

contingent procurement contract.

To see how the allocation of the production quotas to the suppliers may affect the external failure costs to the buyer, one can examine the expected value of $\omega D + c_E D^2$ below (see the appendix for the derivation):

$$E[\omega D + c_E D^2] = \omega\mu[\textstyle\sum_{i=1}^{n} \delta_i Q_i] + c_E\mu^2[\sum_{i=1}^{n} \delta_i Q_i]^2 + c_E\sigma^2[\sum_{i=1}^{n} \delta_i^2 Q_i^2].$$

Interestingly, although the expected external failure cost is a second-order polynomial of Q_i's, sharing the production target Q among the suppliers cannot reduce the cost unless $\sigma^2 > 0$. If instead the "random yield losses" R_i's are deterministic, the third term in $E[\omega D + c_E D^2]$ would vanish. In that case, assigning all the Q to the supplier with the lowest δ_i would minimize the expected external failure costs.

Summarized below is the event sequence of the model:

1. The buyer outsources the production of a component part of its product by offering each supplier a procurement contract (Q_i, T_i).
2. Each supplier decides whether to accept the buyer's offer. If supplier i rejects the offer, it will exit from the game and earn zero profit; if no supplier accepts the offer, the game terminates.
3. Suppliers who have accepted the buyer's offers proceed to the production stage. Production costs, c_iQ_i's, are borne by the suppliers.
4. The contracted amounts of parts, Q_i's, are delivered to the buyer. Out of the amount Q_i, $D_i = R_i\delta_iQ_i$ are defective parts. (The values of D_i's are not observable to any parties until later.)
5. The buyer uses the parts to manufacture the finished products and sells them to end customers.
6. Customers who purchased the $D = \sum_{i=1}^{n} D_i$ units of defective products eventually experience field failures and return the products for warranty repair. The values of D_i's become known to the buyer and suppliers. The buyer incurs warranty related expenses of ωD and suffers reputation damage equivalent to a dollar cost of $C_E(D) = c_E D^2$.
7. The buyer makes payments T_i's to the suppliers.

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resulting from the supplier failure risk is diversifiable. This last point will become clear shortly below.

In principle, the procurement payment T_i to supplier i can be contingent on the amount of defective parts, D_i, manufactured by the supplier. However, given that there is no asymmetric information in the model and the suppliers are risk-neutral, whether T_i can be contingent on D_i is not crucial. For notational simplicity, I therefore do not make an explicit distinction between T_i and $T_i(D_i)$, with the former understood to be a random variable derived from D_i. Consequently, any expectation taken on T_i is an expectation taken on the underlying D_i upon which T_i may depend.

3. Analysis of the Model

Because this is a complete-information model, the buyer and suppliers are supposed to understand fully the model structure and have complete knowledge of all the parameters involved, e.g., the buyer's design quality level characterized by μ. Let $E[T_i|Q_i]$ denote the expected value of the procurement payment to supplier i given that Q_i is the assigned production quota. The buyer's problem is to choose procurement contracts (Q_i,T_i)'s to minimize the expected total cost below:

$$\sum_{i=1}^{n} E[T_i|Q_i] + \omega\mu[\sum_{i=1}^{n} \delta_i Q_i] + c_E\mu^2[\sum_{i=1}^{n} \delta_i Q_i]^2 + c_E\sigma^2[\sum_{i=1}^{n} \delta_i^2 Q_i^2].$$

If supplier i accepts the buyer's offer, (Q_i,T_i), its expected profit will be

$$E[T_i|Q_i] - c_i Q_i.$$

To induce participation, the contract must give a non-negative expected profit to the supplier, which means $E[T_i|Q_i] \geq c_i Q_i$. In addition, the contract must specify production quotas that sum to the required procurement quantity, i.e., $\sum_{i=1}^{n} Q_i = Q$.

The optimization program below summarizes the buyer's sourcing problem:

[SB] $$\min_{Q_i, T_i(\cdot)} \sum_{i=1}^{n} E[T_i|Q_i] + \omega\mu[\sum_{i=1}^{n} \delta_i Q_i] + c_E\mu^2[\sum_{i=1}^{n} \delta_i Q_i]^2 + c_E\sigma^2[\sum_{i=1}^{n} \delta_i^2 Q_i^2]$$

subject to

PC: $E[T_i|Q_i] \geq c_i Q_i$ for all $i = 1,\ldots,n$,

QC: $\sum_{i=1}^{n} Q_i = Q$.

Constraints PC and QC are referred to as the *participation* and *quota constraints*, respectively. Clearly, the participative constraint must be binding at optimum. This implies $\sum_{i=1}^{n} E[T_i|Q_i] = \sum_{i=1}^{n} c_i Q_i$. Substituting this back to the buyer's objective function, the optimization problem becomes one concerning the choice of production quotas Q_i's only. This choice indirectly determines the *supply base* $B = \{ i \in N \mid Q_i > 0 \}$, i.e., the set of suppliers selected by the buyer. Let b denote the *size* of the supply base, i.e., the number of selected suppliers. The type of a supply base, B, is said to be *single sourcing* if $b = 1$ and *multiple sourcing* if $b \geq 2$. In particular, if $b = 2$, B is also said to be of the *dual sourcing* type.

To simplify the expression of the buyer's expected total cost, I define the following notations:

$$\eta \equiv (\mu/\sigma)^2$$

$$s_i \equiv (c_i + \omega\mu\delta_i)/c_E\sigma^2$$

The parameter η is the *squared standardized mean* of the "random yield loss" R_i, which means η^{-1} is the *squared coefficient of variation*. The parameter s_i is a ratio representing the relative unimportance of the quadratic other external failure cost, characterized by c_E, in constituting the buyer's expected total cost. With these notations, the buyer's expected total cost can be expressed as

$$c_E\sigma^2[\sum_{i=1}^{n} s_i Q_i + \eta(\sum_{i=1}^{n} \delta_i Q_i)^2 + \sum_{i=1}^{n} \delta_i^2 Q_i^2].$$

Whether multiple sourcing has advantages over single sourcing, or the other way around, depends on the optimal choice of Q_i's and the resulting size of B. The following result tells us when the protection advantage of multiple sourcing may fail to exist.

PROPOSITION 1 (CONDITIONS FOR NON-EXISTENCE OF PROTECTION ADVANTAGE OF MULTIPLE SOURCING): *Suppose the ascending ranking of the suppliers based on s_i also has supplier 1 ranked highest. Or, alternatively, suppose that for any distinct j and k with $(\delta_k - \delta_j)(s_k - s_j) \leq 0$,*

$$(s_j - s_k)/(\delta_k - \delta_j) < 2\eta\delta_1 Q.$$

Then multiple sourcing has no advantage over single sourcing if one of the following holds:

(a) *The variance of the "random yield loss" is negligible, i.e., $\sigma^2 \to 0$;*

(b) *The marginal other external failure cost is negligible, i.e., $c_E \to 0$.*

The result of this proposition needs one of two preconditions: either that both the s_i-based and δ_i-based rankings have supplier 1 ranked highest, or that whenever they differ in ranking suppliers j and k, the cardinal difference $s_j - s_k$, relative to $\delta_k - \delta_j$, is not too large. When one of these preconditions holds, the reason for multiple sourcing to be advantageous comes solely from the protection against supplier failure risk due to latent defects. Multiple sourcing becomes unattractive if there is little risk to protect against, or the benefit (i.e., external failure cost saved) from the protection is tiny.

Two model elements contributing to the protection advantage of multiple sourcing are highlighted by the proposition. The first is about the nature of the supplier failure risk. If the "risk" is not due to variation in the yield but merely about not knowing which unit is defective (e.g., the fully predictable yields assumed in Burke et al. 2007), there might not be room for risk diversification by multiple sourcing.

The second element is related to the nature of the external failure costs considered here. With genuine random yields, the quadratic other external failure cost induces a desire for spreading the supplier failure risk by multiple sourcing. If the marginal other external failure cost is negligible, the buyer will remain "risk-neutral" to the risk. The protection advantage of multiple sourcing therefore cannot exist.

Below I will characterize the unique optimal production quota allocation for the buyer's sourcing problem, identify some intuitive features of the allocation, and derive necessary and sufficient conditions for determining the exact size of the optimal supply base and whether single or multiple sourcing is optimal. These results are stated in terms of the *quality-adjusted cost-based scoring index* defined as follows:

$$S_i\,(W^*) \equiv \left[\frac{c_i + (\omega + 2c_{\mathrm{E}}\mu\; W^*)\mu\delta_i}{c_{\mathrm{E}}\sigma^2}\right].$$

It will be clear shortly that the μW^* in the index is simply the expected number of external failures given the optimally allocated production quotas.

If suppliers' qualities are nearly identical (i.e., δ_i's are almost the same), the ranking by $S_i(W^*)$ is not much different from that by c_i. Alternatively, if production costs are equal, $S_i(W^*)$ and δ_i give the same ranking. Even when the costs are unequal, the ranking by $S_i(W^*)$ and by δ_i can still be the same, provided c_i's are "not too unequal." Specifically, this means

$$\max_{i\in\{2,\dots,n\}}\left(\frac{c_{i-1} - c_i}{\delta_i - \delta_{i-1}}\right) \le (\omega + 2c_{\mathrm{E}}\mu Q)\mu\delta_1 .$$

In words, the condition requires that the cost saving from using a lower-quality supplier is not too attractive given the loss in quality.

The first major result below characterizes the optimal quota allocation without imposing the condition of "not too unequal" costs. Subsequently, it is added to put more structure on the optimal quota allocation.

PROPOSITION 2 (OPTIMAL QUOTA ALLOCATION): *A quantity vector* $\mathbf{Q}^* = (Q_i^*)_{i\in N}$ *is the unique optimal quota allocation for the buyer's sourcing problem if and only if for some* $\theta^* > 0$*, the following marginal conditions are satisfied:*

$$Q_i^* \geq \left[\frac{\theta^* - S_i(W^*)}{2\delta_i^2}\right] \qquad \text{for all } i \in N$$

with the equality holding for all i's in the supply base $B^* \equiv \{\, i\in N \mid Q_i^* > 0 \,\}$, *where*

$$S_i\ (W^*) \equiv \left[\frac{c_i + (\omega + 2c_{\mathrm{E}}\mu\ W^*)\mu\delta_i}{c_{\mathrm{E}}\sigma^2}\right]$$

is a quality-adjusted cost-based scoring index and θ^* *and* W^* *are given by the following formulas*:

$$\theta^* \quad = \quad \frac{(\eta^{-1}+b^*)[2Q+\sum_{B^*}(s_l/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)][\sum_{B^*}(s_l/\delta_l)]}{(\eta^{-1}+b^*)[\sum_{B^*}(1/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)]^2} \tag{1}$$

$$W^* \quad = \quad \frac{[\sum_{B^*}(1/\delta_l)][2Q+\sum_{B^*}(s_l/\delta_l^2)] - [\sum_{B^*}(1/\delta_l^2)][\sum_{B^*}(s_l/\delta_l)]}{2\eta[\,(\eta^{-1}+b^*)[\sum_{B^*}(1/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)]^2\,]}\ , \tag{2}$$

with $b^* \equiv |B^*|$ *denoting the size of the supply base,* $\eta \equiv (\mu/\sigma)^2$ *denoting the squared standardized mean of the "random yield loss"* R_i*, and* $s_i \equiv (c_i+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2$. *Moreover,* $W^* = \sum_{B^*}\delta_l Q_l^* = \sum_{i=1}^{n}\delta_i Q_i^*$.

This proposition provides a closed-form characterization of the unique optimal quota allocation $\mathbf{Q}^*$. Once the supply base B^* is determined, the optimal quota for a selected supplier i can be computed with the simple formula

$$Q_i^* = \left[\frac{\theta^* - S_i(W^*)}{2\delta_i^2}\right]$$

whose key constituents, θ^* *and* W^*, are given by another two formulas specified in the proposition. Although the procedure is straightforward, determining the optimal combination of suppliers to constitute the supply base can be prohibitively complex. This is especially so when the number of available suppliers is large. In a different but related setting, Federgruen and Wang (2008) show that a similar combinatorial optimization problem of supplier selection is NP-complete.

However, suppose that the ranking of the suppliers by the *quality-adjusted cost-based scoring index* $S_i(W^*) = [c_i + (\omega+2c_{\mathrm{E}}\mu W^*)\mu\delta_i]/c_{\mathrm{E}}\sigma^2$ is the same as that by δ_i. This would be the case if the differences among the costs c_i's are sufficiently small and the marginal other external failure cost c_{E}, or the production target Q and hence $W^* = \sum_{i=1}^{n} \delta_i Q_i^*$, is sufficiently large. Under such circumstances, the weight attached to the second component of $S_i(W^*)$ will be large enough to let δ_i dominate this scoring index. Consequently, the unique optimal quota allocation $\mathbf{Q}^*$ will have some simple, intuitive properties.

The first property is a positive relation between the quality of a selected supplier and the quota assigned to it. That is to say, the higher the quality of a supplier, the larger the quota assigned to it. As a result, the supplier selection is *quality-driven*: if a supplier is selected into the supply base, any suppliers of higher quality (i.e., with lower δ_i's) will also be selected. These two intuitive properties are formally stated as the result below.

PROPOSITION 3 (QUALITY-DRIVEN SUPPLY BASE AND LARGER QUOTAS FOR HIGHER-QUALITY SUPPLIERS): *Suppose* $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \leq \omega\mu + 2\delta_1 Q c_{\mathrm{E}}\mu^2$. *Then the supply base* B^* *of the optimal quota allocation* $\mathbf{Q}^*$ *is quality-driven, i.e.,* $B^* = \{1, 2, \ldots, b^*\}$. *Additionally, higher-quality suppliers will be assigned larger quotas, i.e.,* $Q_j^* > Q_{j+1}^*$ *for all* $j \in B^*\backslash\{b^*\}$.

Despite no fixed costs for selecting more suppliers, expanding the supply base can be costly because it means using suppliers of lower quality than the incumbent ones. The increase in this cost as lower-quality suppliers are included into the supply base eventually may limit its size. An interesting question to ask is when it will stop at the size of one (i.e., single sourcing is optimal) and when it will stop at a size larger than one (i.e., multiple sourcing is optimal). The following results shed some light on this question.

PROPOSITION 4 (SUFFICIENT AND NECESSARY CONDITION FOR DETERMINING THE SIZE OF THE OPTIMAL SUPPLY BASE): *Suppose* $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \leq \omega\mu + 2\delta_1 Q c_E \mu^2$. *Then the size of the (smallest) optimal supply base is j (i.e.,* $b^* = j$*), where* $j \in N\backslash\{n\}$*, if and only if there exist positive* θ_j *and* W_j *defined by formulas (1) and (2) with* B^* *substituted by* $B_j \equiv \{1, \ldots, j\}$ *such that*

$$[\theta_j - S_j(W_j)]/2\delta_j^2 > 0 \geq [\theta_j - S_{j+1}(W_j)]/2\delta_{j+1}^2,$$

where $S_i(W) \equiv [c_i + (\omega+2c_E\mu W^*)\mu\delta_i]/c_E\sigma^2$*. If the condition above is not satisfied by any* $j < n$, $b^* = n$.

This proposition provides a characterization of b^*, the size of the optimal supply base. Determining b^* can be rather complex owing to the combinatorial nature of the supplier selection problem. However, if the precondition of the proposition holds, i.e., the differences between consecutive c_i's are "not too large," then the problem of determining the size of the optimal supply base can be reduced to simply comparing the n quality-driven supply bases, i.e., $B_j \equiv \{1, \ldots, j\}$ for $j \in N$. This comparison only requires solving for each j a linear equation system with two unknowns, i.e., θ_j *and* W_j, and then search for the j with the lowest positive value of $[\theta_j - S_j(W_j)]/2\delta_j^2$. Such a problem takes much less time to solve than the original problem.

Suppose one only needs to determine whether single sourcing or multiple sourcing is optimal, without actually identifying the size of the optimal supply base for the latter case. Then the problem can be simplified even without a precondition. Why? Proposition 2 already provides the necessary and sufficient condition for checking whether any given supply base B is optimal. For the case of single sourcing, there are only n such candidate supply bases to check. If none of them is optimal, the optimal arrangement must be multiple sourcing. To check whether a singleton supply base is optimal, it does not require the remaining unselected suppliers to be lined up in certain orders. One only needs to ensure that the buyer cannot be better off by shifting some quota away from a candidate single-sourcing supplier. This result is the next proposition.

PROPOSITION 5 (SUFFICIENT AND NECESSARY CONDITION FOR SINGLE SOURCING TO BE OPTIMAL): *Let* $\theta^h \equiv s_h + 2(1+\eta)Q\delta_h^2$ *and* $W^h \equiv \delta_h Q$ *for all* $h \in N$*. Then single sourcing is optimal if and only if there exists* $h \in N$ *such that*

$$\max_{i\in N\backslash\{h\}} [(\theta^h - S_i(W^h))/2\delta_i^2] \leq 0,$$

where $S_i(W) \equiv [c_i + (\omega + 2c_E\mu W^*)\mu\delta_i]/c_E\sigma^2 = s_i + 2\eta W\delta_i$ and $s_i \equiv (c_i + \omega\mu\delta_i)/c_E\sigma^2$. *The* h *satisfying the condition above is the only selected supplier of the single-sourcing supply base.*

In the necessary and sufficient condition of this proposition, there is no counterpart of the $[\theta_j - S_j(W_j)]/2\delta_j^2 > 0$ required in Proposition 4. The reason is that such a requirement is automatically satisfied for the case of single sourcing. Using the definitions of θ^h and W^h, it is easy to verify that $[\theta^h - S_h(W^h)]/2\delta_h^2 = Q > 0$, regardless of the $h \in N$ in concern. The requirement of $0 \geq [\theta_j - S_{j+1}(W_j)]/2\delta_{j+1}^2$ in Proposition 4 as well as its precondition $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \leq \omega\mu + 2\delta_1 Q c_E \mu^2$ are substituted by the condition that $\max_{i\in N\setminus\{h\}} [(\theta^h - S_i(W^h))/2\delta_i^2] \leq 0$ for some h. While this looks more complicated, it does not impose any ordering on the c_i's or restrictions on their differences. In terms of computational complexity, checking the condition requires calculating $n-1$ values of $[(\theta^h - S_i(W^h))/2\delta_i^2]$ for each $h \in N$. This amounts to a total of $n(n-1)$ calculations, a more complicated task but still manageable within a reasonable time.

4. Discussions: Importance of Quality Costs and Related Recent Studies

Quality, in particular product safety, is an important factor to consider in supplier selection decisions. Supporting this view, 78% of the senior executives participating in a survey answered that product safety is among the greatest risks in relation to the integrity of their supply chains. In addition, 68% of them cited quality as the main risk of global sourcing (PricewaterhouseCoopers 2008).

Recently, product safety concerns related to global sourcing have hit the headlines repeatedly. The most noticeable cases are recalls of unsafe products manufactured by suppliers in China (more details given in the appendix). For instance, Mattel recalled 436,000 Chinese-made toy cars covered with lead paint in 2007 (Story and Barboza 2007). Other recent recalls for unsafe products include a drug, namely heparin, and dairy products. In both cases, the products were tainted by contaminants structurally so similar to the real ingredients that they could not have been distinguished by routine tests. Such latent defects, like the use of lead paint, are flaws affecting voluminous amounts of products and yet unforeseeable or undiscoverable at the time of buying from the suppliers. By the time the flaws are noticed, the defective products have reached the hands of end customers, or may even have caused irreversible damages. Quality costs to the buying companies of such defective products are no doubt an essential factor to consider in forming the supply base.

Quality problems, however, are not exclusive to sourcing from low-cost countries like China. Alongside with the recall for lead-paint toys, Mattel also recalled 18.2 million magnetized toys made in China following Mattel's design specification. In other words, any high-quality supplier would have made these unsafe toys designed by Mattel, the world's largest toy company.

Other well-known companies in the western world have also recalled defective products for safety reasons. For instance, in 2001 Bridgestone/Firestone recalled 6.5 million tires that seemed to have an unusually high risk of tread failures. Tires with this defect were linked to crashes killing over 250 people and causing more than 3,000 serious injuries. Many of the tires were installed on Ford Motor's vehicles because Firestone is a long-term major supplier of Ford. To restore customer confidence, Ford later announced the replacement of about 13 million Firestone tires installed on Ford vehicles by non-Firestone brands. This move ended the nearly century-long buyer-supplier relationship between Ford and Firestone.

Regardless of lower-cost suppliers in China or well-known western suppliers, latent defects may exist in their products. The quality costs to the buying companies are enormous. Mattel estimated that the cost of recalling 1.5 million toys coated with toxic levels of lead paint could amount $30 million (Barboza 2007). Ford expected to spend $3 billion in a nine-month program to replace millions of Firestone tires causing customer safety concerns (Bradsher 2001, Hakim 2004). Besides the loss of sale and other quality costs that can be estimated, damage claims in defect liability lawsuits are difficult to assess.

Despite the importance of the product safety issue arising from latent defects, models analyzing the protection advantage of multiple sourcing have concentrated on supplier failures due to unreliably supply. In this paper, I use a latent-defect model with non-linear external failure costs to capture the sort of supplier failure risk relevant to the product safety issues discussed above. To highlight the benefit from risk diversification as a reason for multiple sourcing, the model assumes away other reasons such as capacity constraints, cost information asymmetry, etc. Moreover, certainty in lead time and delivery is assumed to avoid overlapping with prior studies' emphases.[7] To focus on the choice of single versus multiple sourcing, the model also abstracts away from other aspects of supply chain management already extensively studied in the literature, such as coordination for information sharing. (See Kouvelis, Chambers, and Wang 2006 for a review of the supply chain management literature and Cachon 2003 for a review specifically on supply chain coordination with incentive contracts.)

I take the perspective of viewing the choice of single versus multiple sourcing as a supply base composition problem. The question asked is about what combination of suppliers can diversify the risk of latent defects most efficiently, in terms of the incremental quality cost to pay as a sacrifice. Instead of formulating a very general model that requires combinatorial mathematical techniques to solve, I structure the model in a tractable stylized fashion, yet rich enough to capture the fundamental economic tradeoff.[8]

Recently, Federgruen and Yang (2008) has examined a general setting of the supplier selection problem that also emphasizes the optimal combination of suppliers. They show that this is an NP-complete combinatorial optimization problem. Their focus is to develop an accurate approximation method to overcome the computational complexity of the problem. Dada et al. (2007) and Federgruen and Yang (2009) have also considered similar general settings, with a focus on characterizing and solving the supplier selection problem with random yields and uncertain demand.

In contrast, I consider a simple setting with no random yields nor uncertain demand but only latent defects. My analysis focuses on non-linear external failure costs leading to a desire for risk diversification. Complementing prior studies focusing on unreliable supply due to random yields, this model emphasizes the linkage between supplier selection and external failure costs. The model allows a closed-form

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[7] Reasons for favoring single or multiple sourcing that have been studied include supplier capacity constraints, saving in outgoing order/incoming inspection/transportation costs, saving in inventory holding costs by shortening the delivery lead time, quantity discounts due to production scale economies, saving in purchasing costs by supplier competition (with or without information asymmetry), and encouraging investment by suppliers to reduce production costs or to improve product quality. These reasons have been examined extensively in economic and operations research/management science (OR/MS) studies. Concise reviews of related OR/MS studies can be found in Berger and Zeng (2006) and Mishra and Tadikamalla (2006). A review of related studies in economics is provided in the appendix.

[8] Other studies in the quality cost reduction literature have focused on incentive contracting related to a variety of quality improvement arrangements like vender certification, incoming inspection, and product recall/warranty cost sharing (e.g., Balachandran and Radhakrishnan 2005, Hwang et al. 2006, Chao et al. 2009, Baiman et al. 2000, 2001). These models have only one supplier without considering multiple sourcing and supplier failure risk diversification.

characterization of the optimal quota allocation through which the fundamental economic driving forces can be clearly seen.

5. Concluding Remarks

The model formulated in this paper is simple. Despite this, the fundamental economic intuition captured by the stylized model appears to be rather general and applicable to some other contexts. For example, an analogy can be drawn between suppliers in a supply base and stocks in a financial portfolio. It is difficult to form a fully diversified portfolio without degrading the mean return. One reason is that there is only a limited choice of high-quality stocks that may be included in a portfolio given the incompleteness of financial markets in reality. This tension between forming a diversified portfolio and maintaining its mean return is similar to the economic tradeoff highlighted in this paper concerning supply base composition.

There are several interesting directions for extending the model here. A possible extension is the buyer's endogenous choice of the total procurement quantity. Another extension is to incorporate the buyer's quality improvement effort to raise the design quality level. A third extension might involve asymmetric information about suppliers' quality levels and an analysis of the optimal procurement contracts and their relations with the optimal choices of quality improvement effort and total procurement quantity. Given the limited space here, these interesting extensions are left for future research.

Quality cost concepts have been introduced into managerial accounting textbooks for some years (e.g., Horngren, Foster, and Datar 1994, Garrison, Noreen, and Brewer 2008). Although attempts have been made to investigate TQM and JIT operations (e.g., Wruck and Jensen 1994, Alles, Datar, and Lambert 1995, Ittner and Larcker 1995, Cremer 1995, and Barron and Gjerde 1996), there remain a lot to explore in relation to quality costs. This paper represents another step towards this direction with regard to quality cost considerations in the supply base composition decision.

Appendix

DERIVATION OF THE EXPECTED TOTAL EXTERNAL FAILURE COST: To derive $E[\omega D + c_E D^2]$, first note that $E(D^2) = \text{var}(D) + E(D)^2$. Hence,

$$E[\omega D + c_E D^2]$$

$$= \omega E[D] + c_E E[D^2]$$

$$= \omega E[D] + c_E E[D]^2 + c_E \text{var}[D]$$

$$= \omega E[\textstyle\sum_{i=1}^{n} D_i] + c_E(E[\sum_{i=1}^{n} D_i])^2 + c_E \text{var}[\sum_{i=1}^{n} D_i]$$

$$= \omega \textstyle\sum_{i=1}^{n} E[D_i] + c_E(\sum_{i=1}^{n} E[D_i])^2 + c_E \sum_{i=1}^{n} \text{var}[D_i]$$

$$= \omega \textstyle\sum_{i=1}^{n} \mu\delta_i Q_i + c_E[\sum_{i=1}^{n} \mu\delta_i Q_i]^2 + c_E \sum_{i=1}^{n} \sigma^2(\delta_i Q_i)^2$$

$$= \omega\mu[\textstyle\sum_{i=1}^{n} \delta_i Q_i] + c_E\mu^2[\sum_{i=1}^{n} \delta_i Q_i]^2 + c_E\sigma^2[\sum_{i=1}^{n} \delta_i^2 Q_i^2].$$

PROOF OF PROPOSITION 1 (CONDITIONS FOR NON-EXISTENCE OF PROTECTION ADVANTAGE OF MULTIPLE SOURCING): When one of the two conditions holds, i.e., either $\sigma^2 \to 0$ or $c_E \to 0$, the buyer's expected total cost becomes $c_E\mu^2[\sum_{i=1}^{n} \Delta_i Q_i + (\sum_{i=1}^{n} \delta_i Q_i)^2]$ or simply $\sum_{i=1}^{n} (c_i+\omega\mu\delta_i)Q_i$, where $\Delta_i \equiv (c_i+\omega\mu\delta_i)/c_E\mu^2$. Suppose the ascending ranking of the suppliers based on Δ_i's also has supplier 1 ranked highest. Then obviously setting $Q_1 = Q$ minimizes $\sum_{i=1}^{n} \delta_i Q_i$ as well as $\sum_{i=1}^{n} \Delta_i Q_i$ individually. Consequently, the expected total cost must also be minimized when $Q_1 = Q$. Thus, multiple sourcing cannot be better than single sourcing.

Alternatively, suppose that for any distinct j and k with $(\delta_k - \delta_j)(s_k - s_j) \leq 0$, $(s_j - s_k)/(\delta_k - \delta_j) < 2\eta\delta_1 Q$. Then if multiple sourcing is better than single sourcing, the supply base must not contain any j and n with the property above. Otherwise, assuming without loss of generality that $\delta_j < \delta_k$, I can rearrange the allocation by shifting some amount of Q_k to Q_j and thereby reducing the sum $\sum_{i=1}^{n} \Delta_i Q_i + (\sum_{i=1}^{n} \delta_i Q_i)^2$ in the expected total cost.

To see this, simply differentiate the sum with respect to Q_i to get the derivative $\Delta_i + 2\delta_i(\sum_{i=1}^{n} \delta_i Q_i)$.

Note that $s_i \equiv (c_i+\omega\mu\delta_i)/c_E\sigma^2 = \eta\Delta_i$. So for $\delta_j < \delta_k$, $(s_j-s_k)/(\delta_k-\delta_j) < 2\eta\delta_1 Q$ implies $(\Delta_j-\Delta_k) < 2(\delta_k-\delta_j)\delta_1 Q < 2(\delta_k-\delta_j)(\sum_{i=1}^n \delta_i Q_i)$. Hence,

$$\Delta_j + 2\delta_j(\textstyle\sum_{i=1}^n \delta_i Q_i) < \Delta_k + 2\delta_k(\sum_{i=1}^n \delta_i Q_i),$$

implying that shifting some amount of Q_k to Q_j will reduce the expected total cost further. This leads to the conclusion that any multiple-sourcing supply base must include only suppliers with δ_i's and s_i's showing exactly the same ranking.

However, with such a ranking of the selected suppliers, the expected total cost can be minimized with Q assigned solely to the supplier ranked highest in the supply base, i.e., the one with the lowest baseline defect rate among the suppliers selected. This contradicts the initial supposition that multiple sourcing can be better than single sourcing if $(s_j - s_k)/(\delta_k - \delta_j) < 2\delta_1 Q$ for any distinct j and k with $(\delta_k - \delta_j)(s_k - s_j) \leq 0$. *Q.E.D.*

PROOF OF PROPOSITION 2 (OPTIMAL QUOTA ALLOCATION): The existence of an optimal quota allocation is guaranteed because any feasible allocation must be from the closed and bounded domain $[0,Q]^n$ and the objective function and constraints of the optimization problem are concave and linear, respectively. The following is the Lagrangian of program SB (with $\sum_{i=1}^n E[T_i|Q_i] = \sum_{i=1}^n c_i Q_i$ substituted into the objective function and constraint QC decomposed into two inequality constraints):

$$\boldsymbol{L} = -c_E\sigma^2[\textstyle\sum_{i=1}^n s_i Q_i + \eta(\sum_{i=1}^n \delta_i Q_i)^2 + \sum_{i=1}^n \delta_i^2 Q_i^2] + \bar{\theta}(\sum_{i=1}^n Q_i - Q) + \underline{\theta}(Q - \sum_{i=1}^n Q_i)$$

with $\eta \equiv (\mu/\sigma)^2$ and $s_i \equiv (c_i+\omega\mu\delta_i)/c_E\sigma^2$. Since $\boldsymbol{L}$ is strictly concave in $\mathbf{Q} = (Q_i)_{i\in N}$, a $\mathbf{Q}^*$ is the unique optimal quota allocation for the program if and only if the first-order conditions of the program are satisfied (see Takayama 1985, Chapter 1, Section D).

Differentiating the Lagrangian with respect to Q_i yields the first-order partial derivative below:

$$\boldsymbol{L}_i = \bar{\theta} - \underline{\theta} - c_E\sigma^2[s_i + 2\eta\delta_i(\textstyle\sum_{h=1}^n \delta_h Q_h) + 2\delta_i^2 Q_i].$$

The first-order conditions require that if $\mathbf{Q}^*$ has some $Q_i^* > 0$, then $\mathbf{Q}^*$ has to satisfy the equation $\boldsymbol{L}_i = 0$ for some $\bar{\theta} \geq 0$ and $\underline{\theta} \geq 0$. These $\bar{\theta}$ and $\underline{\theta}$ must be the same for all such i's with $Q_i^* > 0$. In addition, if $\mathbf{Q}^*$ has some $Q_j^* = 0$, then $\mathbf{Q}^*$ has to satisfy the inequality $\boldsymbol{L}_j \leq 0$ for any such j's for the same $\bar{\theta}$ and $\underline{\theta}$. Moreover, $\mathbf{Q}^*$ must satisfy constraint QC.

As some Q_i^* has to be positive, so must the difference $\bar{\theta} - \underline{\theta}$. Define $\theta^* = (\bar{\theta} - \underline{\theta})/c_{\mathrm{E}}\sigma^2$. The first-order conditions are equivalent to the following ones:

$$\sum_{i=1}^{n} Q_i^* = Q$$

and some $\theta^* > 0$ exists such that

$$[\mathrm{MC}_i]: \qquad \theta^* \leq s_i + 2\eta\delta_i(\sum_{h=1}^{n} \delta_h Q_h^*) + 2\delta_i^2 Q_i^* \qquad \forall\ i \in N$$

with the equality holding for all $i \in B^* \equiv \{\ i{\in}N \mid Q_i^* > 0\ \}$. Another expression of the marginal condition MC_i is as follows:

$$Q_i^* \geq \left[\frac{\theta^* - S_i(W^*)}{2\delta_i^2}\right]$$

with $S_i(W^*) = [c_i + (\omega{+}2c_{\mathrm{E}}\mu W^*)\mu\delta_i]/c_{\mathrm{E}}\sigma^2$ and $W^* \equiv \sum_{i=1}^{n} \delta_i Q_i^* = \sum_{B^*} \delta_l Q_l^*$.

To determine the values of θ^* and W^*, I divide MC_i by $2\delta_i$ and then sum over the equality marginal conditions, i.e., MC_i's $\forall i{\in}B^*$. This yields the following equation of θ^* and W^*:

$$\theta^* [(1/2)\sum_{B^*} (1/\delta_l)] = [(1/2)\sum_{B^*} (s_l/\delta_l)] + W^* [1 + \eta b^*],$$

where $b^* \equiv |B^*|$ is the size of the supply base. Similarly, divide MC_i by $2\delta_i^2$ and sum over the equality marginal conditions. Then incorporate the quota constraint, QC. This gives a second equation of θ^* and W^*:

$$\theta^* [(1/2)\sum_{B^*} (1/\delta_l^2)] = [Q + (1/2)\sum_{B^*} (s_l/\delta_l^2)] + W^* [\eta\sum_{B^*} (1/\delta_l)].$$

The solution of the two equations is as follows:

$$\theta^* = \frac{(\eta^{-1}{+}b^*)[2Q{+}\sum_{B^*} (s_l/\delta_l^2)] - [\sum_{B^*} (1/\delta_l)][\sum_{B^*} (s_l/\delta_l)]}{(\eta^{-1}{+}b^*)[\sum_{B^*} (1/\delta_l^2)] - [\sum_{B^*} (1/\delta_l)]^2}$$

$$W^* = \frac{[\sum_{B^*} (1/\delta_l)][2Q{+}\sum_{B^*} (s_l/\delta_l^2)] - [\sum_{B^*} (1/\delta_l^2)][\sum_{B^*} (s_l/\delta_l)]}{2\eta[\ (\eta^{-1}{+}b^*)[\sum_{B^*} (1/\delta_l^2)] - [\sum_{B^*} (1/\delta_l)]^2\ ]}\ .$$

In summary, the first-order conditions imply the conditions specified in this proposition. The reverse also holds with $\bar{\theta}$ set to $c_E\sigma^2\theta^*$ and $\underline{\theta}$ set to zero. *Q.E.D.*

PROOF OF PROPOSITION 3 (QUALITY-DRIVEN SUPPLY BASE AND LARGER QUOTAS FOR HIGHER-QUALITY SUPPLIERS): For any multiple-sourcing supply base B^*, let supplier j be a selected supplier other than the highest-quality supplier in the supply base. Suppose

$$\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \le \omega\mu + 2\delta_1 Q c_E \mu^2.$$

Because $W^* = \sum_{B^*} \delta_l Q_l^* \ge \delta_1 Q$,

$$(c_{j-1} - c_j)/(\delta_j - \delta_{j-1}) \le \omega\mu + 2\delta_1 Q c_E \mu^2 \le \omega\mu + 2W^* c_E \mu^2.$$

Hence, $c_j + (\omega\mu + 2W^* c_E\mu^2)\delta_j \ge c_{j-1} + (\omega\mu + 2W^* c_E\mu^2)\delta_{j-1}$, or equivalently,

$$S_j(W^*) \ge S_{j-1}(W^*),$$

where $S_i(W^*) = [c_i + (\omega+2c_E\mu W^*)\mu\delta_i]/c_E\sigma^2$. By Proposition 2,

$$Q_j^* = [\theta^* - S_j(W^*)]/2\delta_j^2 > 0 \text{ and}$$

$$Q_{j-1}^* \ge [\theta^* - S_{j-1}(W^*)]/2\delta_{j-1}^2.$$

Thus, $Q_{j-1}^* \ge [\theta^* - S_{j-1}(W^*)]/2\delta_j^2 \ge [\theta^* - S_j(W^*)]/2\delta_j^2 = Q_j^* > 0$. Since $b^* \equiv |B^*|$, it has to be that $B^* = \{1, 2, \ldots, b^*\}$. *Q.E.D.*

PROOF OF PROPOSITION 4 (SUFFICIENT AND NECESSARY CONDITION FOR DETERMINING THE SIZE OF THE OPTIMAL SUPPLY BASE): Suppose $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \le \omega\mu + 2\delta_1 Q c_E\mu^2$. If the size of the optimal supply base is $j \in N\backslash\{n\}$, Proposition 3 implies $B^* = \{1, \ldots, j\}$. Consequently, Proposition 2 implies the existence of positive θ^* and W^*, as defined by formulas (1) and (2), such that

$$Q_j^* = [\theta^* - S_j(W^*)]/2\delta_j^2 > 0 \text{ and}$$

$$0 = Q_{j+1}^* \ge [\theta^* - S_{j+1}(W^*)]/2\delta_{j+1}^2.$$

Define $\theta_j = \theta^*$ and $W_j = W^*$. The condition of this proposition is thus satisfied.

For the "if" part, suppose there exist positive θ_j and W_j defined by formulas (1) and (2) with B^* substituted by $B_j \equiv \{1, \ldots, j\}$ such that $[\theta_j - S_j(W_j)]/2\delta_j^2 > 0 \ge [\theta_j - S_{j+1}(W_j)]/2\delta_{j+1}^2$. Define $\theta^* = \theta_j$ and $W^* = W_j$. Then θ^* and W^* by construction satisfy formulas (1) and (2) for $B^* = B_j$. Moreover, define a quota allocation $\mathbf{Q}^*$ with $Q_i^* = [\theta^* - S_i(W^*)]/2\delta_i^2$ for all $i \le j$ and $Q_i^* = 0$ for all $i > j$. Because $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \le \omega\mu + 2\delta_1 Q c_{\mathrm{E}}\mu^2$, a procedure similar to the proof of Proposition 3 will show that $S_{i+1}(W^*) \ge S_i(W^*)$ for all $i \in N\backslash\{n\}$. Consequently, $[\theta^* - S_j(W^*)]/2\delta_j^2 > 0$ implies $Q_i^* = [\theta^* - S_i(W^*)]/2\delta_i^2 > 0$ for all $i \le j$. Similarly, $0 \ge [\theta^* - S_{j+1}(W^*)]/2\delta_{j+1}{}^2$ implies $Q_i^* = 0 \ge [\theta^* - S_i(W^*)]/2\delta_i^2$ for all $i \ge j+1$. Thus, the marginal conditions in Proposition 2 are satisfied by the $\mathbf{Q}^*$ defined above. This means it is the unique optimal allocation for the buyer's sourcing problem, and B_j is the optimal supply base. Hence, $b^* = j$. *Q.E.D.*

PROOF OF PROPOSITION 5 (SUFFICIENT AND NECESSARY CONDITION FOR SINGLE SOURCING TO BE OPTIMAL): The "only if" part follows directly from Proposition 2. For the "if" part, it is straightforward to verify that for each $h \in N$, the θ^h and W^h defined in this proposition satisfy the formulas (1) and (2) of Proposition 2 for the single-sourcing supply base $B^h \equiv \{h\}$. Define for each h a quota allocation $\mathbf{Q}^h$ with $Q^h{}_h \equiv [\theta^h - S_h(W^h)]/2\delta_h{}^2$ and $Q^h{}_i \equiv 0$ for all $i \in N\backslash\{h\}$. Because $\theta^h \equiv s_h + 2(1+\eta)Q\delta_h^2$ and $W^h \equiv \delta_h Q$, clearly $Q^h{}_h = Q > 0$.

To see if one or none of the B^h's is the optimal supply base, it suffices to check whether one or none of them meets the remaining marginal conditions of Proposition 2, i.e., for any given h,

$$Q^h{}_i \ge [(\theta^h - S_i(W^h))/2\delta_i^2 \text{ for all } i \in N\backslash\{h\}.$$

Some h will meet this last requirement if the condition of the proposition is fulfilled. When this is the case, the $\{h\}$ is the optimal supply base and single sourcing is optimal. *Q.E.D.*

RECENT CASES OF MASSIVE PRODUCT RECALLS: The most noticeable cases are recalls of unsafe products manufactured by suppliers in China. These include

(i) *Toys*. Mattel recalled 436,000 Chinese-made toy cars covered with lead paint in 2007 (Story and Barboza 2007). This incident raised concerns about the insufficient enforcement of existing laws banning lead paint. Such concerns forced other U.S. manufacturers to recall over a million toy ovens, trains, dolls, and other popular toys.

According to Egan, Campbell, and Vogel (2009), "[t]he purported culprit was a Chinese supplier that had subcontracted its work to another Chinese company that had coated the toy cars with lead paint without the knowledge of the U.S. manufacturer. … The foreseeability of these events by any U.S. manufacturer is doubtful. In China, as in the United States, lead paint is illegal. Nevertheless, it appears that a number of Chinese companies began using lead paint because it is more resistant to corrosion and dries faster, thereby, in part, decreasing production time.

Until the 2007 toy recalls, the Chinese government was seemingly unaware that lead paint was widely used in its manufacturing sector, and therefore, did not sufficiently enforce lead paint prohibitions. Similarly the U.S. Consumer Product Safety Commission was also unable to enforce existing laws banning lead paint because it had only around 100 field investigators who were responsible for inspecting $22 billion in toys." For further details of this incident, see also Barboza (2007) and Lipton and Barboza (2007).

(ii) *Drugs*. Heparin is a blood thinner widely used in surgery and dialysis. In early 2008, heparin sold by Baxter International was linked to at least 19 deaths and hundreds of allergic reactions in the U.S.. After recalling nine lots of the drug, problems continued. So the company suspended the manufacturing of the drug associated with the problems.

Investigations later discovered that the heparin at issue, with its raw components bought from a Chinese plant, contained a contaminant mimicking heparin. By that time, Baxter had expanded the recall to cover almost all its heparin products. Because the company supplies about half the U.S.'s heparin, the production suspension and the widespread recall caused worries about shortage problems in the short and long runs. The impact was so huge that some even warned that "many more patients would be likely to experience significant blood loss during dialysis." For further details of the incident, see Harris (2008a,b), Bogdanich (2008a,b), and Barboza (2008a).

(iii) *Dairy products*. In July 2008, infant milk formula produced by Sanlu Group, the largest milk power maker in China for 15 years in a row, were found to contain a toxic industrial chemical called melamine. Follow-up investigations discovered the same problem in the products of 21 other companies. Melamine was added to milk to raise the protein count artificially and fool safety tests for protein content. The contamination caused at least six infants dying from kidney stones and other complications and sickened over 50,000 children.

Exports of food ingredients from China have been growing in recent years. Because milk powder is an ingredient to many dairy products, the milk scandal scared the international community. Tests showed that many products of international brands were also tainted by melamine, leading to worldwide recalls of contaminated products. Affected brands include Nabisco, Kraft Foods, Heinz, Mars, Cadbury, Lipton, and Nestles.

The scope of the contamination later spread to eggs traced back to the use of melamine-tainted animal feed, even though the chemical had been banned as an animal feed additive since July 2007. In reacting to the scandal, over 25 countries banned imports of dairy and other affected food products from China. For further details of the scandal, see Barboza (2008b,c) and Fuller (2008).

REVIEW OF RELATED STUDIES IN ECONOMICS: Before giving a brief account of related economic studies, let me clarify some confusion on the usage of the term "multiple sourcing." In management studies, multiple sourcing generally means *split* procurement arrangement, i.e., relying on two or more suppliers in procuring an item. The sourcing literature in economics, however, also uses multiple sourcing to mean *sharable* procurement arrangement, i.e., having two or more suppliers compete for a share in supplying an item without precluding the one-supplier-take-all outcome. The key difference is that multiple sourcing of the sharable sense (hereafter, *sharable* multiple sourcing) refers to an *ex ante* arrangement concerning the *degree* of supplier competition, i.e., at least two *competitors*, whereas multiple sourcing of the split sense (hereafter, *split* multiple sourcing) refers to an *ex post* arrangement concerning the *outcome* of supplier competition, i.e., at least two *winners*. In other words, *sharable* multiple sourcing can end up having only one selected supplier, as long as multiple suppliers have competed for being the one selected. Some studies (e.g., Laffont and Tirole 1988, Riordan and Sappington 1989) claim to analyze multiple sourcing but are better described as studying *source switching*, i.e., about examining the effects of introducing additional suppliers to compete with an incumbent in *one-supplier-take-all* settings.[9]

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[9] More precisely, Laffont and Tirole (1988) uses the term "second sourcing," which is a special case of multiple sourcing when there are only two suppliers. Second sourcing also has the connotation of finding a second supplier in addition to an incumbent, or to replace the incumbent when used in the source switching sense. Such usages are particularly suitable for describing the sourcing decision in a multiple-period or multiple-stage setting, as in Demski, Sappington, and Spiller (1987),

The sourcing literature in economics is largely based on auction models (e.g., Myerson 1981, Laffont and Tirole 1987). It typically concludes that *split* multiple sourcing is undesirable unless suppliers have increasing marginal costs with sufficiently low fixed costs (e.g., the third case studied by Dasgupta and Spulber 1989/90 and the second case analyzed by Auriol and Laffont 1992). As noted by McMillan (1990), "the disadvantage of multiple sourcing is that economies of scale may be forgone." So for multiple sourcing to be better than single sourcing, scale diseconomies have to be sufficiently great.

When suppliers have constant marginal costs with positive fixed costs (e.g., the first case analyzed by Auriol and Laffont 1992 and the settings studied by Demski, Sappington, and Spiller 1987 and Riordan 1996), split multiple sourcing usually is undesirable because of duplication of fixed costs.[10] However, it is always good to have more suppliers competing for a procurement contract as this increases the chance of finding a lower-cost supplier. This *sampling effect* makes sharable multiple sourcing desirable, regardless of the suppliers' cost structure.

Some studies have examined the effects of incomplete information about suppliers' types on the dis/advantages of multiple sourcing and the sourcing decision. Auriol and Laffont (1992) find that the sampling effect is higher under incomplete information than under complete information, provided the consumer demand is sufficiently price-inelastic. If the consumer demand is sufficiently price-elastic, the contrary instead holds. Riordan (1996) studies a setting with an exogenous procurement quantity requirement, which resembles a price-inelastic demand. He finds that incomplete information biases the choice of the market structure in his model towards sharable multiple sourcing, a result consistent with Auriol and Laffont's.

By contrast, Dana and Spier (1994) draw a different conclusion about split multiple sourcing. Because the split-award outcome is a weaker penalty to a lying supplier than the no-award outcome (i.e.,

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Laffont and Tirole (1988), and Riordan and Sappington (1989). In a single-period setting, dual sourcing seems to be a better term for describing the two-supplier case of multiple sourcing.

[10] Dana and Spier (1994) and McGuire and Riordan (1995) have studied settings with constant marginal costs and positive fixed costs yet still found a split-award outcome (i.e., the duopoly market structure in their models) sometimes desirable. Dana and Spier's result is driven by smaller efficiency loss resulting from Cournot competition by the suppliers in the case of dual sourcing, as compared to unregulated monopoly in the case of single sourcing. McGuire and Riordan's result is due to the spatial competition model embedded in their model: a product differentiation benefit, playing the same role as increasing marginal costs, arises when aggregating the social value of the treatment to clients uniformly distributed on a line of unit length.

single sourcing from a rival supplier), split multiple sourcing is less powerful in discouraging suppliers from lying about their types. Thus, it should be used less often under incomplete information than under complete information. McGuire and Riordan (1995) obtain a similar result for some parameter values of their model, and the contrary for some other values.

The research discussed so far follows a normative approach to study the optimal sourcing decision. Some other research by contrast uses a positive approach to investigate when multiple sourcing can arise in equilibrium. Anton and Yao (1989) consider a setting with two suppliers playing a sharable procurement auction. Interestingly, the buyer is indifferent among all equilibria, which include single-sourcing from the lower-cost supplier. So although dual sourcing can arise in equilibrium, it brings no benefit to the buyer. Extending their model to include incomplete information, Anton and Yao (1992) show that dual sourcing can arise in equilibrium if a technical condition is fulfilled. This condition ensures that dual-source production is less expensive than sole-source production.

Seshadri, Chatterjee, and Lilien (1991) study a procurement auction model with endogenous choice of participation. They point out that specifying in advance a greater number of winners to split the procurement contract may increase the chance of winning. This encourages more suppliers to send in bids and stimulates supplier competition. The downside of expanding the supply base is the higher production costs of the marginal winner, which are borne by the buyer.[11]

From this review, it is clear that the sourcing literature in economics has focused only on the competition advantage of multiple sourcing, with no attention given to the protection advantage of multiple sourcing. Neither has this advantage received sufficient attention in OR/MS studies, as explained in the introduction.

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[11] Seshadri, Chatterjee, and Lilien's approach has some normative favor. Although the winner selection and award splitting rules are exogenously specified, they discuss the implications of expanding the supply base as if it were a design instrument of the buyer.

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Table 1: Notations

N	=	$\{1, 2, \ldots, n\}$ is the index set of the n suppliers ($n \geq 2$).
Q_i	=	the production quota specified in the procurement contract for supplier i ($Q_i \geq 0$).
T_i	$\geq$	0 is the payment to supplier i for the component parts supplied according to the quota specified in the procurement contract.
Q	=	$\sum_{i=1}^{n} Q_i$ is the total procurement quantity ($Q > 0$).
$\mathbf{Q}$	=	$(Q_i)_{i\in N}$ is the production quota allocation.
c_i	=	supplier i's constant marginal cost of production.
D_i	=	$R_i\delta_iQ_i$ is the amount of defective parts manufactured by supplier i.
R_i	$\geq$	0, or more precisely $R_i\delta_i$, is referred to as the *random yield loss* of supplier i. The random variables R_i's are independently and identically distributed with mean $E(R_i) = \mu$, where $0 < \mu \leq \bar{\mu} << 1$, and variance $\mathrm{var}(R_i) = \sigma^2 > 0$.
η	=	$(\mu/\sigma)^2$ is the *squared standardized mean* of the "random yield loss" R_i, or equivalently, η^{-1} is referred to as the *squared coefficient of variation*.
δ_i	>	0 is a parameter affecting the random yield loss of supplier i ($\delta_i \leq 1$). The value $1-\delta_i$ is referred to as the *quality-based scoring index* of the supplier, or simply its *quality* level. It is assumed that $\delta_1 < \delta_2 < \ldots < \delta_n$.
D	=	$\sum_{i=1}^{n} D_i$ is the total amount of defective products sold to end customers by the buyer. It becomes observable after the customers have experienced field failures of the products and take them back for warranty repair.
ω	>	0 is the constant marginal cost of warranty repair.
$C_E(D)$	=	c_ED^2 is the *other external failure cost* (e.g., reputation damage) borne by the buyer, in addition to the warranty-related cost, as a result of the defective products sold to customers (where $c_E > 0$).
s_i	=	$(c_i+\omega\mu\delta_i)/c_E\sigma^2$ is a ratio representing the relative unimportance of the quadratic other external failure cost, characterized by c_E, in constituting the buyer's expected total cost.
$S_i(W^*)$	=	$[c_i + (\omega+2c_E\mu W^*)\mu\delta_i]/c_E\sigma^2 = = s_i + 2\eta W^*\delta_i$ is referred to as the *quality-adjusted cost-based scoring index* for supplier i, evaluated at $W^* = \sum_{i=1}^{n} \delta_iQ_i^*$ based on the optimal quota allocation $\mathbf{Q}^* = (Q_i^*)_{i\in N}$.
B	=	$\{ i\in N \mid Q_i > 0 \}$ is the set of selected suppliers constituting the supply base.
b	=	$\lvert B\rvert$ is the size of the supply base B.