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Abstract

In this paper we study a new factor that matters for fertility and consumption decisions: the risks associated with having and raising a child. We analyze a real options model with incomplete markets to explicitly model both children as a risky investment and the parental option to time fertility. We focus on CRRA preferences and uninsurable shocks to future parental income and to the costs of raising a child. We obtain several results that are new relative to the standard Beckerian fertility framework where children are deterministic goods: i) Independently of wealth, higher child cost volatility diminishes fertility. ii) Consumption is decreasing in higher cost volatility but the slope flattens as wealth increases. iii) Wealth alters the way in which the agent's risk tolerance impacts the fertility and consumption decisions. For low wealth levels, risk aversion speeds up fertility and lowers consumption with children serving as an utility insurance mechanism. iv) Fertility is increasing in the correlation between income and child cost shocks. v) The sign of this correlation determines if higher income volatility speeds up or delays fertility. vi) Fertility is U-shaped in the income over wealth ratio. Finally, we use regression analysis to provide empirical support for the theoretical results.

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1 Introduction

Having children is a risky investment because it is not possible to exactly know ex-ante the costs associated with raising a child (including the opportunity cost of the parents' time and the effects on their career paths) or the benefits that they will provide. For example, no parent may know in advance how often their children will get ill and how much money and time this will cost;[1] if their children will need extra support at school, get fellowships that lower education costs or even become child actors and bring millions of dollars to the family. Markets are incomplete and do not insure most of these contingencies, especially those related to the time costs for the parents. Hence, children risks are at most partially insurable, and childbearing brings a new source of shocks to the parents.

In this paper we study the consequences for fertility and consumption of taking into account that children are a source of risk that interacts with other risks borne by the parents. By doing so, we make two contributions. First, we contribute to the literature on the economics of fertility. This literature has followed Gary Becker's seminal work in modelling children as deterministic normal goods, without considering that children are also a stochastic asset.[2] Our study enlarges the set of effects that drive fertility decisions.

Second, we contribute to the literature on the real options approach to investment in incomplete markets. We study the problem of an infinitely-lived household who has an initial level of assets and every period gets some stochastic income, that we assume exogenous for simplicity. She gets utility from consumption and can save at the risk free rate. The decision to have a child is like the decision to exercise an option. She can decide to have a child or to postpone the decision. If she has a child then she is acquiring an irreversible, durable and non-tradable asset that gives her utility, but implies some stochastic exogenous costs.[3] We focus on the uncertainty about children's costs and assume that the utility flow is deterministic, as people can anticipate how much they will enjoy being parents. We assume CRRA preferences because this allows us to study the effects of wealth in the decisions. Markets are incomplete, because only the risk-free asset is available for investment.

[1] The 2008 National Health Interview Survey, reports that roughly 30% of the children missed 1 or 2 school days, 30% missed 3-5 days and 10% more than 6 days. These numbers are quite robust to differences in income, race or education of the parents. These results indicate that there is cross-sectional heterogeneity in children's health due to unpredictable shocks.

[2] Dehejia and Lleras-Muney (2004) and Manuelli and Seshadri (2009) are examples of recent work within the Beckerian framework. Jones et al. (2008) and Tamura (2000) survey the literature on the economics of fertility.

[3] These are realistic assumptions given that once a child is born is very costly to get rid of her because of sentimental reasons and legal constraints. Moreover, children do not depreciate and it takes several years before they can live independently.

Our main theoretical results are the following: i) Independently of wealth, higher cost volatility diminishes fertility. ii) Consumption is decreasing in higher child cost volatility but the slope flattens as wealth increases.[4] iii) Wealth alters the way in which the household's risk attitude impacts the fertility and consumption decisions. For low wealth levels, risk aversion speeds up fertility and lowers consumption. iv) Fertility is increasing in the correlation between income and child cost shocks. v) The sign of this correlation determines whether higher income volatility speeds up or delays fertility. vi) Fertility is U-shaped in the income over wealth ratio.

Moreover, we confirm two well known results since Becker's pathbreaking work. First, if there is no quality-quantity trade-off and children are normal goods then higher income growth increases fertility.[5] This is the standard income effect. Second, higher expected costs of raising a child speed up fertility. This is the standard substitution effect in an intertemporal dimension.

Our main results come from the interplay of four effects that differ from the standard income and substitution effects: i) As in any real options problem, having the option to time investment implies asymmetric convex payoffs ("in bad times do not exercise and wait for good times to come"). This pushes the household to delay fertility as volatility increases. ii) If the fertility option is exercised the child costs can be thought as an income shock. Incomplete markets and preferences displaying precautionary savings push the household to delay fertility to avoid new sources of risk. iii) Having a child is an alternative source of utility that can hedge fluctuations in goods consumption. This is an important characteristic of children when we think of them as an asset class. This channel pushes the household for early fertility to enjoy the children as an insurance mechanism. iv) When the income and cost shocks are correlated, the sign of the correlation determines if the shocks hedge or add up each other. Fertility speeds up when the shocks hedge each other.

Our analysis offer new insights on the determinants of fertility that may help both in explaining facts and in the design of pronatalist policies. For example, Stetsenko (2010) documents that U.S. fertility changed from being countercyclical to being procyclical as female labor supply increased. This is consistent with our model, because the increase in female labor supply implies a negative correlation between child and income shocks. On the policy side, our theory provides support for policies that reduce the uninsurable uncertainty from raising a child (such as child care programs) and the negative correlation between income and child costs shocks (as

[4] If preferences were CARA, instead of CRRA, the decision boundaries would be independent of wealth. Hence wealth would have no effect on how risk aversion affects the agent's decision.

[5] For simplicity we did not model this tradeoff because it would imply that children's costs are endogenous. This assumption does not seem problematic since our goal is to study how changes in the uncertainty associated with the cost of a child, and the correlation of this uncertainty with the income uncertainty, affect both the decision to have a child and the savings patterns. Moreover, we express the optimal fertility decision as a fraction of wealth.

State-paid leaves of absence to insure the parent's career from children health shocks).

Our solution method does not allow for quantitative results. Hence, to bring the theory to the data, we use regression analysis.[6] To do so, we identify a variable that can serve as a proxy for the uninsurable volatility in the costs associated with raising a child. We focus on the distance of the grandparents to the parents. There are two main arguments to support this choice. First, a large fraction of the uninsurable risks of child rearing seems related to the time costs for the parents. This was especially true in the U.S. in the late 1980s, when concerns that child care was in short supply, not of good enough quality, and too expensive for many families prompted the enactment of legislation in 1990 that expanded Federal support for child care (U.S. House, Committee on Ways and Means 2000). Second, Cardia and Ng (2003) and Rupert and Zanella (2010) have documented that grandparents make substantial time transfers in the form of care of their grandchildren. Hence, we assume that households whose parents live closeby face less uncertainty from the time costs shocks associated with raising a child. We use this variable as a proxy for child cost risk.

Our data are the 1987-88 National Survey of Families and Households (NSFH). We focus on fertility decisions made in the last year. We obtain several results consistent with our theory: i) The higher the distance to parents the smaller the likelihood of childbearing. Distance is significant in all our probit specifications that also control for variables commonly associated with the decision to have a child or not, such as income, wealth, income over wealth, age of the householder, race, religion and education. We interpret this result as evidence that higher cost volatility reduces fertility. ii) Wealth plays an important role in the fertility decision. Less wealthy households are less affected by high levels of child costs risk. This is consistent with our theory, in which children can serve as an insurance mechanism. iii) The marginal effect on fertility of risk tolerance (proxied by the ratio of stock holdings relative to wealth) is negative for low wealth households and positive for medium/high wealth households. iv) The sample correlation between income and parental distance is significantly positive across households who had a child in the last year, while it is negligible across those who did not.

This paper is related and contributes to two different literatures:

A) To the literature on the economics of fertility. We believe that ours is the first model in which a child is a stochastic asset associated with an investment option. Our innovation is to study how the risks coming from children themselves affect fertility and savings. More-

[6] It is not possible to obtain a closed form solution of the model. We solve it by extending to a real options framework the approximation method proposed by Kogan and Uppal (2002) and Bhamra et al. (2007). It consists in a power series expansion of the fertility decision boundary with respect to relative risk aversion, around the logarithmic case.

over, our work complements both the literature that has studied the effects of income uncertainty on fertility (see Kreyenfeld 2005 for a survey, and Sommer 2009 for a recent contribution), and the literature studying children as an insurance mechanism (Portner 2005 surveys theoretical and empirical work). We show that the effects of income uncertainty on fertility depend on how this uncertainty correlates with child rearing risks. And we show that children can be used by low wealth households as an insurance asset even if children do not provide another source of household income, neither take care of the parents (these are the main arguments in models where children provide insurance). In our model children hedge utility fluctuations because they provide a safe flow of utility.

B) To the real options approach to investment. This literature has become a workhorse in finance to study investment decisions that are partially or completely irreversible, imply uncertain future costs or rewards, and have a flexible investment timing.[7] This literature studies the opportunity to invest in a real project as an American call option on the underlying investment project, that can be analyzed using no-arbitrage relationships as in the Black-Merton-Scholes model. However, these techniques do not work when the assets (the children in our case) are nontradable and financial markets are incomplete, i.e. markets do not provide assets to replicate the payoffs of having a child.[8] In this case the model has to be solved using dynamic programming. Few papers have studied real options under incomplete markets. We are only aware of Chen et al. (2010), Henderson (2007), Hugonnier and Morellec (2007) and Miao and Wang (2007). Miao and Wang (2007) is the paper more closely related to our work because they study simultaneously the consumption/savings decision and the option to invest. They focus on CARA preferences and analyze a real options problem in which the decision maker faces uninsurable idiosyncratic risk from his investment opportunity. We differ on two dimensions: i) we focus on CRRA preferences. This allows us to study how the household's wealth affects fertility and consumption decisions. ii) In our setup, the investment payoffs come both as deterministic flows of altruistic utility and as stochastic monetary costs, while in their model the household pays a deterministic cost to receive a stochastic flow payoff. In their case, stronger precautionary savings motives implied by higher risk aversion always delay investment, while we show that this implication can be reversed because of wealth effects and the insurance associated with the deterministic altruistic utility.

The paper proceeds as follows. Sections 2 describes the model and the solution method.

[7] An American call option is a financial contract in which the buyer of the call option has the right, but not the obligation, to buy an agreed quantity of a particular commodity, or financial instrument, from the seller of the option at any time during the life of the option for a certain price.

[8] Dixit and Pindyck (1994) is a textbook survey.

Section 3 discusses the theoretical results. Section 4 provides a regression test of the model. Section 5 concludes. Proofs and details on the solution method are in the Appendix.

2 Model and Solution Method

2.1 Model

We analyze a continuous-time, partial equilibrium economy, populated by an infinitely-lived household whose utility function depends on her consumption of goods (c_t) and on having or not a child (I_t). The household starts without a child and has the option to have it at any moment. We denote by τ the time at which the household decides to have a child. I_t is an indicator function that captures the fertility status: it takes the value one if the household has had a child

$$I_t = \begin{cases} 1 & \text{for} \quad t \geq \tau \\ 0 & \text{for} \quad t < \tau \end{cases} \tag{1}$$

We assume that the utility function is separable into a felicity component arising from goods' consumption and a felicity component arising from the fertility status. We assume Constant Relative Risk Aversion for the consumption component

$$u(c_t, I_t) = \left(\frac{c_t^{1-\gamma} - 1}{1-\gamma} \right) - \frac{I_t - 1}{1-\alpha}, \qquad \alpha > 1, \gamma > 0 \tag{2}$$

where γ is the relative risk aversion coefficient and $\alpha > 1$ is a parameter that captures the utility benefit from having a child. Higher levels of α imply smaller utility benefits.

Every period the household earns a stochastic labor income stream (y_t). Moreover, if she has had a child she must pay the stochastic costs (q_t) associated with child rearing. We model the dynamics of these two processes as Geometric Brownian motions

$$\frac{dy_t}{y_t} = \mu_y dt + \sigma_y dB_t^y \tag{3}$$

$$\frac{dq_t}{q_t} = \mu_q dt + \sigma_q dB_t^q \tag{4}$$

$$\mathbb{E}[dB_t^y dB_t^q] = \rho dt \tag{5}$$

Both income and costs are denominated in units of the consumption good that we take as numeraire. B_t^y and B_t^q are two Wiener processes whose correlation coefficient is ρ. The parameters μ_y and μ_q denote the deterministic growth rates of income and the cost of a child. The parameters σ_y and σ_q denote the volatilities of these processes.

We assume that markets are not complete and the household cannot hedge against child cost or labor income risk because she can only transfer consumption over time investing in a riskless asset. We denote by r the constant instantaneous risk-free rate. Hence the wealth (W) of the household evolves as

$$dW_t = (rW_t + y_t - q_t I_t - c_t)dt \tag{6}$$

The household maximizes time-additive expected utility over consumption and her parental status, i.e. she decides two things: if and when to have a child; and how much to consume and save every period. Assuming a discount rate equal to the interest rate, the household problem is

$$\begin{aligned} H(W_0, q_0, y_0) = \max_{c,\tau} \mathbb{E}\left[\int_0^\infty e^{-rt} u(c_t, I_t) dt\right] \\ s.t. \\ (3) - (6) \end{aligned} \tag{7}$$

2.2 Solving the model

The problem has a recursive nature. Conditional on having or not having a child, the household's decisions depend on her wealth, income and the cost of the child. These are the state variables of the problem. We solve the household's decision problem backward by dynamic programming. Let $H(W, q, y)$ and $J(W, q, y)$ denote the household's value functions before and after the decision to have a child, respectively. After the fertility decision, the value function is

$$\begin{aligned} J(W, q, y) = \max_{c} \mathbb{E}\left[\int_0^\infty e^{-rs} u(c_s, 1) ds\right] \\ \text{s.t.} \quad (3) - (6) \end{aligned} \tag{8}$$

By a dynamic programming argument, the function J solves the following Hamilton-Bellman-Jacobi equation (HBJ)

$$\begin{aligned} rJ(W, q, y) = \max_{c} u(c, 1) + (rW + y - q - c)J_W + \\ + J_y y\mu_y + J_q q\mu_q + \frac{J_{yy}}{2} y^2\sigma_y^2 + \frac{J_{qq}}{2} q^2\sigma_q^2 + J_{qy} yq\sigma_q\sigma_y\rho \end{aligned} \tag{9}$$

subject to the transversality condition

$$\lim_{T\to\infty} \mathbb{E}[J(W_T, q_T, y_T)] = 0 \tag{10}$$

As in the option pricing literature, we define a continuation region, $\{(W, q, y) : H(W, q, y) > J(W, q, y)\}$, as the set of realizations of the state variables at which the household is better off without children and the fertility option is worth more alive. Because of (2), we can write the value function in the continuation region, before the decision to have a child, in the form

$$H(W, q, y) = V(W, q, y) + \frac{1}{(1-\alpha)r} \tag{11}$$

where $V(W, q, y)$ is the value function before the decision to have a child but net of the utility that the child would provide if the option is exercised. $V(W, q, y)$ satisfies the following HBJ equation

$$\begin{aligned} rV(W, q, y) = \max_c \frac{c^{1-\gamma}-1}{1-\gamma} + (rW + y - c)V_W + \\ + V_y y\mu_y + V_q q\mu_q + \frac{V_{yy}}{2} y^2\sigma_y^2 + \frac{V_{qq}}{2} q^2\sigma_q^2 + V_{qy} yq\sigma_q\sigma_y\rho, \end{aligned} \tag{12}$$

where V_W, V_y and V_q are first order partial derivatives, and V_{yy}, V_{qq}, V_{qy} second order partial derivatives.

$V(W, q, y)$ also satisfies the following no-bubble condition

$$\lim_{q\to\infty} V(W, q, y) = V^0(W, y), \tag{13}$$

where

$$V^0(W, y) = \max_c \mathbb{E}\left[\int_0^\infty e^{-rs} u(c_s, 0) ds\right] \tag{14}$$

$$\text{s.t.} \quad (3) - (6)\,. \tag{15}$$

Condition (13) states that if the cost of having a child approaches infinity the household will never exercise the option, thus her value function must converge to the value function of a consumption/savings problem with stochastic income and no fertility option (V^0).

The complement of the continuation region is the fertility region: $\{(W, q, y) : H(W, q, y) = J(W, q, u)\}$. It is the set of realizations of the state variables at which the household wants to have a child. This set is closed,[9] and its boundary is the region of immediate childbearing, or simply the fertility boundary. To characterize it, note that at the instant of investment, the following value-matching condition must hold

$$V(W,q,y)+\frac{1}{(1-\alpha)r}=J(W,q,y) \tag{16}$$

This equation implicitly defines a three-dimensional (in income, wealth and child cost) decision boundary. Since the fertility decision implies that costs are going to be incurred, we can think of this decision as being determined by monitoring the cost level. The household has a child once the stochastic cost process is equal to or smaller than the decision boundary expressed in terms of a critical cost that is a function of current income and wealth ($\overline{q}(y,W)$).

Moreover, the optimal fertility boundary ($\overline{q}(y,W)$) must satisfy the following smooth-pasting conditions. They state that the marginal change in income, wealth and cost of a child must have the same marginal effect on the household's value functions just before and immediately after exercising the option

$$\left.\frac{\partial V(W,q,y)}{\partial y}\right|_{q=\overline{q}(y,W)}=\left.\frac{\partial J(W,q,y)}{\partial y}\right|_{q=\overline{q}(y,W)} \tag{17}$$

$$\left.\frac{\partial V(W,q,y)}{\partial q}\right|_{q=\overline{q}(y,W)}=\left.\frac{\partial J(W,q,y)}{\partial q}\right|_{q=\overline{q}(y,W)} \tag{18}$$

$$\left.\frac{\partial V(W,q,y)}{\partial W}\right|_{q=\overline{q}(y,W)}=\left.\frac{\partial J(W,q,y)}{\partial W}\right|_{q=\overline{q}(y,W)} \tag{19}$$

Given the lack of closed-form solutions for our problem, we use an approximate but analytical solution method proposed by Kogan and Uppal (2002) and Bhamra et al. (2007). It exploits the fact that if the household has logarithmic preferences ($\gamma=1$), then an exact solution is available. This approximation method consists of a power series expansion of the fertility decision boundary with respect to relative risk aversion around the logarithmic case. We do a second-order expansion, therefore the accuracy of the solution decays rapidly if the household is very risk averse. The next Proposition characterizes optimal fertility and consumption as a function of income, wealth, and model parameters. The value function and the details of the derivation are reported in the Appendix.

Proposition 1 *To the second order in γ, the household has a child whenever the cost over*

[9] Assuming that the value function $V(w,q,y)$ is continuous with respect to its arguments.

wealth ratio, $\hat{q} = q/W$, crosses the following decision boundary from above:

$$\overline{q} = \left(\frac{K_3}{\beta \overline{K}_3}\right)^{\frac{1}{\beta-1}} + (\gamma-1)(A + B\hat{y}) + (\gamma-1)^2(AA + BB\hat{y} + CC\hat{y}^2) \tag{20}$$

where $\hat{y} = y/W$. Coefficients $(A, AA, B, BB, CC, \beta, K_3, \overline{K}_3)$ are functions only of the model parameters $(\mu_y, \mu_q, \sigma_y, \sigma_q, r, \rho, \alpha)$ and they are reported in the Appendix. Moreover, to the second order in γ, the consumption rule of the household before the decision to have a child is

$$\begin{aligned} \hat{c} &= r + (\gamma-1)(U_0 r^2 - r log(r)) + \\ &(\gamma-1)^2 \left[-U_0 r^2 + U_1 r^2 + \frac{U_0^2 r^3}{2} + r \log r - U_0 r^2 \log r + \frac{1}{2} r (\log r)^2 \right] \end{aligned} \tag{21}$$

where $\hat{c} = c/W$. Functions $U_0(\hat{q}, \hat{y})$ and $U_1(\hat{q}, \hat{y})$ are defined, respectively, in equations (44) and (60) of the Appendix.

The homotheticity property of the CRRA utility of consumption implies that what the household needs to monitor is cost relative to wealth. As we show in the Appendix, the value function is monotonically decreasing in the cost over wealth ratio, therefore the childbearing decision is taken when this ratio crosses the boundary from above.

The first term in expression (20), corresponds to the zero-th order of the power series. It is the optimal exercise boundary of an investor with logarithmic utility of consumption ($\gamma = 1$). This term depends on some constants reported in the Appendix that only depend on the expected (logarithmic) growth rate of the child rearing cost financed at the risk-free rate. This term is independent of income, implying that this investor is willing to accept a cost that increases linearly in the level of available wealth. The household has a child whenever the benefit deriving from altruistic utility matches the opportunity cost in terms of foregone consumption.[10] This can be seen by analyzing her consumption rule, which is the first term in expression (21). The household with log preferences consumes a constant fraction of wealth, r, which is the marginal opportunity cost of one unit of wealth consumed instead of saved and invested. This short-sighted consumption policy implies that the opportunity cost of fertility does not involve income considerations, but only the *immediate* consumption loss rq incurred because child rearing is subtracting available wealth at a rate q.

The behavior of the non-logarithmic CRRA preferences ($\gamma \neq 1$) is substantially more com-

[10] In the portfolio choice literature, it is well known that log preferences imply myopic investment behavior, in the sense that the optimal intertemporal portfolio allocation is a sequence of single-period portfolio allocations. The agent ignores the risk of time-varying investment opportunities. See Brandt (2009).

plicated. The first and second order terms in the consumption rule (21) tell us that the childbearing option influences the present consumption rule through the value of the fertility option. Wealth effects are nontrivial, far from the linear monotonic pattern of the logarithmic investor, and matter through the first and the second order terms of the approximation. The next Section characterizes numerically the childbearing behavior of the household.

3 Theoretical predictions

In this Section we analyze the qualitative predictions of the model. To do so, we fix a reasonable set of parameter values, and we discuss how changes in fundamentals and parameters affect the household's optimal fertility and consumption decisions.

3.1 Parameterization

Table 1 summarizes our benchmark parameterization. We assume the relative risk aversion coefficient (γ) equal to 2. This value preserves the accuracy of our solution method, and it is consistent with the literature which models preferences as additively separable in consumption and the utility derived from the child.[11] We set initial income and wealth to the median 2007 U.S. household income and family net worth, as reported by the U.S. Census Bureau (2010). We assume that the discount rate equals the interest rate and set the continuously compounded interest rate at 5% annual. We did not have many references for the preference parameter α, which governs the utility from having a child. We calibrated it in such a way that the annual utility stream from the fertility decision equals 50% of the annual utility stream from consumption, assuming a constant consumption equal to the annuity value of wealth (rW).

Concerning the cost of raising a child, the literature is thin and it has focused on the deterministic part assuming a tradeoff between time and market expenditures in child rearing. We assume that the costs of rasing a child grow at approximately 2% annually ($\mu_q = 0.02$), which is also the inflation target in many countries. We take as a benchmark that the correlation (ρ) between income and costs is zero and perform comparative statics in the next section. We set for convenience $\mu_y = \mu_q$, $\sigma_y = \sigma_q$ and set the volatility of cost growth to 5% ($\sigma_q = 0.05$), which give us a steady state in which the critical cost that triggers fertility, given the rest of parameters, is roughly 40% of income. This is consistent with Lino (1998), who using the Consumer Expenditure Survey (CEX), estimates that an average dual-earner household with

[11] See, for example, Becker et al. (1990), or Jones et al. (2008).

two children between ages 0 and 17 spends roughly 40 percent of the household income on direct expenses connected with child rearing (e.g., food, housing, education, transportation, baby-sitting, and day-care).[12]

Insert Table 1 about here

3.2 Results

In our model the decision to have a child is like the decision to exercise an option. If the option is exercised the household receives a deterministic stream of altruistic utility from being a parent and has to pay the stochastic costs associated with raising a child. If the household exercises the option she foregoes the value of waiting and perhaps exercising the option under better circumstances. The certainty equivalent of the fertility option (x) provides an assessment of the value of the fertility option for the household. The certainty equivalent is the additional initial wealth that the household must be provided to be indifferent between having the fertility option or not.[13]

$$V^0(W + x, y) = H(W, q, y) \tag{23}$$

Given the CRRA preferences, the value of the fertility option is a function of the household's wealth, income and cost of having a child. The certainty equivalent behaves conversely to the fertility boundary $\overline{q}(y, W)$.[14] In particular, since fertility is triggered when the cost crosses the boundary from above, an increase in $\overline{q}$ means an increase in fertility, because the household is willing to accept a higher cost in order to have a child. This condition is always associated with a smaller certainty equivalent, which justifies the diminished incentive to wait and postpone the decision. In the following, we will concentrate on the behavior of the fertility boundary and consumption described in Proposition 1.

We solve the model with a second order perturbation with respect to relative risk aversion around the logarithmic utility case ($\gamma = 1$). The accuracy of the results rely on how close we are to log utility. We focus on the qualitative patterns.[15]

[12] The CEX collects only limited data on expenditures directly attributable to children or childrearing. In particular, it collects information on children's clothing, toys, playground equipment, babysitting and daycare.

[13] Due to the wealth effects of CRRA preferences, this certainty equivalent is related, but does not coincide with the notion of utility-based option value, x^o, usually employed in the asset pricing literature, see Davis (1997)

$$V^0(W, y) = V(W - x^o, y, q). \tag{22}$$

[14] As we discuss in the Appendix the value function is increasing in wealth and decreasing in costs, hence certainty equivalent and exercise boundary must behave conversely.

[15] All the parameterizations that we use satisfy the transversality condition (10).

We start by analyzing the influence of cost volatility. Figures 1 and 2 report the fertility boundary and consumption respectively as a function of cost volatility for different levels of risk aversion and wealth.

Insert Figure 1 about here

Insert Figure 2 about here

In panels A we set wealth at the benchmark level, while in panels B we consider a high wealth value that corresponds to the highest percentile of the wealth distribution. All panels report the decision boundary for three values of the risk aversion parameter. We find the following results: i) Independently of wealth, higher cost volatility diminishes fertility. ii) Wealth alters the way in which the household's risk attitude impact the fertility and consumption decisions. iii) Consumption is decreasing in higher cost volatility but the slope flattens as wealth increases.[16] These results come from the interplay of three channels:

1) As in any real options problem, having the option to time investment implies asymmetric convex payoffs ("in bad times do not exercise and wait for good times to come"). This makes the value of the option increasing in the cost volatility. And it pushes the household to delay fertility as volatility increases. More risk averse households value more the option to "time fertility" because they are more sensitive to the consequences of investing in the bad state of the world.

2) Once the option is exercised the household starts bearing the uninsurable risk of the costs of raising a child. The problem of the household becomes a standard incomplete-markets consumption problem with stochastic income.[17] We can read this from equation (6) thinking on $(y_t - q_t)$ as income net of children costs. In a complete markets setup Friedman's permanent-income hypothesis would apply and the household would consume the annuity value of her expected present and future wealth. However, given that we assume incomplete markets, and that the household has a precautionary savings motive $(\gamma > 0)$, then she will reduce consumption for precautionary reasons.[18] This precautionary savings channel pushes the household to delay fertility as cost volatility increases. The delay will be larger the higher the risk aversion. Moreover, wealth will play a role since

[16] If preferences were CARA instead of CRRA the optimal decisions would be independent of wealth. Hence wealth would have no effect on how risk aversion affects the agent's decision.

[17] See for example Caballero (1991) and Wang (2006).

[18] The precautionary savings effect will be present both before and after the option exercise but it has a larger impact after the option exercise because the option gives the flexibility of timing the fertility decision, what is an insurance against the stochastic child costs.

precautionary savings are decreasing in wealth (they are less needed when wealth is high).

3) Exercising the option gives a deterministic flow of altruistic utility that is separable from the utility of goods' consumption. Hence, a child is an alternative source of utility that can hedge fluctuations in goods consumption. This is an important characteristic of children when we think of them as an asset class. This channel pushes the household for early fertility to enjoy the children as an insurance mechanism.

Channels 1 and 2 explain why, for all levels of wealth and risk aversion, in Figure 1 the fertility decision boundary is decreasing in the volatility of the cost of having a child. Channel 3 explains why for medium or low wealth levels risk aversion speeds up fertility (Figure 1 panel A). This effect is reversed for high levels of wealth (Figure 1 panel B). Intuitively, when the household has little wealth to buffer negative income shocks, having a child is a way to hedge future consumption uncertainty. It assures a constant flow of altruistic utility that offsets low consumption levels. Thus, when wealth is medium or low, more risk averse households trigger fertility sooner, they tradeoff utility from fertility against utility from consumption, as reflected in the lower consumption levels in Panel A of Figure 2. However, channel 3 weakens when wealth is high because the utility flow brought by the child is smaller relative to the utility flow of consumption. Hence, as channels 1 and 2 predict, in Panel B of Figures 1 and 2 both the consumption and the fertility are higher the lower the risk aversion.

Figure 3 shows that when income and cost shocks are negatively correlated fertility is lower, i.e. the decision boundary is monotonically increasing in this correlation. The household is reluctant to give birth when positive cost shocks come together with bad income shocks. The reluctance is more pronounced when risk aversion is high. A pronatalist government may encourage fertility by altering this correlation. For example, a negative correlation may be the outcome of child illnesses having negative effects on the parents' careers, policies of State paid leaves in periods of high probability of child illnesses may break the correlation.

Insert Figure 3 about here

Figures 4 and 5 analyze how volatility of income growth affects the childbearing decision and consumption, respectively. Panel A plots the benchmark case when the correlation between risks is zero ($\rho = 0$). Panel B reports the same graphs when this correlation is positive (dashed line) or negative (solid line).

Insert Figure 4 about here

Insert Figure 5 about here

Panels A show that absent any effect on the covariance with child rearing costs, higher income volatility implies higher fertility and lower consumption. The intuition for this result is the channel 3 discussed before: if wealth is not too high, as the amount of risk increases the household wants to have a child to hedge uncertainty about future utility from consumption with the deterministic stream of utility provided by the child. As discussed in Figure 1 this effect vanishes for high levels of wealth. Moreover, the precautionary effect discussed as channel 2 pushes for lower consumption, as it is reflected in the downward slopping curve of Figure 5 panel A.

The Panels B of Figures 4 and 5 show that when the income and cost shocks are correlated, the sign of the correlation is key to understand if higher income volatility speeds up or delays fertility. When cost shocks hedge income shocks ($\rho > 0$) fertility increases in income volatility, and it decreases when the shocks add to each other ($\rho < 0$). This is consistent with the intuition of Figure 3. Moreover, Figure 5 panel B shows that the reaction of consumption to income volatility depends on the sign of the correlation between child costs and income. For positive correlation the shocks hedge each other, hence the precautionary savings demand is smaller and consumption higher.

Figures 6 and 7 show the reaction of fertility and consumption to the income over wealth ratio. The fertility reaction is not monotonic. There is a critical level of income-wealth ratio such that the fertility decision boundary is decreasing before it, and increasing after it. To understand this result it is useful to start when the ratio is very low, close to zero. In this case income is very small relative to wealth, and the household has a lot of wealth to easily buffer income shocks. This translates into very high consumption, as can be seen in Figure 7. Moreover, similarly to the result of Figure 1 panel B, the wealthy household triggers fertility easily since the risks associated with it are less of a concern. As we move to the right in the x-axis, the household has less wealth to hedge income fluctuations, and this translates into higher precautionary savings (consumption decreases in Figure 7 although at a slow rate) as the household starts to be more concerned with assuming children's risk (the fertility boundary decreases). However, there is a critical level when the income/wealth ratio is too high, the household has low wealth relative to her income and channel 3 enters into action. The household trades off children against consumption because childbearing helps her to smooth future utility, hence the fertility boundary becomes increasing. But this pushes her to save more to absorb more volatile cost shocks in the future, thus consumption starts to decrease at a higher rate (the slope in Figure 7 becomes more negative). The households at the right hand extreme of the x-axis illustrate the tradeoff: they have the larger fertility levels in Figure 6 and the lowest consumption in Figure 7.

Insert Figure 6 about here

Insert Figure 7 about here

Figure 8 shows that higher expected income growth increases fertility. This is the wealth effect first highlighted by Becker (1960) when children are normal goods and there is not quality-quantity trade-off. Figure 9 shows that the same intuition applies to wealth.

Insert Figure 8 about here

Insert Figure 9 about here

Finally, Figure 10 displays another traditional Beckerian effect: higher expected growth in the costs of raising a child encourages earlier fertility. It is the standard price substitution effect. It offsets the precautionary savings motive because when expected costs get higher, having a child and enjoying altruistic utility gets expensive relative to future consumption and cheaper relative to present consumption. Thus the increasing pattern of the decision boundary.

Insert Figure 10 about here

4 Empirical support for the theory

In this section we test if the theoretical results discussed before are consistent with the data. To do so we identify a variable that can serve as a proxy for the uninsurable volatility in the costs associated with raising a child. We focus on the distance of the grandparents to the parents. There are two arguments to support this choice. First, a large fraction of the uninsurable risks of child rearing seems related to the time costs for the parents. This was especially true in the U.S. in the late 1980s, when concerns that child care was in short supply, not of good enough quality, and too expensive for many families prompted the enactment of legislation in 1990 that expanded Federal support for child care (U.S. Congress 2000). Second, Cardia and Ng (2003) and Rupert and Zanella (2010) have documented that grandparents make substantial time transfers in the form of care of their grandchildren. Hence, we assume that households whose parents live closeby are better insured face to time costs shocks from raising a child. We use this variable as a proxy for child cost risk.

Our data are the 1987-88 National Survey of Families and Households (NSFH). The NSFH consists of a national probability sample of persons ages 19 and over who resided in the United States in 1987 and 1988. It comes with weights which reflect unequal probability sampling, poststratification and nonresponse adjustment and allow us to represent the U.S. population on age, sex, and race.[19] We use the household as our unit of analysis.

We focus on fertility decisions made in the last year. We construct a dummy variable F which takes the value one if the household had a child in the previous year to taking the NSFH survey. This dependent variable resembles the dichotomic fertility decision of the model better than the fertility rate. Moreover, in accordance with the theory, the decision is contemporaneous to economic indicators such as income, age, race and wealth that are also reported in the NSFH survey.[20]

In Table 2 we analyze the marginal influence of child cost risks on fertility. We estimate the following probit model

$$Prob(F = 1|dis, X) = \Phi(\alpha + \beta_{dis}\, dis + \beta' X) \tag{24}$$

where the explanatory variables are the distance of the householder from her parents (dis) and a set of control variables (X) commonly associated with the decision to have a child or not. We control for income, wealth, income over wealth, age of the householder, race (Latinos and African-American), religion and education. Φ is the standard normal density.

Insert Table 2 about here

Table 2 reports the control variables and the marginal effect on the fertility decision of each explanatory variable. The higher the distance to parents the smaller the likelihood of childbearing, consistent with the pattern of Figure 1. The marginal effect of distance is negative and significant in all probit specifications, each one including an increasing number of controls. This result supports that higher cost volatility reduces fertility.

In Table 3 we test another prediction of Figure 1, that wealth plays an important role in the

[19] For a brief overview of the NSFH, see ftp://elaine.ssc.wisc.edu/pub/nsfh/c1intro.002 and ftp://elaine.ssc.wisc.edu/pub/nsfh/README

[20] For our work the only drawback of the NSFH is the lack of data on consumption.

fertility decision. Less wealthy households are less affected by high levels of cost risk, as it is shown by the convex relation of fertility versus cost risk displayed in Panel A of Figure 1. Panel B in Figure 1 showed the opposite theoretical result (concave shape). Moreover, the higher the household's risk aversion the more (less) likely is that the less (more) wealthy household will have a child. We test these predictions by estimating the following probit model

$$Prob(F = 1|dis, RT) = \Phi(\alpha + \beta_{dis}\, dis + \beta_{RT} RT + \beta_{dis,RT} dis * RT) \tag{25}$$

The variable RT tries to capture the degree of household risk tolerance. It is the ratio of investments in stocks, bonds and mutual funds relative to the household's total wealth. Higher values of RT denote high risk tolerance, i.e. a lower degree of risk aversion, because a substantial percentage of the household's wealth is invested in risky assets

Insert Table 3 about here

As a robustness check, Table 3 also reports the estimates of a linear probability model (LPM)

$$Prob(F = 1|dis, RT) = \alpha + \beta_{dis}\, dis + \beta_{RA} RT + \beta_{dis,RA} dis * RT \tag{26}$$

Panel A of Table 3 reports the estimation for the low wealth sample, that we define as households in the lower 5% percentile of the wealth distribution. Panel B reports marginal effects for medium/high wealth households (those belonging to the top 50% percentile). Consistently with Figure 1, the marginal effect on fertility of risk tolerance, is positive for medium/high wealth households and negative for low wealth households. As predicted by the theoretical model the less (more) wealthy households are more (less) likely to have a child if they are highly risk averse. Both effects are highly statistically significant. The LPM model confirms the results obtained with the probit specification. On the other hand, Table 3 does not allow to claim that the marginal effect on fertility of distance is different by wealth group.

Ai and Norton (2003) emphasize the difficulty in interpreting interactions in nonlinear models. The interaction effect cannot be evaluated by looking at the sign, magnitude, or statistical significance of the coefficient on the conventional interaction term. The interaction effect is conditional on the independent variable, and therefore both the magnitude and statistical significance of the interaction term can vary across observations. We follow Ai and Norton (2003) to compute the corrected marginal effects and their significance at every predicted probability.

In Figure 11 we study the effect on fertility of the interaction between distance to parents and risk tolerance. Figure 11 reports the marginal effect of the interaction across the medium/high-wealth and the low-wealth sample, together with the corresponding z-statistics. In Figure 1 our model predicts a positive marginal effect of the interaction for both wealth classes: the fertility boundary of lower-wealth individuals is less negatively affected by their risk tolerance when cost volatility increases, while the fertility boundary of higher wealth-individuals is more positively affected. The empirical evidence reported in Figure 11 seems consistent with the model for both wealth groups, but in both cases the estimates are not statistically significant, as the z-statistics are low in absolute value for most observations.

We now turn to the following probit specification which is concerned with the income effect and its interaction with distance to parents:

$$Prob(F = 1|dis, I) = \Phi(\alpha + \beta_{dis}\, dis + \beta_I I + \beta_{dis,I} dis * I), \tag{27}$$

where I denotes income. Estimation results are reported in Table 4.

Insert Table 4 about here

The marginal effect of income on fertility is negative and significant. This could be consistent with the theory discussed in Figure 6 because most surveyed households display a low income over wealth ratio. In Figure 12 we analyze the interaction effect between income and parental distance following again the methodology of Ai and Norton (2003). The z statistics show that the marginal effect on fertility of this interaction is positive and statistically significant for a significant portion of the sample. While higher-income households are less prone to child-bearing *per se*, they are also less concerned with the risk of uninsurable child costs when making their fertility decision. According to our model, this could be due to the fact that households who raise a child face a positive correlation between income and child costs, and they can achieve a partial offset of the two. Indeed, as Table 5 shows, the sample correlation between income and parental distance is significantly positive across households who had a child in the last year, while it is negligible across those who did not.

Insert Table 5 about here

In Section 3 we argued that the elasticity of fertility to the income-over-wealth ratio changes sign according to the level of income-over-wealth, giving rise to the U-shaped pattern of the fertility boundary observed in Figure 6. We have not identified this pattern in the 1987-88 NSFH Survey. In the literature, the empirical support for this prediction is at best mixed. Using data from Switzerland and Singapore between 1970 and 1990, Hsing and Rios (1995) estimates a significant quadratic relationship between fertility rates and per capita GDP. Calibrating his model to a Brazilian household survey, Veloso (1999) also finds that the cross-section relationship between fertility and wealth displays a U-shaped pattern, reflecting differences in wealth composition between poor and rich families. Willis (1973) analyzes US survey data of 1960 and finds evidence in favor of a U-shaped dependence of fertility on income: in populations in which wives' education levels are low, the effect of income on fertility tends to be negative, and it becomes positive as these levels grow. More recently, Kremer and Chen (2002) argue that the fertility differential between uneducated and educated women is most pronounced in middle income countries and weak in both low and high income countries. This suggests that an income increase reduces fertility mostly for low income levels.

5 Concluding Remarks

In this paper we have studied the joint consumption and fertility decisions of a household who faces uninsurable shocks to both income and the cost of raising a child, and who can decide when to have a child. To do so we solved a real options model with incomplete markets and CRRA preferences.

While in general fertility is negatively affected by child rearing cost volatility, the extent to which this occurs and the extent to which higher risk aversion postpones or anticipates fertility depend on household's wealth. For low wealth households the utility flows from fertility provide an insurance mechanism that mitigate the negative impact of cost volatility, inducing more risk averse households to increase fertility. We also find that the correlation between income and child rearing costs substantially alters the fertility decisions. Moreover, our model is consistent with the nonlinear relation between fertility and income that part of the empirical literature has identified. In particular, our model implies a U-shaped dependence of fertility on the income over wealth ratio.

We find empirical support for most of our theoretical predictions using regression analysis on data from the U.S. National Survey of Families and Households.

The analysis of this paper can be generalized in a number of directions. For simplicity, we have not considered life-cycle effects in the household's problem. This is a relevant factor in the timing of fertility as, for example, the ability to be fertile is age dependent. We have also abstracted from the fact that income is endogenous and fertility alters labor supply. Introducing this into the model would allow to study career and fertility choices together. In the empirical analysis, we focused on a proxy for the uninsurable risks associated with child rearing (the distance of the grandparents to the parents as a measure of insurance against time costs shocks). Further empirical work may explore alternative ways to measure the risks associated with child rearing and the effects on fertility. We leave these extensions for future research.

References

Ai, C. and Norton, E.: 2003, "Interaction terms in logit and probit models", *Economics Letters* **80**(1), 123–129.

Becker, G.: 1960, "An economic analysis of fertility", *Demographic and economic change in developed countries* **11**, 209–231.

Becker, G., Murphy, K. and Tamura, R.: 1990, "Human capital, fertility, and economic growth", *Journal of Political Economy* **98**(5), 12–37.

Bhamra, H., Ferretti, R., Trojani, F. and Uppal, R.: 2007, "Solving for portfolio policies and asset prices using perturbation methods".

Brandt, M.: 2009, "Portfolio choice problems", *Handbook of financial econometrics: Tools and techniques* .

Caballero, R.: 1991, "Earnings uncertainty and aggregate wealth accumulation", *The American Economic Review* **81**(4), 859–871.

Cardia, E. and Ng, S.: 2003, "Intergenerational time transfers and childcare", *Review of Economic Dynamics* **6**(2), 431–454.

Chen, H., Miao, J. and Wang, N.: 2010, "Entrepreneurial Finance and Non Diversifiable Risk", *forthcoming in Review of Financial Studies* .

Davis, M.: 1997, *Option pricing in incomplete markets. Mathematics of Derivative Securities*, Editors: M.A.H. Dempster and S.R. Pliska. Cambridge University Press.

Dehejia, R. and Lleras-Muney, A.: 2004, "Booms, Busts, and Babies' Health", *Quarterly Journal of Economics* **119**(3), 1091–1130.

Dixit, A., Pindyck, R. and Davis, G.: 1994, *Investment under uncertainty*, Princeton University Press.

Henderson, V.: 2007, "Valuing the option to invest in an incomplete market", *Mathematics and Financial Economics* **1**(2), 103–128.

Hsing, Y. and Rios, R.: 1995, "Testing the Quadratic Relationship between Economic Development and the Fertility Rate: A Comment [with Reply]", *The American Economist* pp. 95–97.

Hugonnier, J. and Morellec, E.: 2007, "Corporate control and real investment in incomplete markets", *Journal of Economic Dynamics and Control* **31**(5), 1781–1800.

Jones, L., Schoonbroodt, A. and Tertilt, M.: 2008, "Fertility theories: can they explain the negative fertility-income relationship?".

Kogan, L. and Uppal, R.: 2002, "Risk aversion and optimal portfolio policies in partial and general equilibrium economies".

Kremer, M. and Chen, D.: 2002, "Income distribution dynamics with endogenous fertility", *Journal of Economic Growth* **7**(3), 227–258.

Kreyenfeld, M.: 2005, "Economic uncertainty and fertility postponement. Evidence from German panel data".

Manuelli, R. and Seshadri, A.: 2009, "Explaining International Fertility Differences", *Quarterly Journal of Economics* **124**(2), 771–807.

Miao, J. and Wang, N.: 2007, "Investment, consumption, and hedging under incomplete markets", *Journal of Financial Economics* **86**(3), 608–642.

Portner, C.: 2005, "Risk and Household Structure: Another Look at the Determinants of Fertility".

Rupert, P. and Zanella, G.: 2010, "Grandparenting and Labor Supply".

Sommer, K.: 2009, "Fertility Choice in a Life Cycle Model with Idiosyncratic Uninsurable Earnings Risk".

Stetsenko, S.: 2010, *"Essays on the Macroeconomics of Labor Markets"*, University of Pennsylvania Dissertations.

Tamura, R.: 2000, "Growth, fertility and human capital: A survey", *Spanish Economic Review* **2**(3), 183–229.

U.S. Census Bureau: 2010, "The 2010 Statistical Abstract".

U.S. House, Committee on Ways and Means: 2000, "Child Care", *House Ways and Means Green Book* .

Veloso, F.: 1999, "Wealth composition, endogenous fertility and the dynamics of income inequality".

Wang, N.: 2006, "Generalizing the permanent-income hypothesis: Revisiting Friedman's conjecture on consumption", *Journal of Monetary Economics* **53**(4), 737–752.

Willis, R.: 1973, "A new approach to the economic theory of fertility behavior", *The Journal of Political Economy* **81**(2), 14–64.

Appendix: Solution Approach

First, we consider the value function after the decision to have a child and its HBJ equation (9). We guess the following functional form for the value function J

$$J(W,q,y) = \frac{W^{1-\gamma}}{r(1-\gamma)} e^{r(1-\gamma)G(\hat{q},\hat{y})} \tag{28}$$

where $\hat{q}$ and $\hat{y}$ are, respectively, the ratio between child cost and income over wealth ($\hat{q} = q/W$ and $\hat{y} = y/W$). G is a function to be determined. The decision boundary, denoted by $\overline{q}$, is a critical level of the cost over wealth ratio. Substituting the guess (28) into the HBJ equation (9), performing the changes of variable $q \to \hat{q}$, $y \to \hat{y}$ and simplifying, we realize that G solves the nonlinear, non-homogeneous PDE[21]

$$\begin{aligned}\left(r^{\frac{1-\gamma}{\gamma}} e^{\frac{\gamma-1}{\gamma} rG} \frac{\gamma}{1-\gamma} - \frac{1}{1-\gamma}\right) + \left(1 + \frac{\hat{y}}{r} - \frac{\hat{q}}{r}\right) + G_{\hat{q}}\mu_q \hat{q} + G_{\hat{y}}\mu_y \hat{y} + G_{\hat{q}\hat{q}}\frac{\sigma_q^2}{2}\hat{q}^2 + G_{\hat{y}\hat{y}}\frac{\sigma_y^2}{2}\hat{y}^2 \\ + G_{\hat{y}\hat{q}}\sigma_y\sigma_q\hat{y}\hat{q}\rho - r(\gamma-1)G_{\hat{q}}^2\frac{\sigma_q^2}{2}\hat{q}^2 - r(\gamma-1)G_{\hat{y}}^2\frac{\sigma_y^2}{2}\hat{y}^2 - r(\gamma-1)G_{\hat{y}}G_{\hat{q}}\sigma_q\sigma_y\hat{q}\hat{y}\rho = 0\end{aligned} \tag{29}$$

Second, we apply a similar guess for the function V which appears in equation (11)

$$V(W,q,y) = \frac{W^{1-\gamma}}{r(1-\gamma)} e^{r(1-\gamma)U(\hat{q},\hat{y})} \tag{30}$$

Replacing the guess (30) into the HBJ equation (12) we conclude that U must satisfy the following partial differential equation

$$\begin{aligned}\left(r^{\frac{1-\gamma}{\gamma}} e^{\frac{\gamma-1}{\gamma} rU} \frac{\gamma}{1-\gamma} - \frac{1}{1-\gamma}\right) + \left(1 + \frac{\hat{y}}{r}\right) + U_{\hat{q}}\mu_q \hat{q} + U_{\hat{y}}\mu_y \hat{y} + U_{\hat{q}\hat{q}}\frac{\sigma_q^2}{2}\hat{q}^2 + U_{\hat{y}\hat{y}}\frac{\sigma_y^2}{2}\hat{y}^2 + U_{\hat{y}\hat{q}}\sigma_y\sigma_q\hat{y}\hat{q}\rho \\ - r(\gamma-1)U_{\hat{q}}^2\frac{\sigma_q^2}{2}\hat{q}^2 - r(\gamma-1)U_{\hat{y}}^2\frac{\sigma_y^2}{2}\hat{y}^2 - r(\gamma-1)U_{\hat{y}}U_{\hat{q}}\sigma_q\sigma_y\hat{q}\hat{y}\rho = 0\end{aligned} \tag{31}$$

The solution of these equations cannot be explicitly characterized. We then follow a perturbation approach proposed by Kogan and Uppal (2002) and Bhamra et al. (2007). It consists on a power series expansion of the value function with respect to the relative risk aversion coefficient γ around the logarithmic utility case ($\gamma = 1$). We represent both value functions J

[21] The transversality condition (10) must also hold.

and V, and the decision boundary $\overline{q}$, as power series

$$J(W,\hat{y},\hat{q}) = \sum_{i=0}^{\infty}(\gamma-1)^i J_i(W,\hat{y},\hat{q}) \tag{32}$$
$$V(W,\hat{y},\hat{q}) = \sum_{i=0}^{\infty}(\gamma-1)^i V_i(W,\hat{y},\hat{q}) \tag{33}$$
$$\overline{q}(W,\hat{y}) = \sum_{i=0}^{\infty}(\gamma-1)^i \overline{q}_i(W,\hat{y}) \tag{34}$$

Because of our guesses (28) and (30), and because of (32) and (33), G and U also have power series representations

$$G(\hat{y},\hat{q}) = \sum_{i=0}^{\infty}(\gamma-1)^i G_i(\hat{y},\hat{q}) \tag{35}$$
$$U(\hat{y},\hat{q}) = \sum_{i=0}^{\infty}(\gamma-1)^i U_i(\hat{y},\hat{q}) \tag{36}$$

Proposition 2 *We can check that,*[22] *up to the second-order, the following relations hold between the power series of $J(.)$ and $G(.)$*

$$J_0(W,\hat{y},\hat{q}) = \frac{\log(W)}{r} + G_0(\hat{y},\hat{q}) \tag{37}$$
$$J_1(W,\hat{y},\hat{q}) = G_1(\hat{y},\hat{q}) - \frac{1}{2r}\left(rG_0(\hat{y},\hat{q}) + \log(W)\right)^2 \tag{38}$$
$$J_2(W,\hat{y},\hat{q}) = G_2(\hat{y},\hat{q}) - G_1(\hat{y},\hat{q})(G_0(\hat{y},\hat{q})r + \log(W)) + \frac{1}{6r}\left(rG_0(\hat{y},\hat{q}) + \log(W)\right)^3 \tag{39}$$

The power series of $V(.)$ and $U(.)$ have a similar relation up to the second-order.

Now we discuss how we characterize the functions G_i and U_i for $i = 0, 1, 2$. The case $i = 0$

[22] To check this, we develop the right hand side of (28) into a power series around $\gamma = 1$. Then we substitute

$$G = G_0 + (\gamma-1)G_1 + (\gamma-1)^2 G_2$$

and group terms proportional to $(\gamma-1)$ and $(\gamma-1)^2$

corresponds to $\gamma = 1$, the logarithmic utility case. G_0 solves equation (29) when $\gamma \to 1$. It is

$$G_0(\hat{y}, \hat{q}) = K_1 + K_2\hat{y} + K_3\hat{q} \tag{40}$$

$$K_1 = \frac{\log r}{r} \tag{41}$$

$$K_2 = \frac{1}{r(r - \mu_y)} \tag{42}$$

$$K_3 = \frac{1}{r(\mu_q - r)} \tag{43}$$

The transversality condition implies $\mu_y < r$ and $\mu_q < r$.

U_0 solves equation (31) when $\gamma \to 1$. It is

$$U_0(\hat{y}, \hat{q}) = \overline{K}_1 + \overline{K}_2\hat{y} + \overline{K}_3\hat{q}^{\beta} \tag{44}$$

$$\overline{K}_1 = \frac{\log r}{r} \tag{45}$$

$$\overline{K}_2 = \frac{1}{r(r - \mu_y)} \tag{46}$$

where β is the negative solution of the quadratic equation[23]

$$\beta(\beta - 1)\sigma_q^2/2 + \beta\mu_q - r = 0 \tag{47}$$

Once G_0 and U_0 are known we can use relation (37) to get J_0 and V_0. Then, substituting J_0 and V_0 into the value matching and smooth-pasting conditions gives the constant $\overline{K}_3$ and the decision boundary $\overline{q}_0$

$$\overline{q}_0 = \left(\frac{K_3}{\beta\overline{K}_3}\right)^{\frac{1}{\beta-1}} \tag{48}$$

$$\overline{K}_3 = [(\alpha - 1)(1 - \beta)]^{\beta-1}\left(\frac{\beta}{K_3}\right)^{-\beta} \tag{49}$$

Equation (48) states that the decision boundary is constant for the household with logarithmic preferences.

To characterize the first-order terms G_1, U_1, and $\overline{q}_1$, we substitute

$$G = G_0 + (\gamma - 1)G_1 \tag{50}$$

[23] By direct substitution, we realize that (44) solves the HBJ equation (31) for the logarithmic case provided that the quadratic equation (47) holds. Selecting β as the negative solution of this equation insures that the no-bubble condition (13) is satisfied.

and

$$U = U_0 + (\gamma - 1)U_1 \tag{51}$$

into equations (29) and (31), respectively, and take into account the HBJ equation solved by the solutions G_0 and V_0 of the logarithmic case. We conclude that G_1 solves the following linear, non-homogeneous PDE

$$\begin{aligned} rG_1 = {} & G_{1\hat{q}}\mu_q\hat{q} + G_{1\hat{y}}\mu_y\hat{y} + G_{1\hat{q}\hat{q}}\frac{\sigma_q^2}{2}\hat{q}^2 + G_{1\hat{y}\hat{y}}\frac{\sigma_y^2}{2}\hat{y}^2 + G_{1\hat{y}\hat{q}}\sigma_y\sigma_q\hat{y}\hat{q}\rho \\ & - rG_{0\hat{q}}^2\frac{\sigma_q^2}{2}\hat{q}^2 - rG_{0\hat{y}}^2\frac{\sigma_y^2}{2}\hat{y}^2 - rG_{0\hat{y}}G_{0\hat{y}}\sigma_q\sigma_y\hat{q}\hat{y}\rho - \frac{1}{2}\left(-G_0 r + \log(r)\right)^2 \end{aligned} \tag{52}$$

The function V_1 solves an identical PDE. We guess the following functional forms for the solutions to these equations and for the first order exercise boundary $\overline{q}_1(W, \hat{y})$

$$G_1 = C_4\hat{y}\hat{q} + C_5\hat{q}^2 + C_6\hat{y}^2 \tag{53}$$

$$U_1 = \overline{C}_4\hat{y}\hat{q}^{\beta} + \overline{C}_5\hat{y}^2 + \overline{C}_6\hat{q}^{2\beta} + \overline{F_1}\hat{q}^{\beta_1} + \overline{F}_2\hat{y}\hat{q}^{\beta_2} \tag{54}$$

$$\overline{q}_1 = A + B\hat{y} \tag{55}$$

where constants A, B, $\overline{F}_1$ and $\overline{F}_2$ are identified by substituting (54)-(55) into the value-matching and smooth pasting conditions, by taking into account the value-matching and smooth-pasting conditions of the household with logarithmic preferences,[24] and by matching coefficients that are proportional to the powers of $\hat{y}$. The remaining constants are identified by the differential equation (52) and its counterpart for V_1.

The second order terms G_2, U_2 and $\overline{q}_2$ are obtained applying a similar procedure. In general, the $n-$th order exercise boundary is a $n-$degree polynomial in $\hat{y}$. The next propositions summarize our power series approximation to the value functions J and V [25]

Proposition 3 *After the decision to have a child, the value function of the household is, to the second-order in γ,*

$$\begin{aligned} J(W,\hat{y},\hat{q}) = {} & \frac{\log(W)}{r} + G_0 + (\gamma-1)\left[G_1 - \frac{1}{2r}\left(rG_0 + \log(W)\right)^2\right] \\ & + (\gamma-1)^2\left[G_2 - G_1(G_0 r + \log(W)) + \frac{1}{6r}\left(rG_0 + \log(W)\right)^3\right] \end{aligned} \tag{56}$$

[24] I.e., the value-matching and smooth-pasting conditions satisfied by the the zero-th order value function (U_0), and fertility boundary ($\overline{q}_0$) .

[25] We do not report coefficients appearing in first and second-order terms, but they are available upon request.

where G_0 is given in (40), while G_1 and G_2 are

$$G_1(\hat{y},\hat{q}) = C_4\hat{y}\hat{q} + C_5\hat{y}^2 + C_6\hat{q}^2 \tag{57}$$

$$G_2(\hat{y},\hat{q}) = D_4\hat{y}\hat{q} + D_5\hat{q}^2 + D_6\hat{y}^2 + D_7\hat{q}^2\hat{y} + D_8\hat{y}^2\hat{q} + D_9\hat{q}^3 + D_{10}\hat{y}^3 \tag{58}$$

Proposition 4 *Before the decision to have a child, the value function of the household is $V + \frac{1}{(1-\alpha)r}$, where V is, to the second order in γ,*

$$\begin{aligned} V(W,\hat{y},\hat{q}) = &\frac{\log(W)}{r} + U_0 + (\gamma-1)\left[U_1 - \frac{1}{2r}\left(rU_0 + \log(W)\right)^2\right] \\ &+ (\gamma-1)^2\left[U_2 - U_1(U_0 r + \log(W)) + \frac{1}{6r}\left(rU_0 + \log(W)\right)^3\right] \end{aligned} \tag{59}$$

where U_0 is given in (44) while U_1 and U_2 are

$$U_1(\hat{y},\hat{q}) = \overline{C}_4\hat{y}\hat{q}^{\beta} + \overline{C}_5\hat{y}^2 + \overline{C}_6\hat{q}^{2\beta} + \overline{F}_1\hat{q}^{\beta} + \overline{F}_2\hat{y}\hat{q}^{\beta_1} \tag{60}$$

$$\begin{aligned} U_2(\hat{y},\hat{q}) = &\overline{D}_4\hat{y}\hat{q}^{\beta} + \overline{D}_5\hat{y}^2 + \overline{D}_6\hat{q}^{2\beta} + \overline{D}_7\hat{q}^{2\beta}\hat{y} + \overline{D}_8\hat{y}^2\hat{q}^{\beta} + \overline{D}_9\hat{q}^{3\beta} + \overline{D}_{10}\hat{y}^3 + \overline{D}_{12}\hat{y}^2\hat{q}^{\beta_2} \\ &+ \overline{D}_{13}\hat{q}^{\beta_1}\hat{y} + \overline{D}_{15}\hat{q}^{\beta_1+\beta} + \overline{D}_{16}\hat{q}^{\beta_2+\beta}\hat{y} + \overline{L}_1\hat{q}^{\nu_1} + \overline{L}_2\hat{q}^{\nu_2}\hat{y} + \overline{L}_3\hat{q}^{\nu_3}\hat{y}^2 \end{aligned} \tag{61}$$

where

$$\nu_1 = \beta \tag{62}$$

$$\nu_2 = \beta_1 \tag{63}$$

while β_1 and ν_3 are, respectively, the negative roots of the following quadratic equations

$$\beta_1(\beta_1 - 1)\frac{\sigma_y^2}{2} + \beta_1(\mu_q + \sigma_y\sigma_q\rho) + \mu_y - r = 0 \tag{64}$$

$$\frac{1}{2}\nu_3(\nu_3 - 1)\sigma_q^2 + \nu_3(\mu_q + \sigma_q\sigma_y\rho) - r + 2\mu_y + \sigma_y^2 = 0 \tag{65}$$

To obtain the second order representation of the household's consumption rule we apply the same methodology that we have used to obtain J and V. Basically, before the childbearing decision, from equation (12) the household's optimal consumption must satisfy

$$c = V_W^{-\frac{1}{\gamma}} \tag{66}$$

If we denote by $\hat{c} = c/W$ the consumption over wealth ratio, then our guess (30) leads to

$$\hat{c}(\hat{q}, \hat{y}) = \frac{1}{r^{-\frac{1}{\gamma}}} e^{r \frac{\gamma - 1}{\gamma} U(\hat{q}, \hat{y})}. \tag{67}$$

Expanding (67) to the second-order in γ around $\gamma = 1$, substituting

$$U = U_0 + (\gamma - 1)U_1 + (\gamma - 1)^2 U_2 \tag{68}$$

and grouping terms proportional to $(\gamma - 1)$ and $(\gamma - 1)^2$, we obtain the household's consumption rule before the childbearing decision reported in Proposition 1. I.e.

$$\begin{aligned}\hat{c} &= r + (\gamma - 1)(U_0 r^2 - r log(r)) + \\ &(\gamma - 1)^2 \left[-U_0 r^2 + U_1 r^2 + \frac{U_0^2 r^3}{2} + r \log r - U_0 r^2 \log r + \frac{1}{2} r (\log r)^2 \right]\end{aligned}$$

To obtain the consumption rule after the childbearing decision we follow the same steps but now using equations (9) and (28). We obtain

$$\begin{aligned}\hat{c} &= r + (\gamma - 1)(G_0 r^2 - r log(r)) + \\ &(\gamma - 1)^2 \left[-G_0 r^2 + G_1 r^2 + \frac{G_0^2 r^3}{2} + r \log r - G_0 r^2 \log r + \frac{1}{2} r (\log r)^2 \right]\end{aligned} \tag{69}$$

It remains to show that the value function before the childrearing decision is decreasing in the current cost over wealth ratio, that is

$$H(W_0, q_0, y_0) \geq H(\tilde{W}_0, \tilde{q}_0, y_0) \text{ for } q_0/W_0 \leq \tilde{q}_0/\tilde{W}_0$$

By a dynamic programming principle,[26] the value function $H(W_0, q_0, y_0)$ can be written as

$$H(W_0, q_0, y_0) = \sup_{c, \tau} \mathbb{E} \left[\int_0^{\tau} e^{-rs} \frac{c_s^{1-\gamma} - 1}{1 - \gamma} ds + J(W_\tau, q_\tau, y_\tau) \right] + \frac{1}{r(1 - \alpha)} \tag{70}$$

Let τ^* denote the optimal time to have a child and c^* the optimal consumption policy. Since $q_0/W_0 \leq \tilde{q}_0/\tilde{W}_0$, either we have $q_0 \leq \tilde{q}_0$, or $W_0 \geq \tilde{W}_0$, or both. In either case, the policies τ^* and c^* are feasible also when the consumption-over wealth ratio is q_0/W_0. In particular, by standard comparison theorems for stochastic differential equations, we have $q_{\tau^*} \leq \tilde{q}_{\tau^*}$ and

[26] This argument is adapted from Miao and Wang (2007).

$W_{\tau^*} \geq \tilde{W}_{\tau^*}$. It then follows that

$$H(W_0, q_0, y_0) > \mathbb{E}\left[\int_0^{\tau^*} e^{-rs} \frac{c_s^{*1-\gamma} - 1}{1-\gamma} \, ds + J(W_{\tau^*}, q_{\tau^*}, y_{\tau^*})\right] + \frac{1}{r(1-\alpha)} >$$
$$\mathbb{E}\left[\int_0^{\tau^*} e^{-rs} \frac{c_s^{*1-\gamma} - 1}{1-\gamma} \, ds + J(\tilde{W}_{\tau^*}, \tilde{q}_{\tau^*}, y_{\tau^*})\right] + \frac{1}{r(1-\alpha)} = H(\tilde{W}_0, \tilde{q}_0, y_0) \quad (71)$$

The last inequality follows from the fact that the value function after childrearing (J) is increasing in wealth and decreasing in cost, while the first follows from the optimality principle (70).

Tables

Table 1: Benchmark calibration

$\mu_q = 0.02$	$\mu_y = \mu_q$	$\sigma_y = 0.05$
$r = 0.5$	$\gamma = 2$	$\sigma_q = \sigma_y$
$\alpha = 3$		$\rho = 0$

Notes: This table shows the parameters used to numerically solve the model and construct the figures of Section 3. The parameter values are discussed in page 11.

Table 2: Probit Regressions on the Determinants of Fertility

Dependent Variable: Dummy variable for having had a child last year					
	[1]	**[2]**	**[3]**	**[4]**	**[5]**
Distance to the parents	-0.00005**	-0.00005**	-0.00004**	-0.00004**	-0.00004*
	(0.00002)	(0.00002)	(0.00002)	(0.00002)	(0.00002)
Income		-0.0828**	-0.0032	0.0058	0.0085
		(0.0423)	(0.0387)	(0.0384)	(0.0375)
Wealth		0.0017	0.0024	0.0025	0.0024
		(0.0040)	(0.0032)	(0.0031)	(0.0031)
Income/Wealth		0.00032	0.00032	0.00032	0.00032
		(0.00028)	(0.00020)	(0.00025)	(0.00024)
Age of householder			-0.009***	-0.00923***	-0.00953***
			(0.00100)	(0.00152)	(0.00151)
Dummy for being latino				0.0288	0.02079
				(0.05689)	(0.05630)
Dummy for being black				0.0516*	0.0459
				(0.03077)	(0.03090)
Dummy for nonreligious person					-0.06694**
					(0.026)
Education Level					-0.00213
					(0.00525)
Observations	1056	1056	1056	1055	1048

Notes: This table shows the marginal effects arising from the estimation of Equation (24) on 1987-88 NFSH household data. The marginal effect is calculated as dF/dx at the sample mean of x for continuous variables. When x is a dummy variable dF/dx is a discrete change from 0 to 1. The dependent variable F, is an indicator function that takes the value 1 if the household had a child in the last year. Income and wealth are measured as multiples of US$ 100,000. The estimation included a constant. Robust standards errors are in parenthesis. The symbols *, **, and *** mean statistically different from zero at the 10, 5 and 1% level of significance.

Table 3: Effects of Risk Tolerance on Fertility by Wealth Level

Dependent Variable: Dummy variable for having had a child last year

A. Low Wealth

	LPM	Probit
Distance to the parents	-0.0001**	-.00021
	(0.000045)	(0.0021)
Risk Tolerance (RT)	-2.34401***	-16.8461**
	(0.6073)	(7.7682)
Distance*RT	0.00345	0.14679
	(0.00389)	(0.1095)
Observations	52	52

B. Medium/High Wealth

	LPM	Probit
Distance to the parents	-0.000319*	-0.000248
	(0.0000171)	(0.00016)
Risk Tolerance (RT)	0.19644**	0.6831**
	(0.01)	(0.3086)
Distance*RT	0.0000428	0.00033
	(0.00013)	(0.00046)
Observations	515	515

Notes: This table shows the marginal effects arising from the estimation of Equations 25 and 26 on 1987-88 NFSH household data. The estimation included a constant. We proxy risk tolerance with the ratio between investments in stocks, bonds and mutual funds relative to household wealth. Robust standards errors are in parenthesis. The symbols *, **, and *** mean statistically different from zero at the 10, 5 and 1% level of significance. Low wealth means below 5% wealth percentile in the sample, while medium/high wealth is the upper 50% percentile. Figure 11 reports the interaction effects and corresponding z-statistics of the interaction variable computed according to Ai and Norton (2003).

Table 4: **Effects of Income on Fertility**

Dependent Variable: Dummy variable for having had a child last year	
Distance to the parents	-0.00044**
	(0.00019)
Income	-0.415**
	(0.207)
Distance*Income	0.0006
	(0.000044)
Observations	1048

Notes: This table shows the marginal effects arising from the estimation of Equation 27 on 1987-88 NFSH household data. The estimation included a constant. Income is measured as multiples of US$ 100,000. Robust standards errors are in parenthesis. The symbols *, **, and *** mean statistically different from zero at the 10, 5 and 1% level of significance. Figure 12 reports the interaction effects and corresponding z-statistics of the interaction variable computed according to Ai and Norton (2003).

Table 5: Correlation of Income and Distance to Parents

Did the household have a child during the last year?	**Yes**	**No**
Correlation of Income and Distance to the parents	0.226	0.0083
Observations	188	860

Notes: This table shows the correlation between income and the distance to parents. The column on the left reports the correlation for those households who had a child in the last year, while the column on the right reports the correlation for those households who did not have a child. Data are from the 1987-88 NFSH Survey.

Figures

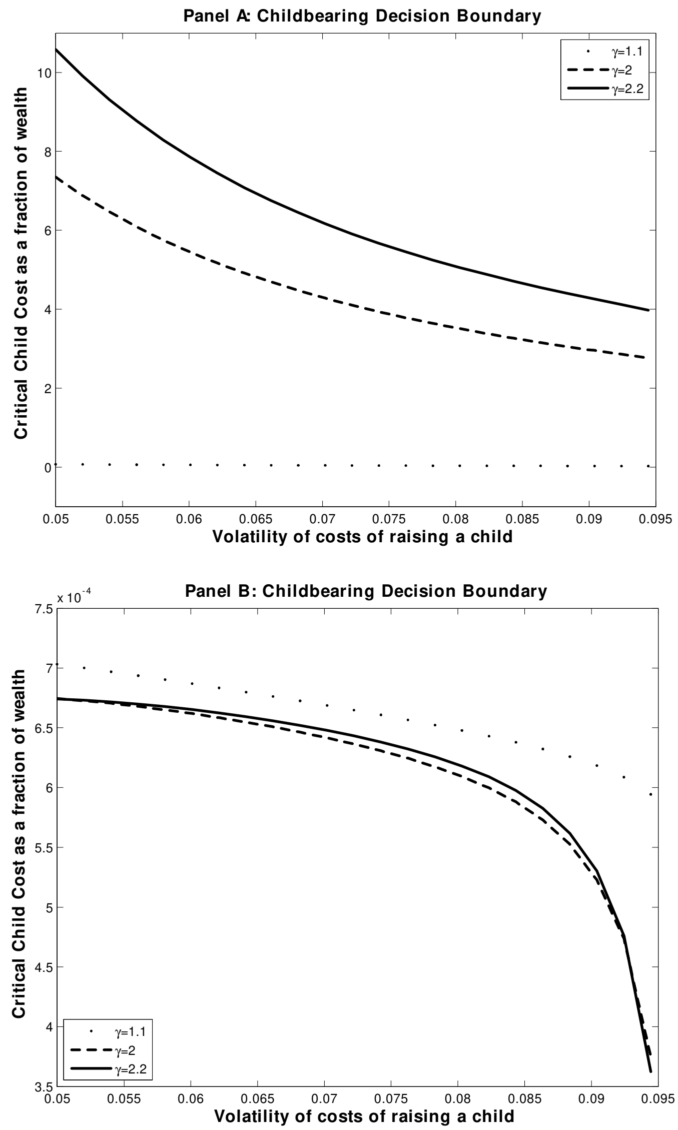


Figure 1: *Fertility as a function of cost volatility.* Panel A reports the fertility decision boundary (critical cost over wealth ratio at which the household takes the fertility decision) as a function of cost volatility, when the household's wealth is the 2007 median U.S. household as reported by the U.S. Census Bureau (2010). The solid, dashed, and dotted line correspond to a relative risk aversion coefficient γ of 2.2, 2, and 1.1, respectively. Panel B reports the boundary as a function of volatility, when household's wealth is in the highest percentile of the 2007 Households net worth distribution (U.S. Census Bureau 2010).

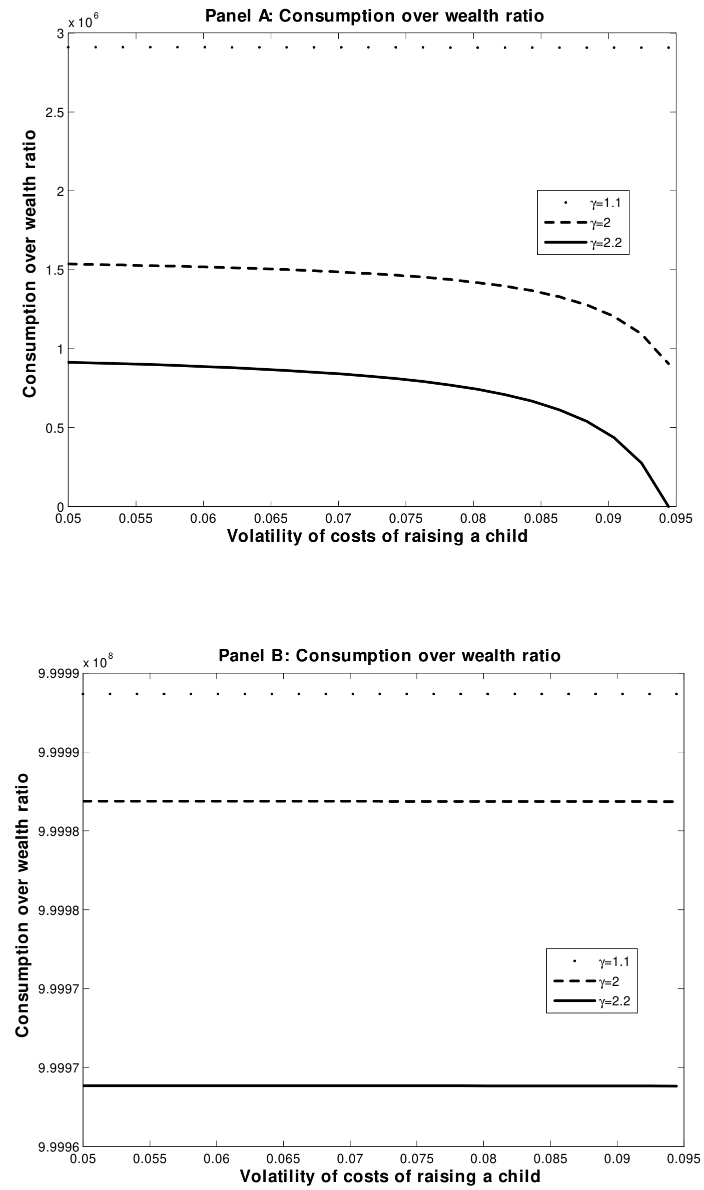


Figure 2: *Consumption as a function of cost volatility.* Panel A reports the consumption over wealth ratio as a function of cost volatility, when the household's wealth is the 2007 median U.S. household as reported by the U.S. Census Bureau (2010). The solid, dashed, and dotted line correspond to a relative risk aversion coefficient γ of 2.2, 2, and 1.1, respectively. Panel B reports the consumption over wealth ratio as a function of volatility, when household's wealth is high, namely in the highest percentile of the 2007 Households net worth distribution according to the U.S. Census Bureau (2010).

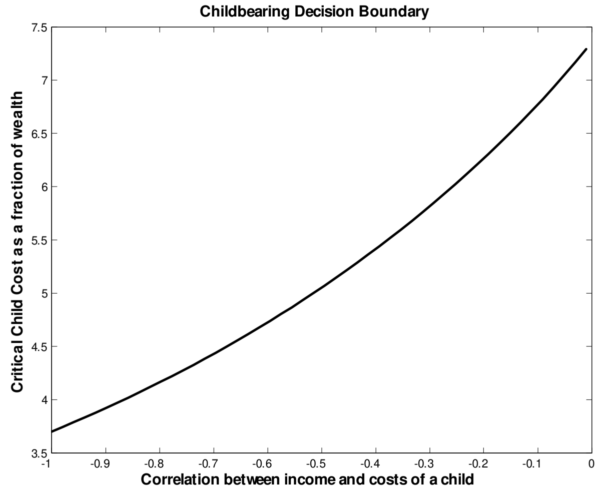
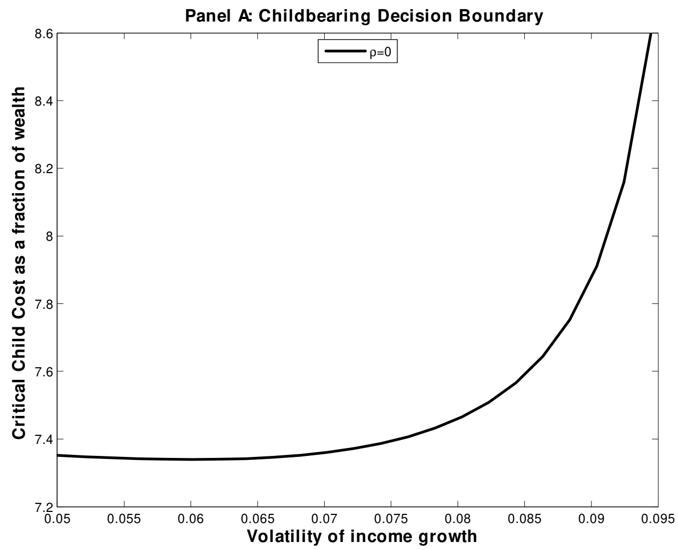


Figure 3: *Fertility as a function of the correlation between income and cost growth.* This Figure reports the fertility decision boundary (critical cost over wealth ratio at which the household takes the fertility decision) as a function of the correlation between shocks to income growth (dy/y) and to cost growth (dq/q).



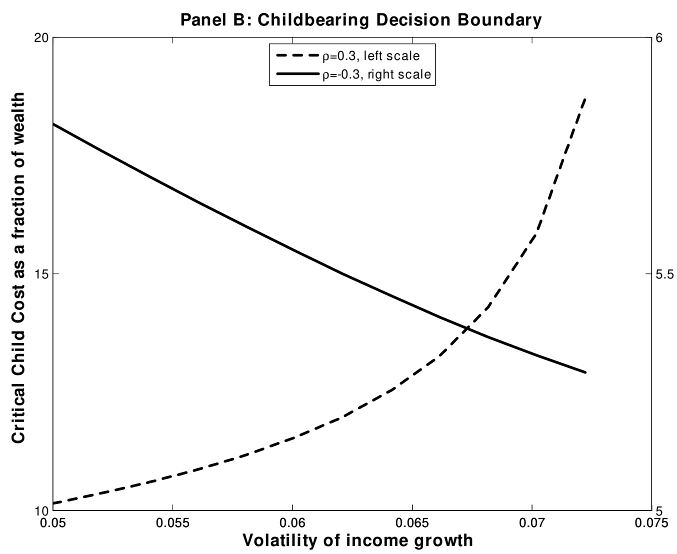
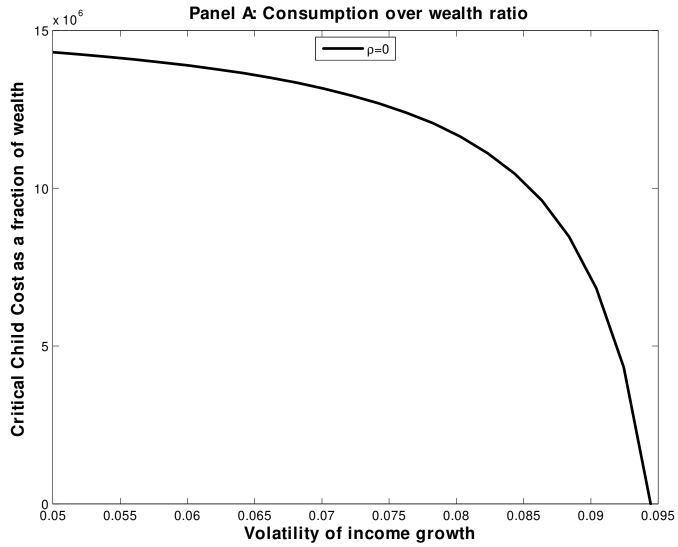


Figure 4: *Fertility as a function of volatility of income growth.* Panel A reports the fertility decision boundary (critical cost over wealth ratio at which the household takes the fertility decision) as a function of volatility of income growth, when the correlation between income and child rearing costs is zero. Panel B reports the same graph when this correlation is positive (dashed line) or negative (solid line).



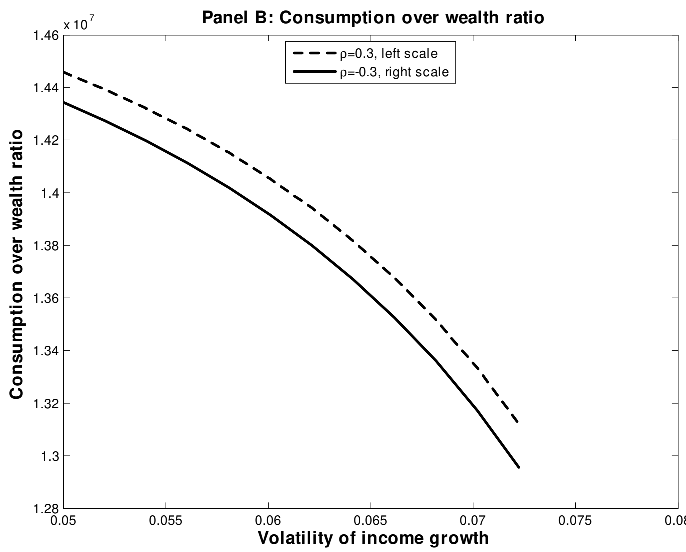


Figure 5: *Consumption as a function of volatility of income growth*. Panel A reports the consumption over wealth ratio as a function of volatility of income growth, when the correlation between income and child rearing costs is zero. Panel B reports the same graph but when this correlation is positive (dashed line) or negative (solid line).

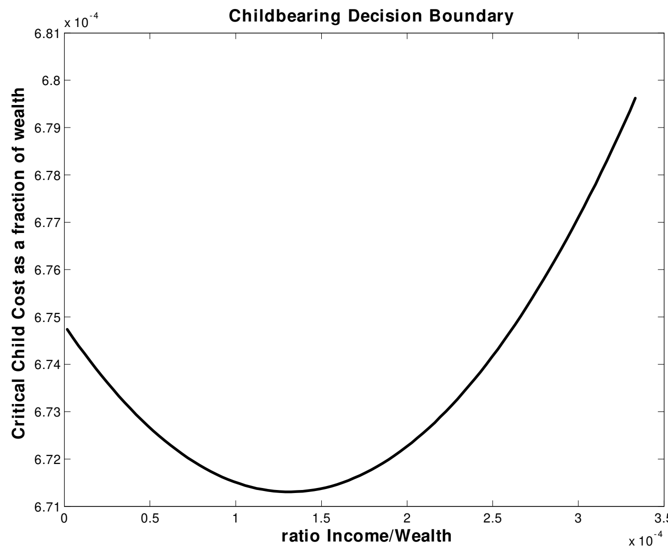


Figure 6: *Fertility as a function of the income to wealth ratio.* This figure reports the fertility decision boundary (critical cost over wealth ratio at which the household takes the fertility decision) as a function of the current income level relative to wealth.

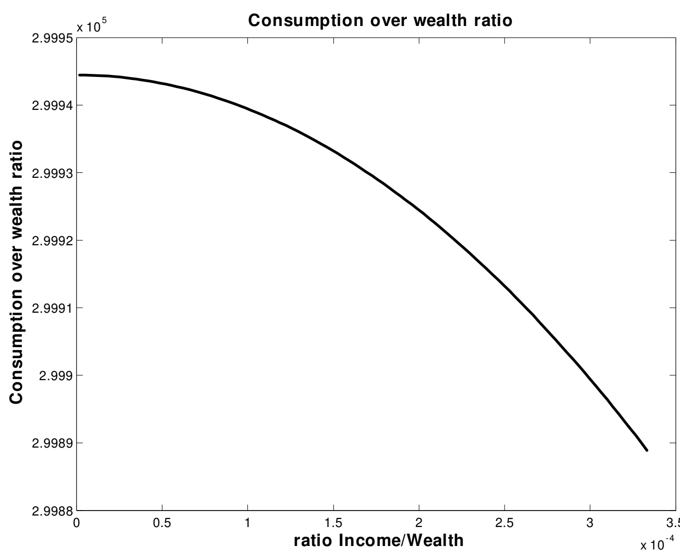


Figure 7: *Consumption as a function of the income to wealth ratio.* This figure reports the consumption over wealth ratio as a function of the current income level relative to wealth.

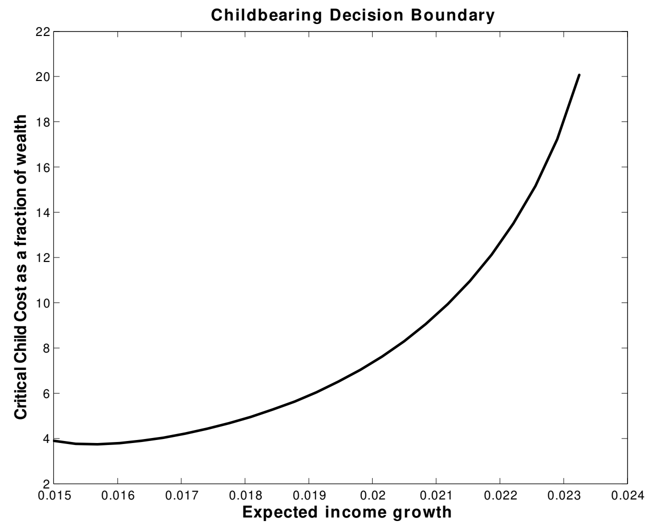


Figure 8: *Fertility as a function of expected income growth.* This Figure reports the fertility decision boundary (critical cost over wealth ratio at which the household takes the fertility decision) as a function of expected income growth.

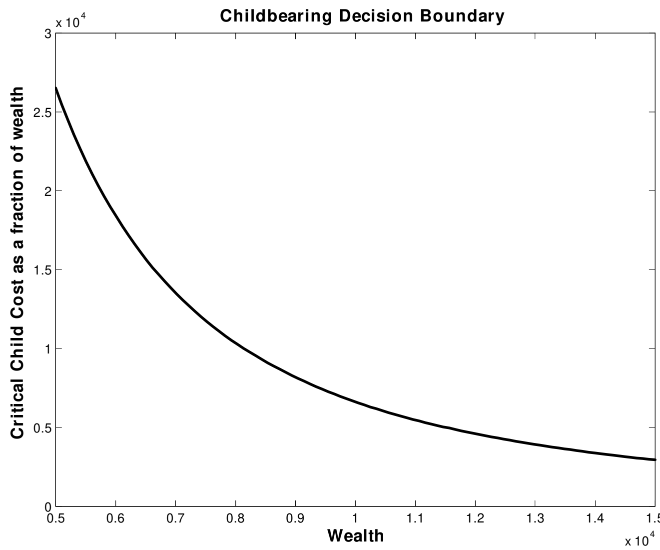


Figure 9: *Fertility as a function of wealth.* This figure reports the fertility decision boundary (critical cost over wealth ratio at which the household takes the fertility decision) as a function of current wealth.

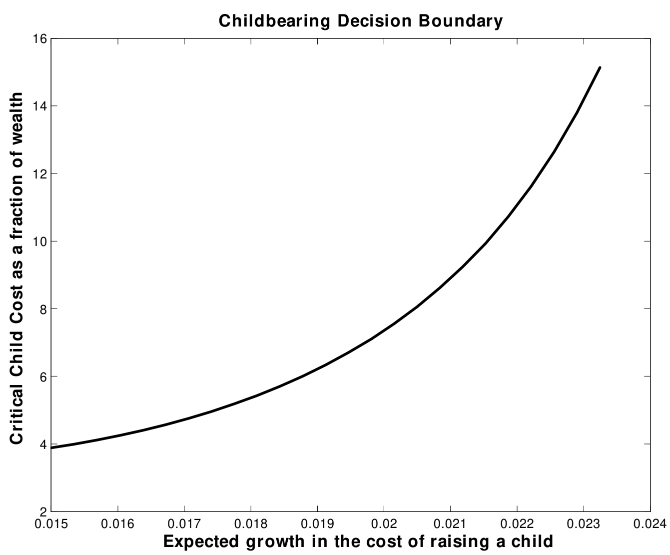


Figure 10: *Fertility as a function of expected cost growth.* This figure reports the fertility decision boundary (critical cost over wealth ratio at which the household takes the fertility decision) as a function of the expected growth in the costs of child rearing.

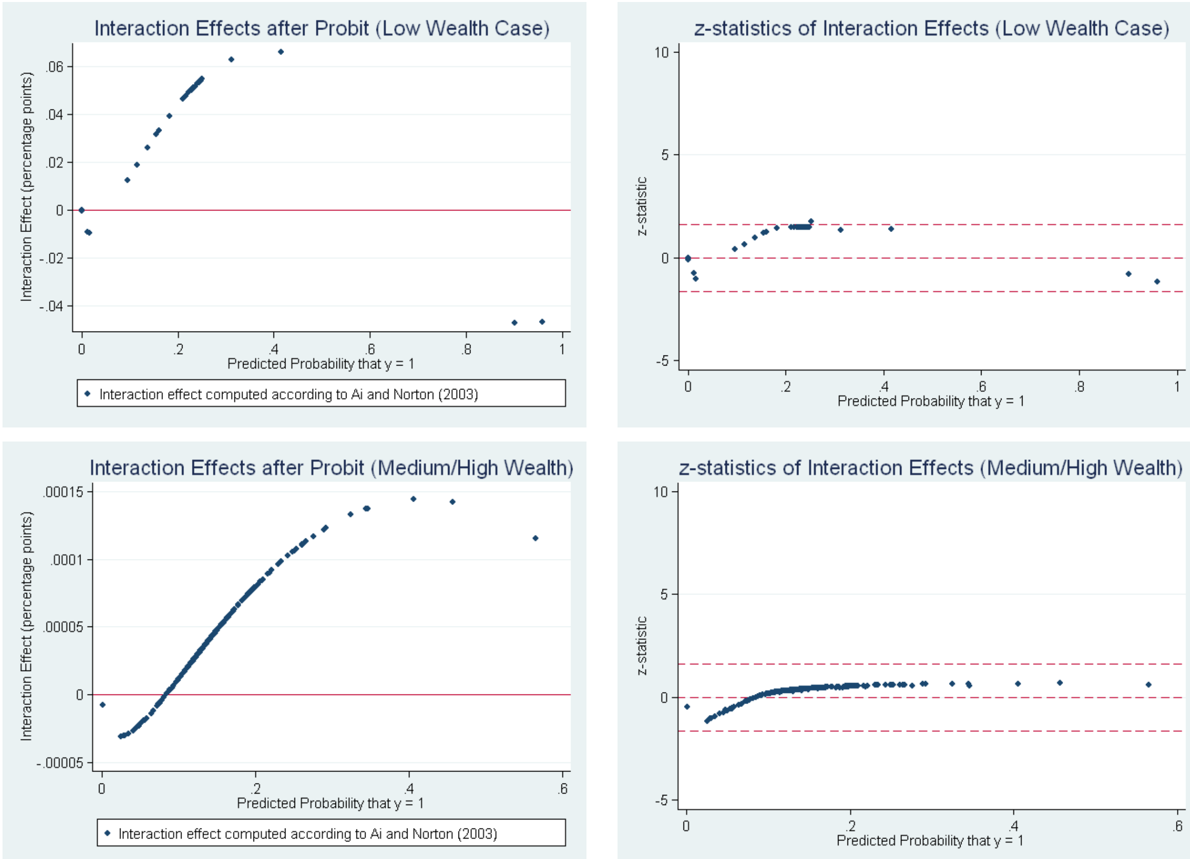


Figure 11: *Marginal effect of the interaction between distance and risk tolerance on fertility after probit.* This figure complements Table 3. It reports the marginal effect and corresponding z-statistics of the interaction variable (parental distance*risk tolerance) on the dummy variable for having had a child in the last year. The interaction is estimated following the methodology proposed by Ai and Norton (2003). The lines above and below zero in the figures located on the right side represent the 10% significance levels. The upper level row has the results for the low wealth case (below 5% wealth percentile in the sample). The row at the bottom has the results for the medium/high wealth case (upper 50% wealth percentile in the sample).

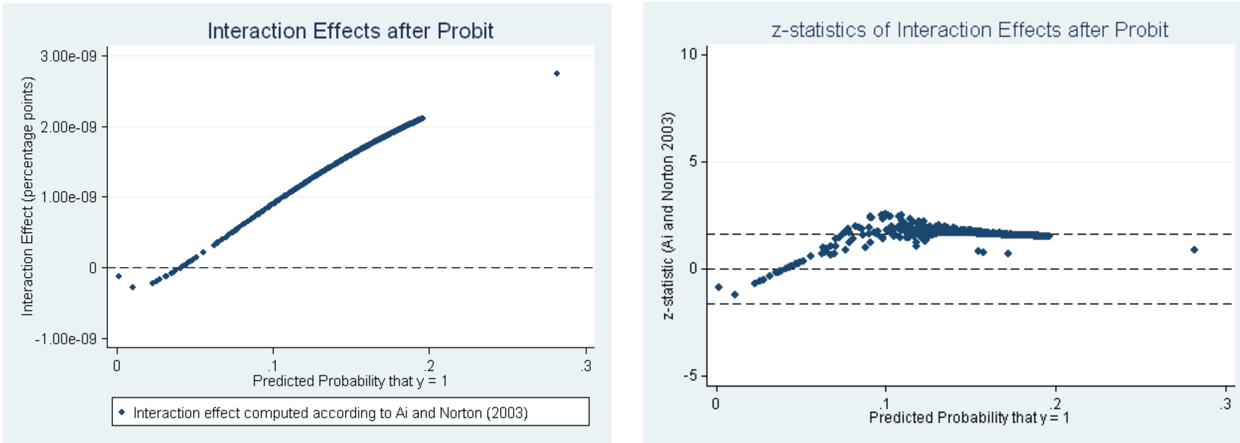


Figure 12: *Marginal effect of the interaction between distance and income on fertility after probit.* This figure complements Table 4. It reports the marginal effect and corresponding z-statistics of the interaction variable (distance*income) on the dummy variable for having had a child in the last year. They are estimated following the methodology proposed by Ai and Norton (2003). The lines above and below zero in the figures located on the right side represent the 10% significance levels.