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# Stationarity of time series and the problem of spurious regression

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## Abstract

*The goal of this paper was to introduce some general issues of non-stationarity for practitioners, students and beginning researchers. Using elementary techniques we examined the effect of non-stationary data on the results of regression analysis. We further showed the effect of larger sample sizes on the spuriousness of regressions and we also examined the well known “rule of thumb” of how to identify spurious regressions. We also demonstrated the problem of spurious regression on a practical example, using closing prices of stock market indices from CEE markets.*

## Keywords

stationarity, time series data, various unit root tests, spurious regression, the R-squared and the Durbin – Watson statistics “rule of thumb”, CEE stock markets

**JEL Classifications:** C15, G15

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## Introduction

There is a group of papers, started by Granger – Newbold (1974), which cover the topics of non-stationarity of time series and when not handled properly, its impact on the spuriousness of regressions. Most of these papers are technically driven showing how different types of non-stationary data effect regression results. However, from the practical point of view, the conclusions are comparable. When all (dependent and independent) time series are non-stationary, the regression results are simply misleading. This alone underlines the importance of this topic.

While not being too technical, the goal of this paper was to introduce some general issues of non-stationarity for practitioners, students and beginning researchers. Using standard methodology of data generating processes (DGP) and simulations we demonstrated how diametrically opposing results can be obtained when time series are not handled properly. We examined following issues: Is there a difference between results when using stationary or non-stationary data? What is the effect of the different sample sizes? What is the difference in regressions of various types of non-stationary data? Does the common “*rule of thumb*” of high adjusted  $R^2$  and low Durbin – Watson statistics hold? Further on, by the means of a case study, we demonstrated the problem of spurious regression using stock market indices.

This paper is organized as follows. In the first section we define basic terms and concepts important for the remainder of the text. The second section is dedicated to a short review of tests for stationarity. The third section describes the design of our simple experiment and the fourth presents the results. In the last, fifth section we analyze stock market indices as stationary and as non-stationary data, thus again underlining the interesting differences.

## 1 Stationarity of time series

We say that stochastic process (which generates the time series) is stationary in a weak form when following conditions holds:

$$E[y_t] = \mu \wedge |\mu| < \infty \quad (1)$$

$$\text{var}(y_t) = E[(y_t - \mu)^2] = \sigma^2 \wedge |\sigma^2| < \infty \quad (2)$$

$$\text{cov}(y_t, y_{t+k}) = \text{cov}(y_t, y_{t-k}) = \gamma_k \wedge |\gamma_k| < \infty \quad (3)$$

In other words,  $\{y_t\}_{t=1}^T$  is stationary (or more precisely *covariance stationary*) if its mean and variance are constant over time, and the value of the covariance between the two time periods depends only on the distance  $k$  (lag) between the two time periods and not the actual time  $t$  itself. The first requirement simply says that the expected value of the time series should be constant and finite. If this requirement is not met, we regard data generated from this stochastic process to be from different population of processes. When these are handled like data from the same population, our results are dubious. The same is true if the second requirement is not met, where we require having constant variance over time. The

last requirement says that the relationship between two equidistant observations stays the same regardless of whether we compare the first observation with the tenth, or the second with eleventh and so on. To sum it up, the very basic idea of these restrictions is that one should not analyze time series data with different statistical properties, because it makes no sense.

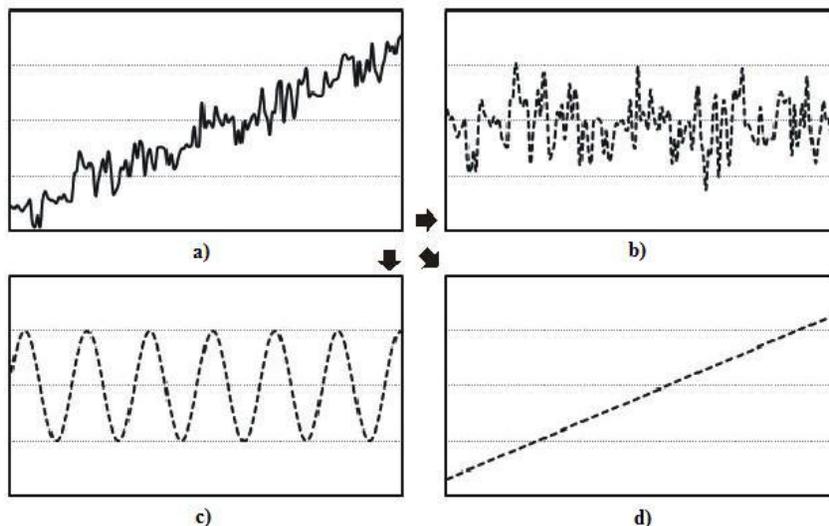
Unfortunately, most of the economic time series is non-stationary and this fact is often neglected by students and beginning researchers. The consequence leads to inaccurate results or so called *spurious regression* problem (first mentioned in Granger – Newbold, 1974). A good “*rule of thumb*” of identifying incorrect regression results is a high coefficient of determination and a low Durbin – Watson statistic of autocorrelation.

One way of decomposing the time series is to assume that every time series contains three components:

1. *An irregular pattern* which is the point of interest in univariate time series modeling, e.g. ARMA, (see Figure 1b). For our purpose consider the following pattern:  $IP_t = 0,5IP_{t-1} + \varepsilon_t$ ,  $\varepsilon_t \sim N(0,4)$ .
2. *A seasonal pattern* which is typical for economic data, which are reported in given period (monthly or quarterly), e.g. macro data such as GDP, inflation, unemployment rate, as well as the company financial reports also available on quarter base, (see Figure 1c). For our purpose consider the following pattern:  $SP_t = \sin\left(\frac{t\pi}{12}\right)$ .
3. *A deterministic trend*, in most cases linear or quadratic. We can also deal with stochastic trend, but the most convenient approach is to handle it as an irregular pattern (see Figure 1d). For our purpose we consider the following pattern:  $T_t = 3 + 0,2t$ .

Taking these three components together, we obtain the following time series, which is obviously non-stationary, (see Figure 1a):  $y_t = T_t + SP_t + IP_t$ .

## Decomposition of a time series



We do not want to supplement econometric textbooks by focusing on trends, seasonality and irregular patterns. Rather our goal here was to distinguish the central point of our interest from other issues, which we do not discuss in as much detail. For attentive reader, we recommend e.g. Gujarati (2004), Mills (1999), Davidson – MacKinnon (2003) or Kočenda – Černý (2007).

There is a simple way how to deal with non-stationary processes, using differences. In most cases by differencing  $\Delta y_t = y_t - y_{t-1}$ , where  $\Delta y_t$  is called the first difference, we obtain a stationary process. If a time series becomes stationary, we say that it is “integrated of order one”, and denote it as  $I(1)$ . Sometimes it is necessary to make higher differences. In general, if we need  $p$  differences to produce a stationary time series, it is denoted as  $I(p)$ , where  $p \in \mathbb{N}$  by definition.

Before differencing it is common to take a natural logarithms of the data, to deal with possible non linear trends. In some cases logarithmic differences have their own reasonable interpretation, e.g. when we are interested in growth rates or assets returns. A good example (mentioned in Kočenda – Černý, 2007) of this extra benefit is price versus inflation issue. If we are analyzing inflation, then we want to transform prices in levels into inflation first, i.e. taking logarithmic differences and getting stationary time series by different purpose.

In this paper we will employ daily closing prices of various stock market indices ( $p_t$ ). After the logarithmic transformation and taking the first differences, we will get *returns* ( $r_t$ ), which should be stationary<sup>1</sup>:

<sup>1</sup> This property will be properly tested.

$$r_{t+1} = \ln\left(\frac{p_{t+1}}{p_t}\right) = \ln(p_{t+1}) - \ln(p_t) \quad (4)$$

where  $r_{t+1}$  are daily returns in time  $t+1$ ,  $p_{t+1}$  are closing prices in time  $t+1$  and  $p_t$  are closing prices in time  $t$ ,  $t=1,2,\dots,T-1$ , where  $T$  is the number of all observations. Daily returns on the close-to-close basis are therefore also a good example of transforming the data with some natural interpretation. After such transformations, it is always good to ask, whether the analysis of resulting variables still accounts for the phenomena of our interest, or whether we can interpret possible results.

## 2 Tests for stationarity

The basic test for stationarity is the Augmented Dickey – Fuller (1979, 1981) test which is based on a unit root testing. First, we will discuss a general Dickey – Fuller test (DF henceforth). Consider following AR(1) process:

$$y_t = \rho y_{t-1} + u_t \quad (5)$$

where  $u_t$  is a stationary error process. The time series contains a unit root if  $|\rho|=1$  and it is stationary if  $|\rho|<1$ . Clearly, one sided  $t$ -test could be employed, nevertheless under the null hypothesis ( $H_0 : |\rho|=1$ ) the  $t$ -ratio does not have a  $t$ -distribution (Verbeek, 2008). With respect to these limitations, authors computed critical values for the test statistic via Monte Carlo simulation, which is called the  $\tau$  statistics. Moreover, they specify three test variations: a) without intercept and trend included, b) with intercept, c) with intercept and trend.

If we subtract  $y_{t-1}$  from both sides of equation (5), we will obtain  $\Delta y_t = \delta y_{t-1} + u_t$ , where  $\delta = 1 - \rho$ . Testing for a null hypothesis  $|\rho|=1$  is equivalent to a null  $\delta = 0$ .

Obviously, the assumption of AR(1) generating process is quite simplifying. That is why the Augmented Dickey – Fuller test (ADF henceforth) is used broader than simple DF test. ADF test allows testing of higher orders of autoregressive processes. Autocorrelation of residuals is controlled by  $m$  lagged values of dependent variable:

$$\Delta y_t = \beta_0 + \beta_1 t + \delta y_{t-1} + \sum_{i=1}^m \alpha_i \Delta y_{t-i} + u_t \quad (6)$$

Similar to simple DF test, its augmented form also allows to test for level stationarity or trend stationarity, as it is stated in equation (6). ADF test is easy to understand and easy to use, but it is a well known fact, that it has low power and a high chance of an error of the second type, i.e. the probability of not rejecting a false  $H_0$  (for further discussion see Kočenda – Černý, 2007). Thus it is not surprising that many variations of ADF have been proposed (e.g. Dickey – Bell – Miller, 1986; Dickey – Pantula, 1987; Phillips – Perron, 1988; Hylleberg et al.,

1990; and others). Also as it is stated in Davidson – MacKinnon (2003), some advantage over the standard ADF in terms of power may be achieved by using ADF-GLS test proposed by Elliott – Rothenberg – Stock (1996).

It is beyond the scope and range of this paper to deal with these tests precisely. Nevertheless, we would like to offer some references for further reading.

Table 1

### Tests for stationarity – an overview

Reference:	Brief description:
Sargan – Bhargava (1983)	based on the Durbin - Watson statistic
Dickey – Bell – Miller (1986)	seasonal unit roots
Dickey – Pantula (1987)	more than one unit root is suspected
Phillips – Perron (1988)	no IID assumption on disturbances, allows autocorrelated residuals
Perron (1989)	structural change; known break point
Hylleberg et al. (1990)	cyclical movements at different frequencies
Kwiatkowski et al. (1992) [KPSS test]	near unit root times series; higher power than ADF; transposition of the null hypothesis
Zivot – Andrews (1992)	structural change; break estimated at unknown point
Elliott – Rothenberg – Stock (1996)	higher power than ADF

Source: authors

### 3 Methodology

By the help of a computer and using simple equations for generating non-stationary data, we can observe some characteristics of spurious regression. Let's assume to have a simple linear regression model:

$$y_t = \alpha + \beta x_t + u_t \quad (7)$$

where for this case, it is important to note, that  $u_t$  is the error term, which is assumed to be  $N \sim (0, \sigma^2)$ . If we a priori know, that both,  $y_t$  and  $x_t$  are *independent* and *non-stationary*, the estimated regression coefficient  $\hat{\beta}$  should be non-significant and with  $t \rightarrow \infty$  converge to zero. These characteristics can be well observed using a simple simulation methodology. We will follow the methodology of Noriega – Ventosa-Santaularia (2006) and standard procedures for testing spurious regressions.

The basic idea is to generate time series data, which are known to be non-stationary and independent, that is not necessarily statistically independent but independent by their design. For this purpose, we have used data generating

equations<sup>2</sup>. For this teaching article, we wanted to analyze two types of time series. The pure random walk (PRW):

$$y_t = y_{t-1} + u_t \quad (8)$$

and random walk with drift  $\mu$  (RWD):

$$y_t = \mu + y_{t-1} + u_t \quad (9)$$

The PRW is a non-stationary process, because with increasing number of observations, the variance increases. This is a good example of a case, where the second requirement (see section 1) does not hold:  $y_t - y_{t-1} = u_t$ ,  $\Delta y_t = u_t$ ,  $VAR[\Delta y_t] = E[(\Delta y_t - E[\Delta y_t])^2]$ ,  $E[\Delta y_t] = 0 \Rightarrow VAR[\Delta y_t] = E[(\Delta y_t)^2]$ . The RWD is a special case of PRW, where the time series has a stochastic trend, see Gujarati (2004). The PRW is a I(1) process, and RWD a I(1) process with drift.

The DGP is as follows: the error terms  $u_t$  are generated from  $N \sim (0, \sigma^2)$  using a random number generator<sup>3</sup> and initial values of  $y_t$  are set to be zero, i.e.  $y_0 = 0$ . For every spurious regression, we have calculated and recorded the following variables:

- value of the  $\hat{\beta}$
- $t$ -statistics for the  $\hat{\beta}$ ,
- $DW$  statistics,
- adjusted coefficient of determination  $R^2$ ,
- results of Phillips – Perron test for both,  $y_t$  and  $x_t$ .

Together, we had 18 groups of different types of data, which were formed as follows. First, we used various types of regressions (TR):

- The type 1 - were the cases with  $y_t$  and  $x_t$  being I(1) processes.
- The type 2 - were the cases with  $y_t$  being I(1) processes and  $x_t$  being I(1) + drift processes.
- The type 3 - were both  $y_t$  and  $x_t$  I(1) + drift processes.

Secondly, because we were interested in the possible dependence of recorded variables upon the number of observations, we analyzed samples with following sizes:  $n = 50, 200, 1000$ . We also replicated these simulations using time series with differences. By using level variables and differences, three types of sample sizes and three TR, the above mentioned 18 groups were formed. In every group, we performed 500 regressions (replication).

Additionally, in the type 3 regressions, we fixed the drift value in  $y_t$  and increased the drift value in  $x_t$ . The question we are trying to answer is, whether

<sup>2</sup> Or the so called “data generating process” (DGP henceforth).

<sup>3</sup> Even if we are aware of the limitation of MS Excel’s random number generator, this is a teaching article, so we found it sufficient for the purpose given. This fact also implies, that all the results may be effected by this.

there is a systematical effect of increased drift on the recorded variables. Rather than answering this question analytically, we incorporated it into the design of type 3 regressions.

## 4 Results

The results are presented in the following next two tables. The first table reports the type I error of falsely rejecting the null hypothesis  $H_0 : \hat{\beta} = 0$ . As can be seen, in all types of processes the error of rejecting the null hypothesis is high<sup>4</sup>. For example, in the type 3 regressions, where both the dependent and independent variables were non-stationary and with drift, we have rejected the null hypothesis in 94,4% from 500 cases. Special attention should be addressed to the type 3 regressions, where independent variables had different drift parameters. Our results suggest that this had no effect on the results. The relationship between the difference of drifts between independent and dependent variables were not significant.

Table 2

### Results from the simulations

DGP Sample	Type 1 regressions			Type 2 regressions			Type 3 regressions		
	n=50	n=200	n=1000	n=50	n=200	n=1000	n=50	n=200	n=1000
	<b>Type I Error (rejection rate of <math>H_0</math>)</b>								
Rejected	57,6%	79,2%	89,2%	59,6%	79,4%	88,2%	76,0%	90,6%	94,4%
Rejected *	1,4%	0,6%	0,4%	0,6%	1,0%	0,8%	1,2%	0,8%	1,2%
	<b>Adjusted R-squared</b>								
Mean	0,24	0,25	0,24	0,25	0,23	0,26	0,42	0,43	0,48
St. dev.	0,25	0,23	0,23	0,24	0,22	0,24	0,30	0,30	0,31
Mean*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
St. dev.*	0,03	0,01	0,00	0,03	0,01	0,00	0,03	0,01	0,00
	<b>Durbin-Watson statistic</b>								
Mean	0,33	0,09	0,02	0,34	0,09	0,02	0,39	0,10	0,02
St. dev.	0,19	0,06	0,01	0,21	0,05	0,01	0,18	0,06	0,01
Mean*	2,01	2,00	2,00	1,99	2,01	2,00	1,99	2,00	2,00
St. dev.*	0,27	0,14	0,07	0,30	0,14	0,06	0,27	0,14	0,06

Note: symbol \* denotes those results, where time series in differences was applied.

<sup>4</sup> This is of course not surprising as this was already shown in numerous papers using various spurious non-stationary data, e.g. Noriega – Ventosa-Santaularia (2006).

In contrast to the results of non-stationary data, the regressions with stationary data<sup>5</sup> had a very low rejection rate at about 1% of all the time. This is a good example of how spurious regressions can mislead beginning researchers and students. A similar result may be observed looking at the adjusted  $R^2$ .

Table 3

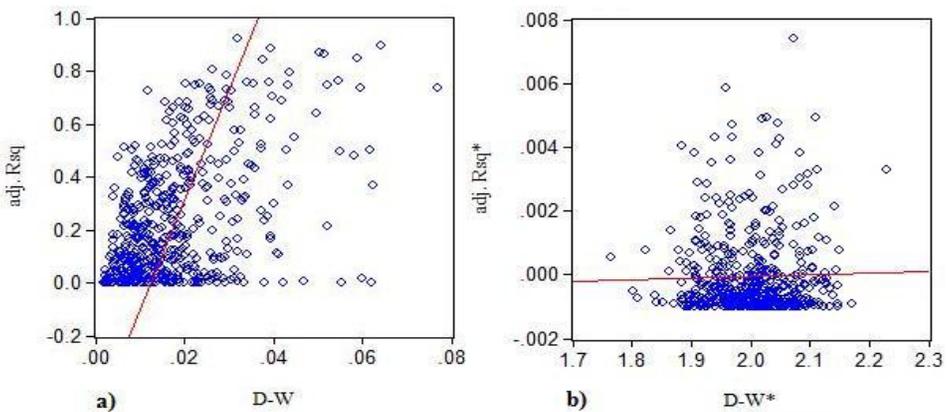
**Results from the PP test – rejection rate of  $H_0$  in %**

DGP Sample	Type 1 regressions			Type 2 regressions			Type 3 regressions		
	n=50	n=200	n=1000	n=50	n=200	n=1000	n=50	n=200	n=1000
y	0,6	1,6	1,6	0,1	2,2	0,1	1,2	0,6	1,0
x	1,8	1,4	1,4	1,2	1,0	0,0	0,0	0,0	0,2
y*	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0
x*	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0

Note: symbol \* denotes those results, where time series in differences was applied.

Figure 2

**Scatter plot of  $R^2$  and DW statistics**



The second phenomenon of our interest was the increasing sample sizes. The observed results suggest that the effect is different with regard whether we regress stationary or non-stationary data. In the first case it seems, that the rejection rate and the adjusted  $R^2$  are not affected (see Table 2). On the contrary, the reverse seems to be true when regressing non-stationary data. With the increase of sample sizes the rejection rate increases regardless of the TR used in the regression. There can be various statistical explanations for this effect. An intuitive non-statistical explanation may be that increasing the number of spurious observations increases

<sup>5</sup> The stationary data were obtained after making simple differences, and the stationarity was tested using Phillips – Perron test (PP test henceforth), see Table 3.

the spuriousness of the dataset, thus making the phantom relationships more convincing. The more “bad” data are used, the more are we fooled.

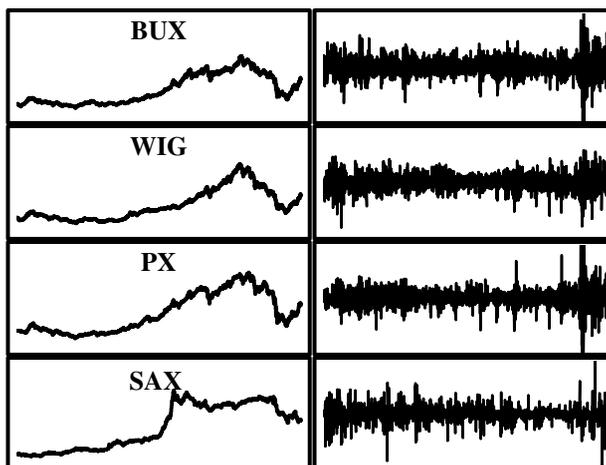
One last and interesting fact had been observed. We were interested whether the “*rule of thumb*” mentioned above was present also in our short study. In the Figure 2, we compared the ordered pairs of adjusted  $R^2$  and Durbin-Watson statistics for the type 3 regressions with the sample size of 1000. As can be clearly seen, the “*rule of thumb*” holds. In cases where the spurious regression was present (see Figure 2 a) where we utilized variables in their levels), we observed much higher values of adjusted  $R^2$  and much lower values of Durbin-Watson statistics (close to zero), than in the case of non-spurious regression, where Durbin-Watson statistics were close to 2 (see Figure 2 b) where differenced time series was applied).

## 5 An illustrative example: Stock market indices

By the means of real case studies, our goal in this section is to demonstrate how misleading can be handling non-stationary time series as stationary. We will employ daily closing prices from several stock market indices covering period from 1<sup>st</sup> September 1999 to 1<sup>st</sup> September 2009. Our sample contains indices from CEE markets (also known as Vysegrad Group, or V4) namely, Hungarian BUX, Polish WIG, Czech PX and Slovakian SAX. Instead of descriptive statistics we decided to present chosen time series in the following figures.

Figure 3

### Stock market indices in levels and logarithmic differences



Source: authors, data retrieved from stooq.com

From the above stated figure it can be seen that closing prices of indices are apparently not stationary. However the opposite could be true with their first logarithmic differences. Of course we need to run some tests to preserve such statement. We have applied standard ADF test with critical values tabulated by

MacKinnon (1996). To compare the result we decided to choose unit root test proposed by Elliott – Rothenberg – Stock (1996), abbreviated as ADF-GLS test. Further, Zivot – Andrews (1992) test (ZA henceforth) and Phillips – Perron (1988) test (PP henceforth). It is a convention in economic literature to provide results of at least two tests. Most frequently ADF, PP test and KPSS test are used, which are also incorporated in the most statistical or econometric software. Since KPSS includes transposed null hypothesis (claims of stationarity against alternative of a unit root), we decided not to apply this test as the results could appear as mixed.

In the following table we present results from selected tests for stationarity. Calculations were made in R software, along with an “urca” package. The level of significance is 1 % in the case of rejecting the null hypothesis (no unit root is present), but in the not rejecting the null cases we were more benevolent and have chosen 10 % significance level. To maintain our results easy to read, following table contains only statements “rejected” and “not rejected” (the null hypothesis of a unit root). More detailed results are available upon request.

Table 4

### Testing for stationarity

	LEVELS		LOGDIFF		LEVELS		LOGDIFF	
	<b>ADF test</b>				<b>ADF-GLS test</b>			
Index	c	ct	c	ct	c	ct	c	ct
BUX	NR	NR	R	R	NR	NR	R	R
WIG	NR	NR	R	R	NR	NR	R	R
PX	NR	NR	R	R	NR	NR	R	R
SAX	NR	NR	R	R	NR	NR	R	R
	<b>ZA test</b>				<b>PP test</b>			
Index	c	ct	c	ct	c	ct	c	ct
BUX	NR	NR	R	R	NR	NR	R	R
WIG	NR	NR	R	R	NR	NR	R	R
PX	NR	NR	R	R	NR	NR	R	R
SAX	NR	NR	R	R	NR	NR	R	R

Note: a) “c” stands for constant included, “ct” stands for constant and trend included; b) NR stands for „not rejected“ the null hypothesis, R stands for „rejected“ the null.

As we can see, time series are non-stationary in their levels (i.e. closing prices), but they are stationary at first logarithmic differences (i.e. returns). So in our case it is easy to decide about stationarity of time series, but still remember that all results in statistical testing have probabilistic nature. It would be much harder to resolve the question about stationary or non-stationary character of time series, when applied tests would provide mixed results. In such doubtful cases, it is upon the researcher to decide which test to believe.

Let's proceed to the problem of a spurious regression. To fulfill our goal, we will estimate simple linear regression model again (Eq. (7)). It is estimated using closing prices as variables and logarithmic differences afterwards (OLS method with HAC applied to deal with autocorrelation problem). Obtained results are presented in the following table.

Table 5

**Results from the regressions**

		DEPENDENT VARIABLE							
		IN LEVELS				IN LOGARITHMIC DIFFERENCES			
		BUX	WIG	PX	SAX	BUX	WIG	PX	SAX
BUX	<i>t</i> -test		0,0000	0,0000	0,0000		0,0000	0,0000	0,2867
	R <sup>2</sup>	-	0,9222	0,9835	0,8286	-	0,3124	0,3411	0,0013
	DW		0,0126	0,0643	0,0117		2,0063	2,0578	2,0584
WIG	<i>t</i> -test	0,0000		0,0000	0,0000	0,0000		0,0000	0,7280
	R <sup>2</sup>	0,9222	-	0,9293	0,6771	0,3124	-	0,3446	0,0001
	DW	0,0131		0,0115	0,0043	2,0026		1,9879	2,0640
PX	<i>t</i> -test	0,0000	0,0000		0,0000	0,0000	0,0000		0,7537
	R <sup>2</sup>	0,9835	0,9293	-	0,8293	0,3411	0,3446	-	0,0001
	DW	0,0646	0,0114		0,0095	2,0158	1,9495		2,0646
SAX	<i>t</i> -test	0,0000	0,0000	0,0000		0,2927	0,7257	0,7577	
	R <sup>2</sup>	0,8286	0,6771	0,8293	-	0,0013	0,0001	0,0001	-
	DW	0,0127	0,0049	0,0102		1,9226	1,932	1,9709	

Note: a) standard *t*-test is applied to test the significance of regression parameter; b) R<sup>2</sup> denotes the coefficient of determination; c) DW stands for Durbin-Watson statistic

When analyzing relationships between closing prices of indices, all regression parameters are significant at 1 % significance level and moreover, high coefficient of determination is observed. Reported Durbin-Watson statistic close to zero implies the presence of autocorrelation, but since we applied HAC covariance matrix, it has no effect on the significance of regression coefficients (asymptotically).

Nevertheless, we already know that these time series are non-stationary, which makes the results misleading. One way to interpret these highly significant spurious results is to say, that what we actually measured was the trend of both indices, not the relationship between closing prices. As it was stated above, a good “*rule of thumb*” in identifying the spurious regression problem is to look at the high R<sup>2</sup> and low DW statistic.

Everyone who is aware of a special position of Slovakian stock market (special in the way of its inefficiency) would expect very weak relationships with SAX and

any other stock market indices, even from the same region. Evidence of this is observable when time series are analyzed in their logarithmic differences. At this point no coefficient is statistically significant (whether SAX is considered as dependent or independent variable) and  $R^2$  is close to zero.

In other relationships the coefficients remains significant and moreover  $R^2$  decreased to more intuitively expected level.

It is worth to mention, that we do not consider in our analysis (nor in the simulations) the presence of cointegration. Various textbooks may be useful for further readings about this phenomenon (e.g. Maddala – Kim, 1998 or Gujarati, 2004).

## Conclusion

Our aim was an introductory approach to the issues of stationarity of time series. We wanted to cover this topic rather broadly, without much technical depth. From our restricted analysis some interesting questions came into attention. We have used only two different types of non-stationary data, one generated through I(1) DGP, the second with I(1) + drift DGP. From these two types of time series, we formed three types of regressions. The error of rejecting the null hypothesis  $H_0 : \hat{\beta} = 0$  in a simple linear regression model seemed to be clearly higher in the type 3 regressions (dependent is I(1) + drift DGP and independent I(1) + increasing drifts). This was probably not due to the increasing drift of the independent variable. This raises the question, of whether the more complicated non-stationarity (more requirements from section 1 are violated) time series are more “*spurious*”.

Further on, as it seemed that the higher samples sizes contributed again to the “*spuriousness*” of the regression results. This is a dangerous issue, because generally if one has a larger sample size, one tends to have greater trust in statistical results. Apart from other possible topics here, like sampling, this confidence is dangerous.

Finally we were interested in commonly presented “*rule of thumb*” that spurious regressions are accompanied by low values of DW statistics and high adjusted  $R^2$ . Using our simulation we can descriptively conclude this to be true and the differences to be very significant.

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